COMMUNICATIONS RESOURCE ALLOCATION -
A GAME OF STRATEGY

by

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THESIS

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ABSTRACT

This thesis looks at optimally allocating communication resources from a game theory point of view. Three basic models are presented and then expanded upon. A prime objective is to briefly review representative examples of work previously done in the attack-defense area showing how it can be applied to the optimal allocation problem for communication resources and indicate the possible direction for future research.

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I. INTRODUCTION

Communication systems represent an integral part of progress in the social, business, and technical fields. Certainly this statement needs no qualification when applied to the military. With the increase in destructive power and improvement of accuracy in modern weapons, large-scale dispersion of troops on the battlefield has become part of the doctrine of warfare. Wider separation has imposed difficulty in controlling and directing forces at all levels of command from squad leader to force commander. Rapid, secure and reliable communication is indispensable in marshalling resources to obtain the military and political objective. Indeed, quality and quantity of communication systems often provides the margin of victory.

Military communication systems encompass a wide range of sophistication; the spectrum runs from the hand-held, short range, single channel radio to the most modern orbiting satellite system. By far the most important in terms of volume of traffic and ease of control is the multi-channel, wide band radio system in the Marine Division. This system closely parallels the long-haul, inter-city trunking system operated nationally by the Bell System and independent telephone companies. Both of these systems have a network of radio links interconnecting nodal or terminal points.

It, then, is a recognized fact in the military that effective communication is an essential element for success of a command. The military spends vast sums of money annually in an effort to procure the equipment, personnel, and other resources needed by a commander to establish effective communications. Without effective communications it would be difficult for a military unit to survive in today's combat environment. However, even considering the importance of communications, the majority of officers who work in the field of communication often rely on past experience and knowledge of the situation as a basis for their recommendations and decisions affecting the allocation of communication resources. Of course, these Communication Officers back up their experience with reconnaissance and area research. A brief examination of past history shows that this method of deciding upon the quantity of resources to be allocated has many times been most successful. However, it would seem possible that some purely analytical, and somewhat less subjective, techniques could be applied to the problem of optimally allocating communication resources. One suggested approach is game theory.

It is the purpose of this thesis to examine possible applications of certain game theory models to the problem of optimally allocating communication equipment. It is not feasible in this thesis to cover all possible applications of Game Theory to problems of resource allocation, rather it is hoped that a broad overview of such applications can be presented so as to act as a basis for future work in this area.

It is assumed that readers have a background in mathematics and hopefully are acquainted with Game Theory. As an aid to those readers who are not familiar with Game Theory, a brief introduction to the theory of games is given in the following section. Some of the more important definitions and concepts are summarized. For a more detail discussion the reader should consult the references listed in the bibliography (for example, see [4], [5], [6], [7], [8], [11], [14], [15]).

II. GAMES OF STRATEGY

The theory of games of strategy is a mathematical theory of competitive decision-making. Games of strategy and games of pure chance differ in that only in the former can the participant bring any influence to bear on the outcome of an event. As a result, intelligence and skill can be useful in the play of games of strategy. Examples of parlor games of strategy are such games as chess, bridge, and poker, where the various players can make use of their ingenuity in order to outwit each other. The theory of games is, in general, applicable to situations which involve conflicting interests, and in which the outcome is controlled partly by one side and partly by the opposing side of the conflict [11].

Note that several of the examples of games of strategy given above involve the element of chance, but, none the less, in each case the participants in these games are allowed, under the rules of the game, to make certain decisions which are completely independent of chance. Games which depend completely on chance and which do not allow the participants an opportunity to exercise any influence, such as dice, are not considered in the Theory of Games of Strategy.

It should be mentioned, finally, that the theory of games of strategy can be expected to find practical application in all kinds of situations in which various people have opposing goals and in which each of them,

although he may exert some influence on the outcome, cannot completely dominate the course of events [11].

Now to Game Theory itself.

RULES AND PLAYERS

First of all, a game of strategy is described by its set of rules. The rules specify what each participant, called a player is allowed, or required, to do under all possible circumstances. Further, rules determine the amount of information, if any, each player receives. If the game involves the use of chance devices, or if chance occurrences are an integral part of the situation establishing the game, the rules specify how the chance events shall be interpreted. Finally, the rules define when the game ends, the amount each player pays or receives, and the objective of each player. As applied to poker, the rules govern how the cards are to be doled out, who may bet and when, how the various hands are to be judged in the showdown, and what happens to the pot.

NUMBER OF PLAYERS

One of the fundamental distinctions in Game Theory is the number of players - distinct sets of interest - that are present in the game. The form of analysis and the entire character of the situation depend on this number. There are three values, for the number of players, which have special significance: one, two, and more-than-two (n, where n = 3, 4, 5 . . .).

Solitaire is an example of a one-person game when played for recreation, for your interests are the only ones present. One-person games are uninteresting, from the Game Theory point of view, and therefore are not really looked at here. In one-person games you simply select the course of action that yields the most and do it. If there are chance elements, you usually select the action which yields the most on the average, and do it. However, one person games (including Solitaire) may be regarded as a special kind of two-person game in which you are one of the players and Nature is the other.

The true two-person game is very interesting. It occurs frequently and its solution is often within our present means, both conceptual and technological. This is the common conflict situation. You have an opponent who, you must assume, is intelligent and trying to undo you. If you choose a course of action which appears favorable, he may discover your plans and set a trap which capitalizes on the particular choice you have made. Many situations which are not strictly two-person games may be treated as if they were; a five man Poker game is an example of this, where you could assign the interests present at the table to two "persons", yourself and everybody-not-you. Most of the work done in Game Theory deals with the two-person game.

THE PAYOFF

It has been indicated that the number of persons/players involved is one of the important items for classifying and studying games,

"person" meaning a distinct set of interests. Another criterion has to do with the payoff: What happens at the end of the game? Say at the end of a hand in Poker? Well, in Poker there is usually just an exchange of assets. If there are two persons, say (Blue) and (Red), then if Blue should win \$10, Red would lose \$10. In other words,

$$\text{Blue winnings} = \text{Red losses}$$

or, stated otherwise,

$$\text{Blue winnings} - \text{Red losses} = 0$$

We may also write it as

$$\text{Blue payoff} + \text{Red payoff} = \$10+(-\$10) = 0$$

by adopting the convention that winnings are positive numbers and that losers are negative numbers. The sum of the payoff need not be zero. For instance, if the person who wins the pot has to contribute 10 per cent toward the drinks and other incidentals, then the sum of the payoffs is not zero; in fact

$$\text{Blue payoff} - \text{Red payoff} = \$9 - \$10 = -\$1$$

The above two cases illustrate a fundamental distinction among games: It is important to know whether or not the sum of the payoffs, counting winnings as positive and losses as negative, to all players is zero. If it is, the game is known as a Zero-Sum game. If it is not, the game is known as a Non-Zero-Sum game.

STRATEGIES

Just as the word person or player has a meaning in Game Theory somewhat different from everyday usage, the word strategy does too.

This word, as used in its everyday sense, carries the connotation of a particularly skillful or adroit plan, whereas in Game Theory it designates any complete plan. A strategy is a plan so complete that it cannot be upset by enemy action or Nature; for everything that the enemy or nature may choose to do, together with a set of possible actions for yourself, is just part of the description of the strategy.

So the strategy of Game Theory differs in two important respects from the conventional meaning: It must be utterly complete and it may be utterly bad; for nothing is required of it except completeness. Thus, in Poker, all strategies must make provision for your being dealt a Royal Flush in Spades, and some of them will require that you fold instantly. Note that a player's plan of action, his strategy, is complete and ready to use before the commencement of the game.

A strategy which guarantees a player the best he can expect regardless of what the other players do is called an optimal strategy.

FINITENESS

There are critical values in the number of strategies; and it turns out to be important to distinguish two major categories. In the first are games in which the player having the greatest number of strategies still has a finite number. The second major category is that in which at least one player has infinitely many strategies.

A PLAY OF THE GAME

The expression "a play of the game" has been used several times in this thesis. However, the exact nature of a play of the game may or may not be apparent. In the Theory of Games the choosing of a particular strategy by each player, along with the exchange of payoffs which possibly result, is defined as a play of the game.

VALUE OF THE GAME

The value of the game is the expected payoff transferred between the players when each player employs his optimal strategy.

Usually the opposing players are placed into one of two categories, either maximizing or minimizing. In an unfair game, a game in which the value is some number greater than zero, the maximizing player, or group of players, will realize a positive expectation. Therefore, the maximizing player will select an optimal strategy so as to maximize his winnings. On the oter hand in an unfair game the minimizing player, or groups of players, will expect to lose the value of the game. Therefore, the minimizing player will choose an optimal strategy so as to minimize his losses. Of course if the value of the game is negative, then the maximizing player will have negative expected winnings and the minimizing player will have negative expected losses. In this case the maximizing player will continue to select strategies which will maximize his expected winnings. But since his expected winnings are negative, in this case, he is in effect

minimizing his losses. Likewise, the minimizing player will continue to choose strategies which will minimize his expected losses. But since his expected losses are negative he is actually maximizing his winnings. In either case, and also in the case of a fair game, one in which the value of the game is zero, all players select strategies which will maximize their individual utilities.

THE GAME MATRIX

Now it is possible to complete the description of games, i.e., conflict situations, in the form required for Game Theory analysis. Remarks will primarily apply to finite, zero-sum, two-person games.

The players are Blue and Red. Each has several potential strategies, which we assume are known; let them be numbered just for identification. Blue's strategies will then bear names, such as Blue 1, Blue 2, and so on; perhaps in a specific case up to Blue 5, and Red's might range from Red 1 through Red 3. This would be a five-by-three game and we would write it as "5 X 3 game".

The rules for the play of the game would be specified and they would contain information from which we can determine what happens at the end of any play of the game: What is the payoff when, say, Blue uses strategy Blue 3 and Red uses Red 2. There will be 5 X 3 = 15 of these pairs and hence that number of possible values for the payoff; and these must be known. Whatever the values are, it is surely possible to arrange the information on this kind of bookkeeping form:

		R E D		
		1	2	3
B L U E	1			
	2			
	3			
	4			
	5			

Such an array of boxes, each containing a payoff number is called a game matrix. We shall adopt the convention that a positive number in the matrix represents a gain for Blue and hence a loss for Red, and vice versa.

When the original problem has been brought to this form, a Game Theory analysis may begin, for all the relevant information is represented in the description of the strategies whose signatures border the matrix and in the payoff boxes. This is the Game Theory model of the conflict, and the applicability of the subsequent analysis will depend completely on the adequacy of this form or representation -- a set of strategies and a payoff matrix.

IMPLICIT ASSUMPTIONS

Perhaps the last statement should be expanded. Two complicated objects are involved: One is the real conflict situation in which Blue

and Red are involved. This includes the rules, regulations, taboos, or whatnots that are <u>really</u> operative; it includes the true motive of the players, the geography, and in fact everything that is significant to the actual game. The second object is also real, but much more simple: It is the rules of the model that have been written, the strategies that have been enumerated and described on paper, and the game matrix that has been written. There is a relationship -- a significant one, we trust -- between these two objects. The second object -- the marks on the paper - is an abstraction from the first. Some non-obvious properties of this second object can be discovered by making a Game Theory analysis, and these properties may have some validity in connection with the first object -- the real world. <u>It will depend on the adequacy of the abstraction</u>. (Will apply only in accordance with how much the model applies).

THE CRITERION

What is the criterion in terms of which the outcome of the game is to be judged. Generally speaking, criterion - trouble is the problem of what to measure and how to base behavior on the measurements. Game Theory has nothing to say on the first topic, but it advocates a very explicit and definite behavior-pattern based on the measurements.

It takes the position that there is a definite way that rational people should behave, if they believe in the game matrix. The notion that there is some way people ought to behave does not refer to an

obligation based on law or ethics. Rather it refers to a kind of mathematical morality, or at least frugality, which claims that the sensible object of the player is to gain as much from the game as he can, safely, in the face of a skillful opponent who is pursuing an antithetical goal. This is our model of rational behavior. Apply the consequences of this model to a zero-sum game in which all the payoffs are positive; this means that the strategy options available to the players only affect how many valuables Red must give to Blue at the end of the game. This then is an unfair game for Red.

Now the viewpoint in Game Theory is that Blue wishes to act in such a manner that the least number he can win is as great as possible, irrespective, of what Red does; this takes care of the safety requirement. Red's comparable desire is to make the greatest number of valuables that he must relinquish as small as possible, irrespective of Blue's action. This philosophy, if held by the players is sufficient to specify their choices of strategy. If Blue departs from it, he does so at the risk of getting less than he might have received; and if Red departs from it, he may have to pay more than he would have otherwise.

The above argument is the central one in Game Theory. There is a way to play every two-person game that will satisfy this criterion. However, this is not the only possible criterion; for example, by attributing to the enemy various degrees of ignorance or stupidity, one could devise many others. Since Game Theory does not attribute these attractive qualities to the enemy, it is a conservative theory.

Note the apparent disparity in the aims of Blue and Red as stated above; Blue's aims are expressed in terms of winning and Red's in terms of losing. This difference is not a real one, as both have precisely the same philosophy. Rather, it is a consequence regarding the meaning of positive and negative numbers in the game matrix. The adoption of a uniform convention, to the effect that Blue is always the maximizing player and Red the minimizing player, will reduce technical confusion (once it becomes fixed in your mind); but let's not pay for this mnemonic by coming to believe that there is an essential lack of symmetry in the game treatment of Blue and Red. There is not actually any lack of symmetry [15].

SOLUTION

Finally, the solution of a game is an optimal strategy for each player and a real number which represents the value of the game.

FORMULATION OF THE GAME

In the formulation which follows the terminology developed above will be used freely. Additional concepts, definitions and notations will be introduced and defined as it is needed.

III. BASIC ALLOCATION MODELS

Rapid, secure, and reliable communication is indispensable in marshalling resources to obtain the military and political objective. Quality and quantity of communication systems often provide the margin of victory. It, then, is a recognized fact in the military that effective communication is an essential element for success of a command.

The military spends vast sums of money annually in an effort to procure the equipment, personnel, and other resources needed by a commander to establish effective communications. Without effective communications it would be difficult for a military unit to survive in today's combat environment.

One of the main objectives, then, of the commands engaged in a battle is to establish as effective a communication system as possible. Obviously, they will, in general, attempt to attain this goal through the proper use of communication resources. On the contrary, the enemy will attempt to distribute their anti-communication resources in such a way as to maximize their gain (i.e. minimize the effectiveness of the communication system or maximize their anti-communication efforts).

A conflict situation has now been described in which the participants are able to influence the outcome by selecting various allocations of communication (anti-communication)resources. Each of the competing commands must make a decision as to how much resources to allocate,

their objectives being conflicting. The conditions necessary to examine the problem of optimal allocation of resources by using Game Theory have now been established.

In the presentation of MODELs to follow bear in mind that the prime objective of this paper is to briefly review representative examples of work previously done in the attack-defense area and show how it can be applied to the optimal allocation problem for communication resources. The general procedure to be followed will be to formulate the allocation of communication resources as a Game Theory problem and then to mention/discuss the following properties of the model; anti-communicator (defense) analysis, questions answered, description (of model), strategies, payoff, solution. Mathematical proofs of theorems and mathematical manipulations to derive solutions will be referenced (from the attack-defense literature) but will not be repeated here. It is considered more advantageous for the purposes of this paper to merely state theorems which already have been proved and to discuss the character of the solutions that already have been obtained.

A. MODEL I

1. Problem

Suppose the communication system of command A must deliver messages from a rear area to an advanced area by one of N independent methods (routes) subject to interruption by enemy action (i. e. action

of Command B). (By "independent routes" is meant routes such that a single enemy action cannot interrupt more than one.) If the route must be selected in ignorance of the interruption plans of the enemy and the enemy must use his anti-communication resources without knowing the route over which the messages will be transmitted, the situation described may be regarded as a Blotto game in which A selects the route (battlefield) for the transmission of messages (A's forces) while B distributes his resources (B's forces) to interrupt transmissions over the different possible routes (battlefields). It is assumed that each route will be controlled by the command allocating the most resources to that route. Both A and B have a fixed amount of resources, X and Y respectively, that must be allocated among the possible routes. It is also assumed that the value of the message traffic, t_i, over each route will go completely to the command using the most resources on that route.

Let N = total number of routes

Each route, i, represents an amount of traffic t_i, where i = 1, 2, 3, ...N

$\sum_{i=1}^{N} t_i =$ T, where T represents the total message traffic value

$X_i =$ Amount of communication resources allocated by Command A to route i

$\sum_{i=1}^{N} x_i =$ X, $x_i \geq 0$

$y_i =$ Amount of anti-communication resources allocated by Command B to route i.

$$\sum_{i=1}^{N} y_i = Y \quad , \; y_i \geq 0$$

The structure of the stated problem allows for construction of the following pay off table.

Relation of resource allocation	A's payoff	B's payoff
$x_i > y_i$	t_i	0
$x_i < y_i$	0	t_i
$x_i = y_i$	$1/2\ t_i$	$1/2\ t_i$

TABLE I

Since both Command A and Command B are trying to secure as large a portion of the message traffic value as possible, the difference between the two commands message traffic value has been chosen as the objective function. Further, Command A is assumed to be the maximizing player and Command B is assumed to be the minimizing player. Therefore, if we define D to be the difference between A's and B's message traffic value then we have

$$D = \sum_{i=1}^{N} t_i \qquad \text{where } t_i \begin{cases} \text{is positive for } x_i \quad y_i \\ \text{is negative for } x_i \quad y_i \\ \text{is zero for } x_i = y_i \end{cases}$$

and where;

$$D \leqq T$$

$$\sum_{i=1}^{N} x_i = X , \; x_i \geqq 0$$

$$\sum_{i=1}^{N} y_i = Y , \; y_i \geqq 0$$

A will select a strategy to maximize D, and B will select a strategy to minimize D.

The problem has now been formulated as a zero-sum, two-person game. The formulation of this game is similar to the well known Colonel Blotto Game and may be summarized as follows:

Two players (A and B) contending on N independent battlefields (labeled 1, 2, ... N) must distribute their forces (X and Y units, respectively) to the battlefields before knowing the opposing deployment. The payoff (a numerical measure of the gain of A or equivalently of the loss of B) on the i^{th} battlefield is given by a function P_i (x, y) depending only on the battlefield and the opposing forces x and y committed to that battlefield by A and B. The payoff of the game as a whole is the sum of the payoffs on the individual battlefields.

In order to thoroughly examine the game we have developed, it is necessary to inspect two cases. In the first case both Command A and Command B have an equivalent amount of resources. In the second case one of the two commands has a larger amount of resources than his adversary. In summary:

Case I:	Both commands have an equivalent amount of resources $X = Y$
Case II:	Command A has a larger amount of resources $X > Y$

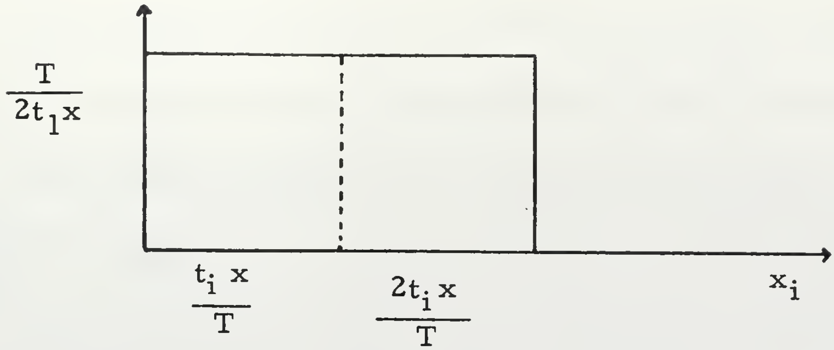
2. Case I : $X = Y$.

This case, where $x = y$, has the property of symmetry. It corresponds very closely to the symmetric case of the Colonel Blotto Game.

It can be shown that the solution to this case lies in a mixed strategy for each of the two players, and the mixed strategies used by each player are identical. Note that a new term, mixed strategy, has been introduced. It has already been explained that in a game situation the players have a number of alternative actions available to them. In some cases a player would find that a pure strategy was his optimal strategy, which means he would follow the same course of action at each play of the game. On the other hand if a player's optimal strategy is a mixed strategy, then he chooses different courses of action at each play of the game. The player's choice of action would be determined by a probability distribution over all his possible courses of action. [6, 9].

The solution for A, which is identical to the solution for B, is as follows:

Command A selects an allocation for the i^{th} route with equal probability (uniform) from the rectangular distribution shown below.



3. <u>Case II : $x > y$.</u>

This case, where $x > y$, unlike Case I, is non-symmetric. It also corresponds to a case of the Colonel Blotto Game, the non-symmetric case.

a. Solution for Command A: Allocate an amount x_i to customer i chosen at random from a rectangular distribution on the interval (0, $\frac{2t_i\, x}{T}$). This is identical to the strategy employed by both Command A and Command B in Case I.

b. Solution for Command B: Since B has less resources to allocate than A, B cannot allocate resources to each potential route, i. If he did, then A with the greater amount of resources available to allocate would be able to match B's effort and, in addition allocate an additional amount ϵ to each potential route. A would obtain the total message traffic value, T. Obviously, this would not be in B's best interest. B wants to minimize A's payoff (or maximize his own (B's) payoff). In this situation B would use a mixed strategy, which would be a probability of $\frac{y}{x}$ of allocating resources to any given route. The

probability that B does not allocate resources to a given route is then $(1 - \frac{y}{x})$. To those routes to which B does decide to allocate resources, B allocates an amount y_i at random on the interval $(0, \frac{2 t_i x}{T})$. (Note: B varies his strategy as to which routes he will allocate resources to, but to those routes to which he allocates resources, he uses the same allocating strategy as A.)

The value of the game in both Case I and Case II is;

$$D = T \left(1 - \frac{y}{x} \right), \text{ where in the symmetric case, } D = 0. \quad [10]$$

4. Further Study.

Friedman [9] discusses advertising expenditures using similar models. Peisakoff [13] examined the general case of the Colonel Blotto Game using similar model. Interested readers desiring to further study this type model would profit by reviewing these references.

B. MODEL II

1. Problem

The problem is the same as for Model I. Changes in assumptions and the like will be mentioned explicitly.

2. Anti-Communicator (Command B - Red) Analysis

The problem is a distribution of anti-communication resources among message traffic routes of differing value.

3. Questions to be Answered

Should all the anti-communication resources be allocated to all routes?

If anti-communication resources are only allocated to some routes, which ones?

Should a/c allocations be in proportion to message traffic route value?

Should common resources be allocated to all routes?

If communication resources are only allocated to some routes, which ones?

Should communication resources be allocated in proportion to message traffic route value?

4. Model

Anti-Communicator, Red, had D units of resource. There are n number of message traffic routes numbered $r_1, r_2, \ldots r_n$. Message traffic route values are $v_1, v_2, \ldots v_n$ where

$$0 < v_1, \leqq v_2 \leqq \ldots \leq v_n$$

Communicator, Blue, has A units of communication resources.

$A \geqq D$ (Covers both Case I and Case II of Model I)

Strategies

(1) A strategy for the Communicator is the allocation of his resources, A, among the n routes. Thus a strategy for Blue is a set of numbers

$$x_1, x_2, \ldots, x_n \ni x_i \geqq 0 \text{ and } \sum_{i=1}^{n} x_i = A$$

Each x_i represents the amount of communication resources allocated to message traffic route r_i.

(2) A strategy for the anti-Communicator, Red, is a set of numbers $y_1, y_2, \ldots, y_n \ni y_i \geqq 0 \; \sum_{i=1}^{n} y_i = D$

Each y_i represents the amount of anti-communication resources allocated to message traffic route r_i.

5. Payoff

Assume that; (1) (1 unit of communication resource) = (1 unit anti-communication resource)

(2) Availability of message traffic route is proportional to the number of communication resources in excess of anti-communication resources for each route.

$$M(x, y) = \sum_{i=1}^{N} v_i \max(0, x_i - y_i)$$

6. Properties of Solution

Communicator (Blue)

(1) Never assign resources to low valued routes

(2) Use a mixed strategy for high-valued routes (use entire amount of resources on one route selected at random).

Anti-Communicator (Red)

(1) Do not assign anti-communication resources to low valued message traffic routes

(2) Use pure strategy for high valued message traffic routes.

Proportions

The proportions of ; (a) value of routes, (b) allocation of anti-communication resources, and (c) probability of allocation of communication resources are not necessarily the same.

7. Model Reduces to

$$\max_{x_i} \; \min_{y_i} \; \sum_{i=1}^{n} v_i \; \max(0, x_i - y_i)$$

$$\text{subject to} \qquad \sum_{i=1}^{n} x_i = A \;, \quad x_i \geqq 0$$

$$\sum_{i=1}^{n} y_i = D \;, \quad y_i \geqq 0$$

$$A \geqq D$$

8. Further Study

Dresher [6 & 7] discusses the attack-defense game with many targets of different values giving proofs and detailed development of solutions for model that is mathematically similar to Model II above.

C. MODEL III

1. Problem

The problem is basically the same as for Model I and Model II. However, in this model each player is assumed to have several different types of resources to be divided in an optimal fashion among a fixed set of message traffic routes. The payoff function of the game

is convex. The "no soft-spot" of Dresher, and the concept of the generalized inverse of a matrix are used to determine optimal strategies for each player and the value of the game.

Specifically, the communicator and the anti-communicator each have a fixed amount of resources that are to be allocated among a set of targets. The resources are divided into a fixed number of types. The communicator (Blue) has A resource units divided into S types and the $m^{\underline{th}}$ type consisting of a_m units with;

$$\sum_{m=1}^{s} a_m = A$$

2. Model

Anti-Communicator, Red, has D units divided into r types;

$$\sum_{j=1}^{n} d_j = D$$

There are n routes labeled $R_1, R_2, \ldots, R_n$

The route value is $\gamma_i > 0$ where $\gamma_1 \geq \gamma_2 \geq \ldots \geq \gamma_n$.

Communicator, Blue, has A units divided into S types;

$$\sum_{m=1}^{s} a_m = A$$

$A \geq D$

Strategies

(1) Communicator - A strategy for the communicator is a set of numbers;

$$x_{im}\ , \quad i = 1, 2, \ldots, n \quad m = 1, 2, \ldots, s$$

such that;

$$x_{im} \geqq 0$$

$$\sum_{i=1}^{n} x_{im} = a_m \quad \text{and} \quad \sum_{m=1}^{s} a_m = A$$

(2) Anti-Communicator - A strategy for the anti-communicator is a set of numbers;

$$y_{ij}\ , \quad i = 1, 2, \ldots, n \quad j = 1, 2, \ldots, r$$

such that;

$$y_{ij} \geqq 0$$

$$\sum_{i=1}^{n} y_{ij} = d_j \quad \text{and} \quad \sum_{j=1}^{r} d_j = D$$

(3) x_{im} and y_{ij} denote, respectively, the communication resources of type m and the anti-communication resources of type j assigned by each command (player) to the $i^{\underline{th}}$ route. Clearly, each payoff function is convex in y for each x and convex in x for each y.

3. Payoff

The $m^{\underline{th}}$ type of resource unit, if unopposed, can earn for the communicator a unit payoff ϵ_m, independent of the message traffic route. Further, each route R_i ($1 \leqq i \leqq n$) has a unit value $\gamma_i = 0$. That is each unopposed communication resource unit of type m at the

i^{th} route will earn for Blue a payoff $\gamma_i \epsilon_m$. Finally, the attacker is at least as strong as the defender ($A \geqq D$), and the targets are ordered so that

$$\gamma_1 \geqq \gamma_2 \geqq \ldots \geqq \gamma_n$$

Introducing distinct types of communication and anti-communication resource units requires the anti-communicator to determine what percentage of his resources at each route will be expended on each type of communication resource unit. To simplify the present analysis, we will assume that this decision process is defined by a matrix $\Lambda = (\lambda_{mj})$, where λ_{mj} denotes that fraction of the allocated anti-communication resource units of type j to be used against a communication resource unit of type m at any route. This definition implies that

$$0 \leqq \lambda_{mj} \leqq 1 \qquad (1 \leqq j \leqq r,\ 1 \leqq m \leqq s),$$

$$\sum_{m=1}^{s} \lambda_{mj} = 1.$$

The types of resource units available to the players will partially determine the assignment of values to the λ_{mj}'s. For example, anti-radio communication resources cannot be used to interrupt wire communications. Since the anti-communicator's optimal strategy and the game value depend strongly on the elements of Λ, a proper choice of these values may greatly decrease the communicator's payoff.

For the sake of definiteness, we will assume henceforth that $s \leqq r$. Then the column vectors of Λ are linearly independent. If they were not, then two or more types of anti-communication units could be combined into a single type without loss of generality. Therefore, the rank of Λ equals s. If $s > r$, we must work with the transpose of Λ. In this case, the row vectors of Λ^1 are linearly independent and the rank of Λ^1 equals r.

4. Payoff Function

Case I - interested only in transmitting messages over message route. Want superiority for each message route (communication resource) type.

$$M(x, y) = \sum_{i=1}^{n} \gamma_i \sum_{m=1}^{s} \max \left[0, E_m(x_{im} - \sum_{j=1}^{r} \lambda_{mj}\, y_{ij})\right]$$

Case II - concerned with preventing the anti-communicator from achieving an offensive role. Want overall superiority on each message traffic route.

$$M(x, y) = \sum_{i=1f}^{n} \gamma_i \max \left[0, \sum_{m=1}^{s} E_m\, (x_{im} - \sum_{j=1}^{r} \quad _{mj}\, y_{ij})\right]$$

5. Assumptions

(1) Game consists of a single move during which the players act simultaneously.

(2) Routes are independent of one another.

(3) Resources of equal amounts neutralize each other.

(4) Commitment of resources by a player, once made, can't be changed.

6. Properties of the Solution

(1) The communicator (Blue) has an optimal mixed strategy which consists of allocating his entire resources to a single message traffic route chosen by means of a probability-distribution function.

(2) The anti-communicator (Red) has the optimal pure strategy (the y_o that minimizes $\max_x M(x, y)$) of allocating each type of anti-communication resource over the n routes.

(3) The value of the game is the $\min_y \max_x M(x, y)$.

(4) Cohen [2] theorem of section 3 completely characterizes the solution of the game. However, it does not provide any practical means for determining either the optimal strategies or the value of the game. We achieve this by using Dresher's "No Soft-Spot Principle" [3, 4], which states that an optimal strategy for Red is to allocate only to those routes which, under a concentrated allocation by Blue, would yield to Blue the value of the game. Conversely, Blue should allocate only to those routes which Red chooses to allocate resources to.

7. Model Reduces to

Case I - Interested only in transmitting messages over message traffic route (communication resource) type

$$\max_{x} \min_{y} \sum_{i=1}^{n} \gamma_i \sum_{m=1}^{s} \max \left[0, \epsilon_m (x_{im} - \sum_{j=1}^{r} \lambda_{mj} y_{ij})\right]$$

$$\text{subj to} \sum_{m=1}^{s} a_m = A, \quad x_{im} \geqq 0, \; y_{ij} \geqq 0$$

$$\sum_{j=1}^{r} d_j = D$$

$$\sum_{m=1}^{s} \lambda_{mj} = 1, \quad 0 \leqq \lambda_{mj} \leqq 1$$

$$, (1 \leqq j \leqq r, \; 1 \leqq m \leqq S)$$

Case II - Want overall superiority on each message traffic route

$$\max_{x} \min_{y} \sum_{i=1}^{n} \gamma_i \max \left[0, \sum_{m=1}^{s} \epsilon_m (x_{im} - \sum_{j=1}^{r} \lambda_{mj} y_{ij})\right]$$

sub to

$$\sum_{m=1}^{s} a_m = A, \quad x_{im} \geqq 0, \; y_{ij} \geqq 0$$

$$\sum_{j=1}^{r} d_j = D$$

$$\sum_{m=1}^{s} \lambda_{mj} = 1, \quad 0 \leqq \lambda_{mj} \leqq 1$$

$$, (1 \leqq j \leqq r, \; 1 \leqq m, \leq s)$$

8. Further Study

Cohen [2] generalizes results of Karlin and Dresher for an attack-defense model similar to Model III. Detailed proofs and developments of solution are given therein.

IV. USE OF MODELS

Like any model, including the most complex simulation or war game, these models are only an abstraction of the real world. Thus, what is important is not the specific numerical results, but rather the insight obtained by the model into a qualitative description of the structure of the logistics allocation decision. This can provide the basis for more detailed analysis of resource allocation and help mold the intuitive assessment of the real world problem [12].

A. SPECIFIC USE

The use of the specific models illustrated in this paper is rather limited. One can easily be misled if what is learned from one model or three models is just summarized and then that information is used for all situations. One must know that other models exist and that "danger" also exists if you apply a model to the wrong problem. How much a model (an abstraction of the real world conflict) applies will depend upon the adequacy of the particular model in describing the real world conflict.

B. GENERAL USE

In general, it is apparent that practically all Game Theory Models; attack-defense, resupply, logistics allocation, and the like can be used in Game Theory Analysis of the problem of allocation of

communication resources. This was the important "discovery" of author early in his research. This is also the main point the author desires to highlight for the reader (i. e. practically all Game Theory Analysis/models can be used in various military applications if the problem can be formulated.). The crucial point is to formulate the problem correctly, and in the solution to bear in mind what the assumptions, capabilities, and limitations are of the mathematical model that you are using to analyze the real world conflict.

C. EXTENSION OF MODELS

This paper discusses three models. There are many, many different models that will fit almost any situation [6 & 7]. One group of games worth mentioning and describing are the continuous games.

In a finite or discrete game previously considered each player selected a strategy from a finite set of strategies. The number of such strategies may be larger, as in chess, but finite. A natural generalization is to consider games in which each player has available an infinite number of strategies over a closed interval. In particular, we shall assume that each player has a continuum of strategies from which to select a strategy to play the game.

Such a game is called a continuous game. There is no loss of generality if we assume that the strategies are represented by points on the closed interval [0, 1]. For if S is the set of strategies, then by relabeling the elements of S, we can get a game in which the selection of a strategy is made from the closed interval [0, 1].

There are two reasons for developing a theory of continuous games. First, many military and economic problems actually involve an infinite number of strategies. For example, a military budget can be thought of as being divisible in an infinite number of ways between offense and defense. A commodity can have an infinite number of price possibilities. Secondly, computations using a continuous variable are generally easier than those using a large number of discrete variables. For development and application see Dresher [6 & 7] and Peisakoff [13].

V. CONCLUSIONS

The extensive work that has been done in the attack-defense area can be applied profitably to the problem of allocation of communication resources. For treatment of attack-defense game see Blackett [1], Cohen [2], Cooper and Restrepo [3], Moglewer and Payne [12], and Peisakoff [13].

One should be aware of the many models that exist and more importantly, aware of when they can profitably be applied to a particular problem always realizing the assumptions, the capabilities, and the limitations.

VI. FUTURE STUDY

As previously mentioned there are some important things to be done in Game Theory. One is to develop further the theory itself, so that more difficult and varied problems can be solved. Another is to find situations to which existing theory can be profitably applied.

Applying Game Theory to the allocation of resources is not new. This became apparent to the author early in his research. However, applying Game Theory specifically to the allocation of communication resources as discussed in this paper was new to the author. Many other areas to which Game Theory can be profitably applied are omnipresent and worthy of future study. (i. e. A concept now aware to me is that Game Theory can profitably be applied to any conflicting situation).

S. Moglewer and C. Payne [12] developed a model which is felt to be significant in formulating a <u>new</u> point of view for military logistics decisions. This paper is of very recent vintage. It is felt that many, many military areas and situations are available for <u>new analysis</u> by Game Theory. Significant contributions could be made in this area by future endeavor.

BIBLIOGRAPHY

1. D. Blackett, "Some Blotto Games", Naval Research Logistics Quarterly, Vol. 1, pg. 55-60, March 1954.

2. N. D. Cohen, "An Attack Defense Game with Matrix Strategies," Naval Research Logistics Quarterly, Vol. 13, pg. 391-402, December 1966.

3. J. Cooper and R. Restrepo, "Some Problems of Attack and Defense", SIAM Review, Vol. 9, pg. 680-691, October 1967.

4. J. M. Danskin, "The Theory of Max-Min, with Applications", SIAM Journal on Applied Mathematics, Vol. 14, pg. 641-664, July 1966.

5. J. M. Danskin, The Theory of Max-Min, and its Applications to Weapons Allocation Problems, Springer-Verlag, New York, N.Y., 1967.

6. M. Dresher, Games of Strategy, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1961.

7. M. Dresher, Theory and Applications of Games of Strategy, RAND Report R-216, 1 December 1951.

8. M. Frechet and J. von Neumann, "Commentary on the Borel Notes", Econometrica Journal of the Econometric Society, Vol. 21, pg. 118-127, January 1953.

9. L. Friedman, "Allocation of Advertising Expenditures," Opns. Res., Vol. 6, pg. 703-709 (1958).

10. D. Hilbert and S. Cohn-Vossen, Geometry and the Imagination, Chelsea Publishing Company, New York, 1952.

11. J. McKinsey, Introduction to the Theory of Games, McGraw-Hill Book Company, Inc., New York, 1952.

12. S. Moglewer and C. Payne, "A Game Theory Approach to Logistics Allocation", Naval Research Logistics Quarterly, Vol. 17, pg. 87-98, March 1970.

13. M. P. Peisakoff, "Continuous Blotto," RAND Memorandum 736, 4 December 1951

14. J. von Neumann and O. Morgenstern, Theory of Games and Economic Behavior.

15. J. D. Williams, The Compleat Strategyst, McGraw-Hill Book Company, Inc., New York, 1952.

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13. ABSTRACT

This thesis looks at optimally allocating communication resources from a game theory point of view. Three basic models are presented and then expanded upon. A prime objective is to briefly review representative examples of work previously done in the attack-defense area showing how it can be applied to the optimal allocation problem for communication resources and indicate the possible direction for future research.

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