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BY A COMPUTER WAR GAME METHOD

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by

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Submitted in partial fulfillment of
the requirements for the degree of

BACHELOR OF SCIENCE

United States Naval Postgraduate School

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William A. Van Train, Jr.

This work is accepted as fulfilling

the thesis requirements for the degree of

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ABSTRACT

The computer war game is emerging as a vital tool for finding near-optimal solutions to current military problems. A computer war game designed to permit parametric analysis of a submarine barrier is developed. Simulation techniques, both mathematical and computer, are discussed. The effects of assumptions inherent in the computer war game are described. Illustrative analyses conducted through use of this computer war game are exhibited. Potential uses and methods for improvement of the developed war game are discussed.

SUMMARY

The work presented in this paper was undertaken with the objective of developing a computer war game which would provide a basis for the parametric analysis of submarine barrier design. This war game was contrived in a manner which would allow its application to a wide variety of barrier forms.

Initially, a hypothetical submarine barrier and its environment are defined. Characteristic parameters are isolated to provide a basis for war game simulation. In order to stay within practicable programming limits, some non-sensitive parameters are eliminated from consideration, and others are combined into a single parameter which effectively represents all elements of the group. In addition, assumptions inherent in this war game simulation of a submarine barrier are enumerated. Both war gaming and computer programming techniques are discussed in connection with simulating the hypothetical barrier. The techniques discussed cover both geometric and functional aspects of barrier design.

Two sample analyses were conducted by the use of this computer war game. One is concerned with an estimate of the optimal design-speed for the barrier submarine, and the other looks at the relative effectiveness of different geometric barrier forms as a function of detection capability. Neither of these analyses can be considered complete. They are presented herein to illustrate analytical use of the computer war game. Finally, some potential uses for this war game are

set forth, along with some brief descriptions of procedures which might be used in modifying the present computer program to achieve greater realism and accuracy.

The author wishes to express his appreciation to Mrs. J. L. Ramos for her clerical assistance and careful proofreading of this paper.

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LIST OF ILLUSTRATIONS

TABLE OF SYMBOLS

Symbol	Meaning
B	Effective firing range of the weapons installed in the barrier submarines.
I_i	Distance between the left extreme of the barrier front and the beginning of the first submarine zone in the ith barrier line.
L	Length of the barrier front.
M	Number of barrier lines.
N_i	Number of submarines in the ith barrier line.
P(D)	Probability of detection of a transiting submarine by the entire submarine barrier. This value is received as an output from the thesis program.
P(I)	Conditional probability of interception of a transiting submarine by the entire submarine barrier, given a detection. This value is received as an output from the thesis program.
P(K)	Conditional probability of kill of a transiting submarine by the entire submarine barrier. This value is received as an output from the thesis program.
R	Range of detection capability of the barrier submarines.
S	Distance between barrier lines.
V	Maximum speed capability of the barrier submarines.
Vr	Speed required of the barrier submarine in order to effect an interception of a transiting submarine.
W	Speed of the transiting submarine.
Z_i	Length of the submarine zone in the ith barrier line.
θ	Angle, relative to the barrier line, at which the transiting submarine is detected by the barrier submarine.
ϕ	Angle, relative to a perpendicular to the barrier line, at which the transiting submarine will cross the barrier.

CHAPTER I[1,2]

Introduction

The field of Operations Research is daily faced with new
problems which become increasingly complex. The upward accel-
eration of the level of complexity in current national and
world problems has been brought about, in large part, by a
rapid general expansion in the level of technology and a rapid
increase in the complexity of inter-relations between individ-
uals, organizations, and nations. Decisions made in this
environment can have wide-spread effects which go far beyond
the immediate problem-area in which the decisions were made.
At the national level, the effect of faulty decisions can
have a devastating effect on national security. The complex-
ity of the decision environment and the vital need for arriv-
ing at the best decisions possible within the constraints
imposed by this environment imply the need for a technique
of analysis which is capable of handling, realistically and
rapidly, the large number of important variables which seem
to constitute a characteristic of modern Operations Research
problems.

That advanced technology which represents a barrier to
the effective solution of current problems has, within the
past few years, evolved a tool which can do much to assist
in solving these problems. The combination of this tool
with a time-honored analytical technique provides a system

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for aiding in the solution of complex problems. The tool evolved is the high-speed, electronic, digital computer, and the old technique is that of war gaming.

The objective of this thesis is to demonstrate the use of such a war-game/computer combination for solving decision problems in connection with an ASW submarine barrier. The problem area associated with submarine barriers has the high level of complexity inherent in most present-day problems, and the proper solution of these problems is vital to the national security. Thus, the area in which this thesis works falls within the general category of important problems referred to above. Before proceeding with the detailed thesis, it would be well to present some of the historical background which has led to the synthesis of war games and the digital computer as an analytical tool.

War gaming is an ancient technique. In its crudest form, it found expression as early as 1000 B.C. as the forerunner of the modern game of chess. In essence, chess is a game of war which has been divested of its physical aspects and remains as a purely intellectual struggle between two individuals. Nearly all of the historical development of war gaming, until the nineteenth century, was based on various attempts to evolve a more complex and sophisticated form of chess. The objective was to devise a game which would more closely approximate the engagement of forces in the field.

Historical variations in the chess-type war game took on many forms. The earliest of these was developed by Christopher Weikhmann in 1664. In his game each opponent had 30 pieces; each piece had a name representing some military function. There were fourteen different kinds of fixed moves. This game remained as an intellectual contest between individuals, and as such was simply a complex extension of the basic chess game.

In 1780 a game called Helwig's War Chess appeared on the war gaming scene. It was played on a board consisting of 1,666 squares, tinted to represent variations in terrain. In the playing of this game, mathematical processes were used toward the objective of reducing a fortress. This objective was synonymous with the major military objective of the day, and the game was used in the early training of potential military officers.

Over a period of many years, a large number of attempts was made to develop a game which would contain elements simulating those normally encountered in the battlefields. It was thought that such a game would provide an economical means of furnishing some battlefield training in the classroom for potential military officers. The number of squares on the chessboard was increased progressively to 3,600. Several different schemes were utilized to provide representations of terrain variation on the board. The number of pieces in the game was increased, and these pieces, along

with their possible moves, began to take on military signif-
icance in a more accurate sense. The methods of making
moves during a game changed considerably. Mathematical
processes were utilized to determine appropriate moves
under a given set of circumstances. Chance elements were
included. For example, in one game a die toss was used to
determine the effect of a particular application of fire-
power.

It became apparent, in the 19th century, that further
modifications of the basic chess game would not greatly en-
hance the training to be derived from this technique. As a
consequence, fundamental changes were made. First, a sand
table and, ultimately, a space the size of a drill floor
were utilized in creating terrain models on which to conduct
war games. The pieces used in the game took on a closer
representation of actual forces in the field. In addition
to its training function, this type of game was utilized
in planning contemplated military actions.

Another trend which became evident during this same
period was the use of charts, laid out to a specific scale,
to serve as the field of play for a war game. A form of
this technique was used by Napoleon; however, he used it as
a personal scheme for planning operations and, as far as is
known, never utilized the technique as a training device.

The ultimate development of this sort of war game has
led to the use of actual forces participating in mock wars

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or, more accurately, mock battles. This technique is used to the present. It is used extensively by the Navy in its fleet exercises. This technique has an advantage in providing military forces with practice in the execution of current tactical procedures. However, it has several disadvantages when used as a device for analysis of a problem-area. It is extremely expensive and requires an inordinate amount of time to acquire sufficient data to render the results significant. This technique completely breaks down as a possibility for testing proposed weapons systems, since the weapons inherent in the proposed system are not available for field test.

Fortunately, at this stage in the history of mankind when technology is advancing at a rate which makes "new" weapons become obsolescent very rapidly, technology has also provided a system for conducting easily controlled, comparatively accurate, widely variable, relatively inexpensive, and rapidly played war games. This device is the high-speed, electronic, digital computer.

The full capabilities of the digital computer are not known at present. However, even with the relatively minor inroads which have been made into the field by current practitioners, the system has demonstrated its worth for the extension of the war gaming technique. The computer, programmed to simulate an operational situation, provides a system for testing new tactics and proposed weapons systems

which is unequalled in speed and effectiveness by any other technique.

Nearly any military situation can be simulated in the computer. It is true that, due to lack of available computer memory space, time, and/or other factors, basic assumptions must ordinarily be made which tend to degrade the realism of the computer simulation. However, careful control over the assumptions which are built into the program can ensure effective results.

The advent of the high-speed digital computer has brought a new, previously unattainable, capability into the war gaming field. By use of this technique, a complex weapons system, which exists only as a parametric mental concept, can be tested for feasibility and operational effectiveness prior to the time that funds must be expended on the technical development of this weapons system. In the present world situation, where limited research and development funds must be expended in the most efficient way possible, this technique can be of inestimable value in the development of a new system. Optimal values for appropriate parameters can be established, and various means of tactical and/or strategic utilization can be explored. The establishment of non-sensitivity conditions with respect to some parameters may lead to savings in the expenditure of development funds. Thus, the application of computer techniques to the old art of war gaming is providing an extremely valuable,

even essential, tool for the test and evaluation of new ideas in this present-day world of soaring technology.

In summary, the war game began as a relatively simple game of chess. As the conduct of military maneuvers and the weapons in use became more complex, the need for a device to train personnel, test tactical concepts, and plan military campaigns became apparent. Until the mid-20th century, various, sometimes quite involved, modifications of the basic chess game served these purposes. In the environment of the modern exponential rate-of-advance of technology, the classical systems of war gaming serve only limited objectives. The new technology establishes a need for a modernized war gaming technique which will provide for rapid and effective analysis of proposed concepts. The high-speed digital computer has filled this need. The potential of this technique has barely been tapped to date. It promises a rapidly increasing utility for some time to come.

In this thesis, computer and war gaming techniques are combined to form a game which simulates the conflict between submarines in a barrier and an unfriendly submarine attempting to transit that barrier. The remainder of this writing sets forth the simulation techniques used, functional operations of the computer program, and illustrative analyses.

Detailed programming techniques will not be covered. This war game was programmed for the 1604 Computer manufactured by The Control Data Corporation. A copy of the program,

along with pertinent operating instructions, is kept on file
in the Mathematics Department of the U. S. Naval Postgraduate
School, Monterey, California.

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Chapter II

War Game Simulation of Submarine Barrier

The war gaming technique as applied to a digital computer is an admirable device for conducting an analysis of optimal submarine barrier design. A simulation of this type serves to illustrate the many advantages of a war game approach to the problem.

The war game which was evolved and programmed for the 1604 computer in the development of this thesis (hereinafter referred to as the thesis program) simulates a hypothetical ASW submarine barrier and the environment in which it functions. Our first step is to establish the geographic environment of the simulated barrier and to determine those parameters inherent in such a barrier which must be incorporated in its war game simulation.

The barrier contemplated by the computer simulation of this thesis has certain specific characteristics. These characteristics are not intended to represent any particular barrier design. A specific attempt has been made to keep the geometry of the barrier quite general and elementary. The purpose in doing so is to provide a program which is suitable for basic analysis of a wide variety of barrier situations.

The barrier under consideration might be classified as an "open-ocean" type. It is positioned in ocean areas which are unobstructed by land masses, and in which transiting submarines

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are not restricted, for one reason or another, to passage through a narrow channel. An example of this sort of barrier would be one which was positioned defensively along a national coast. It might also be positioned across an expanse of ocean.

The barrier under consideration is a straight-line barrier. There is no provision in this program for either a curved-line barrier or one composed of two or more straight-line segments. The barrier formation may consist, in depth, of one, two, or three lines of submarines. Each submarine in each barrier line is assigned to a specific zone, and it must remain within its assigned submarine zone throughout the existence of the barrier. This zonal restriction remains effective in an attack situation. The submarine zones are rectangular in shape and are of the same dimensions for all submarines in a given barrier line. The dimensions of these zones may, however, vary between barrier lines in cases where the barrier being studied is more than one line deep. Each submarine is required to maintain position in the center of its assigned zone. This restriction is, of course, waived during periods when an intercept of a transiting submarine is being conducted.

The development of a computer simulation of the barrier described above must be done in terms of its characteristic parameters. Obviously, complete and realistic simulation involves a large number of these. The following list of parameters might be used to characterize the simulated barrier:

1. Length of barrier front.

2.   Number of barrier lines.

3.   Number of submarines in each barrier line.

4.   Length of submarine zone in each barrier line.

5.   Width of submarine zone in each barrier line.

6.   Distance between barrier lines.

7.   Relative positioning of submarines between barrier lines.

8.   Barrier submarine speed capability.

9.   Barrier submarine endurance at various operating speeds.

10.   Operating depth of barrier submarines.

11.   Geometric pattern of detection capability.

12.   Water conditions which influence detection capability.

13.   Other environmental factors which influence detection capability.

14.   Effective range of installed weapons.

15.   Weapon delivery characteristics.

16.   Accuracy of weapon fire control system.

17.   Lethality of installed weapons.

18.   Human operator-factors which relate to submarine con-

trol, detection, target classification, target fixing, weapon
handling, and fire control.

19.   Inter-submarine communication capability.

The foregoing paragraphs constitute a physical description
of the hypothetical submarine barrier which is simulated by
the thesis program. The simulation techniques used in the
development of the thesis program do not constitute a com-
pletely realistic representation of this hypothetical barrier.

In preparing the thesis program, some of the parameters
listed above were not considered. In addition, simplifying
assumptions were made with respect to some of those which were
considered. The fact that some parameters were either ignored
or modified does <u>not</u> imply that computer processes are not
capable of dealing with them. The list of pertinent parameters
was modified in effecting simulation for one of two reasons:
either time available for thesis preparation was not adequate
to allow consideration of some parameters or a single, com-
posite parameter could represent a group of parameters with an
acceptable degree of realism.

Those parametric characteristics which are used by the
thesis program to effect simulation are set forth below. The
data in parentheses following the description of each para-
meter are its representative symbol and the maximum value of
that parameter which can be entered in the program. In most
cases, this maximum value is dictated by the availability of

data-entry space on the console of the computer. Other systems for data entry than that used in the thesis program would remove this limitation.

1. Length of barrier front. (L) (24,576 miles) The limiting value of this parameter is dictated by the amount of computer memory space reserved for entry of the geographic plot of the barrier.

2. Number of barrier lines. (M) (3).

3. Length of submarine zone in each barrier line. (Z_i; i = 1, 2, 3) (511 miles).

4. Distance between left extreme of barrier front and beginning of first submarine zone. (I_i; i = 1, 2, 3) (511 miles). This parameter is used to establish the relative positioning of barrier submarines between barrier lines.

5. Distance between barrier lines. (S) (511 miles)

6. Number of submarines in each barrier line. (N_i; i = 1, 2, 3) (63).

7. Barrier submarine speed capability. (V) (511 knots) This represents the upper limit of the speed which a barrier submarine is capable of making in a particular run of the computer program.

8. Detection Range. (R) (511 miles).

9. Range of installed weapons. (B) (63 miles).

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10. Conditional probability of kill of transiting submarine, given a detection and interception. $[P(K)]$ (1.00).

Thus far, only a one-sided game has been discussed. Any game involves some sort of conflict between opponents. In the war game in question, the opponent to the barrier submarines is an unfriendly submarine which attempts to transit the barrier. This target would have roughly the same set of characteristic parameters as a barrier submarine, plus freedom of movement. However, due to lack of available time for programming, only the following two parameters are used in the simulation of the transiting submarine:

1. Transit submarine speed. (W) (63 knots) In the thesis program, the transiting submarine moves constantly at the speed set by the program operator on the computer console.

2. Transit track angle. (ϕ) ($+90°$ to $-90°$) The reference ray and directions of measurement for this parameter are indicated in Figure 1.

In addition to those parameters set forth above, there are two which are generated in the computer as a function of game play:

1. Detection angle. (θ) (0° to 180°) The reference ray and direction of measurement are indicated in Figure 1.

2. Interception speed requirement. (Vr) (no effective limit) This parameter represents that speed required by the

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barrier submarine in order to intercept the target following detection.

A diagramatic representation of the geometry and pertinent parameters of the barrier simulation is contained in Figure 1.
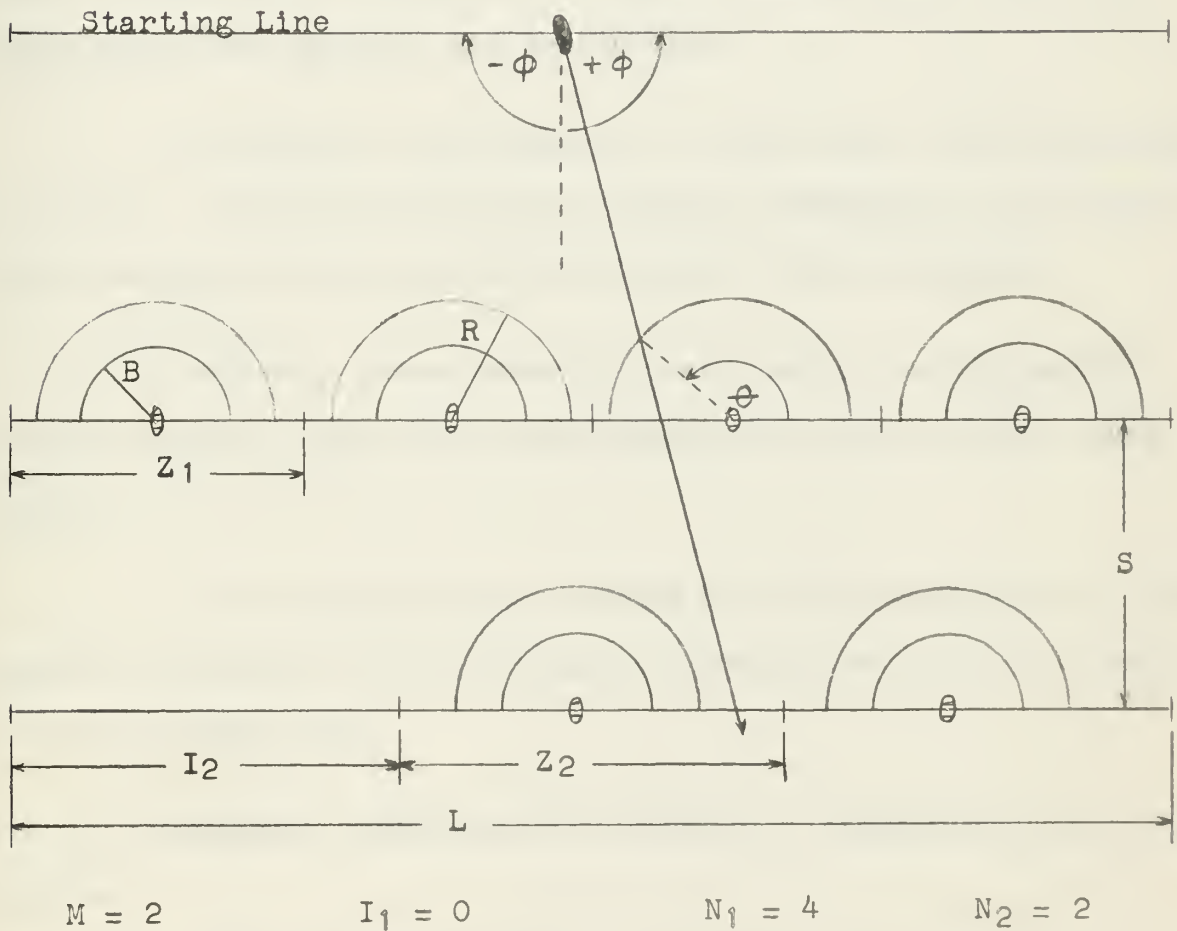
Diagram of Submarine Barrier

Figure 1

In order to understand and, more importantly, to evaluate results, one must know what assumptions have been built into the computer simulation of the submarine barrier. Some of the assumptions inherent in the thesis program have been made in order to reduce the magnitude of the programming effort. The computer is capable of handling the problem stripped of these assumptions, but time was not available for the necessary programming. Procedures for eliminating some of the assumptions will be discussed in Chapter V. Those assumptions which are implicit in the program are as follows:

1. The barrier environment is essentially two-dimensional in nature. Depth of the water, barrier submarine, and transiting submarine are not considered in the thesis program.

2. Weather, ocean currents, and similar environmental factors are not important considerations in the barrier simulation.

3. Due to navigational errors and other influences, each barrier submarine will be normally distributed about the center of its assigned zone.

4. Barrier submarines are unable to communicate with one another.

5. The zone of detection capability for each barrier submarine can be represented as a semi-circular area.

6. The transiting submarine will be detected with

probability 1.0 if it enters the effective detection area of any barrier submarine.

7.   Only one submarine in each barrier line will attempt an attack on any target, and that submarine will be the one which is closest to the point at which the transiting submarine will cross the barrier.

8.   The barrier submarine which is closest to the transit crossing point will always attempt an attack. In other words, the commanding officer of the barrier submarine will never intentionally allow the target to pass through the barrier.

9.   The barrier submarines are constrained to move only on a track parallel to the barrier line while attempting an interception.

10.   Any barrier submarine which is capable of interception will always fire on the target.

11.   An attacking barrier submarine will always fire as soon as it comes within effective firing range of the target.

12.   An attacking barrier submarine will fire on the target only once.

13.   There are no friendly units transiting the barrier. Target classification is not a consideration in the simulation.

14.   The points from which enemy submarines commence their transits will be either uniformly distributed along a reference

line ahead of and parallel to the barrier front or normally distributed about a point on this reference line opposite the center of the barrier front.

15.   The tracks of transiting submarines will be normally distributed about a perpendicular to the barrier line.

16.   Transiting submarines will always travel at their assigned speed.

17.   The transiting submarine has no knowledge of the existence of the barrier. It therefore takes no evasive action.

18.   The transiting submarine will not attack any barrier submarine, and the barrier is not under attack from other outside sources. Therefore, there is no attrition of barrier submarines.

Of the above list of 18 assumptions, there are six which are considered as being unrealistic to the extent that they adversely affect the realism of the output data. These six are numbered: 1, 9, 12, 13, 17, and 18.

Assumptions 9 and 12, while they do not represent those actions one would normally expect to find in the hypothetical barrier, do not seriously affect data derived from the program. In effect, these assumptions represent a conservative simulation of hypothetical barrier action. Their ultimate effect is to generate output data which represents a lower bound on that which might be expected without them.

The remaining critical assumptions tend to generate overly optimistic output data. They serve to oppose the lack of realism in the two assumptions cited above. However, the exact composite effect of all six assumptions is undetermined. Their overall effect is to reduce the degree of resolution which may be obtained in a sensitivity analysis of barrier parameters. With a recognition that these assumptions are implicit in the program, an appropriate estimate of the level of significance can be determined for the results of a particular analysis.

Elements of data for use in analyses are obtained from a series of plays of the war game contained in the thesis program. The number of game plays in a series may be determined by the program operator. A single play of the game consists of the following sequence:

1. The barrier submarines and their detection sensors are entered in a geographic plot of the barrier. Each submarine is positioned in accordance with a normal distribution about the center of its assigned zone.

2. A uniformly or normally distributed starting point along a reference line ahead of the barrier is established for the transiting submarine.

3. The course on which the target will penetrate the barrier is selected in accordance with a normal distribution centered on a perpendicular to the barrier line.

4. The transiting submarine is moved through the barrier.

and the outcome of the game, in terms of detection, intercep-
tion, and kill, is recorded.

The barrier submarine war game described in this chapter
must also be explained in terms of its operation in the computer
program. Pertinent computer simulation techniques are presented
in the next chapter.

Chapter III

Computer Simulation of Submarine Barrier

The thesis program is of modular construction. This sort of program construction, in general, results in inefficient use of computer space and, to some extent, additional running time for the program. However, it has a decided advantage. Since the individual program modules are nearly independent of one another, this type of construction provides for relatively easy later modification of the program.

There are several elements of sophistication which, if added to the thesis program, would heighten the realism of the simulation and/or provide for additional areas of analysis with respect to submarine barriers. Some of these elements will be mentioned in Chapter V, along with descriptions of some general processes by which their incorporation might be accomplished.

In addition to that portion of the thesis program which actually produces the barrier simulation and game play, there are several supporting sub-routines which are used throughout the program. These are not directly concerned with the submarine barrier war game. In general their presence is dictated by needs which arise in connection with either mathematical models or arithmetic processes inherent in a digital computer. The functions of these supporting program modules will be covered, but not the detailed programming techniques, since these techniques are more or less standard.

The supporting sub-routines utilized in this program are as follows:

1. A random number generator which produces random samples from the uniform distribution on the interval $[0,1)$.[3]

2. A random number generator which produces random samples from the standardized normal distribution.[4]

3. A routine which computes the sine, cosine, and tangent of any integer-valued angle from $1°$ to $89°$, inclusive. This routine contains a table of sine values for all angles in the above range. Other trigonometric values are computed as functions of the sine values. One section of the computer program involves a range of angles from $0°$ to $180°$. Angles greater than $90°$ are handled elsewhere in the program by entering this sub-routine with the supplement of the desired angle and affixing the appropriate sign.

4. A routine which converts positive and negative numbers in floating-point format to their nearest integer equivalent.

5. A routine which converts positive and negative integers to their equivalent floating-point format.

6. A routine which converts positive fractional numbers from the interval $[0,1)$ to their equivalent floating-point format.

7. A routine which computes the sample mean and

variance of the output data from the program.

With the exception of the two random number generation sub-routines and the statistical analysis sub-routine, all of the above are entered by a return jump, with the number to be treated in the A-register. The two random number generators are also entered by a return jump, but they function independently of any data input from the main program. The output from all of these sub-routines, except for that which performs the statistical analysis, is entered in the A-register prior to return to the main program. The statistical analysis sub-routine goes into operation automatically upon completion of the specified number of game plays. This sub-routine computes the sample mean and variance of detections, interceptions, and kills for each group of 100 transits through the barrier. It then computes the mean-of-means and variance-of-means for the specified number of 100-transit runs made during an individual running of the program. All of this data is stored in the space in the computer reserved for output data.

All of the above described sub-routines are supporting elements of the main barrier simulation program. The main program itself is broken into several modules, each of which performs a more or less independent function in the process of the overall war game simulation of the submarine barrier. These modules divide the barrier war game into more or less natural components. Figure 2 is a simplified flow-diagram which illustrates both the general war gaming procedure and the functional positions of the main program modules.
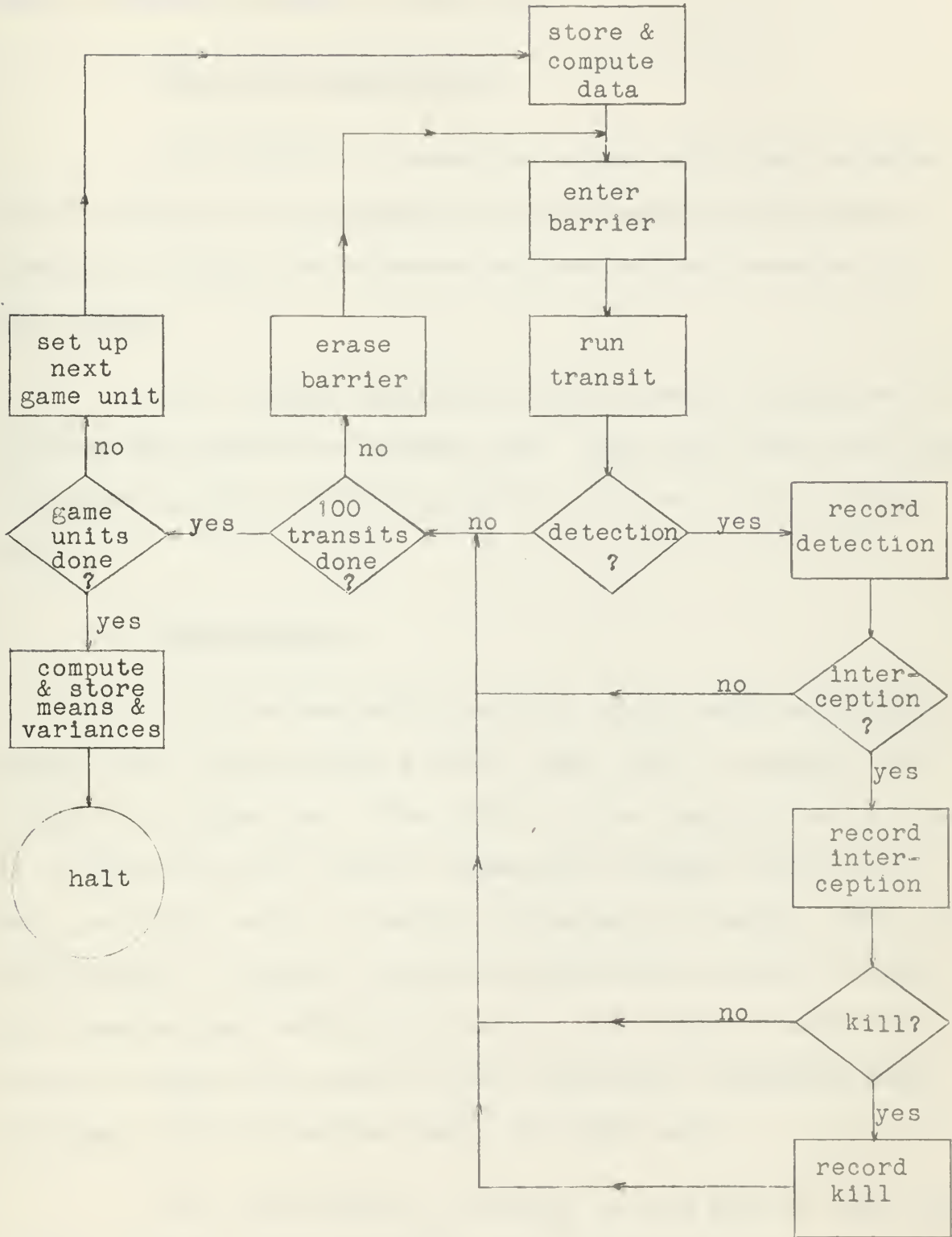
Thesis Program Flow Diagram

Figure 2

The following sub-paragraphs contain brief descriptions of the functions performed by the various modules of the computer program indicated in Figure 2:

1.   Store and Compute Data.

(a)   The input parametric values which were entered on the console of the computer by the program operator are placed in storage for reference and use during operation of the program.

(b)   The mean position of each barrier submarine is computed and stored for further use.   These mean positions are expressed in terms of miles from the left end of the barrier line.

2.   Enter Barrier.

(a)   The geographic position which each barrier submarine will occupy during a single game play is computed and stored for further use.   The effect of the computation involved is to distribute the barrier submarines normally about their mean positions, with a standard deviation of 8 miles.   This is accomplished by taking a random normal deviate from its generator sub-routine, multiplying this by the 8-mile standard deviation, adding the result to the appropriate submarine mean position, and storing the result for later use.

(b)   The diameter, in miles, of the area of detection capability for barrier submarines is computed and stored.   A set of 1-bits, equivalent to the above computed length and

centered at the normally-distributed position of each barrier submarine, is then entered in the space in the computer reserved for the geographic plot of the barrier. The remaining bit positions in the barrier line(s) are left zero.

3. Run Transit.

(a) The starting point for the transiting submarine is established along a reference line ahead of the first barrier line. In order to provide for different environmental barrier conditions, two alternative means of determining this entry point are provided in the program, at the option of the program operator. In normal usage, transit entry points will be uniformly distributed over the full length of the first barrier line. If Jump Key 1 on the computer console is set, the transit starting points will be normally distributed about the center of the first barrier line, with a standard deviation of 104 miles. These options are provided to differentiate between (1) the case wherein a truly open-ocean barrier is concerned and transiting submarines are equally-likely to enter at any point along the barrier front, and (2) the case wherein transiting submarines are expected to travel along a comparatively narrow channel.

(b) The track angle of the transiting submarine is computed. The thesis program provides for normal distribution of this track angle about a perpendicular to the barrier line, with a standard deviation of 8 degrees.

(c) The effective detection front along the barrier

line is increased as a function of the transit track angle (See Appendix A). This effective increase in detection front is now entered at the appropriate end of the detection front for each submarine in the barrier.

(d) A single 1-bit is positioned in the Q-register, and an index register is appropriately set, as a function of the length of the first barrier line, the selected distribution of starting points, and the transit track angle, so that a single bit can be extracted from the appropriate point along the geographic plot of each barrier line. The extracted bit represents that point at which the transiter crosses the barrier line (See Appendix B).

4. Detection.

(a) Whether or not detection occurs is determined by the bit extracted from the geographic plot of the submarine barrier. If a 1-bit is extracted, detection has occurred; if a 0-bit is extracted, there has been no detection.

(b) If no detection occurs during a complete barrier transit, the thesis program erases the geographic plot of the barrier, re-enters the barrier plot with new, normally-distributed positions for the barrier submarines, and runs the next submarine transit through the barrier, in the manner outlined above.

(c) If a detection occurs, the detection is recorded, and the program checks to see whether or not the transiting

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submarine will pass, on its present track, within weapon range of the barrier submarine. If it will, the program moves to the module which determines whether or not the transiter is killed when fired upon. If the transiter will not pass within weapon range of the barrier submarine, the thesis program computes the detection angle and moves to the module which determines whether or not the barrier submarine is capable of intercepting the transiter. (See Appendix C)

5. Interception.

(a) The speed required of the barrier submarine in order to effect interception is computed (See Appendix D). If the speed required is greater than the speed capability of the barrier submarine (as established by the operator prior to starting the program), the geographic plot of the barrier is erased, and the next transiter is run through the barrier, or the next barrier line is transited if the barrier is composed of more than one line. If the speed required is less than or equal to the speed capability of the barrier submarine, the interception is recorded, and the program moves to the module which determines whether or not the transiting submarine is killed when fired upon.

6. Kill.

(a) Determination of whether or not the transiting submarine is killed is accomplished by taking a random number from the uniform distribution on the interval $[0,1)$ and comparing this with the P(K) value which was entered on the con-

sole as an input parameter by the program operator. If the
random number is less than or equal to the input probability,
a kill is recorded, and the next transit is run. If the random
number is greater than the input probability, the program
moves directly to run the next transit.

7.   100 Transits Done.

(a)   The procedure through step 6 above is followed,
with new normally-distributed positions for all barrier sub-
marines on each transit, until 100 transits have been run.

(b)   Upon completion of 100 transits, all values of
input parameters used and the recorded values for detections,
interceptions, and kills are stored in the space provided for
recording of output data from the program. The program then
moves to check whether or not the specified number of game
units (100-transit runs) have been completed.

8.   Game Units Done.

(a)   As each 100-transit run is completed, the values
of input parameters and the output data from the program are
stored in sequence in the space allocated for storage of out-
put data.

(b)   Upon completion of the specified number of game
units, the thesis program moves to the statistical analysis
section.

9.   Compute and Store Means and Variances.

(a) In this part of the program, the mean and variance of the data for detections, interceptions, and kills is computed for each 100-transit run and stored in the output data space in the computer. When this sequence is complete, the mean-of-means and variance-of-means for each of the three types of recorded data is computed with respect to the specified number of game units completed. These are stored as the final set of data in the computer space reserved for output.

The foregoing provided a brief general description of the manner in which the thesis program functions as a submarine simulation. Some additional comments should be made with respect to the way in which two-dimensionality and other features are handled in the thesis program.

The geographic plot of the submarine barrier, as it is entered in the computer, is one-dimensional. The 1-bits which are entered as sensors for the barrier submarines represent their detection capability only along the barrier line itself. The second dimension is incorporated into the program by the use of mathematical models in its "Detection" and "Interception" modules (See Appendices C and D). In a sense, when a detection occurs the program stops for a moment and reconstructs the history of detection and movement of the transiting submarine which would have had to occur in order that the transiting submarine cross the barrier line at the point established during game play. By applying the speed of the transiting submarine to this history, the speed required by the barrier submarine in order to effect interception is computed.

Details of the submarine barrier war game played by the thesis program have been presented in this and the preceding chapter. The game is played in order to isolate and gain insight into the critical features of submarine barrier design. The following chapter explains the output data generated by the thesis program and its use in conducting analyses.

Chapter IV

Output Data from Thesis Program

Our next points of interest are the output data from the
program and the use of this data in conducting analyses. The
data generated and stored by the thesis program was designed
to provide that information needed for thesis development. In
order to serve other purposes or to improve on the accuracy of
generated data, it might be well to alter any or all of the
following output data characteristics: type, quantity, numer-
ical format, and decimal accuracy. Some possible changes in
the output data will be discussed in the next chapter.

All output from this program is expressed in octal integ-
ers. This data form was selected because it provided the best
adequate type of data expression to meet the needs of this
thesis.

The data print from each running of the program contains
the following information:

1. The input parametric values entered on the console by
the program operator.

2. The number of detections, interceptions, and kills
generated by each set of 100 enemy submarine transits through
the barrier.

3. The mean and variance of the generated data for
detections, interceptions, and kills for each set of 100

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enemy submarine transits through the barrier.

4.   The mean-of-means and variance-of-means of the total data generated during the running of the specified number of game units. These values are computed for detections, interceptions, and kills and represent that portion of the data from each program run which is usable for analysis.

As was mentioned before, the output data is expressed in integers. However, since the integers represent proportionate parts of 100, they are immediately expressable in probability terms, as follows:

$$P(D) = \frac{\text{number of detections}}{100}$$

$$P(I) = \frac{\text{number of interceptions}}{100}$$

$$P(K) = \frac{\text{number of kills}}{100}$$

The output data generated for the conditional probability of kill, P(K), are valid as probabilities for all barrier configurations. However, the data generated for P(D) and P(I) are valid as probabilities only when the barrier being simulated consists of a single line of barrier submarines. When multiple lines are involved in the barrier simulation, a single transiting submarine may be detected and intercepted more than once without being killed. Under certain parametric combinations, actual computer runs of the thesis program have regularly resulted in more than 100 detections per 100 transits.

This cannot occur in the case of the P(K) data, since each transiting submarine is eliminated from further game play as soon as a kill occurs. In order to obtain valid P(D) and P(I) data in the case of multiple barrier lines, the methods of both data storage and probability computation would have to be altered. P(K) data was sufficient for the purposes of this thesis, and therefore no modification was made in the program.

During the initial stages of thesis development, it was believed that, for a given set of input parameters, the variation in generated output data between any two 100-transit runs would be negligible. When the program had reached an operational stage of development, it was discovered that the difference between extreme values of the output probabilities could be expected to vary by a value of about 0.10. Thus, it became evident that the data from a single set of 100 transits could only be considered as a sample of size 1. This development gave rise to the need for the statistical program addendum which is now an integral part of the program.

The remainder of this chapter will be devoted to exposition of three sample analyses conducted by use of the thesis program. Two of these analyses demonstrate ways in which the thesis program may be used to conduct a parametric analysis of submarine barrier design. The other sample analysis illustrates a means for selecting optimal sample size for the output from a computer war game.

Optimal Barrier Submarine Speed Capability

One of the important parameters of submarine barrier design is the speed capability of the barrier submarines themselves. It would be desirable for the barrier submarines to have sufficient speed capability to permit them to intercept any target detected. This cannot be, however, since the ability to intercept a target that has been detected just before leaving the barrier zone would require an infinite speed capability on the part of the barrier submarine. In addition to other considerations, high speed capability in a submarine is very expensive. Since available funds must be distributed over many elements of defense, it becomes essential that some optimal speed be determined which will provide the barrier submarine with an acceptable level of interception ability and still represent a properly proportionate expenditure of defense funds.

This particular analysis attempts to look at the above problem. At this point in the proceedings, a word of caution should be injected with regard to this and the succeeding analysis. This caution has to do with the effects of assumptions inherent in the program on the validity of the output data. As was stated in Chapter II, the thesis program does not simulate real operational conditions in any complete sense. Therefore, any analysis conducted by use of this program, at its present level of sophistication, can only lead to conclusions relating to gross aspects of submarine barrier design.

No fine parametric analysis should be attempted with the thesis program in its present state of development. The data in the succeeding analyses will be referred to in definite quantitative terms, but the reader should bear carefully in mind the statements made above.

The output data acquired during this first analysis has been converted to a graphic display in Figure 3 on the following page. This curve shows the probability of kill versus barrier submarine speed and is termed simply a performance curve. Each point on the graph represents the output from a single run of the program, wherein the mean $P(K)$ for 1,500 enemy submarine transits was computed. Thus, the graph presented in Figure 3 portrays the output from 15 program runs of 1,500 transits each. The total running time on the computer for this analysis was two hours. All input parameters, except the barrier submarine speed, were held constant during this analysis. Barrier submarine speed capability was varied, in successive program runs, from 0 to 80 knots. The selection of uneven speed increments, which are evident on the graph, was prompted by a pre-conceived notion that the lower speed values would be most critical. This preconception is borne out by the generated data. The parametric values used in this analysis were as follows:

$M = 1$ barrier line $R = 225$ miles

$I_1 = 50$ miles $B = 10$ miles

$Z_1 = 500$ miles $W = 30$ knots

$N_1 = 6$ submarines $V = 0, 1, 2, 3, 4, 5, 10,$
15, 20, 30, 40, 50, 60, 70, & 80 knots.

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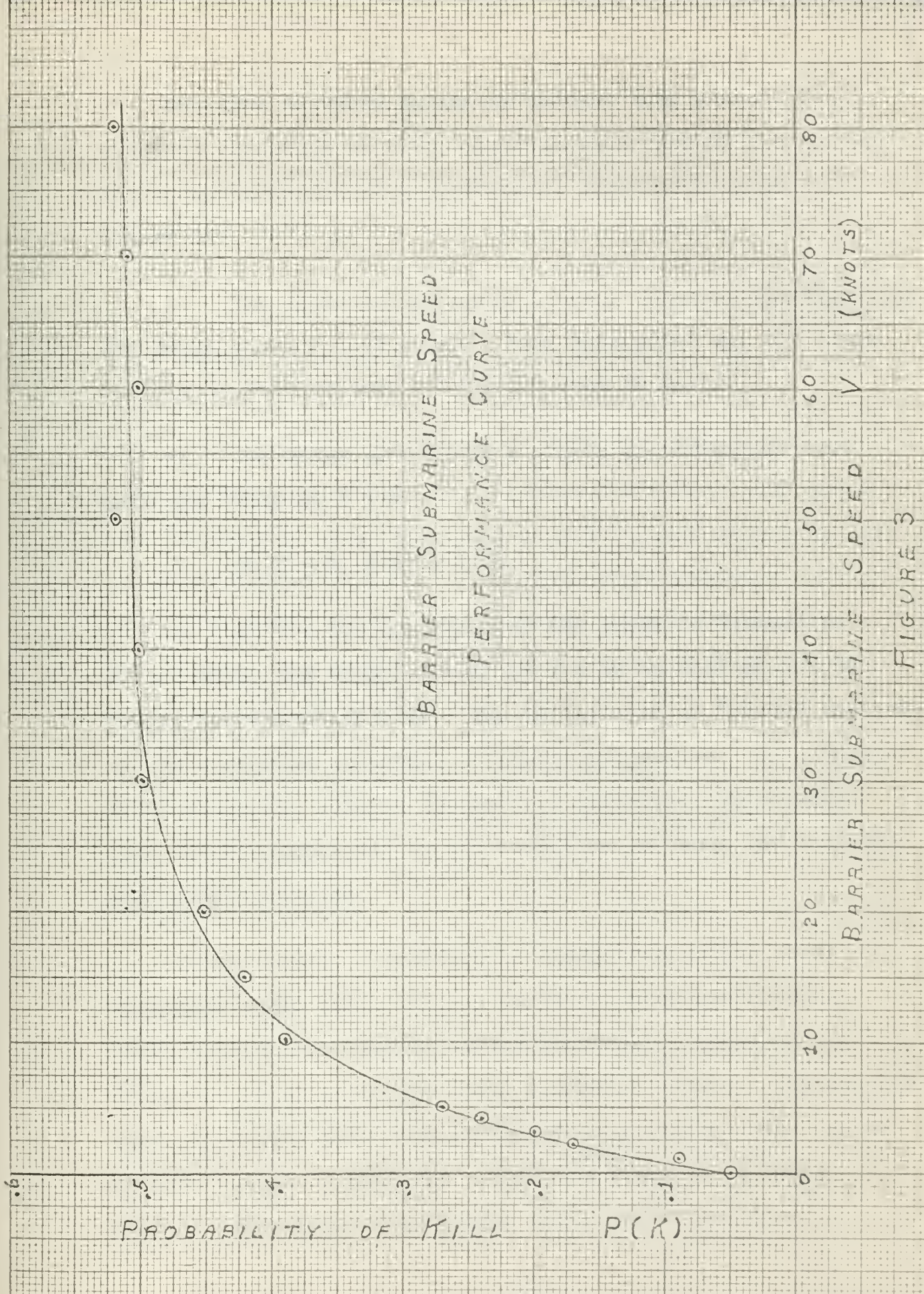
BARRIER SUBMARINE SPEED

PERFORMANCE CURVE

FIGURE 3

PROBABILITY OF KILL   P(K)

BARRIER SUBMARINE SPEED   V   (KNOTS)

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Relative to these parameter values, some general con-
clusions regarding barrier submarine speed can be drawn from
the characteristics of a performance curve such as that in
Figure 3:

1.   The slope of the curve is very steep to approximately
the point where V = 10 knots. From this point on, the slope
rapidly approaches zero as V approaches 80 knots.

2.   The value of P(K) at V = 5 is approximately one-
half its value at V = 80.

3.   At V = 0, P(K) = 0.05.

4.   At V = 30, the slope of the curve is "close" to
zero. Also, the ratio of P(K) at V = 30 to P(K) at V = 80
equals approximately 0.94. The definitiion of "close" implied
by the foregoing statement is purely subjective. In a "real-
life" analysis, this is the sort of definition which would
have to be established by the executive for whom the analysis
was being conducted.

We are now in a position to state conclusions about
submarine speed capability in terms of these performance
curve characteristics; as stated previously, these conclu-
sions are relative to the input parameter values. The conclu-
sions discussed below are presented to illustrate the use of
program output in analysis. They are advanced as being
logical outgrowths of the above curve characteristics.

a.   Characteristic 1.   Barrier potential is most sensitive to the speed capability of its component submarines in the range from 0 to 10 knots. In this range the rate of increase of potential is such that, at $V = 10$, the barrier has attained 71% of its potential. By employing a linear approximation to the curve over this range, it is evident that the rough rate-of-gain in potential is 0.071/knot. By the same rough approximation, the rate-of-gain over the remaining portion of the speed range is only 0.004/knot. The degree of approximation represented by the above procedure is so rough that the derived rates can only be used to gain a feeling for the general behavior of the submarine speed parameter over its entire range. They cannot be used, for instance, as justification for establishment of the optimal level of submarine speed at 10 knots.

b.   Characteristic 2.   The conclusion to be derived in this section is really just an extension of the above. However, it is quite interesting to note that about one-half of the barrier potential, as a function of submarine speed capability, is attained at the 5-knot level. By a linear approximation similar to the above, the rate-of-gain in barrier potential over this range is approximately 0.10/knot.

c.   Characteristic 3.   Conclusions hereby derived are related to the barrier parameter of weapon range rather than submarine speed. Since the speed capability of the barrier submarines was fixed at 0 for the computer run which generated

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this P(K), kills of transiting submarines were achieved only when the target crossed the barrier within the weapon range of some barrier submarine. A logical extension of this conclusion would tend to indicate that the entire barrier potential curve of Figure 3 could be raised as some function of increasing weapon range. The existence and nature of this functional relationship could be established by utilizing the output from a series of computer runs during which barrier submarine speed was varied over the range of this analysis, while weapon range was allowed to vary over some appropriate range. It should also be noted here that the value of B used in this analysis has a positive effect on the quantitative conclusions reached under Characteristics 1 and 2 above.

d. Characteristic 4. This characteristic is not one which may be established by the analyst. It is incumbent upon the executive sponsoring the analysis to establish the level of barrier submarine speed at which the slope of the curve is "close" to 0. The fixing of that point represents, in the terms of this analysis, a compromise between acceptable defense potential and the cost of achieving this potential. An additional analysis of submarine cost as a function of speed capability would greatly assist the executive in making an effective decision. In terms of the 30-knot level selected under this curve characteristic, the executive might reach his decision by the following reasoning process:

1. At V = 30, the barrier submarine has reached 94% of its potential.

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2.   P(K) = 0.49 at this 94% potential level represents an acceptable level of defense.

3.   The rate of gain of potential from V = 30 to V = 80 is 0.0006/knot (The linear approximation used is fairly good over this range of the curve).

4.   The above rate of gain in potential, when compared with the incremental increase in submarine cost over this speed range, does not represent efficient expenditure of defense funds.

5.   Therefore, the optimal level of submarine speed capability should be established at 30 knots.

Choice Between Single and Multiple Barrier Lines

The result of this analysis is interesting. It indicates that a more extensive investigation would be desirable than was possible within the scope of this thesis. The idea which prompted this analysis involved an intuitive feeling that optimal geometric barrier design might be a function of the detection capability of its component submarines. The general operational situation being simulated is one in which the assigned task is to place a submarine barrier across a given expanse of water, utilizing a given number of submarines.

During this analysis, the total number of barrier submarines and the length of the barrier front were held constant. All other parameters except detection range were

also invariant. Total computer running time for this analysis was $4\frac{1}{2}$ hours. For each value of the detection range, a computer run was made for each of the three geographic barrier formations illustrated in Figure 4. The input parametric values used in this analysis were as follows:

General

L = 2,500 miles W = 30 knots

B = 10 miles V = 80 knots

R = 11, 23, 46, 93, 186, 279, & 372 miles.

Single Line

M = 1 barrier line Z_1 = 207 miles

I_1 = 8 miles N_1 = 12 submarines

Double Line

M = 2 barrier lines $Z_1 = Z_2$ = 382 miles

I_1 = 9 miles $N_1 = N_2$ = 6 submarines

I_2 = 200 miles

Triple Line

M = 3 barrier lines $Z_1 = Z_2 = Z_3$ = 511 miles

$I_1 = I_3$ = 100 miles $N_1 = N_2 = N_3$ = 4 submarines

I_2 = 356 miles

Barrier Formations

Single Line

Double Line
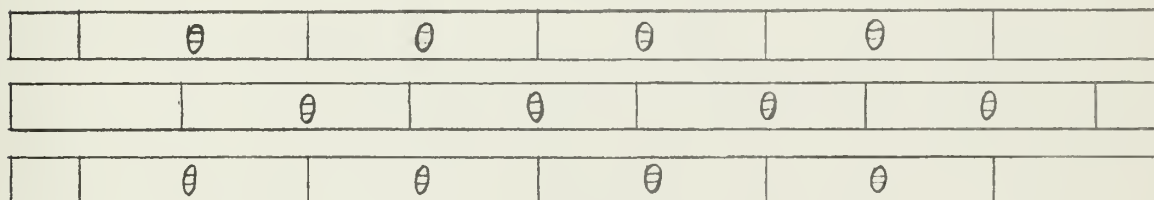
Triple Line

Figure 4

The output data from twenty-one of the above-described computer runs is displayed graphically in Figure 5 on the following page. The variable parameter in this analysis, R, was combined with N and L to form a new parameter, Q, where Q = 2NR/L. The parameter Q provides an expression of that relative detection coverage along the barrier front which is provided by the barrier submarines. For Q = 1, the aggregate detection coverage of the barrier submarines is exactly equal to the length of the specified barrier front.

GEOMETRIC BARRIER FORM
PERFORMANCE CURVE

FIGURE 5

DETECTION FRONT COVERAGE $Q = 2NR/L$

PROBABILITY OF KILL   P(K)

TRIPLE LINE
DOUBLE LINE
SINGLE LINE
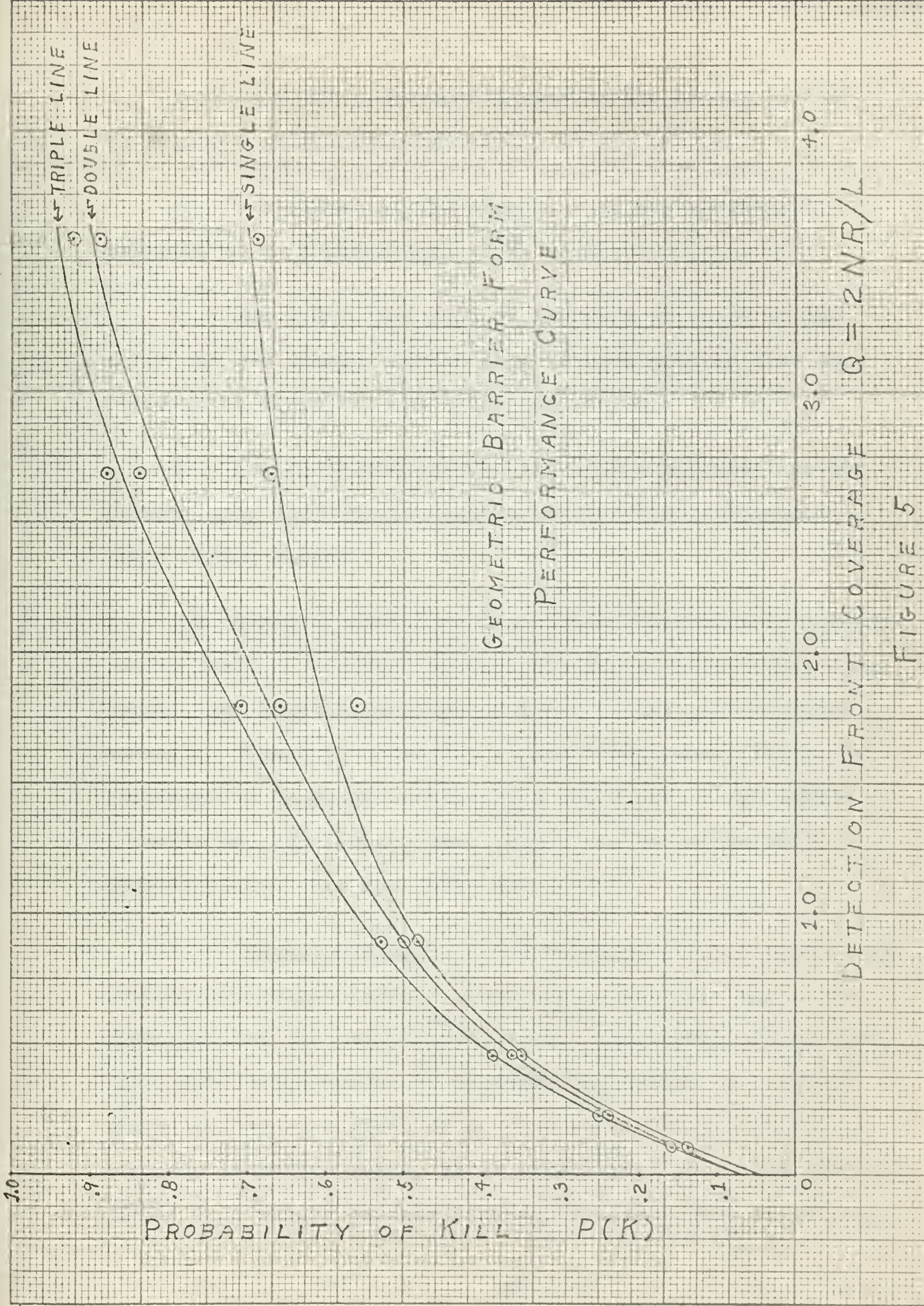
There are three characteristics of the curves plotted in Figure 5 which are pertinent:

1. The slopes of the three curves are essentially the same up to a point just short of $Q = 1$.

2. For $Q > 1$, the rate of decrease in slope of the curve for the single barrier line is much greater than the decrease in slope for the other two curves.

3. The P(K) function for the triple barrier line is greater than that for the double barrier line throughout their entire ranges.

The number of sample points obtained during this analysis is too small to provide for accurate curve fitting and, as a result, for exact definition of the critical points of the curves in Figure 5. However, within the accuracy limits of the data, at least two conclusions would seem to be valid. Comments and conclusions with respect to the above curve characteristics are set forth below:

a. Characteristic 1. The defense potentials of single, double, and triple barrier lines are essentially equal to approximately the point where $Q = 1$. The thesis program provides for no indication of multiple detections of a single target by the submarines of any one barrier line. Given an adequate detection range, it is likely that more than one submarine in a single barrier line would detect that same target.

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In view of this, it would seem advantageous to maintain a single barrier line up to the point where $Q = 1$, to provide for some overlap in detection areas.

b. <u>Characteristic 2</u>. The sharp divergence between the curve for the single barrier line and those for the double and triple lines strongly indicates the use of multiple barrier lines in cases where $Q > 1$. This is a most interesting outcome. Intuition does not lead one to this conclusion, since the individual submarine has a much larger zone to cover in the multiple barrier lines and the value of Q for the individual line drops radically.

c. <u>Characteristic 3</u>. Because of the small number of sample points taken for these curves, the comparatively large variances (0.0022 to 0.0032) for some of the points, and the proximity of sample points for the two curves, little significance can be attached to the fact that one curve dominates another throughout its range. This feature would be a worthy subject for further investigation.

<u>Optimal Run Sample Size</u>

As set forth in Chapter III, a run of the computer program which generates an element of output data is composed of three parts. A single play of the game is a single enemy submarine transit through the barrier; a game unit is composed of 100 such transits; and a single data run may be composed of any given number of game units. Computer runs conducted for

the analyses of this thesis contained 15 game units (1,500 transits).

The objective of this analysis was to determine the optimal number of game units which would provide output data with acceptable accuracy at a minimum cost in computer running time. This analysis is by no means as extensive as it should be for a complete determination of output data accuracy In order to do the latter, much additional output from the computer program would be required, and some attempt should be made to determine the distribution of the output data. Output distribution should be investigated with respect to the random number generators in the program. Such an analysis is beyond the scope of this thesis.

The general procedure followed was to make several computer runs using the same input parameters. The number of game units was varied on each run. The data obtained during that sequence appears in Table 1 on the following page.

The following features are to be noted in connection with the data in Table 1:

1. The mean of P(D) rapidly settles on a constant value, and its variance undergoes a slow, steady increase over the entire sequence.

2. Over the sequence of the first four sample sizes, the means of P(I) and P(K) tend to settle down to a constant value, and their variances decrease monotonically throughout

the sequence.

3.   For sample sizes of 25 and 50, both the means and
variances of P(I) and P(K) increase from the level reached
at sample size 20.

Optimal Sample Size Data

Number of Game Units	P(D)	P(D) Variance	P(I)	P(I) Variance	P(K)	P(K) Variance
5	0.93	0.0003	0.57	0.0061	0.48	0.0044
10	0.92	0.0005	0.56	0.0035	0.48	0.0029
15	0.92	0.0006	0.56	0.0033	0.47	0.0027
20	0.92	0.0006	0.56	0.0027	0.47	0.0022
25	0.92	0.0007	0.57	0.0031	0.48	0.0024
50	0.92	0.0008	0.58	0.0031	0.49	0.0031

Table 1

Feature 1 is to be expected from this program, though it
violates the intuitive feeling that variance should decrease
with increased sample size. Extensive experience in running
this program has indicated that the large majority of game-
unit outputs of P(D) group closely about the mean value, but
occasional output values may deviate from the mean by as much
as 0.10. With the larger sample sizes, more of these large-
deviate values show up, and there is a slow, largely insignif-
icant, tendency for the variance to increase.

Feature 2 illustrates those statistical characteristics

of output data which would be expected with increasing sample size. The selection of sample size for the two preceding analyses was based on this portion of the data. It was aimed at the objective of obtaining acceptably reliable data with minimum running time on the computer. The problem reduced to a selection between sample sizes of 15 and 20. Both of these had the same mean and their variances were comparable. The ultimate selection of sample size 15 was based on computer running time. A good estimate of running time for any given set of input data is 4 seconds per barrier submarine per game unit. A 12 submarine barrier (as in one of the above sample analyses) requires 12 minutes running time for sample size 15, this same barrier requires 16 minutes for sample size 20. Sample size 15 was selected, since it was thereby possible to complete 4 runs during a normal one-hour computer period.

Feature 3 represents a somewhat disturbing trend. The means should not reverse their direction of modification, and the variances should not increase, with increasing sample size. It is felt that the cause of this tendency is probably similar to that expressed in the discussion of Feature 1. A complete analysis of this result would require extensive investigation of the random number generators in the program, and time was not available for such a project in connection with this thesis.

The material presented to this point pretty well covers the objective of this thesis: to present and exemplify the

use of a computer war game in conducting parametric analyses
of submarine barrier design. However, the author, at least,
is convinced of the basic worth and utility of the thesis
program. It is hoped that this program, or a modification
thereof, may be used in the future for conducting a detailed
analysis of barrier parameters. Therefore, the following
chapter is being added to cover the basic utility potential
of the thesis program and some of the more important areas
in which it could be improved.

Chapter V

Computer Program Utility and Improvements

The computer war game developed for this thesis has, in its present form, several limitations which tend to preclude its use as a definitive source of data for the parametric analysis of submarine barrier design. Most of these limitations have either been stated or implied previously. However, the existence of these limitations does not imply that valuable information cannot be derived from the program in its present form. The thesis program does provide a means for illuminating large-scale effects of parametric sensitivity, and it does this in an environment in which the large number of variables virtually prohibits the attainment of a closed, analytic solution to the problem.

The simulation techniques and output data of this war game do provide sufficient accuracy for basic research in barrier design. Relative sensitivity of parameters can be established, thereby isolating those areas in which more definitive analysis will tend to be fruitful. Within rough limits, optimal values for some parameters can be established. The relative worth of various geographic barrier forms can be established. This would appear to be a major potential use for the program, since the entry of various barrier forms can be accomplished easily by the entry of appropriate parametric values on the computer console. Providing careful consideration were given to the effects of inherent assumptions, the thesis

program could be used to obtain an estimate of the expected performance of any proposed, open-ocean barrier. The computer space reserved for the geographic plot of the submarine barrier allows for a maximum front of approximately 25,500 miles, which is far in excess of that in any potential operational barrier.

While these capabilities do provide a valuable source of information, it would be desirable to improve both the realism of the simulation and the accuracy of the output data. Doing so would provide for a much more definitive determination of critical areas; more reliance could be placed on the establishment of optimal parametric values; additional features of submarine barrier design could be investigated; and relatively accurate testing of proposed operational barriers could be achieved.

The thesis program can be modified to achieve these ends. It was composed in modular form for this, and other, purposes. Most of the desirable elements of program sophistication can be accomplished by modification of existing modules. The remainder of this chapter will be devoted to a discussion of some of the more critical areas in need of modification, with brief general descriptions of procedures by which greater realism, utility, or accuracy could be incorporated into the program. Discussion of these critical areas is presented below under general area-identification headings.

Barrier Submarine Interception Track

In the thesis program, the barrier submarines are con-
strained to move only on a track parallel to the barrier line
while attempting an interception of an enemy submarine. The
effect of this condition on the output interception data is
to minimize interception probabilities. The current intercep-
tion output data represents something of a lower bound on that
which would be generated in a program not involving the inter-
ception track constraint. This condition could be eliminated
by modification of the model contained in Appendix D and the
addition of one simple mathematical model.

The model of Appendix D could be modified by the tempor-
ary addition of one new parameter: Interception Track Angle.
In a manner similar to the current model, V could be expressed
in terms of the old parameters and interception track angle.
By techniques of differential calculus, V could be minimized
with respect to the interception track angle. This value
would then be used to determine the ability of the barrier
submarine to effect an interception.

There is, however, one additional problem. Since the
barrier submarine would be allowed two-dimensional movement
under this new model, another mathematical model would be re-
quired to ensure that the barrier submarine did not leave its
assigned zone while making the interception. Essentially,
this model would only have to compute the distance the bar-
rier submarine could move from its game position, along the

optimal interception track, before reaching the boundry of
its assigned zone. The parameters required in this model are
as follows: length and width of the submarine zone, position
(normally-distributed game position) of the barrier submarine
within the zone, and optimal interception track angle.

Dimensionality of Simulation

Conversion of the thesis program from a two-dimensional
to a three-dimensional simulation would both increase the
realism of the output data and provide new fields for analy-
sis. The effect of environmental conditions on detection
range could be incorporated in a three-dimensional program.

The incorporation of three-dimensionality would involve
major changes in the thesis program. Computer memory space is
inadequate to allow the entry of a geographical representation
of the submarine barrier similar to that now used, even if
confined to a two-dimensional plot. Therefore, a system would
be required wherein the positions and/or movement of game
elements were expressed in terms of 3-space coordinates. The
boundries of detection zones would have to be expressed as
3-dimensional mathematical models, and distances between game
elements would have to be computed as distances between points
in 3-space. The magnitude of effort involved in such a mod-
ification makes it an impracticable undertaking for a solo
thesis.

Evasion Tactics by Transiting Submarine

An assumption inherent in the present program is that the transiting submarine has no knowledge of the existence of the opposing barrier and takes no evasive action, even after being fired upon. It goes without saying that this is unrealistic.

One simple method for eliminating a substantial portion of this lack of realism would be to allow the transiting submarine to make one evasive change of course for each barrier line transit. This provision is reasonably realistic if we assume that the transiting submarine is committed to reaching an objective on the other side of the barrier and will attempt to transit each barrier line midway between two adjacent barrier submarines. A fairly simple mathematical model could be devised to compute the course angle required for the transiting submarine to move from its position of initial detection to a point midway between the two barrier submarines closest to the projected barrier transit point on the target's original track. In order to allow for navigational and control errors on the part of the transiting submarine, the actual transit point along the barrier should then be computed under the influence of a probability distribution over the computed evasion track.

Counter-attack by Transiting Submarine

The extant computer simulation provides for no attrition

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of barrier submarines. This is a fairly unrealistic assumption, to the extent that a transiter could normally be expected to fire defensively on opposing barrier submarines. Elimination of this feature could be accomplished by the use of models similar to those in the thesis program. Upon being killed, the barrier submarine should be removed from the geographic computer plot for that number of enemy transits which would represent the time required to effect replacement.

Classification of Transiter

The realism of the current computer simulation is seriously degraded by the inherent assumption that all transiting submarines are fair game. Much of this lack of realism could be eliminated by a fairly extensive, but functionally simple, modification to the program.

The first requirement would be to provide a probabilistic scheme for selection of the type (friendly or unfriendly) of transit for each play of the game. A two-element code would have to be devised to define the type of transit in progress. One such possibility is the use of a double 1-bit extractor for a friendly transiter and a single 1-bit for an unfriendly transiter. The simplest, and fairly realistic, scheme for determining, during game play, the classification performance of the detecting submarine would be to provide for additional input parameters which would express conditional probabilities of correct classification on the part of the barrier submarines. In addition to the above functional considerations, provisions

would have to be made for storage of the following data:
number of friendly transits, number of unfriendly transits,
correct classifications in each category, incorrect class-
ifications in each category, kills of both friendly and
unfriendly units.

Decimal Accuracy of Output Data

In the thesis program all output mean values are rounded
off to the nearest hundredth and all variances to the nearest
ten-thousandth. This degree of variance accuracy is adequate
for the purpose of establishing relative reliability of output
data. However, the provision of one additional decimal-place
of accuracy in the computed mean values would enable more
accurate curve-fitting during analysis of the output data.

Random Normal Deviate Generator

In this area it would seem desirable to change our pre-
vious emphasis and advocate a reduction in the level of simu-
lation realism in order to achieve another worth-while goal.
The use of a computer routine to generate random normal
deviates, as is done in the thesis program, would seem to be
a desirable procedure. However, this routine is used exten-
sively in the program, and it consumes an inordinate amount
of the total running time for a given set of data. In a pro-
gram run which involves 1,500 transits through a barrier
consisting of 12 submarines, 19,500 random deviates are gen-
erated, and their generation takes about 80% of the 13-minute

running time of the program. This would seem to be an exces-
sive price to pay for what may be a questionable level of
realism.

In the usual program run, random deviates are used to
distribute the barrier submarines about their assigned barrier
positions and to distribute the transit tracks about a perpen-
dicular to the barrier line. It may well be that some simpler
probability distribution function (triangular, for instance)
would serve just as well for these purposes and simultaneously
reduce the running time of the program by an appreciable amount.

It is also possible that the normal deviate generator
might be programmed in another way which would involve less
computation time. In the current program, deviates are com-
puted from the following formula:[4]

$$U = \left[-2 \ln(X_1)\right]^{\frac{1}{2}} \cos(2\pi X_2)$$

where (a) U is a random deviate from a N(0,1) distribu-
tion

(b) X_1 and X_2 are random numbers from the uniform
distribution on $[0,1)$.

The ln and cos terms in the above formula are computed by
the use of infinite series, wherein most of the time consump-
tion occurs. Other computational schemes might reduce the
computing time to an acceptable level.

Thus, the simulation program of this thesis has several
areas of potential improvement. In its present form it can

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still be a valuable instrument for analysis of the sensitivity of parameters in a submarine barrier. It also has other allied capabilities, some of which were covered early in the chapter.

The sample analyses conducted (Chapter IV) illustrate the ability of this sort of computer war game to illuminate sensitive areas in barrier design and provide a means of optimizing the effectiveness of the submarine barrier. It does this in a situation involving such a large number of variables that the attainment of closed, analytic solutions is virtually impossible. It provides a system for determining expected performance of a proposed barrier, thereby possibly providing a means of detecting and correcting large-scale design faults prior to the operational use of the barrier. All of these abilities tend to promote the efficient expenditure of defense funds. Even more importantly, it can provide a means of optimizing the level of defense itself, within the constraints imposed by the availability of resources and the level of extant or extrapolated technology. Computer war games of this type are thus an essential tool of modern Operations Research in handling the extreme complexity of today's military problems.

Effective Detection Range Increase Model

During that part of the thesis program which enters the geographic barrier simulation in computer memory, a sequence of 1-bits, equal to 2R + 1 and centered on the submarine location, is inserted in computer memory to simulate the barrier submarine and the projection of its detection area on the barrier line. Since the required projection is one made parallel to the transit track, the above entry is only valid in the case of a transit made perpendicular to the barrier line. The length of the detection area projection must be increased as a function of transit track angle. If $\phi > 0$, detection bits must be added to the right end of the barrier simulation entry for each submarine; if $\phi < 0$, the detection bits must be added to the left end. The number of bits to be added is a function of the detection range and the track angle. Derivation of the mathematical model used in the computer program to determine the magnitude of the effective increase in detection range is set forth on the following page.
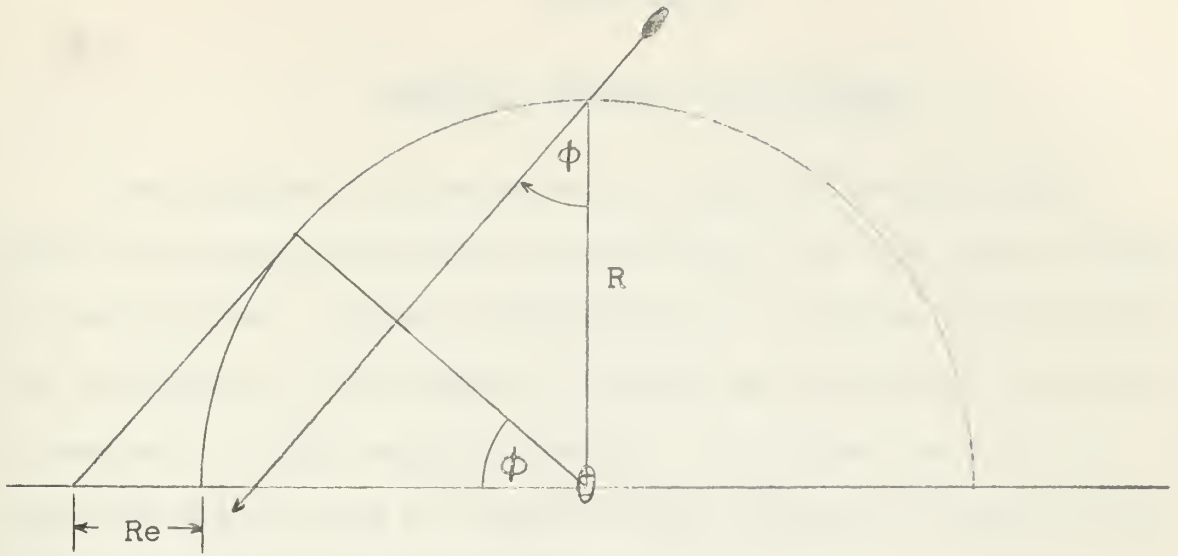
Let Re = Extended detection range due to ϕ.

Re = Re(ϕ, R)

$$\cos(\phi) = \frac{R}{R + Re}$$

$$Re = \frac{R}{\cos(\phi)} - R$$

$$Re = \frac{R[1 - \cos(\phi)]}{\cos(\phi)}$$

Appendix B

Barrier Transit Point Model

The computer program selects transit starting points
with a uniform probability distribution over the total length
of the barrier. Normal distribution of starting points about
the mid-point of the barrier line may be elected by the use of
a jump key on the computer console. In either case, this
starting point is on a "starting line" which is located geo-
graphically ahead of the barrier.

The point at which the transiting submarine will actually
cross the barrier line is a function of this starting point,
the distance between the "starting line" and the barrier line,
and the transit track angle. The derivation of the mathemat-
ical model used in the computer program to determine the bar-
rier transit point is set forth on the following page. A
similar procedure is used to establish transit points on
succeeding barrier lines where a multiple-lined barrier is
concerned.
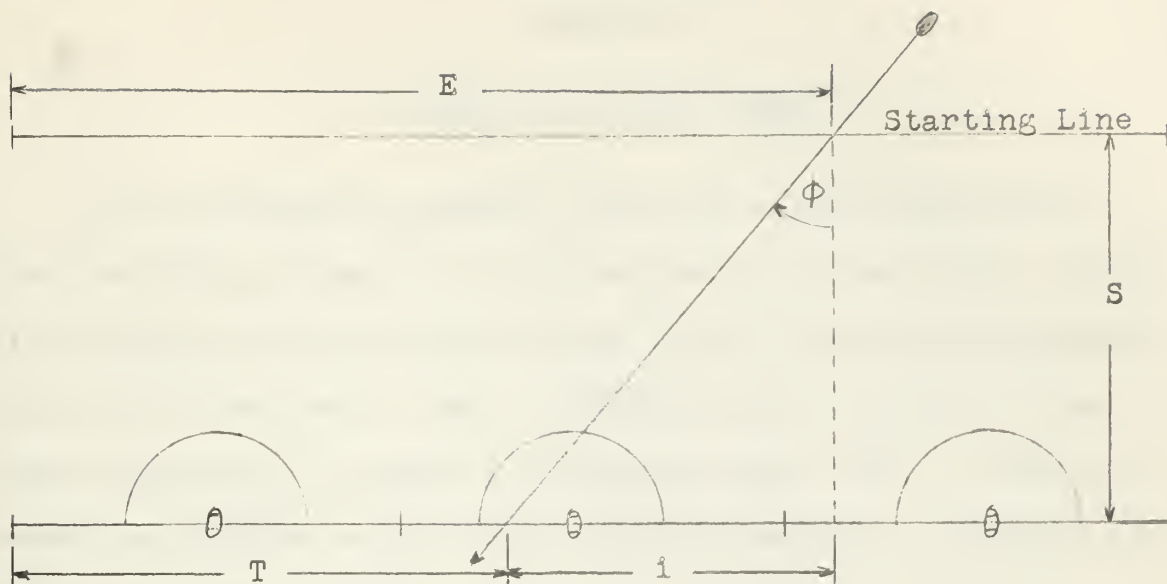
Let E = Distance of transiter's starting point from
        the left end of the starting line.

    T = Distance of barrier transit point from left end
        of barrier line.

    i = Distance along barrier line between transit
        point and projection of transiter's starting
        point on the barrier line.
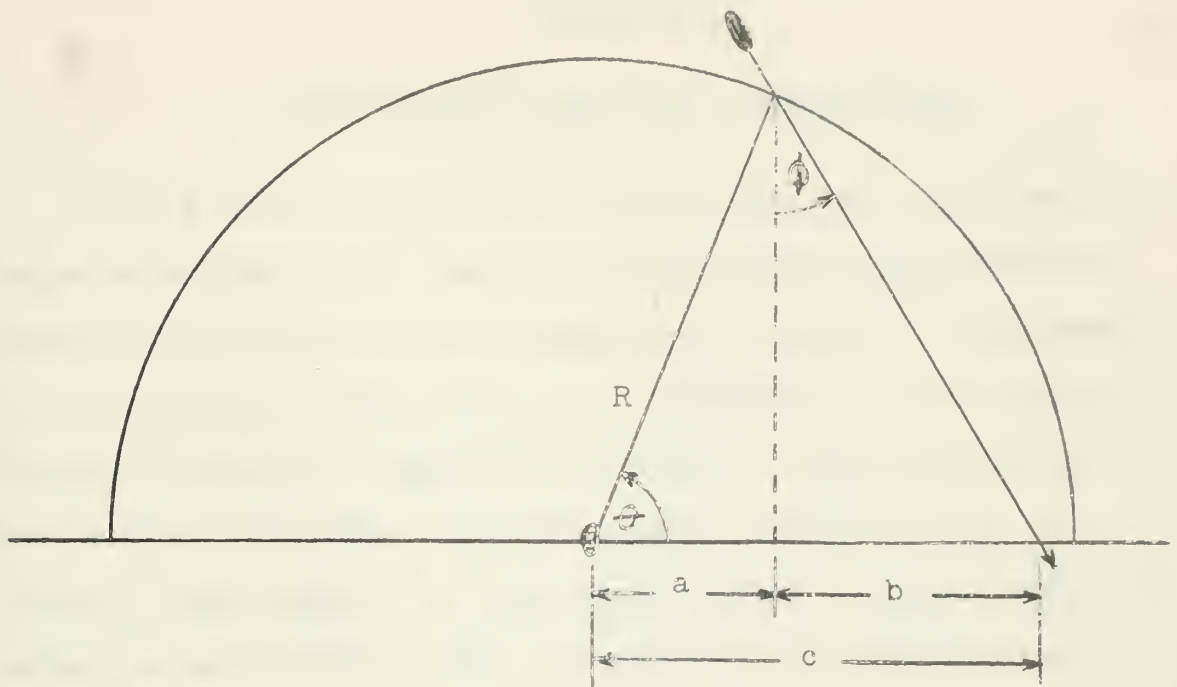
$$T = E - i$$
$$i = S \tan(\phi)$$
$$T = E - S \tan(\phi)$$

Appendix C

Detection Angle Model

In the computer program, detection or non-detection of
the transiting submarine is determined by extracting a single
bit position at the transit point on the simulated geographic
plot of the barrier. When a detection has occurred, it be-
comes necessary to know the detection angle, Θ, in order to
determine whether or not the barrier submarine is capable of
intercepting the target.

At that stage of computer proceedings wherein a detection
has been established, the following data has been generated
and stored: the position of each barrier submarine, the point
at which the target crosses the barrier, the detection range,
and the transit track angle. The positions of all barrier
submarines and the transit point are generated and stored in
terms of miles from the left end of the barrier line. Both
the distance and direction of the transit point from the bar-
rier submarine is established mathematically by subtracting
the barrier submarine position from the transit point. The
difference thus determined ("c" on the following page) repre-
sents the distance desired and the sign of this difference
provides directional information. The detection angle can be
computed as a function of the above data. Derivation of the
mathematical model used in the computer program for this
purpose is set forth on the following page.

$a = R \cos(\Theta)$

$b = R \sin(\Theta) \tan(\phi)$

$c = a + b = R \left[\cos(\Theta) + \sin(\Theta) \tan(\phi)\right]$

$\cos(\Theta) + \sin(\Theta)\tan(\phi) = c/R$

$\cos(\Theta) + \left[1 - \cos^2(\Theta)\right]^{\frac{1}{2}} \tan(\phi) = c/R$

$\left[1 - \cos^2(\Theta)\right] \tan^2(\phi) = c^2/R^2 - (2c/R)\cos(\Theta) + \cos^2(\Theta)$

$\cos^2(\Theta)\tan^2(\phi) + \cos^2(\Theta) = \tan^2(\phi) + (2c/R)\cos(\Theta) - c^2/R^2$

$\left[1 + \tan^2(\phi)\right] \cos^2(\Theta) - (2c/R)\cos(\Theta) + \left[c^2/R^2 - \tan^2(\phi)\right] = 0$

$\Theta = \cos^{-1} \left\{ \dfrac{2c/R - 2\tan(\phi)\left[1 - c^2/R^2 + \tan^2(\phi)\right]^{\frac{1}{2}}}{2\left[1 + \tan^2(\phi)\right]} \right\}$

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Appendix D

Interception Speed Requirement Model

Once a detection has been established and the detection angle determined, the computer program must next determine whether or not the barrier submarine is capable of intercepting the target. This ability to intercept is a function of several variables: speed of the target, speed capability of the barrier submarine, detection angle, weapon range, and transit track angle. By this stage in the program, both the detection and transit track angles have been computed and stored, and values of the remaining variables, which were entered on the console as input data, have been stored for reference.

The ability of the barrier submarine to effect an interception is determined in the computer program by comparing the speed capability of the barrier submarine with that speed which is required to intercept the target. The derivation of the mathematical model by which this speed requirement is computed is set forth on the following pages.
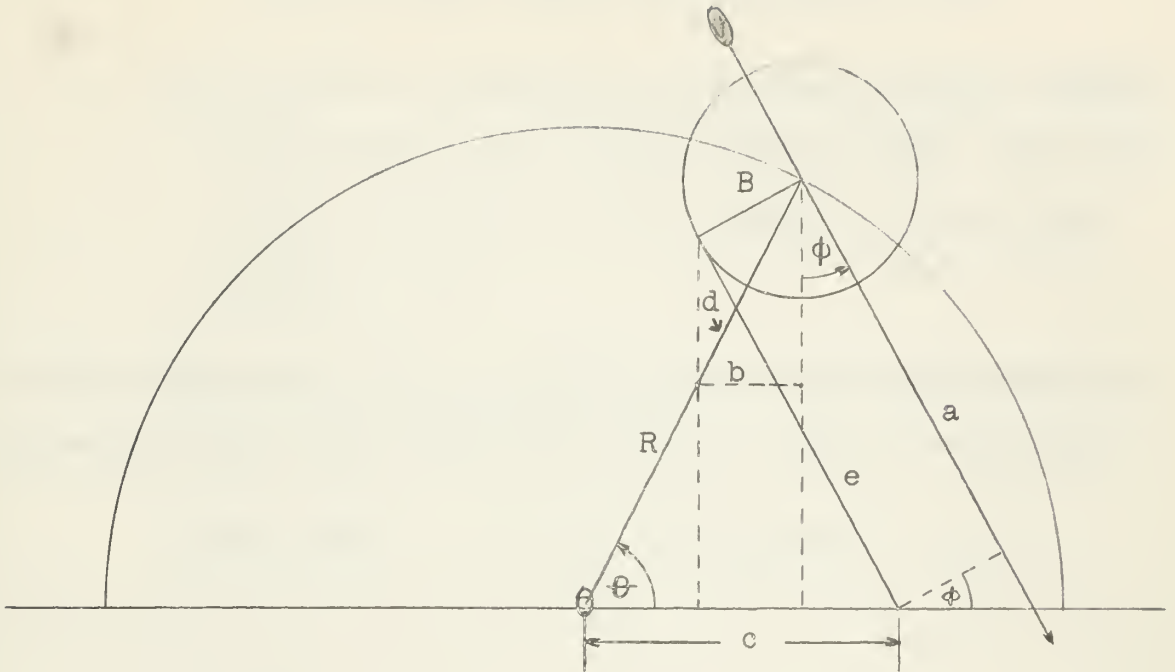
Let a = Distance between the point at which the target is
        detected and the point at which it crosses the barrier
        line.

    b = Distance between the lines of projection on the barrier
        line of (1) the transiting submarine's point of detec-
        tion and (2) the point of intersection of the weapon-
        range circle and the tangent (e) to it.

    c = Distance between the barrier submarine and the point
        of intersection of the tangent line, e, and the
        barrier line.

    d = Distance between the transiter's detection point and
        the point of intersection of the detection angle ray
        and the projection line from the tangent point on the

weapon range circle to the barrier line.

e = Distance, parallel to the transit track, between the tangent point on the weapon range circle and the intersection of the tangent line with the barrier line.

In order that the barrier submarine be capable of reaching a firing position, the following condition must be satisfied:

$$c/Vr = e/W \qquad \text{or} \qquad Vr = Wc/e$$

- -

$$e = a - B \tan(\phi)$$

$$a = R \sin(\theta)/\cos(\phi)$$

$$e = \frac{R \sin(\theta) - B \sin(\phi)}{\cos(\phi)}$$

$$c = e\sin(\phi) + (R - d)\cos(\theta)$$

$$d = b/\cos(\theta) = B \cos(\phi)/\cos(\theta)$$

$$c = \left[\frac{R \sin(\theta) - B \sin(\phi)}{\cos(\phi)}\right] \sin(\phi) + \left[R - \frac{B \cos(\phi)}{\cos(\theta)}\right] \cos(\theta)$$

$$c = \frac{R \sin(\theta)\sin(\phi) - B \sin^2(\phi) + R \cos(\theta)\cos(\phi) - B \cos^2(\phi)}{\cos(\phi)}$$

$$c = \frac{R \cos(\theta - \phi) - B}{\cos(\phi)}$$

$$Vr = \frac{W\left[R \cos(\theta - \phi) - B\right]}{R \sin(\theta) - B \sin(\phi)}$$

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