THE STRESS ANALYSIS OF AN ICEBREAKER
BOW

by

Richard L. DeVries

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RICHARD LEE DE VRIES
LIEUTENANT, UNITED STATES COAST GUARD

B.S., United States Coast Guard Academy (1963)

Submitted in Partial Fulfillment
of the Requirements
for the Master of Science Degree in
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at the

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TECHNOLOGY

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ABSTRACT

The topic of this paper is the stress analysis of an icebreaker bow. The purpose in writing this paper was to familiarize myself with the various methods that have been developed in determining bow loads in ice breaking and the methods that have been developed to compute the stresses in a ship's structure. From a study of many methods, I wanted to select one that would be most useful to me in my day to day work when I depart from M.I.T. I selected as the ship to model the United States Coast Guard Icebreaker WESTWIND.

In the first section, the various methods of determining bow loads in ice breaking was studied. The various methods that were put forth by scholars and engineers over the past half-century gave bow loads ranging from one hundred and fifty-four tons to values as high as twenty-nine hundred and fifty tons of force at the bow in a vertical direction. I chose as the best results to use in loading the bow, those values which are determinable from the computer program developed by Dr. White in the doctoral thesis, "The Dynamically Developed Force at the Bow of an Icebreaker." The computer program developed by Dr. White gives the vertical and horizontal forces acting on the bow as function of the length between perpendiculars, the beam at the waterline, mean draft, displacement, bow angle, spread angle complement, impact velocity, water plane coefficient, longitudinal position of the center of floatation, the longitudinal position of the center of gravity, the height of the center of gravity, height of the thrust line above the baseline, bollard thrust that would be obtained for RPM being used during the crushing phase of the ice, longitudinal metercentric height, kinetic friction coefficient of ice and the compressive failure stress of ice. The vertical load determined by his computer program for the WESTWIND ramming solid ice at fifteen knots gave a peak value of two thousand tons.
The next section of this paper covers the various methods available for determining the stress loading inside the structure. The most used computer programs that were reported upon were the STRESS and FRAN programs. Both of these programs being strictly a frame analysis of the structure leave much to be desired in the complete knowledge of the stresses present in a complicated structure such as an icebreaker.

The most promising and most exact analysis that is available at the present time, utilizes the finite element method. A program developed and presently being revised by NASA that quickly and efficiently analyzes almost any structure is the "Structural Analysis and Matrix Interpretive System" (SAMIS). The remainder of the paper gives a short summary of the program, the data required by the analyst, and finally the necessary steps required for the analysis of the icebreaker WESTWIND.
ACKNOWLEDGEMENTS

Much credit for the completion of this thesis must go to Professor Alaa E. Mansour for his ever willing assistance. I would also like to thank LCDR Melburg for his assistance and guidance over these past two semesters.

Words cannot express the thanks for the patience my wife and family had with me while blueprints and books cluttered every corner of our house.
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INTRODUCTION

In the Shipbuilder by John Ruskin, 1819-1900, he states, "Take it all in all a ship of the line is a most honorable thing that man as a gregarious animal has ever produced. Into that he has put as much of his human patience, common sense, forethought, experimental philosophy, self control, habits of order and obedience, thoroughly wrought handiwork, defiance of brute elements, careless courage, careful patriotism, and calm expectation of the judgement of God as can be put into a space 300 feet long and forty feet broad," (Ref. 1)*

Since that time, many ships have made history--some of them by going to the bottom of the sea; others, in the defiance of the brute elements have trespassed the Arctic waste. This type of ship is what this thesis is all about.

The icebreaker hull at its birth was merely a standard hull with extra reinforcement in the bow. As time progressed, changes in design were based largely on the success and failure of past designs. Efficient ice breaking (or what was believed to be efficient ice breaking) led to the thirty degree angle bow. Removal of the unsuccessful

*References are listed at the end of this report.
bow propeller on the wind class breaker produced the step which prevented the vessel from riding completely up on the ice. With the advent of the computer, more efficient and better designs have been produced.

Although very few papers have been written on the subject of icebreaker design, those that have been produced have made far reaching effects on the design of the icebreaker hull.

METHODS OF DETERMINING BOW LOADS

The earlier works were done by the Russians. M. K. Tarshis in his paper, "Ice Loads Acting on Ships," (Ref. 2) put forth a formula to determine the impact load on the vessel. This formula considers the speed, the angle of blow, and the square root of the relative mass and the relative rigidity of the ship. He uses as an example a 655 ton displacement vessel in contact with an ice floe that is 25 meters in diameter and one meter in thickness. From his formula he deduces that the impact load is 220 tons. He assumed in this work a crushing strength of ice of 570 pounds per square inch.

The general idea of this paper is that the speed of the vessel and the angle of blow in the area of contact with the ice floe are the major contributors to the impact load on the vessel; hence, if you double the speed of the vessel you double the impact load on the bow of the vessel.
"The paper has a high theoretical content and requires many parameters difficult to determine. The relative angles of the hull form at the point of impact are necessary and these can vary quite appreciably in a relatively short distance."
(Ref. 2, pg. 5)

L. M. Nogid in his paper, "Impact of Ships with Ice," (Ref. 3) attempts to determine the reduction of speed of the icebreaker as it contacts the ice. An interesting sidelight of this paper is that the theoretical amount of force required to initiate the crack in the ice is quite small, but that the force required to propagate that crack might be quite large. It also goes on to show that the load required to break off pieces of the ice is more than three times the load required to initiate a crack.

In his paper, Mr. Nogid gives a method to determine the speed reduction a vessel will have when coming into contact with an ice floe of given diameter if you know the maximum load the ship could withstand at the point of impact and the angle formed with the ice at the point of contact. The area can also be determined if you know what the strength of the ice is in the area in which the ice breaking operation is being carried out.

He divides the forces and the strength requirements of the hull into two different areas. One is impact and the other is compression.
Under the impact force, he determines that this force is equal to the mass of the ship times the velocity of the ship, modified by parameters which are dependent upon the angle of the hull at the point of impact, the area of impact and other similar items.

Under compression, the author assumes that the loading is directly related to the crushing strength of the ice. He also suggests some relationships between the changes of length and the strength of the side frames. His formula suggests that the strength of side frames is related to the cube root of the ratio prototype to the proposed length. He also goes further in an attempt to relate hull strength to the frame spacing of the vessel. This paper, in general, is the theoretical approach to the old idea of taking a successful design and changing the dimensions to make a more capable design.

U. N. Raskin in his paper, "Method of Determining the Stresses in Decks and Transverse Bulkheads Caused by Ice Loads," (Ref. 4) uses a compressive ice load on the side framing of a ship to determine what the stresses are in the decks and bulkheads. He divides the decks into strips with the only load carrying area being the deck itself and those beams in direct contact with the deck. The stringer is considered to be in simple compression, while the remaining
strips act as rigid members to bending and carry only shear loads. In between these strips are imaginary elastic bands which allow relative movement back and forth between the bands. The bulkheads are considered as simple plates in end compression.

Using these models, he presents a method to determine stresses in these decks and bulkheads. I believe it would be rather difficult to accurately model a complicated structure, such as an icebreaker, in this form and achieve accurate results.

In September of 1965, R. M. White completed his doctoral thesis, "Dynamically Developed Force at the Bow of an Icebreaker." (Ref. 5) In this thesis, Dr. White developed a computer program for the prediction of the dynamically developed force under the bow of an icebreaker while ramming solid ice.

The solution is based primarily on Newton's Law of Motion. The problem was broken down into two basic phases. The crushing phase represents the local crushing of the ice to accommodate the bow. The sliding phase represents the sliding up of the bow without further crushing. "The final state represents (temporary) equilibrium when motion has stopped; the vertical force at the bow at this state is relatively sustained and is the most effective in breaking the ice." (Ref. 5, pg. 3)
Dr. White gives one of the earlier developed force equations which is attributed to R. Ruineberg in 1888 (Ref. 6).

\[ F_{BZ} = \frac{T_{IB}(\cos i_B \cos \beta - f_k \sin i_B)}{(\sin i_B \cos \beta + f_k \cos i_B)} \]

where

- \( F_{BZ} = \) Downward force
- \( T_{IB} = \) Thrust available for ice breaking.
- \( f_k = \) Coefficient of friction (suggested as .05).
- \( \beta = \) The angle with respect to the center line plane of a normal to the shell.
- \( i_B = \) The angle the stem makes with the base plane.

Dr. White states that Ruineberg assumed no momentum effects and no forward motion, so that all thrust is applied to ice breaking and is applied horizontally at all times. The direction of friction force remains the same during forward horizontal progress and the trim does not effect the solution. For the icebreaker WESTWIND, this would give a downward force of four hundred and fifty-four tons for this continuous mode of ice breaking.

The next equation Dr. White discussed was one developed by A. Kari in 1921 (Ref. 7).

\[ F_{BZ} = \frac{4480 \Delta CL \sin \theta}{H} \]
where

\[ \Delta \text{ is displacement in tons} \]
\[ \theta \text{ is change in trim} \]
\[ L \text{ is the length between perpendiculars} \]
\[ H \text{ is the draft in feet} \]

and

\[ C = \frac{BM_L \times H}{L^2} \approx \frac{GM_L \times H}{L^2} = .07 \]

where

\[ BM_L \text{ is assumed to be equal to } GM_L. \]
\[ GM_L \text{ is longitudinal metacentric height in feet.} \]

Kari makes the following assumptions: (1) no momentum effect, (2) the bow rises the thickness of the ice, (3) the distance from point of contact to the center of flotation and the center of gravity are the same, (4) effective displacement and the draft is not changed by load force, (5) the center of flotation and metacenter remain fixed, (6) \( C = .07 \), and (7) there is no frictional force. This would give a downward force of approximately seven hundred and fifty-eight for the icebreaker WESTWIND.

The third method discussed by Dr. White was one done by E. R. Simonson in 1936. (Ref. 8)

\[ F_{BZ} = \frac{T_{IB}}{\tan (iB + \theta)} \]
In this method, Simonson assumed there are no momentum effects, friction is negligible, thrust is directed horizontally, the coefficient of friction serves as a pivot point, and there is no change in displacement. Dr. White states, "... Simonson's equation is limited to being a good approximation for the stopped equilibrium position." (Ref. 5, pg. 46) This would give a downward force of one hundred and seventy-three tons for the icebreaker WESTWIND.

The fourth work discussed by Dr. White on the downward force developed in ice breaking is one by L. V. Vinogradov which was published in 1946. (Ref. 9) Dr. White states that this is the first time that force due to ramming was put into useable mathematical form.

\[ F_{BZ} = XT + \left( x^2 T^2 + \frac{V}{A} W^2 \cdot \frac{V_0^2 [1 - (1 - e^2 \sin^2 i_B) - v_1^2]}{gD} \right) \]

where

\[ X = \frac{\cos \beta}{\cos \beta} \tan i_B \]
\[ 1 + \frac{\cos \beta}{\cos \beta} \cot i_B \]

\[ T = \text{thrust in tons} \]

\[ Y = \frac{\cos \beta}{\cos \beta} \cot i_B \]

**Symbols have been changed to previously defined symbols.**
\[ \omega = \text{displacement} \]
\[ D = \text{draft in ft.} \]
\[ A = \frac{C_B}{C_W} \left[ 1 + \frac{k_1}{k_2} \frac{1}{4C_W} \right] \]
\[ C_B = \text{block coefficient} \]
\[ C_W = \text{waterline coefficient} \]
\[ k_1 = \frac{a}{L/2} \]
\[ q = \text{distance from point of impact to center of flotation} \]
\[ k_2 = \frac{1}{C_WL} GM_L \cdot H \cdot C_B \]
\[ V_o = \text{speed prior to impact} \]
\[ V_1 = \text{speed while sliding up} \]

Vinogradov assumed that thrust is always horizontal, change in trim and draft is small enough so as not to affect the waterplane characteristics and the metacentric height, and \( GM_L = BM_L \).

In Vinogradov's equation the force downward is related to the frictional force, the angle of the bow, the spread angle of the bow, the coefficient of friction, the block coefficient, the waterline coefficient, the distance of impact from the center of flotation, the speed prior to impact, and the speed while sliding up. This would give a downward force of approximately two thousand nine hundred and sixty tons for the icebreaker WESTWIND.
The fifth equation discussed by Dr. White is one by F. Richardson in 1959 in some personal correspondence to Dr. White. "The development was almost identical to Vinogradov's but did modify some of his weaknesses to some extent. For example, Richardson uses a term for the loss of energy due to wave and frictional resistance (not ice) from the instant of contact up to the moment the ice breaks or motion ceases. He also recognizes an effective increase in the mass of the icebreaker due to entrained water." (Ref 5, p 49)

The final equation discussed by Dr. White was one developed by V. R. Milano in 1962 which is a modification of Vinogradov's equation. Dr. White states, "One of the main contributions was to express thrust as a function of 'Bollard Pull'." (Ref. 5, p. 51)

Dr. White in his analysis assumes:

1. The force normal to the plating is represented by the product of the area of contact and the compressive failure stress of the ice.

2. There is a friction force acting in the plane of the plating.

3. The icebreaker is treated as a "solid body."

4. The radius of gyration of an icebreaker can be assigned 0.266.
5. Crushing has ceased when velocity is in the direction of the angle of the bow plus the angle of trim.

6. During sliding phase, the point of contact is fixed and is at the level of the waterline.

Results show that the peak load occurs during the crushing phase.

In his computer program, the downward force is determined as a function of

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<th>Description</th>
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<tr>
<td>L</td>
<td>Length between perpendiculars, ft.</td>
</tr>
<tr>
<td>B</td>
<td>Beam at waterline, ft.</td>
</tr>
<tr>
<td>H</td>
<td>Mean draft, ft.</td>
</tr>
<tr>
<td>Δ</td>
<td>Displacement, lbs.</td>
</tr>
<tr>
<td>iB</td>
<td>Bow angle (from base line to stem), radians</td>
</tr>
<tr>
<td>SA</td>
<td>Spread angle complement (normal to bow plating with respect to center line plane), radians</td>
</tr>
<tr>
<td>Vl</td>
<td>Impac[e velocity, ft/sec.</td>
</tr>
<tr>
<td>a = CW</td>
<td>a, Waterplane coefficient. dimensionless</td>
</tr>
<tr>
<td>LCF</td>
<td>Longitudinal position of the center of flotation (- if aft of amidships, + if forward), ft.</td>
</tr>
</tbody>
</table>

Computer Symbol
CG  LCG, Longitudinal position of the center of gravity (- if aft of amidships, + if forward), ft.

KG  Height of center of gravity above base line, ft.

d  Height of thrust line above base line near center of gravity, ft.

\( T_{BOL} \)  Bollard thrust which would be obtained for rpm used during crushing and sliding, lbs.

\( G_{ML} \)  Longitudinal metacentric height, ft.

\( f_k \)  Kenetic coefficient of friction of ice/ship.

\( \sigma \)  Compressive failure stress of ice, \( \#/ft^2 \).

The following pages are Dr. White's complete program for developing this downward force. (Ref. 5, pp. 385-393).
M4045-3564, FMS, TEST, 5, 5, 5000, 0 DYNAMIC ICEBREAKING R.M. WHITE

READ 5, BP, B, H, DIS, BA, SA, V1, AL, BF, CG, D, TB, GM, FK, FS, SIG

FORMAT (4F15.3/4F15.3/4F15.3/4F15.3/F15.3)

PRINT 41, BP, B, H, DIS, BA, SA, V1, AL, BF, CG, D, TB, GM, FK, FS, SIG

FORMAT (6H BP=, F15.3, FH B=, F15.3, FH H=, F15.3, 7H DIS=, F15.3
1/6H BA=, F15.3, 6H SA=, F15.3, 6H V1=, F15.3, 6H AL=, F15.3/
26H CF=, F15.3, 6H CG=, F15.3, 6H GK=, F15.3, 5H B=, F15.3/6H TB
3=, F15.3, 6H GM=, F15.3, 6H FK=, F15.3, 6H FS=, F15.3/7H SIG=,
4F15.3/)

XM= (3.36E-2) * DIS
ZA = (5.75E-2) * DIS
RG = 0.22 * BP
THM = (5.0E-2) * (RG ** 2) * DIS
DP = (1.76E-2) * DIS * BP ** 1.5
DH = (5.25E-1) * DIS / BP ** 0.5
TF = (64.2) * BP * B * AL
SIBA = SINF (BA)
COBA = COSF (BA)
TABA = SIBA / COBA
SISA = SINF (SA)
COSA = COSF (SA)
P1 = (SIG * TABA / SISA) * (SIBA * (COBA * (COSA + FK * SISA) - FK * SIBA))
P3 = P2 * (BP / 2 - CG) + P1 * (H - GK)
A1 = THM
B1 = BP
C1 = DIS * GM
D1 = V1 ** 2 * P3
AL1 = -B1 / (2 * A1)
DISC1 = 4 * C1 / AL1 - (B1 ** 2) / (A1 ** 2)
IF (DISC1) 11, 2, 2

BB1 = 0.5 * SQRT (DISC1)
AA1 = (2 * D1 / C1 ** 2) * (A1 - B1 ** 2 / C1)
AA2 = (2 * D1 / ((C1 ** 2) * BB1)) * (B1 - AL1 * (A1 - (B1 ** 2) / C1))
A2 = ZM
B2 = DH
C2 = TF
D2 = -P2 * V1 ** 2
AL2 = -B2(2.&A2)
DISC2 = 4.*C2/A2-(B2**2)/(A2**2)
IF (DISC2) 12,3,3
3 BE2 = 0.5*SQRTF(DISC2)
   BB1 = (2.*D2/C2**2)*(A2-B2**2/C2)
   BB2 = (2.*D2/((C2**2)*BE2))*((B2-A2*(A2-B2**2)/C2))
   PRINT 4, XM,ZM,GM,THM,DP,DM,TF,PLP2,AL,B1,C1,P3,D1,AL1,BE1,AA1,
   AM2,AL2,CL2,AL2,BL1,BL2,SL1,SL2
   T = -0.05
1 T = T+0.05
   EAL1T = EXPF(AL1*T)
   C0B1T = COSF(BE1*T)
   SIB1T = SINF(BE1*T)
   TH = EAL1T*(AA1*COB1T+AA2*SIB1T)+D1*(T**2)/C1-2.*B1*D1*T/(C1**2)
1-AM1
   THD = AA1*EAL1T*(AA1*COB1T+AA2*SIB1T)+EAL1T*(-AA1*BE1*SIB1T+AA2
   1*BE1*COB1T)+2.*B1*T/C1-2.*B1*D1/C1**2
   THDD = (AA1**2-2.*BE1**2)*EAL1T*(AA1*COB1T+AA2*SIB1T)+2.*AA1*BE1*
   EAL1T*(-AA1*SIB1T+AA2*COB1T)+2.*D1/C1
   EAL2T = EXPF(AL2*T)
   C0B2T = COSF(BE2*T)
   SIB2T = SINF(BE2*T)
   Z = EAL2T*(BB1*COB2T+BB2*SIB2T)+D2*(T**2)/C2-2.*B2*D2*T/(C2**2)-BB1
   ZD = AL2*EAL2T*(BE1*COB2T+BB2*SIB2T)+EAL2T*(-BE1*BE2*SIB2T+BB2
   1*BE2*COB2T)+2.*D2*T/C2-2.*B2*D2/C2**2
   ZDD = (AL2**2-2.*BE2**2)*EAL2T*(BB1*COB2T+BB2*SIB2T)+2.*AL2*BE2*
   EAL2T*(-BB1*SIB2T+BB2*COB2T)+2.*D2/C2
   X = V1*(T-P1*X**3/(12.*XM))
   XD = SQRTF(V1**2-2.*P1*X**3/(3.*XM))
   FXC = P1*X**2
   FZC = P2*X**2
   GAX = (BP/2.-CG)-((H-GK)+(BP/2.-CG)/TABA)*TH+Z/TABA
   GAZ = H-GK+Z
   TAGA = (GAX*THD-ZD)/(XD-GAZ*THD)
   DIF = SINF(BA+TH)/COSF(BA+TH)-TAGA
   PRINT 6,T,TH,THD,THDD,Z,2D,ZDD,X,JD,XX,FXC,FZC,TAGA,GAX,GAZ,GIF
FORMAT (F11.5/3F11.5/3F11.5/3F11.5/2F12.5/4F11.5//)
IF (XD) 38,38,37
PRINT 39, FZC
GO TO 36
FORMAT (44H SHIP STOPPED DURING CRUSHING PHASE, FZC2=,E12.5//)
IF (DIF) 14,14,7
TL = T
THL = TH!
THDL = THD
THDDL = THDD
ZL = Z
ZDL = ZD
ZDDL = ZDD
XL = X
XDL = XD
XDDL = XDD
FXCL = FXC
FZCL = FZC
TAGAL = TAGA
GAXL = GAX
GAZL = GAZ
DIFL = DIF
GO TO 1
14 TERP = DIFL/(DIFL-DIF)
T2 = TL+TERP*(T-TL)
TH2 = THL+TERP*(TH-THL)
THD2 = THDL+TERP*(THD-THDL)
THDD2 = THDDL+TERP*(THDD-THDDL)
Z2 = ZL+TERP*(Z-ZL)
ZD2 = ZDL+TERP*(ZD-ZDL)
ZDD2 = ZDDL+TERP*(ZDD-ZDDL)
X2 = XL+TERP*X-XL)
XD2 = XDL+TERP*(XD-XDL)
XDD2 = XDDL+TERP*(XDD-XDDL)
FXC2 = FXCL+TERP*(FXC-FXCL)
FZC2 = FZCL+TERP*(FZC-FZCL)
TAGA2 = TAGAL+TERP*(TAGA-TAGAL)
GAX2 = GAXL+TERP*(GAX-GAXL)
GAZ2 = GAZL+TERP*(GAX-GAZL)
DIF2 = DIFL+TERP*(DIF-DIFL)

10 PRINT 15, T2,TH2,THD2,THDD2,Z2,ZD2,ZDD2,X2,XD2,XDD2,FXC2,FZC2,
      ITAGA2,GAX2,GAZ2,DIF2
15 FORMAT (6H T2=,F11.5/7H TH2=,F11.5,8H THD2=,F11.5,
      26H X2=,F11.5,7H XD2=,F11.5,8H XDD2=,F11.5/8H FXC2=,E12.5,
      38H FZC2=,E12.5/9H TAGA2=,F11.5,8H GAX2=,F11.5,8H GAZ2=,
      4F11.5,8H DIF2=,F11.5//)
      GO TO 16
11 PRINT 13,DISC1
13 FORMAT (E12.4)
      GO TO 36
12 PRINT 13,DISC2
      GO TO 36

C ICEBREAKER SLIDING PHASE SOLUTION R. M. WHITE

16 AS = COSA*SIBA+FK*COSA
ES = COSA*COSA-FK*SIBA
XDD2 = -1.0
ZDD2 = 0.0
THDD2 = 0.0
P4 = GAX2+X2
P5 = 1.+((AS/ES)**2
HGK = H-GK
CGCF = CG-CF
GKD = GK-D
A11 = -XN
B11 = -(TB/V1)*((1.+AS*TH2/BS+P5*TH2**2)
C11 = 0.0
A12 = XM*(AS/BS+P5*TH2)
B12 = DH*(AS/BS+P5*TH2)
C12 = TF*(AS/BS+P5*TH2)
A13 = 0.0
B13 = 0.0
C13 = TB*(AS/BS-AS*XD2/(BS*V1)+2.*P5*TH2-2.*P5*XD2*TH2/V1)
1+TF*(AS*CGCF/BS+P5*Z2+2.*P5*CGCF*TH2)+P5*DH*ZD2+P5*Z2M*ZDD2
D1 = -TF*(1.+AS*XD2*TH2/(BS*V1)-P5*TH2**2+2.*P5*XD2*TH2**2/V1)
1+TF*(P5*Z2*TH2+P5*CGCF*TH2**2)+P5*DH*ZD2*TH2+P5*Z2M*ZDD2*TH2
A21 = 0.0
B21 = TB*(P4*TH2/V1-TH2*X2/V1+AS*HGK/TH2/(BS*V1)+P5*HGK/TH2**2/V1
1+AS*TH2*Z2/(BS*V1)+P5*Z2*TH2**2/V1-GKD/V1)
C21 = TB*(TH2-XD2*TH2/V1)+TF*(Z2+CGCF*TH2)+DH*Z2+ZM*ZDD2
A22 = 2M*(-P4*X2-AS*HGK/BS-P5*HGK/TH2-AS*Z2/BS-P5*TH2*Z2)
B22 = DH*(-P4*X2-AS*HGK/BS-P5*HGK/TH2-AS*Z2/BS-P5*TH2*Z2)
C22 = TF*(X2-AS*HGK/BS-P5*HGK/TH2-P4-2.*AS*Z2/BS-AS*CGCF/TH2/BS
1-2.*P5*Z2*TH2-P5*CGCF*TH2**2-AS*GM*TH2)+TB*(-AS*TH2/BS+AS*X2D*TH2/
2(BS*V1)-P5*TH2**2+P5*XD2*TH2**2/V1)-DH*(-AS*Z2D/BS-P5*Z2D*TH2)
3+ZM*(-AS*ZDD2/BS-P5*ZDD2*TH2)
A23 = -THM
B23 = -DP
C23 = TB*(-P4+P4*XD2/V1+X2-XD2*X2/V1-AS*HGK/BS+AS*HGK*XD2/(BS*V1)
1-2.*P5*HGK/TH2+2.*P5*HGK*XD2/V1-AS*Z2/BS+AS*XD2*Z2/(BS*V1)
2-2.*P5*Z2*TH2+2.*P5*XD2*TH2*Z2/V1)+TF*(-P4*CGCF+CGCF*X2-AS*CGCF*
3HGK/BS-P5*HGK/TH2-2.*P5*CGCF+HGK/TH2-AS*CGCF*Z2/BS-P5*Z2**2-2.*P5*
4*CGCF*PH2Z2-GM*Z2-2.*GM*CGCF/TH2)+DH*(-P5*HGK/BD2-P5*ZD2*Z2)+ZM*
5(-P5*HGK/ZD2-P5*ZDD2*Z2)-DIS*GM
D2 = TD*(P4*XD2/TH2/V1+TH2*X2-2.*XD2/TH2X2/V1+AS*HGK/TH2-2*TH2/(BS*
1V1)-P5*HGK/TH2**2+2.*P5*HGK*XD2/PI2**2/V1-AS*TH2*Z2/BS+2.*AS*XD2
2*TH2/Z2/(BS*V1)-2.*P5*PH2**2*Z2+3.*P5*ZD2*Z2+TH2**2/V1-GKD)+TF*
(32*Z2+CGCF/TH2*X2-P5*HGK/TH2-P5*CGCF+HGK/TH2**2-AS*Z2/BS-AS
4*CGCF*TH2*Z2/BS-2.*P5*TH2**2-2.*P5*CGCF*Z2*TH2**2-GM*TH2*Z2-GM*
5CGCF*TH2**2)+DH*Z2D2+X2-P5*HGK*ZD2*TH2-AS*Z2Z2/BS-2.*P5*ZD2*TH2*
6Z2)+ZM*(ZDD2*X2-P5*HGK*ZDD2/TH2-AS*ZDD2*Z2/BS-2.*P5*ZDD2*TH2*Z2)
A31 = 0.0
B31 = 0.0
C31 = 1.0
A32 = 0.0
B32 = 0.0
C32 = 1./TABA-TH2/SIBA**2
A33 = 0.0
B33 = 0.0
C33 = -GAX2/TABA+GAX2*TH2/SIBA**2+GAXZ2
D3 = X2-GAX2*TH2/TABA+GAX2*TH2**2/SIBA**2+Z2/TABA-22*TH2/SIBA**2
1+GAZ2*TH2
PRINT 17,AS,BS,P4,P5,A11,B11,C11,A12,B12,C12,A13,B13,C13,D1,
1A21,B21,C21,A22,B22,C22,A23,B23,C23,D2,A31,B31,C31,A32,B32,C32,
2A33, B33, C33, D3

17 FORMAT (4E14.6//3E14.6/3E14.6/3E14.6/3E14.6//3E14.6/3E14.6/3E14.6//
1E14.6/3E14.6/3E14.6/3E14.6/3E14.6//)
D11 = A11*XD2+D11*X2+A12*ZD2+B12*Z2
D12 = A11*X2+A12*Z2
D13 = D1
D21 = B21*X2+A22*ZD2+B22*Z2+A23*TidD2+B23*TH2
D22 = A22*Z2+A23*TH2
D23 = D2
D33 = D3
DD4 = A11*A22*C33+A12*A23*C31-A11*A23*C32
DD3 = A11*B22*C33+B11*A22*C33+A12*B23*C31+B12*A23*C31-A11*B23*C32
1-B11*A23*C32-A12*B21*C33
DD2 = A11*C22*C33+B11*B22*C33+A12*C23*C31+B12*B23*C31+C12*A23*C31
1-A11*C23*C32-B11*B23*C32-A12*C21*C33-B12*B21*C33-C13*A22*C31
DD1 = B11*C22*C33+B12*C23*C31+C12*B23*C31+C13*B21*C32-B11*C23*C32
1-B12*C21*C33-C12*B21*C33-C13*C22*C31
DD0 = C12*C23*C31+C13*C21*C32-C12*C21*C33-C13*C22*C31
U13 = D12*A22*C33+A12*A23*C33-D12*A23*C32-A12*B22*C33
U12 = D11*A22*C33+D12*B22*C33+A12*B23*B33+B12*A23*D33-D11*A23*C32
1-D12*B23*C32-A12*D21*C33-B12*D22*C33
U11 = D11*B22*C33+D12*C22*C33+D13*A22*C33+A12*C23*D33+B12*B23*D33
1+C12*A23*D33+C13*D22*C32-B11*D23*C32-D12*C23*C32-D13*A23*C32
2-A12*A23*C33-B12*D21*C33-C12*D22*C33-C13*A22*D33
U10 = D11*C22*C33+D13*B22*C33+B12*C23*D33+C12*B23*D33+C13*D21*C32
1-D11*C23*C32-D13*B23*C32-B21*D23*C33+C12*D21*C33-C13*B22*B33
U09 = D13*C22*C33+C13*C23*D33+C13*D23*C32-D13*C23*C32-C12*D23*C33
1-C13*C22*D33
U23 = A11*D22*C33+D12*A23*C31-A11*A23*D33
U22 = A11*D21*C33+B11*D22*C33+D11*A23*C31+D12*B23*C31-A11*B23*D33
1-B11*A23*D33-D12*B21*C33
U21 = A11*D23*C33+B11*D21*C33+D11*B23*C31+D12*C23*C31+D13*A23*C31
1-A11*C23*D33-B11*B23*D33-D11*B21*C33-D12*C21*C33-C13*D22*C31
U20 = B11*D23*C33+D11*C23*C31+D13*B23*C31+C13*B21*D33-B11*C23*D33
1-D11*C21*C33-D13*B21*C33-C13*D21*C31
U19 = D13*C23*C31+C13*C21*D33-D13*C21*C33-C13*D23*C31
U33 = A11*A22*D33+A12*D22*C31-A11*D22*C32-D12*A22*C31
U32 = A11*D22*D33+B11*A22*D33+A12*D21*C31+B12*D22*C31+D12*B21*C32
1-A11*D21*C32-B11*D22*C32-A12*B21*D33-D11*A22*C31-D12*B22*C31
U31 = A11*C22*D33+B11*B22*D33+A12*B32+D12*D21+C31+B12*D21+C31+C12*D22*C31
1+D11*B21*C32+D12*C21*C32-A11*D23*C32-B11*D21*C32-A12*C21*D33
2-B12*B21*D33-D11*B22*C31-D12*C22*C31-D13*A22+C31
U30 = B11*C22*D33+B12*D23*C31+C12*D21*C31+D11*C21*C32+D13*B21*C32
1-B11*D23*C32-B12*C21*D33-C12*B32+D12*D33-D11*C22*C31-D13*B22*C31
U29 = C12*D23*C31+D13*B21*C32-C12*C21*D33-D13*B22*C31
PRINT 13, D11, D12, D13, D21, D22, D23, D33, DD4, DD3, DD2, DD1, DD0, U13, U12, U11, U10, U09, U23, U22, U21, U20, U19, U33, U32, U31, U30, U29

WB4 = DD3/DD4
WB3 = DD2/DD4
WB2 = DD1/DD4
WB1 = DD0/DD4
W6 = -64.*WB4**6
W5 = 96.*WB4**6
W4 = (WB4**4)*(-48.*WB4**2-32.*WB3)
W3 = (WB4**3)*(32.*WB3*WB4+8.*WB4**3)
W2 = (WB4**2)*(16.*WB1-4.*WB3**2-4.*WB2*WB4-8.*WB3*WB4**2)
W1 = (WB4)*(-3.*WB1*WB4+2.*WB3**2*WB4+2.*WB2*WB4**2)
W0 = WB1*WB4**2-WB2*WB3*WB4+WB2**2
C = 0

19 CL = C
TOTL = TOT
C = C+0.001
TOT = W6*C**6+W5*C**5+W4*C**4+W3*C**3+W2*C**2+W1*C+W0
PRINT 13, C, TOT
IF (TOT) 19, 20, 20
20 C = CL-TOTL*0.001/(TOT-TOTL)
PRINT 13, C
AL3 = C*WB4
DISC3 = (-WB2-3.*WB4*(AL3**2)+4.**(AL3**3)+2.*WB3*AL3)/(4.*AL3-WB4)
IF (DISC3) 21, 22, 22
21 PRINT 13, DISC3
GO TO 36
22 BE3 = SQRTF (DISC3)
AL4 = (1.-2.*C)*WB4/2.
DISC4 = WB3 -(BE3**2)-(AL3**2)-4.*AL3*AL4 -(AL4**2)
IF (DISC4) 23, 24, 24
23 PRINT 13, DISC4
GO TO 36
24 BE4 = SQRTF (DISC4)
G3 = AL3**2 + BE3**2
G4 = AL4**2 + BE4**2
PRINT 13, AL3, BE3, AL4, BE4, G3, G4
A4X = U13/DD4
A3X = U12/DD4
A2X = U11/DD4
A1X = U10/DD4
A0X = U09/DD4
PRINT 13, A4X, A3X, A2X, A1X, A0X
C1X = A0X/(G3*G4)
D1X = A4X-C1X
D2X = A3X-C1X*WB4
D3X = A2X-C1X*WB3
D4X = A1X-C1X*WB2
C6X = (2.*(AL3-AL4)*(G4*D3X-D1X*G4**2-2.*AL4*D4X)+(G3-G4)*(-G4*D2X
1+2.*AL4*G4*D1X+D4X))/(2.*(AL3-AL4)*(2.*AL3*G4-2.*AL4*G3)+(G3-G4)
2**2)
C5X = (D4X-G3*C6X)/G4
C4X = (G4*(D2X-2.*AL4*D1X)-D4X+C6X*(G3-G4))/(2.*G4*(AL3-AL4))
C3X = D1X-C4X
P23X = BE3*C3X
P13X = C5X-AL3*C3X
P24X = BE4*C4X
P14X = C6X-AL4*C4X
A4Z = U23/DD4
A3Z = U22/DD4
A2Z = U21/DD4
A1Z = U20/DD4
A0Z = U19/DD4
PRINT 13, A4Z, A3Z, A2Z, A1Z, A0Z
C1Z = A0Z/(G3*G4)
D1Z = A4Z-C1Z  
D2Z = A3Z-C1Z*WB4  
D3Z = A2Z-C1Z*WB3  
D4Z = A1Z-C1Z*WB2  
C6Z = (2.*(AL3-AL4)*(G4*D3Z-D1Z*G4**2-2.*AL4*D4Z)+(G3-G4)*(-G4*D2Z
1+2.*AL4*G4*D1Z+D4Z))/(2.*(AL3-AL4)*(2.*AL3*G4-2.*AL4*G3)+(G3-G4)
2**2)  
C5Z = (D4Z-G3*C6Z)/G4  
C4Z = (G4*(D2Z-2.*AL4*D1Z)-D4Z+C6Z*(G3-G4))/(2.*G4*(AL3-AL4))  
C3Z = D1Z-C4Z  
P23Z = BB3*C3Z  
P13Z = C5Z-AL3*C3Z  
P24Z = BE4*C4Z  
P14Z = C6Z-AL4*C4Z  
A4T = U33/DD4  
A3T = U32/DD4  
A2T = U31/DD4  
A1T = U30/DD4  
A0T = U29/DD4  
PRINT 13, A4T,A3T,A2T,A1T,A0T  
C1T = A0T/(G3*G4)  
D1T = A4T-C1T  
D2T = A3T-C1T*WB4  
D3T = A2T-C1T*WB3  
D4T = A1T-C1T*WB2  
C6T = (2.*(AL3-AL4)*(G4*D3T-D1T*G4**2-2.*AL4*D4T)+(G3-G4)*(-G4*D2T
1+2.*AL4*G4*D1T+D4T))/(2.*(AL3-AL4)*(2.*AL3*G4-2.*AL4*G3)+(G3-G4)
2**2)  
C5T = (D4T-G3*C6T)/G4  
C4T = (G4*(D2T-2.*AL4*D1T)-D4T+C6T*(G3-G4))/(2.*G4*(AL3-AL4))  
C3T = D1T-C4T  
P23T = BB3*C3T  
P13T = C5T-AL3*C3T  
P24T = BE4*C4T  
P14T = C6T-AL4*C4T  
PRINT 13, C1T,D1T,D2T,D3T,D4T,C6T,C5T,C4T,C3T,P23T,P13T,P24T,P14T
T = -0.100
T = T + 0.100
EAL3T = EXPF (AL3*T)
COB3T = COSF (BE3*T)
SIB3T = SINF (BE3*T)
EAL4T = EXPF (AL4*T)
COB4T = COSF (BE4*T)
SIB4T = SINF (BE4*T)
X = C1X + (1./ (BE3*EAL3T)) * (P23X*COB3T + P13X*SIB3T) + (1./ (BE4*EAL4T))
1. (P24X*COB4T + P14X*SIB4T)
XD = (-AL3/ (BE3*EAL3T)) * (B23X*COB3T + P13X*SIB3T) + (1./ EAL3T) * (-P23X
1.*SIB3T + P13X*COB3T) - (AL4/ (BE4*EAL4T)) * (P24X*COB4T + P14X*SIB4T)
2 + (1./ EAL4T) * (-P24X*SIB4T + P14X*COB4T)
XDD = ((AL3**2 - BE3**2) / (BE3*EAL3T)) * (P23X*COB3T + P13X*SIB3T) - (2.*
1AL3/EAL3T) * (-P23X*SIB3T + P13X*COB3T) + ((AL4**2 - BE4**2) / (BE4*EAL4T))
2 * (P24X*COB4T + P14X*SIB4T) - (2.*AL4/EAL4T) * (-P24X*SIB4T + P14X*COB4T)
Z = C1Z + (1./ (BE3*EAL3T)) * (P23Z*COB3T + P13Z*SIB3T) + (1./ (BE4*EAL4T))
1. (P24Z*COB4T + P14Z*SIB4T)
ZD = (-AL3/ (BE3*EAL3T)) * (P23Z*COB3T + P13Z*SIB3T) + (1./ EAL3T) * (-P23Z
1.*SIB3T + P13Z*COB3T) - (AL4/ (BE4*EAL4T)) * (P24Z*COB4T + P14Z*SIB4T)
2 + (1./ EAL4T) * (-P24Z*SIB4T + P14Z*COB4T)
ZDD = ((AL3**2 - BE3**2) / (BE3*EAL3T)) * (P23Z*COB3T + P13Z*SIB3T) - (2.*
1AL3/EAL3T) * (-P23Z*SIB3T + P13Z*COB3T) + ((AL4**2 - BE4**2) / (BE4*EAL4T))
2 * (P24Z*COB4T + P14Z*SIB4T) - (2.*AL4/EAL4T) * (-P24Z*SIB4T + P14Z*COB4T)
TH = C1T + (1./ (BE3*EAL3T)) * (B23T*COB3T + P13T*SIB3T) + (1./ (BE4*EAL4T))
1. (P24T*COB4T + P14T*SIB4T)
THD = (-AL3/ (BE3*EAL3T)) * (P23T*COB3T + P13T*SIB3T) + (1./ EAL3T) * (-P23T
1.*SIB3T + P13T*COB3T) - (AL4/ (BE4*EAL4T)) * (P24T*COB4T + P14T*SIB4T)
2 + (1./ EAL4T) * (-P24T*SIB4T + P14T*COB4T)
THDD = ((AL3**2 - BE3**2) / (BE3*EAL3T)) * (P23T*COB3T + P13T*SIB3T) - (2.*
1AL3/EAL3T) * (-P23T*SIB3T + P13T*COB3T) + ((AL4**2 - BE4**2) / (BE4*EAL4T))
2 * (P24T*COB4T + P14T*SIB4T) - (2.*AL4/EAL4T) * (-P24T*SIB4T + P14T*COB4T)
FBZS = -TB*TH + T*XD + TH/VL - TF*Z - TF*CGCF*TH - DH*ZD*XM*ZDD
WRAT = FBZS / (VL*DIS)
VAX = XD - (HGK + Z)*THD
TT = T + T2
PRINT 26, TT, T, X, XD, XDD, Z, ZD, ZDD, TH, THD, THDD, FBZS, WRAT, VAX
IF (PRAT-1.0) 42,44,44
42 PRINT 43, PRAT
43 FORMAT (46H CAUTION, CRUSHING FORCE / SLIDING FORCE IS ,F8.5//)
44 GAX3 = P4-X3
GAZ3 = HGX+Z3
Q1 = CGCF+GAX3
A4 = GAZ3/(Q1*TF) .GM*CGCF/(TF*Q1**2)-GM*GAX3/(TF*Q1**2)
B4 = GAX3+GAZ3*TH3+GAZ3*Z3/Q1-GAZ3*GAX3*TH3/Q1+DIS*GM/(Q1*TF)
2Q1**2-2.*GM*GAX3*Z3/Q1**2+2.*GM*GAX3*TH3/Q1**2
C4 = DIS*GM*Z3/Q1-DIS*GM*GAX3*TH3/Q1+TF*GM*Z3**2/Q1-TF*GM*Z3*GAX3*
1TH3/Q1-TP*GM*GAX3*TH3*Z3/Q1+TP*GM*GAX3**2*TH3**2/Q1- (TF*GM/Q1**2)*
2(CGCF*Z3**2+CGCF*GAX3**2-2.*CGCF*GAX3*23*TH3+GAX3*Z3**2+3GAX3**3*TH3**2-2.*GAX3**2*Z3*TH3)
PRINT 13,GAX3,GAZ3,Q1,A4,B4,C4
DISC5 = (B4**2)-4.*A4*C4
IF (DISC5) 34,33,33
34 PRINT 13, DISC5
GO TO 36
33 RAD = SQRTF (DISC5)
FBZ4 = (-B4+RAD)/(2.*A4)
WRAT4 = FBZ4/(V1*DIS)
TH4 = -FBZ4/(Q1*TP-Z3/Q1+GAX3*TH3/Q1
Z4 = Z3+GAX3*(TH4-TH3)
X4 = X3
PRINT 35, X4,Z4,TH4,FBZ4,WRAT4
35 FORMAT (17H STATE 4 VALUES/6H X4=,F11.5,6H Z4=,F11.5,
17H TH4=,F11.5/26H VERTICAL FORCE AT BOW =,F12.5/16H WHITE RA
2T10 = ,F10.6//)
COBA5 = CCSE(BA+TH4)
SIBA5 = SINF(BA+TH4)
A7 = COSA*COBA5+FS*SIBA5
B7 = -COSA*SIBA5+FS*COBA5
ET = FBZ4/((A7/B7)*COSF(TH4)-SINF(TH4))
RAT = ET/TH
PRINT 40,ET,RAT
40 FORMAT (22H EXTRACTING THRUST =,E12.5/50H RATIO OF EXTRACTING
1THRUST TO BOLLARD THRUST IS ,F15.3//)
   GO TO 36
END
STRESS ANALYSIS OF SHIPS STRUCTURES

In the analysis of the Coast Guard icebreaker GLACIER, in Lloyd's Register of Shipping Research and Technical Advisory Services Report No. 5095 (Ref.10), the IBM program STRESS was utilized. This is a computer program which performs a linear elastic analysis of a framed structure under the influence of a static load and is capable of solving two or three dimensional problems.

The condition of loading that was utilized was 400 psi ice pressure on the midship section of the icebreaker, and the ice condition was considered to be 10 feet in thickness. There were two conditions imposed with this 10 foot thickness of ice.

(1) Two feet of ice extending above the waterline and eight feet below it.

(2) Assume that all the ice was placed at the weakest point of the structure that was likely to encounter ice load in the ice breaking operation.

Once again in this program, you have a problem of the plating not carrying any load which certainly, in the analysis of an icebreaker where you have 2 1/2 and 3 inches of plating, will lead to large errors.

In the analysis that was carried out, two sets of end constraints were considered:
(1) Supports only at the extreme corners of the structure or to have four joints.

(2) Supports at all inboard ends of keelsons, etc. in addition to the condition above going a total of eight joints.

The bending moments obtained from this program with four and eight supports varied markedly with factors as low as two and with others as high as 60 and none of the variations showed any real relationships. The four support conditions gave values at various positions in the structure that at some points were much greater and sometimes much less than the corresponding points of the eight support conditions.

The results of the paper did not give any real concrete evidence as to what the actual stresses would be in the vessel considered under the 400 psi ice load condition. The conclusions were only that the structure should be of the grid structure instead of the truss structure. Once again, this is nothing more than a selection from the best of the successful designs to be applied to a new design.

Concurrent with the previous report, further investigation was also being done on these amidship frame areas by Paris Genalis. (Ref. 11) He also used the IBM computer program STRESS to analyze the amidship's section.

As he quotes in his report from
In his report, he breaks the analysis down into four stages:

(1) Consideration of part of only one frame.

(2) Consideration of one half of one frame.

(3) Two dimensional analysis of one complete ring frame.

(4) Three dimensional analysis of more than one complete ring frame.

Each of the analyses carried out was used to substantiate the assumptions and methods utilized in the next analysis. In the final three dimensional analysis, the plating was taken into account by replacing it with stiffeners to get away from the problems that Lloyd's of London had in their report.

Each of the problems considered in the three dimensional analysis, took a total of one hour or more of computer time. The number of calculations was very large, hence the probability of error was very large.

Mr. Genalis also went further in his analysis in that he considered machinery weight, steel weight, and the effects of varying amounts of buoyancy in addition to the ice loads. The wide variations in the results of his analysis point out the absolute need for proper modeling of the structure in order to determine realistic stresses.
Further limitations of the STRESS program is that it cannot handle curved members and thus assumes that each member is straight and slender.

Another application of the IBM program STRESS was completed by Lloyd's Register of Shipping, Research and Technical Advisory Service Department in Report No. 5051, (Ref. 12), in which a structure of shell plating subjected to ice pressure was analyzed. "The purpose of this investigation was to design a simple panel of shell plating stiffened in such a way so that the overall and local strength was sufficient to withstand a specified ice pressure uniformly distributed over the whole panel and for a concentrated load acting on one frame only for the full depth of the panel."
(Ref. 12, p. 1)

Here again, we have results that do not take into account the shell plating other than to simulate it as part of the stiffeners. It does give results that compare well with simple beam theory, but does not give the information necessary for the designers who must, in these days of economy, depart from the large factor of safety of five or ten to insure that the vessel will be successful and safe from the environment it operates in.

The IBM STRESS program was also utilized by Consulter, Inc. of 1725 K Street, N.W., Washington, D. C. in "Polar Icebreaker Preliminary Structural Design and Special Studies," (Ref. 13) completed in August of 1968,
and gave a summary of known ice properties and an analysis of possible structures to be utilized in a new type of icebreaker called the M-10 at that time.

The ice loading used was that developed from the analysis by M. K. Tarshis, "Ice Loads Acting on Ships," (Ref. 14) a translation of a Russian text published in Rechnio Transport, Vol. 16, No. 12, 1957, pg. 19.

Using these values of ice loads they determined the capabilities of the proposed design along both elastic limits and plastic hinge limits. They included factors of 1 1/2 for impact loading and two for static condition loading to account for the lack of reliability of the values of the properties of ice.

The results once again lack aspects of reality due to the limitations of the STRESS program.

During recent years a new method called the finite element technique has been developed to give more accurate solutions for structure analysis. The first easily understood, comprehensive, presentation of the method was written by O. C. Zienkiewicz in his boom, "The Finite Element Method in Structural and Continuum Mechanics." (Ref. 15) Development of a computer program capable of analyzing a wide variety of structures was developed by the Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California, called the Structural Analysis and Matrix Interpretive System (SAMIS). (Ref. 16)

- 30 -
THE STRUCTURAL ANALYSIS AND MATRIX INTERPRETIVE SYSTEM

The Structural Analysis and Matrix Interpretive System (Ref. 16) uses elements that are at present restricted to flat triangular facets that are joined together along their edges, and line elements that are joined to the rest of the structure at their ends. All elements are capable of resisting stretching, shearing, bending and twisting loads. Heating, acceleration and pressure loads can also be analyzed. Additional loads can be introduced as energy equivalent concentrated loads at points on the structure.

Structural changes are defined by stresses, deflections, flexibilities and stiffnesses. It also has the capabilities of computing the natural frequencies of the structure and the mode shapes.

The finite element involved utilizes two ideas

1. The structure is divided into small elements
2. The problem is solved by a structural stiffness analysis.

Load deflection relations are taken for each element of the structure. The coefficients of these relations form the stiffness matrix. When the whole system is taken into account, the load deformation relations for the entire system stiffness matrix is developed by summing the stiffness matrices of the pieces composing the system. Where there are common grid points, the forces are simply added. Boundary conditions
are also formulated in matrix notation. The displacements at each of the corners of the facets and the ends of the beam that make up the complete structure are determined by solving systems of equations simultaneously. By taking these displacements, the stresses in each of the elements are determined.

In the Jet Propulsion Laboratory Technical Memorandum No. 33-317, (Ref. 17), the mathematical equations of the structure are given as:

\[
[K]{d} - \lambda[K_i]{d} + [C]{d} = \{P(t)\} - [M_s]{d}\{\sigma\} = [R]{d}
\]

where

\(\{d\} = \) The vector of grid point displacements (dots indicate time derivatives).

\(\{\sigma\} = \) The vector of element stresses.

\([K] = \) The small deflection stiffness matrix.

\([C] = \) The small deflection damping matrix.

\([K_i] = \) The initial stress stiffness matrix

\(\lambda = \) A scalar defining the magnitude of the initial stress distribution.

\([M_s] = \) The mass loading matrix.

\(\{P\} = \) The force loading matrix.

\([R] = \) The matrix of stress coefficients.

Boundary conditions are applied on "d" and "\lambda" to make the above equations solvable.
The static and dynamic displacement response of the structure is given by the first equation and the stress associated with a given set of displacements is given by the second equation.

An assumed displacement function which is continuous along a facet's edge and continuous over the facet is used to develop the stiffness coefficients by minimizing the potential energy of the entire structure. The resulting stiffness coefficients define the forces and moments at the apexes of the triangle which satisfy macroscopic force and moment equilibrium conditions.

The displacements used to develop the stiffness coefficients are also used to develop the loading coefficients under the restraints of minimum potential energy.

The stress coefficients are developed using the same assumed displacements functions and stress-strain and strain deformation relationships. The stress so developed is the mean stress at the facet centroid.

Two coordinate systems are used to implement this program. The local system describes the grid points of each element as to their local relationship, and the common system describes the overall system. In the facet element, the local x-y plane is assumed to be coincident with the midplane of the facet. The displacements of each grid point are defined as a "d" vector which consists of translations in the x, y, and z directions, and the rotation about the x and y axis. Each component of these deflections is assumed to vary linearly.
over the midplane surface. The deflection component at any point is therefore expressed as

\[ d_i = a_i x + b_i y + c_i \]

In the stress-strain relation, it is assumed that the stress in the z direction is zero; assuming that the thickness radius ratio is small compared to one for the shell.

The selection of the triangular elements must be such that the stiffness matrix is positive definite. This can be done as long as the largest angle for any triangle is less than 90 degrees. If an angle is larger than 90 degrees in the facet, the stiffness matrix will be indefinite and thus cannot be used. The most accurate results are obtained if the triangles are all equilateral.

Discrete loads acting at a joint are described by forces acting in the x, y, and z directions and moments acting about the x and y axis. When torque is considered, it must be treated as a couple with forces in the x and y directions.

Local coordinate axis are used in developing the matrices of coefficient and, hence, the need for a transformation from the common axis. This is automatically accomplished if the grid points are given in the data as the common coordinates.

The line elements used are superposition of models for axial elongation, torsional rotation, shearing and bonding.
Where stiffeners provide the resistance to bending, the classical bending element is used in the analysis. When using a facet, the shear bending element is used to provide the necessary stiffness from the supporting frames as this provides deformations which are consistent with those of the facet.

As in the facet element, assumed displacements are used in the development of the stiffness and loading coefficients. The stress coefficients are obtained directly from the stiffness matrix.

In the line element, the local x axis is considered to go down the centroid of the beam. The program automatically makes this transformation if the line element grid points are given in the common coordinates.

Due to the versatility of the program, the input necessary from the analysis for the SAMIS program is extensive. The first section of input for the program is the program control cards called "pseudocode instructions." These cards control tape assignments, matrix naming and matrix manipulation. Card format is divided into alphanumeric names of the matrices, tape assignment instructions, instruction sequence number and program control information.

The card format is broken up into ten fields. The "O" field is the instruction sequence number. The "A", "B", and "C" fields contain the tape storage assignments,
the fields numbered 1, 2, and 3 contain matrix names, the
"code" field contains the subroutine or subprogram name and
the "E" field contains the control information for the
operations to be performed. Card format is as follows:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>A</td>
<td>1</td>
<td>B</td>
<td>2</td>
<td>CODE</td>
<td>C</td>
<td>3</td>
<td>E</td>
</tr>
</tbody>
</table>

Psuedo instructions are:
TABLE 1
(Ref. 16, p. 44)

**OPERATION PSEUDO INSTRUCTIONS**

<table>
<thead>
<tr>
<th>Instructions</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADDS</td>
<td>Form $C_3 = A_1 + B_2$</td>
</tr>
<tr>
<td>BILD</td>
<td>Construct small deflection stiffness, stress, loading, and/or mass matrices as $A_1$, $B_1$, $C_1$, and $F_1$, respectively.</td>
</tr>
<tr>
<td>CHIN</td>
<td>Form $B_2$ such that $B_2 B_2^T = A_1$ and $C_3 = B_2^{-1}$ where $A_1$ is symmetric and positive definite.</td>
</tr>
<tr>
<td>CHOL</td>
<td>Form $C_3 = A_1^{-1} B_2$ using Choleski decomposition.</td>
</tr>
<tr>
<td>CONT</td>
<td>Continuation Card (see pg 43)</td>
</tr>
<tr>
<td>CODE</td>
<td>Transform $A_1$ to coded format as $C_3$</td>
</tr>
<tr>
<td>COLS</td>
<td>Put the $A_1$ matrix in column listing and call it $C_3$.</td>
</tr>
<tr>
<td>DECO</td>
<td>Transform $A_1$ to precoded format as $C_3$.</td>
</tr>
<tr>
<td>FILL</td>
<td>Read $A_1$, $B_2$, and/or $C_3$ into core.</td>
</tr>
<tr>
<td>FLIP</td>
<td>Form $C_3 = A_1^T$</td>
</tr>
<tr>
<td>INKS</td>
<td>Print matrices $A_1$, $B_2$, and/or $C_3$</td>
</tr>
<tr>
<td>MULT</td>
<td>Form $C_3 = A_1 B_2$</td>
</tr>
<tr>
<td>READ</td>
<td>Read matrices $A_1$, $B_2$, and/or $C_3$ from cards.</td>
</tr>
<tr>
<td>ROWS</td>
<td>Put the $A_1$ matrix in row listing and call it $C_3$.</td>
</tr>
<tr>
<td>ROOT</td>
<td>Find latent roots and vectors of $A_1$, a symmetric matrix. Let vectors be $B_2$, roots $C_3$.</td>
</tr>
<tr>
<td>SAVE</td>
<td>Write $A_1$, $B_2$, and/or $C_3$ on tape.</td>
</tr>
<tr>
<td>SORT</td>
<td>Sort a matrix $A_1$ by row or column as $C_3$.</td>
</tr>
<tr>
<td>SUBS</td>
<td>Form $C_3 = A_1 - B_2$</td>
</tr>
<tr>
<td>WASH</td>
<td>WASH $A_1$ elements from $B_2$ to produce $C_3$.</td>
</tr>
</tbody>
</table>
Psuedo instructions called logic instructions provide the capability of loop operations and are also required for transfer of control to the success exit. When logic instructions are used, the "E" field indicates the number of times the logic instruction is to be carried out. Logic instructions are:
**TABLE 2**
(Ref. 16, p. 41)

**LOGIC PSEUDO INSTRUCTIONS**

<table>
<thead>
<tr>
<th>Instructions</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>PREP</td>
<td>Prepare for multiple execution of the following instructions. Execute the instructions between the PREP and the next BACK instruction the number of times specified in the &quot;E&quot; field.</td>
</tr>
<tr>
<td>VARY</td>
<td>Vary matrix or tape numbers in the next instruction by augmenting corresponding field data by the specified integers after one pass.</td>
</tr>
<tr>
<td>BACK</td>
<td>Back up and repeat instructions after PREP.</td>
</tr>
<tr>
<td>ERRS</td>
<td>Disrupt errors can be corrected as follows.</td>
</tr>
<tr>
<td>SKIP</td>
<td>Skip the next &quot;n&quot; pseudo instructions where &quot;n&quot; is specified in the &quot;E&quot; field. (Skip cannot be included between a PREP and BACK instruction).</td>
</tr>
<tr>
<td>STOP</td>
<td>Stop this case and go to HALT.</td>
</tr>
<tr>
<td>PAWS</td>
<td>Pause in the calculations. Operator can restart at any time. (Not operative on IBM 7094-7040 DCS).</td>
</tr>
<tr>
<td>HALT</td>
<td>Halt and indicate a successful exit.</td>
</tr>
</tbody>
</table>
Control cards needed to direct computations for the most general case of static and pseudostatic loading and normal modes is a single analysis would be:
Fig. 1 - Facsimile of Operational Control Cards (including normal modes)
(Ref. 19, p. 9 & 10)
FORTRAN STATEMENT

16.0    FOC1          READ       -1
17.0    FOC1  9002 DFC1  ADDS  12001 FOC2
18.0  11001 KUR1  12001 FOC2  CHOL  9002 DTC1  910
18.1  12002 DDC1
19.0  12002 DDC1  COLS  11006 GCC1
20.0  12001 FOC2  FLIP       FOR2
21.0  12002 DDC1  MULT  11007 GCC2
22.0  9001 WTR1  9002 DTC1  MULT  12003 FOC3
23.0  12003 FOC3  FLIP       FOR3
24.0  12002 DDC1  MULT  11008 GCC3
25.0  11006 GCC1  DECO  12002 DDC1  3
26.0  12002 DDC1  INKS       305
27.0  9002 DTC1  FILL
28.0  10001 SCR2  DTC1  MULT  11009 RCC2  N
29.0  11009 RCC2  DECO  12001 RDC2  N
30.0  12001 RDC2  INKS  N06
31.0  HALT

NOTE: Replace N by the number of structural elements and P by N + 1^N(I6).
Replace N_4 by the number of cards of codes associated with prescribed displacements (I2).
Statement number 9, for example, means take matrix WURL on tape 11, position two and multiply it by matrix TEC1 on tape 9, position two and place the resulting matrix on tape 12, position 3 and call it DCC1. Instructions are performed in the sequence that they are placed in the input. Any similar type program can use the same sequence. Comment cards may be included in the pseudo instructions.

The next section of input defines the model weight at each grid point which is desired, in addition to that automatically modeled by the SAMIS program from given data. The first digits of the row and column numbers correspond to the grid point number, and the last digit gives the corresponding degree of freedom the weight is associated with, i.e. 1, 2, and 3 the x, y, and z direction respectively; 4, 5, and 6 the off center weights about the x, y, and z axis. This matrix is used to introduce concentrated loadings. Card format is as follows:

Card one

<table>
<thead>
<tr>
<th>A3, I3</th>
<th>I6</th>
<th>I6, I6</th>
<th>I6</th>
<th>I6</th>
<th>I6</th>
<th>I6</th>
<th>I6</th>
<th>. . .</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix name &amp; no.</td>
<td>No. of cards of input</td>
<td>Matrix size</td>
<td>Row(-1) or column(1) listed</td>
<td>Precoded (1) or coded (0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Card two

<table>
<thead>
<tr>
<th>I6</th>
<th>I6</th>
<th>I6</th>
<th>I6</th>
<th>I6</th>
<th>. . .</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column codes followed by row codes</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Card three

\[
\begin{array}{ccc}
E12.0 & E12.0 & E12.0 \\
\end{array}
\]

Matric values row or column listed

For coded format

Card two

\[
\begin{array}{cccc}
I6 & I6 & E12.0 & I6 \\
\end{array}
\]

Row or column or Value or
column and row matrix
listed element

The next matrix data input are the nonzero
accelerations associated with the restraints that are not
modeled by the SAMIS program. Corresponding to the assigned
loading conditions, gravity \((g)\) accelerations are supplied
in the \(x\), \(y\), and \(z\) direction, and \((g) / \text{in}\) accelerations
forces are supplied about the \(x\), \(y\), and \(z\) axis. This matrix
is utilized to make equilibrium checks. Card format is the
same as the weight matrix.

The next section of input gives the nonzero
elements of the stiffness matrix that are not part of the
elements listed. This matrix gives the holonomic boundary
conditions imposed. Card format is the same as for the
weight matrix.

Two cards giving the modulus of elasticity and the
modulus of rigidity and a card of zeroes complete this
section of input.

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Tabulated grid point coordinates are then given in ascending order. If the coordinates are given in the common coordinate system, the SAMIS program makes the necessary transformations to local coordinates for generation of the various matrices. One grid point is given on each card with the following format:

<table>
<thead>
<tr>
<th>Il</th>
<th>I3</th>
<th>3X,E7.0,E7.0,E7.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Card No.</td>
<td>Grid point no.</td>
<td>Grid point coordinates</td>
</tr>
</tbody>
</table>

Following this section of input is the element data cards. This input describes each element as to its type, position, size, geometry and the material type it is. The card format for each of the elements is as follows:
**TABLE 3**
(Ref. 16, p. 114)

ELEMENT DATA FORMAT FOR FACET

<table>
<thead>
<tr>
<th>CARD NO.</th>
<th>ELEMENT NUMBER</th>
<th>ELEMENT IDENTITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>Elastic Node No. 1</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>Continuity Node No. 1</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>NOT</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>NOT</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>Coord. First Local Grid Point ( \begin{align*} X_1 &amp; \quad Y_1 &amp; \quad Z_1 \end{align*} )</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>NOT</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>Coord. First Subs. Grid Point ( \begin{align*} X_1 &amp; \quad Y_1 &amp; \quad Z_1 \end{align*} )</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>NOT</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>Coord. First Elastic Grid Point ( \begin{align*} X_1 &amp; \quad Y_1 &amp; \quad Z_1 \end{align*} )</td>
</tr>
<tr>
<td>CARD NO.</td>
<td>ELEMENT NUMBER</td>
<td>ELEMENT IDENTITY</td>
</tr>
<tr>
<td>----------</td>
<td>----------------</td>
<td>------------------</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>Substitute Node No. 3</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>Temperature Change Upper Surface</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>Coord. Third Local Grid Point</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>Coord. Third Local Grid Point</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>Coord. Third Subs. Grid Point</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>Coord. Third Elastic Grid Point</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CARD NO.</td>
<td>ELEMENT NUMBER</td>
<td>ELEMENT IDENTITY</td>
</tr>
<tr>
<td>---------</td>
<td>----------------</td>
<td>------------------</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CARD NO.</td>
<td>ELEMENT NUMBER</td>
<td>ELEMENT IDENTITY</td>
</tr>
<tr>
<td>----------</td>
<td>----------------</td>
<td>------------------</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Moment of I</td>
<td>Shear Area</td>
</tr>
<tr>
<td></td>
<td>About Z Axis</td>
<td>Z Force</td>
</tr>
<tr>
<td></td>
<td>Moment of I</td>
<td>Moment of I</td>
</tr>
<tr>
<td></td>
<td>About Y Axis</td>
<td>Material Identity</td>
</tr>
<tr>
<td>2</td>
<td>Temperature</td>
<td>Temperature</td>
</tr>
<tr>
<td></td>
<td>Change From</td>
<td>Gradient</td>
</tr>
<tr>
<td></td>
<td>Zero. Str.</td>
<td>X-Y Direction</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Coord. Third</td>
<td>Local Grid Point</td>
</tr>
<tr>
<td></td>
<td>X&lt;sub&gt;3&lt;/sub&gt;</td>
<td>Y&lt;sub&gt;3&lt;/sub&gt;</td>
</tr>
<tr>
<td></td>
<td>Z&lt;sub&gt;3&lt;/sub&gt;</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Coord. Second</td>
<td>Substitute Grid</td>
</tr>
<tr>
<td></td>
<td>X&lt;sub&gt;2&lt;/sub&gt;</td>
<td>Point</td>
</tr>
<tr>
<td></td>
<td>Y&lt;sub&gt;2&lt;/sub&gt;</td>
<td>Z&lt;sub&gt;2&lt;/sub&gt;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Coord. Identity</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Coord. Third</td>
<td>Elastic Grid Point</td>
</tr>
<tr>
<td></td>
<td>X&lt;sub&gt;3&lt;/sub&gt;</td>
<td>Y&lt;sub&gt;3&lt;/sub&gt;</td>
</tr>
<tr>
<td></td>
<td>Z&lt;sub&gt;3&lt;/sub&gt;</td>
<td>Coord. Identity</td>
</tr>
</tbody>
</table>
### Table 5
(Ref. 16, pgs. 124, 125, 126 & 127)

**Facet Element Data**

<table>
<thead>
<tr>
<th>Card</th>
<th>Columns</th>
<th>Format</th>
<th>Item*</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>I1</td>
<td>1</td>
<td>Indicates card number = 1.</td>
</tr>
<tr>
<td>1</td>
<td>2-4</td>
<td>I3</td>
<td>K</td>
<td>Element number. $0 &lt; K &lt; 999$.</td>
</tr>
<tr>
<td>1</td>
<td>5-7</td>
<td>IX, I2</td>
<td>31</td>
<td>Specifies the Facet element assumptions. The first digit, 3, indicates three grid points are required.</td>
</tr>
<tr>
<td>1</td>
<td>3-14</td>
<td>E7.0</td>
<td>GA</td>
<td>First, second, and third elastic grid point numbers for the element. If negative,</td>
</tr>
<tr>
<td></td>
<td>15-21</td>
<td>E7.0</td>
<td>GB</td>
<td></td>
</tr>
<tr>
<td></td>
<td>22-28</td>
<td>E7.0</td>
<td>GC</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>29-35</td>
<td>E7.0</td>
<td>--</td>
<td>Data in this field will be ignored.</td>
</tr>
<tr>
<td>1</td>
<td>36-42</td>
<td>E7.0</td>
<td>SA</td>
<td>First, second, and third substitute grid point numbers.</td>
</tr>
<tr>
<td></td>
<td>43-49</td>
<td></td>
<td>SB</td>
<td>If the grid point numbers are negative, solution displacements are produced in the local coordinate system at the grid point.</td>
</tr>
<tr>
<td></td>
<td>50-56</td>
<td></td>
<td>SC</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>57-63</td>
<td>E7.0</td>
<td>M</td>
<td>Facet mass</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$M &lt; 0$, total mass: $\frac{\text{#sec}^2}{\text{in}}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$M &gt; 0$, mass per unit area: $\frac{\text{#sec}^2}{\text{in}^3}$</td>
</tr>
<tr>
<td>1</td>
<td>64-70</td>
<td>E7.0</td>
<td>T</td>
<td>Facet thickness in inches.</td>
</tr>
<tr>
<td>1</td>
<td>71-72</td>
<td>A2</td>
<td>N</td>
<td>Name of the structural material. The first two characters of the material name must match the first two characters of the material name in the material table.</td>
</tr>
</tbody>
</table>

*Numbers given in this column are to be taken literally. The user must substitute appropriate numbers for letters given.*
**TABLE 5 (Cont'd)**

**FACET ELEMENT DATA**

<table>
<thead>
<tr>
<th>Card</th>
<th>Columns</th>
<th>Format</th>
<th>Item*</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>I1</td>
<td>2</td>
<td>Indicates card number = 2.</td>
</tr>
<tr>
<td>2</td>
<td>2-4</td>
<td>I3</td>
<td>K</td>
<td>Element number. $0 \leq K \leq 999$. (may be omitted)</td>
</tr>
<tr>
<td>2</td>
<td>5-7</td>
<td>IX,I2</td>
<td>31</td>
<td>Same as Card 1, columns 5-7. (may be omitted)</td>
</tr>
<tr>
<td>2</td>
<td>8-14</td>
<td>E7.0</td>
<td>CA</td>
<td>Continuity boundary conditions at the first, second, and third grid points. If both substitute points are given, these apply to the substitute points. If only elastic appear, they apply to the elastic.</td>
</tr>
<tr>
<td>2</td>
<td>15-21</td>
<td>E7.0</td>
<td>CB</td>
<td>Same as Card 1, columns 5-7. (may be omitted)</td>
</tr>
<tr>
<td>2</td>
<td>22-28</td>
<td>E7.0</td>
<td>CC</td>
<td>Same as Card 1, columns 5-7. (may be omitted)</td>
</tr>
<tr>
<td>2</td>
<td>29-35</td>
<td>E7.0</td>
<td>--</td>
<td>Ignored</td>
</tr>
<tr>
<td>2</td>
<td>36-42</td>
<td>E7.0</td>
<td>P</td>
<td>Normal pressure: pounds/inch$^2$ Positive in the plus z direction.</td>
</tr>
<tr>
<td>2</td>
<td>43-49</td>
<td>E7.0</td>
<td>T_m</td>
<td>Temperature (Degrees Rankine) of the material (used to define elastic constants).</td>
</tr>
<tr>
<td>2</td>
<td>50-56</td>
<td>E7.0</td>
<td>T_u</td>
<td>Upper surface temperature change (Degrees Rankine) from zero stress temperature.</td>
</tr>
<tr>
<td>2</td>
<td>57-63</td>
<td>E7.0</td>
<td>T_l</td>
<td>Lower surface temperature change (Degrees Rankine) from zero stress temperature.</td>
</tr>
<tr>
<td>2</td>
<td>64-70</td>
<td>E7.0</td>
<td>--</td>
<td>Ignored</td>
</tr>
<tr>
<td>2</td>
<td>71-72</td>
<td>A2</td>
<td>---</td>
<td>Ignored</td>
</tr>
<tr>
<td>3</td>
<td>Omit</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Omit</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Numbers given in this column are to be taken literally. The user must substitute appropriate numbers for letters given.*
**TABLE 5 (Cont'd)**

**FACET ELEMENT DATA**

<table>
<thead>
<tr>
<th>Card</th>
<th>Columns</th>
<th>Format</th>
<th>Item*</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1</td>
<td>I1</td>
<td>5</td>
<td>Indicates card number = 5.</td>
</tr>
<tr>
<td>5</td>
<td>2-4</td>
<td>I3</td>
<td>K</td>
<td>Element number. $0 \leq K \leq 999$. (may be omitted)</td>
</tr>
<tr>
<td>5</td>
<td>5-7</td>
<td>1X,I2</td>
<td>31</td>
<td>Same as Card 1, columns 5-7. (may be omitted)</td>
</tr>
<tr>
<td>5</td>
<td>8-70</td>
<td>9E7.0</td>
<td></td>
<td>Coordinates (inches) in the overall system of the origin of the local coordinate system $(x_1,y_1,z_1)$; a point on the local x axis $(x_2,y_2,z_2)$; and a point in the x-y plane $(x_3,y_3,z_3)$, noncollinear with the first two points are selected to define desired direction of the local z axis.</td>
</tr>
<tr>
<td>5</td>
<td>71-72</td>
<td>A2</td>
<td>--</td>
<td>Ignored</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td>Omit</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>I1</td>
<td>7</td>
<td>Indicates card number = 7.</td>
</tr>
<tr>
<td>7</td>
<td>2-4</td>
<td>I3</td>
<td>K</td>
<td>Element number. $0 \leq K \leq 999$. (may be omitted)</td>
</tr>
<tr>
<td>7</td>
<td>5-7</td>
<td>1X,I2</td>
<td>31</td>
<td>Same as Card 1, columns 5-7. (may be omitted)</td>
</tr>
<tr>
<td>7</td>
<td>3-19</td>
<td>9E7.0</td>
<td></td>
<td>Coordinates (inches) of the substitute grid points corresponding to the first, second, and third elastic grid points.</td>
</tr>
<tr>
<td>7</td>
<td>71-72</td>
<td>A2</td>
<td>C</td>
<td>Coordinate identification. If $C = L$, substitute grid point coordinates are in the local system. If $C \neq L$, coordinates are overall.</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td>Omit</td>
</tr>
</tbody>
</table>

*Numbers given in this column are to be taken literally. The user must substitute appropriate numbers for letters given.*
### TABLE 5 (Cont'd)

**FACET ELEMENT DATA**

<table>
<thead>
<tr>
<th>Card</th>
<th>Columns</th>
<th>Format</th>
<th>Item</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>1</td>
<td>I1</td>
<td>9</td>
<td>Card number = 9.</td>
</tr>
<tr>
<td>9</td>
<td>2-4</td>
<td>I3</td>
<td>K</td>
<td>Element number. 0 ≤ K ≤ 999.</td>
</tr>
<tr>
<td>9</td>
<td>5-7</td>
<td>I12</td>
<td>31</td>
<td>Same as Card 1, columns 5-7. (may be omitted)</td>
</tr>
<tr>
<td>9</td>
<td>8-70</td>
<td>9E7.0</td>
<td></td>
<td>Coordinates (inches) for the first ( (x_1, y_1, z_1) ), second ( (x_2, y_2, z_2) ) and third ( (x_3, y_3, z_3) ) elastic grid points of the Facet in the overall or local system.</td>
</tr>
<tr>
<td>9</td>
<td>71-72</td>
<td>A2</td>
<td>C</td>
<td>Coordinate identification. If ( C = L ) elastic grid point coordinates are in the local system. If ( C \neq L ), coordinates are overall.</td>
</tr>
</tbody>
</table>

*Numbers given in this column are to be taken literally. The user must substitute appropriate numbers for letters given.*
### TABLE 6
(Ref. 16, pgs. 128, 129, 130, & 131)

**LINE ELEMENT DATA**

<table>
<thead>
<tr>
<th>Card</th>
<th>Columns</th>
<th>Format</th>
<th>Item*</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>I1</td>
<td>1</td>
<td>Indicates card number = 1.</td>
</tr>
<tr>
<td>1</td>
<td>2-4</td>
<td>I3</td>
<td>K</td>
<td>Element number. $0 &lt; K &lt; 999$.</td>
</tr>
<tr>
<td>1</td>
<td>5-7</td>
<td>1X,I2</td>
<td>2A</td>
<td>Specifies the line element equations. Rod and tube equations and $(A = 1)$ Elementary beam or $(A = 2)$ shear beam displacement assumptions.</td>
</tr>
<tr>
<td>1</td>
<td>8-14</td>
<td>E7.0</td>
<td>GA</td>
<td>First, second, and third grid point numbers for the element. The grid number of the third grid point may be omitted. If numbers are negative, solution displacements are in the local coordinate system at the grid point.</td>
</tr>
<tr>
<td>1</td>
<td>15-21</td>
<td>E7.0</td>
<td>GB</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>22-28</td>
<td>E7.0</td>
<td>GC</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>29-35</td>
<td>E7.0</td>
<td>$A_x$</td>
<td>Cross-sectional area (axial) in the y-z plane: inches$^2$.</td>
</tr>
<tr>
<td>1</td>
<td>36-42</td>
<td>E7.0</td>
<td>$J_x$</td>
<td>Torsional rigidity against twist about the x axis: inches$.^4$.</td>
</tr>
<tr>
<td>1</td>
<td>43-49</td>
<td>E7.0</td>
<td>$A_y$</td>
<td>Effective area deforming in shear in the x-y plane due to a y force: inches$^2$.</td>
</tr>
<tr>
<td>1</td>
<td>50-56</td>
<td>E7.0</td>
<td>$I_z$</td>
<td>Moment of inertia resisting a moment about the z axis: inches$^4$.</td>
</tr>
</tbody>
</table>

*Numbers given in this column are to be taken literally. The user must substitute appropriate numbers for letters.*
<table>
<thead>
<tr>
<th>Card</th>
<th>Columns</th>
<th>Format</th>
<th>Item</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>57-63</td>
<td>E7.0</td>
<td>(A_z)</td>
<td>Effective shear area for a shear force in the (z) direction: inches(^2).</td>
</tr>
<tr>
<td>1</td>
<td>64-70</td>
<td>E7.0</td>
<td>(I_y)</td>
<td>Moment of inertia resisting a twist about the (y) axis: inches(^4).</td>
</tr>
<tr>
<td>1</td>
<td>71-72</td>
<td>A2</td>
<td>(N)</td>
<td>Name of the structural material. The first two characters of the name must match the first two characters of the material table material name.</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>I1</td>
<td>2</td>
<td>Card number = 2.</td>
</tr>
<tr>
<td>2</td>
<td>2-4</td>
<td>I3</td>
<td>(K)</td>
<td>Element number. (0 \leq K \leq 999.)</td>
</tr>
<tr>
<td>2</td>
<td>5-7</td>
<td>1X,12</td>
<td>2A</td>
<td>Same as Card 1, columns 5-7. (may be omitted)</td>
</tr>
<tr>
<td>2</td>
<td>8-21</td>
<td>2E7.0</td>
<td>CA</td>
<td>Continuity boundary conditions at the first and second grid points. If substitute grid points are given, these apply to the substitute points. If only elastic appear, they apply to the elastic.</td>
</tr>
<tr>
<td>2</td>
<td>22-28</td>
<td>E7.0</td>
<td>--</td>
<td>Ignored</td>
</tr>
<tr>
<td>2</td>
<td>29-35</td>
<td>E7.0</td>
<td>(M)</td>
<td>Line element mass. If (M &lt; 0), is the total mass (#sec(^2)in). If (M &gt; 0), (M) is the mass per unit length (#sec(^2)/in(^2)).</td>
</tr>
<tr>
<td>2</td>
<td>36-42</td>
<td>E7.0</td>
<td>(P)</td>
<td>Normal pressure: pounds/inch (positive in the plus (z) direction).</td>
</tr>
<tr>
<td>2</td>
<td>43-49</td>
<td>E7.0</td>
<td>(T_m)</td>
<td>Temperature (degrees Rankine) of the material (used to define elastic constants).</td>
</tr>
<tr>
<td>2</td>
<td>50-56</td>
<td>E7.0</td>
<td>(T_o)</td>
<td>Mean temperature change (degrees Rankine per unit of cross-sectional area - (A_x)) of the element from the zero stress temperature.</td>
</tr>
</tbody>
</table>

*Numbers given in this column are to be taken literally. The user must substitute appropriate numbers for letters.*
<table>
<thead>
<tr>
<th>Card</th>
<th>Columns</th>
<th>Format</th>
<th>Item*</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>57-63</td>
<td>E7.0</td>
<td>T&lt;sub&gt;y&lt;/sub&gt;</td>
<td>Temperature gradient (degrees Rankine per unit cross-sectional moment of inertia ( I_y )) in the ( z ) direction.</td>
</tr>
<tr>
<td>2</td>
<td>64-70</td>
<td>E7.0</td>
<td>T&lt;sub&gt;z&lt;/sub&gt;</td>
<td>Temperature gradient (degrees Rankine per unit cross-sectional moment of inertia ( I_y )) in the ( y ) direction.</td>
</tr>
<tr>
<td>2</td>
<td>71-72</td>
<td>A&lt;sub&gt;2&lt;/sub&gt;</td>
<td>--</td>
<td>Ignored</td>
</tr>
<tr>
<td>3</td>
<td>Omit</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Omit</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>I1</td>
<td>5</td>
<td>Indicates card number = 5.</td>
</tr>
<tr>
<td>5</td>
<td>2-4</td>
<td>I3</td>
<td>K</td>
<td>Element number. ( 0 \leq K \leq 999 ). (may be omitted)</td>
</tr>
<tr>
<td>5</td>
<td>5-7</td>
<td>1X,I2</td>
<td>2A</td>
<td>Same as Card 1, columns 5-7. (may be omitted)</td>
</tr>
<tr>
<td>5</td>
<td>8-70</td>
<td>9E7.0</td>
<td>--</td>
<td>Coordinates (inches) in the overall system of the origin of the local coordinate system, ((x_1,y_1,z_1)); a point on the local ( x ) axis ((x_2,y_2,z_2)); and a point in the ( x-y ) plane ((x_3,y_3,z_3)), noncollinear with the first two points and located to define desired plus ( z ) direction.</td>
</tr>
<tr>
<td>5</td>
<td>71-72</td>
<td>A2</td>
<td>--</td>
<td>Ignored</td>
</tr>
<tr>
<td>6</td>
<td>Omit</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>I1</td>
<td>7</td>
<td>Card number = 7.</td>
</tr>
<tr>
<td>7</td>
<td>2-4</td>
<td>I3</td>
<td>K</td>
<td>Element number. ( 0 \leq K \leq 999 ).</td>
</tr>
<tr>
<td>7</td>
<td>5-7</td>
<td>1X,I2</td>
<td>2A</td>
<td>Same as Card 1, columns 5-7. (may be omitted)</td>
</tr>
</tbody>
</table>

*Numbers given in this column are to be taken literally. The user must substitute appropriate numbers for letters.*
<table>
<thead>
<tr>
<th>Card</th>
<th>Columns</th>
<th>Format</th>
<th>Item*</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>8-21</td>
<td>2E7.0</td>
<td>SA</td>
<td>First and second substitute grid point numbers. If the grid point numbers are negative, solution displacements are produced in the local coordinate system at the grid point.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>SB</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>22-28</td>
<td>E7.0</td>
<td>--</td>
<td>Ignored</td>
</tr>
<tr>
<td>7</td>
<td>29-70</td>
<td>6E7.0</td>
<td>--</td>
<td>Coordinates (inches) of the substitute grid points corresponding to the first and second elastic grid points: all in the local or overall system.</td>
</tr>
<tr>
<td>7</td>
<td>71-72</td>
<td>A2</td>
<td>C</td>
<td>Coordinate identity. If C = L, substitute grid point coordinates are in the local system. If C ≠ L, coordinates are overall.</td>
</tr>
<tr>
<td>8</td>
<td>Omit</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>I9</td>
<td>9</td>
<td>Card number = 9.</td>
</tr>
<tr>
<td>9</td>
<td>2-4</td>
<td>I3</td>
<td>K</td>
<td>Element number. 0 ≤ K ≤ 999. (may be omitted)</td>
</tr>
<tr>
<td>9</td>
<td>5-7</td>
<td>1X, I2</td>
<td>2A</td>
<td>Same as Card 1, columns 5-7. (may be omitted)</td>
</tr>
<tr>
<td>9</td>
<td>8-70</td>
<td>9E7.0</td>
<td>--</td>
<td>Coordinates (inches) for the first ((x_1, y_1, z_1)), and second ((x_2, y_2, z_2)) elastic grid points of the line element and the grid point ((x_3, y_3, z_3)) defining the principal plane of the cross-section: all in the overall or local systems.</td>
</tr>
<tr>
<td>9</td>
<td>71-72</td>
<td>A2</td>
<td>C</td>
<td>Coordinate identification. If C = L, elastic grid point coordinates are in the local system. If C ≠ L, coordinates are in the overall system.</td>
</tr>
</tbody>
</table>

*Numbers given in this column are to be taken literally. The user must substitute appropriate numbers for letters.
For each material used in the structure two cards of material data are required. These cards give the material identification, the temperature associated with the material properties, the coefficient of material expansion and material stiffness coefficients. The material stiffness coefficients are the coefficients of the stress-strain equations in accordance with the following equations:

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{xy} \\
\sigma_{zz} \\
\sigma_{xz} \\
\sigma_{yz}
\end{bmatrix}
= \begin{bmatrix}
D_{11} & D_{21} & D_{31} & D_{41} & D_{55} \\
D_{21} & D_{22} & D_{32} & D_{42} & D_{65} \\
D_{31} & D_{32} & D_{33} & D_{43} & D_{66} \\
D_{41} & D_{42} & D_{43} & D_{44} & \end{bmatrix}
\begin{bmatrix}
\epsilon_{xx} \\
\epsilon_{yy} \\
\epsilon_{xy} \\
\epsilon_{zz} \\
\epsilon_{xz} \\
\epsilon_{yz}
\end{bmatrix}
\]

The above equations hold true for a monotropic material. If \(D_{31} = D_{32} = D_{43} = D_{65} = 0\) an orthotropic material is described. An isotropic material is described if in addition \(D_{33} = D_{55} = D_{66} = D_{11} = D_{41} = D_{42}\) and \(D_{21} = (\nu/(1 + \nu))(D_{11})\). Then \(D_{55} = E/(2(1 + \nu))\) and \(D_{11} = E(1 - \nu)/((1 + \nu)(1 - 2\nu))\).

Card format is as follows:
### TABLE 7
(Ref. 16, p. 111)

**MATERIAL TABLES INPUT DATA**

<table>
<thead>
<tr>
<th>Card</th>
<th>Field</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>(2X,A2,A4)</td>
</tr>
</tbody>
</table>

The material identification: Each material must have a unique identification number. It is recommended that standard SAE and Aluminum Association numbers be used insofar as possible. Only the leading two characters of the six-character identification number are significant.

<table>
<thead>
<tr>
<th>Card</th>
<th>Field</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>(E8.0)</td>
</tr>
</tbody>
</table>

Rankin temperature for the given material properties.

<table>
<thead>
<tr>
<th>Card</th>
<th>Field</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>(E8.0)</td>
</tr>
</tbody>
</table>

Coefficient of thermal expansion: inches/inch-degrees Rankin.

<table>
<thead>
<tr>
<th>Card</th>
<th>Field</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4-9</td>
<td>(6E8.0)</td>
</tr>
</tbody>
</table>

Material stiffness coefficients; $D_{11}$, $D_{21}$, $D_{22}$, $D_{31}$, $D_{32}$, $D_{33}$, lbs/sq.in.

<table>
<thead>
<tr>
<th>Card</th>
<th>Field</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2-8</td>
<td>(7E8.0)</td>
</tr>
</tbody>
</table>

Material stiffness coefficients; $D_{41}$, $D_{42}$, $D_{43}$, $D_{44}$, $D_{55}$, $D_{65}$, $D_{66}$, lbs/sq.in.

**SAMPLE MATERIAL TABLE INPUT**

(Isotropic Aluminum)

<table>
<thead>
<tr>
<th>FORMAT</th>
<th>(2X,A2,A4)</th>
<th>(E8.0)</th>
<th>(E8.0)</th>
<th>(E8.0)</th>
<th>(E8.0)</th>
<th>(E8.0)</th>
<th>(E8.0)</th>
<th>(E8.0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CARD 3*</td>
<td>00000000</td>
<td>00000000</td>
<td>00000000</td>
<td>00000000</td>
<td>00000000</td>
<td>00000000</td>
<td>00000000</td>
<td>00000000</td>
</tr>
</tbody>
</table>

* Used to indicate that all pairs of material table cards, for all materials, have been used.
The material tables must be followed by one card of zeroes to signal the end of the material tables.

The next input is the nonzero boundary conditions applied to displacements or loads. Displacements are given in inches for the restrained degrees of freedom in the x, y, and z directions and in radians for the restrained degrees of freedom about the x, y, and z axis. Loads applied in the x, y, and z directions are given in pounds and those applied about the x, y, and z axis are given in inch-pounds. The loads are those associated with the unrestrained degrees of freedom. Card format is the same as that for the weight matrix.

The last section of input is the title information cards. There are a total of eleven cards defining the displacements, generalized stiffness and generalized weight matrices. This is written as follows:

DDC 1,2,3 = DISPLACEMENT, GENERALIZED STIFFNESS AND GENERALIZED WEIGHT MATRICES

ROW AND COLUMN CORRESPOND TO MODE, LOAD, OR REACTION DISPLACEMENT OR EXCEPT DDC 1 ROW LEADING DIGITS CORRESPOND TO GRID POINT AND FINAL DIGIT TO THE DEGREE OF FREEDOM AT THE GRID POINT

REACTION NOT DISPLACEMENT AT RESTRAINT

INTERNAL FORCE FOR ELEMENT -X- (RDC X)

ROW LEADING DIGITS CORRESPOND TO GRID POINT, FINAL DIGIT

1 AXIAL FORCE
3 SHEAR ALONG X2
7 SHEAR ALONG X3

4 TORQUE
5 MOMENT ABOUT X2
6 MOMENT ABOUT X3

COL CORRESPOND TO MODE, LOAD, OR REACT. DISP. OR ACC.
The complete computer program is available in the Naval Architecture and Marine Engineering Library.
MODELING OF THE STRUCTURE

The structure analysis of a system as complicated as an icebreaker bow requires considerable preparation in order to use the SAMIS program. The first requirement is to develop a three-dimensional picture of the structure and orient it in relation to the common coordinate system. In order to clearly indicate the necessary details and label the elements and grid points, the scale must be quite large. In Appendix A, the three-dimensional drawing of the icebreaker WESTWIND is in a scale of 1/2" = 1'. Once the three-dimensional table has been drawn, a material table must be set up to give the necessary material parameters required for the input data. Appendix B gives the material table for the icebreaker WESTWIND.

Since a ship's structure is symmetrical about the centerline, only one half of the ship need be drawn and analyzed. The boundary conditions for the nodes on the centerline can be completely described by displacements. Unless actual whipping of the ship occurs, all nodes can be fixed in the athwartship direction. Movement in the fore and aft direction will be sufficiently modeled by the facets edge, and movement in the vertical direction can be modeled as very stiff spring.
The boundary conditions of the hull are simply the average pressure loads for the depth the facet is beneath the waterline. At the bow, shell pressure loads for the ice contact area can be selected as desired. The total area in contact with the ice is that which will give the load response determined by Dr. White (Ref. 5). Shear and bending moment diagrams developed from the loading conditions imposed can be used to give the boundary forces where the structure is cut off. These can be applied as concentrated loads in the matrix data as was described as Matrix WTR1 in the previous section. This will necessitate some hand calculations to determine the section modulus and shear area. Furthermore, it would be necessary to determine the deacceleration at the time of maximum bow load and apply that to the mass of the remainder of the hull to get a deacceleration force. A check is needed to determine if these total forces satisfy equilibrium for the entire system being analyzed. A simpler method would be to assign large stiffnesses to springs connected to fixed points and supply these stiffness as matrix data input as was described by matrix KER1 in the previous section.

It would be desirable to divide the entire section being analyzed into small facets and model each stiffener as a beam. However, due to the limit on the number of elements the program can handle (999), this type of modeling can only
be done in those areas where complete and accurate detail is desired. The IIT Research Institute Technology Center of Chicago, Illinois in IITRI Project J6127 (Ref. 19), developed what is called an orthotropic plate model for stiffened structures so that large facets may be used for the remainder of the structure. Thickness is modeled as satisfying the following equation.

$$t^2 = \frac{12I}{A_f + A_p}$$

where

$$A_p = bh \text{ (the plate area)}$$

$$A_f = \text{the area of the frame}$$

$$I = A_fd^2 \frac{A_p}{A_p + A_f} + I_p + I_f$$

$$d = \text{distance between the centroids of the plating and frame.}$$

$$I_f = \text{Moment of inertia of frame alone about its own centroidal axis.}$$

$$I_p = \frac{bh^3}{12(1-v^2)}$$

$$b = \text{distance between frames}$$

$$h = \text{plate thickness.}$$

The modulus of elasticity in the plane of the plate and in the direction the stiffeners run is
\[
E_y = \frac{E(\lambda_f + \lambda_p)}{bt(1-v^2)} \frac{I_p}{I}
\]

The modulus of elasticity in the plane of the plate and perpendicular to the plate is

\[
E_x = \frac{I_p}{I} E_y
\]

and

\[
E_{xy} = v E_x
\]

The resulting material constants are then

\[
D_{11} = E_y + \frac{E_{xy}^2}{E_y}
\]
\[
D_{21} = E_{xy} + \frac{E_{xy}^2}{E_y}
\]
\[
D_{22} = E_x + \frac{E_{xy}^2}{E_y}
\]
\[
D_{31} = 0
\]
\[
D_{32} = 0
\]
\[
D_{33} = G
\]
\[
D_{41} = E_{xy}
\]
\[
D_{42} = E_{xy}
\]
\[
D_{43} = 0
\]
\[
D_{44} = E_y
\]
\[
D_{55} = G
\]
\[ D_{65} = 0 \]
\[ D_{66} = G \]

where \( G = \frac{E_x E_y}{2(1 + v)} \)

Utilizing the SAMIS program, version one IITRI analyzed a section of the WESTWIND hull from frame seven to thirty-one. Results were not compared with stress data accumulated by the WESTWIND, but showed a structural weakness at frame 25 which has been an area of structural failures in the past.
FUTURE DEVELOPMENTS

The capabilities of this program provide the ships structure analysts with a very valuable tool. Modeling the entire shell of a ship as an unstiffened plate, and then designing the stiffeners to be most effective from the results of the first run can lead to optimal utilization of material in ship construction not only for tough skinned vessels such as an icebreaker, but also for all other types of vessels. At last there is a glimmer of hope that we can depart from the long standing method of using something that worked before in a new ship so we can have hope of some measure of success.
REFERENCES


(All material is medium black steel unless otherwise indicated)

Shell Plating

Main deck to second deck.
20.4# high strength steel plate.

Second deck to platform deck.
66.3# high strength steel plate.

Platform deck to keel.
56.1# high strength steel plate.

Transverse Frames

Stem, Main deck to 30° bow.
15" x 15.3# plate.

30° bow, frame 7 to keel.
40" x 20.4# plate.

30° bow, frame 7 to keel.
20" x 40.3# plate.

Cant frame 02 to frame 61, main deck to second deck.
6" x 4" x 8.25# T

Second deck to keel.
15" x 4" x 15.3# T

Note special framing detail in Appendix C.
Transverse Bulkheads

(All stiffeners on 24" centers)

Bulkhead 7

Plating
7.65# plate,

Stiffeners
6" x 4" x 11# T

Bulkhead 19

Plating
Main deck to second deck.
7.65# plate.
Second deck to platform deck.
12.75# plate.

Stiffeners
6" x 4" x 8.25# T

Bulkhead 31

Main deck to second deck
7.65# plate.
Second deck to platform deck.
10.2# plate.
Platform deck to keel.
12.75# plate.

Stiffeners
Main deck to second deck.
6" x 4" x 11# T.
Bulkhead 43

Plating
Main deck to third deck
7.65# plate.
Third deck to keel.
10.2# plate.

Stiffeners
Main deck to second deck.
5" x 2.69" x 4.48# T
Second deck to shell.
6" x 4" x 11# T

Bulkhead 61

Plating
Main deck to third deck.
7.65# plate.
Third deck to keel
10.2# plate.

Stiffeners
Main deck to second deck.
5" x 2.69" x 4.48# T
Second deck to shell
6" x 4" x 11# T

Longitudinal Bulkheads
Frame 31 to Frame 61.
Main deck to second deck.
10.2# plate
Second deck to shell.
12.75# plate
Stiffeners
Frame 31 to frame 61 on 16" centers
6" x 4" x 11# T.
Decks
Main deck
Plating
Stem to frame 28
10.2# plate.
Frame 28 to frame 61.
7.65# plate.
Transverse Stiffeners
Stem to frame 61
6" x 4" x 8.25# T.
Frame 34, from centerline to 10' on either side of centerline.
12" x 4" x 16.5# T
Longitudinal stiffeners
Frame 7 to frame 19, 1' off centerline at frame 7 and 5' off centerline at frame 19.
12" x 6 1/2" x 25# I
Frame 31 to frame 61, 9' off centerline at frame 31 and 19' off centerline at frame 61.
12" x 6 1/2" x 25# I

Second deck
Plating
Bow to frame 31
7.65# plate.
Frame 31 to frame 61
10.2# plate.
Deck Stiffeners. All
6" x 4" x 11# T.
Transverse beam frame 35
12" x 4" x 25# T.

Longitudinal beams
Bow to frame 7 on centerline.
12" x 6 1/2" x 25# I
1" from centerline at frame 7 to 6.0 from centerline at frame 27.
12" x 6 1/2" x 25# I.
Frame 31 to frame 43 on centerline.
12" x 6 1/2" x 25# I.
Frame 43 to frame 61, 6.0' from centerline.
12 x 6 1/2" x 25# I

Third deck
Plating (All)
10.2# plate.
Transverse stiffeners
Bow to frame 61
7" x 6 3/4" x 15# T.
Frame 45 and frame 49 from centerline to 6' on either side of centerline.
12" x 6 3/4" x 30# T
Longitudinal stiffeners
1' from centerline at frame 7 to 9' from centerline at frame 31.
Frame 31 to frame 43 on centerline.
Frame 43 to frame 61, 6' from centerline.
16" x 7" x 36# I
12.75# brackets, 21" x 10 1/2", used to tie into frame at end of each deck stringer.
Platform Deck
Plating
Frame 7 to frame 31 and frame 43 to frame 61.
10.2# plate.
Transverse stiffeners (All)
7" x 6 3/4" x 30# T.
Frame 44 and frame 48, from centerline to 6' on either side of centerline.
12" x 6 3/4" x 30# T.
Longitudinal stiffeners
Frame 43 to frame 56 on centerline.
16" x 7" x 36# I.
Longitudinal BHD

6" x 4" x 16# I

Equal

Spaces

APPENDIX C
FRAMES
32-42 Incl.
APPENDIX C (Cont'd)

All stiffeners spaced equal distance apart on Shell Frames 40-60 Incl.

6" x 4" x 16 3/4 I (ALL)