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Measurement of the ratio of charge form factors for stable lithium isotopes by electron scattering

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MEASUREMENT OF THE RATIO OF CHARGE FORM FACTORS FOR STABLE LITHIUM ISOTOPES BY ELECTRON SCATTERING

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MEASUREMENT OF THE RATIO OF CHARGE FORM FACTORS
FOR STABLE LITHIUM ISOTOPES BY ELECTRON SCATTERING

by

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ABSTRACT

The ratio of the charge form factor of lithium seven to the charge form factor of lithium six was measured by high energy electron scattering techniques. Using the well known form factor for lithium seven the form factor for lithium six was determined by finding a best fit to the measured ratios at low values of momentum transfer squared ($q^2$). From this manipulation a root mean square radius of 2.65 Fermi was determined for the lithium six nucleus.
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1. Introduction

The scattering of high energy electrons from nuclei has proved to be a very useful method of determining nuclear electromagnetic properties. As electrons interact only with the electric and magnetic properties of the nucleus, the interaction is known and useful information can be resolved from the resulting data. The elastic scattering process leaves the nucleus in the ground, or unexcited, state so that by measuring the deviation of the scattering from that which would be produced by a point charge the spatial distribution of the nucleus can in principle be determined. The measurement of elastically scattered electrons in effect maps out the Fourier transform of the nucleus.

Scattering from Li$^6$ and Li$^7$ is of particular interest since Lithium is the lightest stable nuclei with P-shell nucleons. Several models have been proposed to account for the deviation of these two isotopes. As is usual for this type of experiment, the data is not accurate enough to use to invert the Fourier transform and directly obtain the charge distribution. It is therefore necessary to assume a charge distribution and see if the results calculated from the assumption fit the experimental results.

The data obtained in electron scattering experiments is usually reduced to form factors. This is accomplished by comparing the value of the scattering cross section experimentally obtained with that which would be produced by a point charge (Mott scattering) [1]. The experimental cross section contains scattering produced by
magnetic as well as electric effects. In this investigation only the electric interactions are of concern so the magnetic effects are removed by a mathematical process described in a latter section.

The ratio of charge form factors, that is the part of the form factor contributed by the electric interaction, of Li\textsuperscript{7} to that of Li\textsuperscript{6} is of particular interest at this time since a relation has been proposed by Suelzle [2] to describe this ratio. Suelzle's data neither proves nor disproves this relation, it is therefore worthwhile to attempt to establish the validity of this proposal.

The ratio of the square of the charge form factors of Li\textsuperscript{6} and Li\textsuperscript{7} is given by Suelzle as:

\[
R = \left( \frac{F_{\text{ch7}}(a_o = 1.755, Q = 4.20)}{F_{\text{ch6}}(a^2 = .87, b = 1.70, c^2 = .205)} \right)^2
\]

In this equation the charge form factor for Li\textsuperscript{7} is given by:

\[
F_{\text{ch7}} = \left\{ (1 - \frac{Z^2 - 2}{6Z} \frac{q^2 a_o^2}{a_o^2}) \exp \left[ -q^2 a_o^2 / 4(1-1/A) \right] F_N(q^2) \right\}^2
+ q^2 Q^2 / 324 \left\{ \exp \left[ -q^2 a_o^2 / 4(1-1/A) \right] F_N(q^2) \right\}^2
\]

This expression is for the monopole plus undeformed oscillator potential model for the Li\textsuperscript{7} nucleus. Suelzle finds that this model provides a good fit to his data over a wide range of momentum transfer squared, \( q^2 \).
The expression for the form factor for Li$^6$ is a three parameter phenomenological expression which Suelzle has developed to fit his data:

$$F_{\text{ch}6} = \exp(-a^2 q^2) - c^2 q^2 \exp(-b^2 q^2)$$

The three constants cannot be independently determined and are difficult to determine exactly. The mean square radius $\left\langle r^2 \right\rangle$ for this distribution is given by $6(a^2 + c^2)$. The numbers given for the parameters are those which provide a best fit to Suelzle's data.

This expression seems to provide a reasonable fit to previous data when $q^2$ is greater than $1 F^{-2}$: however, for lower values there seems to be a reasonable doubt whether this is correct. The Naval Postgraduate School linear accelerator is designed to produce the range of energies that is necessary to carefully investigate this area. It is therefore reasonable to expect that additional work at this facility would produce worthwhile results.
Figure I
Layout of Equipment in Scattering Area
2. Experimental Technique and Equipment

The design and operation of the Naval Postgraduate School's linear accelerator has been thoroughly described in a thesis by Barnett and Cunneen [3]. Several additions have been made to the beam handling and scattering equipment which make an experiment of this type possible. The arrangement of the energy defining and scattering areas is depicted in Figure 1. In further explanation each item shown will be briefly described and its function mentioned, as applied to the NPGS LINAC.

The collimator is in its simplest form is a hole which acts as a source point for the beam entering the magnetic deflection system. It has the effect of making a smaller spot on the target but also reduces the amount of beam current. The present collimation makes possible the selection of three different sized holes plus a fluorescent screen for remotely viewing the beam as it emerges from the accelerator. The collimator is operated from the control console. A collimator hole of 5/16 inch was used on most of the data taking runs of this experiment.

The deflection system consists of a pair of matched magnets that are powered in series by a highly regulated power supply. The magnetic field at the center of the first magnet is monitored by a nuclear magnetic resonance probe so that a very accurate measure of the beam energy is available to the operator at all times. There is also a shunt in series with the input to the magnets so that current to
the system can be monitored, and a calibration is provided so that current can be used to determine the energy setting of the system. The shape of the magnetic field of each magnet was measured with a rotating coil gaussometer and an effective radius of 23 inches has been calculated by Spectromagnetics Ind. [4]. Using this system the energy of the beam is determined to an accuracy of about .5%. The monitoring equipment allows a beam energy to be maintained to a very high precision.

The spectrum analyzer consists of a helix of wires protruding from a central drum so that the sequence of wires sample across the beam as the drum is rotated. The drum is placed below the beam and the wires are so connected to the commutator that the wire that protrudes most vertically into the beam is sampled. The output of the commutator is connected to the vertical deflection of an oscilloscope. The horizontal deflection of the scope is powered by the output of a potentiometer connected to the drum which produces a voltage proportional to the horizontal position of the wire being sampled. With this set of inputs the scope displays relative beam intensity as a function of position across the beam. Accelerator pulses appear as spikes extending from the horizontal trace. Since the analyser is placed downstream from the first deflection magnet, the horizontal spread corresponds to energy spread within the beam. With this device the accelerator can be more easily and accurately adjusted to produce the energetically compact beam required for an accurate, efficient
experiment. When the analyser is not used, for instance during data taking runs, the drum is stopped in a position such that the wires do not interfere with the beam entering the energy defining area. This device was not installed until the last portion of this experiment.

The spectrum of energies in the beam arriving at the target is limited to a narrow band by the use of energy defining slits. This system consists of two-inch thick steel jaws that are placed at a point midway between the two deflection magnets. At this point the beam is dispersed in energy because of having passed through the field of the first magnet of the deflection system so that the jaws remove those electrons which are not within the energy spread desired. The jaws are remotely movable and have a turn counter at the control consol which allows the jaw separation to be set from 0 to 2 inches with an accuracy of .005 inch. A slit width of .125 inch was used for the runs of this experiment. This separation corresponds to an energy spread of about .5 percent of incident energy.

Between the slit box and the second deflection magnet is the first of a pair of magnetic quadrupoles. The quadrupoles function together to focus the beam on the target in as small a diameter as possible while maintaining the electron motion parallel to the beam axis. The second of the pair is placed after the second deflection magnet. The quadrupoles have regulated power supplies that are located in the control area. The present system of focusing consists of setting the power supplies to a current recommended by the manufacturer for the beam
energy; then varying the power supply settings slightly while viewing a fluorescent screen at the target position to see that a minimum beam size has been reached. The size of the beam can not be made smaller than the collimator. The problem is further complicated by the short distance between quadrupoles and target that is available due to the limited space in the scattering area.

Just prior to arriving at the target table the beam passes through a 5 foil secondary emission monitor (SEM). This SEM is used to monitor the beam current during the runs so that the accelerator can be adjusted as necessary to provide the required current. At the end of this SEM is an aluminum window 0.0033 inches thick which separates the vacuum of the accelerator from that of the target chamber. Just after the scattering chamber is a larger SEM having 11 foils which is used for beam integration to determine the total charge passing through the target. The second SEM has a separate vacuum system and is windowed with 0.0033 inch thick aluminum. The construction of this type of device is described by Tautfest and Fechter [5]. A Faraday cup for absolute measurements of beam current is located just forward of the beam dump. The cup is placed on a movable table so that it can be remotely moved into and out of the beam when calibration of the SEM is necessary. Since a secondary emission monitor provides only a relative measurement, calibrations are required whenever absolute measurements are made. This experiment did not require SEM calibration since a standardization target was used.
The relative efficiency of the integrating SEM was occasionally checked to insure that long term changes had not taken place. Short term efficiency changes are not likely, according to Bumiller and Dally \[6\], so consistency of sequential integrations over the three targets was assumed.

The measurement of the efficiency of the present Faraday cup has not been possible, however, a new cup with this feature is presently under construction. The Faraday cup produces a great deal of radiation while it is in the beam. Due to very limited space only a minimum amount of shielding can be provided, therefore, the cup is used only when necessary.

The scattering chamber is based on a vacuum tight target table in which the target holding "ladder" can be remotely rotated and moved vertically to select target and proper target angle. The table is covered with a steel chamber which has nine observation ports located at 15 degree intervals from 30 degrees through 150 degrees relative to the beam line. Through these ports scattered electrons pass to the spectrometer. The spectrometer observation ports are windowed with 0.0033 inch thick aluminum. There are three viewing ports on the opposite side of the scattering chamber which are used for lighting and remote television viewing of the targets. The chamber is approximately 22 inches in diameter and has an 8 inch cover which can be easily removed for target replacement and other work on the chamber. The top can also be removed to make an opening about
18 inches in diameter for larger projects. The chamber is evacuated during scattering experiments.

A magnetic spectrometer is located on a rotatable mount which is centered on the target. The spectrometer movement is limited between 0 degrees and 160 degrees due to the beam pipe and space limitations. The spectrometer is powered by a stable, highly regulated supply which is controlled by a rotating coil gaussmeter system first developed by Bumiller [7]. This type of control is described in detail in a NPGS thesis by Kenaston et al [8]. The spectrometer has an entrance plate which defines the solid angle attached to the lower port to insure that a definite measure of solid angle is observed. The spectrometer vacuum chamber is windowed with 0.005 inch aluminum at top and bottom.

At the top focal plane, the position of which was determined by Oberdier [9], is another set of jaws that determine the momentum spread or bite that is observed at a single spectrometer setting. Located just behind this set of jaws are the scintillation detector counters. Two counters are used. One is located behind and slightly separated from the other so that the separate outputs can be used in a coincidence circuit to reduce the likelihood of counting stray radiation rather than the desired scattered electrons. The electronic system for this type of counter is described in a NPGS thesis by Kenaston et al [8]. The scintillation material is a flat plate slightly wider than the beam leaving the spectrometer and long enough to be certain that all
electrons coming from the spectrometer will be counted. The scintillating material is connected to the top of a photomultiplier tube by a short light pipe of plastic whose output is the input of the counting electronics. Difficulty was encountered in reducing the background counting rate during the experiment. The removal of the Faraday cup from the beam produced a marked decrease in the background counting rate. Additional shielding in the form of lead was added to the deflection area, and a study is underway to improve the shielding on the counters. The present conditions allow data of high enough quality so that background is of little problem for elastic scattering such as was done on this investigation.
3. Target Preparation

Targets of lithium metal were used in this investigation. Since lithium metal is highly reactive, combining with many ordinary materials such as oxygen, nitrogen or water, it is essential that care be taken to prevent contamination of the targets. It was found that even the small amount of oxygen and nitrogen contained in reactant grade argon would cause visible surface contamination on lithium metal. Using a more reactive material such as potassium metal to remove some of these impurities from the scattering chamber was partially effective in reducing contamination.

In preparation for scattering, the targets were first cleaned by scraping them clean of surface contamination. This was accomplished in a mineral oil bath which effectively prevented contamination. The thickness of the targets was then determined by measuring them with a micrometer using two aluminum plates of known thickness to protect the soft metal of the targets. Using this technique it is estimated that the thickness was determined to an accuracy of about 1%.

When the targets were clean they were transferred to the target chamber coated with a protective film of mineral oil. The chamber had been prepared by evacuating and then admitting argon. At the same time pieces of metallic potassium were transferred to the chamber. The chamber was then re-evacuated to remove any air that was introduced during the transfer operation, then refilled with argon. The potassium was cleaned by washing with petroleum ether and placed
in screen covered containers about the chamber. The targets were washed with clean petroleum ether and placed in the target ladder. The chamber was then evacuated to a vacuum of about $10^{-6}$ torr to minimize contamination and provide good scattering conditions.

Targets treated in this manner were clean enough so that only a very small elastic peak from heavy elements, probably oxygen and nitrogen, and a slight peak from hydrogen, could be observed. The first excited levels in oxygen and nitrogen are high enough to be out of range of measurements in this experiment, so have no effect on the measurements. The elastic peaks are well enough resolved and small enough to be easily subtracted; thus they contribute very little to the uncertainty of the measurements.

An absolute determination of the cross section could not be made at this time because the efficiency of the spectrometer and counting system is not known to sufficient accuracy and the incoming electron beam cannot be determined absolutely. Thus it was necessary to use a known target for calibration purposes. A carbon target was used for calibration since an accurate phenomenological formula is available to easily determine the correction factor. A carbon target with a thickness in grams per square centimeter about midway between the two targets was selected.

The targets were obtained from the Stanford High Energy Physics Laboratory for this experiment. The targets were produced by Oak Ridge National Laboratory. The lithium six metal had an isotopic purity
of 99.3 percent with lithium seven making up the rest. The lithium seven target was 99.9 percent pure.
4. Data Reduction

The raw data from a scattering experiment, which consists of a series of measurements of counts at different energies, needs to be put into a more meaningful form. This is usually accomplished by calculating a cross section for the scattering. This was done by first making a careful plot of counts versus spectrometer setting, where the spectrometer setting is a linear measure of the energy of the electrons counted. Error flags are attached to the points and a smooth curve drawn to the points. The error flags are determined by assuming that the counting follows Poisson statistics. The error in a single count is just the square root of the number of counts for each point and also for the total number of counts under the curve. The area under the curve was determined by using the trapezoidal rule. This number is then converted to the number scattered, \( N_{sc} \), by dividing by \( \sum p \), the amount of momentum observed by the counting system at one counting point. \( \sum p \) is defined by the relation:

\[
\frac{\sum p}{P_0} = \frac{dA}{DR_0}
\]

Where \( P_0 \) is the momentum at the elastic scattering peak, \( dA \) is the area of the slits at the upper focal plane of the spectrometer, and \( D \) is the dispersion of the spectrometer. The value of the dispersion was measured by Oberdier [9] to be 3.92.

The measured scattering cross section can now be determined using the relation:
\[
\left[ \frac{d \mathcal{J}}{d \Omega} \right]_m = \frac{N_{sc}}{N_{in} \cdot N_t \cdot \Delta \Omega \cdot A_o}
\]

Where \( N_{in} \) is the number of incident electrons, \( N_t \) is the number of target nuclei per square cm., \( \Delta \Omega \) is the solid angle observed by the spectrometer, and \( A_o \) is a correction factor derived from the carbon data. The number of incident electrons is determined by integrating the beam current passing through the target at each counting point, then dividing this charge by the charge of one electron. The number of target nuclei per cm\(^2\), \( N_t \), is determined by dividing the target density by the mass of one atom then multiplying by the effective target thickness. The effective target thickness is determined by correcting the measured thickness for angular displacement of the target normal from the beam line. The solid angle is found by dividing the area of the opening in the spectrometer entrance plate by the square of the distance from the target to the entrance plate.

The correction factor \( A_o \) was used to compensate for deviations in the system, such as integration error and counting losses. It was determined by taking an elastic peak of carbon simultaneously with the lithium then reducing the carbon data and comparing it to the well known cross section for carbon. The form factor for carbon was computed from \( F(q) = (1 - .3062 q^2) \exp(-.7310 q^2) \), \[10\].

Using this measured cross section as a starting point further corrections are now made. The first considered is a correction for the
fact that electrons radiate when they are scattered or when they pass through nuclear material, that is, we must consider that the target is made of a material and is not just a isolated nuclei. This type of correction is referred to as a radiation correction and is caused by two main processes. The first is the effect brought about by the passage of an electron through the strong electric fields of the nucleus and the second is due to the large change in angle that occurs in the scattering process. The first type is called thick target Bremstrahlung and can be corrected by using a multiplicative correction factor defined by

\[ K_b = \exp \left( \int b \right). \]

This method is attributed to Bethe and Ashkin [11].

\[ \int b \] is given by Tsai [13] as:

\[ \int b = \frac{t}{X_o \ln (2)} \left[ \frac{1}{2} \ln \frac{E_o}{\theta^2 \triangle E} + \frac{1}{2} \ln \frac{E_o}{\theta^2 \triangle E} \right] \]

The factor \( t \) is the target thickness, \( X_o \) is the radiation length of the material and \( \theta \) is the nuclear recoil correction \( 1 + \frac{2E_o}{Mc^2} \sin^2 \theta/2 \). The energy change \( \triangle E \) is the difference between energy at the peak and the data cut off energy. This approximation is valid where \( \hbarq/c \) is less that the mass of the target nucleus. This approximation is true for this investigation.

The effect of the second type of radiation is corrected by a factor developed by Schwinger [12]. It has the same form as the previous correction where the \( \int s \), in a form developed by Tsai [13] valid in the same region is given by:
The factor \( \alpha \) is the fine structure constant \((1/137.04)\) and \( m \) is the electron mass.

With the radiative correction completed the experimental cross section should be that which the nucleus produces. As is noted in the introduction this investigation is concerned with determining the ratio of the charge form factors of \( \text{Li}^6 \) and \( \text{Li}^7 \). The experimental elastic cross section contains also a contribution from the magnetic interaction between the electron and the nucleus, this must be subtracted:

\[
\left[ \frac{d \sigma}{d \Omega} \right]_{\text{ch}} = \left[ \frac{d \sigma}{d \Omega} \right]_{\text{exp}} - \left[ \frac{d \sigma}{d \Omega} \right]_{\text{mag}}
\]

The cross section due to the magnetic interaction may be written, according to Pratt et al \([14]\), as:

\[
\left[ \frac{d \sigma}{d \Omega} \right]_{\text{mag}} = \left[ \frac{d \sigma}{d \Omega} \right]_{\text{mott}} \left[ 1 + \tan^2 \left( \frac{\hbar q}{2M_p c^2} \right) \right] \left[ \frac{J+1}{3J} \right] \mu_o^2 F^{2}_{\text{mag}}(q^2)
\]

The first term is the well know Mott cross section which is the effect produced by a point scattering center. \( M_p \) is the proton rest mass. \( \mu_o \) is the nuclear magnetic dipole moment which is given in units of nuclear magnitrons \((\mu_o \text{ Li}^6 = .82200 \text{ and } \mu_o \text{ Li}^7 = 3.25631)\). The magnetic form factor \( F^{2}_{\text{mag}} \) is calculated differently for the two isotopes.
The magnetic form factor for Li$^6$ may be written according to Griffy and Yu [15] as:

$$F_{\text{mag}}^2 = \left\{ \left[ \langle j_0 \rangle + R_1 \langle j_2 \rangle \right]^2 + R_2^2 \left[ \langle j_2 \rangle \right]^2 \right\} F_N^2(q^2)$$

The constant $R_1$ is taken to be .36 from the best fit data of Rand et al [16], who used 180 degree scattering techniques for their determination. $R_2$ is zero in the case of Li$^6$ because of the coupling scheme used.

The expectation values of the spherical Bessel functions $\langle j_0 \rangle$ and $\langle j_2 \rangle$ can be written, using the harmonic well shell model for the nucleus:

$$\langle j_0 \rangle = (1 - \frac{2}{3} x) \exp \left[ -x(1 - 1/A) \right]$$

$$\langle j_2 \rangle = \frac{2}{3} x \exp \left[ -x(1 - 1/A) \right]$$

The factor $x$ is $q^2 a_o^2/4$, where the $a_o$ is the nuclear radius parameter which is allowed to equal 2 fermi. The nuclear form factor $F_N^2$ is defined by:

$$F_N(q^2) = \frac{1}{1 + q^2 a_p^2 / 12}$$

The proton radius $a_p$ is taken to be .80 fermi. A form factor for scattering may be defined by:

$$\left[ \frac{d \sigma}{d \Omega} \right] = Z^2 \left[ \frac{d \sigma}{d \Omega} \right]_{\text{mott}} \quad F^2$$

Using this definition a correction to the form factor, due to the magnetic contribution, can be written (Suelzle [2]) as:

\[ \text{Correction} \]
\[ S_{\text{mag}}^2(q^2, \theta) = \frac{\left[ \frac{d J}{d \Omega} \right]_{\text{mag}}}{Z^2 \left[ \frac{d \sigma}{d \Omega} \right]_{\text{mott}}} \]

The correction for the magnetic form factor for Li\(^7\) is complicated by the existence of an excited level which occurs as 478 KEV above the ground state. This inelastic contribution was not clearly resolved in the scattering process. This may in part be due to the fact that this inelastic level is very broad and therefore difficult to separate. The effect must in this case be removed by mathematical means. It is shown by Rand et al. [16] that this calculation can be divided into two parts, the first of which can be taken into the magnetic form factor which is then written:

\[ F_{\text{mag}}^2 = \left[ A \left\langle j_\theta \right\rangle^2 + B \left\langle j_\varphi \right\rangle \left\langle j_2 \right\rangle + C \left\langle j_2 \right\rangle^2 \right] F_N^2(q^2) \]

The values of the constants, also taken from Rand et al., are \( a_o = 1.72, A = 1.27, B = 0.22, C = 1.21 \).

The longitudinal component of the transition has been calculated by Willey [17] using the odd proton model for the nucleus:

\[ \left[ \frac{d \sigma}{d \Omega} \right]_{C_2} = \left[ \frac{d \sigma}{d \Omega} \right]_{\text{mott}} \left\langle j_\theta \right\rangle^2 g^2 \left[ 1 - Cq^2 \right] \frac{F_N^2(q^2)}{F_N^2(q^2)} \]

The factor \( g \) is \( \exp(q^2/A) \), where \( A \) is the number of nucleons in the nucleus. \( C \) is computed by Willey to be:

\[ C = \left( \frac{n}{M} \right)^2 \left[ 1 - \frac{2}{8} \frac{\sigma}{g} \right] \approx -0.0253 \}

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A $C_2$ transition correction factor can now be constructed from the cross section just calculated:

$$S_{C_2}^2 = \left[ \frac{d \sigma}{d \Omega} \right] C_2'$$

This correction and the one previously found for the magnetic and transverse part of the $C_2$ transition are then subtracted from the experimental form factor for Li$^7$. The data is now corrected so that both Li$^6$ and Li$^7$ have been reduced to charge form factors. In order to make the ratio it is necessary to have the two isotopes at the same value of $q^2$ since it has been indicated that they are both functions of $q^2$. Since there is a slight difference in mass between the two there is a slight difference in $q^2$ for the same scattering conditions. It is therefore necessary to make a small correction. This correction can be closely approximated by taking the first term in the expansion since the $q^2$ difference is very small. A multiplicative correction would therefore be given by:

$$K_{aq} = (1 + 2 \Delta q^2)$$

This final correction produces a ratio of the charge form factor of Li$^7$ to the charge form factor of Li$^6$ which may then be compared with the calculated values.

An alternate method of arriving at this ratio is to take the ratio of radiation corrected experimental cross sections and multiplying by corrections for differences in momentum transfer squared, Mott cross
section, magnetic cross section and inelastic cross section.

\[
\frac{F^2_{\text{ch}}(\text{Li}^7)}{F^2_{\text{ch}}(\text{Li}^6)} = \exp \left[ \begin{array}{c} \text{Li}^7 \\ \text{Li}^6 \end{array} \right] \left( \text{K}_{\Delta M} \text{K}_{\Delta q} \text{K}_{\text{mag}} + C_2' \right)
\]

Where:

\[
\text{K}_{\Delta M} = \frac{\left[ \frac{d}{d \Omega} \right]_{\text{Mott Li}^6}}{\left[ \frac{d}{d \Omega} \right]_{\text{Mott Li}^7}}
\]

\[
\text{K}_{\text{mag}} + C_2' = 1 \left\{ \frac{1}{1 + \left( \text{S}_{\text{mag}}^2 + \text{S}_{C2'}^2 \right)} \right\}
\]

The terms \( \text{S}_{\text{mag}}^2 \) and \( \text{S}_{C2'}^2 \) are the same as previously presented.

This approximation uses the fact that the magnetic contribution of \( \text{Li}^6 \) is small compared to \( \text{Li}^7 \) so can be neglected. Using this method the errors in numerical calculations are smaller since fewer operations are necessary. Both methods were used in the reduction of data in this experiment; however, no difference in ratios was found.
5. Experimental Uncertainty

Taking the ratio of two experimentally determined quantities has the advantage of cancelling out much of the systematic error of the experiment. This is particularly true if the data can be taken under identical conditions. In this experiment the data for the two lithium targets were taken at the same time, that is both targets were measured at each setting of the spectrometer so that any drift or change in operation would have the same effect on both peaks. For most of the runs the carbon target was also run sequentially with the others, however, for a few of the earlier peaks the carbon was taken later at the same energy and angle, and these peaks are so marked in Figure II. The angle of scattering was determined to 1/4 degree and the energy to within .5% for those runs in which the nuclear magnetic resonance probe could be used to determine the central magnetic field of the deflection system.

The uncertainty in the corrections is also of little effect because of the ratio. Test calculations were made by varying the parameters, such as energy and scattering angle, to see how much effect such a variation would have on the final ratio. A change of incident energy of .7% introduced into the calculations made only a .1% change in the corrected ratio. A change of 1 degree in angle had no effect on the ratio. It was also observed that the corrections were very small change in ratio, usually less than 1% with a few points at 2 or 3 percent. From this information it seems that the uncertainties associated with the corrections are not a main factor in the total experimental error of these measurements.
The largest obvious uncertainty in the experiment is the counting error due to the effect of the statistical nature of the scattering processes. The uncertainty indications attached to the plotted results (Figure II) are directly due to the statistical errors, this is in keeping with standard practice in electron scattering reports.

Another counting problem is a probable source of error. The scalers used to record the number of the scattered electrons detected by the counting system were lower speed than would be desirable. The system also had not been standardized against a faster system so that the exact counting error was not determined. During runs the beam current was kept low enough so that the no more than five counts a second were registered. A correction of 1% was made to the count at each point for every count per second over one count per second. This seemed to give a smooth curve but no experimental confirmation of this correction has been made at this time.

The uncertainty of target parameters was also included in Figure II. This is included in view of the fact that the target thickness could not be determined more accurately than 1% and the density of the targets were uncertain by about 1%.
6. Results

The results of measurements of the ratio of the charge form factor for lithium seven to the charge form factor of lithium six are contained in Table I. This information is also plotted as a function of $q^2$ in Figure II. The dashed curve on the plot indicates the best fit of Suelzle's phenomenological expression for the charge form factor of lithium six. The value of the parameters of this expression,

$$F_{\text{ch6}} = \exp(-a q^2) - c q^2 \exp(-b q^2),$$

were found to be, $a^2 = 0.87$, $b^2 = 1.9$, $c^2 = 0.306$. These values of the parameters provide a root mean square radius of 2.66 Fermi for the lithium six radius. These results are compatible with the results of Suelzle for values of momentum transfer squared greater than $1 \text{F}^{-2}$, for values of momentum transfer squared less than $.5 \text{F}^{-2}$ the lithium six charge form factors are slightly smaller.

It is apparent from the data that there is considerable scattering of points around the best fit. This can not be exactly accounted for but is an indication of the lack of calibration of the various equipment. There is also an inherent difficulty in working with lithium targets since lithium is both reactive and soft so that exact determination and maintenance of a target thickness, in grams/cm$^2$, is very difficult.
<table>
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<th>$E_0$</th>
<th>$\Theta$</th>
<th>$q^2 (Li^7) / F^{-2}$</th>
<th>$\left[ \frac{d\sigma}{d\Omega} \text{Li}^7 / \frac{d\sigma}{d\Omega} \text{Li}^6 \right]_{\text{Exp}}$</th>
<th>$\left[ F^2_{\text{chLi}^7} / F^2_{\text{chLi}^6} \right]_{\text{Exp}}$</th>
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13. ABSTRACT

   The ratio of the charge form factor of lithium seven to the charge form factor of lithium six was measured by high energy electron scattering techniques. Using the well known form factor for lithium seven the form factor for lithium six was determined by finding a best fit to the measured ratios at low values of momentum transfer squared ($q^2$). From this manipulation a root mean square radius of 2.65 Fermi was determined for the lithium six nucleus.
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