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DYNAMICALLY DEVELOPED FORCE AT THE BOW
OF AN ICEBREAKER

RODERICK MACLEOD WHITE

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DYNAMICALLY DEVELOPED FORCE AT THE BOW
OF AN ICEBREAKER

by

RODERICK MACLEOD WHITE

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United States Coast Guard Academy, B.S.
(1950)

Massachusetts Institute of Technology, Nav. E.
(1956)

SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF SCIENCE

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
June, 1965

Signature of Author

Department of Naval Architecture and
Marine Engineering, May 14, 1965

Certified by

Thesis Supervisor

Accepted by

Chairman, Departmental Committee on Graduate Students

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Biographical Note

Federick M. White was graduated from the United States Coast Guard Academy in 1950. He was awarded a Bachelor of Science Degree and was commissioned in the United States Coast Guard.

During the period from 1950 to 1953 the author served on a weather patrol ship, first as a deck officer and then later gained qualification in engineering. During the first part of 1953 he attended Merchant Marine Safety School and was assigned to Merchant Marine inspection in Boston for a relatively brief period.

From 1953 to 1956 he studied Naval Architecture and Marine Engineering at Massachusetts Institute of Technology. An M.I.T. Fellowship was awarded to him and as a result he studied shipbuilding in Copenhagen, Denmark during the summer of 1955. In June, 1956 he was awarded full membership in the Society of the Sigma Xi and was graduated with the degree of Naval Engineer.

For a period following graduation from M.I.T. he worked in the Production Department of the United States Coast Guard Yard in Curtis Bay, Maryland. The first portion of the time was spent as "Ship Superintendent" for ship repair; the latter portion of the time he was in charge of construction of 95-foot patrol coasts.

From 1957 to 1959 the author served as Engineer Officer of the U.S. Coast Guard Icebreaker "Westwind". During that time he made four extended ice cruises (in the Antarctic as well as the Arctic). He also gained qualification as an officer-of-the-deck which enabled him to handle the ship in ice on many occasions.

In 1959 Lieutenant Commander White commenced a tour of duty as a member of the faculty of the Department of Applied Science and Engineering at the United States Coast Guard Academy. In 1962 he was appointed to the permanent commissioned teaching staff. He has taught Naval Architecture and most of the undergraduate courses in Applied Science.

In addition to membership in Sigma Xi he is a member of the Society of Naval Architects and Marine Engineers, and a member of the American Society for Engineering Education.

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DYNAMICALLY DEVELOPED FORCE AT THE BOW
OF AN ICEBREAKER

by

RODERICK MACLEOD WHITE

Submitted to the Department of Naval Architecture and Marine Engineering on 14 May 1965, in partial fulfillment of the requirements for the degree of Doctor of Science.

ABSTRACT

The object of this research has been to develop a suitable mathematical model (computer program) for the prediction of the dynamically developed force under the bow of an icebreaker during (or resulting from) encounter with virtually unyielding ice.

The selection of characteristics for polar icebreakers has been primarily based on experience. Some analytical work has been done on uninterrupted progress (steady icebreaking). Essentially there has been only one analysis of the force resulting from ramming, which represents the primary maximum capability of a polar icebreaker. The validity of that particular dynamic analysis is doubtful because of the use of undefined losses and an improper resolution of the impact (crushing phase). No other approach has been made, until now, to the dynamic aspects of icebreaking.

This solution is based primarily on Newton's Laws of motion. The problem was broken down into two basic phases. The crushing phase represents the local crushing of the ice to accommodate the bow. The sliding phase represents the sliding-up of the bow without further crushing. The final state represents (temporary) equilibrium when motion has stopped; the vertical force at the bow at this state is relatively sustained and is the most effective in breaking the ice.

The predictions of ship motions, as well as the forces, are produced by the computer program. These predictions have been compared with observed motions of a full scale polar icebreaker and have been found valid.

As a result of studying the effect on the downward force of the various characteristics of a polar icebreaker, the following selections and uses are recommended if greater downward force is to be attained:

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The predictions of ship motions, as well as the forces, are produced by the computer program. These predictions have been compared with observed motions of a full scale polar icebreaker and have been found valid.

As a result of studying the effect on the downward force of the various characteristics of a polar icebreaker, the following selections and uses are recommended if greater downward force is to be attained:



- Large displacement
- High impact velocity
- Small bow angle
- Small spread angle complement (blunt bow)
- Small block coefficient
- Large waterplane coefficient
- High beam-to-draft ratio
- Low kinetic friction

It is vital to realize that the selection of characteristics to improve downward force leads (in almost all cases) to a worsening of the thrust requirement for extraction.

In order to reduce the problem of extraction, without reducing the downward force, the following selections are recommended:

- Low static friction
- High backing bollard thrust
- Small spread angle complement

Model tests having dynamic similitude may be run using geometrically similar models with a Froude Number equal to that of the ship at impact. It is necessary that the model "ice" have a compressive failure stress equal to that of the ice divided by λ .

Thesis Supervisor: _____
Jacob P. DenHartog

Title: Professor of Naval Architecture and Mechanical Engineering

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Jacob P. DenHartog, Professor of Mechanical Engineering
and Naval Architecture (Dynamics)

Martin A. Abkowitz, Professor of Naval Architecture
(Ship Motions)

Clyde M. Adams, Jr., Professor of Metallurgy (Ice)

Justin E. Kerwin, Associate Professor of Naval
Architecture (General)

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Philip Mandel, Professor of Naval Architecture

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Special appreciation, for patience and encouragement, is due my wife.

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28	...	1968
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33	...	1973
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I INTRODUCTION

General

There exists today an increasing need for icebreakers, icebreakers which are well designed on the basis of good technology as well as experience. There is now an abundance of experience to rely on. However, relatively speaking, there is a shortage of analytical thought and understanding of the basic mechanics of icebreaking. More work along these lines is desired and needed.

Icebreakers can be defined as vessels which are specifically designed to break ice. Frequently they serve many other purposes but for the sake of definition it is best to keep in mind that the primary function is to be able to break ice. Furthermore, icebreakers can be categorized in many different ways. In the simplest sense let us divide them into two categories, polar and sub-polar. It is to be implied from this that the polar icebreakers are for the real heavy-duty work. Operation of this type of icebreaker eventually means encountering ice which cannot be penetrated by the icebreaker. It may be because of rafting, where sheets of ice build on one another due to the pressures of wind, water, and/or ice or it may be that the ice is simply too thick and/or too strong. There can quite easily be the case where the ice is a monolithic sheet extending from shore to shore in which case even moderate thicknesses may be sufficient to stop progress. Snow covering can make penetration similar fighting one's way through a room full of pillows. At any rate, any

CONCLUSIONS

1952

The first part of the report is devoted to a general discussion of the work done during the year. It is followed by a description of the experimental work carried out and the results obtained. The third part is devoted to a discussion of the theoretical aspects of the work and the conclusions drawn therefrom.

The work has been carried out under the supervision of the Director of the Institute.

The results obtained during the year are discussed in detail in the following sections. It is found that the theoretical results are in good agreement with the experimental results. The work has been carried out in accordance with the programme of work approved by the Council of the Institute. It is hoped that the results obtained will be of value to other workers in the field.

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polar icebreaker will eventually face the day when it will not be able to force its way through the ice - or has already faced that day!

Maximum Capability

Naturally it is very desirable to attain the greatest capability for a given investment. What is this greatest capability? It can be measured in many different ways depending upon the purpose for which a given ship is intended. Capability for a passenger ship would be measured by different standards from those of a tanker. Likewise the most important criterion for a successful polar icebreaker is different from most ships; it is primarily the ice it is able to break through. Other items naturally take on importance too such as the breadth of the channel formed and even the size of the broken pieces of ice left in its wake. Most important though is its ability to impart a relatively sustained force to the ice in the vertical direction.

For illustration let it be assumed that two polar icebreakers exist, icebreakers A and B. Assume that they each are about the same general size, have similar propulsion means, and represent equal investments. Each of them performs an identical mission. Each of them can open up a harbor in Greenland in late spring. Each of them succeeds in escorting supply ships to Arctic or Antarctic bases. Each of them is costing about the same to operate and each is earning its keep, so to speak. In other words, up to this point each is performing its mission. Then one day they are assigned to the task of opening a polar harbor in mid spring. The ice

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conditions are severe. Icebreaker A can break through even though it is necessary to ram the ice, back off and ram again, breaking away large sections each time. Even though the progress is not smooth and steady, there is progress and icebreaker A accomplishes its mission. On the other hand, icebreaker B approaches these same ice conditions and it is found that, even by ramming, icebreaker B cannot break off any ice and makes no progress. Icebreaker B has failed in this particular mission. It is not a question of partly succeeding in this mission; it is simply a question of success or failure.

One may say that icebreaker A was designed better since it obviously performed better. What made its design better? It was able, under ramming conditions, to develop a greater downward force under the bow. It may seem obvious but it must be stated that since the illustrated ice conditions, whatever they may have been, were identical, the difference in the ability was inherent in the ship.

If one were to design an icebreaker at this point he would naturally duplicate or improve the design of icebreaker A, thereby quite rightfully utilizing the experience gained. Along with this it would be desirable to understand why icebreaker A was better. To do this it would be necessary to understand the mechanics of what is happening. If one were to analyze the mechanics of the problem then it would be possible to predict the best performance for a given investment.

Note that the thickness of ice to be broken is not a necessary part of the answer. As any who have been engaged in icebreaking know, a given icebreaker may be able to break through ten feet of ice in one location where there is not complete coverage or where there may be some deterioration. The same icebreaker may not be able to break through five feet of ice the same day in a different location where there may be complete coverage and the ice may be land-fast. So it is quite misleading to indicate that a certain icebreaker can break through a certain number of feet of ice. To compare the ability of one icebreaker to another it is of much more value to state what magnitude of relatively sustained downward force can be generated at the bow as a result of ramming. Although other items are of importance also, ultimately the most important answer lies in that value.

Parameters

It is necessary to determine how the parameters involved effect this answer, the downward force resulting from ramming. The problem is complex and is a function of the form of the ship, the displacement, the thrust, the location of the center of gravity, physical properties of ice, and perhaps other variables. It would seem that the angle the stem makes relative to the ice and the angle of spread of the waterlines at the entrance would be important parameters of ship form. Some answers would appear obvious at first glance. Increasing thrust and displacement would increase this downward force, but to what degree do they effect it?

The first part of the paper is devoted to a general discussion of the
 various methods which have been employed in the study of the
 structure of the atom. It is shown that the methods of
 spectroscopy, diffraction, and the study of the
 scattering of particles, all lead to the same
 conclusion, namely, that the atom is not a
 simple sphere of positive charge with
 negative electrons embedded in it, but
 that it is a complex structure in which
 the electrons are arranged in shells
 around a central nucleus of positive
 charge. The nucleus is shown to be
 composed of protons and neutrons, and
 is shown to be the source of the
 positive charge of the atom. The
 paper concludes with a discussion of
 the various models which have been
 proposed for the structure of the atom,

References

It is necessary to include here the references involved in this
 paper, the references being: the papers of Rutherford,
 Bohr, and the papers of the present author, the papers of
 the present author on the subject of the structure of the atom,
 the papers of the present author on the subject of the structure of the
 nucleus, the papers of the present author on the subject of the
 structure of the atom, the papers of the present author on the
 subject of the structure of the atom, the papers of the present
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 the present author on the subject of the structure of the atom,

Ultimately other questions must be considered. Let it be assumed that the bow angle and other parameters are chosen such that a maximum downward force would exist. If the ice does not yield, except locally, the velocity will become zero, the ship will have reached its farthest point of sliding up on the ice. Is the static friction at this point so great that the ship cannot back off? This is obviously an important consideration and limitation to free choice of parameters.

Table I lists most of the icebreakers constructed. Although not all of them are to be considered polar icebreakers, it is interesting to note the relatively large variation in the selection of parameters. In recent years there has been a strong tendency to set the bow angle at or near 30° . The bow angle is the angle from the design waterline to the stem. This choice is based on one, and only one, good reason; it has worked. It is interesting and significant to note that there has never been an analytical attempt to justify this choice.

Mechanics of Icebreaking; Terms

Some discussion of terms to be used is in order. The methods of breaking ice with an icebreaker can be expressed fundamentally in two ways. The bow of an icebreaker is sloped so that there is a downward component of force produced on the ice. It is this vertical component which is effective in breaking the ice since the ice is significantly more vulnerable to a force applied in this direction, particularly when sustained. The horizontal component, even the horizontal wedging action is no where

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Section 10

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TABLE I CHARACTERISTICS OF VARIOUS ICE-BREAKERS

Name	Murtaga	Stadt Reval	Isbrytaren	Trouvor	Nadeshnij	Gaidamak	Sampo	Baikal	Apu	Ermack	Tarmo	Feodor Litke	Chief Wawatam
Home port	Helsingfors	Reval	Gothenburg	Kronstadt	Vladivostok	Blzerte	Helsingfors	Lake Baikal	Helsingfors	Lenin-grad	Helsingfors	Archangel	Muskegon Mich.
Year built	1890	1895	1895	1896	1897	1898	1898	1899	1899	1899	1907	1909	1911
Where built	Stockholm	Stettin	Gothenburg	Copenhagen	Copenhagen	Kiel	Newcastle	Newcastle	Kiel	Newcastle	Newcastle	Barrow-in-Furness	Toledo, O.
CHARACTERISTICS													
Gross tonnage, registered	676	576	568	999	1212	724	1339	...	568	4955	1574	2375	2990
Net tonnage, registered	...	43	290	176	258	99	91	...	326	2357	173	930	1793
Length, overall	156'-0"	148'-8 1/2"	139'-9 1/2"	166'-0"	192'-0"	198'-7"	202'-0"	290'-0"	144'-0"	320'-0"	220'-1"	265'-0"	338'-10"
Waterline length	137'-5 1/2"	147'-3 3/4"	134'-0"	161'-5"	180'-0"	163'-0"	191'-6"	...	141'-9 1/4"	310'-7"	210'-5"	250'-5"	...
Maximum beam	36'-0 0/3"	38'-11"	35'-6"	40'-3"	42'-11"	42'-0"	43'-0"	...	35'-6"	71'-6"	47'-0 1/2"	47'-11"	...
Waterline beam	35'-4"	37'-1"	39'-0"	42'-0"	41'-6"	42'-0"	42'-0"	57'-0"	34'-0"	70'-1 1/2"	42'-6"	47'-6"	62'-0"
Depth, molded to weather deck	24'-11"	18'-7 1/2"	...	22'-3"	24'-5"	18'-0"	29'-5"
Normal draft, d	15'-7"	14'-0"	18'-10"	18'-9"	17'-9"	12'-2"	18'-3"	...	18'-0"	24'-0"	18'-2"	16'-9"	...
Maximum draft	18'-1"	21'-0"	16'-0"	28'-0"	...	22'-6"	...
Normal displacement, tons	825	865	720	1450	1700	1030	1850	4200	800	7875	2300	2570	...
Maximum displacement, tons	900	1600	1300	2000	2170	1065	900	10,000	...	4600	...
Stem angle to waterline	33°	30°	31°	25°	30°	30°	2.2°	...	20°	22°	20°	52°/70°	...
Angle of normal to bow plating and E plane	11°
Flare amidships at waterline	10°	20°	...	3 1/2°	...
Complement	28	...	21	38	6	39	43	112	43
Speed, knots	12.5	11.5	12/12.6	9.5/13	10/14.4	11.5/13	10/13	...	23	14	...	13	35
HULL COEFFICIENTS													
Block	0.382	0.396	0.410	0.430	0.443	0.438	0.441	...	0.323	0.527	0.454	0.452	...
Midship
Waterplane
Longitudinal
L.O.A./B, maximum	4.33	3.82	3.94	4.12	4.48	4.73	4.70	5.09	4.06	4.48	4.68	5.53	...
L.W.L./B, waterline	3.89	3.97	3.94	4.14	4.29	3.93	4.56	...	4.17	4.43	4.53	5.26	...
d/B, waterline	0.441	0.378	0.554	0.481	0.422	0.296	0.435	...	0.529	0.342	0.391	0.353	...
$\Delta / (\frac{L}{100})^3$	317.4	270.7	299.2	344.9	291.5	237.8	263.4	...	280.8	262.9	246.9	164.5	...
PROPELLING MACHINERY													
No. shafts	1	1	1	1	1	1	1	2	1	1	1	1	3
No. forward	0	0	0	0	0	0	1	3	0	4	2	2	2
No. aft	1	1	1	1	1	1	1	1	1	1	1	0	1
FORWARD ENGINES													
Horsepower, a normal	1200	2500	1300
Horsepower, a forced	1417	3000	1660
R.P.M.	90/97	89/102
AFTER ENGINES													
Horsepower, a normal	1200	1600	1200	2000	2475	1300	1300	...	1350	7500	2200	7000	...
Horsepower, a forced	1793	...	1449	2600	3530	1940	1635	...	1600	9000	2190	7580	...
R.P.M., normal/maximum	83	...	102	110	95/106	83/94	80/86	...	84/90	105	89/102	125	...
Total horsepower, normal	1200	1600	1200	2000	2475	1300	2500	...	1350	10,000	3500	7000	...
Total horsepower, forced	1793	...	1449	2600	3530	1940	3052	...	1600	12,000	3850	7580	4500
Cruising radius	2300	...	1100/1750	1100/2200	1560/2040	...	1404	10,650	1250	1665/2775	...
Ballast pumps	1	1	1	1	1	1	1	1	...	1	...
Total capacity, tons per hr.	1150	1000	...	1000	1000	1000	700	600	...	250	...
Capacity of trimming tanks	400	4000	502
Thickness of ice belt plating, in.	1 1/4"
Frame spacing	12"
LCF aft of FP
BM _L

a - Indicated horsepower unless otherwise stated

TABLE I(cont) CHARACTERISTICS OF VARIOUS ICE-BREAKERS

Name	Jaakarhu	N. B. McLean	Store Bjorn	Ymer	Gota Lejon	J. Stalin	Sisu	Raritan	Cactus	Storis	North-wind	Mackinaw	Canadian Car Ferry
Home port	Helsingfors	Ottawa, Canada	Copenhagen	Stockholm	Gothenburg	Leningrad	Helsingfors	Philadelphia, Pa.	Boston, Mass.	Boston, Mass.	Boston, Mass.	Cheboygan, Mich.	Charlottetown, P. E. I.
Year built	1926	1929	1931	1932	1933	1937	1938	1939	1941	1941	1944	1945	1945
Where built	Rotterdam	Halifax	Aalborg	Malmö	Gothenburg		Helsingfors	Bay City, Mich.	Duluth, Minn.	Toledo, O.	San Pedro, Cal.	Toledo, Cal.	Sorel, Que.
CHARACTERISTICS													
Gross tonnage, registered	2622	3254	1393	3053	1355	...	1610	7000
Net tonnage, registered	...	1171	509	535	151	4500
Length, overall	263'-0"	277'-0"	196'-10"	257'-10 5/8"	183'-8 3/4"	350'-0"	210'-6"	110'-0"	180'-0"	230'-0"	269'-0"	290'-0"	372'-6"
Waterline length	246'-0"	260'-0"	180'-5 1/4"	246'-0 1/4"	173'-10 3/4"	335'-0"	194'-9"	105'-0"	170'-0"	220'-0"	250'-0"	280'-0"	372'-6"
Maximum beam	63'-1"	60'-4"	49'-4"	63'-4"	49'-5"	76'-0"	47'-6"	26'-5"	37'-0"	43'-0"	63'-6"	74'-4"	348'-0"
Waterline beam	60'-2 1/2"	...	46'-10 3/4"	61'-0 1/2"	46'-11"	74'-6"	46'-6"	25'-0"	35'-0"	41'-0"	62'-0"	70'-0"	61'-0"
Depth, molded to weather deck	31'-10"	31'-0"	23'-11 1/2"	33'-0"	28'-2 5/8"	30'-0"	18'-6"	14'-11 5/8"	17'-4 3/4"	19'-3 1/2"	37'-9 1/2"	28'-0 3/4"	24'-9"
Normal draft, d	21'-10"	19'-6"	18'-2 1/2"	21'-0"	20'-0"	26'-3"	16'-0 1/2"	10'-6"	12'-0"	14'-0"	25'-9"	19'-0"	19'-0"
Maximum draft	10'-0 1/4"	29'-8"	...	11'-0"	12'-8"	14'-10"	29'-1"
Normal displacement, tons	4850	5034	2500	3465	1900	9300	2000	328.6	940	1760	5300	5140	...
Maximum displacement, tons	4900	4350	...	11,000	...	354	1025	1864	6515
Stem angle to waterline	25°-30°	...	24°	25°	23°	30°	24°	30°	30°	30°	30°	30°	...
Angle of normal to bow plating and L plane	47.8
Flare amidships at waterline	20°	...	11°	18°	20°	20°	20°	20°	20°	20°	...
Complement	47	...	45	42	30	142	100	16	47	103	145	132	...
Speed, knots	13.5	...	13.5	14/15.87	...	15.5	16	12.3	13	14	16	16	...
HULL COEFFICIENTS													
Block	0.525	...	0.568	0.384	0.408	0.498	0.482	0.417	0.457	0.478	0.465	0.497	...
Midship	0.789	...	0.848	0.801	0.728	0.817	0.835	0.752	0.812	...
Waterplane	0.741	...	0.734	0.725	0.738	0.728	0.761	0.724	0.728	...
Longitudinal	0.645	...	0.628	0.593	0.578	0.558	0.572	0.618	0.612	...
L.O.A./B, maximum	4.17	4.59	3.99	4.07	3.75	4.61	4.43	4.16	4.87	5.35	4.24	3.90	6.11
L.W.L./B, waterline	4.09	...	3.85	4.03	3.71	4.50	4.18	4.20	4.86	5.37	4.03	4.00	...
d/B, waterline	0.363	...	0.388	0.344	0.426	0.352	0.345	0.420	0.343	0.341	0.415	0.271	...
$\Delta / (\frac{L}{100})^3$	325.8	286.4	425.6	232.6	361.3	247.4	270.8	283.9	191.3	165.3	339.2	234.1	...
PROPELLING MACHINERY													
No. shafts	3	2	3	3	2	3	3	1	1	1	3	3	4
No. forward	1	0	1	1	1	...	1	0	0	1	1	1	2
No. aft	2	2	2	2	1	...	2	1	1	1	2	2	2
FORWARD ENGINES													
Horsepower, ^a normal	2530	...	1800	3000 shp	1300	...	1335 shp	3333/0	3333/0	5600/2400
Horsepower, ^a forced	3000	3300 shp	1500
R.P.M.	130	...	125	155	140	...	160	140-210	175-200	...
AFTER ENGINES													
Horsepower, ^a normal	5310	6500	3600	6000 shp	2500	10,000	2670 shp	1000 shp	1000 shp	1800 shp	6666/10,000	6666/10,000	5600/7200
Horsepower, ^a forced	6000	6600 shp	2500
R.P.M., normal/maximum	117	105	100	140	120	125	140	236
Total horsepower, normal	7840	6500	5400	9000 shp	3800	10,000	4005 shp	1000 shp	170 ^b -208 ^c	160 ^b -195 ^c	105 ^b -145 ^c	136 ^b -170 ^c	...
Total horsepower, forced	9000	9900 shp	4000	1000 shp	1800 shp	10,000 shp	10,000/10,000	11,200/9600
Cruising radius	2950	1800
Ballast pumps													
Total capacity, tons per hr.	1900	900	...	1	1	4	...	1	1	1	full power ^{3d}	full power ^{4d}	...
Capacity of trimming tanks	750	200	1800	1755	...	57	17	17	934 ^d	12,459 ^d	...
Thickness of ice belt plating, in.	330	283	1791	150	20	22	72	717	1541	...
Frame spacing	1"	...	1"	1"	3/4"	3/4"	7/8"	1 5/8"	1 5/8"	...
	24"	...	20"	15 3/4"	20"	21"	21"	16"	16"	15"

^a Indicated horsepower unless otherwise stated

^b Ice-breaking or towing

^c Running free.

^d Heeling pumps

TABLE I (cont'd) CHARACTERISTICS OF VARIOUS ICE-BREAKERS

Name	Petr Velikiy (Sunk in 1915)	Sainte Marie Muskegon, Mich.	Suur Tool Reval	Isbry taren II Stockholm	Pollux French (mine layer)	Mikula Selianinovitch Cherbourg	Leonid Krasin Kron- stadt	Stepan Makarow Arch- angel	Lenin Kron- stadt	Vioma Helsing- fors	Isbjorn Copen- hagen	Krisjanis Valdemars Riga	Atle Stock- holm
Year built	1912	1913	1914	1914	1915	1916	1917	1917	1917	1917-1924	1923	1925	1926
Where built	Gothenburg	Toledo, O.	Stettin	Stock- holm	Newcastle	Montreal	New- castle	New- castle	New- castle	Helsing- fors	Copen- hagen	Dalmuir	Gothen- burg
CHARACTERISTICS													
Gross tonnage, registered	1267	2383	2417	1651	1613	3165	5105	2372	3828	1510	978	1932	1777
Net tonnage, registered	427	1620	822	329	653	2042	2246	885	1310	...	358	757	257
Length, overall	182'-1"	267'-4"	247'-4 1/2"	200'-0"	210'-0"	292'-0"	323'-3"	248'-0"	281'-0"	210'-7"	170'-7"	196'-6"	204'-7"
Waterline length	170'-7"	252'-0"	236'-4 1/4"	190'-7"	198'-1 1/2"	275'-0"	297'-0"	236'-0"	273'-0"	200'-10"	156'-2"	185'-0"	194'-7"
Maximum beam	51'-0"	62'-0"	57'-1"	55'-10"	50'-6"	57'-5"	71'-0"	57'-0"	71'-0"	46'-7"	40'-2 1/2"	55'-10"	55'-9"
Waterline beam	48'-6 5/8"	54'-8"	56'-1 1/4"	52'-6"	70'-6 1/2"	...	41'-4 1/2"	45'-10 7/8"	39'-4 1/2"	54'-0"	53'-2"
Depth, molded to weather deck	27'-5"	25'-0"	22'-10"	28'-8 1/2"	27'-6"	32'-0"	41'-4 1/2"	30'-4"	31'-11"	...	21'-0"	28'-6"	28'-9"
Normal draft, d	19'-1"	14'-0"	18'-7 5/8"	20'-8 5/8"	19'-10"	19'-3"	26'-0 1/4"	22'-0"	19'-0"	16'-9"	18'-6"	22'-0"	19'-8"
Maximum draft	21'-4"	...	18'-9"	21'-6"	30'-0 1/2"	...	20'-6"	...	20'-6"	...	20'-8"
Normal displacement, tons	1610	2567	3562	2350	3100	...	8730	4570	5074	2070	1330	2800	2464
Maximum displacement, tons	1953	...	3622	...	4830	...	10,620	4600	5620	2180	1670	...	2740
Stem angle to waterline	30°	...	18°	20°	25°	18°	18°	23°	20°	33°	24°
Angle of normal to bow plating and B plane	24°	20°	15°-20°	15°-18°	15°	...	17°	16°
Flare amidships at waterline	20°	41	100	90	190	...	118	44	30	43	45
Complement	60	27	65	13.4	14	15.5	15	15	16	14	12.5	15.2	15.5
Speed, knots	14.4	...	12/14	13.4	14	15.5	15	15	16	14	12.5	15.2	15.5
HULL COEFFICIENTS													
Bloch	0.357	0.479	0.505	0.397	0.562	0.469	0.409	0.446	0.424
Midship	0.708	0.755	0.733
Waterplane	0.723	0.662
Longitudinal	0.533	0.530	0.571
L.O.A./B, maximum	3.57	4.31	4.33	3.58	4.16	5.09	4.55	4.35	4.40	4.52	4.37	3.52	3.66
L.W.L./B, waterline	3.51	4.61	4.21	3.63	4.21	4.37	3.97	3.43	3.66
d/B, waterline	0.393	0.256	0.332	0.395	0.369	0.365	0.470	0.407	0.370
$\Delta / (\frac{L}{100})^3$	324.4	152.8	269.8	339.5	333.2	255.5	350.6	442.2	334.4
PROPELLING MACHINERY													
No. shafts	2	2	3	2	2	2	3	3	3	2	1	2	2
No. forward	1	1	1	1	0	0	0	1	1	2	1	2	2
No. aft	1	1	2	1	2	2	3	2	2	1	1	1	1
FORWARD ENGINES													
Horsepower, ^a normal	1000	...	1500	800	1550	2500	1100	...	1500	1400
Horsepower, forced	1255	...	2300	1200	1900	2600	1280	...	1500	2000
R.P.M.	220/286	...	120	220	115/140	...	146	...	263	85/105
AFTER ENGINES													
Horsepower, ^a normal	2200	...	3000	1800	4000	8000	10,000	4000	5000	2600	2500	3700	2600
Horsepower, forced	2660	...	4600	2500	4300	4700	5400	2870	3000	4000	4000
R.P.M., normal/maximum	130/143	...	100	130	105	100	...	90/105	...	111	90	122	100/125
Total horsepower, normal	3200	2500	4500	2600	4000	8000	10,000	5550	7500	3700	2500	5200	4000
Total horsepower, forced	3915	...	6900	3700	4300	6600	8000	4150	3000	5500	6000
Cruising radius	1600	...	2920	1600	4250/8700	1600/4900	2150/3800	1625	2000
Ballast pumps	2	2 ballast 1 salvage	5	...	1	1	...	2	1
Total capacity, tons per hr.	2400	925	1000	...	700	500	...	1800	3600
Capacity of trimming tanks	700
Thickness of ice belt plating, inc.	...	11/16"	1 1/4"	...	1 1/4"	1"	...
Frame spacing	...	15"	12"	...	12"	12"	...

TABLE I (cont'd) CHARACTERISTIC DATA OF VARIOUS ICE-BREAKERS

Name	Diberville Canada	Capitan. Class Canada	Glacier USSR	Ierin USSR	Macdonald Canada	McKenzie USSR
Home port	Canada	USSR	USSR	USSR	Canada	USSR
Year built	1952	1954	1955	1955	1960	1960-62
Where built	Lanzon	Helsinki	Pasca- goda		Lanzon	Helsinki
CHARACTERISTIC DATA						
Gross tonnage, registered						
Net tonnage, registered						
Length, overall	310'	275'	300'-"	440'	310'	390'
Waterline length	300'	270'	300'	420'	307'-3"	383'-4"
Maximum beam	60'-6"	55'-"	64'	90'-"	70'	80'-"
Waterline beam	50'	65'	72'-"	90'	69'	79'
Depth, molded	40'-3"	45'-2"	40'		41'	46'
Normal draft						
Maximum draft	30'-3"	35'	30'	30'-3"	29'	34'-"
Normal displacement, tons						
Maximum displacement, tons	10,030	10,000	10,000	18,000	10,000	10,000
Stem angle to waterline	30°	25°	30°	20°	30°	26°
Angle of normal to bow plating and E plane, deg.			0.1°			44.0°
Flare amidships at waterline	0°	0°	0°		0°	0°
Complement	1	1	1	200	77	
Speed, knots	14.5		14.0	14.0	12.0	
HULL COEFFICIENTS						
Block	0.79		0.73	0.49		0.72
Midship	0.84					0.85
Waterplane			0.8			0.8
Longitudinal						
L.O.A./B, maximum	5.6	5.0	4.7	4.9	4.4	4.8
L.W.L./B, waterline						
d/B, waterline						
$\Delta \left(\frac{\tau}{100}\right)^3$						
PROPELLING MACHINERY						
	Steam Recip.	Diesel Elect.	Diesel Elect.	Nuclear Turbo-elect.	Diesel Elect.	Diesel Elect.
No. shafts						
No. forward	-	2	-	-	-	-
No. aft	2	2	2	1	2	3
FORWARD ENGINES						
Horsepower ^a , normal						
Horsepower ^a , forced						
R.P.M.						
AFTER ENGINES						
Horsepower ^a , normal	10,800		21,000	19,000		
Horsepower ^a , forced						
R.P.M., normal/maximum	145/	120/	170/	1500/2000	136/170	
Total horsepower, normal	10,800	21,000	21,000	19,000	15,000	
Total horsepower, forced						
Cruising radius	12,000			3000	10,000	10,000
Ballast pumps						
Total capacity, tons/hr.						
Capacity of trimming tanks						
Thickness of ice belt plating, in.						
Frame spacing						
LCF aft of FP			14.7'	22 feet		
M_L			27'	14 feet		

This table is based on references (1), (2), (3), and (4).

near as effective since the structural shape of the ice is such that it can withstand tremendous forces in this direction. As an illustration of this, if one desired to break a pane of glass he would apply a force normal to the plane (causing a bending moment) rather than apply a force against the edge in the plane of the glass.

In the simpler condition of uninterrupted progress, steady state, the icebreaker moves along maintaining a relatively constant velocity. Except for minor variations it can be considered that there are no accelerations involved. The other fundamental method is ramming. This is where the icebreaker backs away from any contact with solid ice, proceeds forward so that there is a forward velocity at the time of initial contact and strikes the ice with its sloped bow. The bow rides up on the ice and a force is generated acting against the ice. Some of this force is the result of the thrust being applied by the propulsion; the rest of it is the result of converting the kinetic energy before impact to potential energy. The two methods then are uninterrupted progress, where any acceleration is negligible, and ramming, where the acceleration (negative) is extremely important. All actual icebreaking is done by one of these two methods or by something in between these two extremes. It should be apparent that ramming will lead to the greater force development since uninterrupted progress is, in a sense, a minimum ramming situation where the accelerations have reduced to zero.

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Methods

Many methods of icebreaking have been utilized with varying degrees of success. The earliest account of deliberate icebreaking was contained in remarks made by a British Rear Admiral in 1865. (5)^{*} "I have had two years experience in ramming the ice. Our vessels had long oblique overhanging stems to lift the bow over the ice. We struck the floe ice of about six feet in thickness, end-on; a man at the bowsprit end dropped down on the ice and placed a boarding pike as a mark where the blow was given. The vessel backed astern, and then ran directly for the mark which had been placed on the ice; the man who was standing by the crack thus made picked up his boarding pike and placed it on the edge of the crack, so that the vessel might be steered directly for it again, and the third time the ice opened and the steam tender towed the ship through; such was the constant practice. ----- We ran the vessel's nose dead onto the ice and did the ice more injury than the vessel, for the vessel never was injured during several years of such service." It is interesting to note that this very first account was of ramming. Perhaps the object of this present research could be stated more succinctly in the words Belcher used, "Do the ice more injury than the vessel"!

✓ Prior to breaking ice using iron-clads another method had been used on occasion. (6) Ships would become beset in harbor ice and it became necessary to free them. Men would be recruited from the city in gangs of fifty to two hundred and they would be equipped with pikes and saws. With

* Number indicates literature citation of Appendix D.

THEOREM

Let \mathcal{A} be a finite-dimensional algebra over a field F . Let \mathcal{B} be a subalgebra of \mathcal{A} . Let \mathcal{C} be the centralizer of \mathcal{B} in \mathcal{A} . Then $\mathcal{B}\mathcal{C} = \mathcal{C}\mathcal{B}$ and $\mathcal{B}\mathcal{C}$ is a subalgebra of \mathcal{A} . Moreover, \mathcal{B} and \mathcal{C} are mutual centralizers of each other in \mathcal{A} .

Proof. Let $b \in \mathcal{B}$ and $c \in \mathcal{C}$. Then $bc = cb$ because c commutes with every element of \mathcal{B} . Thus $\mathcal{B}\mathcal{C} = \mathcal{C}\mathcal{B}$. To show that $\mathcal{B}\mathcal{C}$ is a subalgebra, let $x, y \in \mathcal{B}\mathcal{C}$. Then $x = \sum b_i c_i$ and $y = \sum b_j c_j$ for some $b_i, c_i, b_j, c_j \in \mathcal{A}$. Then $x + y = \sum (b_i + b_j) c_i$ and $xy = \sum \sum b_i c_i b_j c_j = \sum \sum b_i b_j c_i c_j = \sum (b_i b_j) c_i c_j$. Since $b_i b_j \in \mathcal{B}$ and $c_i c_j \in \mathcal{C}$, we have $xy \in \mathcal{B}\mathcal{C}$. Thus $\mathcal{B}\mathcal{C}$ is a subalgebra.

To show that \mathcal{B} and \mathcal{C} are mutual centralizers, let $b \in \mathcal{B}$ and $c \in \mathcal{C}$. Then $cb = bc = cb$, so c commutes with b . Thus \mathcal{C} is the centralizer of \mathcal{B} . Conversely, let $c \in \mathcal{C}$ and $b \in \mathcal{B}$. Then $bc = cb$, so b commutes with c . Thus \mathcal{B} is the centralizer of \mathcal{C} . Therefore, \mathcal{B} and \mathcal{C} are mutual centralizers of each other in \mathcal{A} .

Corollary. Let \mathcal{A} be a finite-dimensional algebra over a field F . Let \mathcal{B} be a subalgebra of \mathcal{A} . Then \mathcal{B} is a simple algebra if and only if \mathcal{B} is a simple algebra and \mathcal{B} is a simple algebra.

* See the definition of simple algebra in the next section.

these tools they would clear a path from the ship out to open water.

Even today one of the more obvious methods of getting from one side of an ice field to the other is used. Go around, or at least minimize the contact with ice by following leads in the ice field. Even the largest of icebreakers is operated with that discretion.

Another method has been used with some success, particularly on icebergs. Sections of the ice have been painted black using soot or some form of paint. These colored sections will absorb the heat very quickly and melting takes place relatively rapidly in these areas.

Although quite expensive, it has been found that firing torpedoes under the ice will either break it completely or at least make it relatively easy for an icebreaker to penetrate. (7).

Most polar icebreakers carry explosives to use on the ice. Their effect on hummocks or very solid ice is actually quite limited. However, when an icebreaker becomes stuck after ramming the ice an explosive charge may have the very beneficial effect of jarring the ship and the ice enough to allow the ship to break free from the grasp of static friction and back off.

The Russians have utilized streams of water at the bow at great pressure to break and destroy the ice. They have found this more suitable than torpedoes. (8)

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One of the most unusual and interesting approaches to the problem has been tried by the Russians and was published in an official magazine of the U.S.S.R.. (7) "An underwater explosion accompanied by an exceptionally bright beam of light acts particularly strongly on ice. Very strong light, arising through water into ice, produces in it many tiny cracks. This was observed by the well-known English Physicist Tyndall more than a hundred years ago. The little cracks lower the solidity of the ice to such an extent that, passing through the ice after the light, the shock wave of the explosion relatively easily magnifies the cracks and destroys the ice."

Although seemingly irrelevant, other less scientific approaches have been tried. In the winter of early 1959 an icebreaker was attempting to escort a small ice-protected tanker into a harbor in Newfoundland. The ice was quite solid and reached from shore to shore across the bay. The icebreaker was repeatedly rammed into and onto the ice. Heeling tanks, trimming tanks, and explosives were used. After one full day of frustration the progress could easily be measured in inches. The Commanding Officer decided to stop and relax for the night. The tanker was brought up astern and all hands joined in one massive bingo game, which lasted for most of the night. When light appeared the following morning it was quickly noticed that during the night a lead had opened up all the way from the ship to the dock. The icebreaker and the tanker continued their trip without further opposition. However, it seems difficult to justify bingo

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as a scientific method.

Certainly the most commonly used method for polar icebreaking is to use a well designed icebreaker with the bow sloped such that it is possible to generate a significant downward force under the bow, particularly as a result of ramming.

History of Icebreakers

Although 1851 is given as the first year a vessel was built specifically for breaking through ice, very little was ever published concerning it. Generally it is regarded that the first successful icebreaker built for that express purpose was constructed in 1871 and was named appropriately "Eisbrecher I". (9)

It was constructed for the purpose of keeping the channel open from Cuxhaven to Hamburg throughout the winter season. This icebreaker was 130 feet long and had an engine of 300 ihp. The concept of design and operation was then much the same as it is now. There was a sloping stem in order to get a downward component. It was intended that the vessel progress as constantly as possible, but when pack ice was encountered the icebreaker was to be backed down and then ram against the ice at full speed. It had a rather full bow with a sloping stem. This was frequently copied in the years to follow and later it was modified to a spoon-shaped bow. A disadvantage with the full spoon-shaped bow was quickly discovered; if there was snow on the ice it would pile up ahead and impede or stop progress. Small entrance angles and small stem

On the basis of the information received from the source, it is believed that the source is reliable and that the information is accurate. The source has provided information regarding the activities of the group and the individuals involved. It is recommended that the information be used for intelligence purposes only.

Summary of Information

The information received from the source indicates that the group is active in the area of [redacted]. The source has provided information regarding the activities of the group and the individuals involved. It is recommended that the information be used for intelligence purposes only.

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The information received from the source is reliable and that the information is accurate. The source has provided information regarding the activities of the group and the individuals involved. It is recommended that the information be used for intelligence purposes only. The source has provided information regarding the activities of the group and the individuals involved. It is recommended that the information be used for intelligence purposes only. The source has provided information regarding the activities of the group and the individuals involved. It is recommended that the information be used for intelligence purposes only.

angles were advocated but not tried as many felt that this would be poor from a structural point of view. Propellers at the bow were also advocated at that time and were actually in use before the turn of the century. It is to be noted then that very large steps in thought and application were taken in the last thirty years of the last century.

Mathematical Expression of Icebreaking

In order to see what has been done concerning the prediction of the downward force under the bow, one need only to look back to the same period of time mentioned above. From then until the present only four men have left a deep impression by the development of a mathematical expressions for the mechanics of icebreaking. As will be seen, the first three did not develop an expression suitable for ramming; they developed equations for uninterrupted progress.

R. Runeberg was the first to analyze the mechanics of the icebreaking process. (10) Particularly considering that he was unable to base any of his work on previous developments, he did a remarkable amount. Some of this takes in the concept of ramming but unfortunately no useable equation for the downward force during ramming was developed.

Even in 1888 he recognized that "the vertical component should be as large as possible" since this does the breaking. His equation for the downward force is redeveloped completely in Appendix A.

His equation states that the downward force under the bow, F_{BZ} , for uninterrupted progress is a function of the following:

... it is a condition of the contract that the contractor shall not assign or subcontract the work under the contract to any other person without the prior written consent of the Engineer. It is further provided that the contractor shall be responsible for the performance of the contract by his subcontractors or assignees.

Assignment of Contract

It is hereby agreed that the contractor shall not assign or subcontract the work under the contract to any other person without the prior written consent of the Engineer.

... It is further provided that the contractor shall be responsible for the performance of the contract by his subcontractors or assignees. It is further provided that the contractor shall be responsible for the performance of the contract by his subcontractors or assignees.

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T_{IB} = Thrust available for icebreaking, LB.

i_B = Stem angle with reference to base plane, deg.

β = Angle with respect to the \bar{E} plane of a normal to the shell at the bow, deg.

f_k = Coefficient of kinetic friction between ice and hull.

The downward force is expressed as follows:

$$F_{BZ} = \frac{T_{IB} (\cos i_B \cos \beta - f_k \sin i_B)}{(\sin i_B \cos \beta + f_k \cos i_B)} \quad (A12)^*$$

Runeberg suggests the use of 0.05 for f_k .

The following assumptions were used for the development of this equation:

1. There are no momentum effects.
2. The forward motion through the water is effectively non-existent so that the thrust can therefore all be applied to icebreaking.**
3. Thrust was directed horizontally at all times.**
4. The direction of friction force (along the direction defined by the slope of the stem) remained the same during forward horizontal progress.**
5. Trim, although it exists, is not great enough to affect the solution.**

* Numbers in parentheses refer to equations of the appendix.

** These assumptions were used but not stated.

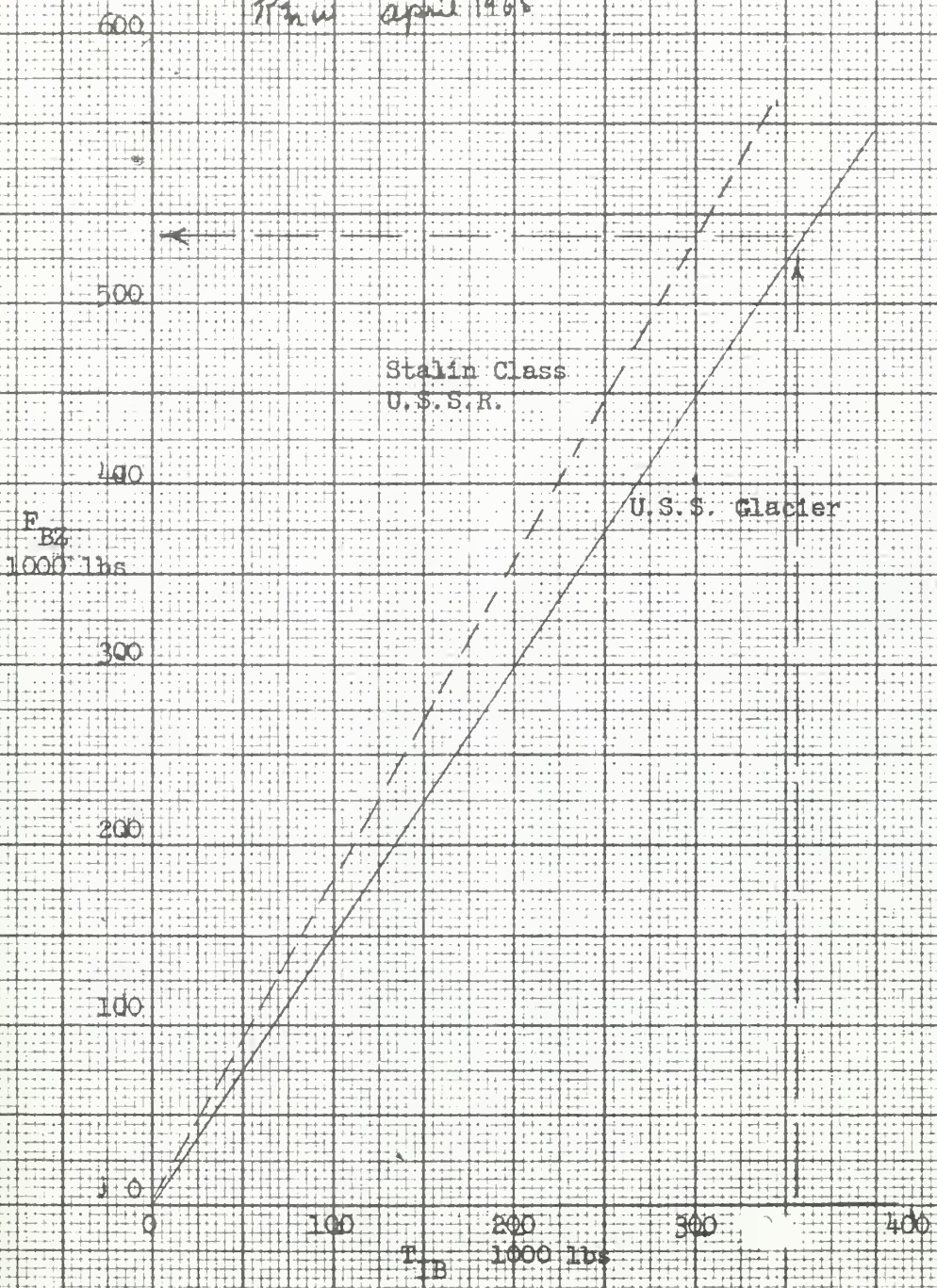
The first part of the problem is to find the value of x such that $x^2 + 1 = 2x$.
 This is a quadratic equation, which can be rearranged to $x^2 - 2x + 1 = 0$.
 This equation can be factored as $(x - 1)^2 = 0$, which implies $x = 1$.
 The second part of the problem is to find the value of y such that $y^2 + 4 = 4y$.
 This is also a quadratic equation, which can be rearranged to $y^2 - 4y + 4 = 0$.
 This equation can be factored as $(y - 2)^2 = 0$, which implies $y = 2$.

Thus, $\frac{(x^2 + 1) + (y^2 + 4)}{x + y} = \frac{2 + 8}{1 + 2} = \frac{10}{3}$.

- The following are the solutions to the given problems:
1. The value of x is 1.
 2. The value of y is 2.
 3. The value of the expression is $\frac{10}{3}$.
 4. The value of the expression is 10.
 5. The value of the expression is 20.
 6. The value of the expression is 30.
 7. The value of the expression is 40.
 8. The value of the expression is 50.
 9. The value of the expression is 60.
 10. The value of the expression is 70.

Figure I
 Downward Force for Uninterrupted
 Progress vs Thrust based on
 Runeberg's Equation

Rnw April 1965



U.S.S. Glacier

i_B 30.0°
 β 40.3°
 f_k 0.05

Stalin Class, U.S.S.R.

25.0°
 42.2°
 0.05

His equation was developed on the basis of the ship sliding up on the ice very slowly but it was intended to be used as a good approximation for an icebreaker making uninterrupted progress in the horizontal direction.

Figure I shows a plot of the downward force under the bow versus the icebreaking thrust based on Rimeberg's equation for both the U.S.S. Glacier and the Stalin class of the U.S.S.R. For example, if a thrust useable for icebreaking of 358,400 lb were developed by the U.S.S. Glacier, a downward force of 537,600 lb would be generated under the bow. This represents the maximum available force downward for this given value of thrust.

A. Kari was the second to analyze the mechanics of the icebreaking process. (11) Both the statics and dynamics of icebreaking was discussed in his paper "The Design of Icebreakers", but his equations for the downward force under the bow are of use only for uninterrupted progress. A complete redevelopment of his equations for the downward force is given in Appendix A.

His equations state that the downward force under the bow, F_{BZ} , for uninterrupted progress is a function of the following:

Δ = Displacement, tons.

θ = Change of trim, deg.

L = Length between perpendiculars, ft.

and for equation (A25).

GM_L = Longitudinal metacentric height, ft.

or for equation (A26).

The following information is given for the purpose of illustrating the use of the method of least squares in the case of a single variable.

Table 1 shows the data for the first 10 years of the life of a certain machine. The number of hours of use is given in column 1 and the corresponding cost in column 2. The total cost for the 10 years is \$10,000.00 and the total number of hours is 10,000. The average cost per hour is \$1.00.

The method of least squares is used to determine the best fit line for the data. The equation of the line is $y = ax + b$, where y is the cost and x is the number of hours. The values of a and b are determined by the method of least squares. The result is $a = 0.10$ and $b = 0.00$. The equation of the line is $y = 0.10x$. This means that the cost of the machine is \$0.10 per hour.

The following table shows the data for the first 10 years of the life of a certain machine.

Year	Hours of Use	Cost
1	1000	100
2	2000	200
3	3000	300
4	4000	400
5	5000	500
6	6000	600
7	7000	700
8	8000	800
9	9000	900
10	10000	1000

H = Draft, ft.

$$C = \frac{GM_L \times H}{L^2} = \frac{GM_L \times H}{L^2} = 0.07$$

The downward force is expressed as follows:

$$F_{BZ} = \frac{4480 \Delta GM_L \sin \theta}{L} \quad (A25)$$

and

$$F_{BZ} = \frac{4480 \Delta C L \sin \theta}{H} \quad (A26)$$

The following assumptions were used in the development of these equations:

1. There are no momentum effects.*
2. The vertical rise of the bow is equal to the thickness of the ice.* (This assumption fortunately has no bearing on the development of equations (A25) and (A26) but it is used later in his work to determine the thickness of ice which can be broken.)
3. The distance from the point of contact with the ice to the center of flotation is equal to the distance from the point of contact with the ice to the center of gravity.
4. The effective displacement is not affected by the force at the bow nor is the draft.*

* These assumptions were used but not stated.

$$x^2 + 2x + 1 = (x+1)^2$$

The square root of the left-hand side is

$$\sqrt{x^2 + 2x + 1} = \sqrt{(x+1)^2} = |x+1|$$

and

$$\sqrt{x^2 + 2x + 1} = x+1$$

The following conditions must be satisfied in the derivation of these

equations:

1. There are no terms other than $x^2 + 2x + 1$.
2. The radical sign of the left-hand side is equal to the radical sign of the right-hand side (the expression $\sqrt{x^2 + 2x + 1}$ is equal to $|x+1|$ and not $x+1$).
3. The expression $|x+1|$ must be written as $x+1$ or $-(x+1)$ (and not as $|x+1|$).
4. The radical sign of the left-hand side is equal to the radical sign of the right-hand side (the expression $\sqrt{x^2 + 2x + 1}$ is equal to $|x+1|$ and not $x+1$).

5. As a result of 4, the center of flotation and the longitudinal metacenter remain fixed.*
6. $GM_{L_b} \approx GM_L$
7. The value of C is 0.07.
8. There is no frictional force.*

At one point in the development he set the summation of moments equal to zero but failed to do the same with the summation of forces. If he had done so a discrepancy would have been apparent. The equations were developed for an icebreaker having its bow slide slowly up onto the edge of the ice but he intended that the equations be used for an icebreaker making uninterrupted progress in the horizontal direction. At best they are a good approximation only for the stopped equilibrium position.

Figure II represents an illustrative plot of the downward force for uninterrupted progress versus the change in trim in degrees using Kari's equation (A26). Unless an arbitrary limit for the change in trim, θ , is given, the maximum force under the bow cannot be obtained from the equation directly. One would have to solve for it separately using an equation such as the one developed by Runeberg based on T_{IB} .

D. R. Simonson, a Coast Guard Lieutenant, was the third to analyze the mechanics of the icebreaking process. (12) The purpose of his work was to determine a bow profile which would represent an equilibrium condition

* These assumptions were used but not stated.

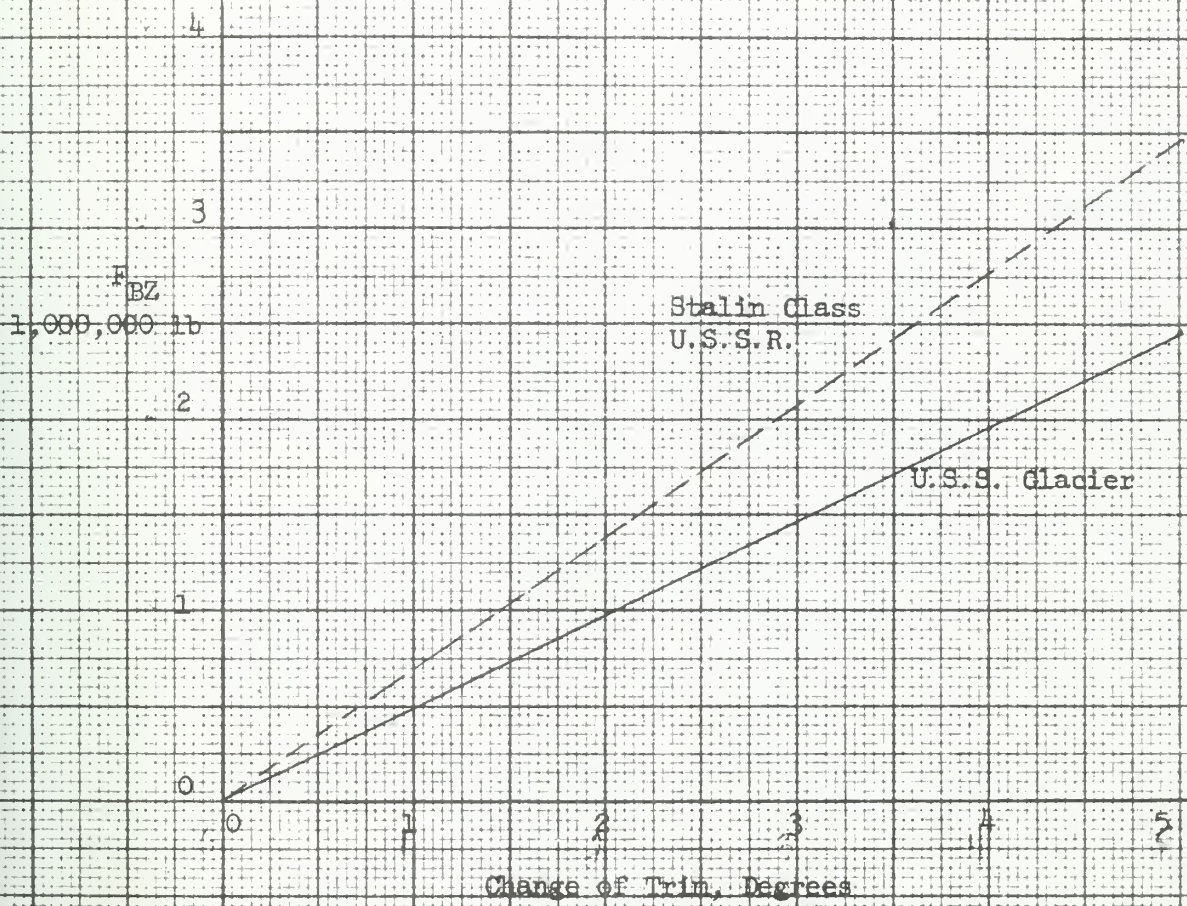
In the first part of the paper, the authors discuss the general theory of the...
 The second part is devoted to the...
 In the third part, the authors discuss the...
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 The eighteenth part is devoted to the...
 In the nineteenth part, the authors discuss the...
 The twentieth part is devoted to the...

* These equations were used but not solved.

Figure II

Downward Force for Uninterrupted Progressive Degrees Change of Trim based on Mari's Equation

R.W.W. 4/65



	<u>U.S.S. Glacier</u>	<u>Stalin Class, U.S.S.R.</u>
A	8,640 tons	11,000 tons
L	290 ft.	335 ft.
H	28 ft.	29.5 ft.
C	0.07	0.07

regardless of trim if the other factors were held constant. This lead to a mathematical description of the stem contour which turned out to be somewhat spoon-shaped. As a necessary step toward that determination he developed an equation for uninterrupted progress. A complete redevelopment of his work is given in Appendix A.

All of his work is statical since he felt that "momentum should be neglected as it is desirable to break ice without charging or ramming."

His equation states that the downward force under the bow, F_{BZ} , for uninterrupted progress is a function of the following:

T_{IB} = Thrust available for icebreaking, LB

i_B = Stem angle with reference to base plane, deg.

θ = Change of trim, deg.

The downward force is expressed as follows:

$$F_{BZ} = \frac{T_{IB}}{\tan (i_B + \theta)} \quad (A43)$$

The following assumptions were used for the development of this equation:

1. There are no momentum effects.*
2. Friction with the ice is negligible.*
3. Thrust is directed horizontally at all times.*

*These assumptions were used but not stated.

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$$I_1 = \frac{1}{2} \pi r^2$$

$$I_2 = \frac{1}{2} \pi r^2$$

$$I_3 = \frac{1}{2} \pi r^2$$

The ...

(14)

$$\frac{I_1}{I_2} = \frac{r_1^2}{r_2^2}$$

The ...

...

1. ...
2. ...
3. ...

...

4. The center of flotation serves as a pivot point.*
5. There is no change in displacement.*

Figure III represents an illustrative plot of the downward force for uninterrupted progress versus the icebreaking thrust based on Simonson's equation for both the U.S.S. Glacier and the Stalin Class of the U.S.S.R. For example, if a thrust useable for icebreaking of 358,400 lb were developed by the U.S.S. Glacier, a downward force of 595,000 lb would be generated under the bow. This represents the maximum available force downward for this given value of thrust. Note that it is necessary to solve for the trim independently or make a suitable assumption. The forces indicated in Figure III are all in excess of those indicated by Runeberg's equation which are illustrated by Figure I. This is due to the fact friction is neglected by Simonson.

In fact, Simonson's equation is limited to being a good approximation for the stopped equilibrium position, not really uninterrupted progress.

During 1946 a book was published in Russia entitled "Vessels for Artic Navigation" written by I. V. Vinogradov. (13) It contained the development of an equation for the downward force under the bow of an icebreaker which resulted from ramming. The work was significant in that it represented the first time that this force due to ramming was put into useable mathematical form.

* These assumptions were used but not stated.

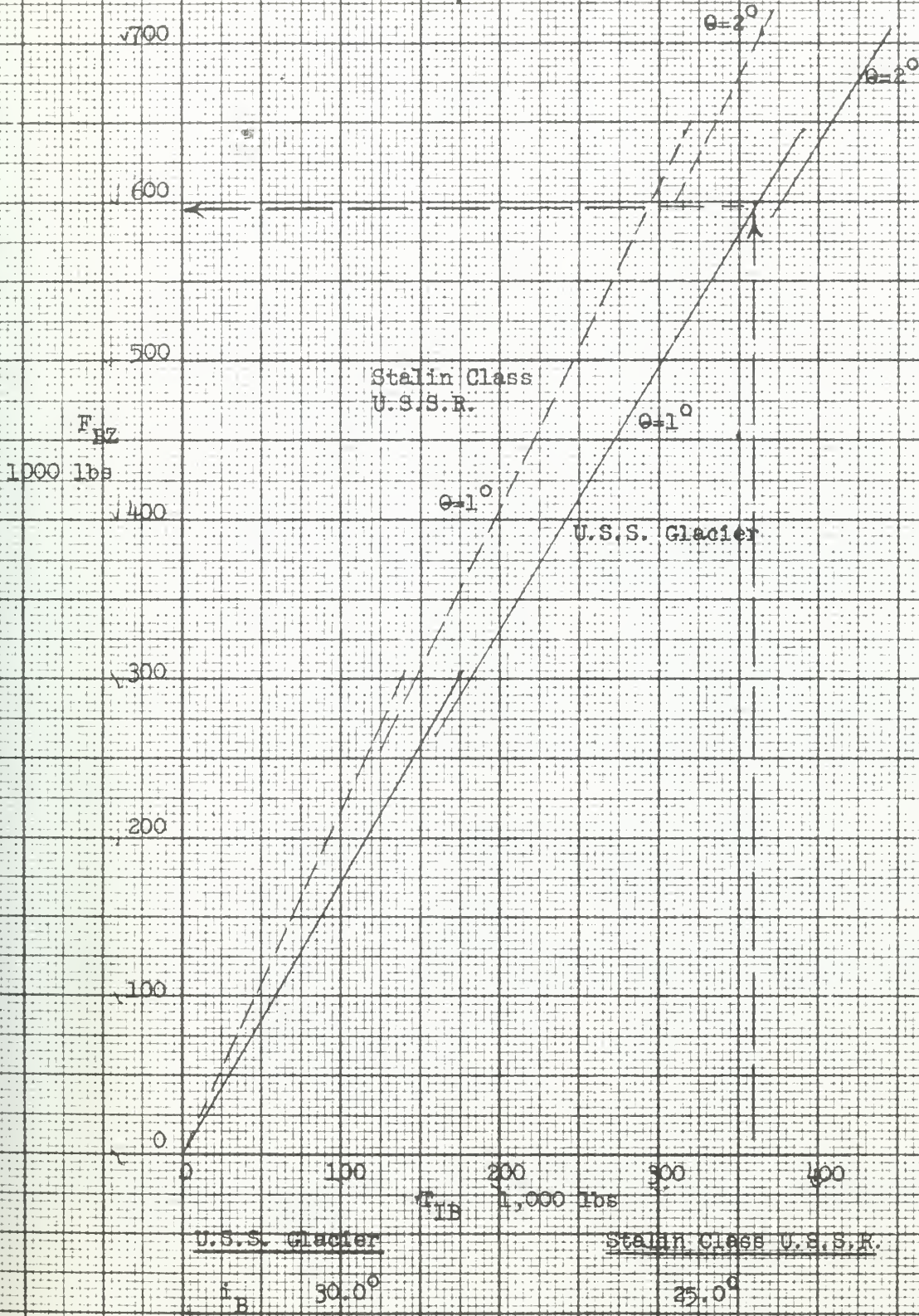
The first of these is the fact that the
 development of the Soviet Union as a
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 industrial revolution which began in
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 capabilities.

* These examples were used but not listed

Figure III

Downward Force for Uninterrupted Progress vs Thrust based on
Simonsen's Equation

R_W 4/85



L. W. Ferris has paraphrased Vinogradov as follows (14): "The analysis is based on the following concept: An icebreaker moving with known velocity strikes a uniform ice shelf and the bow of the ship glides up until the downward pressure reaches a magnitude which causes the ice shelf to collapse. While the ship is climbing the ice shelf, the propellers continue to push. In general, the forward motion of the ship is not reduced to zero at the instant when the ice collapses."

"The quantity which is to be determined is the maximum value of the vertical force P developed on the stem of the icebreaker."

Since ramming is taken into account by use of the principle of the conservation of energy, a much larger number of terms (i.e. parameters of ship form) will be necessary than have appeared in previously mentioned solutions for uninterrupted progress.

Vinogradov's equation states that the downward force under the bow, F_{BZ} , (P in his equations) for the ramming condition is a function of the following*:

F = Coefficient of sliding friction. (f_k)

φ = Angle of stem, deg. (i_B)

β = Angle of normal to shell plating with respect to ξ
plane, deg. (β)

δ = Block coefficient (δ)

* The symbols used are those of Vinogradov. Symbols in parentheses are those used commonly.

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* The details and the form of the ...

- α = Waterline coefficient (α)
 Q = $L/2$ plus the distance aft from to the center of flotation, ft. ($L/2 - LCF$)
 L = Length between perpendiculars, ft. (L)
 D = Draft, ft. (H)
 M = Longitudinal metacentric height, ft. (GM_L)
 T = Thrust, tons. (T)
 W = Displacement, tons. (Δ)
 E = Coefficient of resistution. (e)
 V_0 = Speed just prior to impact, ft/sec. (V_0)
 V_1 = Speed while sliding up (normally taken as zero to get maximum P), ft/sec. (V_1)

This downward force under the bow is expressed as follows:

$$P = XT + \left\{ X^2 T^2 + \frac{Y}{A} W^2 \cdot \frac{v_0^2 [1 - (1 - e^2) \sin^2 \phi] - v_1^2}{g D} \right\} \quad (A65)$$

where

$$X = \frac{1 - \frac{f}{\cos \beta} \tan \phi}{1 + \frac{f}{\cos \beta} \cot \phi} \cot \phi \quad (A63)$$

Let μ be the mean of the distribution of X and σ^2 be the variance of X .

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$f'(x) = -\frac{x-\mu}{\sigma^2} f(x)$$

$$f''(x) = \left[\frac{(x-\mu)^2}{\sigma^4} - \frac{1}{\sigma^2} \right] f(x)$$

$$E[f''(X)] = \left[\frac{E[(X-\mu)^2]}{\sigma^4} - \frac{1}{\sigma^2} \right] E[f(X)]$$

$$= \left[\frac{\sigma^2}{\sigma^4} - \frac{1}{\sigma^2} \right] \frac{1}{\sigma\sqrt{2\pi}}$$

$$= \left[\frac{1}{\sigma^2} - \frac{1}{\sigma^2} \right] \frac{1}{\sigma\sqrt{2\pi}} = 0$$

$$E[f''(X)] = 0$$

$$E[f''(X)] = 0$$

Let $Y = \frac{X-\mu}{\sigma}$ be the standardized variable. Then $f_Y(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}$.

$$f_Y'(y) = -y f_Y(y)$$

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$$(10) \quad \left\{ \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \right\} \left[\frac{(x-\mu)^2}{\sigma^4} - \frac{1}{\sigma^2} \right]$$

where

$$(11) \quad \frac{1 - \frac{1}{2} \frac{1}{\sigma^2} \frac{1}{\sigma}}{1 + \frac{1}{2} \frac{1}{\sigma^2} \frac{1}{\sigma}}$$

$$Y = \frac{1}{1 + \frac{f}{\cos \beta}} \cot \phi \quad (A64)$$

$$A = \frac{\delta}{\alpha} \left[1 + \left(\frac{k_1}{k_2} \right) \frac{1}{4\alpha} \right] \quad (A61)$$

$$k_1 = \frac{q}{(L/2)}$$

$$k_2^2 = \frac{m D \delta}{\alpha^2 L^2} \quad k_2 = \frac{1}{\alpha L} \sqrt{m D \delta}$$

The following assumptions were used by Vinogradov in the development of this equation:

1. Thrust was directed horizontally at all times*.
2. Change in trim and draft do not seriously affect properties of the waterplane or the longitudinal metacentric height.
3. $GM_L = EM_L$

In addition to the assumptions listed above, many expedients were taken and these deserve some comment or criticism so that the reader may have a better idea of the validity of Vinogradov's equation. The most necessary of comments or criticisms follow:

* This assumption was used but not stated.

(10)

$$= \frac{1}{\sqrt{100}} \cdot 100$$

(11)

$$\left| \frac{d}{dt} \left(\frac{1}{t} \right) \right| = \left| \frac{-1}{t^2} \right| = \frac{1}{t^2}$$

$$\frac{d}{dt} \left(\frac{1}{t} \right) = -\frac{1}{t^2}$$

$$\frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}} \quad \frac{d}{dx} \frac{1}{\sqrt{x}} = -\frac{1}{2x^{3/2}}$$

The following table shows the derivatives of the functions in the exercises.

Table 1.1

1. Derivative of the constant function $f(x) = c$ is $f'(x) = 0$.

2. Derivative of the identity function $f(x) = x$ is $f'(x) = 1$.

3. Derivative of the power function $f(x) = x^n$ is $f'(x) = nx^{n-1}$.

$$f'(x) = nx^{n-1}$$

In addition to the functions listed above, there are many other functions whose derivatives can be found. For example, the derivative of the exponential function $f(x) = e^x$ is $f'(x) = e^x$. The derivative of the natural logarithm function $f(x) = \ln x$ is $f'(x) = \frac{1}{x}$. The derivative of the sine function $f(x) = \sin x$ is $f'(x) = \cos x$. The derivative of the cosine function $f(x) = \cos x$ is $f'(x) = -\sin x$. The derivative of the tangent function $f(x) = \tan x$ is $f'(x) = \sec^2 x$. The derivative of the secant function $f(x) = \sec x$ is $f'(x) = \sec x \tan x$. The derivative of the cosecant function $f(x) = \csc x$ is $f'(x) = -\csc x \cot x$. The derivative of the cotangent function $f(x) = \cot x$ is $f'(x) = -\csc^2 x$.

Table 1.2

* This derivative was not yet derived.

1. Trim was taken into account during the solution for movement of the icebreaker (in order to determine the distance a force moved in doing work), but it is disregarded (or considered negligible) in the solution for the resultant perpendicular to the stem.
2. Thrust, T , was kept as a constant representing total thrust. There would be an improvement in the use of his equation if one were to consider this as the thrust applied to breaking ice, T_{IB} , since some of the total thrust is used in the resistance of the water.
3. Although kinetic energy and work are used for the basis of his work, there is no mention of the possibility of forces due to the acceleration. One of his key equations, (A54), sets the summation of forces equal to zero when in fact there is a large deceleration.
4. There is no mention of the fact that some of the kinetic energy while sliding up may be in the form of rotational energy as well as translational energy.
5. The change of trim is based on the original displacement using the equation for a couple when actually the effective displacement is changed.

1. The first step in the process of developing a business plan is to conduct a market analysis. This involves identifying the target market, understanding the needs and preferences of the customers, and assessing the competitive landscape. A thorough market analysis provides valuable insights into the opportunities and challenges of the industry, which are essential for formulating a realistic and profitable business strategy.

2. Once the market analysis is complete, the next step is to define the business model. This involves determining the products or services to be offered, the pricing strategy, the distribution channels, and the revenue streams. The business model should be designed to meet the needs of the target market while ensuring a sustainable and profitable operation. A clear and concise business model is crucial for attracting investors and securing financing.

3. The third step in the process is to develop a financial plan. This involves projecting the revenues, expenses, and cash flows of the business over a period of time, typically three to five years. A detailed financial plan provides a clear picture of the financial viability of the business and helps to identify potential risks and opportunities. It is also an essential tool for monitoring the performance of the business and making informed decisions about its future.

4. The fourth and final step in the process is to implement the business plan. This involves putting the business model into action, launching the products or services, and establishing the necessary infrastructure and support systems. Successful implementation requires a strong commitment to the business plan, effective communication, and a willingness to adapt to changing circumstances. Regular monitoring and evaluation of the business plan's progress are essential for ensuring long-term success.

5. In conclusion, developing a business plan is a critical and ongoing process that requires careful planning, execution, and evaluation. By following these five steps, entrepreneurs can increase their chances of success and build a sustainable and profitable business.

6. Q is used exclusively as a constant representing the distance from the center of flotation to the forward perpendicular, which is assumed to be the original point of contact. For the geometrical determination of certain distances this is proper; however, it is not proper when this length is used as a moment arm from the point of contact. That particular distance is a variable. In fact, even using Vinogradov's equation as a basis, it can be shown (14) that the distance travelled after initial impact, for the U.S.S. Glacier ramming at six knots with $T = 160$ tons, is 28.7 feet. This means that Q should approximately decrease from the original 150 feet to 121 feet, a decrease of approximately 19 o/o.
7. The expression for the loss of energy on impact is based on direct central impact. In other words, it is assumed that the loss is the same as if a perpendicular to the stem passed through the center of gravity of the icebreaker. (Later in the introduction much more can be found on this expedient. It is common to some other recent developments and comment will be reserved until these others have been mentioned.)
8. Although the sliding velocity is contained in the final equation, the equation is only valid when this sliding velocity, V_1 , is equal to zero. This is not only because of the use of equilibrium in the solution, it is also because while sliding up there

The first part of the paper is devoted to a general discussion of the
 various methods of determining the velocity of light. It is shown that
 the most accurate method is the method of Fizeau, which is based on
 the measurement of the time taken for light to travel a certain distance.
 This method is superior to the method of Michelson, which is based on
 the measurement of the interference of light. The method of Fizeau is
 more accurate because it does not depend on the wavelength of the light.
 The velocity of light is found to be 3×10^{10} cm per second.
 This value is in good agreement with the value obtained by other
 methods.

The second part of the paper is devoted to a discussion of the
 various methods of determining the velocity of sound. It is shown that
 the most accurate method is the method of Kundt, which is based on
 the measurement of the length of the tubes in which the sound waves
 are standing. This method is superior to the method of Rüchli, which
 is based on the measurement of the time taken for sound to travel a
 certain distance. The method of Kundt is more accurate because it
 does not depend on the frequency of the sound. The velocity of sound
 is found to be 330 m per second. This value is in good agreement
 with the value obtained by other methods.

Although the velocity of light is constant in the ether, it is not
 constant in the medium. This is not only because of the refraction
 of light, but also because of the absorption of light. The velocity
 of light in a medium is given by $v = \frac{c}{n}$, where c is the velocity
 of light in the ether, and n is the refractive index of the medium.
 The refractive index of a medium is a measure of the optical density
 of the medium. It is found that the refractive index of a medium
 increases with the density of the medium.

is a component due to friction which is acting in opposition to the downward force, P . This term is not included in the final equation; it goes abruptly to zero as soon as the velocity goes to zero (and then it reverses).

9. In addition to the prohibition mentioned in 8, V_1 does not take on (in the equation) all values from V_0 down to zero; this is due to the impact term which indicates that there is an immediate reduction, to some degree, of the velocity.

It must be noted that, in spite of the comments made above, Vinogradov's equation was the first equation that was of any use for the ramming condition. This ramming is quite important and for many years following 1946 this equation was by far the best criterion for the ability of an icebreaker. The development is given in Appendix A.

The result of a calculation which is given in the paper by Ferris (14) showed that this downward force for the U.S.S. Glacier ramming at six knots with a thrust of 160 tons became 3,225,600 pounds. This compares to 537,600 pounds (according to Runeberg) or 595,000 pounds (according to Simonson) for uninterrupted progress. These illustrative results are shown in Figure IV. It is quite readily seen that the order of magnitude of force generated by ramming completely overshadows the force generated during uninterrupted progress.

These four men named above (Runeberg, Kari, Simonson, and Vinogradov) have made the most significant contributions. Table II shows the

The first part of the report is devoted to a description of the experimental apparatus and the method of measurement. It is shown that the results obtained are in good agreement with the theoretical predictions. The second part of the report is devoted to a discussion of the results and to a comparison with the results obtained by other workers in this field. It is shown that the results obtained in this work are in good agreement with the results obtained by other workers in this field.

The third part of the report is devoted to a discussion of the results and to a comparison with the results obtained by other workers in this field. It is shown that the results obtained in this work are in good agreement with the results obtained by other workers in this field.

The fourth part of the report is devoted to a discussion of the results and to a comparison with the results obtained by other workers in this field. It is shown that the results obtained in this work are in good agreement with the results obtained by other workers in this field.

The fifth part of the report is devoted to a discussion of the results and to a comparison with the results obtained by other workers in this field. It is shown that the results obtained in this work are in good agreement with the results obtained by other workers in this field.

Figure IV

Comparison of Magnitudes of Force
Developed Under Bow During Icebreaking
(for the U.S.S. Glacier at 6 knots with
 $T = 160$ tons)

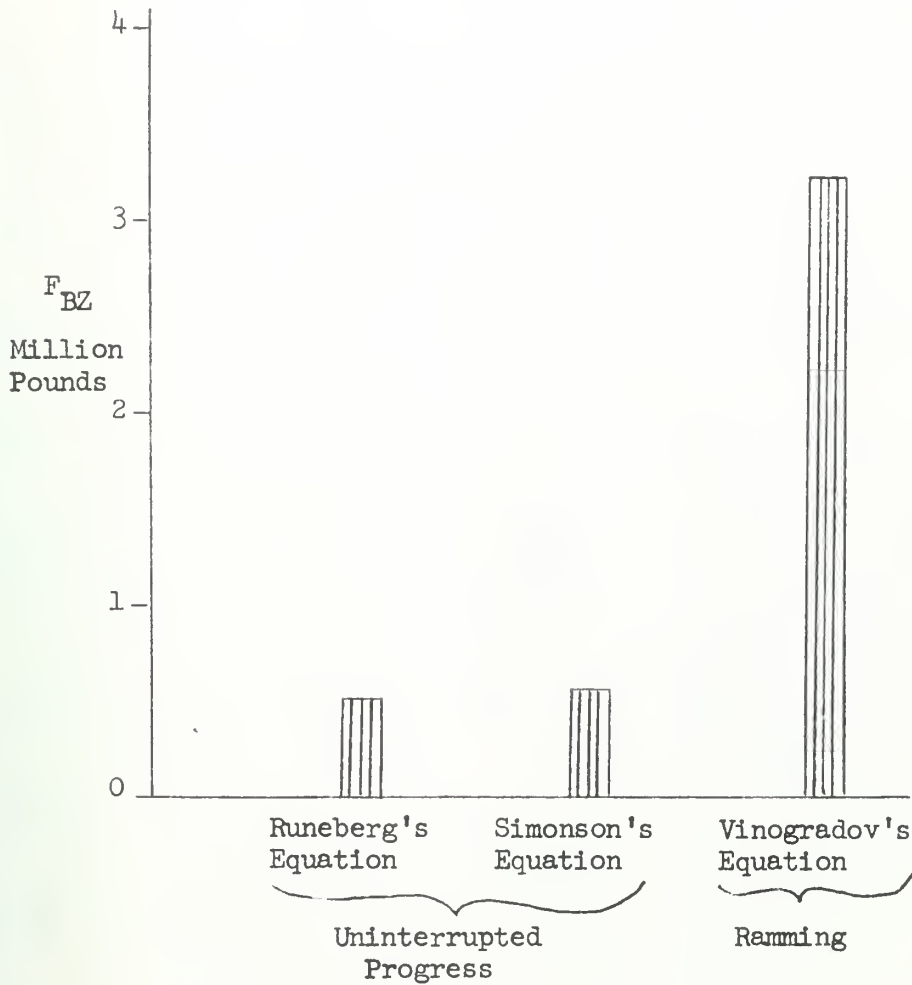


Table II

Use or Reference to Icebreaking Equations in
Relatively Prominent Publications Presented Chronologically

	<u>Runeberg</u>	<u>Kari</u>	<u>Simonson</u>	<u>Vinogradov</u>
1880				
1890	X 1888			
1900				
1910				
1920		X 1921		
1930				
1940	X X X		X 1936	
1950	X X		X	X 1946
1960		X	X X	XXX X X
1970				

1940
 The following table shows the number of persons in the
 various occupations in the United States in 1940.

Occupation	Number	Percentage	Total
Professional	1,000,000	3.0	33,000,000
Managerial	2,000,000	6.0	33,000,000
Technical	3,000,000	9.0	33,000,000
Skilled	10,000,000	30.0	33,000,000
Semi-skilled	10,000,000	30.0	33,000,000
Unskilled	8,000,000	24.0	33,000,000
Homemaker	2,000,000	6.0	33,000,000
Retired	1,000,000	3.0	33,000,000
Unemployed	1,000,000	3.0	33,000,000
Total	33,000,000	100.0	33,000,000

chronological popularity of their developments. Table III illustrates the parameters appearing in their equations, however, there have been others since then who have developed equations for icebreaking, but the equations lack the significance of those mentioned earlier.

In 1956 Jan-Erik Jansson presented an equation for the determination of work utilized in ramming of ice. (15) Unfortunately it does not include any equation for obtaining the downward force during icebreaking. However, his work is quite comprehensive and for that reason is included in Appendix A. He also uses the conservation of energy principle in his development. However, as a convenience he has disregarded loss at initial impact and has neglected friction.

In 1959 C. Richardson presented an equation for the downward force under the bow during ramming. (16) It was developed in conjunction with some model studies of the force system. The equation is similar to the equation presented by Vinogradov and is presented in Appendix A. The development was almost identical to Vinogradov's but did modify some of his weaknesses to some extent. For example, Richardson uses a term for the loss of energy due to wave and frictional resistance (not ice) from the instant of contact up to the moment the ice breaks or motion ceases. He also recognizes an effective increase in the mass of the icebreaker due to entrained water. For the most part, however, he has used the same assumptions and expedients that Vinogradov used and for that reason comments expressed earlier also apply here.

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Table III
 Parameters Appearing in the Equations of
 Downward Force by Various Developers

<u>Parameter</u>	<u>Developer</u>			
	<u>Runeberg</u>	<u>Kari</u>	<u>Simonson</u>	<u>Vinogradov</u>
T_{IB}	X		X	X
i_B	X		X	X
β	X			X
f_k	X			X
Δ		X		X
θ		X	X	
L		X		X
GM_L		X		X
δ				X
α				X
LCF				X
H				X
e				X
V_0				X
V_1				X

Symbols used are those given in Appendix C, Symbols and Their
 Titles.

Table 17

Percentage of total population in various age groups in various years

Year	Age Group	Percentage
1950	0-4	16.1
1950	5-9	15.0
1950	10-14	13.9
1950	15-19	12.8
1950	20-24	11.7
1950	25-29	10.6
1950	30-34	9.5
1950	35-39	8.4
1950	40-44	7.3
1950	45-49	6.2
1950	50-54	5.1
1950	55-59	4.0
1950	60-64	2.9
1950	65-69	1.8
1950	70-74	0.7
1950	75-79	0.6
1950	80-84	0.5
1950	85-89	0.4
1950	90-94	0.3
1950	95-99	0.2
1950	100+	0.1

Source: Data from the Census Bureau, Bureau of Economic Analysis, "U.S. Census of Population and Housing, 1950, Summary Reports, PC80-1A, Washington, D.C., 1953, Table 17.

As part of a report released in 1959 concerning the feasibility of a nuclear icebreaker, an equation concerning the relative magnitude of force "transmitted to the ice" at the bow was presented (12). This concerns uninterrupted progress only; it does not have to do with ramming. However, the development is given in Appendix A since it is of interest. The force under consideration here is the force perpendicular to each side of the bow. This could, of course, be resolved into the downward component, but this would lead right back to Simonson's equation since this work is based on that equation. It is interesting to note though that the equations of this report are based on the assumption that thrust remains parallel to the base line at all times, and not simply horizontal.

In 1962 V. R. Milano presented his modification of Vinogradov's equation (18). One of the main contributions was to express thrust as a function of "Bollard Pull". He also rewrote the equation so that the displacement may be solved based on other parameters including the desired downward force. The equation is given in Appendix A.

All four equations for ramming presented above are based on the principle of the conservation of energy. There is, of course, nothing wrong with the concept; any shortcomings exist only in the developments. Although the situation is obviously dynamic, which is the reason for the use of energy in the calculations, each at some point in his development, uses static equilibrium. They have set the summation of forces at a point equal to zero when there is acceleration involved.

It may be noted that the first of these is the

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which is given by the first of the

equations of the system, and the second

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the seventh is the equation of the

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Furthermore, all developments of the ramming condition force utilize a very important term for the loss of energy at initial impact. (An exception to this is the development by Jansson; he neglects this term.) This loss of energy is determined using the coefficient of restitution, E . As this term has been developed, it is fundamentally in error. The use in their developments means that the impact has been direct central impact. This means that a normal to the stem at the point of contact would have to pass through the center of gravity (which it does not) and no rotation would be imparted. Furthermore, it means that the velocity component tangent (parallel to the inclination of the stem) to the stem is conserved on initial contact. The velocity component normal to the stem inclination would be reversed and be of magnitude equal to $(E) \times$ (initial normal component velocity). In most illustrations E has been set equal to 0.90 or 0.95. Figure V shows the implication of the acceptance of this form of impact energy loss. If one is willing to believe the energy-loss-at-initial-impact term, then one must also be willing to believe that after initial contact the velocity of the center of gravity is only slightly less in magnitude and is upward in direction at an angle which is almost twice the angle of the stem. Even one who has never seen an icebreaker in action would find this hard to believe.

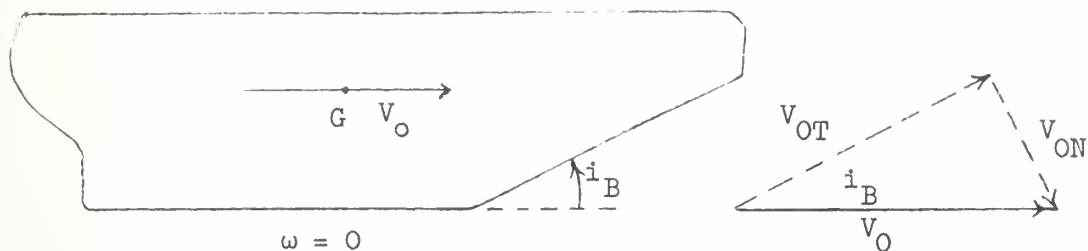
Incidentally, carrying this concept further for the sake of illustration, it can be shown that the term would imply that a ship with a vertical stem hitting the ice at 10.0 knots would bounce so that it ended up going

The first part of the report is devoted to a general
 survey of the situation in the country. It is
 followed by a detailed study of the various
 branches of the economy. The first of these
 is the agricultural sector. It is followed
 by the industrial sector. The third part
 of the report is devoted to a study of the
 social and cultural conditions of the
 country. It is followed by a study of the
 political and administrative system. The
 final part of the report is devoted to a
 study of the foreign relations of the
 country. It is followed by a study of the
 international situation. The report is
 concluded by a summary of the main
 findings and a list of references.

Figure V

Implication of Acceptance of
Impact Energy Loss in Presently-
Used Ramming Equations

Before Initial Contact

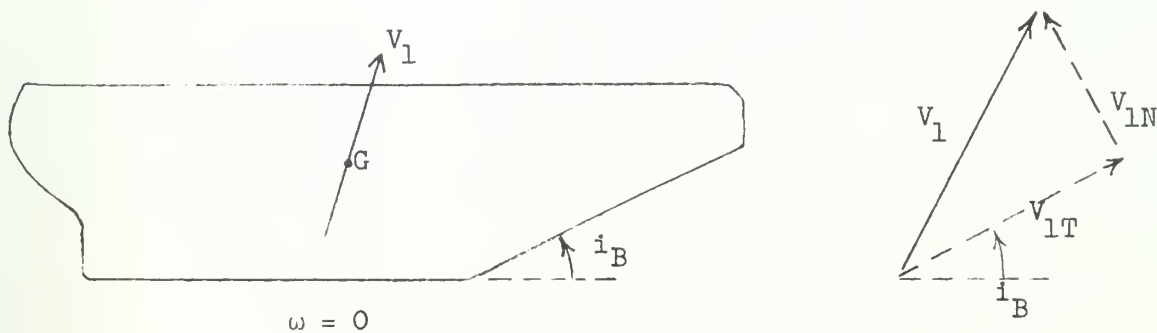


If $e = 0.90$

$$V_{IN} = -0.90 V_{ON}$$

$$V_{IT} = V_{OT}$$

After Initial Contact



astern at 9.0 or 9.5 knots. It is quite apparent this is not the case when a ship encounters ice.

Need for Suitable Analysis

It is interesting to note that many do not yet take full advantage of the equations which do exist. Many references (19), (20), (9), (21), (4), (22), and (23)., indicate the choice of the "optimum bow" or "standard bow" without any further mention of procedure or justification. Some of these simply state that 30° is the best angle.

Presently the downward force resulting from ramming is being used by some. In spite of any weaknesses which exist in the ramming equations, they are the best available and certainly present a more meaningful value than the force developed during uninterrupted progress.

Four criteria are presently used for measuring icebreaking capability.

(4) These are listed by Lank as follows:

1. "Probably the most complete analysis of the action of an ice-breaker in breaking a uniform sheet of ice is the one developed by Vinogradov." The downward force developed by ramming is apparently the most important criterion.
2. "A rough measure of the ability of a ship to force its way into leads or broken ice is the ratio of horsepower to beam."
3. "The horsepower displacement ratio has been widely used for comparing the relative power of icebreakers but probably does little more than express relatively ability to accelerate."

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... ..

It is ... the question ... (SS), and (S), ... "down" ... these ...

Four ... are presently used for ...

(4) These are listed by Rank as follows:

1. "Probably the most complete analysis of the action of ... by Vinogradov." The downward force ...
2. "A ... of the ability of a ... to ..."
3. "The ... ratio ... for ... the relative power of ... but probably does little more than express relatively ability to accelerate."

4. "For large icebreakers of generally similar hull form, the simple value of thrust at zero speed is probably as good a measure of forcing ability as any."

In general it is conceded that the equations for the ramming give the best measure of ability of a polar icebreaker. Just as generally, it is pointed out that more work must be done along these lines since the present equations do not quite lead to a proper representation.

Admiral E. H. Thiele said, in 1959, "icebreakers are relatively expensive to build and maintain. Every effort to make an icebreaker more effective through improvements in design and operating technique will be repaid many times".

The object of this research is to make icebreakers "more effective through improvement in design". The design can be improved by improving the measure of the most important criterion for a polar icebreaker, the downward force generated by ramming. Specifically, the object is to develop a suitable equation for the prediction of the dynamically developed force at the bow of an icebreaker during encounter with virtually unyielding ice.

The first condition of a good government is that it should be

based on the consent of the governed.

It should be based on the consent of the governed.

In general it is assumed that the consent of the governed is

the basis of a good government. It is assumed that the consent of the governed is

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the basis of a good government.

Figure VI

"Wind Class" Icebreaker Kumming Ice



Official U.S. Coast Guard Photo

II Procedure

General

When a polar icebreaker encounters very heavy ice, the icebreaker must resort to "ramming". The object of this technique is to get a large, relatively sustained, downward force under the bow. It is this sustained force which tends to cause the ice to collapse. See Figure VI and VII.

The bow first crushes into the ice until the bow is accommodated and supported sufficiently to allow sliding. The bow rises up to a point where forward progress ceases and the icebreaker settles at this point. It is as if the icebreaker were "grounded" at the bow.

The problem is to predict this downward force (F_{BZ}) as a function of the following parameters:

- L = Length between perpendiculars, ft. (EP)
- B = Waterline beam, ft. (B)
- H = Normal draft, ft. (H)
- Δ = Normal displacement, lb. (DIS)
- i_B = Angle from base line to stem, radians (BA)
- β = Angle of normal to bow plating with respect to the centerline plane, radians. (SA)
- v_1 = Velocity of icebreaker immediately prior to initial contact, ft./sec.
- α = Waterplane coefficient, dimensionless. (AL)
- LCF = Distance from amidships to center of flotation (+ if forward, - if aft), ft. (CF)

1. Introduction

1950

The first section of the report is devoted to a general discussion of the problem. The second section contains a description of the experimental apparatus and the results obtained. The third section is devoted to a discussion of the results and the conclusions drawn therefrom. The fourth section contains a list of references and a summary.

- (a) Length of the specimen, L
- (b) Sectional area, A
- (c) Total strain, ϵ
- (d) Initial displacement, δ_0
- (e) Load from beam line to steel, P
- (f) Load at normal to low dialing with respect to the specimen, P_0
- (g) Velocity of specimen immediately prior to initial contact, V_0
- (h) Strain rate coefficient, $\dot{\epsilon}$
- (i) Strain rate coefficient to order of direction, $\dot{\epsilon}_0$
- (j) Total strain, ϵ

- LCC = Distance from amidships to center of gravity (+ if forward, - if aft), ft. (CG)
- KG = Height of center of gravity above keel, ft. (GK)
- d = Height of thrust line above base line, ft. (D)
- T_{BOL} = Bollard thrust for rpm of sustained approach velocity, lb. (TB)
- GM_L = Longitudinal metacentric height, ft. (GM)
- f_k = Coefficient of kinetic friction between ice and ship, dimensionless. (FK)
- σ = Compressive failure stress of ice, lb/ft². (SIG)
- f_s^* = Coefficient of static friction, dimensionless (FS)

(Note: The symbols to the right in parentheses are those used in the Fortran computer program.)

The complete step-by-step solution is given in the appendix.

Definition of States and Phases

"State 1" is defined as the state of the icebreaker immediately prior to initial contact.

The "Crushing Phase" is that period when the ice is crushing locally to accommodate the bow. The bow is tending to rise and the ship is tending to slow down.

* This is not necessary for the downward force but is used for the extraction thrust.

- (a) ...
- (b) ...
- (c) ...
- (d) ...
- (e) ...
- (f) ...
- (g) ...
- (h) ...
- (i) ...
- (j) ...
- (k) ...
- (l) ...
- (m) ...
- (n) ...
- (o) ...
- (p) ...
- (q) ...
- (r) ...
- (s) ...
- (t) ...
- (u) ...
- (v) ...
- (w) ...
- (x) ...
- (y) ...
- (z) ...

Definition of Order and Type

The order of a point is the number of lines passing through it. The type of a point is the number of lines passing through it which are not concurrent with any other line passing through it.

* This is not necessary for the present purpose but is used for the extension of the theory.

"State 2" is defined as the state of the icebreaker when local crushing has ceased and the bow has a velocity tangent to the ship-ice interface. (In other words, there is no more penetration into the ice.)

The "Sliding Phase" commences at State 2. The bow slides up on the ice without further appreciable penetration. It is assumed the point of contact is fixed relative to the ice.

"State 3" occurs when the velocity of a point on the bow relative to the ice becomes zero. This does not necessarily imply that all velocities (\dot{x} , \dot{z} , and $\dot{\theta}$) are zero.

"State 4" occurs when all velocities have become zero and the icebreaker is in static equilibrium. The downward force under the bow, F_{BZ4} , is the relatively sustained force which is the object of this research.

The coordinates may be seen in Figure B-V.

Bow Forces During Crushing

It is assumed that all forces acting on the bow from the ice act at the intersection of the stem and the waterline. There are three forces acting at the bow. There is a force normal to the plating and it is assumed that this normal force may be represented by the product of the area of contact and the compressive failure stress of the ice. There is a component of friction force acting parallel to the stem in the plane of the plating. During crushing there is another friction force acting perpendicular to the stem in the plane of the plating. See Figure B-I.*

* These figures appear in Appendix B.

The first part of the paper is devoted to the study of the
 conditions under which the motion of a particle is
 periodic. It is shown that the motion is periodic if
 the energy is constant and the potential is a function
 of the distance from a fixed point. The period of
 the motion is then calculated.

$$T = 2\pi \sqrt{\frac{m}{k}}$$

The second part of the paper is devoted to the study of
 the conditions under which the motion of a particle is
 aperiodic. It is shown that the motion is aperiodic
 if the energy is not constant or the potential is not
 a function of the distance from a fixed point.

The conditions are given in Figure 1.

THE PERIOD OF OSCILLATION

It is shown that the period of oscillation is a function
 of the energy and the potential. The period is
 calculated for a harmonic oscillator and for a
 particle in a potential well. The period is
 shown to be independent of the mass of the
 particle. The period is also shown to be
 independent of the amplitude of the
 oscillation.

* These figures refer to Figure 1.

As seen in Figures B-III and B-IV, these forces may be expressed as components in the x-direction and the z-direction.

$$F_{BXC} = N (\cos \beta + f_k \sin \beta) \sin (i_B + \theta) + N f_k \cos (i_B + \theta) \quad (B8)^+$$

$$F_{BZC} = N (\cos \beta + f_k \sin \beta) \cos (i_B + \theta) - N f_k \sin (i_B + \theta) \quad (B3)$$

where N is the force normal to the bow plating.

As mentioned earlier, the normal force N is the product of the area of contact due to penetration and the compressive failure stress of the ice.

$$N = \sigma A = \frac{\sigma}{\sin \beta \tan (i_B + \theta)} \left[x \tan (i_B + \theta) - \left(\frac{L}{2} - LCG \right) \theta + z \right]^2$$

Newton's Laws of Motion During Crushing

Figure B-IX illustrates the free body diagram of the icebreaker during the crushing phase. It is assumed that the icebreaker may be treated as a "solid body".

The forces acting on the bow are included in the equations of motion. Dropping negligible terms, the forces may be expressed as a function of x^2 .

⁺ The numbers in parentheses refer to equations in Appendix B.

The following is a list of the most important results of the theory of the Laplace transform.

$$\text{1. } \mathcal{L}\{f(t)\} = F(s) \text{ if and only if } f(t) = \mathcal{L}^{-1}\{F(s)\}$$

$$\text{2. } \mathcal{L}\{af(t) + bg(t)\} = aF(s) + bG(s)$$

where a and b are constants.

The Laplace transform is a linear operator. It is also a one-to-one mapping from the space of functions to the space of functions. The Laplace transform is a powerful tool for solving differential equations and for analyzing systems.

$$\mathcal{L}\{f(t) + g(t)\} = \mathcal{L}\{f(t)\} + \mathcal{L}\{g(t)\}$$

Properties of the Laplace Transform

The Laplace transform is a linear operator. It is also a one-to-one mapping from the space of functions to the space of functions. The Laplace transform is a powerful tool for solving differential equations and for analyzing systems.

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As a result of summing forces in the x-direction,

$$\ddot{x} + a_1 \dot{x} + a_2 x^2 + a_3 = 0 \quad (\text{B29})$$

where a_1 , a_2 , and a_3 are constants representing the influence of parameters such as T_{BOL} , v_1 , C , i_B , β , f_k , and m_x . (" m_x " will be explained later. It is sufficient for the present to say that it is the mass of the icebreaker.)

Furthermore, by summing forces in the z-direction,

$$\ddot{z} + a_1 \dot{z} + a_2 z + b_1 \theta + c_1 x^2 = 0 \quad (\text{B31})$$

where a_1 , a_2 , b_1 , and c_1 are constants representing the influence of parameters such as T_{BOL} , v_1 , C , i_B , β , f_k , m_z , and k_h . (" k_h " is a heave damping coefficient which will be explained later.

By summing moments around the center of gravity, by linearizing, by dropping negligible terms, and by substitution,

$$\ddot{\theta} + a_1 \dot{\theta} + a_2 \theta + d_1 x^2 = 0 \quad (\text{B51})$$

where a_1 , a_2 , and d_1 are constants representing the influence of parameters such as I_θ (mass moment of inertia, to be explained later), k_p (pitch damping coefficient, to be explained later), Δ , and GM_L .

to find the value of λ such that the system is consistent.

$$(10) \quad \begin{cases} x + y + z = 1 \\ x + 2y + 3z = 2 \\ x + 3y + 4z = 3 \end{cases}$$

The augmented matrix is $[A|b]$ where $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$. The rank of A is 2, and the rank of $[A|b]$ is 3. Therefore, the system is inconsistent for all values of λ .

For the system to be consistent, the rank of $[A|b]$ must be equal to the rank of A .

$$(11) \quad \begin{cases} x + y + z = 1 \\ x + 2y + 3z = 2 \\ x + 3y + 4z = 3 \end{cases}$$

where $\lambda = 1$, the system is consistent. The rank of A is 2, and the rank of $[A|b]$ is 2. Therefore, the system is consistent for $\lambda = 1$. For $\lambda \neq 1$, the system is inconsistent.

$$(12) \quad \begin{cases} x + y + z = 1 \\ x + 2y + 3z = 2 \\ x + 3y + 4z = 3 \end{cases}$$

where $\lambda = 1$, the system is consistent. The rank of A is 2, and the rank of $[A|b]$ is 2. Therefore, the system is consistent for $\lambda = 1$. For $\lambda \neq 1$, the system is inconsistent.

Solution for x During Crushing

It is natural to solve for x first. It is independent of other variables and the solutions for z and θ depend on x.

By dropping negligible terms,

$$\dot{x} = \frac{-k_1}{m_x} x^2 \quad (B39)$$

where k_1 is a constant incorporating the influence of σ , i_B , β and f_k .

By substitution, manipulation, and integration,

$$x = \left(v_1^2 - \frac{2k_1}{3 m_x} x^3 \right)^{1/2} \quad (B40)$$

where the constants are as previously defined.

The equation for \dot{x} cannot be directly integrated. Therefore a series expansion is used. From this it is possible to integrate to find t in terms of x. Negligible terms are then dropped. The expression can be expressed as a function of t by using a reversion of the series. Retaining significant terms,

$$x = v_1 \left(t - \frac{k_1 t^3}{12 m_x} \right) \quad (B41)$$

where k_1 and m_x have been previously defined.

Now that x has been found as a function of t, x' and \dot{x} may also be expressed as functions of t by means of substitution.

In the next section we will see how the Laplace transform can be used to solve differential equations. We will also see how the Laplace transform can be used to solve integral equations.

$$(10.1) \quad \mathcal{L}\{f(x)\} = F(s)$$

where $f(x)$ is a function of x and $F(s)$ is a function of s . The Laplace transform is a linear operator, and it satisfies the following properties:

$$(10.2) \quad \mathcal{L}\{af(x) + bg(x)\} = a\mathcal{L}\{f(x)\} + b\mathcal{L}\{g(x)\}$$

where a and b are constants. The Laplace transform is also a linear operator, and it satisfies the following properties: $\mathcal{L}\{cf(x)\} = c\mathcal{L}\{f(x)\}$ and $\mathcal{L}\{af(x) + bg(x)\} = a\mathcal{L}\{f(x)\} + b\mathcal{L}\{g(x)\}$. The Laplace transform is also a linear operator, and it satisfies the following properties: $\mathcal{L}\{cf(x)\} = c\mathcal{L}\{f(x)\}$ and $\mathcal{L}\{af(x) + bg(x)\} = a\mathcal{L}\{f(x)\} + b\mathcal{L}\{g(x)\}$.

$$(10.3) \quad \mathcal{L}\{f(x)g(x)\} = \int_0^s F(s-u)G(u)du$$

where $F(s)$ and $G(s)$ are the Laplace transforms of $f(x)$ and $g(x)$ respectively. The Laplace transform is also a linear operator, and it satisfies the following properties: $\mathcal{L}\{cf(x)\} = c\mathcal{L}\{f(x)\}$ and $\mathcal{L}\{af(x) + bg(x)\} = a\mathcal{L}\{f(x)\} + b\mathcal{L}\{g(x)\}$.

Solution for θ During Crushing

The summation of moments can be expressed in terms of θ and t by the substitution of equation (B41).

The solution becomes

$$\theta = e^{\alpha_1 t} (A_1 \cos \beta_1 t + A_2 \sin \beta_1 t) + \frac{d_1}{c_1} t^2 - \frac{2b_1 d_1}{c_1^2} t - A. \quad (B53)$$

where α_1 , β_1 , A_1 , A_2 , b_1 , d_1 , and c_1 are constants reflecting the influence of I_{θ} , k_p , A , GM_L , L , LCG , H , KG , v_1 , i_B , β , and f_k .

$$\begin{aligned} \dot{\theta} = & \alpha_1 e^{\alpha_1 t} (A_1 \cos \beta_1 t + A_2 \sin \beta_1 t) + e^{\alpha_1 t} (-A_1 \beta_1 \sin \beta_1 t + A_2 \beta_1 \cos \beta_1 t) \\ & + \frac{2d_1 t}{c_1} - \frac{2b_1 d_1}{c_1^2} \end{aligned} \quad (B55)$$

$$\begin{aligned} \ddot{\theta} = & (\alpha_1^2 - \beta_1^2) e^{\alpha_1 t} (A_1 \cos \beta_1 t + A_2 \sin \beta_1 t) \\ & + 2 \alpha_1 \beta_1 e^{\alpha_1 t} (-A_1 \sin \beta_1 t + A_2 \cos \beta_1 t) + \frac{2 d_1}{c_1} \end{aligned}$$

The function $f(x)$ is defined for $x > 0$ by

$$f(x) = \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \dots$$

Find $f'(x)$.

$$f'(x) = -\frac{1}{x^2} - \frac{2}{x^3} - \frac{3}{x^4} - \dots$$

For $x > 0$, the function $f(x)$ is defined by

$$f(x) = \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \dots$$

$$f'(x) = -\frac{1}{x^2} - \frac{2}{x^3} - \frac{3}{x^4} - \dots$$

(10)

$$\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + \dots$$

$$f'(x) = -\frac{1}{x^2} - \frac{2}{x^3} - \frac{3}{x^4} - \dots$$

$$\frac{1}{x^2} + \frac{2}{x^3} + \frac{3}{x^4} + \dots$$

Solution for z During Crushing

The summation of forces in the z-direction can be expressed in terms of z and t by the substitution of equation (B41).

The solution becomes

$$z = e^{\alpha_2 t} (B_1 \cos \beta_2 t + B_2 \sin \beta_2 t) + \frac{d_2}{c_2} t^2 - \frac{2b_2 d_2}{c_2^2} - B_1 \quad (B62)$$

where α_2 , β_2 , B_1 , B_2 , b_2 , c_2 , and d_2 are constants reflecting the influence of m_z , k_h , T_f (pounds per foot immersion, to be explained later), v_1 , i_B , β , and f_k .

$$\begin{aligned} \dot{z} = & \alpha_2 e^{\alpha_2 t} (B_1 \cos \beta_2 t + B_2 \sin \beta_2 t) + e^{\alpha_2 t} (-B_1 \beta_2 \sin \beta_2 t + B_2 \beta_2 \cos \beta_2 t) \\ & + \frac{2 d_2 t}{c_2} - \frac{2 b_2 d_2}{c_2^2} \end{aligned} \quad (B63)$$

$$\begin{aligned} \ddot{z} = & (\alpha_2^2 - \beta_2^2) e^{\alpha_2 t} (B_1 \cos \beta_2 t + B_2 \sin \beta_2 t) \\ & + 2\alpha_2 \beta_2 e^{\alpha_2 t} (-B_1 \sin \beta_2 t + B_2 \cos \beta_2 t) + \frac{2 d_2}{c_2} \end{aligned} \quad (B64)$$

PROBLEM 10.10

Let \mathbf{r} be the position vector of a particle of mass m moving in a central force field $\mathbf{F} = -\frac{k}{r^2} \hat{\mathbf{r}}$. Show that the angular momentum $\mathbf{L} = \mathbf{r} \times m\dot{\mathbf{r}}$ is constant in time. (Hint: Use the vector identity $\mathbf{r} \times \ddot{\mathbf{r}} = -\dot{\mathbf{r}} \times \dot{\mathbf{r}}$.)

$$(10a) \quad \frac{d}{dt} (\mathbf{r} \times m\dot{\mathbf{r}}) = m (\mathbf{r} \times \ddot{\mathbf{r}} + \dot{\mathbf{r}} \times \dot{\mathbf{r}}) = m (\mathbf{r} \times \ddot{\mathbf{r}}) = m (\mathbf{r} \times (-\frac{k}{r^3} \mathbf{r})) = 0$$

Since \mathbf{L} is constant in time, the motion of the particle is confined to a plane perpendicular to \mathbf{L} . (Hint: Use the vector identity $\mathbf{r} \times \dot{\mathbf{r}} = \frac{1}{2} \frac{d}{dt} (\mathbf{r} \times \mathbf{r}) = 0$.)

$$(10b) \quad \mathbf{r} \times \dot{\mathbf{r}} = \frac{1}{2} \frac{d}{dt} (\mathbf{r} \times \mathbf{r}) = 0$$

$$(10c) \quad \frac{d}{dt} (\mathbf{r} \times \dot{\mathbf{r}}) = \dot{\mathbf{r}} \times \dot{\mathbf{r}} = 0$$

$$(10d) \quad \mathbf{r} \times \ddot{\mathbf{r}} = \mathbf{r} \times (-\frac{k}{r^3} \mathbf{r}) = 0$$

$$\frac{d}{dt} (\mathbf{r} \times m\dot{\mathbf{r}}) = m (\mathbf{r} \times \ddot{\mathbf{r}} + \dot{\mathbf{r}} \times \dot{\mathbf{r}}) = 0$$

Mass and Mass Moment of Inertia

The values of m_x , m_z , and I_θ are needed to solve the previously expressed equations for x , z , and θ . These values of mass and mass moment of inertia (for movement as in pitching about the center of gravity) must be determined or suitably approximated based on the "given" parameters.

The underwater shape of polar icebreakers approximates that of a prolate ellipsoid as given by Saunders (24). Using a typical value for the ratio of $L/B = 4.0$, the following effective masses (body mass plus added mass) and mass moment of inertia are obtained:

$$m_x = 1.08 \frac{\Delta}{g} = 0.0336 \Delta \quad (B66)$$

$$m_z = 1.86 \frac{\Delta}{g} = 0.0578 \Delta \quad (B67)$$

$$I_\theta = 1.61 k^2 \frac{\Delta}{g} = 0.050 k^2 \Delta \quad (B68)$$

where k = radius of gyration.

As indicated by Vossler (25), a reasonable value for radius of gyration for an icebreaker would be

$$k = 0.22 L \quad (B72)$$

Substitution of that value in the equation for I_θ gives a reasonable approximation.

Problem 1. (10 points)

Let $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear map. Suppose that f satisfies the following conditions:

- $f(1, 0) = (2, 1)$
- $f(0, 1) = (1, 2)$

Find the matrix of f with respect to the standard basis of \mathbb{R}^2 .

$$(1) \quad A \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$(2) \quad A \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$(3) \quad A \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

where A is the matrix of f .

To solve this problem, you should use the fact that the matrix of a linear map is determined by its action on the standard basis.

$$(4) \quad A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

Therefore, the matrix of f with respect to the standard basis is $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$.

Damping Coefficients

Values are needed for k_h (heave damping coefficient) and k_p (pitch damping coefficient) in order to solve the equations for x , z , and θ .

A good approximation which is relatively simple to use can be found published by Vossler (25). Based on his coefficients,

$$k_p = \frac{0.10}{g} \Delta L^{3/2} \text{ lb-ft-sec} \quad (\text{B69})$$

$$k_h = \frac{3.0}{g} \frac{\Delta}{L^{1/2}} \text{ lb-sec/ft} \quad (\text{B70})$$

Pounds per Foot Immersion

For sea water,

$$T_f = (64.2) L B \alpha \text{ for sea water} \quad (\text{B71})$$

Solution of Bow Forces During Crushing

In the development of the summation equations two important substitutions were made. These can be used to determine the components of the force at the bow.

$$F_{BAC} = k_1 x^2 \quad (\text{B34})$$

$$F_{B2C} = k_2 x^2 \quad (\text{B14})$$

where k_1 and k_2 reflect the influence of σ , i_B , β , and f_k .

Boundary Conditions

Values are noted for μ (from the coefficient) and μ (from the boundary conditions) in order to solve the equation for μ . A good approximation when it is relatively simple to use the boundary conditions by using (17) based on the coefficient.

$$(18) \quad \mu^2 = \frac{0.111 - 1.111}{1.111 - 1.111} = 0$$

$$(19) \quad \mu^2 = \frac{0.111 - 1.111}{1.111 - 1.111} = 0$$

Results for Part I analysis

For the water:

$$(20) \quad \mu = 0.111 \text{ (from the water)}$$

Definition of the terms in the analysis

In the development of the equation various the important definitions were made. These can be used to determine the components of the force at the base.

$$(21) \quad \mu = 0.111$$

$$(22) \quad \mu = 0.111$$

where μ and μ without the influence of μ and μ .

Termination of the Crushing Phase, State 2.

Point A is defined as a point on the bow of the icebreaker at the waterline (point of force application from the ice). When this velocity has a direction which is forward and upward at an angle equal to the angle of the bow plus the trim there is no more penetration of the ice and crushing has ceased. See Figures B-XVIII and B-XIX.

Let $(GA)_x$ equal the horizontal distance from the center of gravity to the point of contact. Let $(GA)_z$ equal the vertical distance.

If γ is the angle from the horizontal to the velocity direction,

$$\tan \gamma = \frac{(GA)_x \dot{\theta} - \dot{z}}{\dot{x} - (GA)_z \dot{\theta}}$$

It may be seen that $\tan \gamma$ (or γ) is a function of t .

When

$$\tan \gamma = \tan (i_B + \theta)$$

crushing has ceased and State 2 is reached.

At this point sliding will commence (presuming the icebreaker still has forward velocity).

1. The function $f(x)$ is defined by

the rule $f(x) = \frac{1}{x^2}$ for $x > 0$. Find the value of $f(2)$ and $f(3)$.
The function $f(x)$ is defined by the rule $f(x) = \frac{1}{x^2}$ for $x > 0$.
Find the value of $f(2)$ and $f(3)$.
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Find the value of $f(2)$ and $f(3)$.
The function $f(x)$ is defined by the rule $f(x) = \frac{1}{x^2}$ for $x > 0$.
Find the value of $f(2)$ and $f(3)$.

$$\frac{f(2) - f(3)}{f(2) - f(3)} = \frac{1}{1}$$

It can be seen that for $f(x) = \frac{1}{x^2}$, $f(2) = \frac{1}{4}$ and $f(3) = \frac{1}{9}$.

$$f(2) = \frac{1}{4} \text{ and } f(3) = \frac{1}{9}$$

Therefore, the values of $f(2)$ and $f(3)$ are $\frac{1}{4}$ and $\frac{1}{9}$ respectively.
The function $f(x)$ is defined by the rule $f(x) = \frac{1}{x^2}$ for $x > 0$.
Find the value of $f(2)$ and $f(3)$.

$$f(2) = \frac{1}{4} \text{ and } f(3) = \frac{1}{9}$$

Sliding Phase, General

The sliding phase commences at State 2, when local crushing has ceased. The bow forces are no longer a function of the penetration. However, the vertical component and the horizontal component are inter-related.

During the sliding phase, the point of contact is assumed fixed (since further crushing would be negligible) and is assumed to be at the level of the waterline.

Bow Forces During Sliding

Figure B-XX illustrates the forces on the bow during sliding.

The force normal to the plating on each side is $N/2$. The friction

force on each side is then $f_x \frac{N}{2}$.

In order to resolve these forces into components, Figure B-XXII, let

$$a_s = \cos \beta \sin i_B + f_k \cos i_B \quad (B80)$$

$$b_s = \cos \beta \cos i_B - f_k \sin i_B \quad (B81)$$

After linearizing and using trigonometric substitution

$$F_{BZS} = N(b_s - a_s \theta) \quad (B82)$$

$$F_{BXS} = N(a_s + b_s \theta) \quad (B83)$$

Problem 1

The function $f(x)$ is defined as $f(x) = \frac{1}{x^2}$ for $x > 0$.
Find the derivative of $f(x)$ using the power rule.
Verify your answer using the quotient rule.

Using the limit definition, find the derivative of $f(x)$ at $x = 1$.
Compare this result with the derivative found in part (a).

Problem 2

Let $f(x) = \sin(x)$ and $g(x) = \cos(x)$.
Find the derivative of $f(x)g(x)$ using the product rule.

$$\frac{d}{dx} (\sin(x)\cos(x)) = \cos(x)\cos(x) - \sin(x)\sin(x)$$

Use the identity $\cos^2(x) - \sin^2(x) = \cos(2x)$ to simplify the result.

(10) $\frac{d}{dx} (\sin(x)\cos(x)) = \cos(2x)$

(11) $\frac{d}{dx} (\sin(x)\cos(x)) = \cos(2x)$

Verify the result in (10) using the chain rule.

(12) $\frac{d}{dx} (\sin(x)\cos(x)) = \cos(2x)$

(13) $\frac{d}{dx} (\sin(x)\cos(x)) = \cos(2x)$

It follows that

$$F_{\text{BXS}} = F_{\text{BZS}} \left[\frac{a_s}{b_2} + \frac{a_2}{b_2} \left(\frac{a_s^2 + b_s^2}{a_s b_s} \right) \theta \right] \quad (\text{B85})$$

This allows substitution later on.

Icebreaking Thrust

Of the total thrust, part is being used to overcome the resistance of the water and the other part is used against the ice. This latter part, or the thrust produced in excess of the requirement to maintain a velocity in water is called the "icebreaking thrust".

At State 1,

$$T(1 - t) = R_T \quad (\text{B21})$$

where T is thrust, t is the thrust deduction factor, and R_T is total resistance (no ice at State 1).

Then "icebreaking thrust" may be expressed

$$T_{\text{IB}} = T(1 - t) - R_T$$

R_T may be broken down into residual and frictional resistance. After breaking it down and making suitable substitutions based on the assumption that the rotational propeller speed remains constant,

$$T_{\text{IB}} = T_{\text{BOL}} - K_5 v - \left[\frac{T_{\text{BOL}} - K_5 v_1}{v_1^2} \right] v^2 \quad (\text{B27})$$

where T_{BOL} is bollard thrust assuming r p m constant.

the above...

$$(100) \quad \left[\frac{1}{2} \left(\frac{v_1^2}{g} + \frac{v_2^2}{g} \right) + \frac{z_1 - z_2}{g} \right] \rho g = \rho g h$$

The above equation may be written as...

Example 10

A horizontal pipe of diameter 10 cm is connected to a reservoir of water. The water level in the reservoir is 2 m above the center of the pipe. The pipe is 100 m long and the water flows through it with a velocity of 10 m/s. Calculate the head loss due to friction in the pipe.

Sol: Given...

$$(101) \quad h_f = f \frac{L}{D} \frac{v^2}{2g}$$

where f is friction factor, L is the length of pipe, D is the diameter of pipe and v is the velocity of flow.

Substituting the values...

The head loss due to friction is...

$$h_f = 1.5 \text{ m}$$

∴ The head loss due to friction is 1.5 m.

Example 11: A horizontal pipe of diameter 15 cm is connected to a reservoir of water. The water level in the reservoir is 3 m above the center of the pipe. The pipe is 150 m long and the water flows through it with a velocity of 15 m/s. Calculate the head loss due to friction in the pipe.

Sol: Given...

$$(102) \quad h_f = f \frac{L}{D} \frac{v^2}{2g}$$

∴ The head loss due to friction is...

An illustrative plot of equation (B27) is shown in Figure B-XVI. It is seen that State 1 there is no thrust being used against the ice. At the conclusion of sliding all thrust is being used against the ice and none to propel the ship through the water.

The term K_5 in equation (B27) is based on many parameters which would not be known at the early stages of design. It may be seen in Figure B-XVI that a linear approximation is suitable and in order.

$$T_{IB} = T_{BOL} \left(1 - \frac{v}{v_1}\right)$$

where $v = \frac{dx}{dt} = v_1 = \text{impact velocity.}$

Newton's Laws of Motion During Sliding

Figure B-XXIII illustrates the free body diagram for the sliding phase. Newton's laws of motion may be applied to the three types of motion encountered (x-direction, z-direction, and rotationally about the y-axis as in pitching). The three equations resulting are not independent.

In the forward direction,

$$\sum F_x = m_x \frac{d^2x}{dt^2}$$

$$T_{IB} \cos \theta - F_{BXS} = m_x \frac{d^2x}{dt^2} \quad (B89)$$

The first part of the proof is to show that the function $f(x)$ is continuous at x_0 . To do this, we need to show that for any $\epsilon > 0$, there exists a $\delta > 0$ such that if $|x - x_0| < \delta$, then $|f(x) - f(x_0)| < \epsilon$. This is done by using the definition of the function $f(x)$ and the properties of the real numbers.

$$\frac{1}{x} - \frac{1}{x_0} = \frac{x_0 - x}{xx_0} = \frac{x_0 - x}{x_0^2} + \frac{x_0 - x}{xx_0 - x_0^2}$$

$$\left| \frac{1}{x} - \frac{1}{x_0} \right| = \left| \frac{x_0 - x}{xx_0} \right| = \frac{|x_0 - x|}{|xx_0|}$$

Lemma 1.1.1

Let $f(x) = \frac{1}{x}$. Then $f(x)$ is continuous at $x_0 \neq 0$. Proof: Let $\epsilon > 0$ be given. We want to find a $\delta > 0$ such that if $|x - x_0| < \delta$, then $|\frac{1}{x} - \frac{1}{x_0}| < \epsilon$. We have $|\frac{1}{x} - \frac{1}{x_0}| = \frac{|x_0 - x|}{|xx_0|}$. Since $x_0 \neq 0$, we can choose δ small enough so that $|x| > \frac{|x_0|}{2}$ whenever $|x - x_0| < \delta$. Then $|xx_0| > \frac{|x_0|^2}{2}$, and so $|\frac{1}{x} - \frac{1}{x_0}| < \frac{2|x_0 - x|}{|x_0|^2}$. If we choose $\delta = \frac{\epsilon|x_0|^2}{2}$, then $|\frac{1}{x} - \frac{1}{x_0}| < \epsilon$ whenever $|x - x_0| < \delta$.

$$\frac{1}{x} = \frac{1}{x_0} + \frac{x_0 - x}{xx_0}$$

$$\left| \frac{1}{x} - \frac{1}{x_0} \right| = \left| \frac{x_0 - x}{xx_0} \right| = \frac{|x_0 - x|}{|xx_0|} \tag{1.1}$$

This becomes,

$$T_{BOL} - \left(\frac{T_{BOL}}{v_1}\right) \dot{x} - \frac{a_s}{b_s} F_{BZS} - \frac{a_s}{b_s} \left(\frac{a_s^2 + b_s^2}{a_s b_s}\right) F_{BZS} \theta - m_x \dot{x}' = 0 \quad (B90)$$

In the downward vertical direction (z-direction),

$$\sum F_z = m_z \frac{d^2 z}{dt^2}$$

$$-F_{BZS} - T_{IB} \theta - \Delta - T_f h - k_h \frac{dz}{dt} + \Delta = m_z \frac{d^2 z}{dt^2} \quad (B91)$$

This equation may be used to express F_{BZS} following substitution to eliminate T_{IB} and h .

$$F_{BZS} = -T_{BOL} \theta + \frac{T_{BOL}}{v_1} \dot{x} \theta - T_f z - T_f (LCG-LCF) \theta - k_h \dot{z} - m_z \dot{z}' \quad (B92)$$

In order to obtain an equation containing x , z , and θ as the only unknowns, equation (B92) is substituted into equation (B90) and then linearized. This equation is expressed as follows:

$$a_{11} \ddot{x} + b_{11} \dot{x} + c_{11} x + a_{12} \ddot{z} + b_{12} \dot{z} + c_{12} z + a_{13} \ddot{\theta} + b_{13} \dot{\theta} + c_{13} \theta = d_1 \quad (B95)$$

$$y'' + 2y' + 2y = 0 \quad (10)$$

$$(10) \quad y = e^{-x} (C_1 \cos x + C_2 \sin x)$$

Using the initial conditions $y(0) = 1$ and $y'(0) = 0$, we get

$$C_1 = 1, C_2 = 0$$

$$(11) \quad y = e^{-x} \cos x$$

The solution of the differential equation (10) is $y = e^{-x} \cos x$. This solution satisfies the initial conditions $y(0) = 1$ and $y'(0) = 0$.

$$(12) \quad y = e^{-x} \cos x$$

$$(13) \quad y = e^{-x} \cos x$$

In order to obtain an explicit solution, we use the method of variation of parameters. Let $y = u(x)e^{-x}$. Then $y' = u'e^{-x} - ue^{-x}$ and $y'' = u''e^{-x} - 2u'e^{-x} + ue^{-x}$. Substituting these into (10), we get $u'' - u = 0$. The general solution of this equation is $u = C_1 e^x + C_2 e^{-x}$. Thus, the general solution of (10) is $y = C_1 e^{-x} e^x + C_2 e^{-x} e^{-x} = C_1 + C_2 e^{-2x}$. Using the initial conditions $y(0) = 1$ and $y'(0) = 0$, we get $C_1 = 1/2$ and $C_2 = 1/2$. Therefore, the solution of (10) is $y = 1/2(1 + e^{-2x})$.

$$(14) \quad y = \frac{1}{2}(1 + e^{-2x})$$

where c_{11} , a_{13} , and b_{13} are zero. All other coefficients are constant and represent the influence of L , B , α , m_x , m_z , T_{BOL} , i_B , β , k_h , \dot{x}_2 , \dot{z}_2 , $\ddot{\theta}_2$ and f_k .

The summation of moments (counterclockwise) is taken about the center of gravity.

$$\sum M_g = I_\theta \frac{d^2\theta}{dt^2}$$

$$F_{BZS} (GA)_x + F_{BXS} (GA)_z + T_{IB} \cos \theta (KG - d)$$

$$- (\Delta + T_f h) \frac{GM_L \theta}{L} - k_p \frac{d\theta}{dt} - I_\theta \frac{d^2\theta}{dt^2} = 0 \quad (B96)$$

Substitution for F_{BZS} , F_{BXS} , $(GA)_x$, $(GA)_z$, T_{IB} and h leads to an equation containing x , z , and θ . This equation (B98), contains 43 terms and reflects the influence of all 16 parameters. Linearizing produces more terms but eventually the equation becomes,

$$a_{21} \ddot{x} + b_{21} \dot{x} + c_{21} x + a_{22} \ddot{z} + b_{22} \dot{z} + c_{22} z + a_{23} \ddot{\theta} + b_{23} \dot{\theta} + c_{23} \theta = d_2 \quad (B100)$$

where $a_{21} = 0$. All other coefficients are constant and reflect the influence of all 16 parameters.

The matrix A is given by $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ and the vector b is given by $b = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$. The system of linear equations is $Ax = b$. The inverse of A is $A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}$. The solution is $x = A^{-1}b$.

$$\frac{d^2 x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = F \cos(\omega t)$$

$$(A - \omega^2 I) \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} F \cos(\omega t) \\ 0 \end{pmatrix}$$

$$x = \frac{F \cos(\omega t)}{\omega_0^2 - \omega^2} + \frac{F \sin(\omega t)}{\omega(\omega_0^2 - \omega^2)}$$

The solution for x and y is $x = \frac{F \cos(\omega t)}{\omega_0^2 - \omega^2} + \frac{F \sin(\omega t)}{\omega(\omega_0^2 - \omega^2)}$ and $y = 0$. The solution for x and y is $x = \frac{F \cos(\omega t)}{\omega_0^2 - \omega^2} + \frac{F \sin(\omega t)}{\omega(\omega_0^2 - \omega^2)}$ and $y = 0$. The solution for x and y is $x = \frac{F \cos(\omega t)}{\omega_0^2 - \omega^2} + \frac{F \sin(\omega t)}{\omega(\omega_0^2 - \omega^2)}$ and $y = 0$.

$$\frac{d^2 x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = F \cos(\omega t)$$

The solution for x and y is $x = \frac{F \cos(\omega t)}{\omega_0^2 - \omega^2} + \frac{F \sin(\omega t)}{\omega(\omega_0^2 - \omega^2)}$ and $y = 0$. The solution for x and y is $x = \frac{F \cos(\omega t)}{\omega_0^2 - \omega^2} + \frac{F \sin(\omega t)}{\omega(\omega_0^2 - \omega^2)}$ and $y = 0$.

Newton's laws have allowed us to express two equations containing three unknowns, x , z , θ (and their derivatives).

Location Geometry

During the sliding phase the bow is in contact with a fixed point on the ice. Figure B-XXIV illustrates the relationship of $(\theta - \theta_2)$, $(z - z_2)$, and $(x - x_2)$.

It is seen that,

$$(x - x_2) = \frac{(GA)_{x2}(\theta - \theta_2) - (z - z_2)}{\tan(i_B + \theta)} + (GA)_{z2}(\theta - \theta_2)$$

where the subscript 2 indicates the initial condition for sliding, State 2.

Substitutions are made and then all non-linear terms are linearized and this leads to

$$a_{31}\ddot{x} + b_{31}\dot{x} + c_{31}x + a_{32}\ddot{z} + b_{32}\dot{z} + c_{32}z + a_{33}\ddot{\theta} + b_{33}\dot{\theta} + c_{33}\theta = d_3 \tag{E103}$$

where c_{31} , c_{32} , c_{33} , and d_3 are constants reflecting the influence of i_B , $(GA)_{x2}$, $(GA)_{z2}$, θ_2 , z_2 , and x_2 . Other coefficients are zero.

This gives us our necessary third equation.

... (faint text) ...

... (faint title) ...

... (faint text) ...

$$(\dots) \text{ and } (\dots)$$

... (faint text) ...

$$\frac{(\dots) + (\dots)}{(\dots)} = (\dots)$$

... (faint text) ...

... (faint text) ...

... (faint text) ...

... (faint text) ...

$$\dots + \dots + \dots + \dots + \dots + \dots + \dots + \dots + \dots + \dots$$

(2/23)

... (faint text) ...

... (faint text) ...

... (faint text) ...

Simultaneous Equations of Sliding

There are three equations of sliding to be solved simultaneously, equations (B95), (B100), and (B103) respectively. There are three unknowns, x, z, and θ (and their derivatives). These equations are the basis for the solution of sliding motion.

$$a_{11}\ddot{x} + b_{11}\dot{x} + c_{11}x + a_{12}\ddot{z} + b_{12}\dot{z} + c_{12}z + a_{13}\ddot{\theta} + b_{13}\dot{\theta} + c_{13}\theta = d_1$$

$$a_{21}\ddot{x} + b_{21}\dot{x} + c_{21}x + a_{22}\ddot{z} + b_{22}\dot{z} + c_{22}z + a_{23}\ddot{\theta} + b_{23}\dot{\theta} + c_{23}\theta = d_2$$

$$a_{31}\ddot{x} + b_{31}\dot{x} + c_{31}x + a_{32}\ddot{z} + b_{32}\dot{z} + c_{32}z + a_{33}\ddot{\theta} + b_{33}\dot{\theta} + c_{33}\theta = d_3$$

There must be an operation performed on these equations in order to solve them. The method chosen, since it incorporates initial conditions, is that of the LaPlace Transform.

The three equations become, as a result of the LaPlace transformation,

$$(a_{11}s^2 + b_{11}s) \mathcal{L}(x) + (a_{12}s^2 + b_{12}s + c_{12}) \mathcal{L}(z) + (c_{13}) \mathcal{L}(\theta) =$$

$$a_{11}s x_2 + a_{11}\dot{x}_2 + b_{11}x_2 + a_{12}s z_2 + a_{12}\dot{z}_2 + b_{12}z_2 + d_1/s$$

$$(b_{21}s + c_{21}) \mathcal{L}(x) + (a_{22}s^2 + b_{22}s + c_{22}) \mathcal{L}(z) + (a_{23}s^2 + b_{23}s + c_{23}) \mathcal{L}(\theta) =$$

Integration of the wave function

The wave function $\psi(x)$ is a function of position x and time t . It is a complex-valued function. The probability density is given by $|\psi(x)|^2$. The wave function is normalized so that the total probability is 1. The wave function is a solution to the Schrödinger equation.

$$\psi(x) = \frac{1}{\sqrt{2L}} \left[e^{i\pi x/L} + e^{-i\pi x/L} \right]$$

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$$b_{21}x_2 + a_{22}s z_2 + a_{22}\dot{z}_2 + b_{22}z_2 + a_{23}s \theta_2 + a_{23}\dot{\theta}_2 + b_{23}\theta_2 + d_2/s$$

$$(c_{31}) \underline{L}(x) + (c_{32}) \underline{L}(z) + (c_{33}) \underline{L}(\theta) = d_3/s$$

The right hand terms may be grouped into new constants by collecting coefficients of like powers of s . (For example $d_{11} = a_{11}\dot{x}_2 + b_{11}x_2 + a_{12}\dot{z}_2 + b_{12}z_2$).

The three simultaneous equations may now be written in shorter form.

$$(a_{11}s^2 + b_{11}s) \underline{L}(x) + (a_{12}s^2 + b_{12}s + c_{12}) \underline{L}(z) + (c_{13}) \underline{L}(\theta) =$$

$$d_{11} + d_{12}s + d_{13}/s$$

$$(b_{21}s + c_{21}) \underline{L}(x) + (a_{22}s^2 + b_{22}s + c_{22}) \underline{L}(z) + (a_{23}s^2 + b_{23}s + c_{23}) \underline{L}(\theta) =$$

$$d_{21} + d_{22}s + d_{23}/s$$

$$(c_{31}) \underline{L}(x) + (c_{32}) \underline{L}(z) + (c_{33}) \underline{L}(\theta) = d_{33}/s$$

Now we have three equations each containing the same three unknowns, $\underline{L}(x)$, $\underline{L}(z)$, and $\underline{L}(\theta)$.

Each of the unknowns may be resolved starting by using determinants.

$$x^2 + y^2 + z^2 = 1 \quad (1)$$

$$x^2 + y^2 + z^2 = 1 \quad (2)$$

The first two equations are identical, so we can subtract them to get:

$$x^2 + y^2 + z^2 - (x^2 + y^2 + z^2) = 1 - 1$$

$$0 = 0$$

The third equation is independent of the first two, so we can use it to solve for z:

$$x^2 + y^2 + z^2 = 1 \implies z^2 = 1 - x^2 - y^2$$

$$z = \pm \sqrt{1 - x^2 - y^2}$$

$$x^2 + y^2 + z^2 = 1 \implies x^2 + y^2 + (1 - x^2 - y^2) = 1$$

$$1 = 1$$

$$x^2 + y^2 + z^2 = 1 \implies x^2 + y^2 + z^2 = 1$$

Since the first two equations are identical, we can use the third equation to solve for z:

$$z = \pm \sqrt{1 - x^2 - y^2}$$

Thus, the solution set is the union of the two surfaces:

$$\begin{array}{c}
 \begin{array}{|ccc|}
 \hline
 (d_{11} + d_{12}s + d_{13}/s) & (a_{12}s^2 + b_{12}s + c_{12}) & (c_{13}) \\
 (d_{21} + d_{22}s + d_{23}/s) & (a_{22}s^2 + b_{22}s + c_{22}) & (a_{23}s^2 + b_{23}s + c_{23}) \\
 (d_{33}/s) & (c_{32}) & (c_{33}) \\
 \hline
 (a_{11}s^2 + b_{11}s) & (a_{12}s^2 + b_{12}s + c_{12}) & (c_{13}) \\
 (b_{21}s + c_{21}) & (a_{22}s^2 + b_{22}s + c_{22}) & (a_{23}s^2 + b_{23}s + c_{23}) \\
 (c_{31}) & (c_{32}) & (c_{33}) \\
 \hline
 \end{array} \\
 \hline
 \end{array}$$

The determinants for $\underline{L}(z)$ and $\underline{L}(\theta)$ are expressed in similar manner.

Each has the same denominator.

After collecting coefficients of like powers of s , the $\underline{L}(x)$ numerator becomes,

$$N_{13}s^2 + N_{12}s^2 + N_{11}s + N_{10} + N_{09}/s \quad (E108)$$

The numerator for the $\underline{L}(z)$ becomes,

$$N_{23}s^3 + N_{22}s^2 + N_{21}s + N_{20} + N_{19}/z \quad (E152)$$

(s^2)	(s^2)	(s^2)	- (s) \int
(s^2)	(s^2)	(s^2)	
(s^2)	(s^2)	(s^2)	
(s^2)	(s^2)	(s^2)	
(s^2)	(s^2)	(s^2)	

The characteristic equation for (a) and (b) is $s^2 + 2s + 1 = 0$.
 The roots are $s = -1 \pm j$.
 The transfer function for (a) is $\frac{1}{s^2 + 2s + 1}$.
 The transfer function for (b) is $\frac{1}{s^2 + 2s + 1}$.

(1000)
$$s^2 + 2s + 1 = (s + 1 - j)(s + 1 + j)$$

The transfer function for (a) is $\frac{1}{(s + 1 - j)(s + 1 + j)}$.

(1000)
$$s^2 + 2s + 1 = (s + 1 - j)(s + 1 + j)$$

The numerator for the $L(\theta)$ becomes,

$$N_{33}s^3 + N_{32}s^2 + N_{31}s + N_{30} + N_{29}/s \quad (E166)$$

The denominator, which is common to all three, becomes,

$$D_4s^4 + D_3s^3 + D_2s^2 + D_1s + D_0 \quad (E109)$$

Therefore,

$$L(x) = \frac{N_{13}s^4 + N_{12}s^3 + N_{11}s^2 + N_{10}s + N_{09}}{s [D_4s^4 + D_3s^3 + D_2s^2 + D_1s + D_0]} \quad (E110)$$

The $L(z)$ and $L(\theta)$ may be expressed in similar fashion. It is noted that this may be expressed as a proper fraction; the denominator is one power higher than the numerator.

By letting $a_{4x} = N_{13}/D_4$, $a_{3x} = N_{12}/D_4$, $a_{x2} = N_{11}/D_4$,

$$a_{1x} = N_{10}/D_4 \quad , \quad \text{and} \quad a_{0x} = N_{09}/D_4$$

along with

$$b_4 = D_3/D_4 \quad , \quad b_3 = D_2/D_4 \quad , \quad b_2 = D_1/D_4 \quad ,$$

$$\text{and} \quad b_1 = D_0/D_4 \quad ,$$

Let $f(x) = \int_0^x (x-t)^2 g(t) dt$

(1987)

$$f(x) = \int_0^x (x-t)^2 g(t) dt$$

Let $f(x) = \int_0^x (x-t)^2 g(t) dt$

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(1989)

$$f(x) = \int_0^x (x-t)^2 g(t) dt$$

Let $f(x) = \int_0^x (x-t)^2 g(t) dt$ and $f'(x) = \int_0^x (x-t) g(t) dt$.
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$$f(x) = \int_0^x (x-t)^2 g(t) dt$$

$$f(x) = \int_0^x (x-t)^2 g(t) dt$$

$$L(x) = \frac{a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0}{s^5 + b_4s^4 + b_3s^3 + b_2s^2 + b_1s} \quad (B113)$$

The numerator may be written as

$$(s - s_1) (s^4 + b_4s^3 + b_3s^2 + b_2s + b_1)$$

where $s_1 = 0$.

Biquadratic Solution

The biquadratic appearing in the denominator was presumed to have two pairs of complex conjugate roots. (This was after many other attempts proved unmeaningful. Furthermore this would lead to damped oscillatory motion which would appear typical in the physical case if the icebreaker were to slide back if unrestricted by static friction.)

This means that

$$(s^4 + b_4s^3 + b_3s^2 + b_2s + b_1) = \left[(s + \alpha_3)^2 + \beta_3^2 \right] \left[(s + \alpha_4)^2 + \beta_4^2 \right] \quad (B115)$$

Let $\alpha_3 = B_1$, $\beta_3 = A_1$, $\alpha_4 = B_2$, and $\beta_4 = A_2$

Then the right hand side becomes

$$\left[(s_4 B_1)^2 + A_1^2 \right] \left[(s+B_2)^2 + A_2^2 \right] = \left[s^2 + 2B_1s + (A_1^2 + B_1^2) \right] \left[s^2 + 2B_2s + (A_2^2 + B_2^2) \right]$$

(10.1)

$$\frac{2x^2 + 3x - 1}{x^2 - 1} = \frac{2x^2 + 3x - 1}{(x-1)(x+1)} = (1) \quad (10.1)$$

The number can be written as

$$(x^2 + 0x) + \frac{3x - 1}{x^2 - 1} = (x^2 + 0) + \frac{3x - 1}{x^2 - 1}$$

where $x = 1$

Partial Fractions

The technique of partial fractions is used to decompose a rational function into a sum of simpler fractions. This is done by expressing the numerator as a sum of terms whose denominators are factors of the denominator. For example, the fraction $\frac{2x^2 + 3x - 1}{x^2 - 1}$ can be decomposed into partial fractions as follows:

Let us assume that

$$\frac{2x^2 + 3x - 1}{x^2 - 1} = \frac{A}{x-1} + \frac{B}{x+1}$$

(10.2)

$$\left[\frac{2x^2 + 3x - 1}{(x-1)(x+1)} \right] = \left[\frac{A}{x-1} + \frac{B}{x+1} \right]$$

$$\frac{2x^2 + 3x - 1}{x^2 - 1} = \frac{A}{x-1} + \frac{B}{x+1} \quad (10.2)$$

Then the right hand side becomes

$$\left[\frac{2x^2 + 3x - 1}{x^2 - 1} \right] = \left[\frac{A}{x-1} + \frac{B}{x+1} \right] = \left[\frac{A(x+1) + B(x-1)}{(x-1)(x+1)} \right] = \left[\frac{Ax + A + Bx - B}{x^2 - 1} \right] = \left[\frac{(A+B)x + (A-B)}{x^2 - 1} \right]$$

Equation (E115) may be written as

$$\frac{s^4 + b_4 s^3 + b_3 s^2 + b_2 s + b_1}{s^2 + 2B_1 s + (A_1^2 + B_1^2)} = s^2 + 2B_2 s + (A_2^2 + B_2^2)$$

If the division is carried out on the left, that side becomes

$$s^2 + (b_4 - 2B_1)s + \left[b_3 - (A_1^2 + B_1^2) - 2B_1(b_4 - 2B_1) \right]$$

and this must equal

$$s^2 + 2B_2 s + (A_2^2 + B_2^2)$$

Therefore,

$$2B_2 = (b_4 - 2B_1) \tag{E117}$$

$$A_2^2 + B_2^2 = \left[b_3 - (A_1^2 + B_1^2) - 2B_1(b_4 - 2B_1) \right]$$

However, the division carried out above has a remainder, and this remainder must be set equal to zero.

$$\left[b_2 - (b_4 - 2B_1)(A_1^2 + B_1^2) \right] - \left[b_3 - (A_1^2 + B_1^2) - 2B_1(b_4 - 2B_1) \right] \left[2B_1 \right] = 0 \tag{E118}$$

and

$$b_1 - \left[b_3 - (A_1^2 + B_1^2) - 2B_1(b_4 - 2B_1) \right] \left[A_1^2 + B_1^2 \right] = 0 \tag{E120}$$

Let $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$.

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

Write this as $\mathbf{A}\mathbf{u} + \mathbf{B}\mathbf{v} = \mathbf{c}$ where $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$, and $\mathbf{c} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$.

$$\left[\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \right] = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

(1.1)

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix} - \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$\left[\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} - \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \right] = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

Let $\mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$ and write the above as $\mathbf{C}\mathbf{w} = \mathbf{d}$ where $\mathbf{C} = \begin{pmatrix} 1 & 2 & -2 & -1 \\ 3 & 4 & -1 & -3 \end{pmatrix}$ and $\mathbf{d} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$.

Write \mathbf{C} as $\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}$ where $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -2 & -1 \\ -1 & -3 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$, and $\mathbf{D} = \begin{pmatrix} 1 & 3 \\ 3 & 4 \end{pmatrix}$.

$$(1.2) \quad \mathbf{0} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \\ \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} 5 \\ 7 \end{pmatrix} \\ \begin{pmatrix} 5 \\ 7 \end{pmatrix} \end{bmatrix}$$

$$(1.3) \quad \mathbf{0} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \\ \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} 5 \\ 7 \end{pmatrix} \\ \begin{pmatrix} 5 \\ 7 \end{pmatrix} \end{bmatrix}$$

Now there are two equations with two unknowns, A_1 and B_1 . The unknowns are real numbers.

The solution of these two equations for B_1 leads to an equation of the form

$$a_6 B_1^6 + a_5 B_1^5 + a_4 B_1^4 + a_3 B_1^3 + a_2 B_1^2 + a_1 B_1 + a_0 = 0$$

where all coefficients are known constants reflecting the influence of b_4 , b_3 , b_2 , and b_1 .

It is noted that we started to find the roots of a fourth order polynomial and now it is "simplified" to a sixth order. Actually this is simpler; the unknown, B_1 , is a real number. (The roots of the quartic are not.)

It is discovered in the complete development that

$$0 < B_1 < \frac{b_4}{4} \quad \text{or} \quad \frac{b_4}{4} < B_1 < \frac{b_4}{2} \quad (\text{E121})$$

Therefore, B_1 is best solved by a trial and error iterative solution starting with B_1 near zero.

Once B_1 has been determined, the other values may be determined.

$$\alpha_3 = B_1 \quad (\text{E124})$$

$$\beta_3 = \sqrt{\frac{-b_2 - 3b_4\alpha_3^2 + 4\alpha_3^3 + 2b_3\alpha_3^3}{4\alpha_3^3 - b_4}} \quad (\text{E125})$$

$$\alpha_4 = \frac{1}{2} (b_4 - 2\alpha_3) \quad (\text{E126})$$

Let $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x^2}$. Then $f(x) > g(x)$ for $x > 1$ and $f(x) < g(x)$ for $0 < x < 1$.
 The function $f(x) = \frac{1}{x}$ is strictly decreasing on $(0, \infty)$ and $g(x) = \frac{1}{x^2}$ is strictly decreasing on $(0, \infty)$.
 The function $f(x) = \frac{1}{x}$ is strictly increasing on $(-\infty, 0)$ and $g(x) = \frac{1}{x^2}$ is strictly decreasing on $(-\infty, 0)$.

$$f(x) - g(x) = \frac{1}{x} - \frac{1}{x^2} = \frac{x-1}{x^2}$$

Since $f(x) - g(x) = \frac{x-1}{x^2}$, we see that $f(x) > g(x)$ for $x > 1$ and $f(x) < g(x)$ for $0 < x < 1$.
 The function $f(x) = \frac{1}{x}$ is strictly decreasing on $(0, \infty)$ and $g(x) = \frac{1}{x^2}$ is strictly decreasing on $(0, \infty)$.

Let $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x^2}$. Then $f(x) > g(x)$ for $x > 1$ and $f(x) < g(x)$ for $0 < x < 1$.
 The function $f(x) = \frac{1}{x}$ is strictly decreasing on $(0, \infty)$ and $g(x) = \frac{1}{x^2}$ is strictly decreasing on $(0, \infty)$.
 The function $f(x) = \frac{1}{x}$ is strictly increasing on $(-\infty, 0)$ and $g(x) = \frac{1}{x^2}$ is strictly decreasing on $(-\infty, 0)$.

$$(1) \quad \frac{1}{x} > \frac{1}{x^2} \iff \frac{1}{x} > \frac{1}{x^2} \iff x > 1$$

Since $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x^2}$, we see that $f(x) > g(x)$ for $x > 1$ and $f(x) < g(x)$ for $0 < x < 1$.
 The function $f(x) = \frac{1}{x}$ is strictly decreasing on $(0, \infty)$ and $g(x) = \frac{1}{x^2}$ is strictly decreasing on $(0, \infty)$.

$$(2) \quad \frac{1}{x} = \frac{1}{x^2} \iff x = 1$$

$$(3) \quad \frac{1}{x} < \frac{1}{x^2} \iff \frac{1}{x} < \frac{1}{x^2} \iff 0 < x < 1$$

$$(4) \quad \frac{1}{x} < \frac{1}{x^2} \iff \frac{1}{x} < \frac{1}{x^2} \iff 0 < x < 1$$

$$\beta_4 = \sqrt{b_3 - (\beta_3^2 + \alpha_3^2) - 4\alpha_3\alpha_4 - \alpha_4^2} \quad (\text{E127})$$

We now have the solution to the denominator of equation (E113).

Partial Fraction Form

It is necessary to put equation (E113) into the form of partial fractions in order to take the inverse LaPlace.

$$L(x) = \frac{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}{s [(s + \alpha_3)^2 + \beta_3^2] [(s + \alpha_4)^2 + \beta_4^2]} =$$

$$\frac{A_1}{s} + \frac{A_3 s + B_3}{[(s + \alpha_3)^2 + \beta_3^2]} + \frac{A_4 s + B_4}{[(s + \alpha_4)^2 + \beta_4^2]}$$

The right hand side contains five unknowns. The right hand term is put into the form of a polynomial with a common denominator. The coefficients of like terms in the numerator are collected and set equal to the equivalent coefficient of a like term in the numerator of the left side. For example,

$$a_4 s^4 = (\text{coefficients on right of } s^4) s^4$$

$$a_4 = (\text{coefficients of } s^4 \text{ from right side})$$

$$a_4 = A_1 + A_3 + A_4 \quad (\text{E128})$$

$$a_3 = 2\alpha_4 A_1 + 2\alpha_3 A_1 + 2\alpha_4 A_3 + B_3 + 2\alpha_3 A_4 + B_4 \quad (\text{E129})$$

(13.10)

$$\frac{1}{2} \frac{d}{dt} (x^2 + y^2) = \frac{1}{2} \frac{d}{dt} r^2 = r \dot{r}$$

The rate of change of the distance from the origin to the point is

Final Answer

It is required to find the rate of change of the distance from the origin to the point

when the point is at the position

$$\left[\frac{1}{\sqrt{2}} \cos t, \frac{1}{\sqrt{2}} \sin t \right] \quad (13.11)$$

$$\frac{d}{dt} \left(\frac{1}{\sqrt{2}} \cos t \right) = -\frac{1}{\sqrt{2}} \sin t, \quad \frac{d}{dt} \left(\frac{1}{\sqrt{2}} \sin t \right) = \frac{1}{\sqrt{2}} \cos t$$

The point has the Cartesian coordinates (x, y) and the polar coordinates (r, θ) . The rate of change of the distance from the origin to the point is \dot{r} . The rate of change of the angle is $\dot{\theta}$. The rate of change of the distance from the origin to the point is \dot{r} . The rate of change of the angle is $\dot{\theta}$. The rate of change of the distance from the origin to the point is \dot{r} . The rate of change of the angle is $\dot{\theta}$.

$$\dot{r} = \frac{1}{\sqrt{2}} \sin t$$

$$\dot{\theta} = \frac{1}{\sqrt{2}} \cos t$$

(13.12)

$$\dot{r} = \frac{1}{\sqrt{2}} \sin t$$

(13.13)

$$\dot{r} = \frac{1}{\sqrt{2}} \sin t$$

$$a_2 = \text{-----} \quad (B130)$$

$$a_1 = \text{-----} \quad (B131)$$

$$a_0 = (\alpha_3^2 + \beta_3^2)(\alpha_4^2 + \beta_4^2) A_1 \quad (B132)$$

These five equations contain five unknowns. They may be reduced to four equations with four unknowns by the substitution of A_1 (from equation (B132)) into all other equations.

After further substitution involving $d_1, d_2, d_3,$ and d_4 as previously defined, the four equations may be solved by the use of a matrix.

$$\begin{array}{cccc}
 (A_3) & (A_4) & (B_3) & (B_4) \\
 \left[\begin{array}{cccc|c}
 1 & 1 & 0 & 0 & d_1 \\
 2\alpha_4 & 2\alpha_3 & 1 & 1 & d_2 \\
 \varepsilon_4 & \varepsilon_3 & 2\alpha_4 & 2\alpha_3 & d_3 \\
 0 & 0 & \varepsilon_4 & \varepsilon_3 & d_4
 \end{array} \right]
 \end{array}$$

where $\varepsilon_3 = (\alpha_3^2 + \beta_3^2)$

$$\varepsilon_4 = (\alpha_4^2 + \beta_4^2)$$

The solution of the matrix leads to

(10.1)

$$x^2 + 2x + 1 = 0$$

(10.2)

$$x^2 - 2x + 1 = 0$$

(10.3)

$$x^2 + 2x + 1 = 0$$

These three equations involve the unknown x and are to be solved in four equations with four unknowns if the addition of x^2 is made to equation (10.3) and all other equations remain as they are. After further manipulation involving x^2 , x , and 1 as follows, along with the four equations may be solved by the use of a matrix.

x^2	(A)	(B)	(C)	(A)
x	2	0	2	0
1	1	1	1	0
x^2	0	0	0	0
x	0	0	0	0
1	0	0	0	0

$$x^2 + 2x + 1 = 0$$

$$x^2 - 2x + 1 = 0$$

The solution of the matrix leads to

$$B_4 = \frac{2(\alpha_3 - \alpha_4) \left[\varepsilon_4 d_3 - \varepsilon_4^2 d_1 - 2\alpha_4 d_4 \right] + (\varepsilon_3 - \varepsilon_4) \left[-\varepsilon_4 d_2 + 2\alpha_4 \varepsilon_4 d_1 + d_4 \right]}{2(\alpha_3 - \alpha_4) \left[2\alpha_3 \varepsilon_4 - 2\alpha_4 \varepsilon_3 \right] + (\varepsilon_3 - \varepsilon_4)^2}$$

$$B_3 = \frac{d_4 - \varepsilon_3 B_4}{\varepsilon_4} \quad (\text{E142})$$

$$A_4 = \frac{\varepsilon_4 (d_2 - 2\alpha_4 d_1) - d_4 + B_4 (\varepsilon_3 - \varepsilon_4)}{2\varepsilon_4 (\alpha_3 - \alpha_4)} \quad (\text{E143})$$

$$A_3 = d_1 - A_4 \quad (\text{E144})$$

One more substitution is in order.

$$\phi_{23} = \beta_3 A_3 \quad \phi_{24} = \beta_4 A_4 \quad (\text{E146})$$

$$\phi_{13} = B_3 - \alpha_3 A_3 \quad \phi_{14} = B_4 - \alpha_4 A_4$$

Now we have $L(x)$ with all terms known and in useable form.

$$L(x) = \frac{A_1}{s} + \frac{(s + \alpha_3)\phi_{23} + \beta_3\phi_{13}}{\beta_3 \left[(s + \alpha_3)^2 + \beta_3^2 \right]} + \frac{(s + \alpha_4)\phi_{24} + \beta_4\phi_{14}}{\beta_4 \left[(s + \alpha_4)^2 + \beta_4^2 \right]} \quad (\text{E145})$$

$$\frac{1}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}}$$

(10/10)

$$\frac{1}{\sqrt{1-x^2}}$$

(11/10)

$$\frac{1}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}}$$

(12/10)

$$\frac{1}{\sqrt{1-x^2}}$$

QUESTION 12 (10 marks)

(13/10)

$$x^2 - 2x + 1 = (x-1)^2$$

$$x^2 - 2x + 1 = (x-1)^2$$

QUESTION 13 (10 marks)

(14/10)

$$\frac{1}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}}$$

$$\begin{aligned}
 x = \int^{-1} (x) = A_1 + \frac{1}{\beta_3} e^{-\alpha_3 t} (\phi_{23} \cos \beta_3 t + \phi_{13} \sin \beta_3 t) \\
 + \frac{1}{\beta_4} e^{-\alpha_4 t} (\phi_{24} \cos \beta_4 t + \phi_{14} \sin \beta_4 t) \quad (E147)
 \end{aligned}$$

Differentiating with respect to time, t.

$$\begin{aligned}
 \dot{x} = \frac{-\alpha_3}{\beta_3} e^{-\alpha_3 t} (\phi_{23} \cos \beta_3 t + \phi_{13} \sin \beta_3 t) \\
 + e^{-\alpha_3 t} (-\phi_{23} \sin \beta_3 t + \phi_{13} \cos \beta_3 t) \\
 \frac{-\alpha_4}{\beta_4} e^{-\alpha_4 t} (\phi_{24} \cos \beta_4 t + \phi_{14} \sin \beta_4 t) \\
 + e^{-\alpha_4 t} (-\phi_{24} \sin \beta_4 t + \phi_{14} \cos \beta_4 t) \quad (E150)
 \end{aligned}$$

$$\begin{aligned}
 \ddot{x} = \frac{(\alpha_3^2 - \beta_3^2)}{\beta_3} e^{-\alpha_3 t} (\phi_{23} \cos \beta_3 t + \phi_{13} \sin \beta_3 t) \\
 - 2\alpha_3 e^{-\alpha_3 t} (-\phi_{23} \sin \beta_3 t + \phi_{13} \cos \beta_3 t) \\
 + \frac{(\alpha_4^2 - \beta_4^2)}{\beta_4} e^{-\alpha_4 t} (\phi_{24} \cos \beta_4 t + \phi_{14} \sin \beta_4 t) \\
 - 2\alpha_4 e^{-\alpha_4 t} (-\phi_{24} \sin \beta_4 t + \phi_{14} \cos \beta_4 t) \quad (E151)
 \end{aligned}$$

$$(x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2) \frac{d^2}{dt^2} = \frac{1}{2} + \frac{1}{2} - (x_1^2 + x_2^2) - 1 = 0$$

(1995) $(x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2) \frac{d^2}{dt^2} = \frac{1}{2} + \frac{1}{2}$

differentiating both sides w.r.t. t

$$(2x_1 \dot{x}_1 + 2x_2 \dot{x}_2 + 2x_3 \dot{x}_3 + 2x_4 \dot{x}_4 + 2x_5 \dot{x}_5 + 2x_6 \dot{x}_6) \frac{d^2}{dt^2} = \frac{d}{dt} \left(\frac{1}{2} + \frac{1}{2} \right) = 0$$

$$(x_1 \dot{x}_1 + x_2 \dot{x}_2 + x_3 \dot{x}_3 + x_4 \dot{x}_4 + x_5 \dot{x}_5 + x_6 \dot{x}_6) \frac{d^2}{dt^2} = 0$$

$$(2x_1 \dot{x}_1 + 2x_2 \dot{x}_2 + 2x_3 \dot{x}_3 + 2x_4 \dot{x}_4 + 2x_5 \dot{x}_5 + 2x_6 \dot{x}_6) \frac{d^2}{dt^2} = \frac{d}{dt} \left(\frac{1}{2} + \frac{1}{2} \right)$$

(2001)

$$(x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2) \frac{d^2}{dt^2} = 0$$

$$(2x_1 \dot{x}_1 + 2x_2 \dot{x}_2 + 2x_3 \dot{x}_3 + 2x_4 \dot{x}_4 + 2x_5 \dot{x}_5 + 2x_6 \dot{x}_6) \frac{d^2}{dt^2} = \frac{d}{dt} \left(\frac{1}{2} + \frac{1}{2} \right) = 0$$

$$(x_1 \dot{x}_1 + x_2 \dot{x}_2 + x_3 \dot{x}_3 + x_4 \dot{x}_4 + x_5 \dot{x}_5 + x_6 \dot{x}_6) \frac{d^2}{dt^2} = 0$$

$$(2x_1 \dot{x}_1 + 2x_2 \dot{x}_2 + 2x_3 \dot{x}_3 + 2x_4 \dot{x}_4 + 2x_5 \dot{x}_5 + 2x_6 \dot{x}_6) \frac{d^2}{dt^2} = \frac{d}{dt} \left(\frac{1}{2} + \frac{1}{2} \right) = 0$$

(2007)

$$(2x_1 \dot{x}_1 + 2x_2 \dot{x}_2 + 2x_3 \dot{x}_3 + 2x_4 \dot{x}_4 + 2x_5 \dot{x}_5 + 2x_6 \dot{x}_6) \frac{d^2}{dt^2} = 0$$

The equations given above represent the complete sliding motion of the center of gravity in the x-direction. The coefficients used are shown somewhat generalized here in that they do not carry the subscript x. For example,

$$\phi_{23} = P_{23x}$$

$$d_1 = d_{1x}$$

$$a_4 = a_{4x} \quad \text{etc.}$$

In order to solve for z and θ (along with their derivatives) it is only necessary to recognize that

$$L(z) = \frac{N_{23}s^4 + N_{22}s^3 + N_{21}s^2 + N_{20}s + N_{19}}{s[D_4s^4 + D_3s^3 + D_2s^2 + D_1s + D_0]} \quad (\text{E153})$$

and

$$L(\theta) = \frac{N_{33}s^4 + N_{32}s^3 + N_{31}s^2 + N_{30}s + N_{29}}{s[D_4s^4 + D_3s^3 + D_2s^2 + D_1s + D_0]} \quad (\text{E167})$$

Except for the values of the constant coefficients in the respective numerators, these equations are identical to equation (E110). The method of solution for z and θ is identical to that of the solution for x. The resulting equations are identical except for subscript. For example,

The following theorem states that if a function $f(x)$ is continuous on the interval $[a, b]$ and if $f(a) = f(b)$, then there exists at least one point c in the interval (a, b) such that $f(c) = f(a) = f(b)$.

$$f(x) = \cos x$$

$$f(a) = f(b)$$

$$\cos a = \cos b$$

In order to solve for c and b (along with their derivatives) it is only necessary to recognize that

$$(19a) \quad \frac{a_0 x^5 + a_1 x^4 + a_2 x^3 + a_3 x^2 + a_4 x + a_5}{x^5 + a_1 x^4 + a_2 x^3 + a_3 x^2 + a_4 x + a_5} = (19)$$

$$(19b) \quad \frac{a_0 x^5 + a_1 x^4 + a_2 x^3 + a_3 x^2 + a_4 x + a_5}{x^5 + a_1 x^4 + a_2 x^3 + a_3 x^2 + a_4 x + a_5} = (20)$$

Since for the values of the constant coefficients in the respective numerators, these equations are identical or similar (19), the same will be true for the solutions for x . The same reasoning applies to the solutions for x .

$$z = c_{1z} + \frac{1}{\beta_3} e^{-\alpha_3 t} (P_{23z} \cos \beta_3 t + P_{13z} \sin \beta_3 t) + \frac{1}{\beta_4} e^{-\alpha_4 t} (P_{24z} \cos \beta_4 t + P_{14z} \sin \beta_4 t) \quad (B163)$$

\dot{z} and \ddot{z} are obtained by differentiation and are given by equations (B164) and (B165).

$$\theta = c_{1\theta} + \frac{1}{\beta_3} e^{-\alpha_3 t} (P_{23\theta} \cos \beta_3 t + P_{13\theta} \sin \beta_3 t) + \frac{1}{\beta_4} e^{-\alpha_4 t} (P_{24\theta} \cos \beta_4 t + P_{14\theta} \sin \beta_4 t) \quad (B177)$$

$\dot{\theta}$ and $\ddot{\theta}$ are obtained by differentiation and are given by equations (B68) and (B69).

The equations completely describe the motion of the icebreaker during the sliding phase.

Vertical Force on Bow During Sliding

In the previous solution, F_{BZS} , the vertical bow force during sliding, was eliminated by substitution. Equation (B92) gives the substitution. Equation (B92) gives the value of F_{BZS} directly.

$$F_{BZS} = -T_{BOL} \theta + \frac{T_{BOL}}{v_1} \dot{x} \theta - T_f z - T_f (LCG-LCF) \theta - k_n \dot{z} - m_z \ddot{z}$$

$$(2) \quad \frac{d}{dt} \left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 \right) = 0$$

(3)

$$\frac{d}{dt} \left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 \right) = 0$$

where x is the displacement of the mass from its equilibrium position.

(4) and (5)

$$\frac{d}{dt} \left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 \right) = 0 \Rightarrow \dot{x}^2 + \frac{k}{m} x^2 = C$$

(6)

$$\frac{d}{dt} \left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 \right) = 0 \Rightarrow \dot{x}^2 + \frac{k}{m} x^2 = C$$

where C is a constant of integration.

(7) and (8)

The solution of the differential equation is given by

(9)

where C is a constant of integration.

In the above solution, \dot{x} is the velocity of the mass and x is the displacement of the mass from its equilibrium position.

where C is a constant of integration.

where C is a constant of integration.

$$\dot{x}^2 + \frac{k}{m} x^2 = C$$

$$\dot{x}^2 + \frac{k}{m} x^2 = C$$

where θ , \dot{x} , z , \dot{z} , and \dot{z}' are obtained from equations (B67), (E150), (E163), (E164), and (E165) respectively.

Termination of Sliding Phase, State 3

The equations of velocity, (E150), (E164), and (E178), may be combined vectorially to indicate the velocity of a point on the bow in contact with the ice. When this velocity (or likewise the horizontal component) becomes zero the sliding between ship and ice has terminated; State 3 is reached.

The z-component of the bow velocity is

$$v_{az} = \dot{z} - (GA)_x \dot{\theta} \quad (E161)$$

The x-component is

$$v_{ax} = \dot{x} - (GA)_z \dot{\theta} \quad (E162)$$

When the velocity of the bow relative to the ice becomes zero each of the components becomes zero. It is therefore sufficient to use either one to define State 3.

$$v_{ax} = \dot{x} - \left[(H-KG) + z \right] \dot{\theta} \quad (E183)$$

For each value of t (time, during sliding) there is a value of v_{ax} . When, by iteration, $v_{ax} = 0$, that time is assigned the symbol t_3

Let \mathcal{H} be a Hilbert space and let $\mathcal{H} = \mathcal{H}_1 \oplus \mathcal{H}_2$ be a decomposition of \mathcal{H} into two orthogonal subspaces \mathcal{H}_1 and \mathcal{H}_2 . Let $\mathcal{H}_1 = \text{span}\{e_1, e_2, \dots\}$ and $\mathcal{H}_2 = \text{span}\{e_3, e_4, \dots\}$ where $\{e_n\}$ is an orthonormal basis for \mathcal{H} .

Definition of the operator T

The operator T is defined on \mathcal{H} by $T(e_n) = e_{n+1}$ for $n \geq 1$. It is clear that T is a linear operator. We will show that T is an isometry. Let $x = \sum_{n=1}^{\infty} x_n e_n$ and $y = \sum_{n=1}^{\infty} y_n e_n$ be two vectors in \mathcal{H} . Then $\langle Tx, Ty \rangle = \langle \sum_{n=1}^{\infty} x_n e_{n+1}, \sum_{n=1}^{\infty} y_n e_{n+1} \rangle = \sum_{n=1}^{\infty} x_n \overline{y_n} = \langle x, y \rangle$. Thus T is an isometry. The adjoint of T is $T^*(e_n) = e_{n-1}$ for $n \geq 2$ and $T^*(e_1) = 0$.

(10.1) $T^*(e_1) = 0$

The adjoint is

(10.2) $T^*(e_n) = e_{n-1}$ for $n \geq 2$

Now the adjoint of the operator T is the operator T^* defined by $\langle Tx, y \rangle = \langle x, T^*y \rangle$ for all $x, y \in \mathcal{H}$. It is obvious that T^* is the adjoint of T . It is also clear that T^* is a linear operator.

(10.3) $T^*(e_n) = e_{n-1}$ for $n \geq 2$

For each value of n (10.3) shows that $T^*(e_n)$ is a vector in \mathcal{H} . It is also clear that T^* is a linear operator. It is also clear that T^* is a linear operator.

and all other values can be determined using t_3 .

Static Equilibrium, State 4

Presuming the icebreaker does not immediately back off, a slight amount of settling is going to take place with the bow remaining held in the ice at a position defined by State 3. If all velocities and accelerations were zero at State 3 there would be no "settling". However, this seems rather unlikely. For that reason, the static equilibrium problem will be solved using a point of support as defined by State 3. See Figure B-XXVII (Thrust has been dropped from the equilibrium solution. The screws must be stopped at some point anyway and this will lead to the higher value of sustained downward force under the bow.)

$$\sum F_x = 0 \tag{E184}$$

$$\sum F_z = -F_{BZ4} - (\Delta + T_f h_4) + \Delta = 0 \tag{E185}$$

$$\sum M_G = F_{BZ4} (GA_4)_x - (\Delta + T_f h_4) GM_L \theta_4 = 0 \tag{E187}$$

$$\text{where } h_4 = z_3 + (LCG-LCF)\theta_4 + (GA_3)_x (\theta_4 - \theta_3) \tag{E186}$$

Equation (E186) can be substituted into equations (E185) and (E187). This gives us two equations with two unknowns, θ_4 and F_{BZ4} .

Combining these two equations leads to one equation,

Mathematical Analysis

Consider the function $f(x) = x^2 + 2x + 1$. We can write this as $f(x) = (x+1)^2$. The function is always non-negative, and it is zero only at $x = -1$. This is a simple example of a function that is always non-negative. In general, a function $f(x)$ is called non-negative if $f(x) \geq 0$ for all x in its domain. For example, the function $f(x) = x^2$ is non-negative because $x^2 \geq 0$ for all real numbers x . Another example is $f(x) = |x|$, which is also non-negative. The function $f(x) = x^2 + 1$ is non-negative because $x^2 + 1 \geq 1 > 0$ for all x . The function $f(x) = x^2 - 1$ is not non-negative because it is negative for $-1 < x < 1$. The function $f(x) = x^2 - 2x + 1$ is non-negative because it is $(x-1)^2$.

(10a)
$$0 = \sum_{k=1}^n x_k^2$$

(11a)
$$0 = \sum_{k=1}^n (x_k^2 + 1) - \sum_{k=1}^n 1 = n + \sum_{k=1}^n x_k^2 - n = \sum_{k=1}^n x_k^2$$

(12a)
$$0 = \sum_{k=1}^n (x_k^2 + 1) - \sum_{k=1}^n (x_k^2 + 1) = 0$$

(13a)
$$(x_1^2 - 1) + (x_2^2 - 1) + \dots + (x_n^2 - 1) = \sum_{k=1}^n (x_k^2 - 1)$$

Equation (13a) can be substituted into equation (12a) and (11a).

The above two equations with two unknowns, $\sum_{k=1}^n x_k^2$ and $\sum_{k=1}^n (x_k^2 - 1)$.

Combining these two equations leads to one equation.

$$a_4 F_{BZ4}^2 + b_4 F_{BZ4} + c_4 = 0 \quad (E194)$$

where a_4 , b_4 , and c_4 are constants reflecting the influence of $(GA_3)_x$, θ_3 , z_3 and the hydrostatic properties of the icebreaker.

The meaningful solution of equation (E194) is

$$F_{BZ4} = \frac{-b_4 + \sqrt{b_4^2 - 4a_4c_4}}{2a_4} \quad (E195)$$

This value is the object of this research.

Incidentally, the final position, which may be of interest, may be readily determined.

The change in position (from State 3 to State 4) in the x-direction is negligible. The final trim, θ_4 , may be obtained from equation (E193),

$$\theta_4 = \frac{-F_{BZ4}}{d_1 T_f} + \frac{z_3}{d_1} + \frac{(GA_3)_x \theta_3}{d_1} \quad (E193)$$

$$\text{where } d_1 = (LCG-LCF) + (GA_3)_x$$

The final position of the center of gravity may be obtained from equation (E186),

$$z_4 = z_3 + (GA_3)_x (\theta_4 - \theta_3) \quad (E186)$$

(100)
$$D = \frac{1}{2} \omega^2 = \frac{1}{2} \omega^2 \frac{1}{\omega^2} = \frac{1}{2}$$

In the case of the present problem, we have $\omega = \frac{1}{2}$ and $\frac{1}{\omega^2} = 4$.
Therefore, the constant properties of the function
are (100) $D = \frac{1}{2} \times 4 = 2$.

(101)
$$\frac{1}{2} \omega^2 = \frac{1}{2} \times \frac{1}{\omega^2} = \frac{1}{2} \times 4 = 2$$

This value is the same as the value
obtained in the previous problem, which may be
verified by
the change in position (from $x = 0$ to $x = 1$) in the
direction of the x -axis. The final value of x may be obtained from equation (101).

(102)
$$x = \frac{1}{\omega^2} = \frac{1}{\frac{1}{4}} = 4$$

The final position of the center of gravity may be obtained from
equation (102).

(103)
$$x = 4 = \frac{1}{\omega^2} = \frac{1}{\frac{1}{4}} = 4$$

Extracting Thrust

Normally the static friction acting on the bow, once motion has stopped, reaches magnitudes greater than that of kinetic friction. It is possible, in fact probable, that the icebreaker may not slide back of its own accord. In that case backing thrust is necessary.

Actually any movement of the ship relative to the ice may free this static grip. This is where shifting the rudder, using heeling and/or trimming tanks, or setting off jarring blasts on the ice may help.

Most important though is the extracting thrust requirement, based on backing thrust sufficient in itself to free the icebreaker from this static grip.

Figure B-XXIX shows the forces acting on the bow. Figure B-XXX illustrates the free body diagram.

Solution of the free body diagram leads to the required extraction thrust,

$$E_t = \frac{F_{BZ4}}{\left(\frac{a_7}{b_7}\right) \cos \theta_4 - \sin \theta_4} \quad (B202)$$

$$\text{where } a_7 = (\cos \beta) \cos (i_B + \theta_4) + f_s \sin (i_B + \theta_4)$$

$$\text{and } b_7 = - (\cos \beta) \sin (i_B + \theta_4) + f_s \cos (i_B + \theta_4)$$

Introduction

The first section of the report deals with the general situation of the economy in the last few years. It is possible to find many statistics in the appendix of this report. It may be interesting to know that the average rate of increase in the price level is 1.5% per year. This is a very low rate compared with other countries. The reason for this is that the economy is still recovering from the recession of 1980-82. The price level is still below its long-run equilibrium level. This is why the average rate of increase is so low.

It can be seen from the above that the price level is still below its long-run equilibrium level. This is why the average rate of increase is so low. The price level is still below its long-run equilibrium level. This is why the average rate of increase is so low.

(1)
$$\frac{\partial \ln P}{\partial \ln Y} = \frac{1}{\epsilon}$$

where $\epsilon = \frac{\partial \ln P}{\partial \ln Y}$ and $\epsilon = \frac{\partial \ln P}{\partial \ln Y}$

Computer Program

The solution of all previous equations is extremely lengthy and there are several iterative processes involved. The digital computer has made it feasible to solve the entire problem. In fact it has made comparisons and further study possible.

The following is a listing of the input data which must be supplied:

- BP Length between perpendiculars, ft.
- B Beam at waterline, ft.
- H Mean draft, ft.
- DIS Displacement, lbs.
- BA Bow angle (from base line to stem), radians
- SA Spread angle complement (normal to bow plating with respect to centerline plane), radians
- VI Impact velocity, ft/sec.
- AL α , Waterplane coefficient, dimensionless
- CF LCF, Longitudinal position of the center of flotation (- if aft of amidships, + if forward), ft.
- CG LCG, Longitudinal position of the center of gravity (- if aft of amidships, + if forward), ft.
- GK KG, Height of center of gravity above base line, ft.
- D Height of thrust line above base line near center of gravity, ft.
- TB Bollard thrust which would be obtained for rpm used during crushing and sliding, lbs.
- GM GM_L , Longitudinal metacentric height, ft.
- FK Ice/ship kinetic coefficient of friction, dimensionless

SIG Compressive failure stress of ice, lb/ft².

The most important output of the program is the relatively sustained downward force under the bow during State 4.

F_{BZ4} = Vertical Force at Bow, lbs.

In addition other output is available as follows:

$X4$ = Forward motion from initial point of contact, ft.

$Z4$ = Vertical position of the center of gravity relative to the original position at the time of contact, ft.

$TH4$ = θ_4 , Final trim, radians

$WRAT$ = "White Ratio" = $\frac{F_{BZ4}}{(\text{Displacement})(\text{Impact velocity})}$, sec/ft

ET = Extracting thrust, lbs.

RAT = Extracting thrust/Bollard thrust, dimensionless.

Other information is readily available (if desired) as a function of time.

$X, XD, XDD = x, \dot{x}, \ddot{x}$ Forward position and its derivatives
(ft, ft/sec, and ft/sec²)

$Z, ZD, ZDD = z, \dot{z}, \ddot{z}$ Vertical position of the center of gravity and its derivatives (ft, ft/sec, and ft/sec²)

$TH, THD, THDD = \theta, \dot{\theta}, \ddot{\theta}$ Pitch angle and its derivatives
(radians, rad/sec, and rad/sec²)

The following is a list of the

names of the persons who have

been mentioned in the above

list, in the order in which

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been mentioned in the above

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F_{EZ} Downward force under bow during all phases as a function of time. lbs.

Other output is available directly but is only incidental to the solution of the basic problem. This includes total mass, including virtual (in each sense, x , z , θ), radius of gyration, pounds per foot immersion, pitch damping coefficient, heave damping coefficient, and scores of coefficients used in the solution.

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Suitable Simplifications

Some of the information required for the solution may not be known with much accuracy during the design stage. For that reason suitable approximations are in order.

For example

$$BM_L = \frac{C_{il} L^2}{C_b H} \quad (B206)$$

$$\text{and } KB = \frac{\alpha}{C_b + \alpha} H \quad (B207)$$

$$\text{where } C_{il} = 0.030 + 0.1304 (\alpha - 0.65) \quad (B203)$$

C_b = Block coefficient

L = Length between perpendiculars

H = Draft

α = Waterplane coefficient

Other such approximations include KG (and therefore GM_L), and bollard thrust.

Parametric Study

The variation of a parameter certainly has an effect on the sustained downward force. There are sixteen input variables. (The static coefficient of friction is only for the determination of extraction thrust.)

Of the sixteen, the following may not be considered independent:

GM_L , LCF , α , Δ , H , B , LEP

Problem 1

Let X_1, X_2, \dots, X_n be independent random variables with the following probability density functions (pdfs):

(a) $f_1(x) = \frac{1}{2}e^{-x/2}, x > 0$

(b) $f_2(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-x^2/(2\sigma^2)}, x \in \mathbb{R}$

(c) $f_3(x) = \frac{1}{\Gamma(k)\theta^k}x^{k-1}e^{-x/\theta}, x > 0$

- (d) $f_4(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-x^2/(2\sigma^2)}, x \in \mathbb{R}$
- (e) $f_5(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-x^2/(2\sigma^2)}, x \in \mathbb{R}$
- (f) $f_6(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-x^2/(2\sigma^2)}, x \in \mathbb{R}$
- (g) $f_7(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-x^2/(2\sigma^2)}, x \in \mathbb{R}$

Define each random variable X_i in terms of θ_i and σ_i .

Joint pdf:

Parameter Estimation

The estimator of a parameter θ is said to be unbiased if $E(\hat{\theta}) = \theta$. The estimator is said to be efficient if it has the minimum variance among all unbiased estimators. The estimator is said to be consistent if $Var(\hat{\theta}) \rightarrow 0$ as $n \rightarrow \infty$. The estimator is said to be asymptotically normal if $\sqrt{n}(\hat{\theta} - \theta) \rightarrow N(0, I^{-1}(\theta))$ as $n \rightarrow \infty$. The estimator is said to be asymptotically efficient if it is consistent and asymptotically normal.

$E(\hat{\theta}) = \theta, Var(\hat{\theta}) = \frac{1}{nI(\theta)}$

A change in any one of these involves a change in another.

Some of the parameters are relatively independent within reasonable limits. These are as follows:

Bollard thrust, D (Height of thrust line), KG, LCG, β (the spread angle complement of the bow plating), and, perhaps most significant i_B (the bow angle relative to the base line.)

A few of the parameters may be considered completely independent.

They are as follows:

v_1 (impact velocity), f_k (kinetic coefficient of friction),
and (compressive failure stress of ice).

The "independent" variables will be varied over a suitable range to determine the effect on the downward force at the bow. The impact velocity will be varied along with each one. The remaining parameters will be assigned values representing the "Wind" Class Icebreaker.

(Actually, the "Glacier" Class and the "Lenin" Class will be used also but the illustrations of result will be based on the "Wind" Class. Conclusions, unless noted to the contrary, will be valid for all three classes.)

"White Ratio"

For lack of a better name, the ratio is defined as

$$\text{WRAT} = \frac{F_{BZ}^2}{(\text{DIS})(V_1)} \text{ sec/ft} \quad (\text{B214})$$

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(100)

$$\frac{1}{(100)^2} = \frac{1}{10000}$$

It is anticipated that the downward force under the bow will be affected approximately linearly with displacement and impact velocity. The coefficient (White Ratio) may be of use for approximate comparison of the parameter effects.

The "dependent variables" (GM_L , LCF, α , Δ , H, B, LBP) may be varied only by varying other parameters simultaneously. For example, a change in α , the waterplane coefficient, will cause a change in pounds-per-foot-immersion (accounted for automatically in the program), the height of the center of buoyance (KB), and the distance from the center of buoyance to the longitudinal metacenter (BM_L). Keeping displacement length, draft, and beam constant, the resulting change may be examined.

The longitudinal position of the center of flotation may be changed slightly and a change in form would then be necessary to keep displacement, length, draft, and beam constant. This shift is incorporated to find the effect.

The beam-to-draft ratio is varied to investigate the effect. (Displacement is held constant as is length.) A new solution for GM_L is necessary.

The length-to-beam ratio (frequently 4.0 in polar icebreakers) is varied to investigate the effect. (Displacement and draft are held constant.) A new solution for GM_L is necessary.

Displacement effect is investigated three ways. One is simply the comparison of three different classes of icebreakers (Wind, Glacier, Lenin).

It is considered that the proposed scheme will be
of the greatest benefit to the community and
the Government (H.M.S.O. 1974) and the Government
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Government of the United Kingdom. (H.M.S.O. 1974)

A second way is by holding, for a given class, the length, draft, and beam constant while varying the block coefficient (and consequently displacement).

The third displacement comparison is to vary the size of a given class of icebreaker such that geometrically similar ships (geosims) are generated. For example, all lengths are multiplied by the scale factor; volumes (i.e. displacement) are multiplied by the scale factor cubed.

By means of the variations indicated above it is possible to determine what values (i.e. high, low) would lead to the generation of the maximum sustained downward generated by ramming.

Model Parameters

In order to model test it is necessary to multiply all ship linear dimensions by $1/2$. See equations (B222) and (B223). Coefficients are dimensionless and are not changed. Likewise, bow angle and spread angle are not changed.

The ship displacement and the bollard thrust must be multiplied by $1/\lambda^3$. See equations (B226).

The compressive failure stress of the ice must be multiplied by $1/\lambda$. This, of course, implies that a different bow supporting medium must be used in model tests. (Care must be taken to adjust the coefficient of kinetic friction if necessary.)

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Legal Analysis

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Since gravity and dynamics are involved, it is necessary that ship and model be at equivalent Froude Numbers.

$$V_{1m} = V_{1s} / \sqrt{\lambda} \quad (B229)$$

By using the above-mentioned scales, the scaled final position (State 4.) of model and ship will be identical and the downward forces under the bow will be related as follows:

$$F_m = F_s / \lambda^3 \quad (B230)$$

The relationship of time of events for the ship compared to the time of events for the model is

$$\frac{T_s}{T_m} = \sqrt{\lambda} \quad (B231)$$

Let \mathcal{H} be a Hilbert space and T a self-adjoint operator on \mathcal{H} .

Suppose λ is an eigenvalue of T with eigenvector v .

(10a)

$$Tv = \lambda v$$

By taking the inner product of both sides of (10a) with v , we obtain
(10b) $\langle Tv, v \rangle = \lambda \langle v, v \rangle$. Since T is self-adjoint, $\langle Tv, v \rangle = \langle v, Tv \rangle = \lambda \langle v, v \rangle$.
Thus, $\lambda \langle v, v \rangle = \lambda \langle v, v \rangle$, which is true for all v .

(10c)

$$\lambda = \overline{\lambda}$$

The equality of the real and imaginary parts of λ shows that λ is real.
This is true for all eigenvalues of T .

(10d)

$$\lambda = \overline{\lambda}$$

Let λ be an eigenvalue of T with eigenvector v . Then $Tv = \lambda v$.
Taking the inner product of both sides with v , we get
 $\langle Tv, v \rangle = \lambda \langle v, v \rangle$.
Since T is self-adjoint, $\langle Tv, v \rangle = \langle v, Tv \rangle = \lambda \langle v, v \rangle$.
Thus, $\lambda \langle v, v \rangle = \lambda \langle v, v \rangle$, which implies $\lambda = \overline{\lambda}$.
Therefore, λ is real.

III RESULTS

Prediction of Westwind Behavior

The problem of a "Wind Class" icebreaker ramming virtually unyielding ice is solved using the solution indicated in Chapter II. The parameters used are given in Table IV. Note that three different impact velocities are used 11.32 ft/sec (6.7 knots), 13.51 ft/sec, (8.0 knots), and 15.52 ft/sec (9.2 knots).

The solution to the problem includes x , z , and θ (as well as their respective first and second derivatives as functions of time. In addition, the downward force under the bow is determined as a function of time.

The choice of parameters is based on the characteristics of the Westwind at the time of tests run during the summer of 1963. During the period of contact with the ice, full throttle was used so the maximum value of bollard thrust is used in the program.

Figures VIII, IX and X are plots of the prediction of x , \dot{x} , \ddot{x} ; z , \dot{z} , \ddot{z} ; and θ , $\dot{\theta}$, $\ddot{\theta}$ as functions of time for the run (37B) with an impact velocity of 13.51 ft/sec.

Figure XI is a plot of the predictions of \ddot{z} , $\ddot{\theta}$, $\ddot{\theta}$, and \dot{x} as functions of time for an impact velocity of 11.32 ft/sec (Run 36B). Figure XIII is similar but for Run 37B and Figure XV is for Run 38B.

Figure XII is a plot of the prediction of the downward force under the bow as a function of time for Run 36B. (Figure XIV is for Run 37B

Properties of the function

The function $f(x)$ is defined for all real numbers x . It is continuous and differentiable everywhere. The function is periodic with period 2π . The function is even, i.e., $f(x) = f(-x)$. The function is bounded, with $0 \leq f(x) \leq 1$. The function is concave down on $(0, \pi)$ and concave up on $(\pi, 2\pi)$. The function has a local maximum at $x = \pi$ and a local minimum at $x = 0$ and $x = 2\pi$.

The function is the product of the function $\cos(x)$ and a constant 1 . The function is the sum of the function $\cos(x)$ and the function 0 . The function is the difference of the function $\cos(x)$ and the function 0 . The function is the quotient of the function $\cos(x)$ and the function 1 .

The function is periodic with period 2π . The function is even, i.e., $f(x) = f(-x)$. The function is bounded, with $0 \leq f(x) \leq 1$. The function is concave down on $(0, \pi)$ and concave up on $(\pi, 2\pi)$. The function has a local maximum at $x = \pi$ and a local minimum at $x = 0$ and $x = 2\pi$.

Figure 1 shows the graph of the function $f(x) = \cos(x)$ for $x \in [0, 2\pi]$. The function is periodic with period 2π . The function is even, i.e., $f(x) = f(-x)$. The function is bounded, with $0 \leq f(x) \leq 1$. The function is concave down on $(0, \pi)$ and concave up on $(\pi, 2\pi)$. The function has a local maximum at $x = \pi$ and a local minimum at $x = 0$ and $x = 2\pi$.

Figure 2 shows the graph of the function $f(x) = \cos(x)$ for $x \in [0, 2\pi]$. The function is periodic with period 2π . The function is even, i.e., $f(x) = f(-x)$. The function is bounded, with $0 \leq f(x) \leq 1$. The function is concave down on $(0, \pi)$ and concave up on $(\pi, 2\pi)$. The function has a local maximum at $x = \pi$ and a local minimum at $x = 0$ and $x = 2\pi$.

Figure 3 shows the graph of the function $f(x) = \cos(x)$ for $x \in [0, 2\pi]$. The function is periodic with period 2π . The function is even, i.e., $f(x) = f(-x)$. The function is bounded, with $0 \leq f(x) \leq 1$. The function is concave down on $(0, \pi)$ and concave up on $(\pi, 2\pi)$. The function has a local maximum at $x = \pi$ and a local minimum at $x = 0$ and $x = 2\pi$.

and Figure XVI is for Run 38B.)

It must be recalled that the solution is based on an icebreaker identical to the Westwind except that it was assumed the stem was straight and continuous from the waterline to the keel.

Observation of Westwind Trials

In the summer of 1963 trials were run using the C.G.C. Westwind off the northwest coast of Greenland. (37)

The following values pertain to the trial runs of interest (26):

$$BP = 250.0 \text{ ft}$$

$$B = 64.0 \text{ ft}$$

$$\text{Draft frd} = 25.0 \text{ ft}$$

$$\text{Draft aft} = 27.5 \text{ ft}$$

$$\text{Mean draft } H = 26.25'$$

$$RA \text{ (bow angle)} = 0.523$$

$$SA (\beta) = 0.886$$

$$AL (\alpha) = 0.724$$

$$\text{SIG (failure stress of ice)} = 19.6 \text{ kg/cm}^2 \quad (27)$$

in tension

Incidentally, the ice thickness exceeded 580 cm, or 19.0 ft.

By conversion,

$$\text{SIG} = 279.0 \text{ lb/in}^2 = 40,200 \text{ lb/ft}^2 \text{ (in tension)}$$

and Tables III and IV.

It may be recalled that the maximum value of Δ is

obtained for the condition $\Delta = 0$ and is given by

the condition $\Delta = 0$ is satisfied when

Discussion of Results

In the range of Δ values from 0 to 100, the

of the maximum value of Δ is

The following table gives the values of Δ for

$$\Delta = 0.001$$

$$\Delta = 0.01$$

$$\Delta = 0.1$$

$$\Delta = 1.0$$

$$\Delta = 10.0$$

$$\Delta = 100.0$$

$$\Delta = 1000.0$$

$$\Delta = 10000.0$$

It is seen from the above that the values of Δ are

in order

Increasingly, the values of Δ are

By comparison

$$\Delta = 100.0 \text{ for } \Delta = 10000.0 \text{ (in order)}$$

From the "Displacement and Other Curves" for the Westwind (28),

$$\text{Trim} = 2.5 \text{ ft by the stern}$$

$$H = 26.25 \text{ ft}$$

$$\text{DIS} = 5600 \text{ tons} = 12,530,000.0 \text{ lb}$$

From a "Wind Class" inclining experiment (29), for normal load,

$$\text{GK (height of c.g.)} = 23.4 \text{ ft}$$

$$\text{CF (longitudinal position of c.g. of waterplane)} = -1.3 \text{ ft}$$

$$\text{Uncorrected LCB} = -2.4 \text{ ft}$$

$$\text{Moment to change trim 1"} = 18.6 \times 20 = 372 \text{ ft-tons}$$

$$\text{Moment} = (372)(30) = 11,160 \text{ ft-tons}$$

$$\text{Shift of LCB aft} = \frac{11,160}{5,600} = 1.99 \text{ ft aft}$$

$$\text{LCG} = \text{LCB} = -2.40 - 1.99 = -4.39 \text{ ft}$$

$$\text{Bow angle is increased due to trim by } \frac{2.5}{250.0} = 0.010 \text{ rad.}$$

$$\text{BA} = 0.533$$

At the center of gravity, the thrust line is approximately 16 feet above the keel. Therefore,

$$D = 16.0$$

From equation (B20)

From the "Methodology" and "Data Sources" sections (10)

$$1000 = 1000 \times 1.05 = 1050$$

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$$1000 = 1000 \times 1.05 = 1050$$

At the center of gravity, the forces are approximately 1000

From the "Methodology" and "Data Sources" sections (10)

$$1000 = 1000 \times 1.05 = 1050$$

From the "Methodology" and "Data Sources" sections (10)

$$C_b = \frac{DIS}{64.2 LEH} = \frac{12,530,000}{(64.2)(250)(64)(26.25)}$$

$$C_b = 0.464$$

From equation (B207)

$$KB = \frac{\alpha}{C_b + \alpha} H = \frac{(0.724)(26.25)}{(0.464 + 0.724)}$$

$$KB = \frac{(0.724)(26.25)}{(1.188)} = 16.0 \text{ ft.}$$

From equation (B203),

$$C_{il} = 0.030 + 0.1304 (\alpha - 0.65)$$

$$C_{il} = 0.030 + 0.1304 (0.724 - 0.650)$$

$$C_{il} = 0.030 + 0.1304 (0.074) = 0.0396$$

From equation (B206)

$$EM_L = \frac{C_{il} L^2}{C_b H} = \frac{(0.0396)(250)^2}{(0.464)(26.25)} = 203 \text{ ft}$$

From equation (B211),

$$GM_L = KB + EM_L - KG$$

$$GM_L = (16.0) + (203.0) - (23.4) = 195.6 \text{ ft.}$$

Maximum thrust was used during the sliding and crushing phase.

Maximum bollard thrust for the "Wind Class" is 270,000 lbs. (4).

$$\frac{(0.05)(1000000)}{(1.05)^1 + (1.05)^2} = \frac{50000}{1.05 + 1.1025} = 23000$$

$$23000 = P_1$$

(1000) mortgage worth

$$\frac{(0.05)(1000000)}{(1.05)^3 + (1.05)^4} = \frac{50000}{1.1576 + 1.2167} = 22000$$

$$\text{PV} = \frac{(0.05)(1000000)}{(1.05)^5 + (1.05)^6} = 21000$$

(1000) mortgage worth

$$(0.05)(1000000) + (0.05)(1000000) = 100000$$

$$(0.05)(1000000) + (0.05)(1000000) = 100000$$

$$(0.05)(1000000) + (0.05)(1000000) = 100000$$

(1000) mortgage worth

$$\text{PV} = \frac{(0.05)(1000000)}{(1.05)^7 + (1.05)^8} = 20000$$

(1000) mortgage worth

$$23000 + 22000 + 21000 + 20000 = 86000$$

$$\text{PV} = (1.05)^{-1} + (1.05)^{-2} + (1.05)^{-3} + (1.05)^{-4} = 3.17$$

the value of the mortgage is 86000 and the value of the annuity is 3.17

(4) The value of the mortgage is 86000 and the value of the annuity is 3.17

In summary, the values given in Table IV pertain to the three trial runs of interest.

The observed behavior of these three runs is given in Figures XI, XIII and XV. These are plots of \dot{z} , $\dot{\theta}$ (measured by accelerometers) (the one for \dot{z} was mounted near the center of gravity.), θ (measured by gyro), and \dot{x} (as measured using "Raydist". The value of \dot{x} was not considered to be reliable according to DMB personnel.

It is noted that the protrusion of the housing for the formerly installed bow propeller would come in contact with the ice after about 1.4 seconds in Run 36B (about 1.2 seconds for 37B.; about 1.0 seconds for 38B). For that reason, observed results are not plotted much beyond those times.

Figures XII, XIV and XVI are plots of the strain reading in the transverse direction at the lower portion of a forward transverse bulkhead. There is no direct correlation to the magnitude of the load at the bow. However, the strain on that bulkhead is primarily created by the bow load. For that reason it is plotted to show that the maximum peak load occurs about half a second after initial contact rather than when the icebreaker has come to a stop with its bow well up on the ice.

The results of the present study are shown in Figure 1. The results of the present study are shown in Figure 1.

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TABLE IV
PARAMETERS USED FOR FULL SCALE TEST COMPARISON

WESTWIND

HP	=	250.0 ft	B	=	64.0 ft
H	=	26.25 ft	DIS	=	12,530,000 lbs
BA	=	0.533 rad*	SA	=	0.886
AL	=	0.724	CF	=	-1.30 ft
CG	=	-4.39 ft	GK	=	23.4 ft
D	=	16.0	TB	=	270,000 lbs
GM _L	=	195.6 ft	FK	=	0.2
FS	=	0.8	SIG	=	144,000 lbs. (30)

Run 36B Impact velocity, V_I = 11.32 ft/sec

Run 37B Impact velocity, V_I = 13.51 ft/sec

Run 38B Impact velocity, V_I = 15.52 ft/sec

* Slightly greater than 30° to account for initial trim.

TABLE II

MEAN AND STANDARD DEVIATION OF THE DATA

		MEAN	STANDARD DEVIATION
1	100	100	10
2	100	100	10
3	100	100	10
4	100	100	10
5	100	100	10
6	100	100	10
7	100	100	10
8	100	100	10
9	100	100	10
10	100	100	10
11	100	100	10
12	100	100	10
13	100	100	10
14	100	100	10
15	100	100	10
16	100	100	10
17	100	100	10
18	100	100	10
19	100	100	10
20	100	100	10
21	100	100	10
22	100	100	10
23	100	100	10
24	100	100	10
25	100	100	10
26	100	100	10
27	100	100	10
28	100	100	10
29	100	100	10
30	100	100	10
31	100	100	10
32	100	100	10
33	100	100	10
34	100	100	10
35	100	100	10
36	100	100	10
37	100	100	10
38	100	100	10
39	100	100	10
40	100	100	10
41	100	100	10
42	100	100	10
43	100	100	10
44	100	100	10
45	100	100	10
46	100	100	10
47	100	100	10
48	100	100	10
49	100	100	10
50	100	100	10

NOTE: The mean and standard deviation of the data are given in Table II.

Figure VIII
 x , \dot{x} , and \ddot{x} (Predicted)
vs
TIME
Run 37B (Parameters given in Table IV)
R. m. W.
April 1965

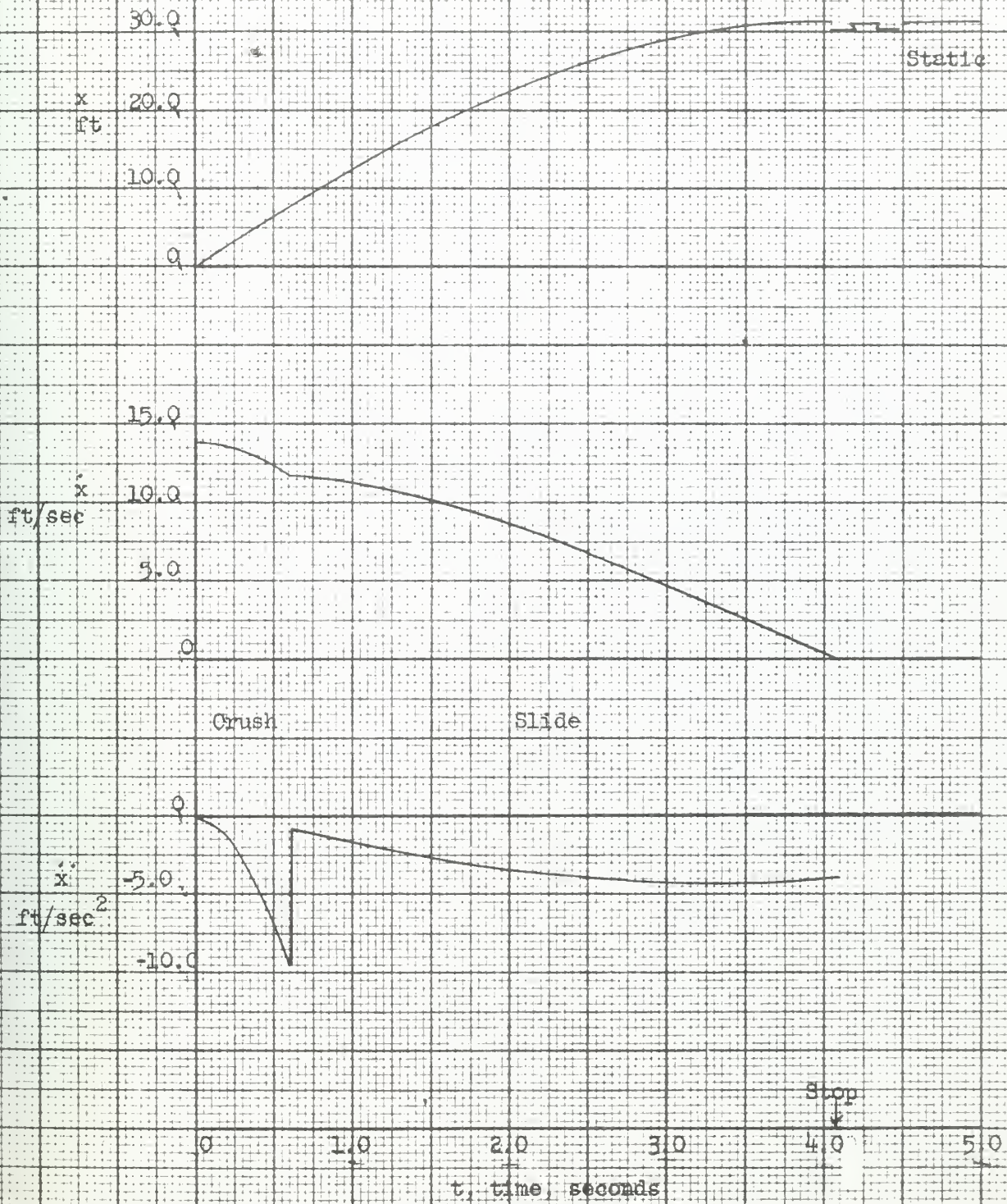


Figure IX
 z , \dot{z} , and \ddot{z} (Predicted)

vs
TIME

Run 37 B (Parameters given in Table IV)

R. m. W. April 1965

R. m. W.

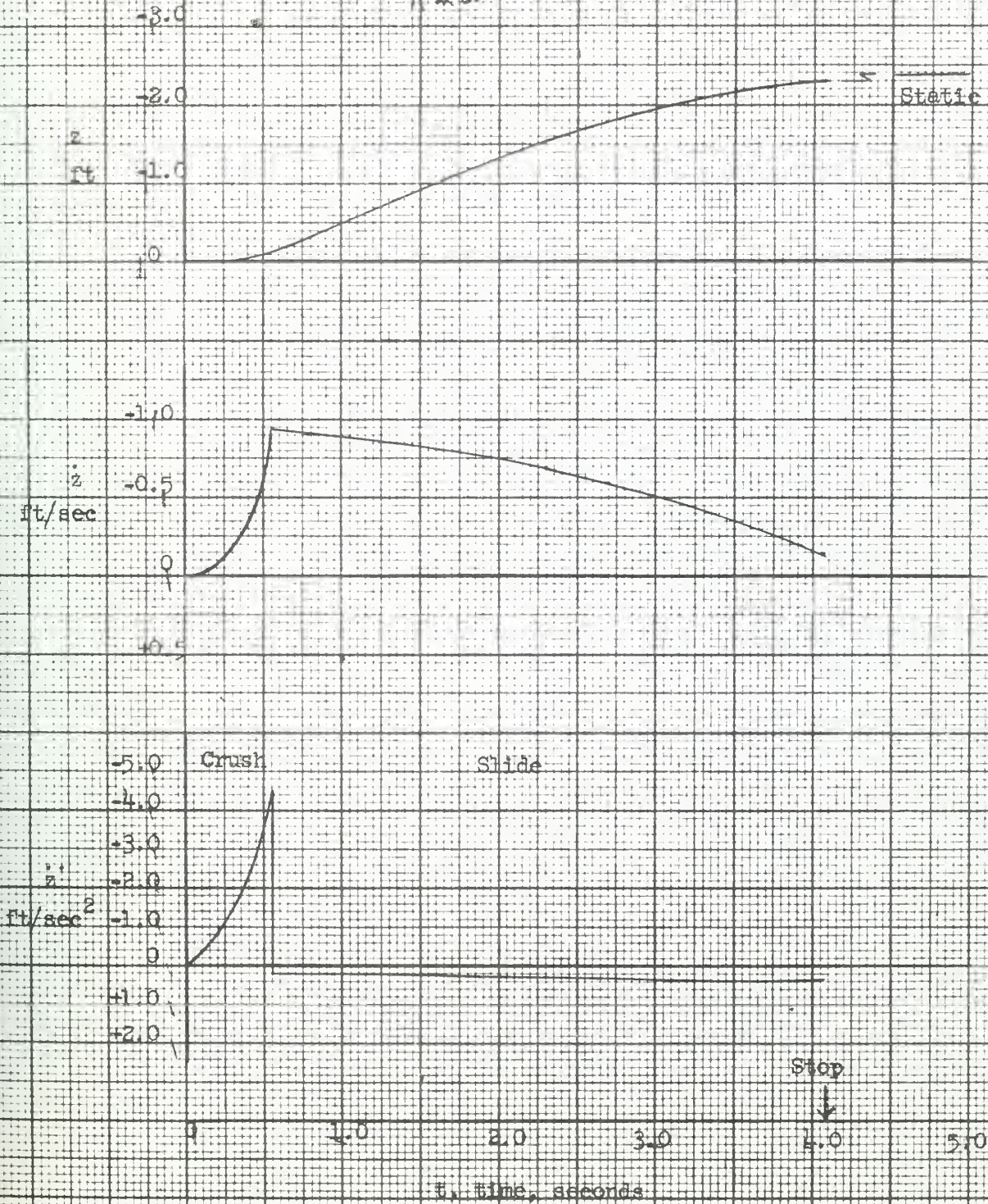


Figure X

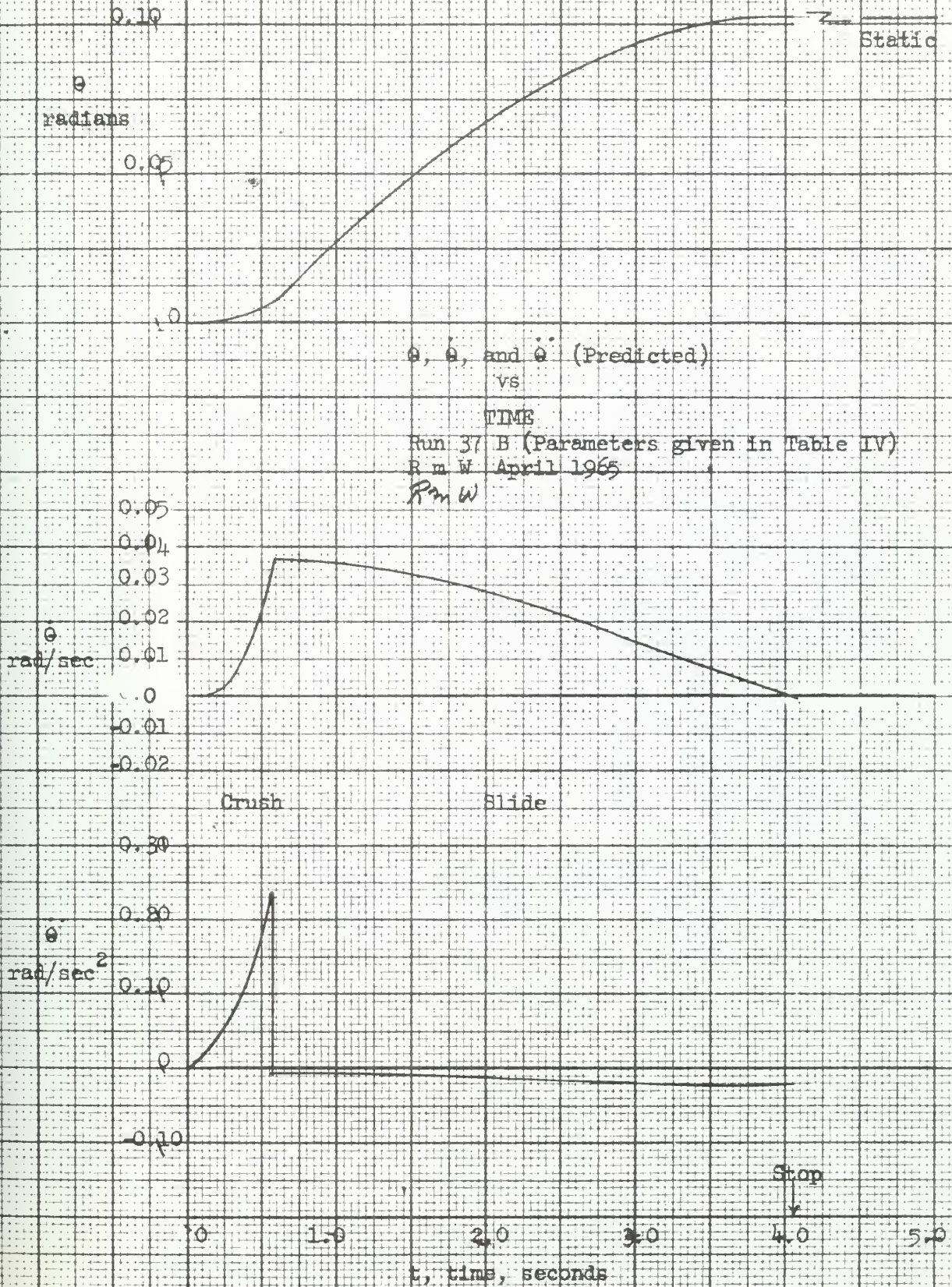


Figure XI

z , θ , $\dot{\theta}$, and \dot{x} (Predicted and Observed)
vs
TIME

Run 36 B (Parameters given in Table IV)

R m W April 1963

R m W

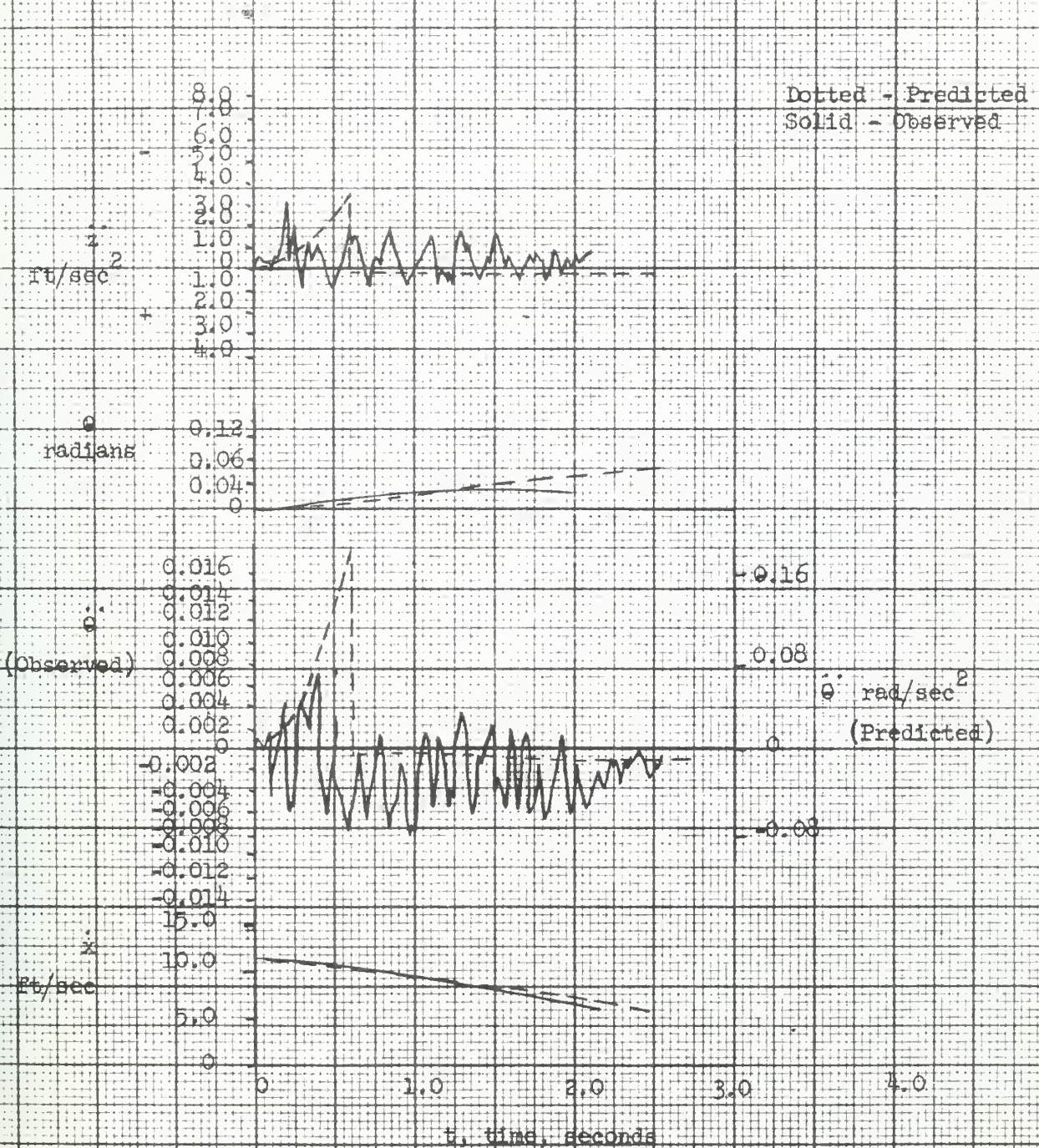


Figure XII
Observed Strain on Bulkhead
and Predicted Force Under Bow

VS
TIME

Run 36B (Parameters given in Table IV)

R m W April 1965

R m W

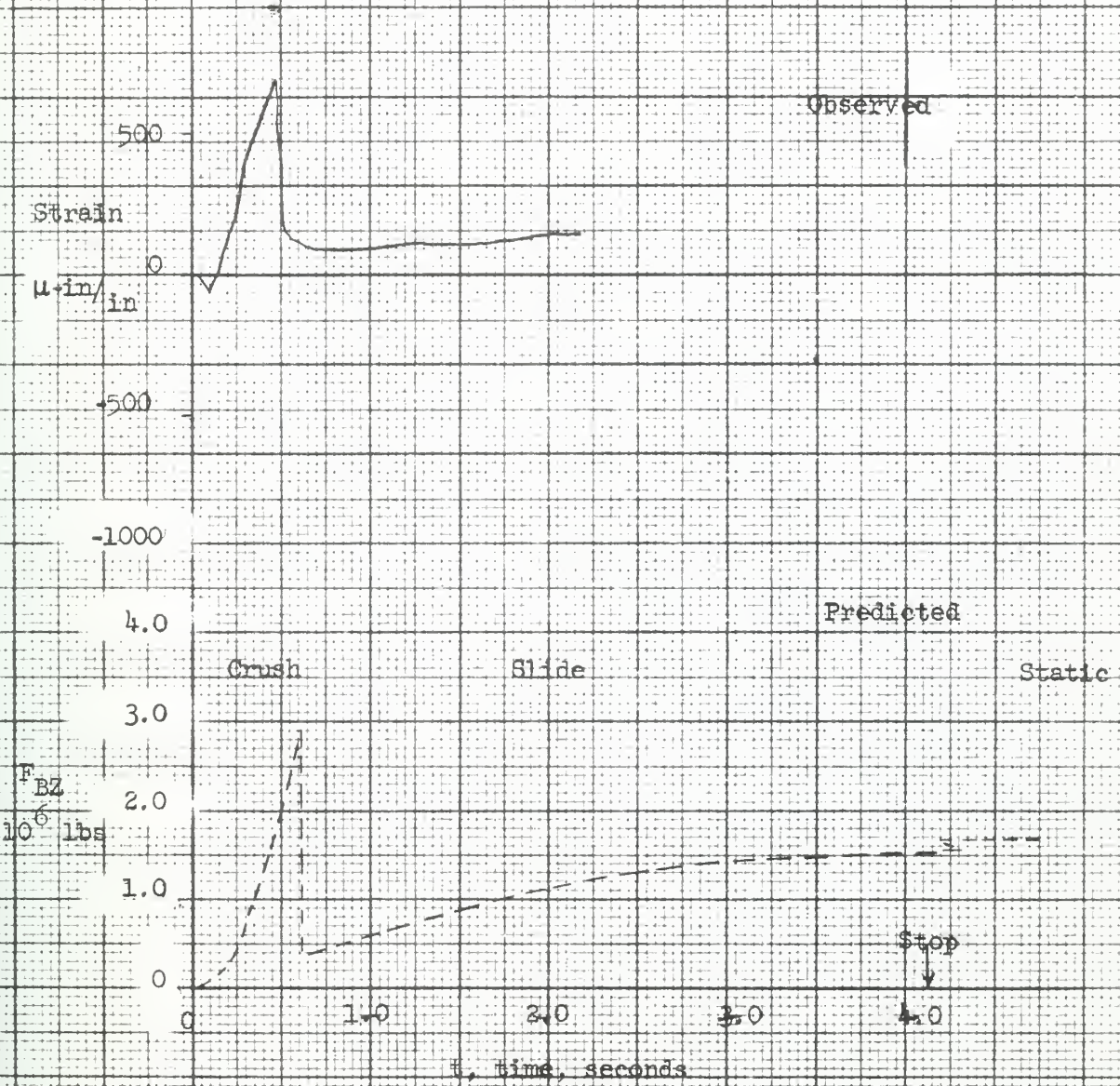


Figure XIII

\ddot{z} , θ , $\ddot{\theta}$, and \dot{x} (Predicted and Observed)
vs
TIME

Run 37B (Parameters given in Table IV)

R m W April 1965

R m W

Dotted - Predicted
Solid - Observed

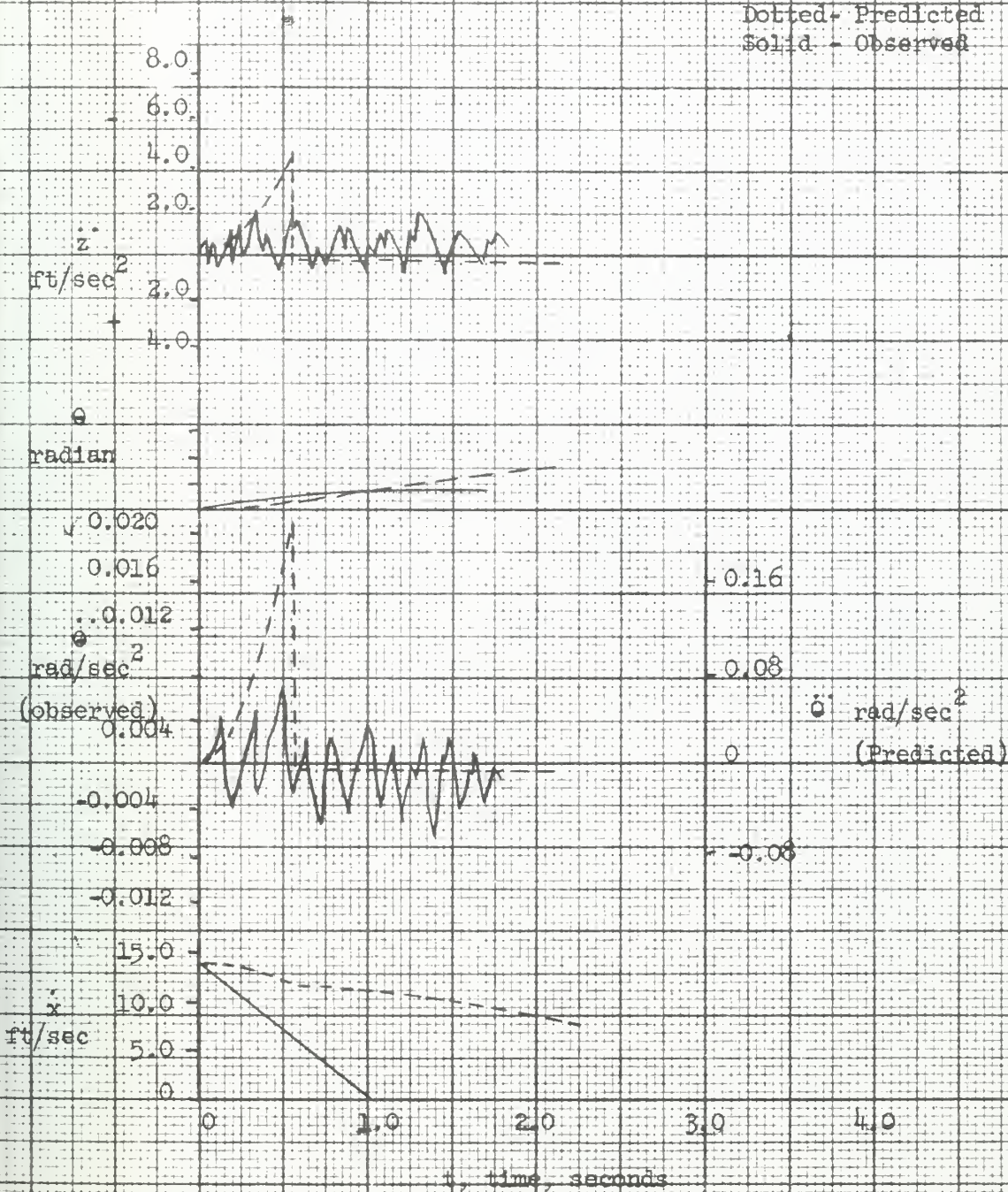


Figure XIV

Observed Strain on Bulkhead
and Predicted Force Under Bow

VE

TIME

Run 37B (Parameters given in Table IV)

run w. April 1965

PMW

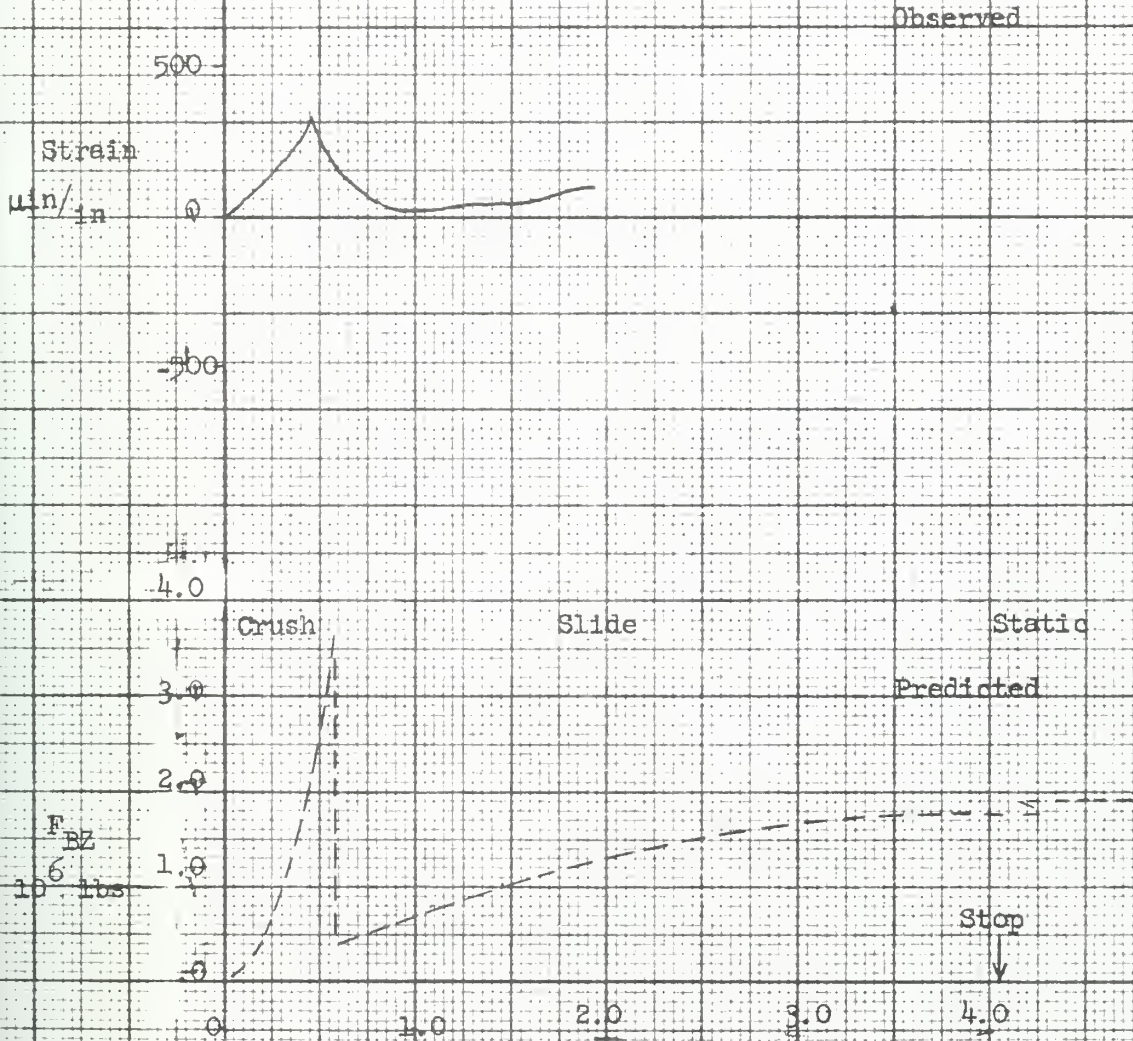


Figure IV

\ddot{x} , θ , $\ddot{\theta}$, and \dot{x} (Predicted and Observed)

vs
TIME

Run 38 B (Parameters given in Table IV)

R m W April 1965

R m W

Dotted - Predicted
Solid - Observed

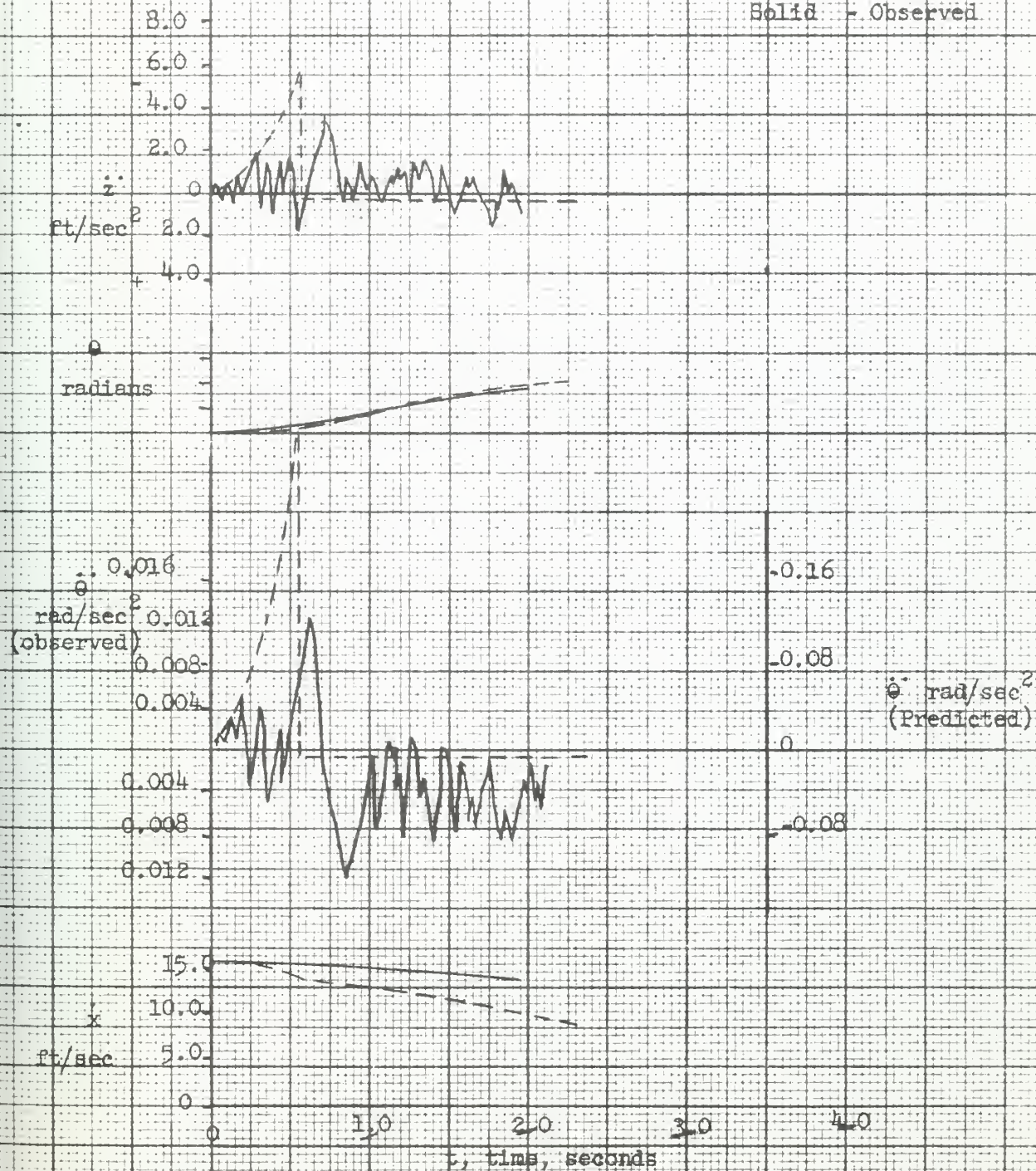
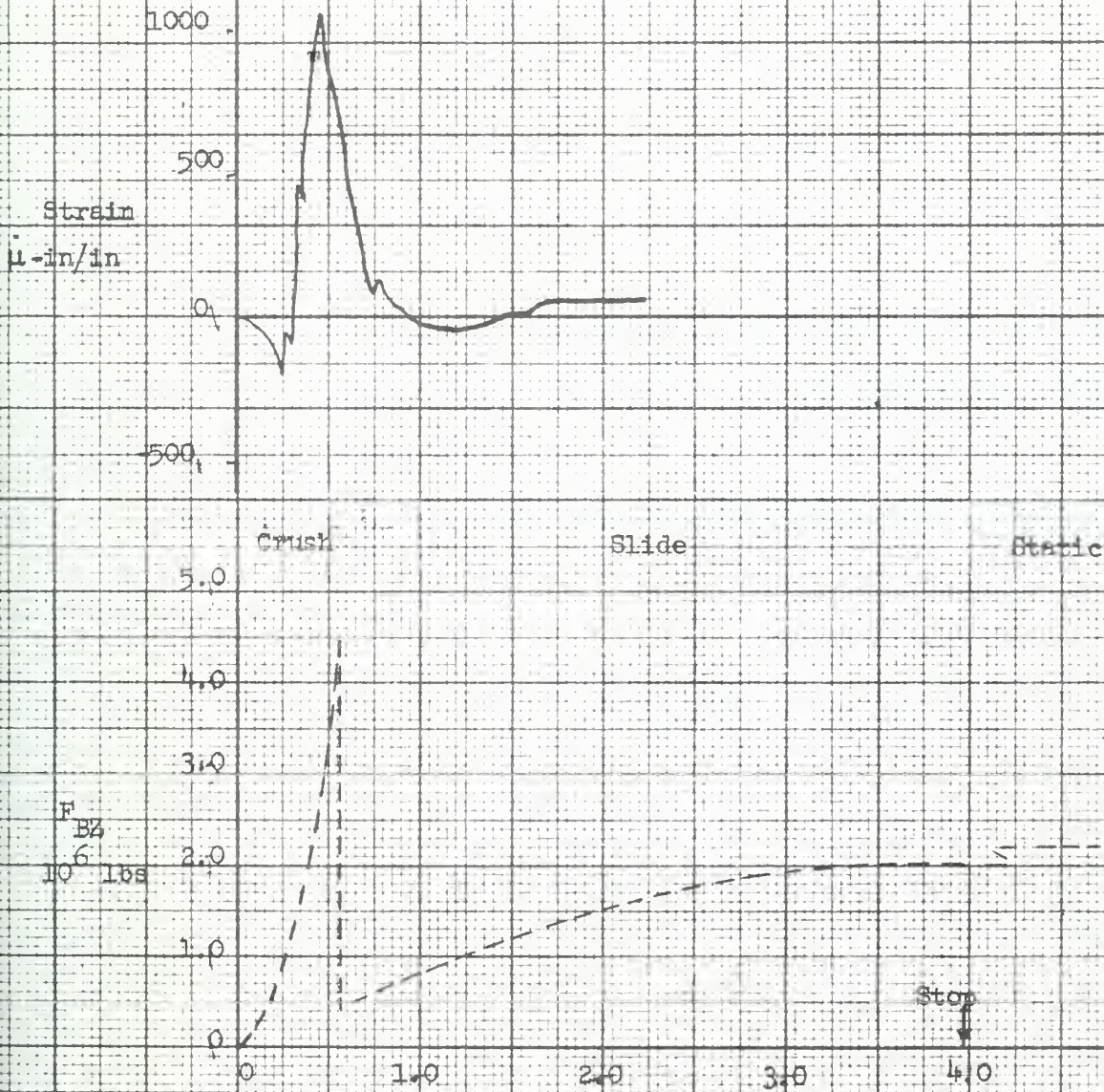


Figure XVI
Observed Strain on Bulkhead
and Predicted Force Under Bow
vs
TIME

Run 38 B (Parameters given in Table IV)
R m W April 1965

Raw



Effect of Variation of Parameters on Bow Force

Figures XVII through XXIX are plots of the icebreaking force (sustained, State 4) as a function of each parameter. A "Wind Class" icebreaker, as indicated in Table V, is used as the parent in each case. The parameters, in many uses, are not independent. The procedure is explained in Chapter II.

In each case the impact velocity is also varied (5, 10, 15, 20, 25 ft/sec) and the plots reflect the effect of three increments of impact velocity (5, 15, 25 ft/sec).

The entering argument for each plot is expressed in dimensionless form.

Effect of a change in temperature on the rate

It was found that the rate of the reaction (the
rate constant k) is a function of the pressure. In
fact, as indicated in Table V, it was found that
the rate constant, k , was not independent of the pressure as
expected in a simple case.

It was also found that the rate constant is a function of the
temperature and the rate constant for the reaction of the
reactants is a function of the temperature.

The following equation for the rate constant is derived from
the data.

TABLE V
PARAMETERS USED FOR ICEBREAKING CALCULATIONS
(UNLESS OTHERWISE NOTED ON FIGURES)

WIND CLASS

BP = 250.0 ft	B = 64.0 ft
H = 25.75 ft	DIS = 12,100,000 lbs.
BA = 0.523 rad.	SA = 0.886 rad.
AL = 0.724	CF = -1.25 ft
CG = -2.40 ft	GK = 23.40 ft
D = 16.0 ft	TB = 270,000 lbs
GM = 195.6 ft	FK = 0.2
FS = 0.8	SIG = 144,000 lbs/ft ²

GLACIER

BP = 290.0 ft	B = 72.5 ft
H = 28.0 ft	DIS = 19,350,000 lbs
BA = 0.523 rad	SA = 0.886 rad
AL = 0.800	CF = -1.45 ft
CG = -2.78 ft	GK = 24.5 ft
D = 16.8 ft	TB = 455,000 lbs
GM = 275.0 ft	FK = 0.2
FS = 0.8	SIG = 144,000 lbs/ft ²

LENIN

BP = 420.0 ft	B = 90.0 ft
H = 30.25 ft	DIS = 35,800,000 lbs
BA = 0.523 rad	SA = 0.886 rad
AL = 0.800	CF = -2.10 ft
CG = -4.04 ft	GK = 27.5 ft
D = 18.8 ft	TB = 730,000 lbs
GM = 545.0 ft	FK = 0.2
FS = 0.8	SIG = 144,000 lbs/ft ²

TABLE 1

STATISTICAL PROPERTIES OF THE DATA SETS
(BASED ON THE ORIGINAL DATA SET)

DATA SET

1. 0.00 = 1
 2. 0.00, 0.00, 0.00 = 2
 3. 0.00, 0.00 = 3
 4. 0.00, 0.00 = 4
 5. 0.00, 0.00 = 5
 6. 0.00, 0.00 = 6
 7. 0.00, 0.00 = 7
 8. 0.00, 0.00 = 8
 9. 0.00, 0.00 = 9
 10. 0.00, 0.00 = 10

11. 0.00, 0.00 = 11
 12. 0.00, 0.00 = 12
 13. 0.00, 0.00 = 13
 14. 0.00, 0.00 = 14
 15. 0.00, 0.00 = 15
 16. 0.00, 0.00 = 16
 17. 0.00, 0.00 = 17
 18. 0.00, 0.00 = 18
 19. 0.00, 0.00 = 19
 20. 0.00, 0.00 = 20

DATA SET

21. 0.00, 0.00 = 21
 22. 0.00, 0.00 = 22
 23. 0.00, 0.00 = 23
 24. 0.00, 0.00 = 24
 25. 0.00, 0.00 = 25
 26. 0.00, 0.00 = 26
 27. 0.00, 0.00 = 27
 28. 0.00, 0.00 = 28
 29. 0.00, 0.00 = 29
 30. 0.00, 0.00 = 30

31. 0.00, 0.00 = 31
 32. 0.00, 0.00 = 32
 33. 0.00, 0.00 = 33
 34. 0.00, 0.00 = 34
 35. 0.00, 0.00 = 35
 36. 0.00, 0.00 = 36
 37. 0.00, 0.00 = 37
 38. 0.00, 0.00 = 38
 39. 0.00, 0.00 = 39
 40. 0.00, 0.00 = 40

DATA SET

41. 0.00, 0.00 = 41
 42. 0.00, 0.00 = 42
 43. 0.00, 0.00 = 43
 44. 0.00, 0.00 = 44
 45. 0.00, 0.00 = 45
 46. 0.00, 0.00 = 46
 47. 0.00, 0.00 = 47
 48. 0.00, 0.00 = 48
 49. 0.00, 0.00 = 49
 50. 0.00, 0.00 = 50

51. 0.00, 0.00 = 51
 52. 0.00, 0.00 = 52
 53. 0.00, 0.00 = 53
 54. 0.00, 0.00 = 54
 55. 0.00, 0.00 = 55
 56. 0.00, 0.00 = 56
 57. 0.00, 0.00 = 57
 58. 0.00, 0.00 = 58
 59. 0.00, 0.00 = 59
 60. 0.00, 0.00 = 60

Figure XVII
Icebreaking Force of Windlass
vs
Compressive Failure
Stress of Ice

(Parameters given in Table V)

R. M. W. April 1965

R. M. W.

R
BZ
 10^6 lbs

$V_1 = 25$ ft/sec

$V_1 = 15$

$V_1 = 5$

5

4

3

2

1

5

5

50

55

100 ft

125

150

175

200

Figure XVIII

Icebreaking Force of Wind Class
VB

Ratio of Height of Thrust Line to Draft
(Parameters given in Table V)

R m W April 1965

RmW

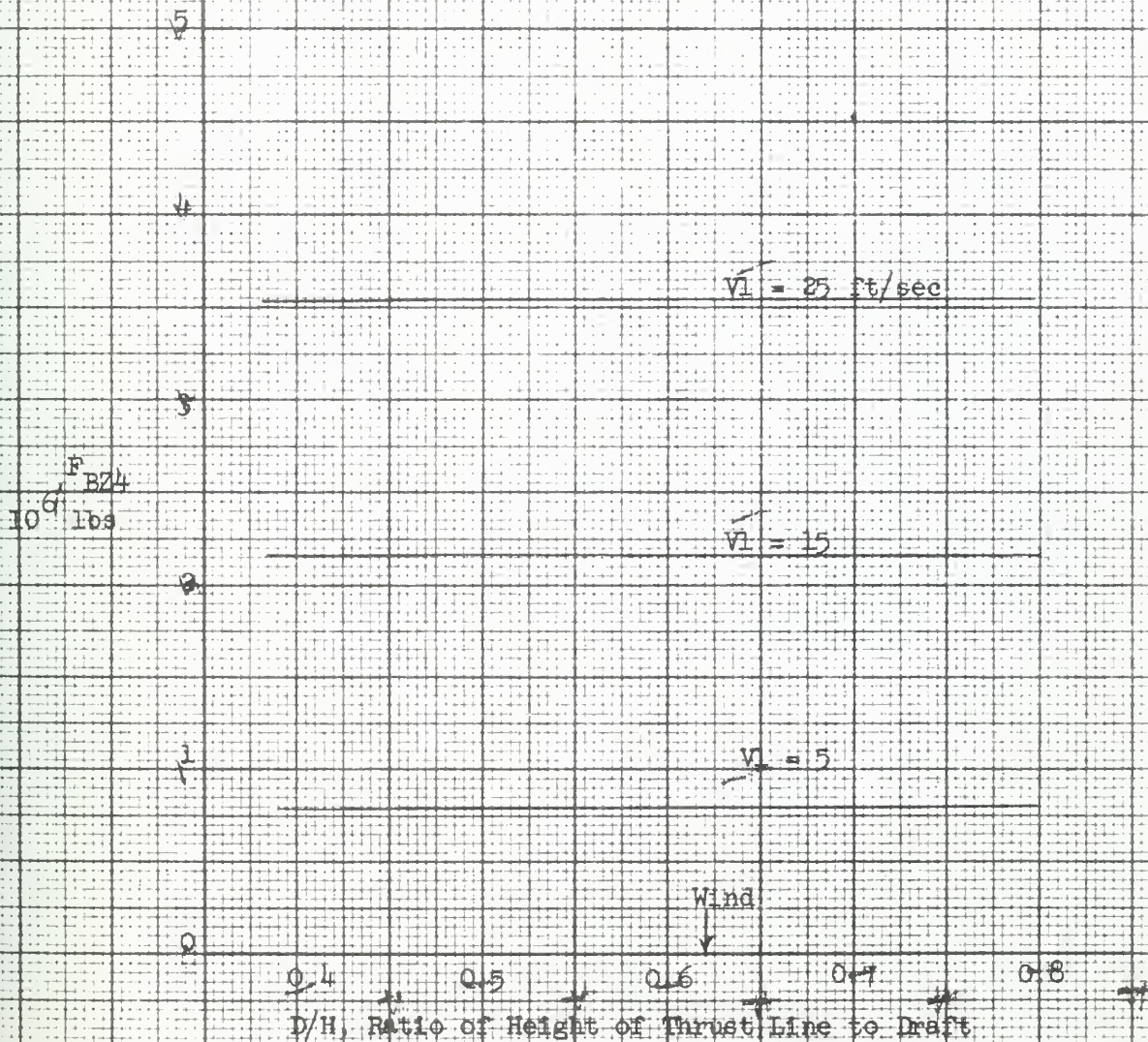


Figure XIX
Icebreaking Force of Wind Class
vs
Ratio of Bollard Thrust (lbs) to
Displacement (lbs)

(Parameters given in Table V)

R m W April 1965

R m W

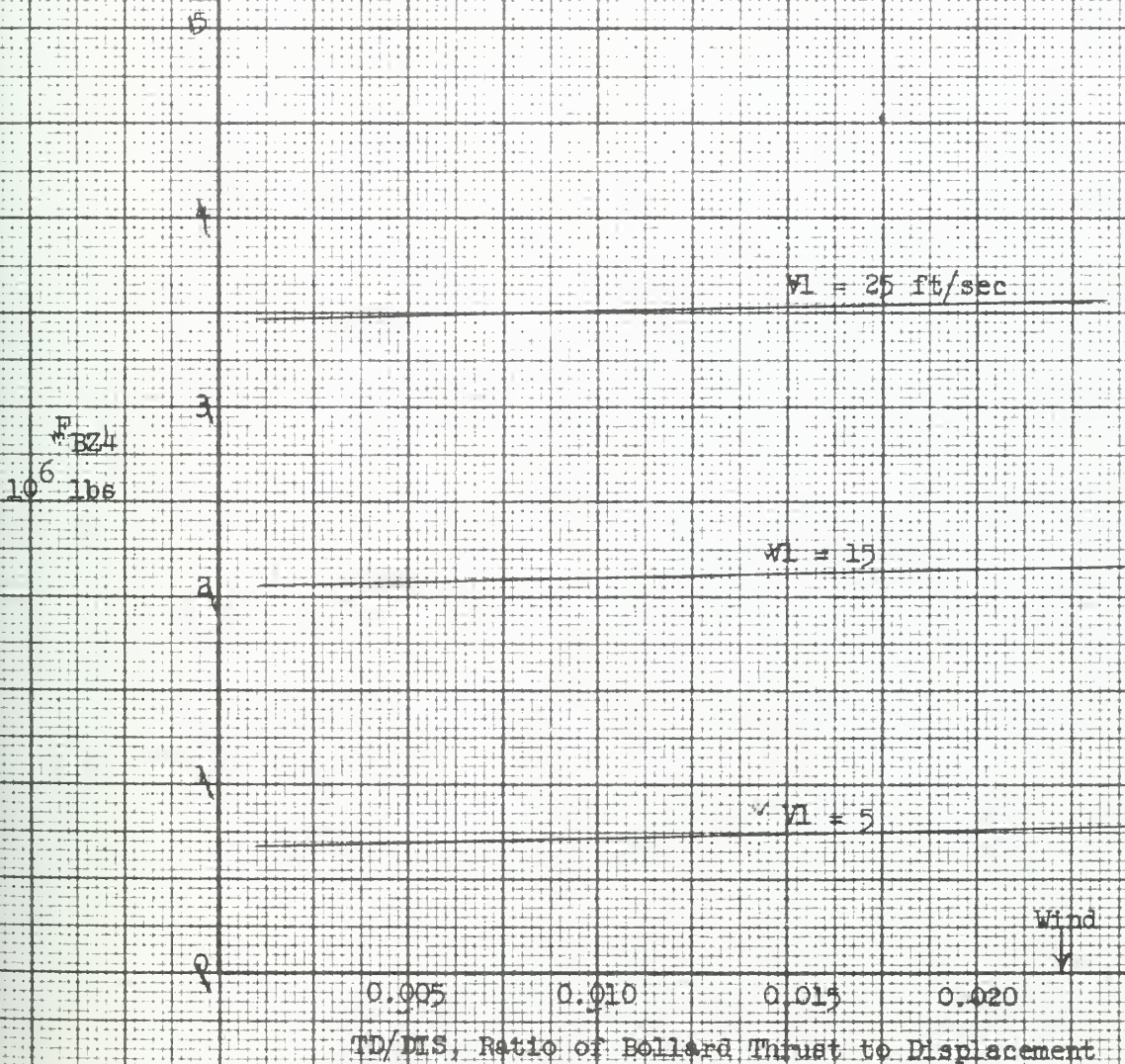


Figure XX
Icebreaking Force of Wind Class

vs

Ratio of Long. Position of Center
of Gravity to Length
(Parameters given in Table V)

$R_{LW} = 4/65$

$V_1 = 25$ ft/sec

$V_2 = 15$

$V_3 = 15$

Wind



-0.018

-0.014

-0.010

-0.006

CG/ BP, Ratio of Long. Position of Center of Gravity to Length

R_{BZ}

10^6 lb

5

4

3

2

1

0

Figure XXI
Icebreaking Force of Wind Class
vs
Ratio of Long. Position of Center
of Flotation to Length
(Parameters given in Table V)

R. W. April 1965

R. W.

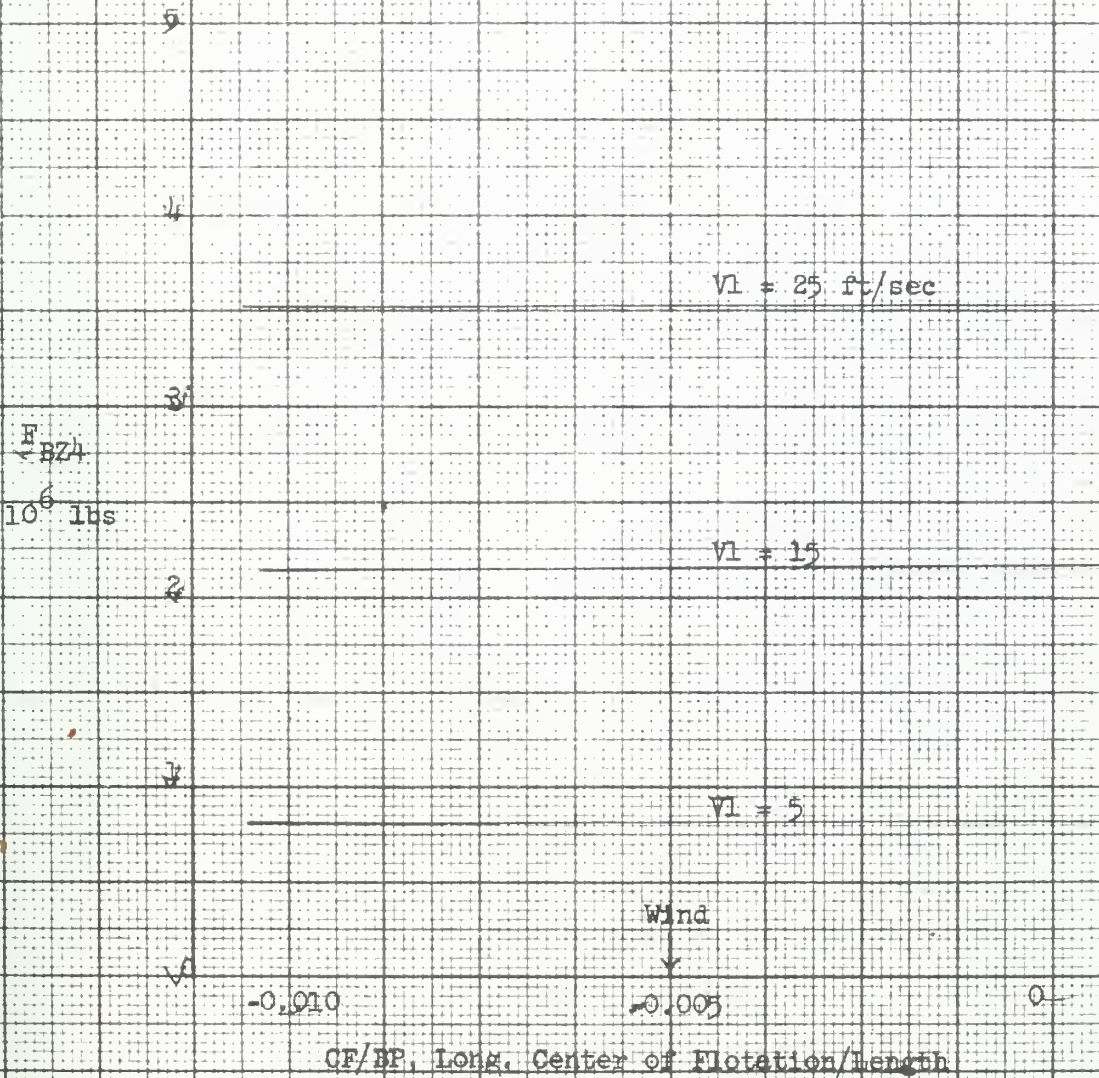


Figure XXII
Icebreaking Force of Wind Class
vs
Length-to-Beam Ratio
(Parameters given in Table V)
R m W April 1965
R m W

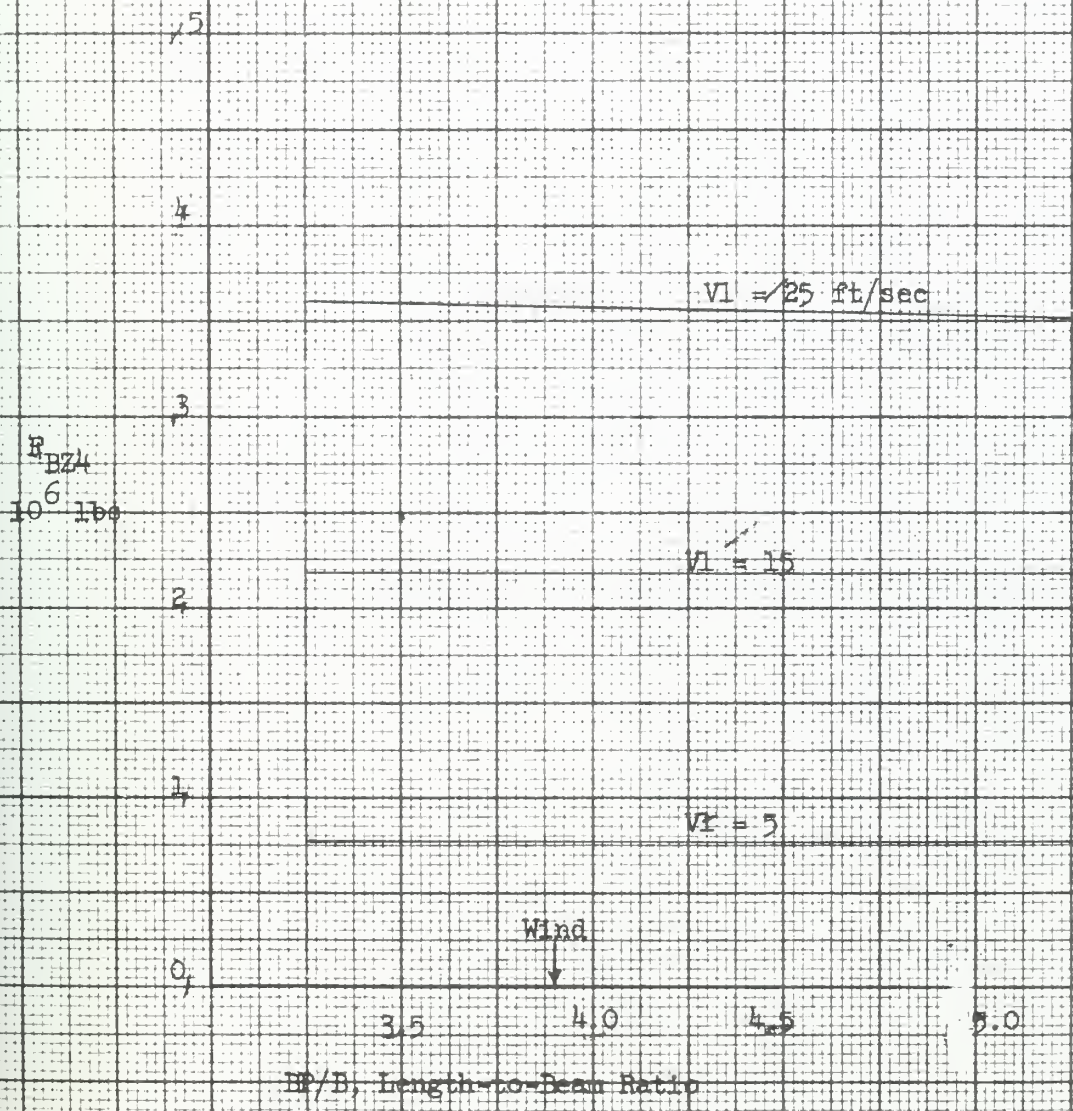


Figure IXIII
Icebreaking Force of Wind Class

vs
Ratio of Height of Center of
Gravity to Draft

(Parameters given in Table V)
R. m. W. April 1965

R_mW

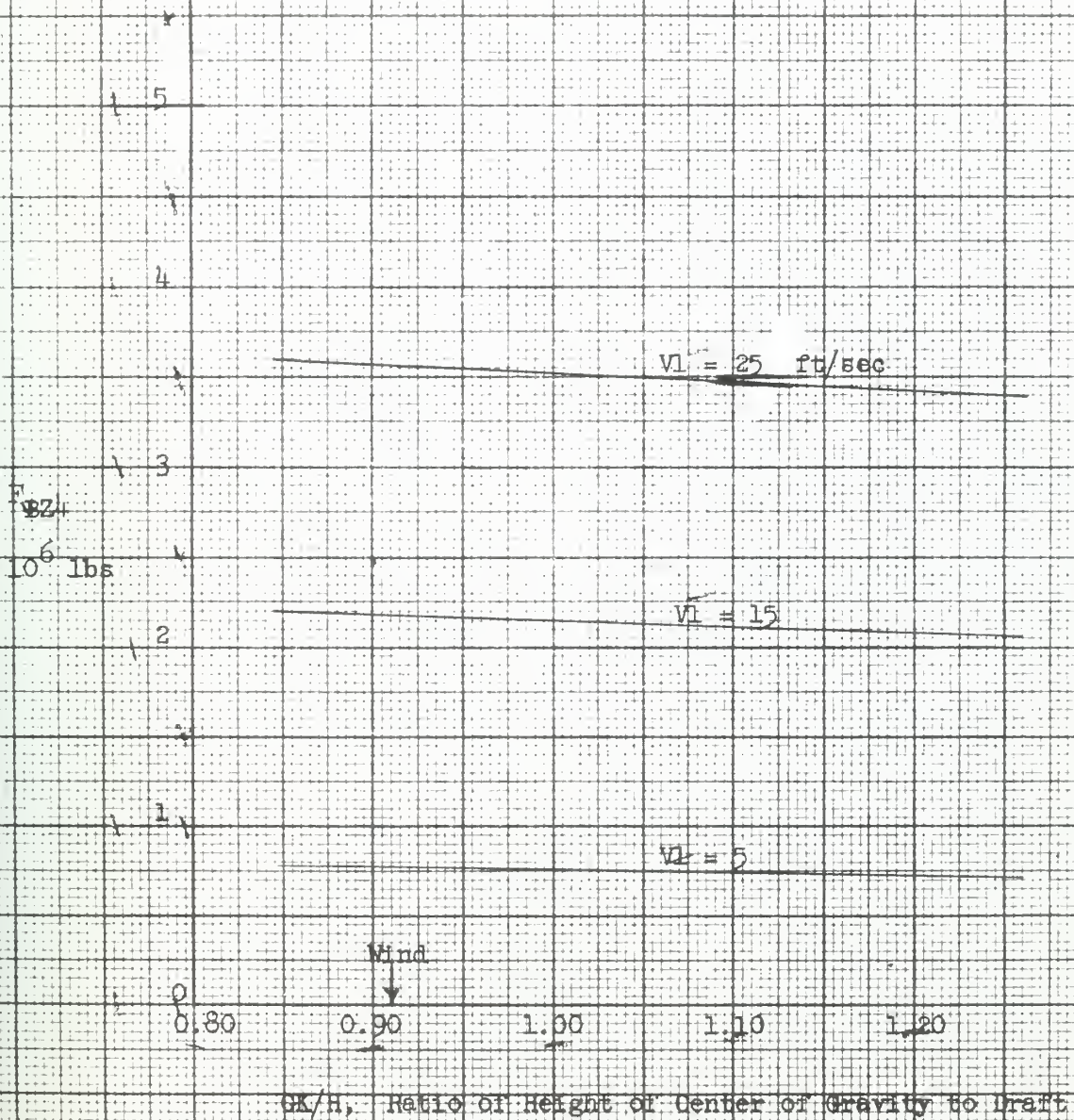


Figure 501V
Icebreaking Force of Wind Class
vs
Beam-to-Draft Ratio
(Parameters given in
Table V)

R. m. W. April 1965

R. m. W.

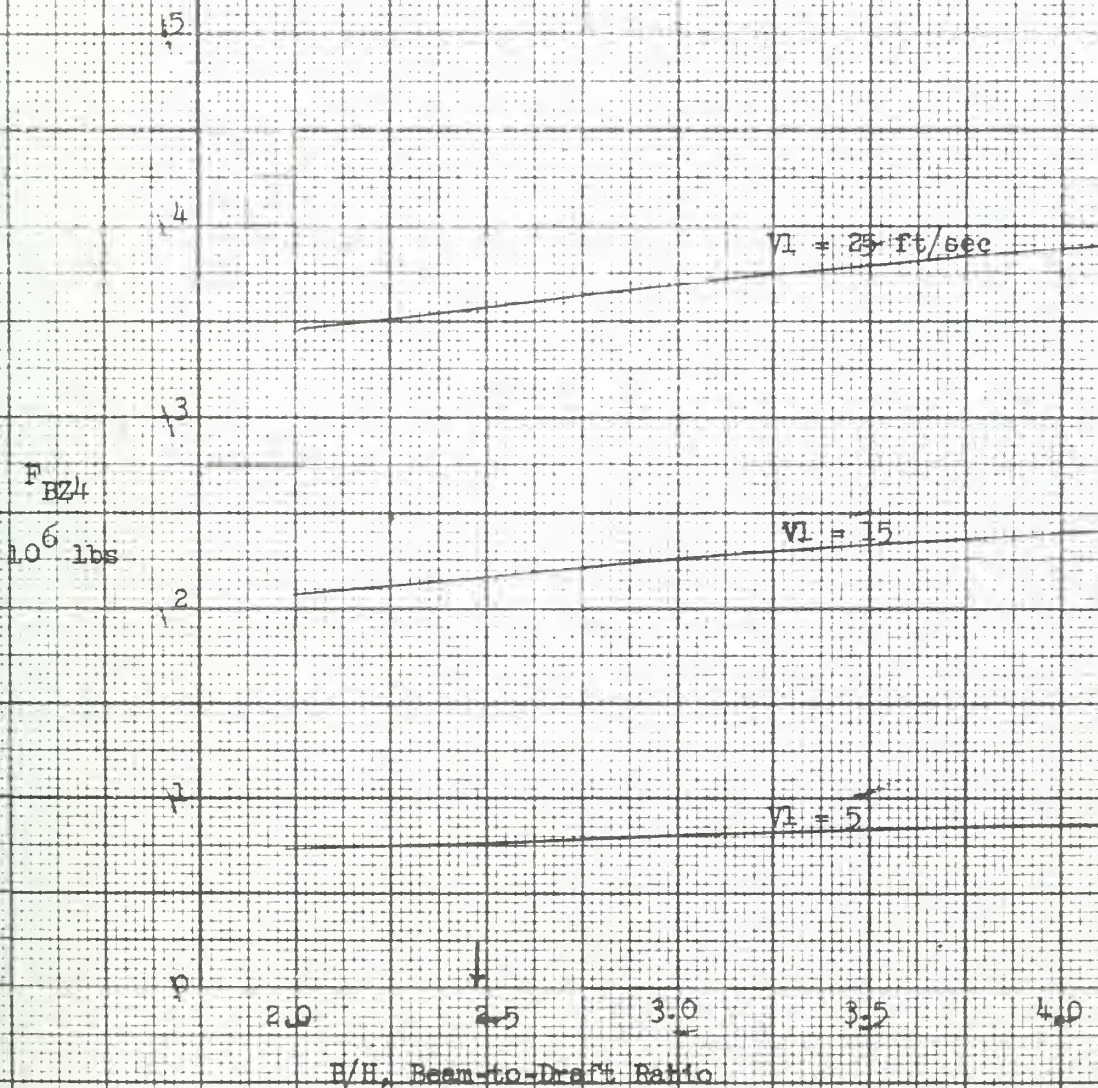


Figure XXV
 Icebreaking Force of Wind Class
 vs
 Waterplane Coefficient
 (and long. GM-to-length Ratio)
 (Parameters given in Table V)
 RMW April 1965
 Rm w

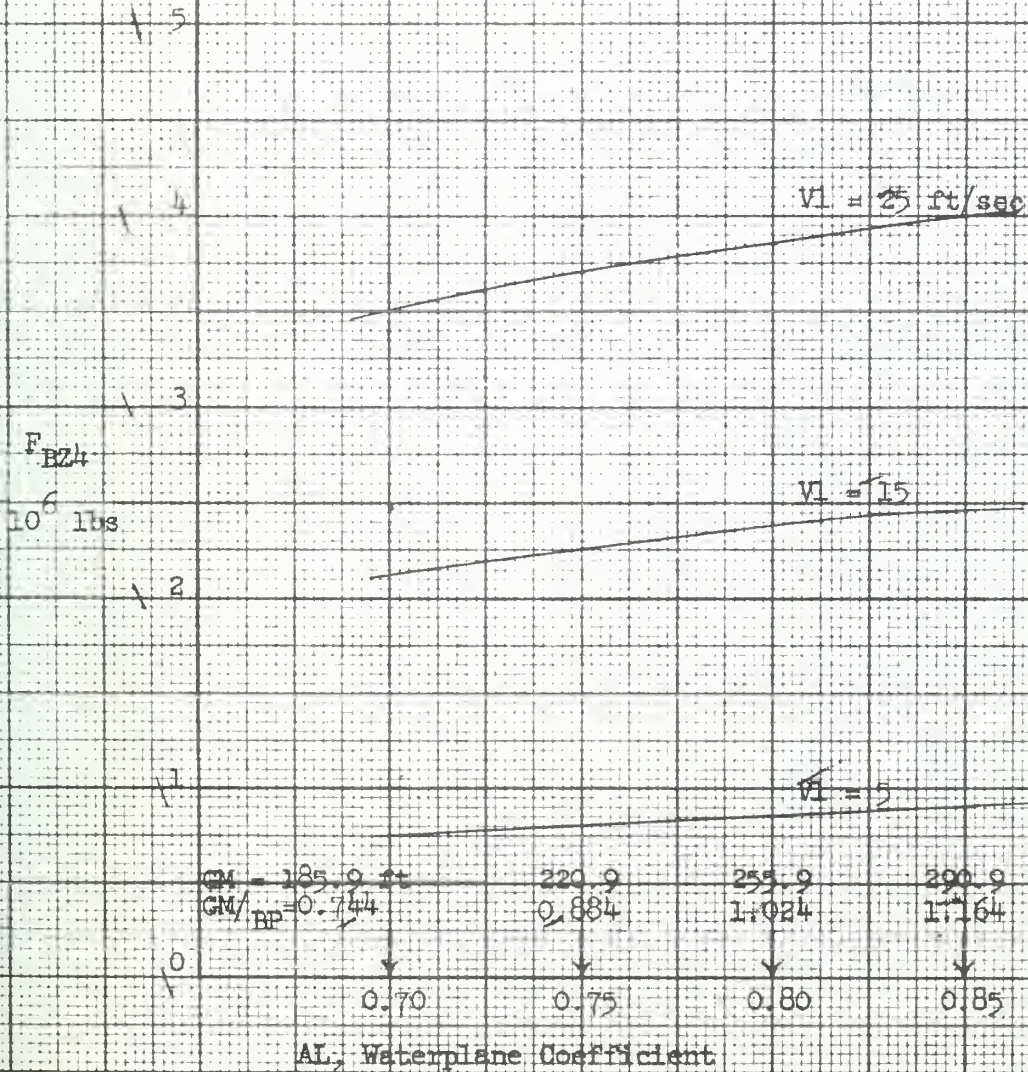


Figure XXVI
 Icebreaking Force of Wind Class
 vs.
 Block Coefficient
 (and Displacement)
 (Parameters given in Table V)
 RMW April 1965

RMW

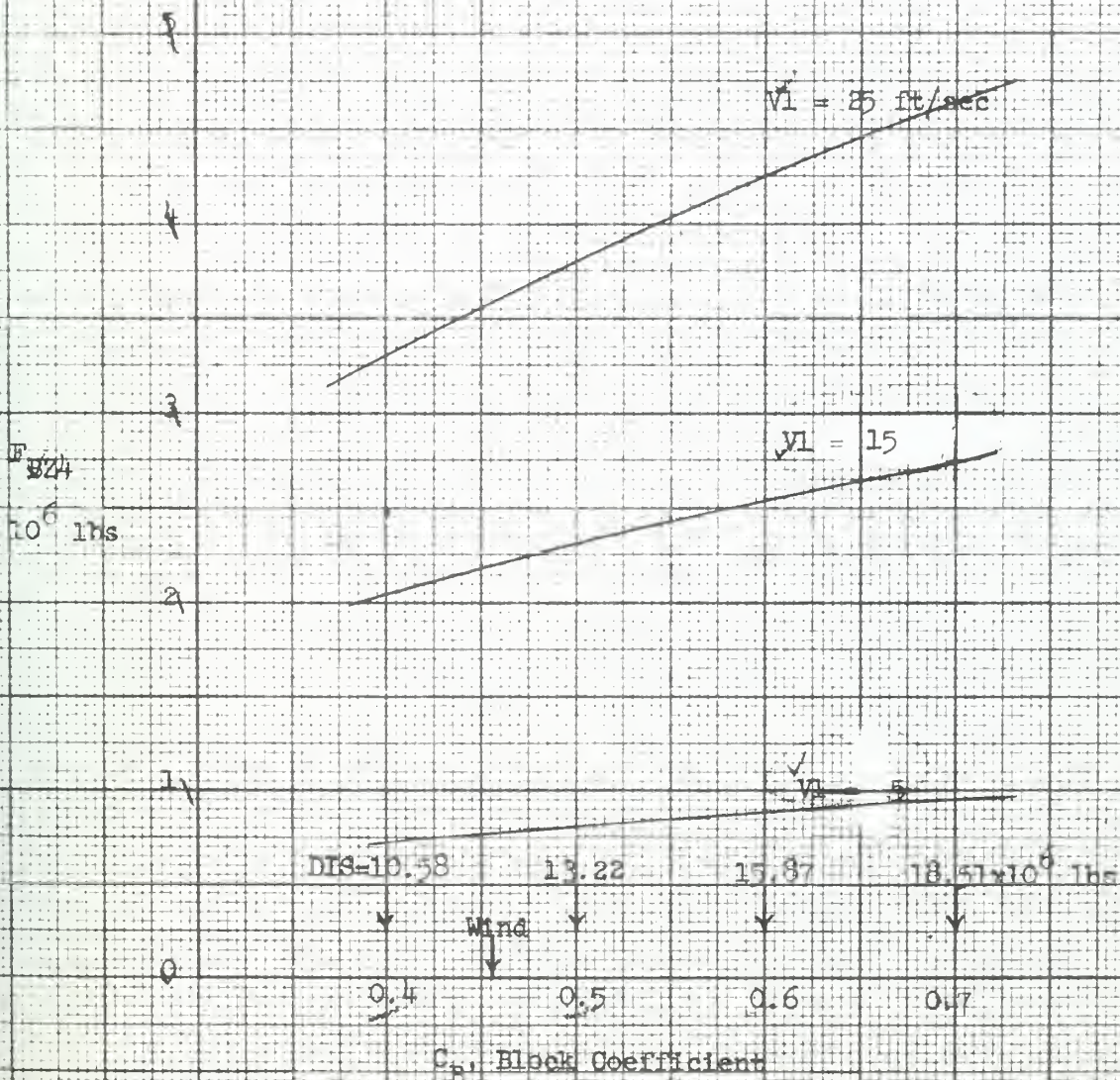


Figure XXVII
Icebreaking Force of Wind Class
vs
Spread Angle Complement
(Parameters given in Table V)
RMW April 1965

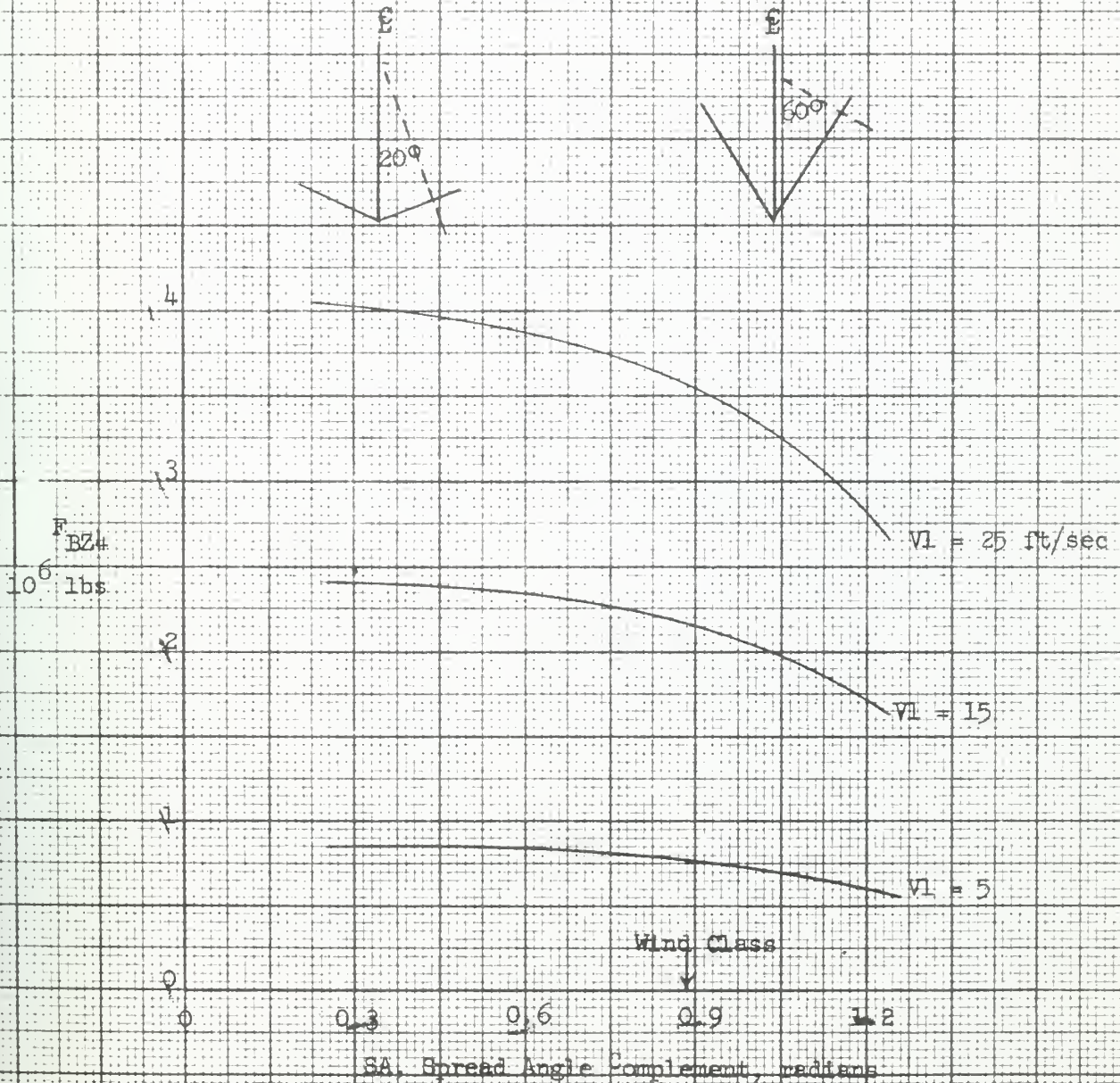


Figure XVIII
Icebreaking Force of Wind Class
vs
Coefficient of Kinetic Friction
(Parameters given in Table V)
RMW April 1965
R.W.W.

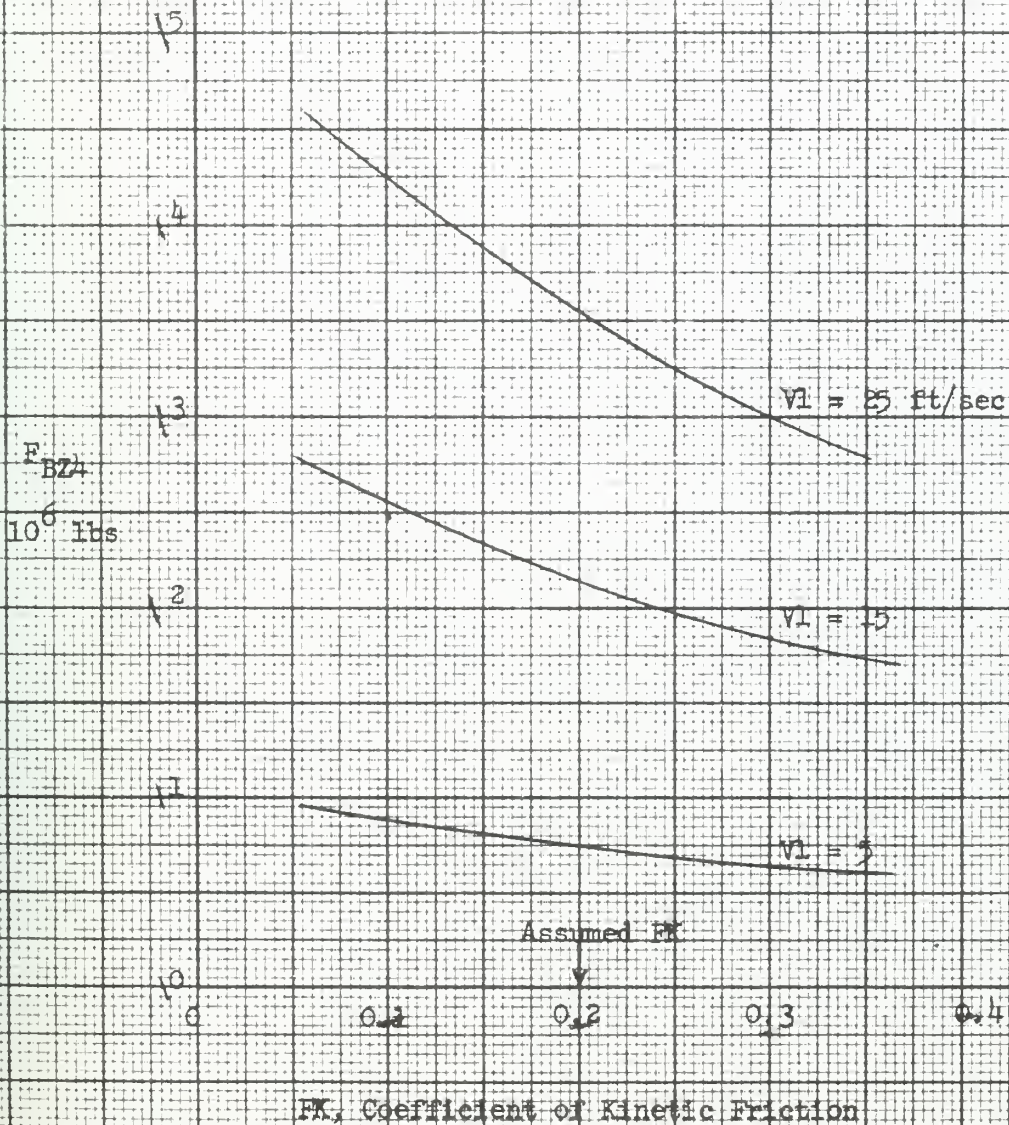


Figure XXIX Icebreaking Force of Wind Class

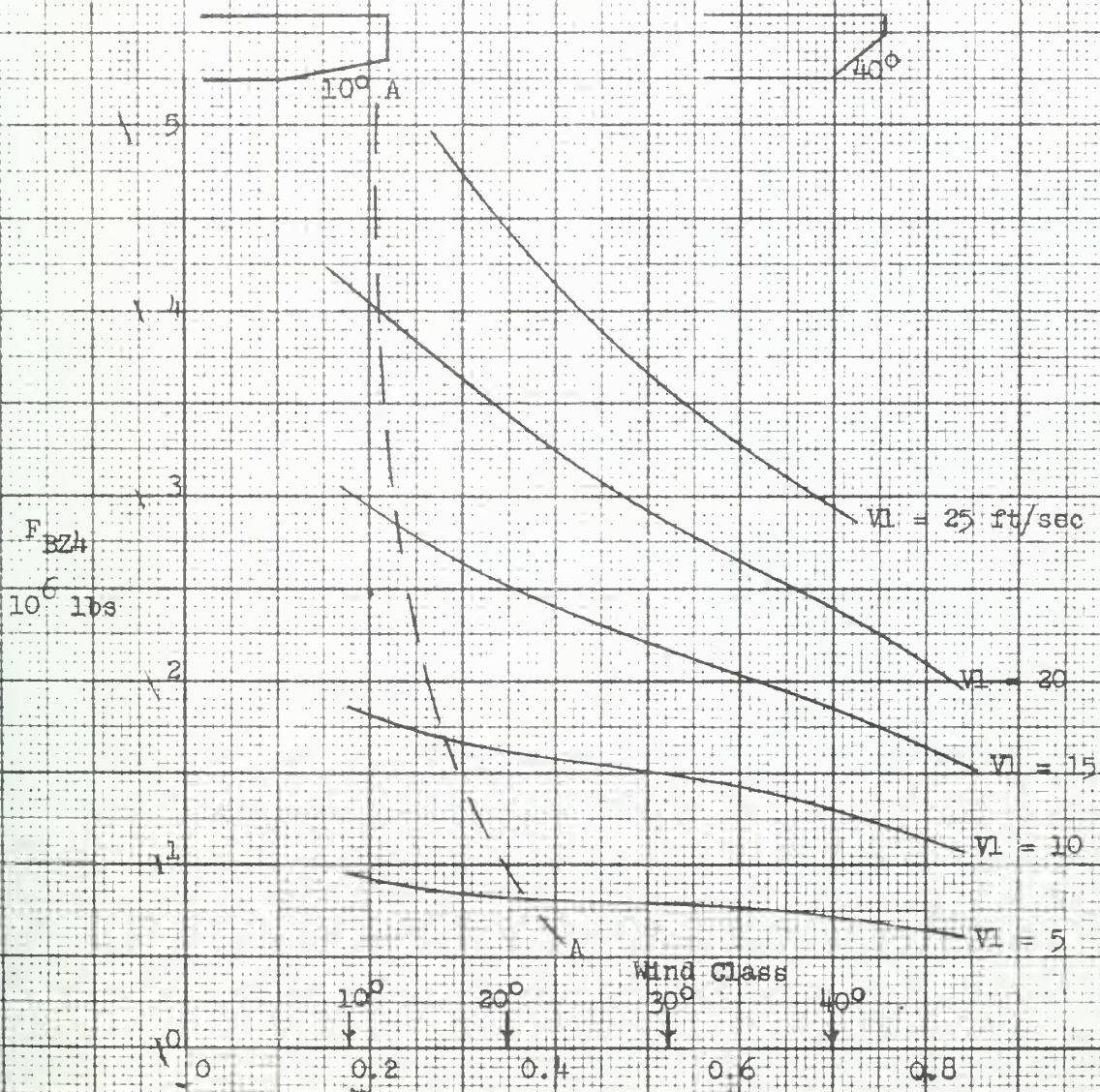
vs

Bow Angle

(Parameters given in Table V)

RMW April 1965

RMW



Effect of Displacement and Impact Velocity on Bow Force

Figure XXX is a plot of icebreaking force as a function of displacement for three different impact velocities. The displacements represent the "Wind Class", "Glacier Class", and "Lenin Class". In reality only the nine points plotted are the direct result of calculation. The curves have been drawn in to represent the trend.

Figure XXXI is a plot of icebreaking force as a function of impact velocity for the three above-mentioned classes of icebreakers.

Figure XXXII is a plot of icebreaking force as a function of displacement for an impact velocity of 15 ft/sec. There are three curves; each curve represents icebreakers which are geometrically similar to the parent icebreaker indicated.

The parameters used for the parent icebreakers are given in Table V.

Effect of Variation of Parameters on "White Ratio"

As indicated in the procedure, since the icebreaking force is approximately linear with respect to impact velocity and displacement, it appears useful to divide the icebreaking force by displacement times velocity (which is the "White Ratio").

Figures XXXIII through XXXX are plots of this ratio as a function of various parameters. These are based on an impact velocity of 15 ft/sec although other velocities give approximately the same value. The three major classes are each plotted so that similar tendencies and magnitudes

TABLE OF CONTENTS

Chapter I is a study of the general theory of the
relativity and the special relativity. The special
relativity is the "rest frame", "moving frame", and "light
velocity". The general relativity is the study of the
curvature of space-time and the motion of particles
in a curved space-time. The general relativity is the
study of the motion of particles in a curved space-time.
The general relativity is the study of the motion of
particles in a curved space-time. The general relativity
is the study of the motion of particles in a curved
space-time. The general relativity is the study of
the motion of particles in a curved space-time.

APPENDIX

As indicated in the introduction, the following table is
a summary of the results of the present investigation. It
appears useful to divide the following table of contents
into two parts, the first part is the "rest frame",
the second part is the "moving frame". The first part
is the study of the motion of particles in a curved
space-time. The second part is the study of the motion
of particles in a curved space-time. The first part
is the study of the motion of particles in a curved
space-time. The second part is the study of the motion
of particles in a curved space-time.

may be illustrated.

The parameters for the three parent icebreakers are given in Table V.

Extracting Thrust

Figures XXXXI and XXXXII are plots of the ratio of extracting thrust to the maximum (forward) bollard thrust available as a function of bow angle, coefficient of static friction, and impact velocity. In a sense, Figure XXXXII is a set of cross-curves of Figure XXXXI.

Figure XXXXIII is a plot of the ratio of extracting thrust to the bollard thrust as a function of the spread angle complement (for various impact velocities of the three major classes).

The parameters used are given in Table V.

Table 11.1

The following table shows the results of the analysis of variance for the data in Table 11.1.

Table 11.1

The analysis of variance for the data in Table 11.1 is shown in Table 11.2. The results are summarized in Table 11.3. The results show that there is a significant difference between the two groups of students on the test of statistical inference for the null hypothesis that the two groups are equal. The results also show that there is a significant difference between the two groups of students on the test of statistical inference for the null hypothesis that the two groups are equal. The results also show that there is a significant difference between the two groups of students on the test of statistical inference for the null hypothesis that the two groups are equal.

The results are given in Table 11.3.

Source	SS	df	MS	F	p-value
Between groups	10.00	1	10.00	1.00	0.32
Within groups	90.00	19	4.74		
Total	100.00	20			

Figure IXK
Icebreaking Force
vs
Displacement of

Lenin
Glacier
Wind

(Parameters given in Table V)
RMW April 1965

RMW

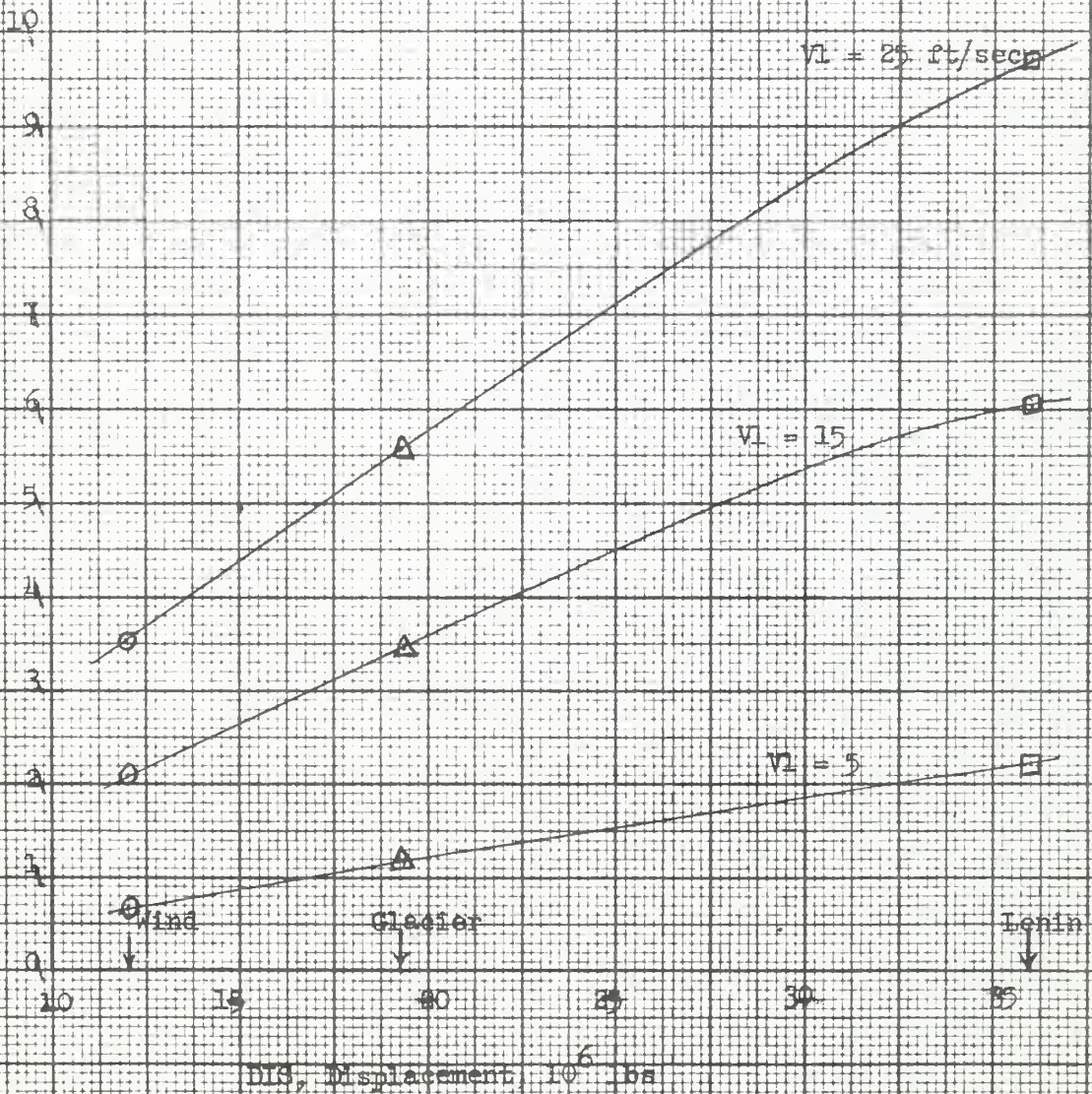


Figure XXXI
Icebreaking Force
vs.
Impact Velocity
(Parameters given in Table V)
RMW April 1965
T.M.W.

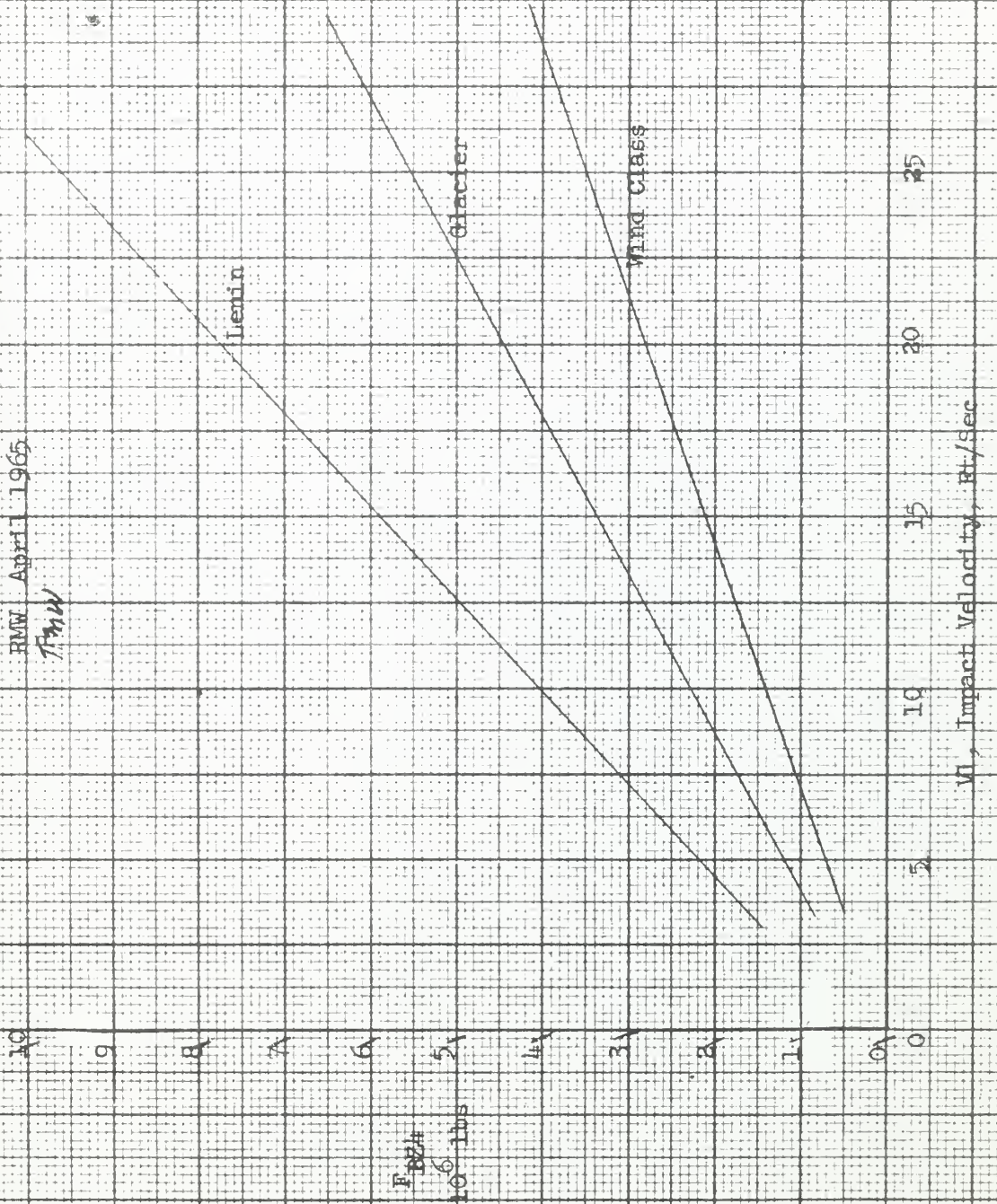


Figure XXXII
Icebreaking Force
vs

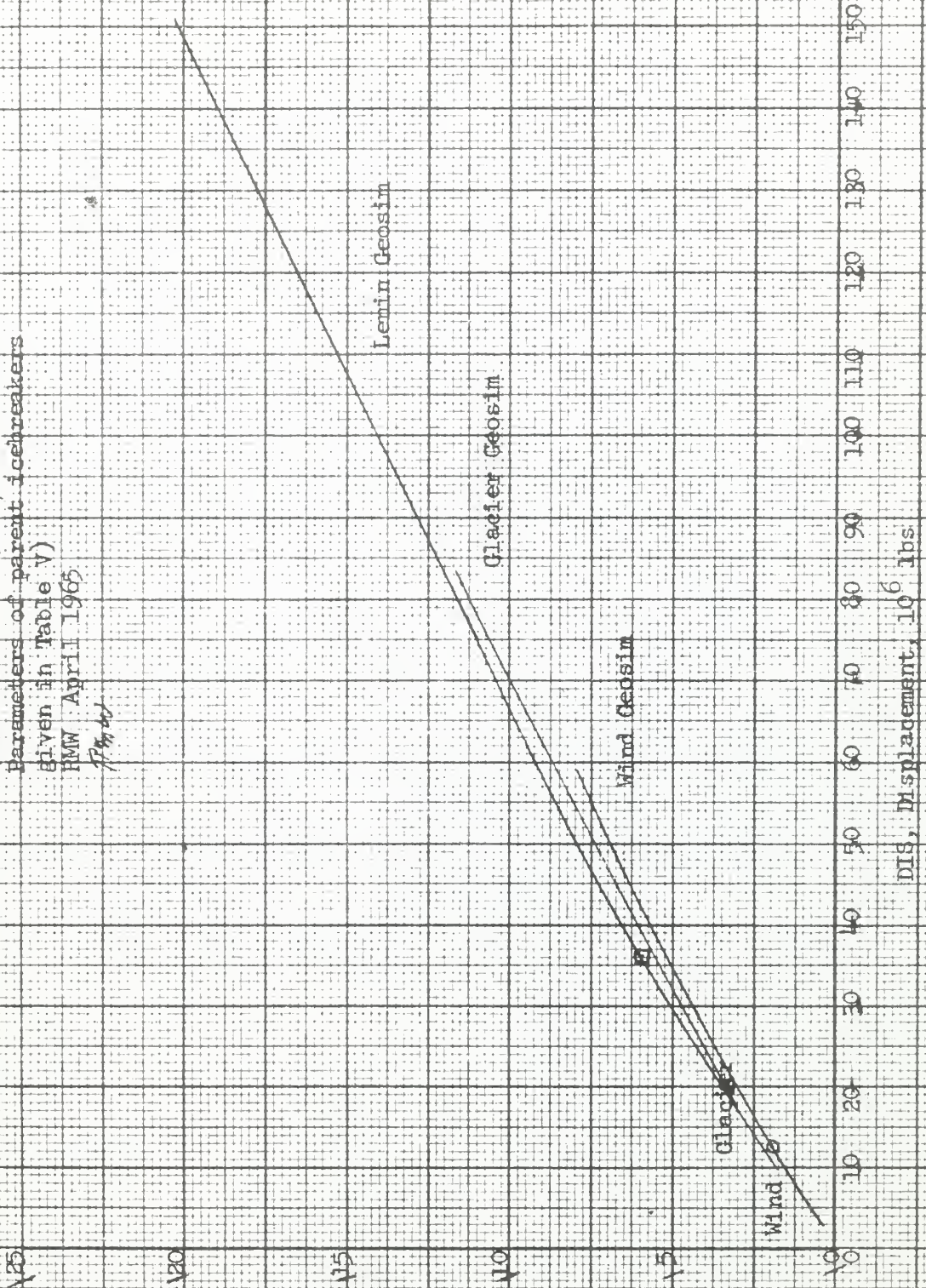
Displacement of
Geometrically Similar
Icebreakers

(Based on $V_1 = 15$ ft/sec)

Parameters of parent icebreakers
given in Table V)

RMW April 1965

RMW



DIS, Displacement, 10⁶ lbs

Figure XXIII
White Ratio
vs
Beam-To-Draft Ratio
(Based on $V_L = 15$ ft/sec)
(Parameters given in Table V)
RMW April 1965

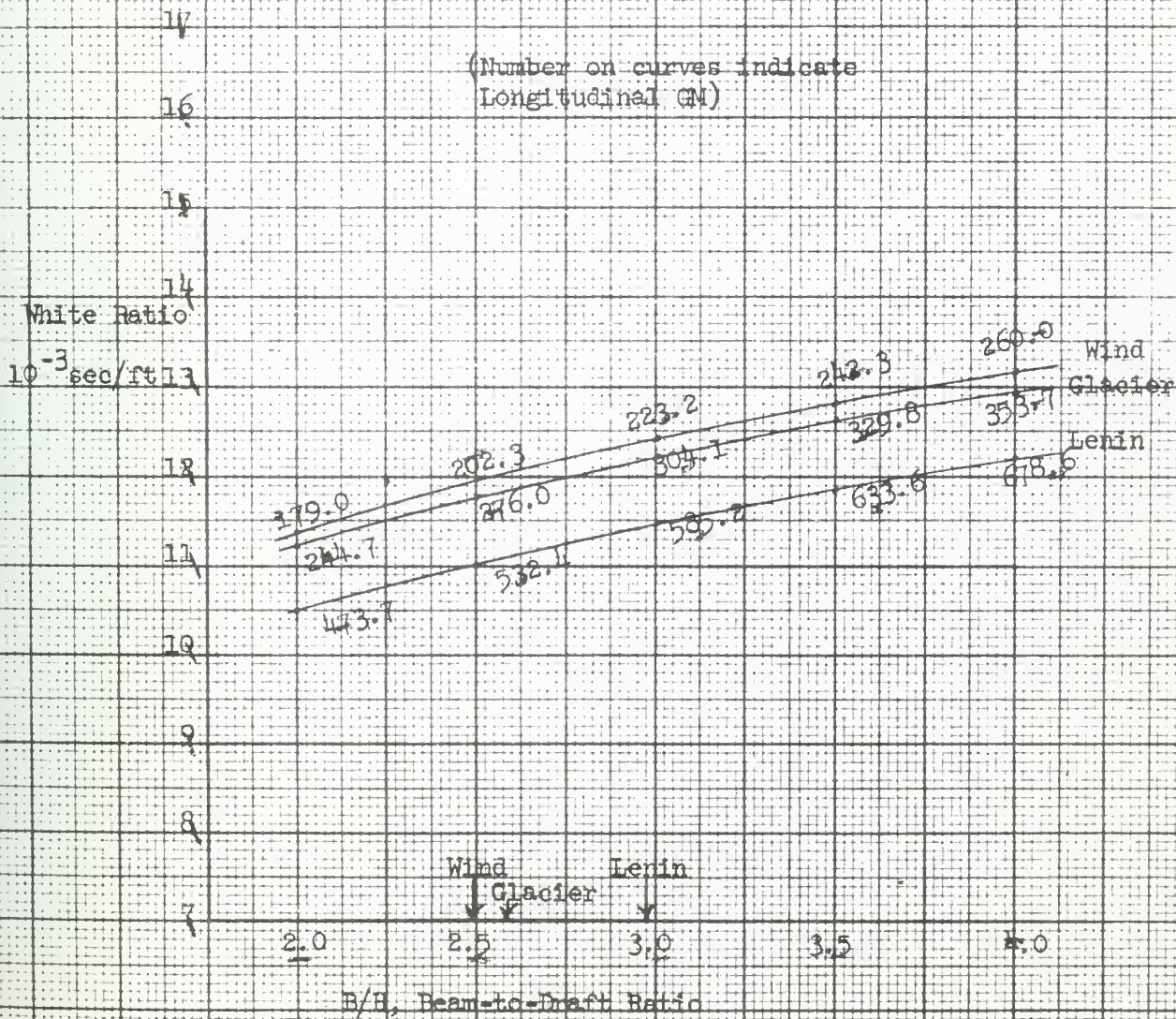


Figure 801V
 White Ratio
 vs
 Waterplane Coefficient
 (and Long. CM-to-Length Ratio)
 (Based on $V_L = 15$ ft/sec)
 (Parameters given in Table V)
 RMW April 1965
 R.W.

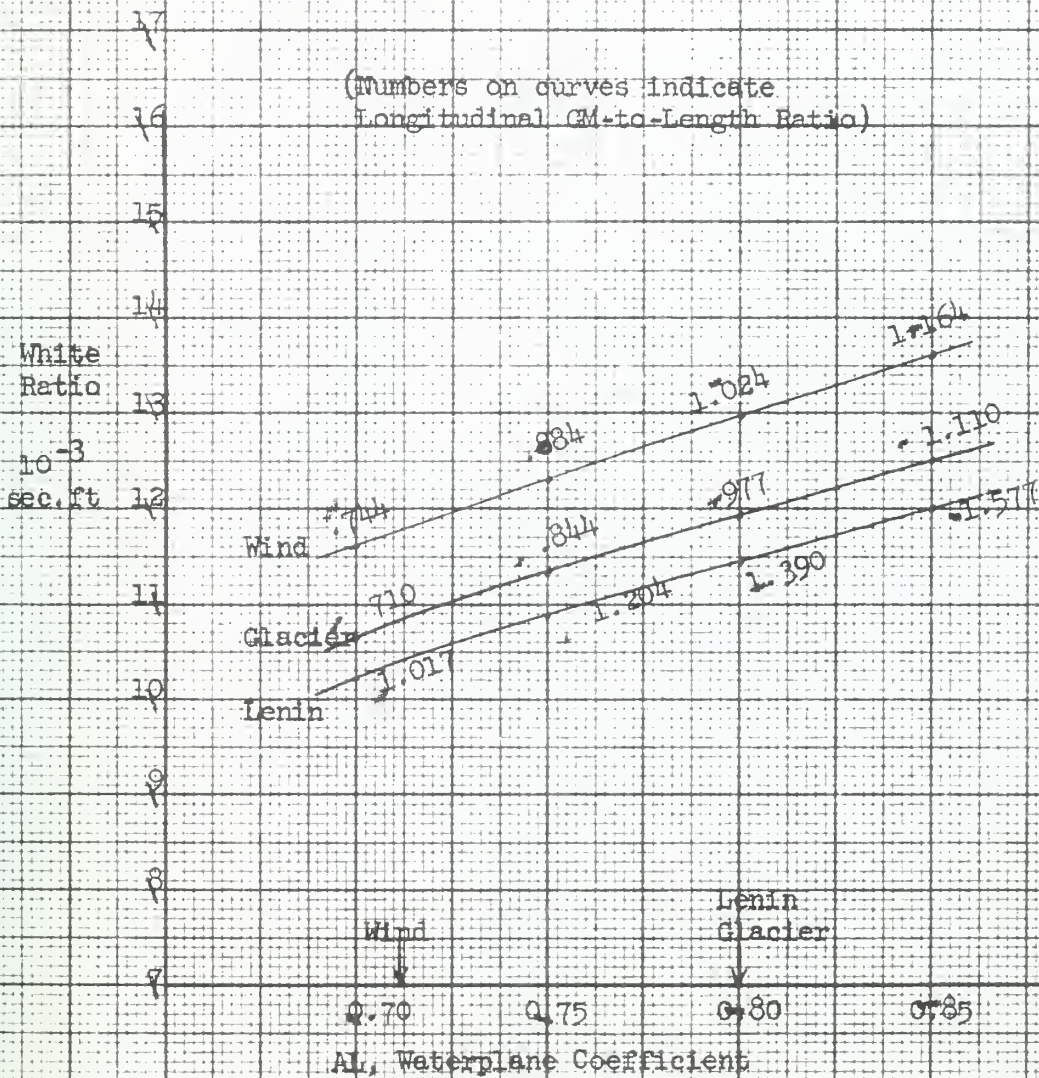


Figure XXXV
White Ratio
vs
Block Coefficient
(And Displacement)
(Based on $V_1 = 15$ ft/sec)
(Parameters given in Table V)
RMW April 1965
R.W.

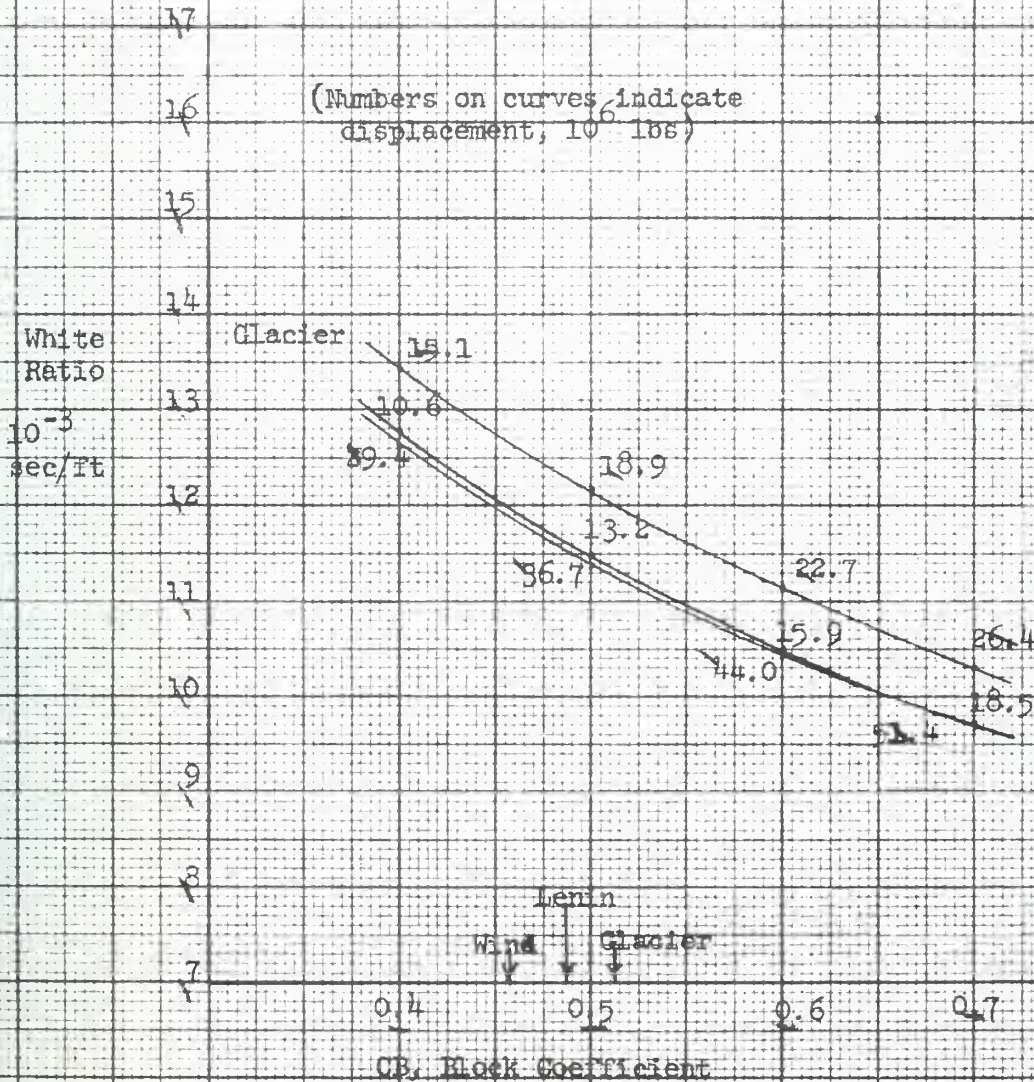


Figure XXXVI

White Ratio

vs.

Spread Angle Complement

(Based on $V_L = 15$ ft/sec)

(Parameters given in Table V)

RMW April 1965

R.W.

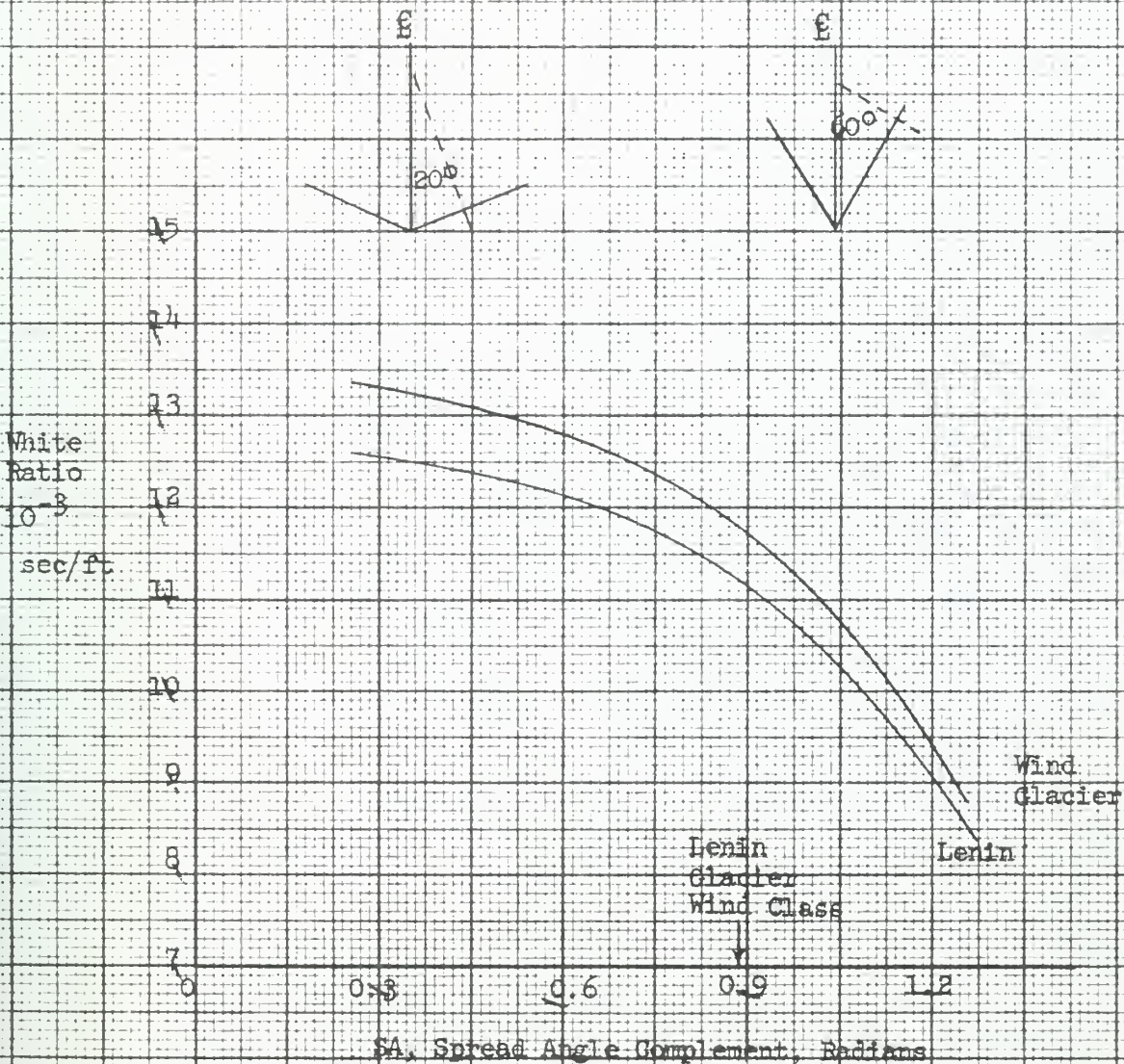


Figure DCCVII
White Ratio
vs
Coefficient of Kinetic Friction
(Based on $V_1 = 15$ ft/sec)
(Parameters in Table V)
BMW April 1965
P.M.W.

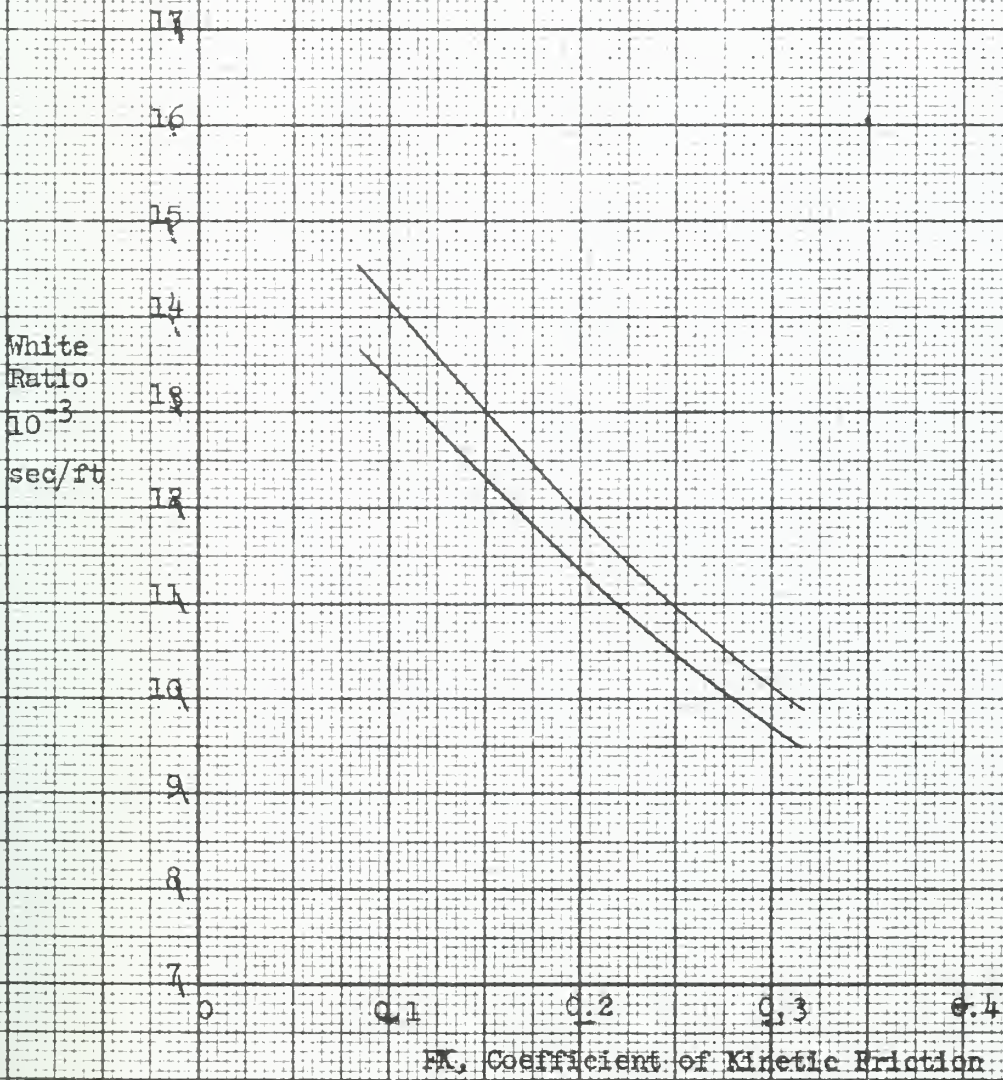


Figure XXXVIII

White Ratio

vs

Bow Angle

(Based on $V_L = 15$ ft/sec)

(Parameters given in Table V)

BMW April 1965

R.W.W.

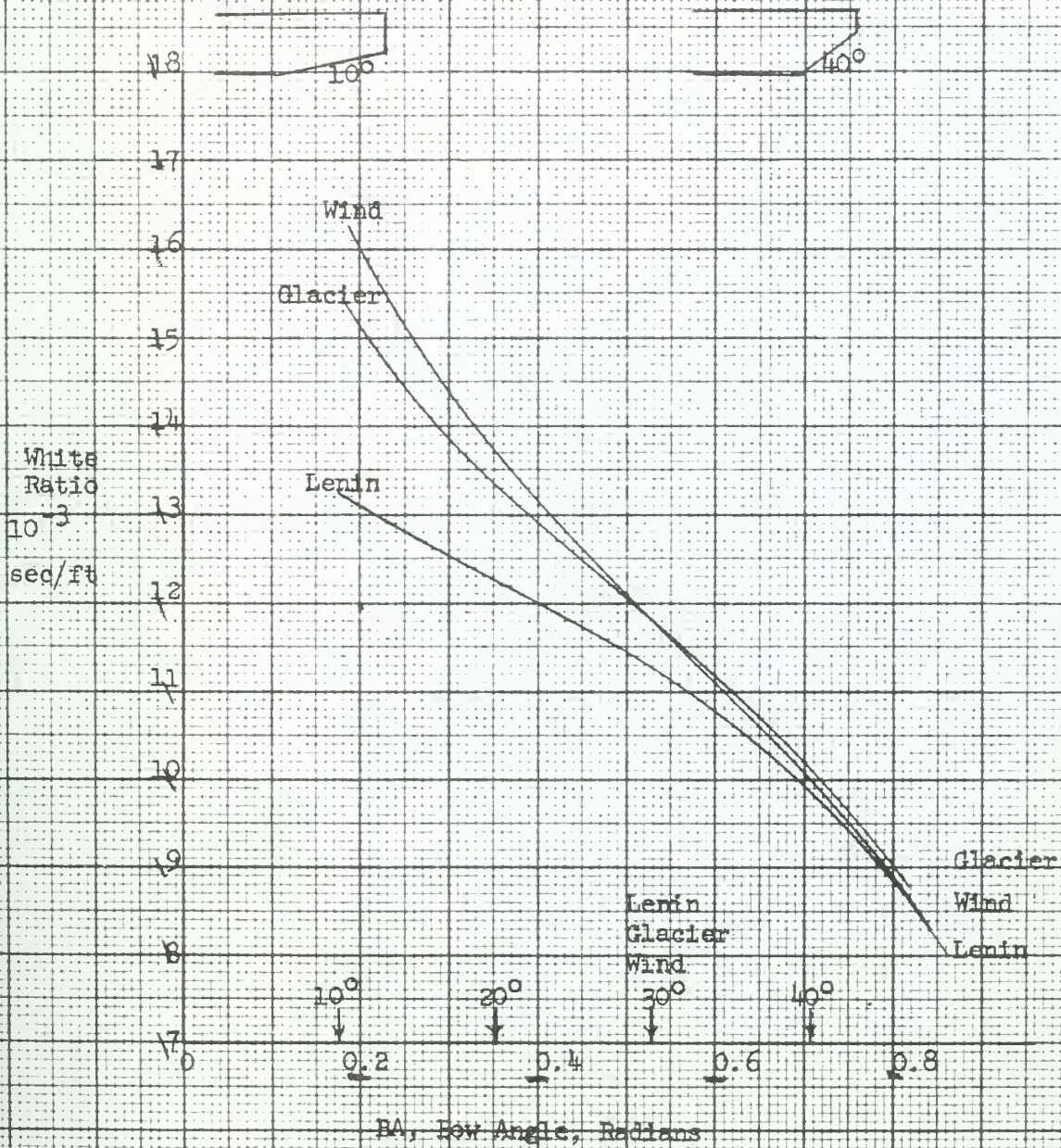


Figure XXXIX
White Ratio for Wind Class
vs
Bow Angle
(Parameters given in Table V)
RMW April 1965
Ran W

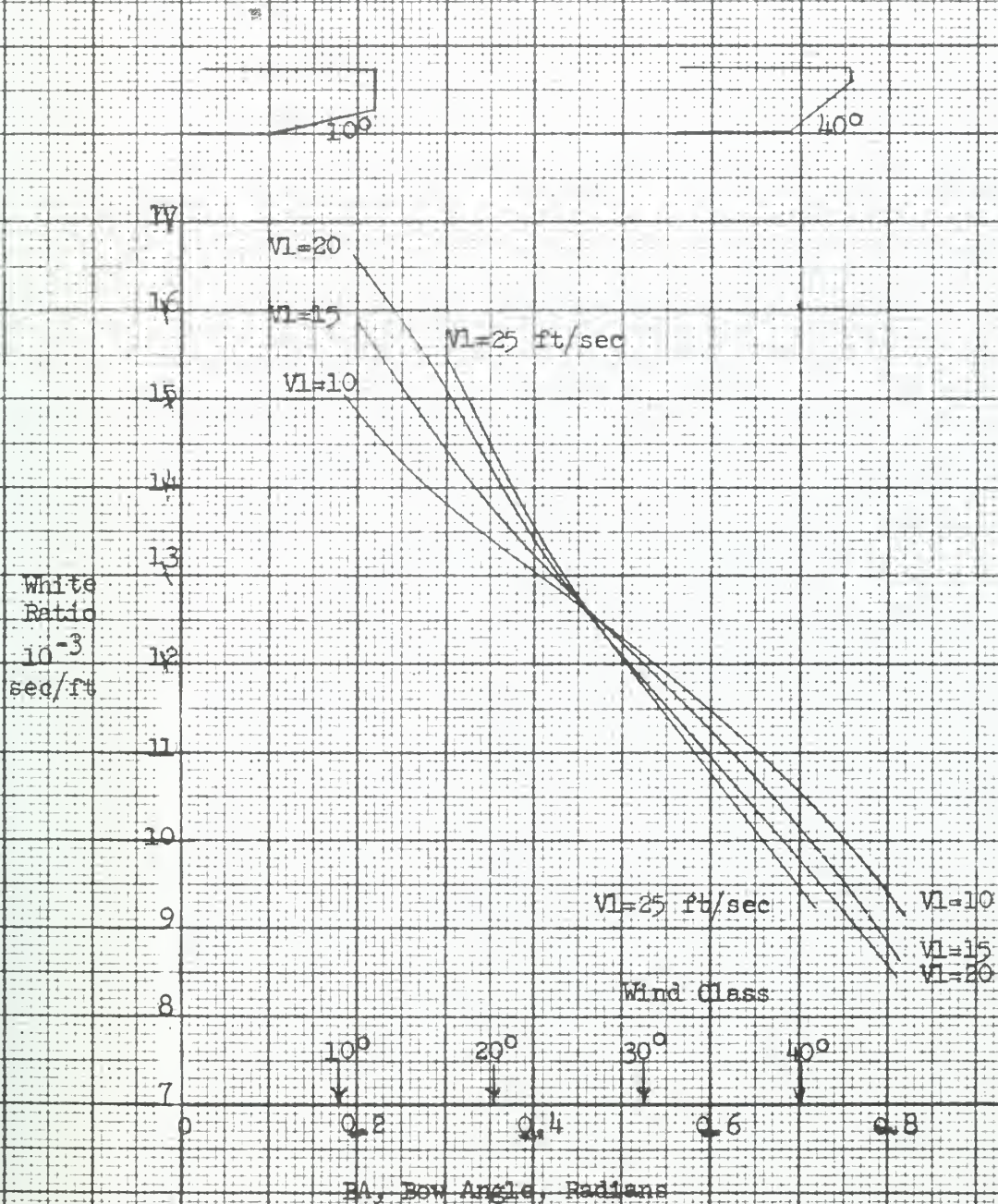


Figure XXX

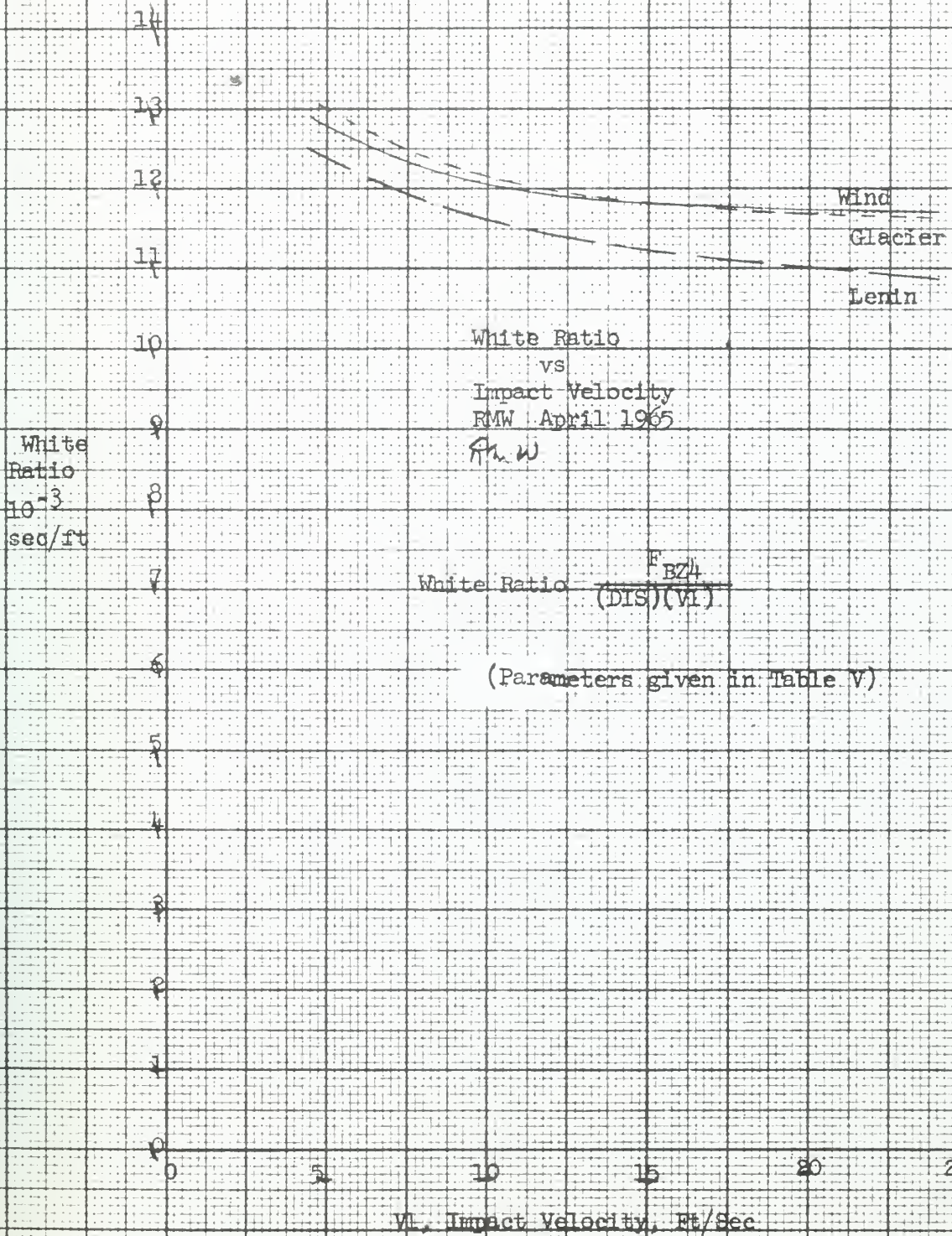


Figure XXXI
Ratio of Extracting Thrust
to Bollard Thrust for Wind Class
vs
Bow Angle
(Based on $V_1 = 15$ ft/sec)
(Parameters as given in Table V)
RMW April 1965

R₂W

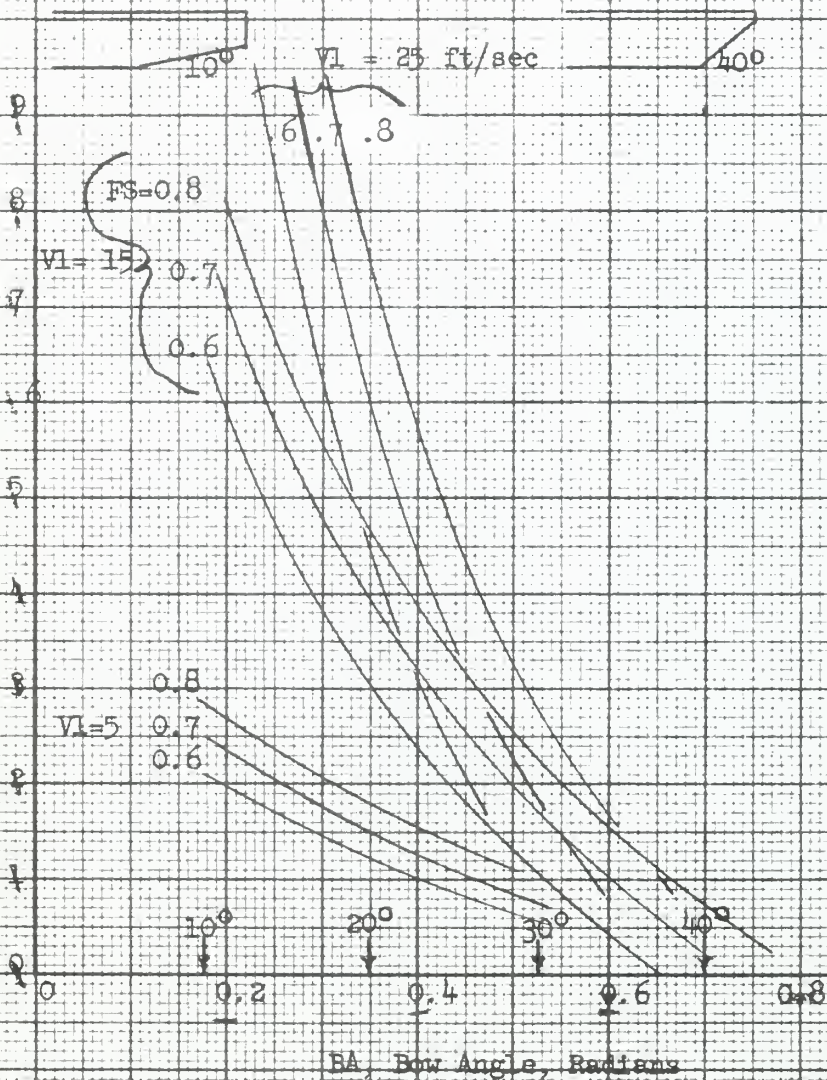


Figure XXXXII
Ratio of Extracting Thrust to
Max. Bollard Thrust for Wind Class
vs.
Coefficient of Static Friction
(Parameters given in Table V)
RMW April 1965
Rm u

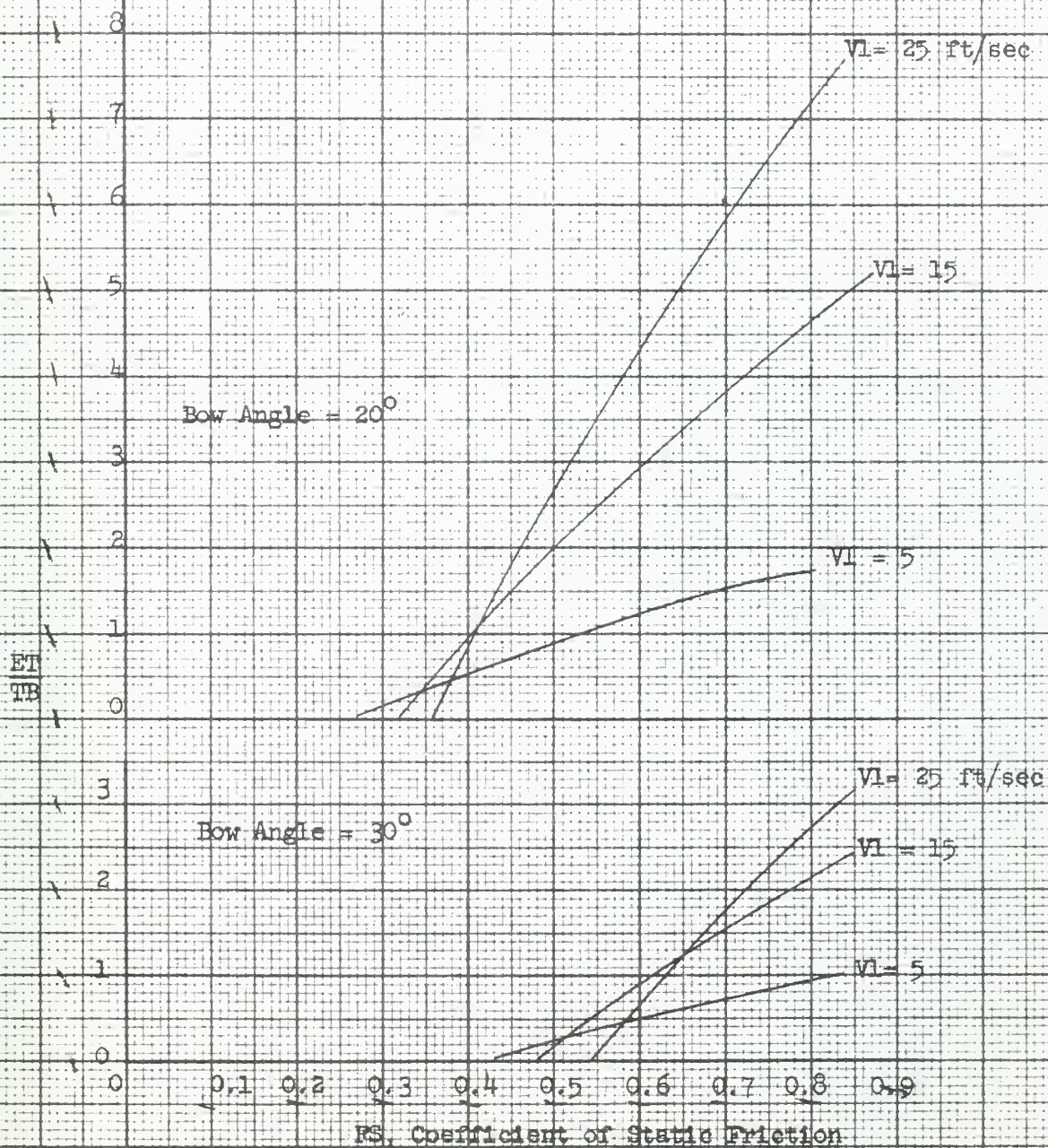
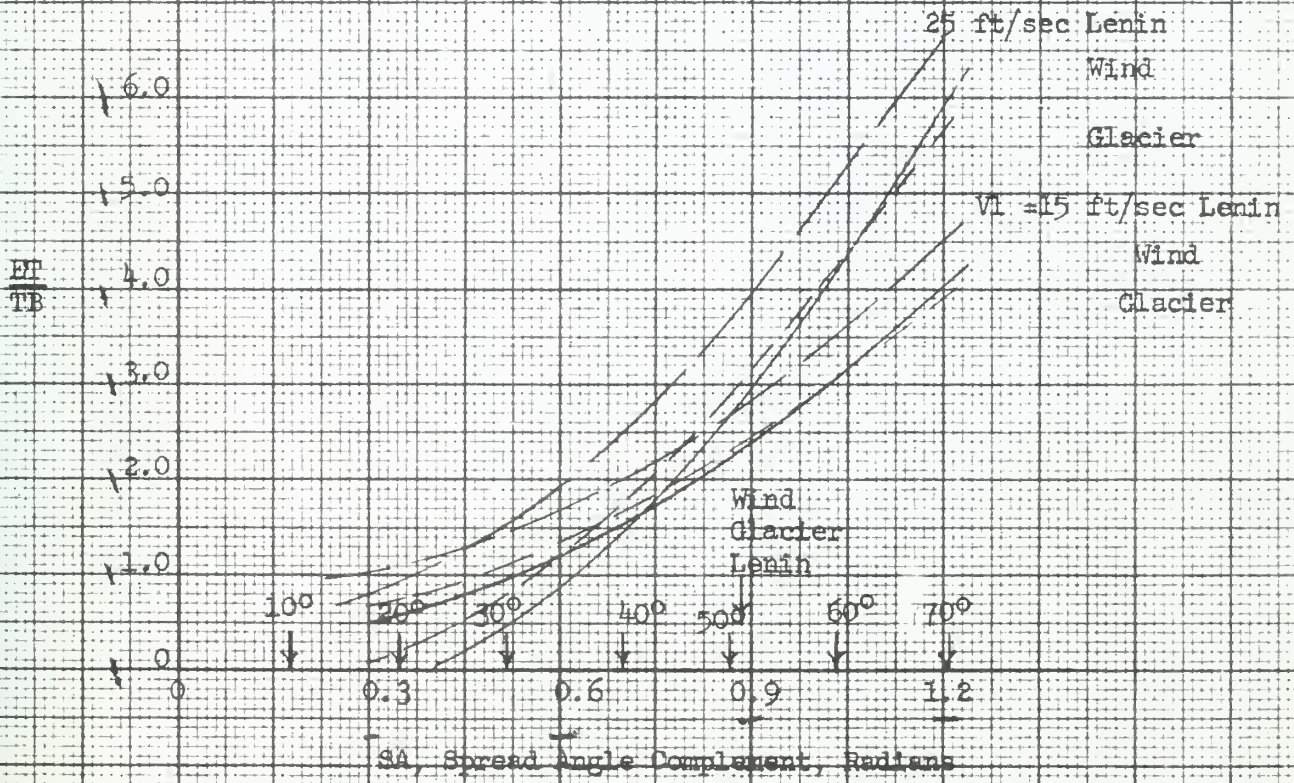
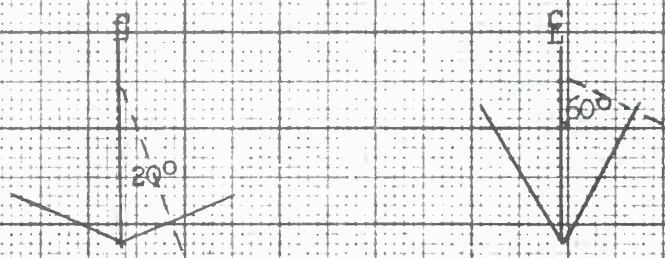


Figure XXXIII
Ratio of Extracting Thrust
to Bollard Thrust
vs
Spread Angle Complement
(Parameters are given or in Table V)
BMW April 1965
 $R_{E,W}$



Model Correlation

The upper portion of Figure XXXIV shows the prediction of ice-breaking force of a model as a function of time. The lower portion shows the prediction of icebreaking force of a geometrically similar ship as a function of time when the scale ratio, λ , is 100:1. The parameters used in the two solutions are given in Table VI.

Table VII gives the predictions of State 2 (end of crushing) and State 4 (static) values for model and ship respectively.

Mathematical

The first part of the paper shows the procedure of the
 solving the problem of the linear system of the
 equations of the form $Ax = b$ where A is a
 matrix of the form of a block-diagonal matrix
 and b is a vector of the form $b = (b_1, b_2, \dots, b_n)^T$.
 The second part of the paper shows the procedure of
 solving the problem of the linear system of the
 equations of the form $Ax = b$ where A is a
 matrix of the form of a block-diagonal matrix
 and b is a vector of the form $b = (b_1, b_2, \dots, b_n)^T$.
 The third part of the paper shows the procedure of
 solving the problem of the linear system of the
 equations of the form $Ax = b$ where A is a
 matrix of the form of a block-diagonal matrix
 and b is a vector of the form $b = (b_1, b_2, \dots, b_n)^T$.

TABLE VI

PARAMETERS USED IN MODEL-SHIP PREDICTION

($\lambda = 100$) (Wind Class)

Model

BP = 2.500 ft	B = 0.640 ft
H = 0.257 ft	DIS = 12.10 lbs
BA = 0.523 rad.	SA = 0.886 rad
VI = 1.00 ft/sec	AL = 0.724
CF = -0.012 ft	CG = -0.024 ft
GK = 0.234 ft	D = 0.16 ft
TB = 0.270 ft	GM = 1.956 ft
FK = 0.20	FS = 0.80
SIG = 1440.0 lb/ft ²	

Ship

BP = 250.0 ft	B = 64.0 ft
H = 25.7 ft	DIS = 12,100,000.0 lbs
BA = 0.523 rad	SA = 0.886 rad
VI = 10.0 ft/sec	AL = 0.724
CF = -1.25 ft	CG = -2.40 ft
GK = 23.4 ft	D = 16.0 ft
TB = 270,000.0 lb	GM = 195.6 ft
FK = 0.20	FS = 0.80
SIG = 144,000.0 lb	

Summary Statement of the Balance Sheet
(as at 31st Dec 1992)

Total

Fixed Assets	100
Current Assets	150
Less: Current Liabilities	(20)
Net Assets	130
Share Capital	100
Reserves	30
Total	130

Fixed Assets	100
Current Assets	150
Less: Current Liabilities	(20)
Net Assets	130
Share Capital	100
Reserves	30
Total	130

Total

Fixed Assets	100
Current Assets	150
Less: Current Liabilities	(20)
Net Assets	130
Share Capital	100
Reserves	30
Total	130

Fixed Assets	100
Current Assets	150
Less: Current Liabilities	(20)
Net Assets	130
Share Capital	100
Reserves	30
Total	130

Figure XXXIV
Predicted Icebreaking Force of Model and Ship

vs
Time

(Parameters given in Table)

$\rho = 100$
RMW May 1965

RMW

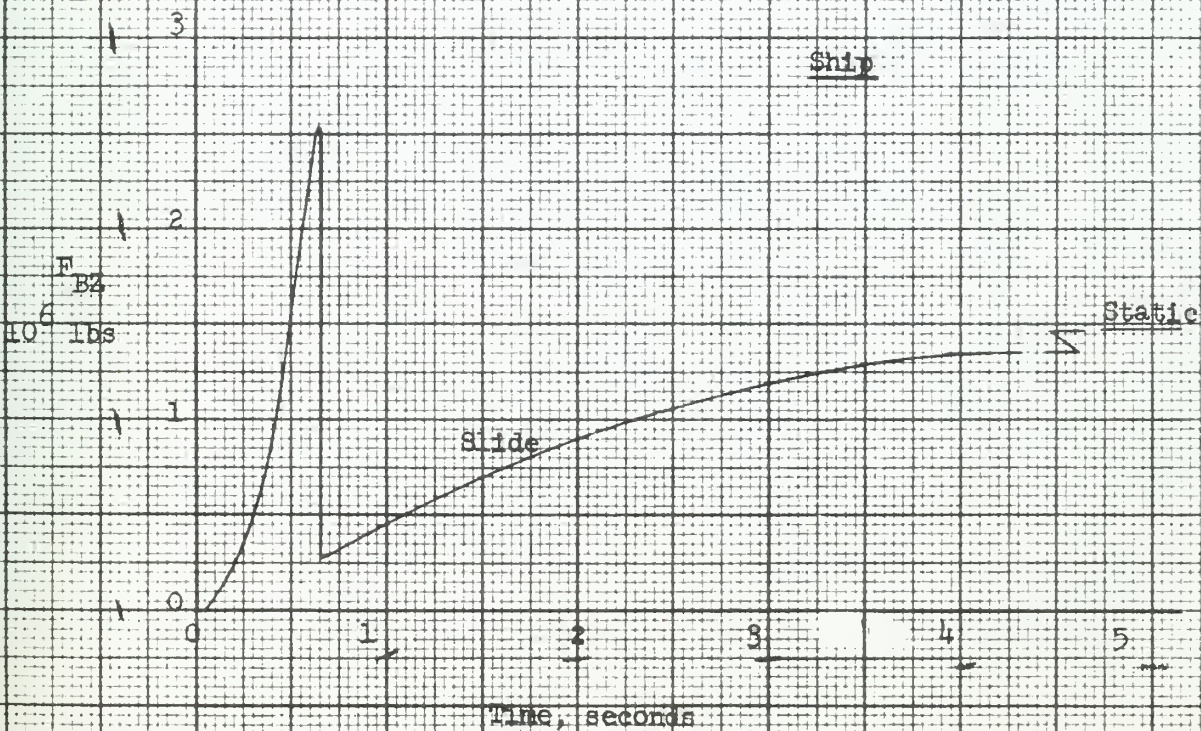
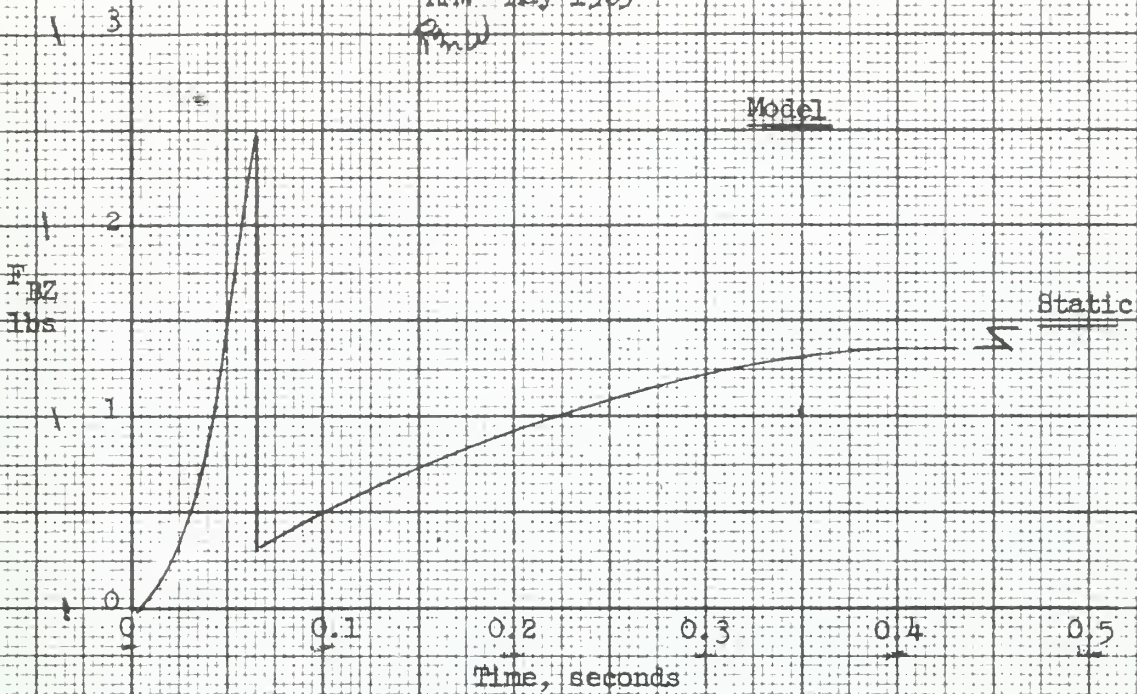


TABLE VII

COMPARISON OF STATES FOR MODEL-SHIP

($A = 100$) (Wind Class)

Model

State 2 $T_2 = 0.06298$ sec.
TH2 = 0.00555 rad THD2 = 0.33924 THDD2 = 15.35228
Z2 = -0.00111 ft ZD2 = -0.06757 ZDD2 = -3.04988
X2 = 0.06298 ft XD2 = 0.86371 XDD2 = -5.95698
FXC2 = 0.24219 x 10¹ lb FZC2 = 0.24890 x 10¹ lb

State 4
X4 = 0.24292 ft Z4 = -0.01871 ft TH4 = 0.07567
Vertical force at bow = 0.14587 x 10¹ lb
White Ratio = 0.120551
Extracting Thrust = 0.46863 lb
Ratio of Extracting Thrust to Bollard Thrust = 1.736

Ship

State 2 $T_2 = 0.63496$
TH2 = 0.00556 rad THD2 = 0.03425 THDD2 = 0.15531
Z2 = -0.11101 ft ZD2 = -0.68218 ZDD2 = -3.08528
X2 = 6.34961 ft XD2 = 8.63136 XDD2 = -6.02285
FXC2 = 0.24486 x 10⁷ FZC2 = 0.25165 x 10⁷

State 4
X4 = 24.35203 ft Z4 = -1.87490 ft TH4 = 0.07568
Vertical Force at bow = 0.14591 x 10⁷ lb
White Ratio = 0.012058
Extracting Thrust = 0.46874 x 10⁶ lb
Ratio of Extracting Thrust to Bollard Thrust = 1.736

PROBLEMS ON THE USE OF THE CALCULATOR

$\log 2 = 0.3010$ $\log 3 = 0.4771$ $\log 4 = 0.6021$
 $\log 5 = 0.6990$ $\log 6 = 0.7782$ $\log 7 = 0.8451$
 $\log 8 = 0.9031$ $\log 9 = 0.9542$ $\log 10 = 1.0000$

Example 1: Find $\log 12$
 $\log 12 = \log (3 \times 4) = \log 3 + \log 4 = 0.4771 + 0.6021 = 1.0792$
 Example 2: Find $\log 15$
 $\log 15 = \log (3 \times 5) = \log 3 + \log 5 = 0.4771 + 0.6990 = 1.1761$
 Example 3: Find $\log 21$
 $\log 21 = \log (3 \times 7) = \log 3 + \log 7 = 0.4771 + 0.8451 = 1.3222$

$\log 11 = 0.1041$ $\log 12 = 0.1771$ $\log 13 = 0.2543$
 $\log 14 = 0.3364$ $\log 15 = 0.4130$ $\log 16 = 0.4914$
 $\log 17 = 0.5691$ $\log 18 = 0.6478$ $\log 19 = 0.7244$

Example 4: Find $\log 22$
 $\log 22 = \log (2 \times 11) = \log 2 + \log 11 = 0.3010 + 0.1041 = 0.4051$
 Example 5: Find $\log 24$
 $\log 24 = \log (3 \times 8) = \log 3 + \log 8 = 0.4771 + 0.9031 = 1.3802$
 Example 6: Find $\log 27$
 $\log 27 = \log (3 \times 9) = \log 3 + \log 9 = 0.4771 + 0.9542 = 1.4313$

IV DISCUSSION OF RESULTS

General

The most significant result of this research is the establishment of a method of solution for determining the downward force under an icebreaker bow; this force is the result of ramming the ice and is a relatively sustained force. The complete computer program is given in Appendix B and yields the force under the bow as a function of 16 inputs. (14 are characteristics of the icebreaker; 2 pertain to the ice.) The object of this research has thus been fulfilled - "a suitable equation for the prediction of the dynamically developed force at the bow of an icebreaker during encounter with virtually unyielding ice".

Validity

Full scale tests were made in 1963 to determine structural strains during ramming. As part of these observations other measurements were made (i.e. \dot{z} , θ , $\dot{\theta}$ and \dot{x}). Figures XI, XIII, and XV show the predicted and observed values as functions of time. Comparison can be made only up to the time when the bow knuckle comes in contact, as indicated in Chapter II. The agreement of prediction and observation is quite obvious with trim angle, θ . The agreement of prediction and observation is very good with velocity, \dot{x} (with the exception of Run 37B where the observed value was known to be in error).

CHAPTER I

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CHAPTER II

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The correlation between predicted and observed values of acceleration (\dot{z} and $\dot{\theta}$) is much better than would seem apparent to a casual observer. It must be recalled that the prediction is for a solid body and that the observations were made on an elastic body. The prediction is essentially an impulse, similar to striking the end of a beam with a sledge hammer. The response (observation) is a vibration of this beam (ship) at its natural frequency. The accelerometers sense and record this vibration and do not feel the impulse that a solid body would have.

Figures VIII, IX, and X show, to a more readable scale, the predictions of x , z , and θ (along with their respective first and second derivatives) for Run 37B. Inspection of these curves reveals a more meaningful representation of the prediction.

Figures XII, XIV, and XVI illustrate one basic idea and observation. The observation shows that there is a peak strain on a forward transverse bulkhead which occurs at about one half second, while the ship doesn't come to a stop until three or four seconds later. This peak is important because it implies that there is a maximum bow force during load crushing. This force may be quite readily seen in the prediction curve for the force under the bow. The time this peak occurs is quite dependent on the compressive failure stress of the ice (although the ultimate value of sustained force is not sensitive to the stress - as will be explained later). For example, using a stress of $144,000 \text{ lbs/ft}^2$ leads to a peak (for Run 37B) at about 0.6 seconds. If $40,000 \text{ lbs/ft}^2$ were used the peak would occur at about 0.9 seconds.

From the above-mentioned comparisons it is realized that the mathematical model of this dynamic motion and the corresponding force under the bow does give a valid representation of real dynamic icebreaking.

Variation of Parameters, Effect on Downward Force*

Compressive Failure Stress of Ice:

As may be seen in Figure XVII the dynamically developed force under the bow is insensitive to the compressive failure stress of the ice. As noted earlier, the impulse peak comes earlier (and is of greater magnitude) when the stress is increased. Obviously the ship designer does not have control of this characteristic so it is indeed fortunate that this parameter is not significant.

Ratio of Height of Thrust Line to Draft:

As used in the calculations, the "height of thrust line" represents the approximate distance from the base line to the shaft line measured at the longitudinal position of the center of gravity.

Figure XVIII illustrates that the downward force is insensitive to the height of the thrust line.

Ratio of Bollard Thrust to Displacement:

It is interesting to note, from Figure XIX, that the application of full power, once initial contact is made, increases the downward force by

* This section of the discussion is based on Wind Class calculations but is valid for the Glacier and Lenin.

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only a few percent. Bollard thrust is very important, but for two reasons not immediately apparent here. High thrust capability is necessary to attain worthwhile impact velocity in a short distance. As will be noted later, backing thrust of large magnitudes is very important.

Ratio of Longitudinal Position of Center of Gravity to Length:

Figure XX illustrates that the downward force is insensitive to the longitudinal position of the center of gravity.

Ratio of Longitudinal Position of Center of Flotation to Length:

Figure XXI illustrates that the downward force is insensitive to the longitudinal position of the center of flotation.

Length-to-Beam Ratio:

Although the beam is important as it affects transverse stability, the width of the channel established, and maneuverability, the length-to-beam ratio has little or no effect on the downward force. This is apparent in Figure XXII.

Ratio of Height of Center of Gravity to Draft:

Figure XXIII indicates that there is a slight advantage in keeping the center of gravity relatively low. There is naturally a gain in transverse stability also. However, this variation should not be considered as significant in the design of an icebreaker since the magnitude of change is only a few percent in a shift of one tenth of the draft.

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Beam-to-Draft Ratio:

As may be seen in Figures XXIV and XXXIII, an increase in the beam-to-draft_{ratio} causes a definite increase in the downward force. Although beam and draft are normally determined on the basis of other considerations, where possible a preference should be given to high beam-to-draft ratios.

Waterplane Coefficient:

If the waterplane coefficient is increased (implying a reduction of the waterplane coefficients of immersed waterplanes) there is an increase of magnitude of longitudinal metacentric height. Consequently there is a greater downward force. This may be seen in Figures XXV and XXXIV.

Block Coefficient:

Figure XXVI indicates that the downward force may be increased by increasing the block coefficient. However, the reason for this increase is that the displacement has been increased correspondingly. It is to be noted from Figure XXXV (where the force has been divided by the product of impact velocity and displacement) that increasing the block coefficient decreases the downward force with respect to displacement.

In substance, this means that where a choice is possible, it is preferable to have a large (by linear dimension) icebreaker than a small full one (large C_p) of the same displacement.

Spread Angle Complement:

As may be seen from Figure XXVII, an increase in the spread angle complement (making the bow "sharper") causes a reduction in the downward force which can be attained by ramming. Figure XXXVI illustrates the same result. It is to be noted that there is a significant reduction if bows were to be "sharper" than those on the three major classes investigated.

A decrease of the spread angle complement (making the bow "blunter") causes an increase in the downward force which can be attained by ramming.

As will be discussed later, it is important to note that making the bow "blunter" also decreases the amount of thrust necessary for extraction.

Coefficient of Kinetic Friction:

As would seem obvious, an increase in the coefficient of kinetic friction causes a reduction in the downward force. This may be observed in Figure XXVIII and in Figure XXXVII.

Unfortunately, the coefficient cannot be readily controlled since it depends on the ice as well as the ship. It is apparent, however, that any reduction of this coefficient would be of value. Smoothness of the bow or the application of a durable low friction coating should certainly be considered. (A reduction of 1/10 in coefficient may lead to a 20 o/o increase in downward force.)

Generalized Equilibrium

As we have seen, the general equilibrium model is a complex one, involving the interaction of many different variables. The model is based on the assumption that the economy is in a state of general equilibrium, and that the variables are interdependent. The model is used to analyze the effects of changes in the economy, and to predict the future course of the economy.

Introduction

The general equilibrium model is a complex one, involving the interaction of many different variables. The model is based on the assumption that the economy is in a state of general equilibrium, and that the variables are interdependent. The model is used to analyze the effects of changes in the economy, and to predict the future course of the economy.

Definition of General Equilibrium

As we have seen, the general equilibrium model is a complex one, involving the interaction of many different variables. The model is based on the assumption that the economy is in a state of general equilibrium, and that the variables are interdependent. The model is used to analyze the effects of changes in the economy, and to predict the future course of the economy.

The General Equilibrium Model

Introduction: The general equilibrium model is a complex one, involving the interaction of many different variables. The model is based on the assumption that the economy is in a state of general equilibrium, and that the variables are interdependent. The model is used to analyze the effects of changes in the economy, and to predict the future course of the economy.

Bow Angle:

Probably the most frequently discussed variable of icebreaker design is the angle the stem makes with the base line. As long as the attaining of maximum downward force as a result of ramming is the main consideration, it is of great desire to have a relatively small bow angle. For example, an icebreaker with a 20° bow angle could exert (by ramming) about 20 o/o more downward force than an equivalent icebreaker with a 30° bow angle. This may be observed quite clearly in Figures XXIX, XXXVIII, and XXXIX.

In Figure XXIX line A-A indicates the condition where the peak load (vertically) during crushing is equal to the final sustained downward force. The area to the right of line A-A is a region where the peak crushing load is greater than the sustained value. For example, at 30° the peak crushing load is about twice the magnitude of the sustained downward force. Therefore it is desirable to reduce the bow angle in order to reduce the relative intensity of this peak load.

Unfortunately, decreasing the bow angle increases the thrust necessary for extraction, as will be explained later.

Displacement:

Figures XXX, XXXI, and XXXII all indicate that an increase in the displacement causes an increase in the downward force, as would be anticipated.

The first of these is the fact that the
 results of the present study are in general
 in line with those of other workers in the
 field. This is particularly true in the
 case of the results obtained from the
 analysis of the data obtained from the
 study of the effect of the concentration
 of the solution on the rate of reaction.
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APPENDIX

TABLE I
 Rate constants for the reaction of
 the various solutions used in the
 present study. The values are given
 in units of min^{-1} . The standard
 deviation is given in parentheses.
 The values are in general in line
 with those of other workers in the
 field. This is particularly true in
 the case of the results obtained from
 the analysis of the data obtained from
 the study of the effect of the concentration
 of the solution on the rate of reaction.

Figure XXX simply shows the effect of displacement by plotting the results of the three major classes of icebreaker investigated. It is significant that the downward force (in the full scale range) is approximately linear with respect to displacement. Figure XXXI is a set of cross curves of the same information.

Figure XXXII shows the effect of increasing displacement by generation of geometrically similar icebreakers. It is clear that the downward force is approximately linear with respect to displacement. It is also clear that "geosims" of the three classes selected produce about the same downward force at any given displacement.

Impact Velocity:

As may be seen in Figure XXXI, the downward force produced as a result of ramming is approximately linear with impact velocity. For example, a Wind Class icebreaker produces a downward force of about 1 1/2 million pounds after impacting at 10 feet per second (about 6 knots); 3 million pounds is produced at 20 feet per second (about 12 knots).

It is also interesting to note that a Wind Class icebreaker can produce, at 15 feet per second (about 9 knots), the same downward that the Glacier produces at 9 feet per second (about 5.3 knots). This is quite significant when one realizes the Glacier has about 60 o/o greater displacement.

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Impacting at higher velocities is probably the most productive way of increasing downward force. However, this means that the peak crushing load will be greater also unless the bow angle is reduced from the present practice of 30° .

As will be seen later, the necessary thrust for extraction will probably (but, oddly enough, not "necessarily") be increased.

Higher impact velocities require more thrust for acceleration - and probably more confidence and courage on the part of a commanding officer.

Figure XXXX shows the relative insensitivity of "White Ratio" with variation of impact velocity. Since the "White Ratio" is the downward force divided by the product of impact velocity and displacement, it is again clear that the downward force varies linearly with impact velocity (and displacement).

Extracting Thrust

General:

Since the extracting thrust necessary to pull the ship off the ice is directly related to the downward force under the bow (and the angle at which the static friction is applied), it may be safely stated that practically any variation of parameter which causes an increase in downward force also causes a corresponding increase in extracting thrust.

The effect of change in some parameters is worthy of mention, particularly because there is one notable exception to the above generality.

Static Coefficient of Friction:

As may be readily seen in Figures XXXXI and XXXXII, and as is intuitively obvious, a decrease in the coefficient causes a decrease in the extracting thrust.

The plots use extracting thrust divided by maximum bollard thrust (ahead) as the ordinate. At first glance it would not seem likely that an icebreaker with a 30° bow angle could extract itself if the coefficient of static friction were 0.8 (as used in most calculations). However, in spite of extraction difficulties, all the icebreakers have managed to break free. This is readily explained when one considers the other factors influencing extraction such as shifting the rudder, using trimming and heeling tanks, and explosive charges on the ice.

Experience would indicate that an extracting thrust to bollard thrust ratio of approximately 2 is not unreasonable for a valid icebreaker design (presuming the coefficient is about 0.8). However, experience has shown that we are not far from the threshold with present designs.

Bow Angle:

It is obvious from Figure XXXXI, a decrease in bow angle causes a very significant increase in extracting thrust. For example let us assume that a value of 2 is a tolerable limit for extracting thrust to bollard thrust ratio (as above). Note that we are approximately in that range (or below) with a 30° bow angle. However, if a Wind Class icebreaker had

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THE EFFECTS OF PRACTICE ON THE
ACQUISITION OF SKILLS

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AND

W. D. COOPER

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a 20° bow angle the ET/TB ratio goes to approximately 7 (for 25 feet per second), an obviously intolerable value.

Yet we have seen that, for reasons of increasing downward force and decreasing the relative peak crushing load, it is desirable to decrease the angle. Apparently reduction of the bow angle (below present 30° practice) cannot be wisely undertaken unless there is a reduction of static friction (or, as will be seen later, a "blunting" of the bow).

Figure XXXIII* shows quite clearly that an icebreaker with a 20° bow angle could be operated as safely (from the point of view of extraction) as an equivalent icebreaker with a bow angle of 30° if there were some way of reducing the static friction two tenths (i.e. from 0.7 to 0.5).

It is recommended that strong consideration be given to some method of reduction of static friction. This could be accomplished to some degree by making the bow smoother. It seems probable that durable, low friction coatings could be used. So-called "no stick" coatings are in common use in other applications. They are even used on snow shovels to prevent sticking. Although the use on snow shovels points out the reduction of static friction it does not necessarily represent the

* The numerical values are also approximately valid for the Glacier and Lenin.

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durability. However, the Air Force uses such a coating (Teflon) on the skis of some of its heavy aircraft to prevent adhering to the ice. Investigations along these lines should prove worthwhile.

It is also interesting to note, in Figure XXXII, that a reversal takes place at low values of static friction. For example, an icebreaker (with a 30° bow angle) requires about half bollard thrust to extract if it has impacted at a low velocity of 5 feet per second (about 3 knots) when the coefficient of static friction is about 0.55. However, little or no backing thrust is required if the impact velocity is 25 feet per second (about 15 knots). This is because it is a somewhat critical region for static friction and the higher impact velocity has given a greater trim angle.

Spread Angle Complement:

As was noted earlier, as an icebreaker bow is made more blunt the downward force is increased. Most significantly however, the necessary extraction thrust is reduced. This may be seen clearly in Figure XXXIII. For example, if a Wind Class icebreaker with the regular (about 50°) spread angle complement rams the ice at 25 feet per second the extracting thrust is about 3 times the bollard thrust. If the same icebreaker had a blunter bow (about 25° spread angle complement) no thrust at all would be required for extraction.

It is to be noted that this is the only variable which can be

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changed and improve downward force and extracting characteristics at the same time. It is recommended that bows for polar icebreakers be designed with a smaller spread angle complement (a blunter bow).

It should be realized that reducing the spread angle complement increases the entrance angle of the bow (measured in the waterplane). However, the entrance angle may be reduced by decreasing the bow angle.

The first part of the report deals with the general situation of the country and the progress of the work done during the year.

The second part contains a detailed account of the work done in the various departments, and the progress of the work done in the various branches of the service.

The third part contains a summary of the work done in the various departments, and the progress of the work done in the various branches of the service.

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The thirteenth part contains a summary of the work done in the various departments, and the progress of the work done in the various branches of the service.

The fourteenth part contains a summary of the work done in the various departments, and the progress of the work done in the various branches of the service.

It is quite apparent (from Figure XXXI) that there would be a considerable gain attained in downward force (and relative decrease of peak load during crushing) by using small bow angles. Yet it is also apparent (from Figure XXXI) that the bow angle, for extraction, should be relatively high. However, the need for this higher bow angle exists only at State 4. Therefore it is recommended that this higher angle exist only at lower sections of the stem, where the stem and bow plating would be in contact with the ice once the forward motion had stopped (State 4).

The result of adopting this idea would be as shown in Figure XXXV. The stem is slightly concave. The initial contact with the ice would come where the bow is at a 15° to 20° angle. The slope would change continuously down to the lower portion of the stem such that the bow angle would be slightly in excess of 30° in the area which would be in contact during State 4. Particularly considering the recommendation for a small spread angle complement (blunter bow), this should lead to higher sustained downward force, relatively smaller peak load during crushing, and elimination of extraction difficulties.

Model Correlation

As seen in Figure XXXXIV, model predictions may be scaled up to ship predictions based on the equations and scaling factors given in the procedure.

Naturally the model must be geometrically similar. This means that linear dimensions are related by λ and that volumes are related by λ^3 . Coefficients remain the same. The compressive failure stress of the ice (or simulating support for the bow of the model) must be related by λ . Model and ship are to be operated at the same Froude Number at impact.

The vertical force at the bow of the ship is λ^3 times the force at the bow of the model. The time-of-ship-event is $\sqrt{\lambda}$ times the time-of-model-event.

The distances and positions are related by λ as may be seen in Table VII. It is noted that extracting forces are related by λ^3 .

Generalization

It is seen that the results of the analysis may be applied to all cases where the conditions are the same as those given in the preceding section.

Turning now to the case of a generalization, it is seen that the results of the analysis may be applied to all cases where the conditions are the same as those given in the preceding section. The results of the analysis may be applied to all cases where the conditions are the same as those given in the preceding section.

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V CONCLUSIONS

General

The mathematical model of icebreaker motion and corresponding downward force under the bow (given in the Procedure) is valid.

Therefore, the computer program may be used for the prediction of dynamically developed force at the bow of an icebreaker during encounter with virtually unyielding ice.

Effect of Parameter Variation

There is no "optimum" value for any one parameter for maximum downward force. In other words, all curves of downward force as a function of a given parameter are without peak or hollow. (The derivative of the curve does not go to zero.)

The following is a list of causes which will give the effect of increasing the downward force developed by ramming:

Increase of displacement (approximately linear relationship).

Increase of impact velocity (approximately linear relationship).

Decrease of the bow angle.*

Decrease of the spread angle complement (blunter bow).

Decrease of the coefficient of kinetic friction.

Decrease of the block coefficient.

Increase of the waterplane coefficient.

Increase of the beam-to-draft ratio.

* This decrease of bow angle also lessens the severity of the peak load at impact relative to the final downward force.

The following parameters have little or no effect on the downward force developed by ramming:

Ratio of height of center of gravity to draft.

Length to beam ratio.

Ratio of longitudinal position of center of flotation to length.

Ratio of longitudinal position of center of gravity to length.

Ratio of bollard thrust to displacement (except as explained in Discussion).

Ratio of height of thrust line to draft.

Compressive failure stress of the ice.

Extracting Thrust

With the exception of the spread angle complement, all variations of parameters which cause an increase in downward force also cause an increase in the extraction thrust.

Decreasing the spread angle complement reduces the extracting thrust markedly while improving the downward force characteristic.

It should also be noted that any technique used for reducing kinetic friction (which would increase the downward force) would probably reduce the static friction (which would decrease the extracting thrust).

A reduction of the coefficient of static friction significantly reduces the extracting thrust.

Model Correlation

In icebreaker model tests the results of force may be scaled by λ^3 if the Froude Number of the model at impact is the same as the ship and if the "virtually unyielding ice" of the model test has a compressive failure stress equal to the failure stress of the ice divided by λ .

The position may be scaled using λ based on the relationship that the time-of-ship-event equals $\sqrt{\lambda}$ times the time-of-model-event.

1944-1945

In January 1945 the British Government announced that it would support the formation of a coalition government in India. This was a significant step towards the realization of the Indian National Congress's demand for a united India. The British had long been reluctant to grant India independence, but the war had weakened their position and the Indian National Congress had become a powerful force. The coalition government was to be formed by the British, the Indian National Congress, and other Indian political parties. The British were to remain in charge of the administration, but the Indian National Congress was to have a say in the government. This was a major step towards the end of British rule in India.

1945-1946

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VI RECOMMENDATIONS

General

It is recommended that the characteristics of any proposed polar icebreaker be used in the mathematical model (computer program) to investigate the downward force developed by ramming. The program should also be used to determine extracting thrust and the peak load of the crushing phase.

Selection of Characteristics

If attainment of the maximum downward force were the prime objective in the design of a polar icebreaker, the following choices would be significant:

Large displacement

High impact velocity

Small bow angle

Small spread angle complement

Low value of kinetic friction (dependent in part on the ice).

Small block coefficient (if displacement is constant)

Large waterplane coefficient

High beam-to-draft ratio

The following characteristics may be disregarded (concerning downward force):

Ratios of

Height of center of gravity to draft

Length to beam

MEMORANDUM

TO: [Illegible]

FROM: [Illegible]

Subject: [Illegible]

1. [Illegible]

[Illegible text]

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Longitudinal position of center of flotation to length

Longitudinal position of center of gravity to length

Bollard thrust to displacement (except as explained in Discussion)

Height of thrust line to draft

The free selection of apparently desirable characteristics is limited by the extracting requirement. It is recommended that extracting thrust requirements be kept in mind (and evaluated) when selecting characteristics.

Decreasing the spread angle complement and reducing friction in general (both static and kinetic) are the only ways of simultaneously increasing downward force and reducing the extracting thrust requirement.

It is recommended that future polar icebreakers have blunter bows (measured in a plane perpendicular to the stem).

It is furthermore recommended that significant efforts be made to reduce friction between the hull and the ice, particularly static friction. One of the most hopeful solutions is in the use of durable "no-stick" coatings as discussed previously. Other techniques may also be possible (i.e. "lubrication" or heating).

If a useful technique for reducing this static friction becomes possible, then it would be recommended that the bow angle be selected from values less than the presently used 30° .

Thrust should be as great as possible commensurate with other considerations. A larger thrust allows higher rates of steady icebreaking. A large thrust allows greater acceleration in a relatively shorter

The first part of the paper is devoted to a general discussion of the problem of the origin of the universe. It is shown that the origin of the universe is a problem which has not yet been solved. The second part of the paper is devoted to a discussion of the origin of life. It is shown that the origin of life is a problem which has not yet been solved. The third part of the paper is devoted to a discussion of the origin of the human race. It is shown that the origin of the human race is a problem which has not yet been solved. The fourth part of the paper is devoted to a discussion of the origin of the human mind. It is shown that the origin of the human mind is a problem which has not yet been solved. The fifth part of the paper is devoted to a discussion of the origin of the human soul. It is shown that the origin of the human soul is a problem which has not yet been solved.

distance to attain a desired impact velocity, which is quite important. Backing thrust is equally important. If higher backing thrusts were available then it would be possible to select characteristics which would increase the downward force under the bow without as severe a limitation imposed by extracting requirements.

Since high thrust at low speed (or zero speed) is extremely desirable, efforts should be made to select (or design) for maximum backing thrust at 100 percent slip, even at the sacrifice of open water efficiency. Although some work has already been done (80) concerning the use of Kort Nozzles, much more investigation is needed and recommended.

Model Testing

It is recommended the model test of ramming be undertaken using a Froude Number for model operation which is the same as the ship at impact.

The model that may be similar to that used by Richardson (16) and by McMahan and Abrahams (40). However, it is necessary that the material used as "ice" have a significantly lower compressive failure stress specifically $1/\lambda$ times the compressive failure stress of ice. This will allow local crushing to accommodate the bow of the model to the same relative degree as the bow of the ship. This will lead to results which may be scaled.

Care must be taken to insure that vibration of the support for the

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"ice" is eliminated or minimized.

Through such tests other effects, such as loss of stability when encountering virtually unyielding ice, may be examined.

Bow Shape

It is recommended that the bow shape of polar icebreakers incorporate the ideas illustrated in Figure XXXXV. The angle at initial entry should be small (i.e. 15° to 20°) and the stem should be concave such that the area in contact with the ice after stopping have a relatively steeper slope (i.e. 30° - 35°). The spread angle complement should be relatively higher (blunter), perhaps in the order of 0.6 radians (34°). Compared to present bow shapes, this recommended shape will lead to greater downward sustained force, relatively smaller peak load during crushing, and elimination of extraction difficulties.

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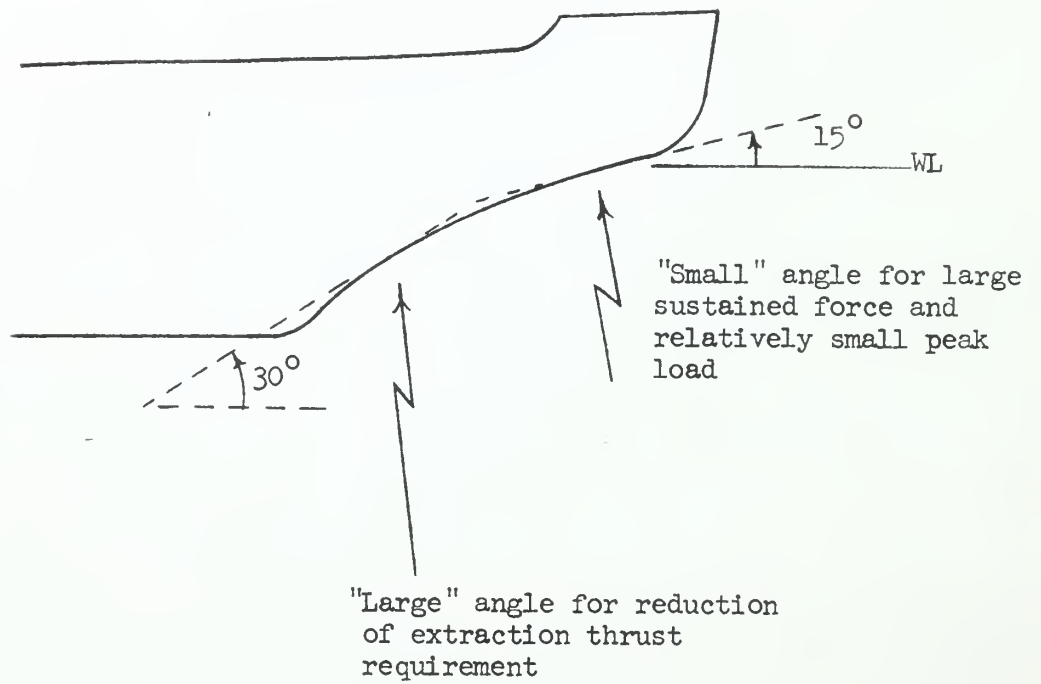
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Figure XXXXV
Recommended Polar Icebreaker
Bow Profile



VII APPENDIX

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A. SUPPLEMENTARY INTRODUCTION

Runeberg's Equation

In 1888 an equation was published (10) for the determination of the vertical component of force produced at the bow of an icebreaker during uninterrupted progression.

Runeberg used the following symbols for his development:

V = "Vertical pressure at bow", Vertical component of force at bow in lb.

R = Thrust of the propeller in lb.

θ = Trim in deg. (change of trim)

Δ = Displacement in t. '

ϕ = Angle of inclination of buttock lines to the waterline, (stem angle) in deg. See Figure A-I.

b = "Inclination of cross sections taken perpendicular to the buttock lines with respect to the waterline".

(His use of this term indicates that it is the complement of the angle from the ξ plane to the hull measured in a plane which is perpendicular to the stem.) See Figure A-II. Expressed in deg.

v = Velocity in ft/sec.

δ = Mean decrease in draft in ft.

Q = "Pressure normal to buttocks". (His use of this term indicates it is the force in the ξ plane normal to the stem.) Expressed in lb.

N = Total force perpendicular to the bow plating. (Note that N/2 acts on each side of bow.) Expressed in lb.

f = Coefficient of friction of bow plating relative to ice while moving.

CHAPTER 1

1.1 Introduction to Physics
1.2 Kinematics
1.3 Dynamics

1.4 Energy and Work

1.5 Momentum and Collisions

1.6 Rotational Motion

1.7 Oscillations

1.8 Waves

1.9 Sound

1.10 Light

1.11 Modern Physics

1.12 Relativity

1.13 Quantum Mechanics

1.14 Atomic Structure

1.15 Nuclear Physics

1.16 Particle Physics

1.17 Astrophysics

1.18 Cosmology

1.19 The Future of Physics

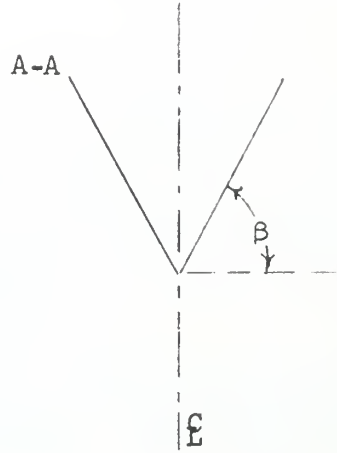
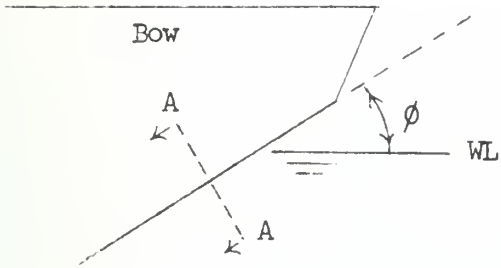
1.20 Appendix

1.21 Glossary

1.22 Index

1.23 Bibliography

Figure A-I
Illustration of Terms Used by Runeberg



P = Propeller pitch in ft.

No = Shaft r p m .

p = "Mean effective pressure on total piston area". (His use of this term indicates that it is the mean effective pressure multiplied by the total area of the pistons.) Expressed in lb.

S = Length of stroke in ft.

Runeberg developed his equation for the vertical component of force at the bow using the equilibrium equation based on Figure A-II. It is to be noted that his figure does not agree with presently accepted standards of notation but still leads to an acceptable result.

V was drawn perpendicular to the waterline. AB represents the line of the stem and the buttocks in the area of contact. Q was drawn perpendicular to AB.

The ship slides up (neglecting momentum) to a point where the force downward along the stem becomes equal to the force pushing the bow upward along the line indicated by the slope of the stem. At that point

$$K = R \cos \phi \quad (A1)$$

$$\text{where } K = V \sin \phi + f N \quad (A2)$$

It follows that

$$R \cos \phi = V \sin \phi + f N \quad (A3)$$

This can be put into the following form:

$$V = \frac{R \cos \phi - f N}{\sin \phi} \quad (A4)$$

He indicates that the thrust provided (by pressure on pistons) is divided into six parts according to Froude.

1 - ...

2 - ...

3 - ...

4 - ...

5 - ...

6 - ...

... developed the equation for the vertical component of force at the box using the collision equation from Section 11.1. It is to be noted that the sign does not agree with generally accepted notation of rotation but still leads to an acceptable result.

... perpendicular to the rotation. It represents the line of the stem and the bottom in the area of contact. A line drawn perpendicular to AB.

The sign is taken as (negative moment) to a point where the force downward along the stem becomes equal to the force pushing the box upward along the line indicated by the slope of the stem. At that point

$$(A1) \quad F = R \cos \phi$$

$$(A2) \quad R \sin \phi = V \sin \theta + F \sin \theta$$

It follows that

$$(A3) \quad R \cos \phi = V \sin \theta + F \sin \theta$$

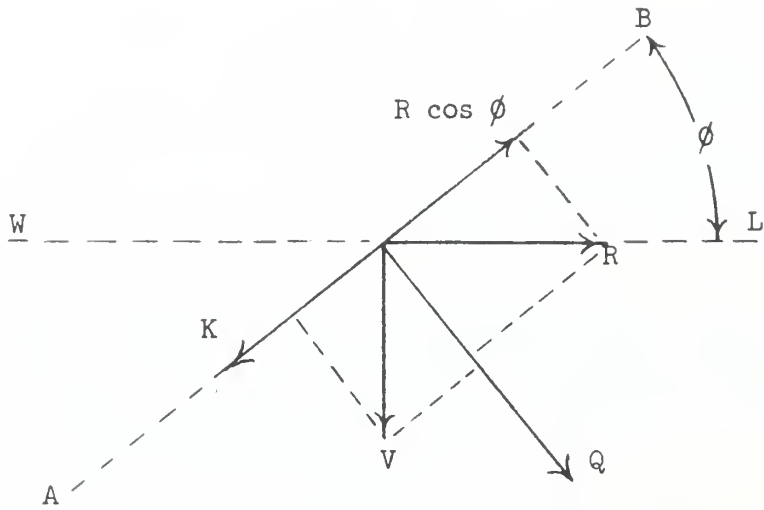
This can be put into the following form

$$(A4) \quad V = \frac{R \cos \phi - F \sin \theta}{\sin \theta}$$

It indicates that the current provided (by pressure on pistons) is

divided into six parts according to Table.

Figure A-II
Bow Equilibrium by Runeberg



1. Useful thrust (normally equal to the ship's own net resistance).
2. Augmented resistance due to action of propeller.
3. Friction of screw blades against water.
4. Slow speed friction of engine.
5. Working load friction of engine.
6. Resistance due to air and feed pumps, etc.

If the ship is pushing against the ice the last five remain unchanged but the ship's own resistance is equal to zero and in its place is the thrust R. (He has assumed no advance through the ice and that all useful thrust can be used against the ice.)

Runeberg assumes that 37.5 o/o of indicated thrust is that portion which goes to "the ship's own net resistance". Therefore he simply transfers this amount to use for ice-breaking.

$$R = \frac{37.5}{100} \times \frac{\text{IHP} \times 33,000}{P \times \text{No}} = \frac{12,375 \text{ IHP}}{P \times \text{No}} \quad (\text{A5})$$

$$\text{where IHP} = \frac{2 p S \text{ No}}{33,000}$$

Rewriting equation (A5),

$$R = \frac{0.75 p S}{P} \quad (\text{A6})$$

As can be seen from Figure A-III,

$$Q = (2) \left(\frac{N}{2}\right) \cos \beta = N \cos \beta \quad (\text{A7})$$

where Q is in the plane and perpendicular to the stem.

Again referring to Figure A-III, it can be seen that

$$Q = R \sin \phi + V \cos \phi \quad (\text{A8})$$

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 (2) ...
 (3) ...
 (4) ...
 (5) ...
 (6) ...
 (7) ...
 (8) ...
 (9) ...
 (10) ...

(11)
$$\frac{100 \times 100 \times 100}{100 \times 100} = \frac{100 \times 100 \times 100}{100 \times 100} = \frac{100 \times 100}{100} = 100$$

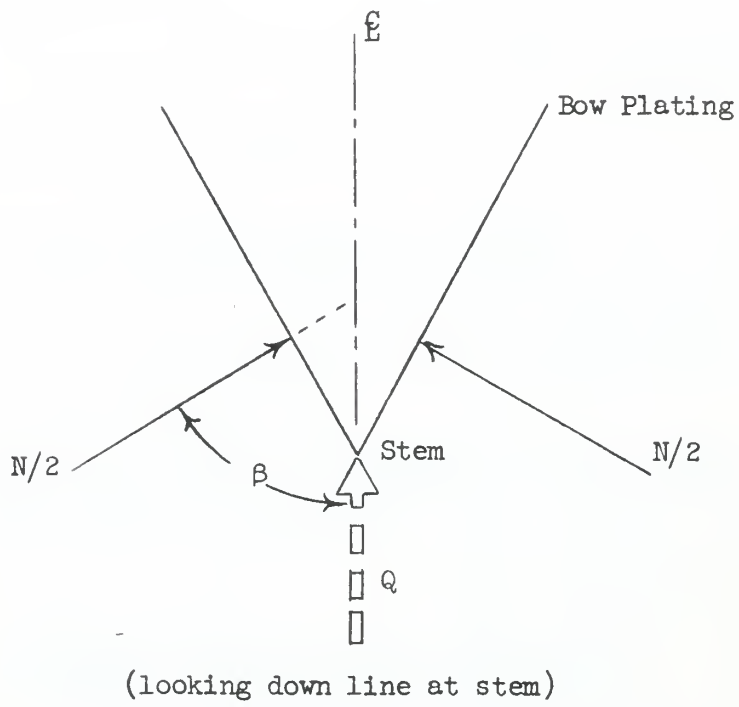
(12)
$$\frac{100 \times 100}{100} = 100$$

(13)
$$100 \times 100 = 10000$$

(14)
$$100 \times 100 = 10000$$

Figure A-III

Resolution of Forces Normal to Bow Plating



Substituting for Q this becomes

$$N \cos \beta = R \sin \phi + V \cos \phi$$

or

$$N = \frac{R \sin \phi + V \cos \phi}{\cos \beta} \quad (A9)$$

By substituting equations (A6) and (A9) into equation (A4) the following equation results:

$$V = \frac{0.75 p S \cos \phi}{P \sin \phi} - \frac{0.75 f p S \sin \phi}{P \sin \phi \cos \beta} - \frac{f V \cos \phi}{\sin \phi \cos \beta} \quad (A10)$$

Then this can be rewritten to

$$V = \frac{12,375 \text{ IHP} (\cos \phi \cos \beta - f \sin \phi)}{No P (\sin \phi \cos \beta + f \cos \phi)} \quad (A11)$$

Converting the symbols used in equation to those used generally in this research, the equation becomes

$$F_{BZ} = \frac{T_{IB} (\cos i_B \cos \beta - f_k \sin i_B)}{(\sin i_B \cos \beta + f_k \cos i_B)} \quad (A12)$$

or

$$F_{BZ} = \frac{12,375 \text{ (ihp)} (\cos i_B \cos \beta - f_k \sin i_B)}{P \text{ (rpm)} (\sin i_B \cos \beta + f_k \cos i_B)} \quad (A13)$$

where Runeberg suggests that $f_k = 0.05$.

The following assumptions were made during this development:

1. There are no momentum effects.
2. The forward motion through the water is effectively non-existent so that the thrust can therefore all be applied to icebreaking*.

* These assumptions were not stated.

...
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(109)
$$\frac{R \sin \theta}{\sin \phi} = \frac{R \sin \theta}{\sin \phi}$$

By substituting equation (109) into equation (108) the following relation results:

(110)
$$\frac{R \sin \theta}{\sin \phi} = \frac{R \sin \theta}{\sin \phi}$$

One may be written as

(111)
$$\frac{R \sin \theta}{\sin \phi} = \frac{R \sin \theta}{\sin \phi}$$

Considering the symbols used in equation (111) and generally in this context, the equation becomes

(112)
$$\frac{R \sin \theta}{\sin \phi} = \frac{R \sin \theta}{\sin \phi}$$

or

(113)
$$\frac{R \sin \theta}{\sin \phi} = \frac{R \sin \theta}{\sin \phi}$$

These findings suggest that $\theta = \phi$.

The following assumptions were made during this development:

1. There are no moment effects.
 2. The forward motion through the water is effectively non-existent.
- so that the thrust and torque will be applied to the propeller.

3. Thrust was directed horizontally at all times*.
4. The direction of friction force (along the line of the stem) remains the same during forward horizontal progress*.

His equation was developed on the basis of a ship sliding up on the ice very slowly*under the influence of its own thrust. It is deliberately approximate and was developed to be used for uninterrupted progress.

Runeberg does go on to develop some ideas concerning the "icebreaking power of a steamer when charging" but of these there is no direct connection to the forces developed at the bow.

If a ship is charging the ice he indicates that it will have "momentum" equal to $\frac{D v^2}{2g}$, where D = displacement in pounds. This "momentum" (actually kinetic energy) will be employed in the following two ways:

1. Elevating the ship
2. Overcoming frictional resistance as the bow glides up on the ice.

Later he mentions the work added by means of thrust while in ice contact. He indicates that there is an increase of frictional resistance due to an increase of normal pressure which is brought about "by the center of gravity of the ship changing direction of motion after the bow has struck the ice". Although he does not use it to advantage, this is the only mention of this particular dynamic force to appear up to this date (1964).

Unfortunately on the other hand, he presumes the loss by concussion is insignificant.

His concern over ramming does not lead to any prediction of force at the bow.

* These assumptions were not stated.

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Kari's Equation

In 1921 a book entitled "The Design of Icebreakers" by A. Kari was published (11). An equation is developed which does give the downward component under the bow during icebreaking. Kari developed this in order to determine certain characteristics the vessel should assume in order to break a given thickness of ice. As written the development leaves much to be desired. It is paraphrased and clarified here somewhat.

The following symbols are used:

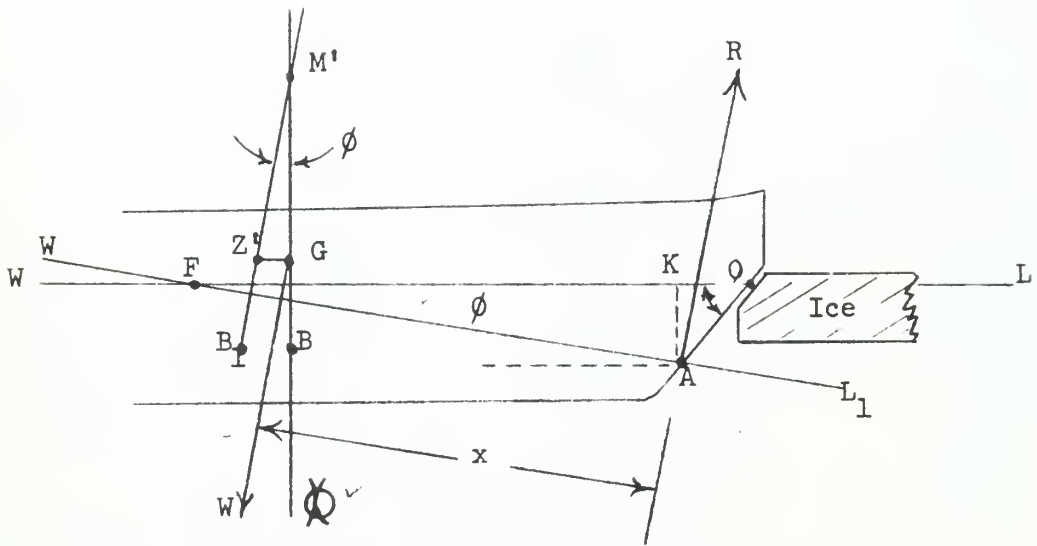
- W = Displacement, tons
- R = Upward ice resistance, tons
- θ = Inclination of stem to horizontal (original), deg.
- ϕ = Maximum permissible angular displacement of LWL, deg.
- $\pm a$ = Distance of the center of flotation forward (+) or aft (-) of amidships, ft.
- L = Length of LWL, ft.
- D = Moulded mean draft, ft.
- GM' = Longitudinal metacentric height, ft.
- t = Maximum thickness of ice to be expected, ft.

Figure A-IV illustrates many of these symbols.

Consider the locus of the point of initial contact; it moves along a somewhat circular path. Kari states, "This is the result of angular oscillation about the center of gravity and the gradually reducing forward motion. A force is produced by the angular displacement of the ship's waterline. The center of buoyancy is shifted aft and a trim moment is provided which, being divided by the separation of the point of contact with the ice from the center of gravity, provides the breaking power".

Figure A-IV

Illustration of Symbols Used by Kari



This statement becomes reasonable if one substitutes "center of flotation" for "center of gravity".

The trimming moment at a trim ϕ can be expressed as

$$W \times G Z' = W \times G M' \sin \phi \quad (A14)$$

Referring to Figure A-IV it can be seen that

$$AK = AF \sin \phi \quad (A15)$$

Without stating the equality or the reason for it, Kari then sets $AK = t$. It is important to note that, in order to continue with any logic, it is necessary to redefine t .

$$t = \text{Rise of point A in ft.}$$

Then

$$t = AF \sin \phi \quad (A16)$$

Kari states that AF is approximately equal to x . Therefore

$$t = x \sin \phi \quad (A17)$$

Summing moments about the center of gravity and setting them equal to zero he gets

$$W \times G M' \times x \sin \phi = Rx \quad (A18)$$

Using equation (A17) x may be eliminated.

$$R \times \frac{t}{\sin \phi} = W \times G M' \times x \sin \phi \quad (A19)$$

or

$$R = \frac{W \times G M' \times x \sin^2 \phi}{t} \quad (A20)$$

Assuming that $G M'$ is approximately $B M'$ and that $B M' = \frac{CL^2}{D}$ one gets

$$R = \frac{W \times C \times L^2 \times x \sin^2 \phi}{D^x t} \quad (A21)$$

where $C = 0.07$.

This statement is true if the function is continuous at the point in question. The function is continuous at a point if the limit of the function as the argument approaches the point is equal to the value of the function at that point.

(11) $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

(12) $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0$

It is important to note that in order to continue this way, it is necessary to assume that the function is continuous at the point in question.

(13) $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$

(14) $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \ln a$

Counting moments about the center of gravity and setting them equal to zero we get

(15) $\sum m_i x_i = 0$

(16) $\sum m_i x_i^2 = I$

(17) $\sum m_i x_i^3 = 0$

(18) $\sum m_i x_i^4 = 0$

From this point he goes on to end up with a rather astonishing result which will not be developed here. The equation gives the necessary length of a ship as a function of bow angle, trim, location of the center of flotation, and the ice thickness.

$$L = 2 t (\cot \theta + \cot \phi) + 2 a \quad (A22)$$

where t in this case reverts to the original definition of "maximum thickness of ice to be expected, ft."

Returning now to the downward force, R, as seen in equations (A20) and (A21), it is necessary for one to use an approximation for the vertical rise of point A which is indicated by "t" in the equations. It will be no less logical than many of the assumptions he has used to substitute $L/2 \sin \phi$ for t in order to obtain a more useful form. Equation (A20) becomes

$$R = \frac{W \times G M' \times \sin \phi}{L/2} \quad (A23)$$

and equation (A21) becomes

$$R = \frac{W \times C \times L \times \sin \phi}{D/2} \quad (A24)$$

where C = 0.07.

Converting the symbols used in equations (A23) and (A24) to those used generally in this research, the equations become

$$F_{BZ} = \frac{2 \Delta G M_L \sin \theta}{L} \quad (A25)$$

and

$$F_{BZ} = \frac{2 \Delta C L \sin \theta}{H} \quad (A26)$$

where C = 0.07.

The first part of the proof is to show that the function $f(x)$ is continuous at $x = a$. Let $\epsilon > 0$ be given. We need to find $\delta > 0$ such that if $|x - a| < \delta$, then $|f(x) - f(a)| < \epsilon$. Since $f(x) = x^2$, we have $|f(x) - f(a)| = |x^2 - a^2| = |x + a||x - a|$. If $|x - a| < \delta$, then $|x + a| < |x - a| + 2|a| < \delta + 2|a|$. We choose $\delta = \min\{\epsilon, \epsilon/(2|a| + 1)\}$. Then $|f(x) - f(a)| < (\delta + 2|a|)\delta < \epsilon$.

(100)
$$f(x) = x^2$$

where f is the function from the real numbers to the real numbers defined by $f(x) = x^2$.

It follows from the definition that f is continuous at $x = a$ if and only if $\lim_{x \rightarrow a} f(x) = f(a)$.

Let $f(x) = x^2$. It is necessary for us to use an approximation for the value of x^2 when x is close to a . We will use the binomial expansion of $(a + h)^2$ where $h = x - a$. This gives $(a + h)^2 = a^2 + 2ah + h^2$. If $|h| < \delta$, then $|2ah + h^2| < 2|a|\delta + \delta^2$. We choose $\delta = \min\{\epsilon/(2|a| + 1), \epsilon\}$. Then $|f(x) - f(a)| < \epsilon$.

(101)
$$f(x) = x^2$$

and equation (101) becomes

(102)
$$f(x) = x^2$$

where $\epsilon > 0$ is given.

Choosing the δ as in equation (102) and (101) we have $|f(x) - f(a)| < \epsilon$ whenever $|x - a| < \delta$. Generally in this case, the function f is continuous at $x = a$.

(103)
$$f(x) = x^2$$

and

(104)
$$f(x) = x^2$$

where $\epsilon > 0$ is given.

The equations for the downward component do not indicate the maximum and are approximately valid only for a motionless case.

In arriving at equations (A20) and (A21) Kari used the following assumptions or expedients:

1. There are no momentum effects.
2. The vertical rise of the bow is equal to the thickness of the ice. (The effect of this assumption was nullified by redefining the symbol t .)*
3. The distance from the final point of contact with the ice to the center of flotation is assumed to be the same as the horizontal distance from the final point of contact to the center of gravity.
4. The effective displacement is not effected by the force at the bow, nor is the draft*.
5. Following 4, the center of flotation and the longitudinal meta-center remain fixed*.
6. The normal assumption is made that $GM_L = B M_L$.
7. The value of C in equation (A21) is 0.07.
8. Friction is neglected*.
9. It is insignificant but $G Z'$ should be shown perpendicular in Figure A-IV.

* These assumptions were made but not stated.

The boundary of the domain is the surface S and the boundary of S is the curve C . In order to apply the divergence theorem we consider the region V bounded by S and the surface S_1 .

1. The surface S_1 is the surface $z = h$.
2. The volume V is the region bounded by S and S_1 .
3. The boundary of V is the surface S and the curve C .
4. The divergence of the vector field F is $\text{div } F = \dots$
5. The volume V is the region bounded by S and S_1 .
6. The surface S_1 is the surface $z = h$.
7. The volume V is the region bounded by S and S_1 .
8. The surface S_1 is the surface $z = h$.
9. The volume V is the region bounded by S and S_1 .

* Some examples are given in the text.

10. The summation of moments was set equal to zero. If forces had been summed there would have been a discrepancy.

In summary, Kari's equation states that the vertical component of force, F_{BZ} , in tons is a function of the following:

Δ , displacement, tons

GM_L , longitudinal metacentric height, ft.

θ , change of trim, deg.

L, length between perpendiculars, ft.

or

H, draft, ft. (instead of GM_L)

The first part of the document discusses the importance of maintaining accurate records of all transactions. It emphasizes that proper record-keeping is essential for the success of any business and for the protection of the interests of all parties involved.

The second part of the document outlines the specific procedures to be followed in the event of a dispute. It states that all disputes should be resolved through a process of mediation and arbitration, and that the parties to the agreement should agree to these procedures in advance.

The third part of the document provides a detailed description of the terms and conditions of the agreement. It includes provisions regarding the duration of the agreement, the responsibilities of each party, and the consequences of breach.

The fourth part of the document contains the signatures of the parties to the agreement, along with their respective titles and contact information.

(Signature of Party A) (Signature of Party B)

This document is a legal agreement between the parties mentioned above. It is intended to define the rights and obligations of each party and to provide a framework for the resolution of any disputes that may arise.

The parties to this agreement are:

- Party A: [Name], [Address], [City], [State], [Zip]
- Party B: [Name], [Address], [City], [State], [Zip]

The agreement is made this [Date] day of [Month], [Year].

Witnessed by: [Name], [Address], [City], [State], [Zip]

Simonson's Equations

In a paper published in 1936 (12) an equation was presented giving the force available for breaking ice. This is the steady state vertical reaction which results when the vessel is forced out of her normal water-plane by the thrust of the propeller.

The following symbols were used in this development:

- W = Vertical reaction at the bow at the point of contact with the ice in lb.
- M = Trimming moment of the vessel to change trimming 1 in. expressed ft-t/in.
- T = Thrust of the propeller in lb.
- Y = Allowable trim in in.
- D = Distance from the center of flotation to portion of the stem in contact with the ice in ft.
- L = Length between perpendiculars in ft.
- Δ = Displacement in t.
- GM = Longitudinal metacentric height in ft.
- ϕ = Change in trim in deg.
- K = Velocity expected through the ice in kt.
- HP = Total horsepower available less the amount necessary to drive the ship at speed K (in open water), hp.
- f = Overall efficiency of power plant and propeller at speed K when developing maximum horsepower; varies between 10 and 25 percent.

Experimental Procedure

In a series of experiments it was found that the rate of reaction between hydrogen peroxide and potassium iodide is first order with respect to hydrogen peroxide and first order with respect to potassium iodide. The rate of reaction is unaffected by the presence of potassium hydroxide.

The following table shows the results of the experiments.

Experiment No.	[H ₂ O ₂] (mole/l)	[KI] (mole/l)	Initial Rate (mole/l.hr)
1	0.01	0.01	0.0001
2	0.02	0.01	0.0002
3	0.01	0.02	0.0002
4	0.01	0.01	0.0001
5	0.01	0.01	0.0001
6	0.01	0.01	0.0001
7	0.01	0.01	0.0001
8	0.01	0.01	0.0001
9	0.01	0.01	0.0001
10	0.01	0.01	0.0001
11	0.01	0.01	0.0001
12	0.01	0.01	0.0001
13	0.01	0.01	0.0001
14	0.01	0.01	0.0001
15	0.01	0.01	0.0001
16	0.01	0.01	0.0001
17	0.01	0.01	0.0001
18	0.01	0.01	0.0001
19	0.01	0.01	0.0001
20	0.01	0.01	0.0001

C_t = Thrust coefficient, (These units must be lb-sec²/ft⁴.)

N = Revolutions per second obtainable at speed K at rated horsepower, rps.

P = Propeller pitch in ft.

d = Propeller diameter in ft.

θ = Angle between the stem and the surface of the ice field in deg.

γ = Angle between stem and waterplane (original) in deg. (This is the designed stem angle.)

The vertical reaction, W , is due to the trimming moment, M , when the vessel is forced out of normal waterplane by the thrust, T . This is expressed in the following approximate equation:

$$W = \frac{2240 M Y}{D} \quad (A27)$$

The moment to change trim one inch can be expressed as

$$M = \frac{\Delta G M}{12 L} \quad (A28)$$

Simonson assumes that $G M$ can be approximated by L . Then

$$M = \frac{\Delta}{12} \quad (A29)$$

He furthermore assumes that when the bow is not cut away too much and when the trim is small (less than 5°), D can be approximated by $L/2$.

By substitution in equation (A28) he gets

$$W = \frac{2240 \Delta 12L \tan \phi}{12 L/2} \quad (A30)$$

$$W = 4480 \Delta \tan \phi \quad (A31)$$

(1) $\frac{1}{x^2} = x^{-2}$
 $\frac{d}{dx} x^{-2} = -2x^{-3} = -\frac{2}{x^3}$

(2) $\frac{1}{x^3} = x^{-3}$
 $\frac{d}{dx} x^{-3} = -3x^{-4} = -\frac{3}{x^4}$

(3) $\frac{1}{x^4} = x^{-4}$
 $\frac{d}{dx} x^{-4} = -4x^{-5} = -\frac{4}{x^5}$

(4) $\frac{1}{x^5} = x^{-5}$
 $\frac{d}{dx} x^{-5} = -5x^{-6} = -\frac{5}{x^6}$

(5) $\frac{1}{x^6} = x^{-6}$
 $\frac{d}{dx} x^{-6} = -6x^{-7} = -\frac{6}{x^7}$

(6) $\frac{1}{x^7} = x^{-7}$
 $\frac{d}{dx} x^{-7} = -7x^{-8} = -\frac{7}{x^8}$

(7) $\frac{1}{x^8} = x^{-8}$
 $\frac{d}{dx} x^{-8} = -8x^{-9} = -\frac{8}{x^9}$

(8) $\frac{1}{x^9} = x^{-9}$
 $\frac{d}{dx} x^{-9} = -9x^{-10} = -\frac{9}{x^{10}}$

Converting the symbols used in this equation to those used generally, the equation becomes

$$F_{BZ} = 4480 \Delta \tan \theta \quad (A32)$$

Note, this equation simply states that if the displacement and the trim (caused by pushing the bow up on ice) are known, the vertical component of the force at the bow can be solved.

Remember F_{BZ} is defined as the force against the bow. Therefore it is positive. Naturally the magnitude downward against the ice is the same.

In arriving at this equation it should be noted that Simonson made the following assumptions:

1. $G M$ (longitudinal) = L
2. $D = L/2$
3. M remains constant*
4. Displacement remains constant*.
5. Longitudinal metacentric height remains constant.*

It is to be emphasized that equation (A32) is intended for steady state icebreaking. However, by itself it does not give the maximum force.

In order to find a maximum it is necessary to determine thrust. The method he uses employs the following equation:

$$T = \frac{HP \times f \times 33,000 \text{ ft.lb./min.} \times 60 \text{ min/hr}}{K \times 6080 \text{ ft/hr.}} \quad (A33)$$

* These assumptions were not stated.

... ..
... ..

$$(57) \quad \dots \dots \dots$$

... ..
... ..
... ..
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... ..

$$1. \quad \dots \dots \dots$$

$$2. \quad \dots \dots \dots$$

$$3. \quad \dots \dots \dots$$

$$4. \quad \dots \dots \dots$$

$$5. \quad \dots \dots \dots$$

... ..
... ..
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... ..

(58)

$$\frac{\dots \dots \dots}{\dots \dots \dots}$$

* These questions were not asked.

$$T = \frac{325.7 \times \text{HP} \times f}{K} \quad \text{lb.} \quad (\text{A34})$$

Other approximations for thrust are as follows:

$$T = 22.40 \times \text{IHP} \quad (\text{A35})$$

(one ton thrust per 100 IHP)

or

$$T = C_x N^2 P^2 d^2 \quad (\text{A36})$$

The angle γ (stem angle) is represented by Figures A-V and A-VI.

In the case of the curved bow shown in Figure A-V, the angles are expressed by tangents to the stem profile.

He assumed that friction in the steady state was negligible although this is not stated. His solution of the equilibrium was based on forces and he assumed that thrust remained horizontal. Since the summation of moments was not introduced, it is irrelevant that he did not mention the line of action of thrust relative to the point of contact.

From Figure A-VI it can be seen that

$$\gamma = (\theta - \phi) \quad (\text{A37})$$

and that

$$\tan \theta = \frac{T}{W} \quad (\text{A38})$$

From this point he goes on to substitute equations (A31), (A34) and (A38) into equation (A37) to get

$$\gamma = \tan^{-1} \left(\frac{0.0727 \times \text{HP} \times f \cot \phi}{\Delta K} \right) - \phi \quad (\text{A39})$$

(10A)

$$y = \frac{1}{2}x^2 - \frac{1}{2}x + \frac{1}{2}$$

Complete the square for the equation above.

(11A)

$$x^2 + 6x + 9 = 2$$

Write the equation in vertex form.

(12A)

$$x^2 + 4x + 4 = 5$$

Write the equation in vertex form.

In the process of completing the square, the equation $x^2 + 6x + 9 = 2$ is transformed into $(x + 3)^2 = 2 - 9$.

What is the value of $2 - 9$?

Write the equation in vertex form.

The equation $x^2 + 6x + 9 = 2$ is transformed into $(x + 3)^2 = 2 - 9$.

What is the value of $2 - 9$?

Write the equation in vertex form.

What is the value of $2 - 9$?

Write the equation in vertex form.

(13A)

$$x^2 + 6x + 9 = 2$$

Write the

(14A)

$$x^2 + 6x + 9 = 2$$

Write the equation in vertex form.

Write the equation in vertex form.

(15A)

$$x^2 + 6x + 9 = 2$$

Figure A-V (12)
Illustration of Symbols Used by Simonson

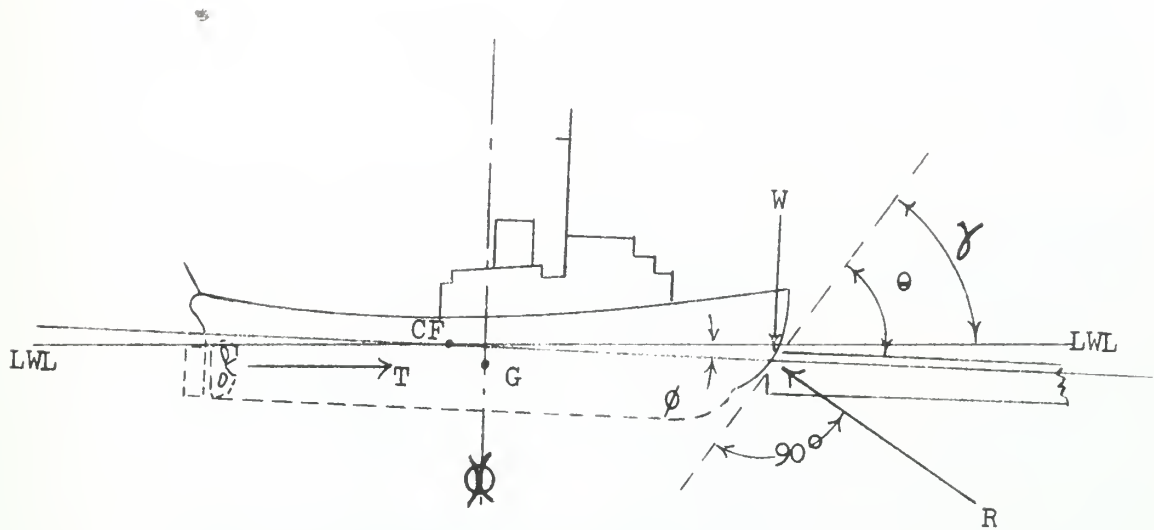
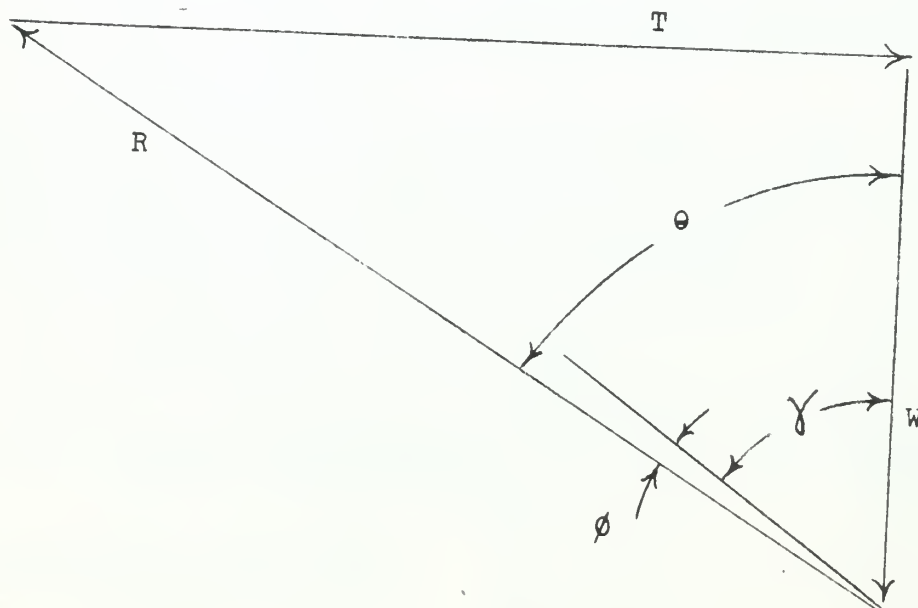


Figure A-VI (12)
Bow Equilibrium by Simonson



γ then simply indicates the stem angle which should exist if T, R, and W are to be in equilibrium. (R is the force perpendicular to the stem.) He uses this to obtain a bow profile which would represent an equilibrium condition regardless of the trim if other factors (i.e. thrust) were held constant.

Incidentally, this equation, although it is of no direct significance concerning this research, is given as follows in order to demonstrate Simonson's goal:

$$X = (6 L + \frac{0.436 \text{ HP} \times f \times L}{\Delta K}) .$$

$$\log \left(\frac{10.5 L^2 \times \text{HP} \times f}{\Delta K} - Y^2 \right) \quad (\text{A40})$$

$$\text{where } \cot \phi = \frac{12 L}{Y} \quad \text{and } \cot \theta = \frac{W}{T}$$

Y = trim in in.

X = distance in inches from center of flotation to stem on waterplane of trim.

Utilizing his equations it is possible to deduce an equation for maximum (limiting) downward force available from a given ship.

Starting with equation (A38)

$$\tan \theta = \frac{T}{W} \quad W = \frac{T}{\tan \theta}$$

$$W_{\text{max}} = \frac{T}{\tan (\gamma + \phi)} \quad (\text{A41})$$

or

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 ... (faint text) ...
 ... (faint text) ...
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 ... (faint text) ...
 ... (faint text) ...
 ... (faint text) ...

$$\frac{1}{2} = \frac{1}{2} + \frac{1}{2}$$

(iii) $\frac{1}{2} = \frac{1}{2} + \frac{1}{2}$

$$\frac{1}{2} = \frac{1}{2} + \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2} + \frac{1}{2}$$

... (faint text) ...

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$$\frac{1}{2} = \frac{1}{2} + \frac{1}{2}$$

(iii) $\frac{1}{2} = \frac{1}{2} + \frac{1}{2}$

$$W_{\max} = \frac{325.7 \times \text{HP} \times f}{K \tan(\gamma + \phi)} \quad (\text{A42})$$

Converting the symbols used in equations (A41) and (A42) to those used generally, these equations become

$$F_{\text{BZ}} = \frac{T_{\text{IB}}}{\tan(i_{\text{B}} + \theta)} \quad (\text{A43})$$

or

$$F_{\text{BZ}} = \frac{325.7 (\text{IHP} \times f_{\text{g}} - \text{EHP})}{V_{\text{IB}} \tan(i_{\text{B}} + \theta)} \quad (\text{A44})$$

Equations (A43) and (A44) indicate the maximum downward force possible under steady state icebreaking conditions. The term in parentheses in the numerator of equation (A44) indicates the horsepower available for breaking ice. It is to be recalled that f_{g} in this equation represents an overall efficiency of plant and propeller and varies between 10 and 25 percent.

In arriving at equations (A43) and (A44) it should be noted that Simonson made the following assumptions:

1. There are no momentum effects.
2. Friction with these was negligible*.
3. Thrust was directed horizontally at all times*.
4. The center of flotation remained a "pivot point".*
5. There is no change in displacement*.

Since friction was disregarded, the spread angle of the bow was not relevant and for that reason does not appear.

* These assumptions were not stated.

$$(19) \quad \frac{1}{\sqrt{1-x^2}} = \sum_{n=0}^{\infty} \binom{2n}{n} \frac{x^{2n}}{4^n}$$

Example (20) Let $f(x) = \frac{1}{1-x}$ and $g(x) = \frac{1}{1+x}$. Then

we have $f(x)g(x) = \frac{1}{1-x^2}$ and $f(x) = \frac{1}{1-x} = \frac{1}{1-x^2} \cdot \frac{1+x}{1+x}$

$$(20) \quad \frac{1}{1-x} = \frac{1}{1-x^2} \cdot \frac{1+x}{1+x} = \sum_{n=0}^{\infty} x^{2n} \cdot \sum_{n=0}^{\infty} (-1)^n x^n$$

$$(21) \quad \frac{1}{1-x^2} = \sum_{n=0}^{\infty} x^{2n} = \sum_{n=0}^{\infty} \frac{x^{2n}}{1}$$

Equations (19) and (21) illustrate the binomial theorem for negative exponents. The fact that $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ is a consequence of the geometric series formula. Equation (21) illustrates the binomial theorem for negative exponents. It is to be noted that $\frac{1}{1-x^2} = \frac{1}{(1-x)(1+x)}$ and the partial fraction decomposition of $\frac{1}{1-x^2}$ is $\frac{1}{2} \left(\frac{1}{1-x} + \frac{1}{1+x} \right)$. In view of equation (20) and (21) it would be noted that

1. $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$
2. $\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n$
3. $\frac{1}{1-x^2} = \sum_{n=0}^{\infty} x^{2n}$
4. $\frac{1}{1-x} = \frac{1}{1-x^2} \cdot \frac{1+x}{1+x}$
5. $\frac{1}{1-x^2} = \frac{1}{(1-x)(1+x)}$

These results are important in the study of the binomial theorem and the binomial expansion.

* This expansion may be found

Furthermore it is interesting to note that Simonson felt "momentum should be neglected as it is desirable to break ice without charging or ramming". (12) His analysis was a basic approximation for the steady state condition.

Vinogradov's Equation

In a book published in 1946 a mathematical analysis of the downward force under the bow created during ramming was presented (13). The development was paraphrased and presented as an appendix to a paper presented by Ferris in 1959. (14)

The paraphrased version is presented here.

"The analysis is based on the following concept. An icebreaker moving with known velocity strikes a uniform ice shelf and the bow of the ship glides up until the downward pressure reaches a magnitude which causes the ice shelf to collapse. While the ship is climbing the ice shelf, the propellers continue to push. In general, the forward motion of the ship is not reduced to zero at the instant when the ice collapses.

The quantity which is to be determined is the maximum value of the vertical force P developed on the stem of the icebreaker. The maximum is reached at the instant when collapse of the ice shelf impends; therefore, the dynamic study will cover events occurring up to this time.

The principle of conservation of energy is applied. Energy expended is a portion of the ship's kinetic energy plus the propeller thrust acting through the distance travelled. The energy expended is diverted into three channels; (a) Energy dissipated by impact of the bow of the ship on the ice shelf; (b) potential energy of the ship due to its being raised and changed in trim; (c) frictional loss caused by running of ship against the ice shelf

Electrostatics

The force between two point charges is given by Coulomb's law. The force is attractive if the charges are of opposite sign and repulsive if they are of the same sign. The force varies inversely as the square of the distance between the charges.

The electric field is defined as the force per unit positive charge. It is a vector quantity. The electric field due to a point charge is radial and its magnitude is given by $E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$. The electric field due to a uniformly charged sphere is zero inside the sphere and is the same as that of a point charge outside the sphere.

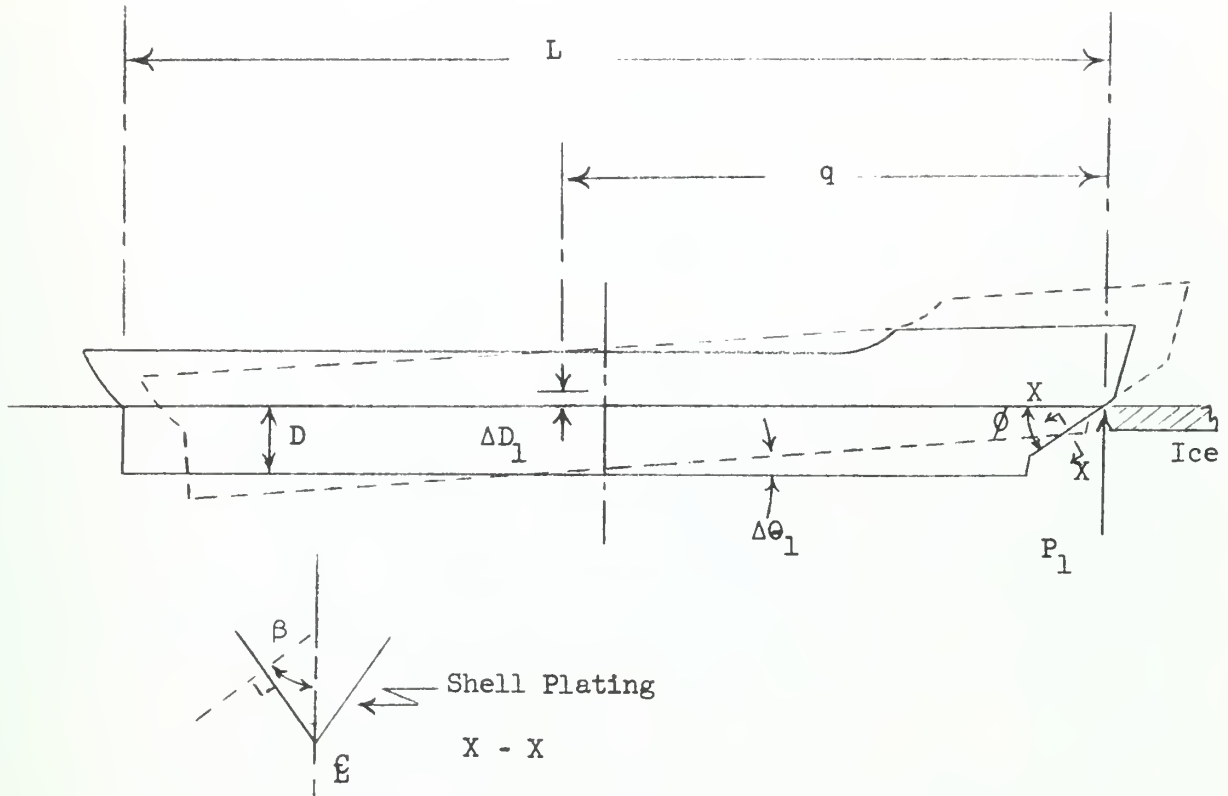
The quantity which is to be determined is the electric field. The electric field is a vector quantity. The electric field is defined as the force per unit positive charge. The electric field due to a point charge is radial and its magnitude is given by $E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$.

The principle of conservation of energy is applied. Energy expended in moving a charge from one point to another is equal to the work done. The energy expended in moving a charge from one point to another is equal to the work done. The energy expended in moving a charge from one point to another is equal to the work done.

for field

Figure A-VII (14)

Illustration of Terms Used in Analysis by Vinogradov



- L = Length between perpendiculars
- B = Beam
- W = Displacement
- q = Distance from stem to center of flotation
- D = Draft
- ΔD_1 = Maximum change in draft
- $\Delta \theta_1$ = Maximum change in trim
- ϕ = Angle of Stem to horizontal
- α = Waterplane area coefficient
- δ = Block coefficient
- P_1 = Maximum value of the vertical reaction
- S = Area of waterplane
- γ = Density of sea water
- m = Longitudinal metacentric height

$$(E_0 - E_1) + E_2 = E_3 + E_4 + E_5 \quad (A45)$$

where

E_0 = kinetic energy of ship when the ice is first touched

E_1 = kinetic energy of ship when ice collapses

E_2 = energy derived from propeller thrust

E_3 = energy dissipated by impact

E_4 = potential energy acquired by ship

E_5 = energy lost by friction

"Let W represent the weight or displacement of the ship, v_0 the velocity when the ice is first touched, and v_1 the velocity at the instant when the ice collapses. The initial kinetic energy is then $E_0 = (W/2g)v_0^2$. The remaining kinetic energy at the instant of collapse is $E_1 = (W/2g)v_1^2$. Kinetic energy absorbed during the operation is

$$E_0 - E_1 = \frac{W}{2g} (v_0^2 - v_1^2) \quad (A46)$$

"The next item considered is the energy delivered by the propellers to the ship while the latter is sliding up on the ice shelf. During this interval there is a reduction in mean draft designated by ΔD_1 , and the ship assumes an angle of trim of $\Delta\theta_1$. Distance from the point of contact on the stem to the center of flotation is designated by q . The stem of the ship is sloped at angle ϕ from horizontal. Then from the instant of first contact until the time when the ice collapses, the linear advance of the ship is

$$\Delta D_1 \cot \phi + q \Delta\theta_1 \cot$$

(134)

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

where

- $k = \text{spring constant}$
- $x = \text{displacement from equilibrium}$
- $v = \text{velocity}$
- $m = \text{mass}$
- $\frac{1}{2}mv^2 = \text{kinetic energy}$
- $\frac{1}{2}kx^2 = \text{potential energy}$

Let us consider the motion of a mass m on a spring. The total energy of the system is constant and is equal to the sum of the kinetic and potential energies. The kinetic energy is $\frac{1}{2}mv^2$ and the potential energy is $\frac{1}{2}kx^2$. The total energy is $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$. Kinetic energy is zero when the displacement is at a maximum.

(135)

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

The next form considered is the energy delivered by the projectile to the side while the latter is tilted up on the ball. During the interval there is a reduction in mass which is represented by Δm , and the side moves an angle of $\Delta \theta$. Distance from the point of contact to the axis of the center of friction is denoted by r . The mass of the side is almost at angle θ from horizontal. When θ is the instant of first contact until the time when the surface, the linear velocity of the side is

$$\Delta L \text{ and } \Delta \theta = \Delta \omega r$$

Let T represent the average value of propeller thrust during this advance, then

$$E_2 = T(\Delta D_1 + q\Delta\theta_1) \cot \varphi \quad (A47)$$

"It is desired that this formula be expressed in different terms so as to include P_1 , the maximum value of the vertical force developed at the stem. Assuming that P_1 is small in comparison with the displacement W , and that the change in draft and trim do not seriously change the properties of the waterplane, $\Delta D_1 = P_1/S$, S being the waterplane area and ρ the density of sea water. The angle of trim $\Delta\theta_1$, depends on the applied moment P_1q , and the longitudinal metacentric height, m ; thus $\Delta\theta_1 = P_1q/Wm$. The energy under consideration can then be expressed as

$$E_2 = T \left[\frac{P_1}{S} + \frac{P_1q^2}{Wm} \right] \cos \varphi \quad (A48)$$

"Waterplane area equals the product of length, beam and waterplane area coefficient, or $S = LB\alpha$. Weight of ship equals the product of length, beam, draft, block coefficient and density of sea water, or $W = LBD\delta$. New non-dimensional coefficients k_1 and k_2 are arbitrarily set up by relationships $q = k_1(L/2)$ and $m = (k_2^2\alpha^2L^2)/(D\delta)$, it being assumed that the longitudinal metacentric height is essentially equal to the height between center of buoyancy and longitudinal metacenter. Substituting these new quantities in the last equation, there results

$$E_2 = \left\{ \frac{\delta}{\alpha} \left[1 + \left(\frac{k_1}{k_2} \right)^2 \frac{1}{4\alpha} \right] \right\} \frac{DP_1}{W} T \cot \varphi$$

or, for abbreviation

$$(100) \quad \gamma = \frac{1}{2} (\sqrt{1 + 4\beta} + \sqrt{1 - 4\beta})$$

$$(101) \quad \gamma = \frac{1}{2} \left[\frac{1 + \beta}{1 - \beta} - \frac{1 - \beta}{1 + \beta} \right] = \beta$$

$$\gamma = \frac{1}{2} \left[\frac{1 + \beta}{1 - \beta} + \frac{1 - \beta}{1 + \beta} \right] = \frac{1}{2} \left[\frac{1 + \beta^2}{1 - \beta^2} + \frac{1 - \beta^2}{1 + \beta^2} \right]$$

$$E_2 = A \frac{DP_1}{W} T \cot \varphi \quad (A49)$$

"According to the theory of impact, when two bodies collide normally there is always a dissipation of energy, whose magnitude depends on relative velocity and a physical constant e known as the coefficient of restitution. Now the stem of the ship does not collide normally with the edge of the ice owing to the fact that the stem is sloped at an angle from the horizontal. The component of initial velocity v_0 which is directed normal to the edge of the ice is $v_0 \sin \varphi$ and the energy dissipated by impact is

$$E_3 = \frac{W}{2g} (v_0 \sin \varphi)^2 (1 - e^2) \quad (A50)$$

"The vertical force P is a variable which keeps increasing as the ship slides up on the ice. The total rise of the point on the stem at which P is first applied equals to reduction in draft ΔD_1 plus the angle of trim, in radians, times the horizontal arm between center of flotation and stem $\Delta \theta_1 q$. The potential energy set up by the force P is therefore

$$E_4 = \int_0^{\Delta D_1} P d\Delta D + \int_0^{\Delta \theta_1} P q d\Delta \theta \quad (A51)$$

"Energy is dissipated by sliding friction between the shell plating and the ice. The coefficient of sliding friction is f and it must be applied to that component of the pressure which is normal to the plating. The resultant frictional force, designated by F , acts in a direction parallel to the stem of the ship and is a variable; half of it acts on one side of the stem and half on the other".

(10)

$$\frac{1}{\sqrt{1-x^2}} = \sum_{n=0}^{\infty} \binom{2n}{n} \frac{x^{2n}}{4^n}$$

The binomial series for $(1-x)^{-1/2}$ is $\sum_{n=0}^{\infty} \binom{-1/2}{n} (-x)^n = \sum_{n=0}^{\infty} \frac{(-1)^n (-1/2)(-3/2)\dots(-2n+1)}{n!} (-x)^n$.
 Simplifying the coefficients, we get $\sum_{n=0}^{\infty} \frac{(2n-1)!!}{2^n n!} x^n$. For even $n=2k$, $(2k-1)!! = \frac{(2k)!}{2^k k!}$.
 Thus, the series becomes $\sum_{k=0}^{\infty} \frac{(2k)!}{2^{2k} (k!)^2} x^{2k} = \sum_{k=0}^{\infty} \binom{2k}{k} \frac{x^{2k}}{4^k}$.

(11)

$$(1-x)^{-1/2} = \sum_{n=0}^{\infty} \binom{2n}{n} \frac{x^{2n}}{4^n}$$

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(12)

$$\int_0^1 \frac{1}{\sqrt{1-x^2}} dx = \int_0^1 \sum_{n=0}^{\infty} \binom{2n}{n} \frac{x^{2n}}{4^n} dx = \sum_{n=0}^{\infty} \frac{\binom{2n}{n}}{4^n} \int_0^1 x^{2n} dx = \sum_{n=0}^{\infty} \frac{\binom{2n}{n}}{4^n} \frac{1}{2n+1}$$

The integral $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$ is $\arcsin(x)$ evaluated from 0 to 1, which is $\frac{\pi}{2}$.
 The series $\sum_{n=0}^{\infty} \frac{\binom{2n}{n}}{4^n} \frac{1}{2n+1}$ converges to $\frac{\pi}{2}$.
 This result is known as the Wallis product for π .

"The energy dissipated by frictional force F acting through a distance which is determined by the changes in draft and trim is given by

$$E = \frac{f}{\sin} \int_0^{\Delta D_1} F d\Delta D + \frac{1}{\sin} \int_0^{\Delta \theta_1} F q d\Delta \theta \quad (A52)$$

"Consider an inclined plane intersecting the bow of the ship in a direction normal to the stem; this section of the bow will appear as a wedge with essentially flat sides and the normal pressure on these sides makes an angle β with the centerline plane."

"As the bow rides up on the ice shelf it forms a wedgelike groove, with pressure developed normal to the faces of the groove and friction along the faces of the groove directed parallel to the sloping stem".

"Let R be the resultant force acting normal to the stem. On each side, then, the force acting normal to the plating is

$$(R/2)(1/\cos \beta)$$

so the resultant frictional force is given by

$$F = fR \left(\frac{1}{\cos \beta} \right) \quad (A53)$$

"The magnitude of force R is related to other forces acting on the ship as follows:

$$R = P \cos \varphi + T \sin \varphi \quad (A54)$$

Equation (A53) is thus rewritten as

$$F = f \left[P \frac{\cos \varphi}{\cos \beta} + T \frac{\sin \varphi}{\cos \beta} \right] \quad (A55)$$

The first part of the problem is to find the Laplace transform of the function $f(t) = t \cos t$. We can use the product rule for Laplace transforms, which states that $\mathcal{L}\{t f(t)\} = -F'(s)$, where $F(s) = \mathcal{L}\{f(t)\}$. In this case, $f(t) = \cos t$ and $F(s) = \frac{s}{s^2 + 1}$. Therefore, $\mathcal{L}\{t \cos t\} = -\frac{d}{ds} \left(\frac{s}{s^2 + 1} \right)$.

$$\mathcal{L}\{t \cos t\} = -\frac{d}{ds} \left(\frac{s}{s^2 + 1} \right) = -\frac{(s^2 + 1) - s(2s)}{(s^2 + 1)^2} = \frac{s^2 - 1}{(s^2 + 1)^2}$$

Next, we need to find the Laplace transform of the function $f(t) = t \sin t$. Using the same product rule, $\mathcal{L}\{t f(t)\} = -F'(s)$, where $F(s) = \mathcal{L}\{f(t)\}$. In this case, $f(t) = \sin t$ and $F(s) = \frac{1}{s^2 + 1}$. Therefore, $\mathcal{L}\{t \sin t\} = -\frac{d}{ds} \left(\frac{1}{s^2 + 1} \right) = \frac{2s}{(s^2 + 1)^2}$.

Finally, we need to find the Laplace transform of the function $f(t) = t^2 \cos t$. Using the product rule twice, $\mathcal{L}\{t^2 f(t)\} = F''(s)$, where $F(s) = \mathcal{L}\{f(t)\}$. In this case, $f(t) = \cos t$ and $F(s) = \frac{s}{s^2 + 1}$. Therefore, $\mathcal{L}\{t^2 \cos t\} = \frac{d^2}{ds^2} \left(\frac{s}{s^2 + 1} \right)$.

$$\mathcal{L}\{t^2 \cos t\} = \frac{d^2}{ds^2} \left(\frac{s}{s^2 + 1} \right) = \frac{d}{ds} \left(\frac{s^2 - 1}{(s^2 + 1)^2} \right) = \frac{2s(s^2 + 1)^2 - (s^2 - 1)2(s^2 + 1)s}{(s^2 + 1)^4} = \frac{2s^3 + 2s - 2s^3 + 2s}{(s^2 + 1)^3} = \frac{4s}{(s^2 + 1)^3}$$

$$(17) \quad \mathcal{L}\left\{t \cos t\right\} = \frac{s^2 - 1}{(s^2 + 1)^2}$$

The Laplace transform of $f(t) = t \sin t$ is $\frac{2s}{(s^2 + 1)^2}$.

$$(18) \quad \mathcal{L}\left\{t \sin t\right\} = \frac{2s}{(s^2 + 1)^2}$$

Finally, the Laplace transform of $f(t) = t^2 \cos t$ is $\frac{4s}{(s^2 + 1)^3}$.

$$(19) \quad \mathcal{L}\left\{t^2 \cos t\right\} = \frac{4s}{(s^2 + 1)^3}$$

"Substituting in equation (A52), there results

$$\begin{aligned}
 E_5 = & f \frac{\cot \varphi}{\cos \beta} \int_0^{\Delta D_1} P d\Delta D \\
 & + f \frac{T}{\cos \beta} \int_0^{\Delta D_1} d\Delta D + f \frac{\cot \varphi}{\cos \beta} \int_0^{\Delta \theta_1} P q d\Delta \theta \\
 & + \frac{fT}{\cos \beta} \int_0^{\Delta \theta_1} q d\Delta \theta
 \end{aligned} \tag{A56}$$

Equations (A51) and (A56) are combined, as follows:

$$\begin{aligned}
 E_4 + E_5 = & (1 + f \frac{\cot \varphi}{\cos \beta}) \int_0^{\Delta D_1} P d\Delta D \\
 & + f \frac{T}{\cos \beta} \int_0^{\Delta D_1} d\Delta D + (1 + f \frac{\cot \varphi}{\cos \beta}) \\
 & \int_0^{\Delta \theta_1} P q d\Delta \theta + \frac{fT}{\cos \beta} \int_0^{\Delta \theta_1} q d\Delta \theta
 \end{aligned} \tag{A57}$$

For P the quantity SAD can be substituted and for Pq the quantity WmΔθ."

"Rewriting (A57) gives

$$E_4 + E_5 = (1 + f \frac{\cot \varphi}{\cos \beta}) S \int_0^{\Delta D_1} \Delta D d\Delta D$$

QUESTION (10) (10 marks)

$$\text{QUESTION (10)} \quad \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

(10)

$$\text{QUESTION (10)} \quad \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

$$\text{QUESTION (10)} \quad \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

QUESTION (11) (10 marks)

$$\text{QUESTION (11)} \quad \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

$$\text{QUESTION (11)} \quad \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

(11)

$$\text{QUESTION (11)} \quad \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

QUESTION (12) (10 marks)

QUESTION (12) (10 marks)

$$\text{QUESTION (12)} \quad \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

$$\begin{aligned}
 & + r \frac{T}{\cos \beta} \int_0^{\Delta D_1} d\Delta D + \left(1 + r \frac{\cot \varphi}{\cos \beta}\right) Wm \\
 & = \int_0^{\Delta \theta_1} \Delta \theta d\theta + r \frac{Tq}{\cos \beta} \int_0^{\Delta \theta_1} d\Delta \theta
 \end{aligned} \tag{A58}$$

Integrating

$$\begin{aligned}
 E_4 + E_5 & = \left(1 + r \frac{\cot \varphi}{\cos \beta}\right) S \left(\frac{\Delta D_1^2}{2}\right) \\
 & + r \frac{T}{\cos \beta} (\Delta D_1) + \left(1 + r \frac{\cot \varphi}{\cos \beta}\right) Wm \left(\frac{\Delta \theta_1^2}{2}\right) + r \frac{Tq}{\cos \beta} (\Delta \theta_1)
 \end{aligned} \tag{A59}$$

"Since

$$\Delta D_1 = \frac{P_1}{S} \quad \text{and} \quad \Delta \theta_1 = \frac{P_1 q}{Wm}$$

$$\begin{aligned}
 E_4 + E_5 & = \left(\frac{1}{S} + \frac{q^2}{Wm}\right) \\
 & \left[\left(1 + r \frac{\cot \varphi}{\cos \beta}\right) \frac{P_1^2}{2} + r \frac{TP_1}{\cos \beta} \right]
 \end{aligned} \tag{A60}$$

"Using the previously established values, $S = LB\alpha$, $q = k_1(L/2)$,

$$W = LBD\delta \quad \text{and} \quad m = (k_2^2 \alpha^2 L^2)/(D\delta),$$

$$\begin{aligned}
 E_4 + E_5 & = \left\{ \frac{\delta}{\alpha} \left[1 + \left(\frac{k_1}{k_2}\right)^2 \frac{1}{4\alpha} \right] \right\} \frac{D}{W} \\
 & \left[\left(1 + r \frac{\cot \varphi}{\cos \beta}\right) \frac{P_1^2}{2} + r \frac{TP_1}{\cos \beta} \right]
 \end{aligned}$$

or

$$= \left(\frac{1}{\sqrt{1-x^2}} \right) + (1+x^2) \left(\frac{-2x}{1-x^2} \right)$$

(8x)

$$\frac{d}{dx} \left(\frac{1}{\sqrt{1-x^2}} + \frac{1+x^2}{1-x^2} \right)$$

Problem 10

$$\left(\frac{1}{\sqrt{1-x^2}} \right) + \left(\frac{1+x^2}{1-x^2} \right) = \frac{1}{\sqrt{1-x^2}} + \frac{1+x^2}{1-x^2}$$

(9x) $\frac{d}{dx} \left(\frac{1}{\sqrt{1-x^2}} \right) = \left(\frac{1}{\sqrt{1-x^2}} \right) \cdot \left(\frac{2x}{1-x^2} \right) = \frac{2x}{(1-x^2)^{3/2}}$ (10x) $\frac{d}{dx} \left(\frac{1+x^2}{1-x^2} \right) = \frac{2x}{(1-x^2)^2}$

Problem 11

$$\frac{d}{dx} \left(\frac{1}{\sqrt{1-x^2}} + \frac{1+x^2}{1-x^2} \right)$$

$$\left(\frac{1}{\sqrt{1-x^2}} \right) + \left(\frac{1+x^2}{1-x^2} \right) = \frac{1}{\sqrt{1-x^2}} + \frac{1+x^2}{1-x^2}$$

(11x)

$$\left[\frac{1}{\sqrt{1-x^2}} + \frac{1+x^2}{1-x^2} \right] \cdot \left(\frac{2x}{1-x^2} \right) = \frac{2x}{(1-x^2)^{3/2}} + \frac{2x(1+x^2)}{(1-x^2)^2}$$

(12x) $\frac{d}{dx} \left(\frac{1}{\sqrt{1-x^2}} + \frac{1+x^2}{1-x^2} \right) = \frac{2x}{(1-x^2)^{3/2}} + \frac{2x(1+x^2)}{(1-x^2)^2}$

$$\frac{d}{dx} \left(\frac{1}{\sqrt{1-x^2}} + \frac{1+x^2}{1-x^2} \right) = \frac{2x}{(1-x^2)^{3/2}} + \frac{2x(1+x^2)}{(1-x^2)^2}$$

$$\frac{2x}{(1-x^2)^{3/2}} + \frac{2x(1+x^2)}{(1-x^2)^2}$$

$$\left[\frac{1}{\sqrt{1-x^2}} + \frac{1+x^2}{1-x^2} \right] \cdot \left(\frac{2x}{1-x^2} \right) = \frac{2x}{(1-x^2)^{3/2}} + \frac{2x(1+x^2)}{(1-x^2)^2}$$

$$E_4 + E_5 = \frac{AD}{W} \left[\left(1 + f \frac{\cot \varphi}{\cos \beta} \right) \frac{P_1^2}{2} + f \frac{TP_1}{\cos \beta} \right] \quad (A61)$$

"Substituting all the values of component energies in equation (A45)

there finally results

$$A \left[\left(\frac{P_1}{W} \right)^2 - 2X \frac{P_1 T}{W} \right] = Y \frac{v_0^2 [1 - (1 - e^2) \sin^2 \varphi] - v_1^2}{gD} \quad (A62)$$

in which

$$X = \frac{1 - \frac{f}{\cos \beta} \tan \varphi}{1 + \frac{f}{\cos \beta} \cot \varphi} \cot \varphi \quad (A63)$$

$$Y = \frac{1}{1 + \frac{f}{\cos \beta} \cot \varphi} \quad (A64)$$

The quantity to be calculated is the downward icebreaking force P_1 . Solving the quadratic equation (A62) gives

$$P_1 = XT \pm \left\{ X^2 T^2 + \frac{Y}{A} W^2 \frac{v_0^2 [1 - (1 - e^2) \sin^2 \varphi] - v_1^2}{gD} \right\}^{1/2} \quad (A65)$$

The positive sign in front of the radical must be used in order for the value of P_1 to come out positive."

(134)

$$\left[\frac{1}{1-x} - \frac{1}{1-x^2} \right] \frac{dx}{x} =$$

(135) ...

(136)

$$\frac{\frac{1}{x} - [0.5 \ln(x^2 - 1) - 1] \frac{1}{x}}{x} =$$

(137)

$$\frac{1}{x} \ln \frac{1+x}{1-x} =$$

(138)

$$\frac{1}{x} \ln \frac{1+x}{1-x} =$$

... the positive sign is taken for the integral ...

... (139) ...

(140)

$$\frac{1}{x} \ln \frac{1+x}{1-x} =$$

... the positive sign is taken for the integral ...

... (141) ...

It is important to note the assumptions and expedients used by Vinogradov in his development.

1. Trim was taken into account for the solution of movement but it was disregarded in his solution for the resultant perpendicular to the stem R based on thrust T and the downward component P_1 . See equation (A54).
2. Thrust T was directed horizontally at all times.
3. Thrust T was kept as a constant instead of attempting to elaborate and make it a function of other parameters such as i h p, propeller area, velocity, and so forth.
4. The friction loss is not correct in that the normal force on the bow plating would be valid only for static equilibrium.
5. There is no mention of the possibility that some of the kinetic energy while sliding up may be in the form of rotation.
6. It was assumed that the change in trim and draft did not seriously affect properties of the waterplane or the longitudinal metacentric height.
7. The change of trim is based on the original displacement using the equation for a couple when actually the displacement is effectively changed.
8. q is used exclusively as a constant representing the distance from the center of flotation to the forward perpendicular which is the original point of contact. For the determination of certain distances this is proper but it is an assumption when dealing with moment arms since the point of contact moves.

2. The distance between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

3. The distance between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

4. The distance between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

5. The distance between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

6. The distance between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

7. The distance between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

8. The distance between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

9. The distance between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

10. The distance between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

11. The distance between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

12. The distance between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

13. The distance between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

14. The distance between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

15. The distance between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

9. The expression for loss of energy on impact is based on direct central impact. In other words it is assumed that the loss is the same as if a perpendicular to the stem passed through the center^s of gravity.
10. The normal assumption was utilized that GM (longitudinal) = BM (longitudinal).
11. The final equation is written including v_1 as the velocity during the sliding. However, in effect the equation is valid only when $v_1 = 0$ since there is not a continuous spectrum of velocity from v_0 to v_1 .
12. A necessary step in his development was the use of static equilibrium, $F_R = 0$. See equation (A54). Acceleration at that point in contact with the ice in the direction of the force may be zero but not the acceleration of the body.

In summary, Vinogradov's equation states that the downward component of force, P_1 , is a function of the following:

- f, coefficient of sliding friction
- ψ , angle of stem, deg.
- β , angle of normal to shell plating with respect to \hat{E} plane, deg.
- δ block coefficient
- α , waterplane coefficient
- q, $L/2$ plus the distance aft from \mathcal{Q} to the center of flotation, ft.
- L, length between perpendiculars, ft.
- D, draft, ft.

The condition for the flow of energy is given by the law of conservation of energy. In other words, it is assumed that the law of conservation of energy is valid in all cases.

The energy conservation law is expressed as follows:

The total energy E is constant in time, i.e. $\frac{dE}{dt} = 0$. This means that the energy of a system is conserved. In other words, the energy of a system is constant in time.

It is necessary to pay attention to the fact that the energy of a system is conserved only if the system is isolated. In other words, the energy of a system is conserved only if there is no exchange of energy with the surroundings.

In summary, the energy conservation law is the statement that the total energy of a system is constant in time.

of laws, E , is a function of the following:

- 1. coefficient of sliding friction
- 2. angle of incline
- 3. mass of object
- 4. height of incline
- 5. initial velocity
- 6. coefficient of static friction
- 7. length of incline
- 8. mass of object
- 9. angle of incline
- 10. coefficient of kinetic friction

- m, longitudinal metacentric height, ft.
- T, thrust, tons
- W, displacement, tons
- e, coefficient of restitution
- v_0 , speed just prior to impact, ft/sec
- v_1 , speed while sliding up, ft/sec (normally taken as zero to get maximum P_1).

Jansson's Equation

In 1956 Jansson presented an equation for the determination of work utilized in the ramming of ice (15). It does not indicate the downward force on the ice but is included here because of it's comprehensive approach.

Jansson used the following symbols for his development.

P = Vertical force between vessel and edge of ice.

T = Thrust of propeller, a function of speed.

X = Trim in deg. (change of trim).

W = Weight of vessel.

v = Speed of vessel.

M = Mass of vessel plus virtual mass of water.

J = Moment of inertia of mass of vessel plus virtual added mass of water, referred to a horizontal axis through the center of gravity and at right angles to the lateral plane.

S = Length coordinate in meters.

Y = Vertical coordinate in meters.

ω = Angular velocity about a horizontal axis at right angles to the lateral plane.

p = Number of tons load for 1 meter immersion.

q = Trim moment in ton-meters for 1 radian trim.

l = Distance from center of gravity of waterline areas to foremost point in the water line in meters.

As can be seen from Figure A-VIII , the following equation can be determined statically:

The first part of the document is a list of items.

1. Item 1: Description of the first item.

2. Item 2: Description of the second item.

3. Item 3: Description of the third item.

4. Item 4: Description of the fourth item.

5. Item 5: Description of the fifth item.

6. Item 6: Description of the sixth item.

7. Item 7: Description of the seventh item.

8. Item 8: Description of the eighth item.

9. Item 9: Description of the ninth item.

10. Item 10: Description of the tenth item.

11. Item 11: Description of the eleventh item.

12. Item 12: Description of the twelfth item.

13. Item 13: Description of the thirteenth item.

14. Item 14: Description of the fourteenth item.

15. Item 15: Description of the fifteenth item.

16. Item 16: Description of the sixteenth item.

17. Item 17: Description of the seventeenth item.

18. Item 18: Description of the eighteenth item.

19. Item 19: Description of the nineteenth item.

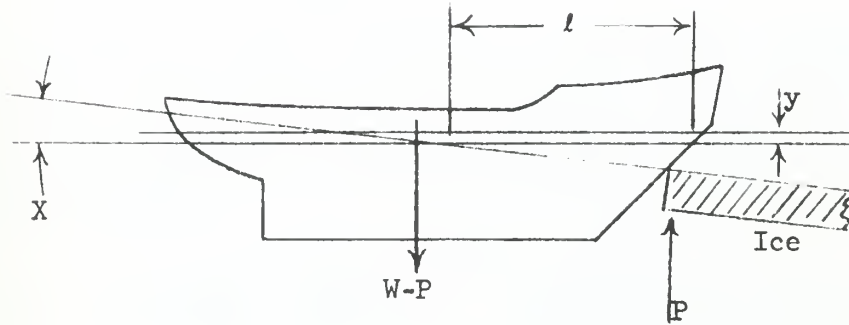
20. Item 20: Description of the twentieth item.

21. Item 21: Description of the twenty-first item.

22. Item 22: Description of the twenty-second item.

Figure A-VIII (15)

An Icebreaker with Stem in Contact with Edge of Ice



$$P = p y \quad . \quad (A66)$$

where y is the change in mean draft in meters.

$$P \cdot l = q \cdot x \quad (A67)$$

where x is in radians.

It is assumed that trim is small enough to assume p , l , and q may be taken as constant.

Equations (A66) and (A67) may be combined to get

$$y = \frac{qx}{pl} \quad (A68)$$

Writing down the energy equation for condition 1 (immediately before ramming) and condition 2 (as the bow slides up the ice) the following equation is obtained.

$$\frac{1}{2} M (v_1^2 - v_2^2) + \int_{s_1}^{s_2} T ds =$$

$$\int_{y_1}^{y_2} p y dy + \int_{x_1}^{x_2} q x dx + \frac{1}{2} J (\omega_2^2 - \omega_1^2) \quad (A69)$$

In his development he deliberately neglected the friction between the ice and the forward end of the vessel. Furthermore, without mentioning it, he has assumed that no energy is lost on impact.

When the maximum vertical force is reached the angle of trim, X , has reached its maximum value and the speed, v , is zero. Thus the angular velocity, ω , is zero. For initial conditions he uses $y_1 = 0$, $X_1 = 0$, and

(100)

$$v = v_0$$

where v_0 is the initial velocity of the particle.

(101)

$$v = v_0 e^{-\lambda t}$$

where λ is the decay constant.

If λ is assumed to be zero, then $v = v_0$ and $x = v_0 t$.

Equation (100) can be written as

$$m \frac{dv}{dt} = -kx$$

(102)

$$\frac{dv}{v} = -\frac{k}{m} x dx$$

Integrating both sides, we get $\ln v = -\frac{k}{2m} x^2 + \ln C$, where C is a constant.

At $t = 0$, $x = 0$ and $v = v_0$. Substituting these values, we get $\ln v_0 = \ln C$.

Therefore, $C = v_0$.

$$v = v_0 e^{-\frac{k}{2m} x^2}$$

(103)

$$\left(\frac{S}{L} - \frac{S_0}{L} \right) \frac{1}{2} = \frac{1}{2} \left(\frac{S}{L} + \frac{S_0}{L} \right) \frac{1}{2}$$

In the derivation of the above equation, we have assumed that the friction between the ice and the forward end of the vessel is negligible, without realizing it.

It is assumed that no energy is lost as heat.

When the medium velocity force is removed, the angle of tilt, X , has

remained the maximum value and the speed, v , is zero. Thus the angular

velocity, ω , is zero. For initial conditions we need $x = 0$, $\dot{x} = 0$, and

and $\omega_1 = 0$. Equation (A69) reduces to

$$\frac{1}{2} M v_1^2 + \int_{S_1}^{S_2} T ds = \frac{py_2^2}{2} + \frac{qx_2^2}{2} \quad (A70)$$

Equation (A68) can be substituted into equation (A70) to obtain the following expression for maximum icebreaking work:

$$\frac{1}{2} M v_1^2 + \int_{S_1}^{S_2} T ds = X_2^2 \left(\frac{q}{2 p l^2} \right) \quad (A71)$$

It is noted that the term in parentheses is constant for a given ship.

Although Jansson does not go further it would be possible to solve this equation for trim, X , if $T(s)$ were known as well as S_1 and S_2 . Substitution of X_2 back into equation (A67) would then yield the maximum downward force.

It is important to reiterate that the result would have neglected friction and impact losses.

Richardson's Equation

In 1959 Richardson presented an equation for the downward force under the bow created during ramming. (75) The development is the most complete to this date (1964) and is part of a model study of the force system.

The complete development will not be reproduced here. The steps are basically the same as those of Vinogradov. It is based on the conservation of energy and is shown as follows:

$$T_V - T_f + T_T = T_S + T_{P, \psi} + T_R + T_D \quad (A72)$$

where

T_V = the kinetic energy at the instant of initial contact with the ice shelf.

T_f = the kinetic energy remaining after the ice splits.

T_T = the energy furnished by the propellers from the instant of contact up to the moment the ice splits or breaks, or as the case may be, the forward motion closes.

T_S = the energy lost at the impact of stem with the ice shelf.

$T_{P, \psi}$ = the energy spent to raise and trim the icebreaker.

T_R = the energy spent in friction between the hull and the ice.

T_D = the energy spent in overcoming the friction and wave resistance from the instant of contact up to the moment the ice splits or breaks M motion ceases.

T_E = the energy lost in elastic vibrations. (This loss is neglected.)

The total energy of the system is conserved. The kinetic energy of the ball is given by $\frac{1}{2}mv^2$ and the potential energy is given by mgh .

At the top of the loop, the ball has a velocity v_1 and is at a height $2R$ from the bottom.

The energy conservation equation is $\frac{1}{2}mv_1^2 + mg(2R) = \frac{1}{2}mv_2^2 + mgR$, where v_2 is the velocity at the bottom.

Solving for v_2 , we get $v_2 = \sqrt{v_1^2 + 2gR}$.

$$(2) \quad v_2 = \sqrt{v_1^2 + 2gR}$$

where

v_1 = the kinetic energy of the ball at the top of the loop.

the result

v_2 = the kinetic energy of the ball at the bottom.

E_p = the energy transferred to the ball as it moves from the top to the bottom of the loop.

up to the bottom of the loop is given by mgR .

the total energy of the ball

E_k = the kinetic energy of the ball at the bottom.

$$E_k = \frac{1}{2}mv_2^2 = \frac{1}{2}m(v_1^2 + 2gR)$$

T = the energy spent to rotate the ball and the loop.

E_p = the energy spent to overcome the friction and air resistance.

from the instant of contact up to the instant the ball leaves.

where R is the radius.

E_f = the energy lost in elastic deformation. (This loss is negligible.)

The use of this approach is quite appropriate. It agrees with Vinogradov except that Richardson has wisely included a term for non-ice friction and wave making resistance. He also takes virtual mass into account.

The equation for downward force is as follows:

$$P_V = P_h + \sqrt{P_h^2 + \frac{\Delta}{Ag} \left\{ v_1^2 \left[(1 + \gamma_x) - (1 - \epsilon^2) \sin^2 \varphi \right] - v_2^2 (1 + \gamma_x) \right\}} \quad (A73)$$

where

$$= \frac{\frac{\mathcal{T}}{P} - \frac{c \tan \alpha}{\cos \beta} - \frac{\overline{R_{rt}}}{P}}{1 + \frac{c \cot \alpha}{\cos \beta}} \cot \alpha \quad (A74)$$

$$\eta = 1 + \frac{c \cot \alpha}{\cos \beta} \quad (A75)$$

$\overline{R_{rt}}$ = arithmetic mean for ship resistance computed for v_1 , and v_2 .

\mathcal{T} = propeller thrust

$$A = \left(\frac{1}{p} + \frac{i^2}{q^2} \right) \quad (A76)$$

γ_x = added virtual mass (percent)

P_h = horizontal component of force produced by the icebreaker.

C = coefficient of friction between the hull and the ice.

D = displacement

L = length at load waterline

r = distance from center of rotation of the waterplane from the point of contact on the ice.

φ = angle of the stem measured from the load waterline.

The first step is to identify the variables and their relationships. In this case, we are dealing with a system of equations involving variables x and y . The goal is to solve for these variables based on the given conditions.

$$\sqrt{x^2 + y^2} + \sqrt{x^2 - y^2} = 2 \quad (1)$$

$$\frac{x}{y} = \frac{1}{2} \quad (2)$$

$$\frac{x^2}{y^2} = \frac{1}{4} \quad (3)$$

$$\sqrt{x^2 + y^2} = \frac{1}{2} \sqrt{x^2 - y^2} + 2 \quad (4)$$

$$\sqrt{x^2 + y^2} - \frac{1}{2} \sqrt{x^2 - y^2} = 2 \quad (5)$$

$$\sqrt{x^2 + y^2} = \frac{1}{2} \sqrt{x^2 - y^2} + 2 \quad (6)$$

$$\sqrt{x^2 + y^2} - \frac{1}{2} \sqrt{x^2 - y^2} = 2 \quad (7)$$

$$\sqrt{x^2 + y^2} = \frac{1}{2} \sqrt{x^2 - y^2} + 2 \quad (8)$$

$$\sqrt{x^2 + y^2} - \frac{1}{2} \sqrt{x^2 - y^2} = 2 \quad (9)$$

$$\sqrt{x^2 + y^2} = \frac{1}{2} \sqrt{x^2 - y^2} + 2 \quad (10)$$

$$\sqrt{x^2 + y^2} - \frac{1}{2} \sqrt{x^2 - y^2} = 2 \quad (11)$$

ψ = angle of trim

α = $\phi + \psi$. (Since ψ is of the order of 1 to 2 degrees, $\alpha = \phi$ will be used in most trigonometric quantities, i.e. $\sin \alpha \approx \sin \phi$)

v_1 = velocity of the icebreaker at instant of impact with the ice.

v_2 = velocity at the close of the cycle, i.e. at the instant the ice breaks or zero if the icebreaker comes to a dead stop before the ice breaks.

p = tons per inch immersion at the load waterline.

q = moment in foot tons per inch trim.

The following assumptions or expedients were used by Richardson:

1. In effect, all steps lead to the final condition of $v_2 = 0$.
The equation is not valid where acceleration may exist.
2. It was assumed that the change in trim and draft did not seriously effect properties of the waterplane or moment to change trim one inch.
3. The distance from the center of rotation to the point of ice contact is assumed constant.
4. The "center of rotation" as he uses it is the center of flotation. In absolute terms this is not actually the case since there is also a change in draft (and effective displacement).
5. Although equations include trim angle in the first part of the development it is effectively dropped when he equates angle of trim plus angle of stem to angle of stem. It is granted that when the cosine is used there would be little difference but this is not necessarily the case for sine and tangent.

6. In the end conditions he uses rotational velocity (ω) equal to zero but this would only be true when all kinetic energy is lost or converted.
7. Although he recognizes that thrust may not always be horizontal only the work developed horizontally by this force is incorporated.
8. The change in the vertical position of the center of gravity is assumed to be the same as the average change in draft.
9. The angle of trim throughout the transition is based on static equilibrium.
10. The expression for loss of energy on impact is based on direct central impact. In other words it is assumed that the loss is the same as if a perpendicular to the stem passed through the center of gravity.
11. The determination of the downward force throughout the transition is based on static equilibrium.
12. The horizontal component of force against the ice must be known to use the equation. As used this is not the same as thrust and it is not clear where this value comes from.

The first of these is the fact that the...

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General Dynamics Equation

As part of a report released in 1959 concerning the feasibility of a nuclear icebreaker, an equation was developed representing the relative magnitude of the force "transmitted to the ice" at the bow. (17) However, this equation does not give the direction of this force and only a component of it is downward.

The following symbols were used in the development:

T = Thrust at zero speed in lb.

ρ = Mass density, lb sec²/ft⁴

α = Angle at the bow in the vertical plane, deg.

ZB = Angle at the bow in the horizontal plane in deg. (Note that this is not the same as 2β).

Δ = Vessel displacement in tons.

P = Shaft horsepower.

A = Propeller disc area, ft².

γ = Change of trim in deg.

W = Weight supported by ice.

R = Force perpendicular to the stem.

The forces acting include thrust at zero speed

$$T = \left[\left(\frac{\rho}{g} \right)^2 P^2 A \right]^{1/3} \quad (A77)$$

(In this form the units are not compatible and this is not explained.)

and that portion of the weight supported by the ice

$$W = 2 \Delta \tan \gamma \quad (A78)$$

(This equation is from Simonson (12).

Figure A-IX (17)
Geometry at an Icebreaker Bow

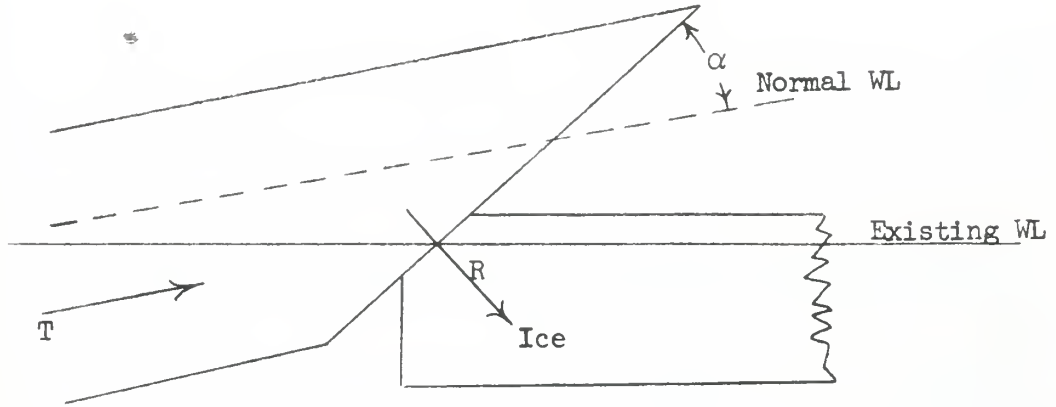
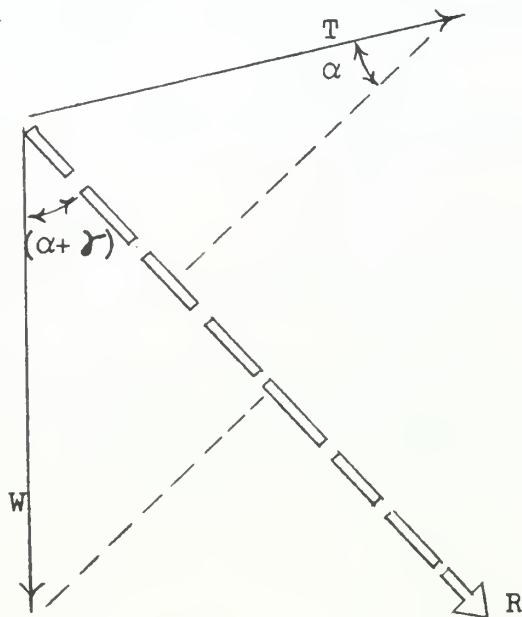


Figure A-X
Forces Acting on Ice from Bow



In substance Figures A-IX and A-X appear in the reference (17). However, they have been illustrated here with a fair amount of clarification and simplification.

It is interesting to note that this is the only development to date (1964) which assumes that thrust remains parallel to the normal waterline (base line) at all times.

Neglecting friction the force, R, perpendicular to the stem can be deduced from Figure A-X.

$$R = T \sin \alpha + W \cos (\alpha + \gamma) \quad (A79)$$

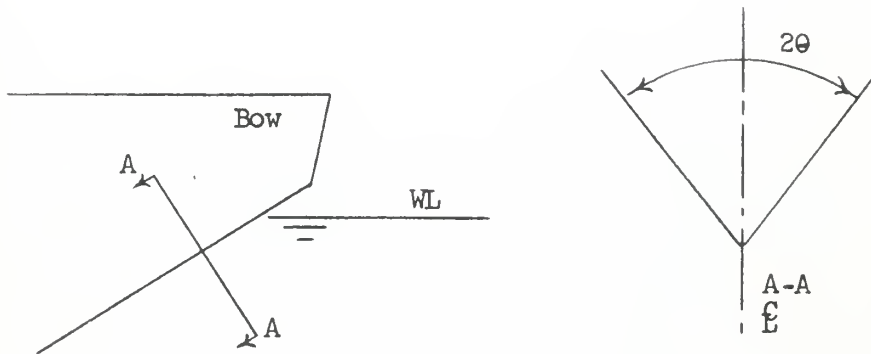
Using the analogy of a wedge being forced into the ice by force, R, (neglecting friction), an equation can be developed using the term (20) as the "wedge angle". Figure 21B in the reference is a three dimensional representation which is quite confusing and for that reason is not shown here. However, the definition of the "wedge angle" is needed. Although it is not explained, it is apparent from its use that it is the spread angle seen as one looks down the stem. See Figure A-XI.

The force transmitted to the ice is perpendicular to each side of the bow and is called R_{\perp} , where

$$R_{\perp} = \left[\left(\rho / g \ 2P^2A \right)^{1/3} \sin \alpha + 2\Delta \tan \gamma \cos (\alpha + \gamma) \right] \left[\frac{\sqrt{\sin^2 \alpha + \tan^2 B}}{2 \tan B} \right] \quad (A80)$$

The reference carries a graph of this force, R_{\perp} , versus bow angle, α . Figure A-XII is not a reproduction of this graph but does illustrate its appearance.

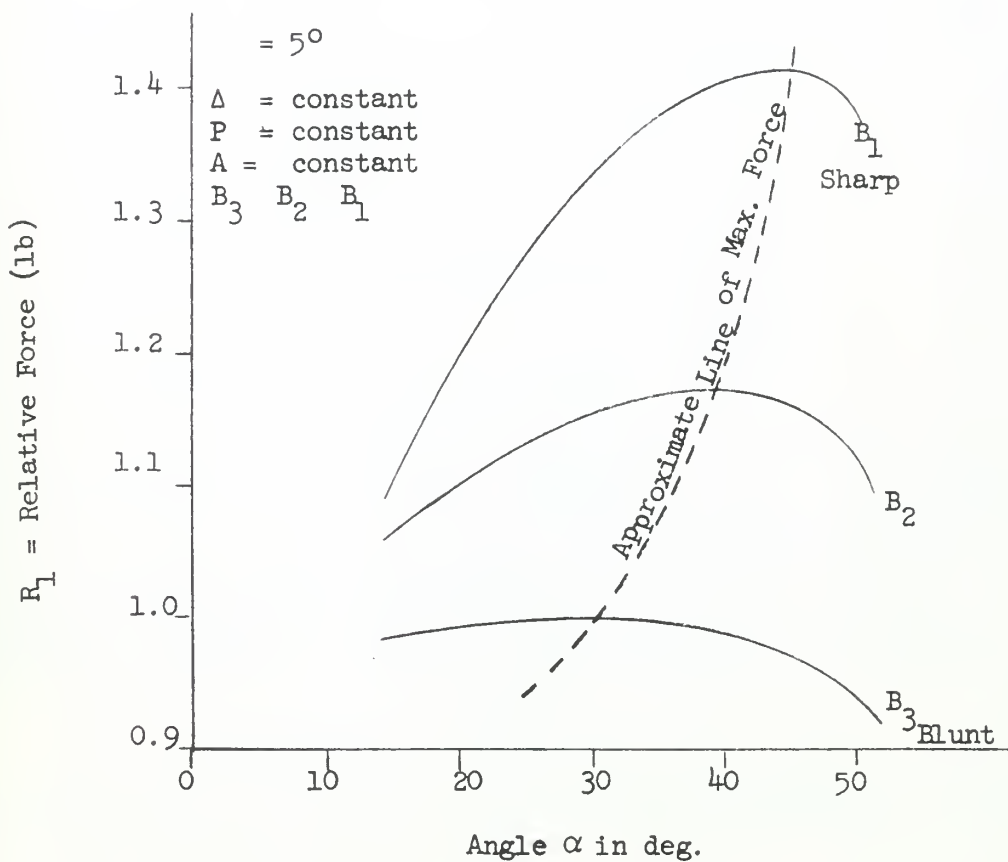
Figure A-XI
Illustration of "Wedge Angle"



"Wedge Angle" $\equiv 2 \theta$

Figure A-XII

Illustration of a Plot entitled
"Variation of Force Exerted on
the Ice as a Function of Bow Design"



A reader must be extremely careful not to jump to any conclusions. The graph is simply a plot of the results of equation (A80) where trim angle δ , displacement Δ , power P , and disc area A are held constant. It simply illustrates that if the bow is made sharper and all other parameters are held constant the force normal to the hull plating will increase. As a matter of interest and fact, the downward force, W , is constant throughout under the conditions used since Δ and γ are held constant!

The first part of the report deals with the general situation in the country and the progress of the work of the Commission. It then goes on to discuss the various aspects of the work of the Commission, such as the work of the various committees and the work of the Commission as a whole. The report concludes with a summary of the work of the Commission and a list of recommendations.

1/1/1950

Milano's Equation

In 1962 an equation was published giving the vertical bow reaction force which an icebreaker can develop climbing onto the ice (18).

The following symbols appear in the equation:

F = Downward vertical force.

μ_0 = Coefficient of friction (dynamic) between steel and ice.

α = Angular rise of the forefoot.

β = Angle between the centerline plane and the normal to the shell at the bow.

X and Z are each a direct function of μ_0 , α , and β .

T_0 = Bollard pull, thrust

W = Vessel displacement

v = Velocity prior to contact with ice.

H = Draft

As presented the equation is as follows:

$$F = 0.91 X T_0 + \sqrt{0.828 X^2 T_0^2 + \frac{W^2 v^2 Z}{4g H}} \quad (A81)$$

This equation originates from Vinogradov (13) but it has been abbreviated by selection of constants and coefficients. For example

$$T_{VINOGRAOV} = 0.91 T_0$$

The equation is based on the same assumptions Vinogradov made and has the same limitations.

Mathematical Model

It is assumed that the system is linear and time-invariant. The input signal is assumed to be a white noise process with a power spectral density of $S_{xx}(\omega) = \frac{1}{2\pi} S_0$. The output signal is assumed to be a stationary process with a power spectral density of $S_{yy}(\omega) = |H(\omega)|^2 S_{xx}(\omega)$. The transfer function of the system is assumed to be $H(\omega) = \frac{1}{1 + j\omega T}$, where T is the time constant of the system. The output signal is assumed to be a stationary process with a power spectral density of $S_{yy}(\omega) = \frac{S_0}{1 + \omega^2 T^2}$. The output signal is assumed to be a stationary process with a power spectral density of $S_{yy}(\omega) = \frac{S_0}{1 + \omega^2 T^2}$.

- 1 - The input signal is assumed to be a white noise process with a power spectral density of $S_{xx}(\omega) = \frac{1}{2\pi} S_0$.
- 2 - The output signal is assumed to be a stationary process with a power spectral density of $S_{yy}(\omega) = |H(\omega)|^2 S_{xx}(\omega)$.
- 3 - The transfer function of the system is assumed to be $H(\omega) = \frac{1}{1 + j\omega T}$, where T is the time constant of the system.
- 4 - The output signal is assumed to be a stationary process with a power spectral density of $S_{yy}(\omega) = \frac{S_0}{1 + \omega^2 T^2}$.
- 5 - The output signal is assumed to be a stationary process with a power spectral density of $S_{yy}(\omega) = \frac{S_0}{1 + \omega^2 T^2}$.

As presented the equation is as follows:

$$(1) \quad \frac{d^2 y}{dt^2} + \frac{dy}{dt} + y = x$$

This equation is derived from the physical model of the system. The equation is derived by taking the Laplace transform of the differential equation and solving for the output signal. The equation is derived by taking the Laplace transform of the differential equation and solving for the output signal.

$$Y(s) = \frac{X(s)}{1 + s + s^2}$$

The equation is based on the new frequency response function and the new transfer function.

B. DETAILS OF PROCEDURE

General

When an icebreaker encounters virtually unyielding ice (rams) it crushes the ice locally to accommodate the bow, the bow then slides up on the ice with decreasing velocity, and then the icebreaker undergoes minor settling after the velocity of the bow relative to the ice has come to zero. At this last point the ship tends to slide back but is frequently held by static friction and/or forward thrust.

The following definitions will be of use for the purpose of constructing a mathematical model:

State 1. Immediately prior to contact with the ice.

$$t = 0$$

$$\dot{x} = 0 \quad \dot{z} = 0 \quad \dot{\theta} = 0$$

$$\dot{x}' = v_1 \quad \dot{z}' = 0 \quad \dot{\theta}' = 0$$

$$x = 0 \quad z = 0 \quad \theta = 0$$

$$F_{BZ} = 0 \quad F_{BZ} = 0$$

Crushing Phase. Ice is being crushed locally to accommodate the bow. (The ice is not collapsing.)

During the crushing phase five equations may be expressed.

Vertical force at bow. (function of penetration)

Horizontal force at bow. (function of penetration)

Summation of Horizontal Forces

Summation of Vertical Forces

Summation of Moments

PHYSICS 551

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$$\begin{aligned}
 \frac{\partial \psi}{\partial t} &= -\frac{\hbar^2}{2m} \nabla^2 \psi - V \psi \\
 \frac{\partial \psi}{\partial x} &= -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} - V \frac{\partial \psi}{\partial x} \\
 \frac{\partial \psi}{\partial y} &= -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial y^2} - V \frac{\partial \psi}{\partial y} \\
 \frac{\partial \psi}{\partial z} &= -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial z^2} - V \frac{\partial \psi}{\partial z}
 \end{aligned}$$

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There are five time-dependent unknowns,

x , z , θ , Vertical force at bow (F_{BZC}), and Horizontal force at bow (F_{BXC}).

State 2. Local crushing has ceased and sliding without crushing commences.

This is reached when the velocity of a point on the bow has a direction which is the same as the slope of the bow plus the trim.

In other words, there is no component of bow velocity perpendicular to the stem.

$$t = t_2 \text{ (for crushing)} \quad t = 0 \text{ (for sliding)}$$

$$x = x_2 \qquad z = z_2 \qquad \theta = \theta_2$$

$$\dot{x} = \dot{x}_2 \qquad \dot{z} = \dot{z}_2 \qquad \dot{\theta} = \dot{\theta}_2$$

$$\ddot{x} = \ddot{x}_2 \qquad \ddot{z} = \ddot{z}_2 \qquad \ddot{\theta} = \ddot{\theta}_2$$

$$F_{BZ} = F_{BZ2} \qquad F_{BX} = F_{BX2}$$

Sliding Phase. The bow slides up on the ice without further penetration.

During the sliding phase four equations may be expressed.

Equation of geometry since point of contact is fixed relative to the ice.

Summation of Horizontal Forces.

Summation of Vertical Forces.

Summation of Moments

There are four time-dependent unknowns, x , z , θ , and the force at the bow (which can be divided into two components).

Chapter 10: The Binomial Distribution

Let X be the number of successes in n trials, each with probability p of success. Then X has a binomial distribution with parameters n and p . The probability mass function is given by

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ is the binomial coefficient. The mean and variance of X are given by

$$E(X) = np \quad \text{and} \quad \text{Var}(X) = np(1-p)$$

$k=0$	$k=1$	$k=2$
$\binom{n}{0} p^0 (1-p)^n$	$\binom{n}{1} p^1 (1-p)^{n-1}$	$\binom{n}{2} p^2 (1-p)^{n-2}$
$\binom{n}{n-k}$	$\binom{n}{n-k-1}$	$\binom{n}{n-k-2}$
$\binom{n}{n-k} p^{n-k} (1-p)^k$	$\binom{n}{n-k-1} p^{n-k-1} (1-p)^{k+1}$	$\binom{n}{n-k-2} p^{n-k-2} (1-p)^{k+2}$

Example 10.1: Let X be the number of heads in 10 tosses of a fair coin. Then X has a binomial distribution with $n=10$ and $p=0.5$.

Find the probability that X is even. Solution: The probability that X is even is given by

$$P(X \text{ is even}) = P(X=0) + P(X=2) + P(X=4) + P(X=6) + P(X=8) + P(X=10)$$

Using the binomial formula, we find that $P(X \text{ is even}) = 0.5$. This result can be generalized to any binomial distribution.

State 3. The velocity of the bow relative to the ice has come to zero.
(Velocities $(\dot{x}, \dot{z}, \dot{\theta})$ may be negligible but they are not necessarily zero.)

$$t_4 = t_3$$

$$x = x_3 \quad z = z_3 \quad \theta = \theta_3$$

$$\dot{x} = \dot{x}_3 \quad \dot{z} = \dot{z}_3 \quad \dot{\theta} = \dot{\theta}_3$$

$$\ddot{x} = \ddot{x}_3 \quad \ddot{z} = \ddot{z}_3 \quad \ddot{\theta} = \ddot{\theta}_3$$

$$F_{BZ} = F_{BZ3} \quad F_{BX} = F_{BX3}$$

State 4. The icebreaker is in static equilibrium. All velocities have become zero.

$$x = x_4 \quad (x_4 = x_3) \quad z = z_4 \quad \theta = \theta_4$$

$$\dot{x} = 0 \quad \dot{z} = 0 \quad \dot{\theta} = 0$$

$$\ddot{x} = 0 \quad \ddot{z} = 0 \quad \ddot{\theta} = 0$$

$$F_{BZ} = F_{BZ4}$$

F_{BZ4} is the relatively sustained downward force under the bow we are seeking.

... ..

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$$\frac{1}{2} \dots$$

$$\begin{matrix} \frac{1}{2} \dots & \frac{1}{2} \dots & \frac{1}{2} \dots \\ \frac{1}{2} \dots & \frac{1}{2} \dots & \frac{1}{2} \dots \\ \frac{1}{2} \dots & \frac{1}{2} \dots & \frac{1}{2} \dots \end{matrix}$$

$$\frac{1}{2} \dots = \frac{1}{2} \dots + \frac{1}{2} \dots$$

... ..

... ..

$$\begin{matrix} \frac{1}{2} \dots & \frac{1}{2} \dots (\dots - \dots) & \frac{1}{2} \dots \\ \frac{1}{2} \dots & \frac{1}{2} \dots & \frac{1}{2} \dots \\ \frac{1}{2} \dots & \frac{1}{2} \dots & \frac{1}{2} \dots \end{matrix}$$

$$\frac{1}{2} \dots = \frac{1}{2} \dots$$

... ..

Bow Forces During Crushing

Assume all forces from the ice act on the bow at point A, the point of contact at the waterline.

If $\frac{N}{2}$ represents the force normal to the plating on each side, then the friction force can be represented by

$$\frac{F}{2} = f_k \frac{N}{2} \quad \text{where } f_k = \text{coeff. of kinetic friction} \quad (B1)$$

Note that the friction forces during crushing are parallel to the stem and perpendicular to the stem (each in the plane of the plating). This is because there is a component of velocity relative to the ice in each direction (parallel and perpendicular). See Figures B-I and B-II.

From Figure B-II it can be seen that

$$P = N \cos \beta + F \sin \beta \quad (B2)$$

where $\beta =$ angle between normal to plating and centerline plane.

Substitution of (B1) into (B2) leads to

$$P = N (\cos \beta + f_k \sin \beta) \quad (B3)$$

From (B1) and (B3) we get

$$N = \frac{F}{f_k}$$
$$N = \frac{P}{\cos \beta + f_k \sin \beta} \quad (B4)$$
$$\frac{F}{f_k} = \frac{P}{\cos \beta + f_k \sin \beta}$$

THE FURTHER STUDY

Consider all linear functions $f(x) = ax + b$ such that $f(1) = 1$ and $f(2) = 2$.
The function $f(x) = x$ is the only one that satisfies these conditions.
The function $f(x) = x$ is the only one that satisfies these conditions.

(10) $f(x) = x$ is the only function that satisfies these conditions.

Let $f(x) = ax + b$ be a linear function. Then $f(1) = a + b = 1$ and $f(2) = 2a + b = 2$.
Subtracting the first equation from the second gives $a = 1$. Substituting $a = 1$ into the first equation gives $b = 0$.
Thus $f(x) = x$ is the only linear function that satisfies these conditions.

(11) $f(x) = x$ is the only function that satisfies these conditions.

Let $f(x) = ax + b$ be a linear function. Then $f(1) = a + b = 1$ and $f(2) = 2a + b = 2$.
Subtracting the first equation from the second gives $a = 1$. Substituting $a = 1$ into the first equation gives $b = 0$.
Thus $f(x) = x$ is the only linear function that satisfies these conditions.

(12) $f(x) = x$ is the only function that satisfies these conditions.

From (11) and (12) we get $f(x) = x$.

(13) $f(x) = x$ is the only function that satisfies these conditions.

Figure B-I
Forces Acting on Bow During Crushing

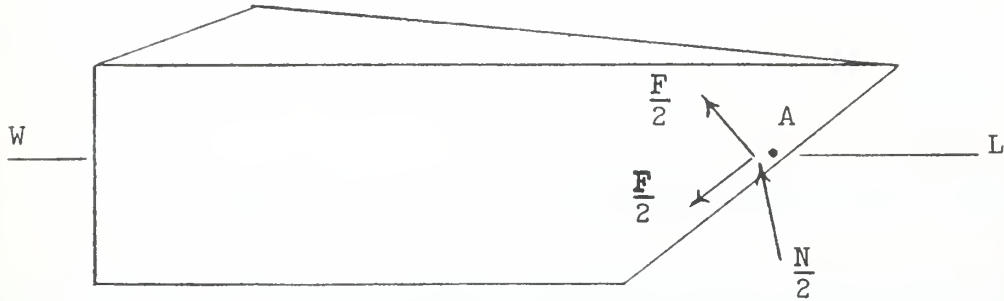
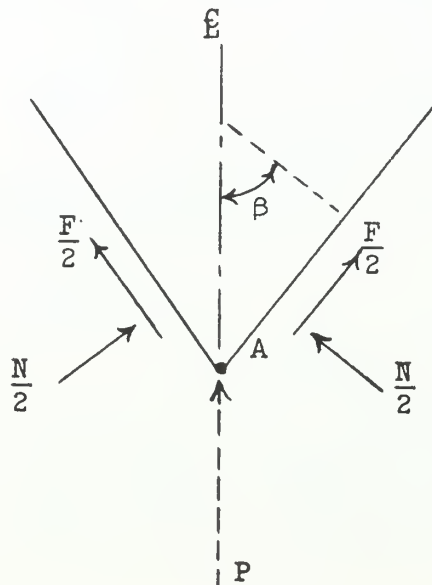


Figure B-II
Resolution of Friction and Normal Forces
During Crushing (Looking Down Stem)



$$\text{Set } k_1 = \frac{f_k}{\cos \beta + f_k \sin \beta} \quad (B5)$$

$$\text{Then } F = k_1 P. \quad (B6)$$

The upward forces at the bow, designated F_{BZC} , can be seen in Figures B-III and B-IV.

$$F_{BZC} = P \cos (i_B + \theta) - F \sin (i_B + \theta) \quad (B7)$$

$$F_{BZC} = N (\cos \beta + f_k \sin \beta) \cos (i_B + \theta) - N f_k \sin (i_B + \theta)$$

The horizontal force to the left, designated F_{BXC} , can also be seen.

$$F_{BXC} = P \sin (i_B + \theta) + F \cos (i_B + \theta)$$

$$F_{BXC} = N (\cos \beta + f_k \sin \beta) \sin (i_B + \theta) + N f_k \cos (i_B + \theta) \quad (B8)$$

While crushing is taking place, assume that the ice is failing in compression over an area in contact with the bow plating. If the area in contact on each side of the bow is $A/2$ and the failing compressive stress of the ice is designated σ , then

$$\frac{N}{2} = \sigma \frac{A}{2} \quad (B9)$$

As can be seen in Figures B-VI and B-VII,

$$\frac{A}{2} = \frac{A'}{2 \sin \beta}$$

$$\frac{A'}{2} = \frac{1}{2} \left[x - (\text{corr. for } z \text{ and } \theta) \right]^2 \tan (i_B + \theta) \quad (B10)$$

Assume that area-triangle remains at point A at bow (intersection of waterline and stem) and is small enough (or that ice is deep enough) to keep

$$(12) \quad \frac{1}{\cos^2 \theta} = \frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta}$$

$$(13) \quad \frac{1}{\cos^2 \theta} = \frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta}$$

The general form of the vector \mathbf{r} can be written as

$$(14) \quad \mathbf{r} = r \cos \theta \mathbf{i} + r \sin \theta \mathbf{j}$$

$$(15) \quad \mathbf{r} = r \cos \theta \mathbf{i} + r \sin \theta \mathbf{j}$$

The partial derivatives of \mathbf{r} with respect to r and θ are

$$\frac{\partial \mathbf{r}}{\partial r} = \cos \theta \mathbf{i} + \sin \theta \mathbf{j}$$

$$(16) \quad \frac{\partial \mathbf{r}}{\partial \theta} = -r \sin \theta \mathbf{i} + r \cos \theta \mathbf{j}$$

While working in polar coordinates, assume that the ice is flowing in a direction over an area in contact with the bow plating. If the ice is in contact on each side of the bow as shown in the figure, the contact area is the ice in contact with the bow plating.

$$(17) \quad \frac{1}{2} = \frac{1}{2}$$

As can be seen in Figures 1-11 and 1-12,

$$\frac{1}{2} = \frac{1}{2}$$

$$(18) \quad \left[\frac{1}{2} - \frac{1}{2} \right] \frac{1}{2} = \frac{1}{2}$$

Assume that cross-sections are taken at point A as bow intersection of waterline and stem) and is small enough (or that the ice is very

Figure B-III
Bow Forces During Crushing

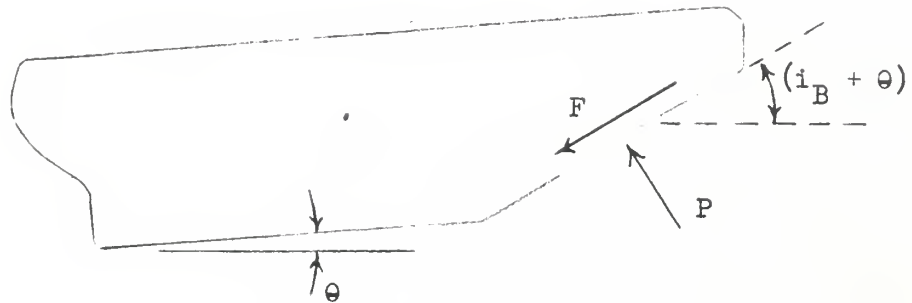


Figure B-IV
Bow Forces During Crushing Resolved
into X and Z Directions

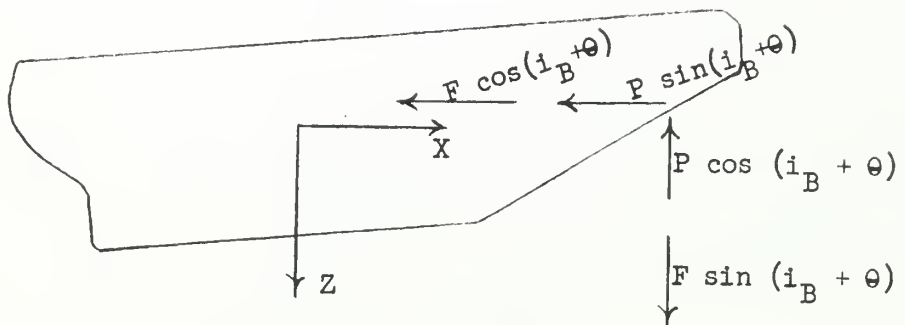


Figure B-V
Coordinate System Defined by Position
when $t = 0$, Immediately Prior to Contact

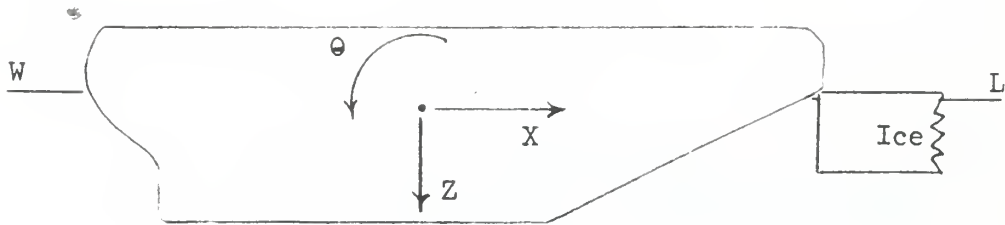
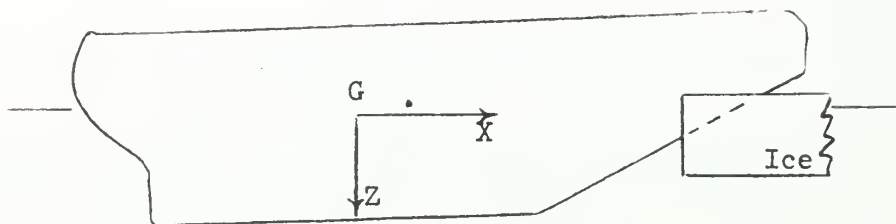


Figure B-VI
Local Crushing of Ice



shape triangular.

In order to correct for trim, θ , it is necessary to define distance from G to A. Assume, for this purpose, that KG (height of center of gravity above keel) is about the same as H (draft),

$$(GA) \text{ Horizontal} = \left(\frac{L}{2} - LCG\right)$$

where

L = Length between perpendiculars.

LCG = Distance from midships to center of gravity,
+ if forward, - if aft.

In Figure B-VIII it can be seen that

$$(\text{Corr. for } z \text{ and } \theta) = \frac{\left(\frac{L}{2} - LCG\right) \theta - z}{\tan (i_B + \theta)} \quad (B11)$$

From Figures B-VII and B-VIII, and from equations (B10 and (B11), it can be seen that

$$\frac{A'}{2} = \frac{1}{2} \left[x - \left(\frac{\left(\frac{L}{2} - LCG\right) \theta - (z)}{\tan (i_B + \theta)} \right) \right]^2 \tan (i_B + \theta)$$

$$\frac{A'}{2} = \frac{1}{2} \left[x \tan (i_B + \theta) - \left(\frac{L}{2} - LCG\right) \theta + z \right]^2 \frac{1}{\tan (i_B + \theta)} \quad (B12)$$

$$N = \sigma A = \frac{\sigma}{\sin \beta \tan (i_B + \theta)} \left[x \tan (i_B + \theta) - \left(\frac{L}{2} - LCG\right) \theta + z \right]^2$$

Substitution of equation (B12) into equation (B3) leads to the force in the upward direction at the bow during crushing.

To obtain the asymptotic behavior of the function

we use the expansion of the function in powers of ϵ

where ϵ is a small parameter

$$f(x) = \sum_{n=0}^{\infty} \epsilon^n f_n(x)$$

where $f_n(x)$ are functions of x

Substituting this expansion into equation (1)

$$L f(x) = g(x)$$

we obtain the following equations

$$L f_0(x) = g(x) \quad (2)$$

From equations (2) and (3) we obtain the asymptotic expansion

of the function

$$f(x) \sim \sum_{n=0}^{\infty} \epsilon^n f_n(x) \quad (4)$$

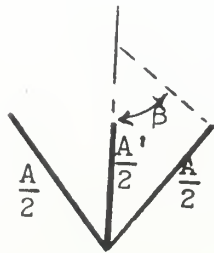
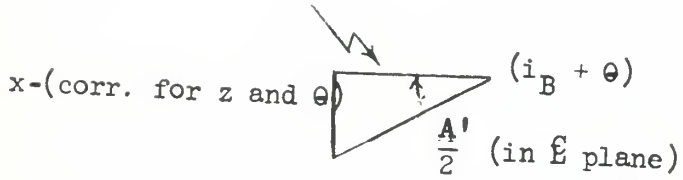
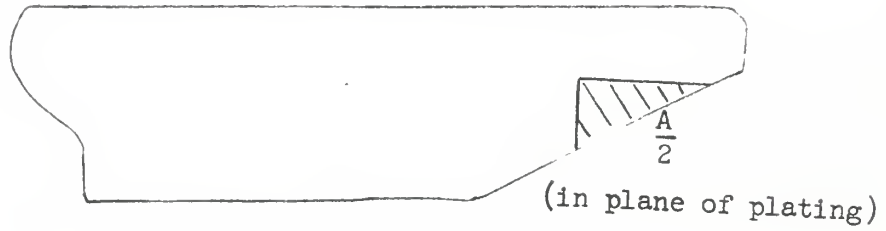
$$L f_1(x) = -\epsilon L f_0(x) \quad (5)$$

$$L f_2(x) = -\epsilon L f_1(x) - \frac{\epsilon^2}{2} L^2 f_0(x) \quad (6)$$

Substituting the expansion (4) into equation (1) leads to the equation

for the function $f_0(x)$

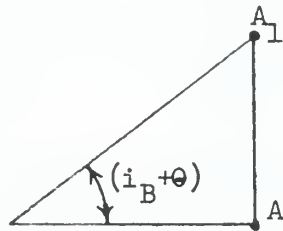
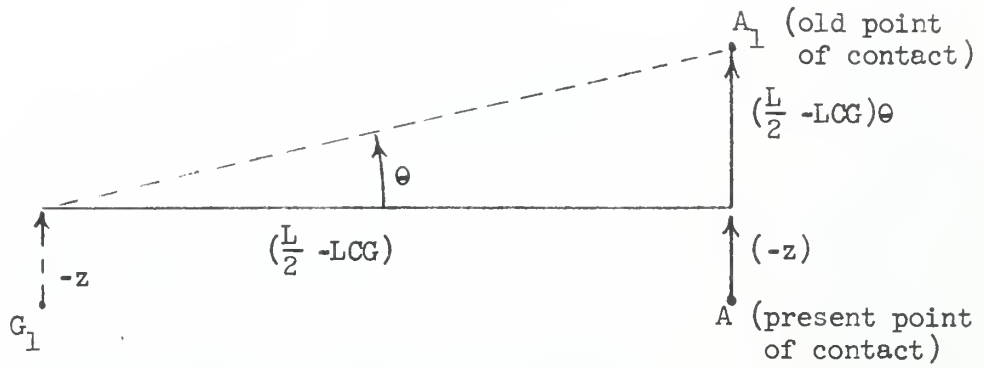
Figure B-VII
Area in Contact During Crushing



Looking Down Stem

Figure B-VIII

Movement of A_1 for Purpose of Area Correction



(Correction for z and θ)

$$F_{BZC} = \frac{\sigma (\cos \beta + f_k \sin \beta) \cos (i_B + \theta)}{\sin \beta \tan (i_B + \theta)} \left[x \tan (i_B + \theta) - \left(\frac{L}{2} - LCG \right) \theta + z \right]^2$$

$$- \frac{\sigma f_k \sin (i_B + \theta)}{\sin \beta \tan (i_B + \theta)} \left[x \tan (i_B + \theta) - \left(\frac{L}{2} - LCG \right) \theta + z \right]^2$$

$$(a - b + c) (a - b + c) = a^2 - ab + ac - ab + b^2 - bc$$

$$+ ac - bc + c^2$$

$$= a^2 - 2ab + b^2 - 2bc + 2ac + c^2$$

$$a = x \tan (i_B + \theta)$$

$$b = \left(\frac{L}{2} - LCG \right) \theta$$

$$c = z$$

$$\left[x \tan (i_B + \theta) - \left(\frac{L}{2} - LCG \right) \theta + z \right]^2 =$$

$$\left[x^2 \tan^2 (i_B + \theta) - 2 \left(\frac{L}{2} - LCG \right) \theta x \tan (i_B + \theta) \right.$$

$$\left. + \left(\frac{L}{2} - LCG \right)^2 \theta^2 - 2 z \left(\frac{L}{2} - LCG \right) \theta + 2 z x \tan (i_B + \theta) + z^2 \right]$$

$$F_{BZC} = \frac{\sigma (\cos \beta + f_k \sin \beta) \cos (i_B + \theta) x^2 \tan^2 (i_B + \theta)}{\sin \beta \tan (i_B + \theta)}$$

$$- \frac{2 \sigma (\cos \beta + f_k \sin \beta) \cos (i_B + \theta) \left(\frac{L}{2} - LCG \right) \theta x \tan (i_B + \theta)}{\sin \beta \tan (i_B + \theta)}$$

$$+ \frac{\sigma (\cos \beta + f_k \sin \beta) \cos (i_B + \theta) \left(\frac{L}{2} - LCG \right)^2 \theta^2}{\sin \beta \tan (i_B + \theta)}$$

$$- \frac{2 \sigma (\cos \beta + f_k \sin \beta) (i_B + \theta) z \left(\frac{L}{2} - LCG \right) \theta}{\sin \beta \tan (i_B + \theta)}$$

$$\frac{1}{(1-x^2)^2} = \frac{1}{(1-x)^2(1+x)^2} = \frac{A}{1-x} + \frac{B}{(1-x)^2} + \frac{C}{1+x} + \frac{D}{(1+x)^2}$$

$$\frac{1}{(1-x)^2(1+x)^2} = \frac{A(1-x)(1+x) + B(1+x) + C(1-x)(1+x) + D(1-x)}{(1-x)^2(1+x)^2}$$

$$1 = A(1-x^2) + B(1+x) + C(1-x^2) + D(1-x)$$

$$1 = A - Ax^2 + B + Bx + C - Cx^2 + D - Dx$$

$$1 = (A+B+C+D) + (B-D)x - (A+C)x^2$$

$$\begin{cases} A+B+C+D = 1 \\ B-D = 0 \\ -A-C = 0 \end{cases}$$

$$A = -C$$

$$B = D$$

$$-A-C = 0 \implies A = -C$$

$$B = D$$

$$1 = \left[A + (A+B+C+D) + (B-D)x - (A+C)x^2 \right]$$

$$1 = \left[A + (A+B+C+D) + (B-D)x - (A+C)x^2 \right]$$

$$\left[\begin{aligned} 1 &= (A+B+C+D) + (B-D)x - (A+C)x^2 \\ 1 &= (A+B+C+D) + (B-D)x - (A+C)x^2 \end{aligned} \right]$$

$$\frac{(A+B+C+D) + (B-D)x - (A+C)x^2}{(1-x)^2(1+x)^2} = \frac{1}{(1-x)^2(1+x)^2}$$

$$\frac{(A+B+C+D) + (B-D)x - (A+C)x^2}{(1-x)^2(1+x)^2} = \frac{1}{(1-x)^2(1+x)^2}$$

$$\frac{1 + (B-D)x - (A+C)x^2}{(1-x)^2(1+x)^2} = \frac{1}{(1-x)^2(1+x)^2}$$

$$\frac{1 + (B-D)x - (A+C)x^2}{(1-x)^2(1+x)^2} = \frac{1}{(1-x)^2(1+x)^2}$$

$$\frac{+2\sigma (\cos \beta + f_k \sin \beta) \cos (i_B + \theta) z \times \tan (i_B + \theta)}{\sin \beta \tan (i_B + \theta)}$$

$$+\frac{\sigma (\cos \beta + f_k \sin \beta) \cos (i_B + \theta) z^2}{\sin \beta \tan (i_B + \theta)}$$

$$-\frac{\sigma f_k \sin (i_B + \theta) z^2 \tan^2 (i_B + \theta)}{\sin \beta \tan (i_B + \theta)}$$

$$+\frac{2\sigma f_k \sin (i_B + \theta) \left(\frac{L}{2} - LCG\right)\theta \times \tan (i_B + \theta)}{\sin \beta \tan (i_B + \theta)}$$

$$-\frac{\sigma f_k \sin (i_B + \theta) \left(\frac{L}{2} - LCG\right)^2 \theta^2}{\sin \beta \tan (i_B + \theta)}$$

$$+\frac{2\sigma f_k \sin (i_B + \theta) z \left(\frac{L}{2} - LCG\right)\theta}{\sin \beta \tan (i_B + \theta)}$$

$$-\frac{2\sigma f_k \sin (i_B + \theta) z \times \tan (i_B + \theta)}{\sin \beta \tan (i_B + \theta)}$$

$$-\frac{\sigma f_k \sin (i_B + \theta) z^2}{\sin \beta \tan (i_B + \theta)}$$

(E13)

It is necessary to linearize equation (E13) as much as possible in order to make it useful for inclusion in simultaneous differential equations.

Throughout this development, since θ is small (around 5° or less),

$$\cos \theta = 1.00$$

$$\tan \theta = \theta \text{ rad.}$$

$$\sin \theta = \theta \text{ rad.}$$

$$\frac{5.10^2 - 2.10 + 1}{(10 - 2)^2} = \frac{50 - 20 + 1}{64} = \frac{31}{64}$$

$$\frac{5.10^2 - 2.10 + 1}{(10 - 2)^2} = \frac{31}{64}$$

$$\frac{5.10^2 - 2.10 + 1}{(10 - 2)^2} = \frac{31}{64}$$

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$$\frac{5.10^2 - 2.10 + 1}{(10 - 2)^2} = \frac{31}{64}$$

(20)

At present we use as (20) which is the result of the

analysis of the data. The result is that the

value of P_0 is 0.125. This is the result of the

$$P_0 = 0.125$$

$$P_0 = 0.125$$

$$P_0 = 0.125$$

<u>Function</u>	<u>Substitution</u>	<u>Limit of θ° if Error $< 1\%$</u>
cos θ	1.00	5.7 $^\circ$
tan θ	θ rad.	9.9 $^\circ$
sin θ	θ rad.	14.0 $^\circ$

Using fundamental trigonometric identities, the following conversions may be used.

$$\sin (i_B + \theta) = \sin i_B \cos \theta + \cos i_B \sin \theta$$

$$\sin (i_B + \theta) = \sin i_B + \theta \cos i_B$$

$$\cos (i_B + \theta) = \cos i_B \cos \theta - \sin i_B \sin \theta$$

$$\cos (i_B + \theta) = \cos i_B - \theta \sin i_B$$

$$\tan (i_B + \theta) = \frac{\sin (i_B + \theta)}{\cos (i_B + \theta)} = \frac{\sin i_B + \theta \cos i_B}{\cos i_B - \theta \sin i_B}$$

In order to check the orders of magnitude of equation (E13), let us examine the following terms:

$$\left(\frac{L}{2} - LCG\right) \theta x^2$$

$$\left(\frac{L}{2} - LCG\right) \theta^2$$

$$\left(\frac{L}{2} - LCG\right) \theta z$$

$$x z$$

$$z^2$$

It is noted that there is an initial velocity in the x-direction.

$$\left(\frac{dx}{dt}\right) = v_1$$

However,

$$\left(\frac{d\theta}{dt}\right) = 0 \quad \text{and} \quad \left(\frac{dz}{dt}\right) = 0$$

1. The function $f(x)$ is defined by

$$f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ x^3 & \text{if } x \geq 0 \end{cases}$$

Find $f'(x)$

Answer

$$f'(x) = \begin{cases} 2x & \text{if } x < 0 \\ 3x^2 & \text{if } x \geq 0 \end{cases}$$

$$f'(x) = \begin{cases} 2x & \text{if } x < 0 \\ 3x^2 & \text{if } x \geq 0 \end{cases}$$

2. The function $f(x)$ is defined by $f(x) = \begin{cases} x^2 \sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

Find $f'(0)$

$$f'(0) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{h^2 \sin(1/h) - 0}{h} = \lim_{h \rightarrow 0} h \sin(1/h) = 0$$

$$f'(0) = 0$$

$$f'(0) = 0$$

$$f'(0) = 0$$

$$\frac{d}{dx} (x^2 \sin(1/x)) = 2x \sin(1/x) - \cos(1/x)$$

3. The function $f(x)$ is defined by $f(x) = \begin{cases} x^2 \cos(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

Find $f'(0)$

$$f'(0) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \rightarrow 0} \frac{h^2 \cos(1/h) - 0}{h} = \lim_{h \rightarrow 0} h \cos(1/h) = 0$$

$$f'(0) = 0$$

$$f'(0) = 0$$

$$f'(0) = 0$$

4. The function $f(x)$ is defined by $f(x) = \begin{cases} x^2 \sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

$$f'(0) = 0$$

Answer

$$f'(0) = 0$$

Up to the point where the velocity at the bow has a direction defined by $(i_B + \theta)$, the following magnitudes would be typically representative:

$$x \approx 10 \text{ ft}$$

$$\left(\frac{L}{2} - LCG\right)\theta + (-z) \approx 0.8 \text{ ft}$$

Assume $\left(\frac{L}{2} - LCG\right)\theta \approx 0.4 \text{ ft}$

$$-z \approx 0.4 \text{ ft}$$

Then, $x^2 = 100 \text{ ft.}$

$$\left(\frac{L}{2} - LCG\right)\theta x = 4 \text{ ft.}$$

$$\left(\frac{L}{2} - LCG\right)^2 \theta^2 = 0.16 \text{ ft.}$$

$$\left(\frac{L}{2} - LCG\right)\theta z = 0.16 \text{ ft.}$$

$$x z = 4 \text{ ft.}$$

$$z^2 = 0.16 \text{ ft.}$$

When it becomes desirable to simplify equation (B13) it is apparent that all terms may be dropped except those containing x^2 . It may furthermore be assumed that

$$\tan (i_B + \theta) \approx \tan i_B$$

during the crushing phase.

The simplifications mentioned above may be used directly to rewrite equation (B12) as follows:

$$N = \frac{\sigma x^2 \tan i_B}{\sin \beta}$$

Equation (B13) may now be written in the following form:

Consider the function $f(x) = \frac{1}{x^2}$ defined on the interval $(0, \infty)$. The function is continuous on this interval and differentiable on $(0, \infty)$ with derivative $f'(x) = -\frac{2}{x^3}$.

$$\begin{aligned}
 f'(x) &= -\frac{2}{x^3} \\
 f'(x) &= -\frac{2}{x^3} \implies f'(x) = -\frac{2}{x^3} \\
 f'(x) &= -\frac{2}{x^3} \\
 f'(x) &= -\frac{2}{x^3} \\
 f'(x) &= -\frac{2}{x^3} \\
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 \end{aligned}$$

Consider the function $f(x) = \frac{1}{x^2}$ defined on the interval $(0, \infty)$. The function is continuous on this interval and differentiable on $(0, \infty)$ with derivative $f'(x) = -\frac{2}{x^3}$.

$$f'(x) = -\frac{2}{x^3}$$

The function $f(x) = \frac{1}{x^2}$ is continuous on the interval $(0, \infty)$ and differentiable on $(0, \infty)$ with derivative $f'(x) = -\frac{2}{x^3}$.

$$\frac{d}{dx} \left(\frac{1}{x^2} \right) = -\frac{2}{x^3}$$

The function $f(x) = \frac{1}{x^2}$ is continuous on the interval $(0, \infty)$ and differentiable on $(0, \infty)$ with derivative $f'(x) = -\frac{2}{x^3}$.

$$F_{BZC} = \left[\frac{\sigma \tan i_B}{\sin \beta} \cos i_B (\cos \beta + f_k \sin \beta) - \frac{\sigma \tan i_B}{\sin \beta} f_k \sin i_B \right] x^2 \quad (E14)$$

Equation (E8) for the horizontal force at the bow may now be written in the following form:

$$F_{BXC} = \left[\frac{\sigma \tan i_B}{\sin \beta} \sin i_B (\cos \beta + f_k \sin \beta) + \frac{\sigma \tan i_B}{\sin \beta} f_k \cos i_B \right] x^2 \quad (E15)$$

Free Body Description During Crushing

Figure B-IX shows the complete free body diagram for an icebreaker during the crushing phase. (It is the same for the sliding phase except for the composition of the bow forces.)

Point A (during crushing) is at the intersection of the waterline and is therefore fixed only in the z-direction; it is not fixed in the x-direction.

Since the origin of the coordinate system is at the position G had just prior to initial contact (See Figure B-V), the vertical moment arm from G to A is

$$(G A)_z = H - K G + z \quad (H16)$$

where H is the initial draft and K G is the height of the center of gravity above the keel.

The horizontal moment arm from G to A, $(G A)_x$ is somewhat more complex since A is not absolutely fixed in the x-direction.

As can be seen in Figures B-XI and B-XII, the horizontal moment arm can be expressed as follows:

$$\begin{aligned} (G A)_x = & \left[\left(\frac{L}{2} - LCG \right) + \frac{z}{\tan i_B} \right] \\ & - \left[(H - KG) + z \right] \theta \\ & - \frac{1}{\tan (i_B + \theta)} \left[\left(\frac{L}{2} - LCG \right) + \frac{z}{\tan i_B} \right] \theta \end{aligned} \quad (H18)$$

Linearize $(G A)_x$. First, linearize $\frac{1}{\tan (i_B + \theta)}$.

PROBLEM 10.10

Let $f(x) = x^2 + 2x + 1$. Find $f'(x)$ using the definition of the derivative.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Substituting $f(x) = x^2 + 2x + 1$ into the definition, we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 + 2(x+h) + 1 - (x^2 + 2x + 1)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 + 2x + 2h + 1 - x^2 - 2x - 1}{h}$$

Simplifying the numerator, we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{2xh + h^2 + 2h}{h}$$

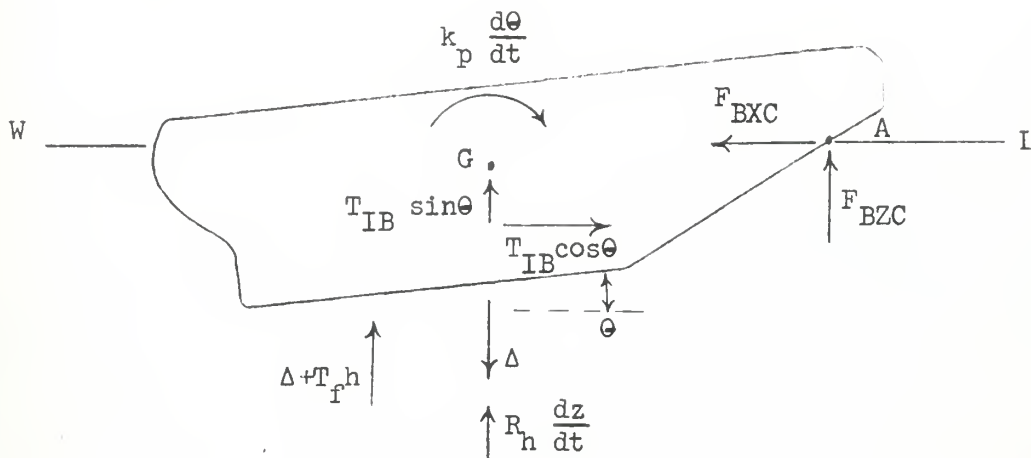
Factoring h out of the numerator, we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{h(2x + h + 2)}{h}$$

$$= \lim_{h \rightarrow 0} (2x + h + 2)$$

$$= 2x + 0 + 2 = 2x + 2$$

Figure B-IX
Free Body Diagram During Crushing Phase



$h \equiv$ increase of draft at LCF

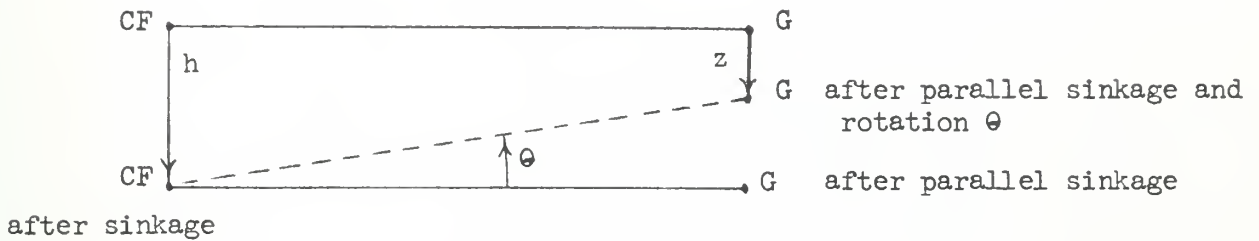
$k_p \equiv$ coefficient of pitch damping

$k_h \equiv$ coefficient of Leave damping

$T_{IB} =$ thrust available against ice

$T_f =$ pounds per foot immersion

Figure B-X
Relationship of h to z and θ



CF = center of flotation on original ship waterline

G = center of gravity

Step 1. Sink ship h in parallel fashion.

Step 2. Trim about CF (which does not effect buoyance magnitude).

Note that LCF and/or LCG are negative if they are aft of amidships.

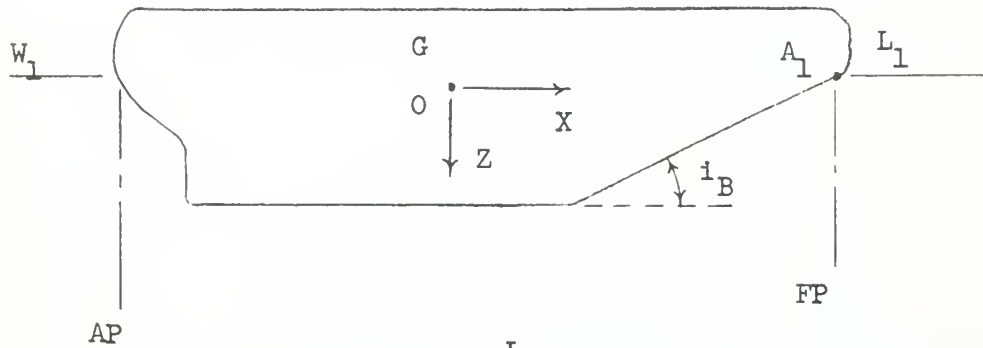
Therefore the radius of rotation is $(LCG-LCF)$.

$$z = h - (LCG-LCF)\theta$$

or

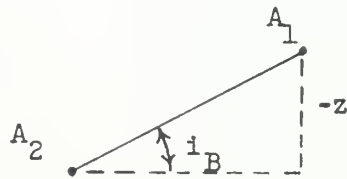
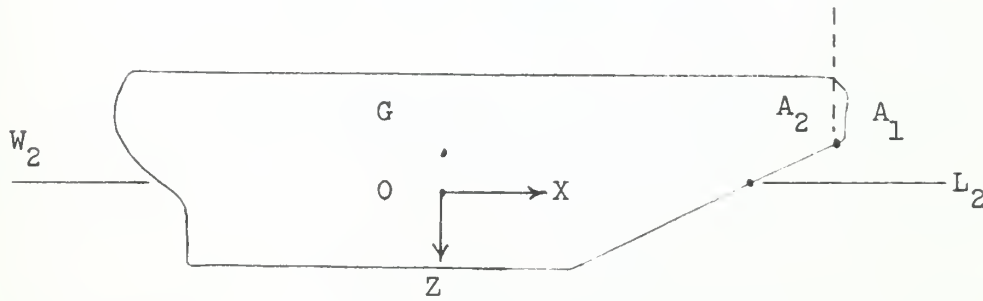
$$h = z + (LCG-LCF)\theta \quad (B17)$$

Figure B-XI
 Change of $(GA)_x$ Caused by Change in z



Original $(GA)_x = \left(\frac{L}{2} - LCG\right)$

Raise ship (-z)

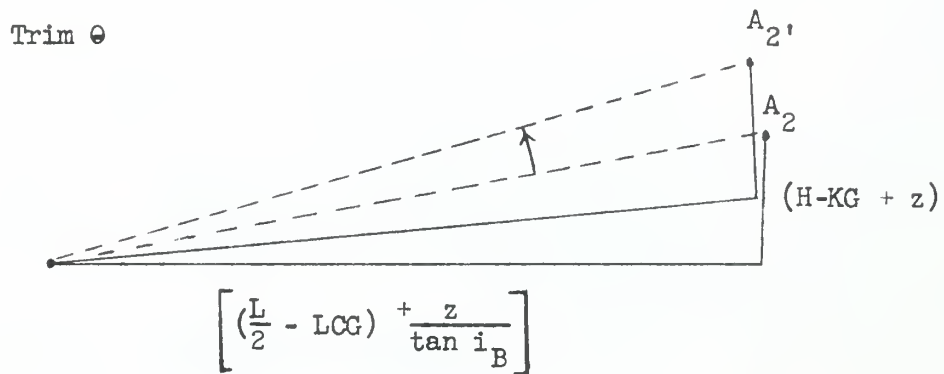


with ship raised,

$$(GA)_x = \left(\frac{L}{2} - LCG\right) - \frac{(-z)}{\tan i_B}$$

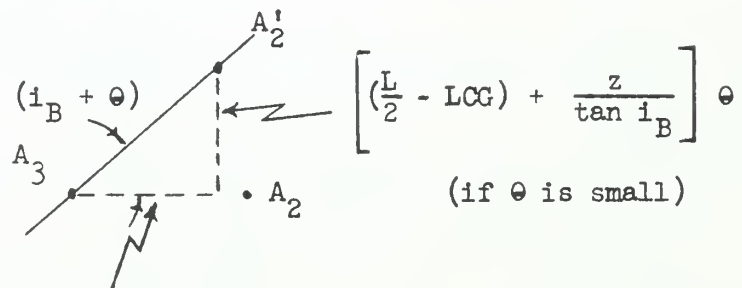
Figure B-XIII

Change of $(GA)_x$ Caused by Change of Trim After Change of z .



$$(GA)_x = \left[\left(\frac{L}{2} - LCG \right) + \frac{z}{\tan i_B} \right] \cos \theta - (H-KG + z) \sin \theta$$

to 2'



$$\frac{1}{\tan (i_B + \theta)} \left[\left(\frac{L}{2} - LCG \right) + \frac{z}{\tan i_B} \right] \theta$$

$$\tan (i_B + \theta) = \frac{\sin i_B + \theta \cos i_B}{\cos i_B - \theta \sin i_B}$$

Set $\sin i_B = c$ and $\cos i_B = e$

$$\text{Then } \frac{1}{\tan (i_B + \theta)} = \frac{e - \theta c}{c + \theta e} = \frac{e}{c} \left(\frac{1 - \frac{c}{e} \theta}{1 + \frac{e}{c} \theta} \right)$$

$$\frac{1}{\tan (i_B + \theta)} = \frac{e}{c} \left[1 - \left(\frac{c}{e} + \frac{e}{c} \right) \theta + \frac{e}{c} \left(\frac{c}{e} + \frac{e}{c} \right) \theta^2 - \dots \right]$$

The term containing θ^2 is negligible and therefore

$$\frac{1}{\tan (i_B + \theta)} = \frac{\cos i_B}{\sin i_B} \left[1 - \left(\frac{\sin i_B}{\cos i_B} + \frac{\cos i_B}{\sin i_B} \right) \theta \right]$$

$$\frac{1}{\tan (i_B + \theta)} = \frac{1}{\tan i_B} - \theta - \frac{\theta}{\tan^2 i_B}$$

$$\frac{1}{\tan (i_B + \theta)} = \frac{1}{\tan i_B} - \left(1 + \frac{1}{\tan^2 i_B} \right) \theta$$

$$\frac{1}{\tan (i_B + \theta)} = \frac{1}{\tan i_B} - \frac{\theta}{\sin^2 i_B} \tag{E19}$$

Substitute equation (E18) into (E19) and expand the equation.

$$\begin{aligned} (GA)_x &= \left(\frac{L}{2} - LCG \right) + \frac{z}{\tan i_B} - (H - KG) \theta - z \theta \\ &- \frac{\left(\frac{L}{2} - LCG \right) \theta}{\tan i_B} - \frac{z \theta}{\tan^2 i_B} + \frac{\left(\frac{L}{2} - LCG \right) \theta^2}{\sin^2 i_B} \\ &+ \frac{z \theta^2}{\tan i_B \sin^2 i_B} \end{aligned}$$

$$\frac{1}{x^2} \frac{d}{dx} \left(\frac{1}{\sqrt{1-x^2}} \right) = \frac{1}{1-x^2} \frac{d}{dx} \left(\frac{1}{\sqrt{1-x^2}} \right)$$

Let $u = \frac{1}{\sqrt{1-x^2}}$ then $\frac{d}{dx} \left(\frac{1}{\sqrt{1-x^2}} \right) = \frac{d}{dx} u$

$$\frac{1}{x^2} \frac{d}{dx} \left(\frac{1}{\sqrt{1-x^2}} \right) = \frac{1}{x^2} \frac{d}{dx} u = \frac{1}{x^2} \frac{du}{dx}$$

$$\left[\frac{1}{x^2} \frac{d}{dx} \left(\frac{1}{\sqrt{1-x^2}} \right) - \frac{1}{x^2} \frac{d}{dx} u \right] \frac{1}{x^2} = \frac{1}{x^2} \frac{d}{dx} \left(\frac{1}{\sqrt{1-x^2}} \right)$$

Therefore, the value of $\frac{1}{x^2} \frac{d}{dx} \left(\frac{1}{\sqrt{1-x^2}} \right)$ is

$$\left[\frac{1}{x^2} \frac{d}{dx} \left(\frac{1}{\sqrt{1-x^2}} \right) - \frac{1}{x^2} \frac{d}{dx} u \right] \frac{1}{x^2} = \frac{1}{x^2} \frac{d}{dx} \left(\frac{1}{\sqrt{1-x^2}} \right)$$

$$\frac{1}{x^2} \frac{d}{dx} \left(\frac{1}{\sqrt{1-x^2}} \right) = \frac{1}{x^2} \frac{d}{dx} u = \frac{1}{x^2} \frac{du}{dx}$$

$$\left(\frac{1}{x^2} \frac{d}{dx} \left(\frac{1}{\sqrt{1-x^2}} \right) + 1 \right) = \frac{1}{x^2} \frac{d}{dx} u = \frac{1}{x^2} \frac{du}{dx}$$

(iii)

$$\frac{1}{x^2} \frac{d}{dx} \left(\frac{1}{\sqrt{1-x^2}} \right) = \frac{1}{x^2} \frac{d}{dx} u = \frac{1}{x^2} \frac{du}{dx}$$

Therefore, the value of $\frac{1}{x^2} \frac{d}{dx} \left(\frac{1}{\sqrt{1-x^2}} \right)$ is

$$\frac{1}{x^2} \frac{d}{dx} \left(\frac{1}{\sqrt{1-x^2}} \right) = \frac{1}{x^2} \frac{d}{dx} u = \frac{1}{x^2} \frac{du}{dx} \quad (10)$$

$$\frac{1}{x^2} \frac{d}{dx} \left(\frac{1}{\sqrt{1-x^2}} \right) + \frac{1}{x^2} \frac{d}{dx} u = \frac{1}{x^2} \frac{d}{dx} \left(\frac{1}{\sqrt{1-x^2}} \right)$$

$$\frac{1}{x^2} \frac{d}{dx} \left(\frac{1}{\sqrt{1-x^2}} \right)$$

Considering the non-linear terms as negligible,

$$(GA)_x = \left(\frac{L}{2} - LCG\right) - \left[(H-KG) + \frac{\left(\frac{L}{2} - LCG\right)}{\tan i_B} \right] \theta + \left[\frac{z}{\tan i_B} \right] z \quad (B20)$$

The buoyance force, $(\Delta + T_F h)$, acts upward through the center of buoyance. When θ is positive B is aft of G by a distance of $GM_L \theta$, where GM_L is the height of the longitudinal metacenter above the center of gravity.

Icebreaking Thrust

It is assumed that the thrust available for icebreaking acts parallel to the base line at a height of (d) above the keel. Therefore, the lever arm for the $(T_{IB} \cos \theta)$ term is $(KG - d)$. See Figure B-IX.

Thrust just prior to impact is utilized in overcoming "non-ice" resistance. At State 0,

$$T(1 - t) = R_T \quad (B21)$$

where t is the thrust deduction factor, R_T is the total "non-ice" resistance, and T is the propeller thrust.

The thrust available for icebreaking, T_{IB} , may be defined as follows:

$$T_{IB} = T(1 - t) - R_T \quad (B22)$$

It is noted that

$$R_T = (C_r + C_f) \rho / 2 S v^2 \quad (B23)$$

where

C_r = Coefficient of residual resistance

C_f = Coefficient of frictional resistance

$$\left[\frac{1}{\sqrt{1-\beta^2}} - \frac{\beta}{\sqrt{1-\beta^2}} \right] = \dots \quad (12)$$

$$\dots \left[\frac{1}{\sqrt{1-\beta^2}} \right] \quad (13)$$

The ... through the ... of ...

... ..

It is ... the ... of ...

$$\dots = \dots \quad (14)$$

where ... is the ... of ...

$$\dots = \dots \quad (15)$$

It is ... that

$$\dots = \dots \quad (16)$$

where ... = coefficient of ...

ρ = Density of water (constant)

S = Wetted surface area (constant)

v = Ship velocity

At low $\frac{V}{\sqrt{L}}$ values (below 0.5), C_r may be considered constant. (31)

Gebers indicates

$$C_r = 0.02058 \left(\frac{\sqrt{L}}{v}\right)^{-\frac{1}{8}}$$

while Prandtl and Von Karman indicate

$$C_r = 0.072 \left(\frac{\sqrt{L}}{v}\right)^{-\frac{1}{5}}$$

where L = Length of ship (constant)

v = Kinematic viscosity (constant)

It may be seen that

$$R_T = K_1 v^2 + K_2 v^{(2 - \frac{1}{8})}$$

(Using Gebers' equation). The first term is for residual resistance while the second is for frictional resistance.

$$R_T = K_1 v^2 + K_2 v^{15/16}$$

This equation may be written in the following approximate form:

$$R_T \approx K_3 v^2 \tag{B24}$$

The total resistance (including ice) which may be opposed is

$$T (i - t)$$

The thrust deduction factor is virtually independent of v.

$$T = K_T \rho n^2 D^4$$

Consider the vector \mathbf{v}

$$\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

$$v_1^2 + v_2^2 + v_3^2 = 1$$

Let \mathbf{u} be a unit vector in the xy -plane, then $\mathbf{u} = \begin{pmatrix} \cos \theta \\ \sin \theta \\ 0 \end{pmatrix}$

where θ is the angle

$$\mathbf{u} \cdot \mathbf{v} = \cos \theta$$

Since \mathbf{u} and \mathbf{v} are both unit vectors

$$|\mathbf{u} - \mathbf{v}|^2 = 2 - 2 \cos \theta$$

where $|\mathbf{u} - \mathbf{v}|$ is the length of the chord

of the circle of radius 1

It can be seen that

$$|\mathbf{u} - \mathbf{v}|^2 = 2 - 2 \cos \theta$$

(Using vectors, equation (1) can be used to find the angle θ)

the angle θ is the angle between

$$\mathbf{u} \cdot \mathbf{v} = \cos \theta$$

This equation may be written in the following form:

$$(1) \quad \cos \theta = \mathbf{u} \cdot \mathbf{v}$$

The angle between \mathbf{u} and \mathbf{v} is θ and we are given that

$$\cos \theta = \frac{1}{2}$$

The three vectors \mathbf{u} , \mathbf{v} and \mathbf{w} are

$$\mathbf{u} = \begin{pmatrix} \cos \theta \\ \sin \theta \\ 0 \end{pmatrix}, \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}, \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix}$$

where n = propeller revolutions per second

Q = propeller diameter (constant)

It will be assumed that n remains constant throughout the crushing phase (and the sliding phase). (See Figure B XIII).

Using the Taylor wake fraction, a relationship between v_o (approach velocity to propeller) and v (ship velocity) may be set up (31).

$$v_o = v(1 - w)$$

where w = Taylor Wake Fraction

The wake fraction is virtually independent of ship velocity over most of the range.

As is illustrated in Figure XIV the thrust varies approximately linearly with ship velocity. Furthermore, since t is virtually constant,

$$T(1 - t) = K_4 - K_5 v \quad (B25)$$

It is to be noted that $T(1 - t)$ at a ship velocity equal to zero this force is commonly known as "bollard pull", T_{BOL} .

Therefore, in equation (B25)

$$K_4 = T_{BOL}$$

and equation (B25) becomes

$$T(1 - t) = T_{BOL} - K_5 v \quad (B26)$$

where v_1 = velocity of ship in the x-direction at State 1, equation (B26) becomes

$$T(1 - t) = T_{BOL} - K_5 v_1$$

and from equation (B21) and equation (B24),

Consider the function $f(x) = x^2 - 2x + 1$.

$$f(x) = x^2 - 2x + 1 = (x-1)^2$$

It is clear that $f(x) \geq 0$ for all x .

Therefore, $f(x) \geq 0$ for all x .

Since $f(x) = (x-1)^2$, we have $f(x) \geq 0$.

Thus, $f(x) \geq 0$ for all x .

$$f(x) = (x-1)^2$$

Therefore, $f(x) \geq 0$.

Since $f(x) = (x-1)^2$, we have $f(x) \geq 0$.

Thus, $f(x) \geq 0$.

As a result, $f(x) \geq 0$ for all x .

Therefore, $f(x) \geq 0$ for all x .

$$(1) \quad f(x) = (x-1)^2$$

It is clear that $f(x) \geq 0$ for all x .

Thus, $f(x) \geq 0$ for all x .

Therefore, $f(x) \geq 0$.

$$f(x) = (x-1)^2$$

Thus, $f(x) \geq 0$.

$$(2) \quad f(x) = (x-1)^2$$

Since $f(x) = (x-1)^2$, we have $f(x) \geq 0$.

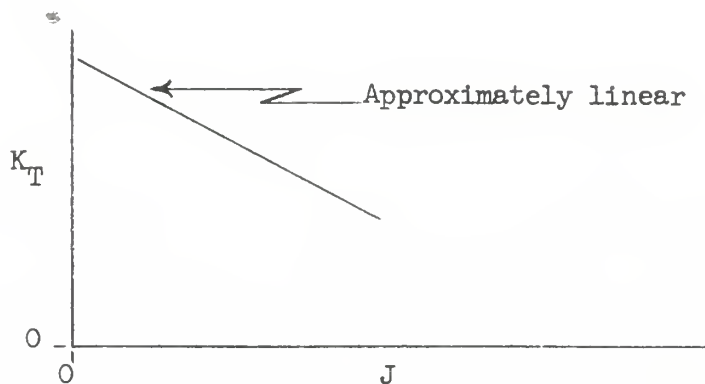
Thus, $f(x) \geq 0$.

$$f(x) = (x-1)^2$$

Therefore, $f(x) \geq 0$.

Figure B-XIII

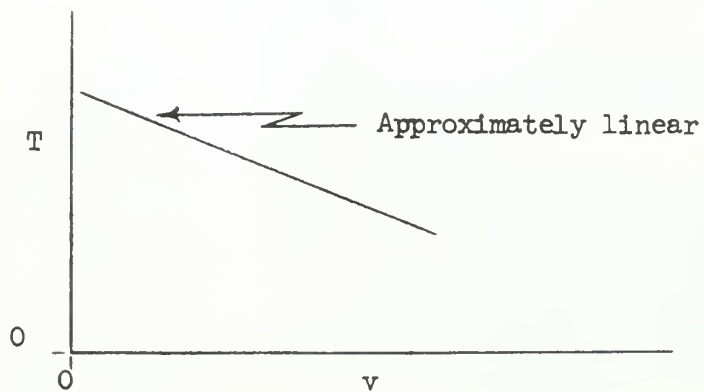
Illustration of Propeller Design Chart (31)
Showing Variation of K_T with J .



where $J = \frac{v_0}{n d}$ (v_0 = approach velocity to propeller)

Figure B-XIV

Variation of T with Ship Velocity



(n = constant)

$$R_T = T (1 - t)$$

$$K_3 v_1^2 = T_{BOL} - K_5 v_1$$

$$K_3 = \frac{T_{BOL} - K_5 v_1}{v_1^2}$$

From equation (B22)

$$T_{IB} = (T_{BOL} - K_5 v) - \left(\frac{T_{BOL} - K_5 v_1}{v_1^2} \right) v^2$$

$$T_{IB} = T_{BOL} - [K_5] v - \left[\frac{T_{BOL} - K_5 v_1}{v_1^2} \right] v^2 \quad (B27)$$

Figure XV shows illustrative plots of equations (B24) and (B26).

Figure XVI illustrates equation (B27).

Note that equation has an unknown constant, K_5 . This can be solved for only by knowing the thrust and resistance characteristics for a wide range of impact speeds. These could only be known if other variables (i.e., ρ , n , d , S , v , C_r , C_f etc.) were known and introduced. Furthermore, the equation is non-linear so it could not be used in linear differential equations even if K_5 were known.

In the crushing phase, a good linear approximation could be made by determining the slope of the curve at v_1 . However, the slope is also a function of K_5 so in spite of the fact this would lead to a linear equation it would be unduly complex.

The next best approximation would be the one illustrated in Figure B-XVI.

$$f(x) = \frac{1}{2} x^2 + \dots$$

$$\frac{d^2 y}{dx^2} + \dots = \dots$$

(100) equation

$$y'' + \dots = \dots$$

(101)

$$y'' + \dots = \dots$$

Figure 10 shows alternative plots of equation (100) and (101).

Figure 101 illustrates equation (101).

For each equation an arbitrary constant, C_1 , will not be defined for only by knowing the initial and boundary conditions for a wide range of these species. These could only be known if other variables

$$(102) \quad y = C_1 e^{ax} + C_2 e^{-ax} + \dots$$

Furthermore, the equation is nonlinear so it could not be used in linear differential equations over its \mathbb{R} range.

In the central portion, a good linear approximation could be made by determining the slope of the curve at x_0 . However, the slope is also a function of x_0 so in spite of the fact this would lead to a linear equation it would be highly complex.

The next best approximation would be the one illustrated in Figure 10-101.

Figure B-XV
Thrust and Resistance Forces vs Ship Speed

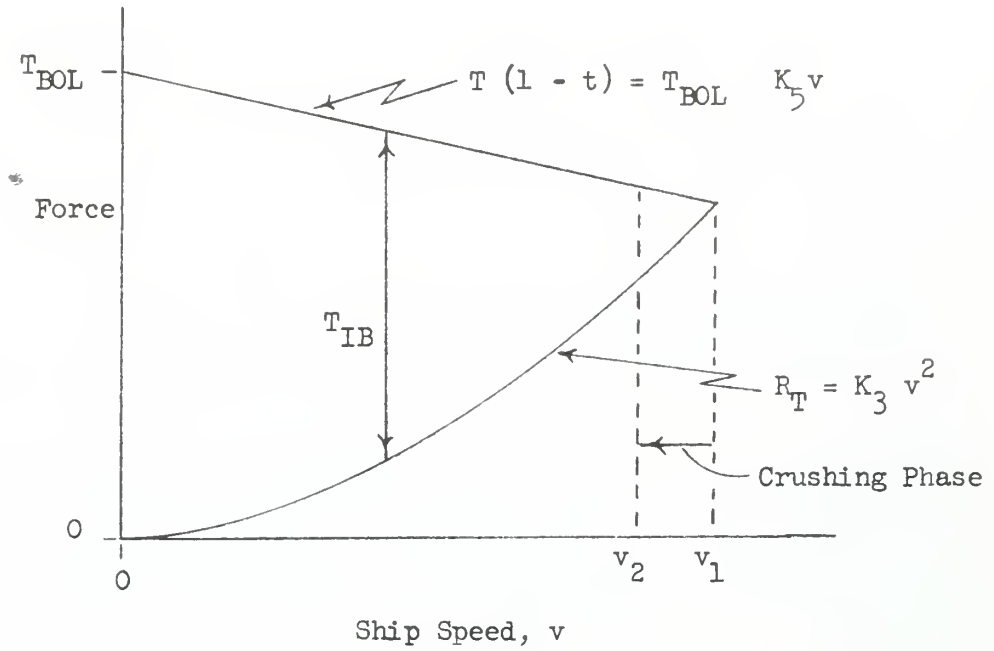
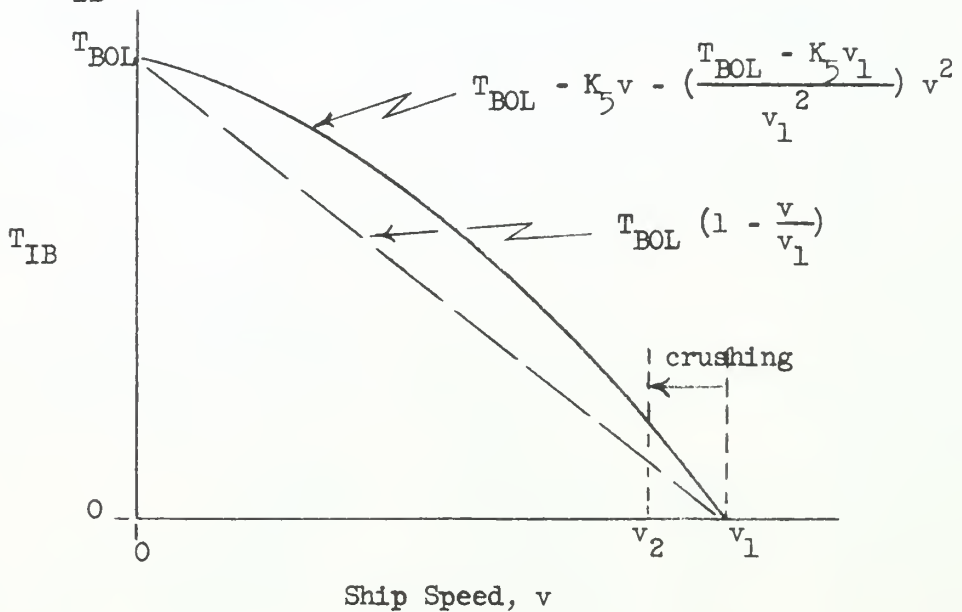


Figure B-XVI
 T_{IB} , Icebreaking Thrust vs Ship Speed



$$T_{IB} = T_{BOL} \left(1 - \frac{v}{v_1}\right) \quad (B28)$$

$$\text{where } v = \frac{dx}{dt}$$

Incidentally, this approximation will be valid during the sliding phase as well as the crushing phase.

It should be reiterated that T_{BOL} is the "bollard pull" generated by using the same rpm that is necessary to maintain v_1 in open water.

Newton's Laws of Motion During Crushing

Newton's Laws of Motion may now be applied for the crushing phase rotationally about the center of gravity, in the x-direction, and in the z-direction.

In the horizontal direction (see Figure B-IX)

$$\sum F_x = m_x \frac{d^2x}{dt^2}$$

where $m =$ Mass of ship plus "virtual" mass in the x-direction

$$T_{IB} \cos \theta - F_{BXC} = m_x \frac{d^2x}{dt^2}$$

Setting $\cos \theta = 1$ and substituting equations (B15) and (B28) this becomes

$$T_{BOL} \left(1 - \frac{v}{v_1}\right) - \left[\frac{\sigma \tan i_B}{\sin \beta} \sin i_B (\cos \beta + f_k \sin \beta) + \frac{\sigma \tan i_B}{\sin \beta} f_k \cos i_B \right] x^2 - m \frac{d^2x}{dt^2} = 0$$

$$- m \frac{d^2x}{dt^2} - \frac{T_{BOL}}{v_1} \frac{dx}{dt} - \frac{\sigma \tan i_B}{\sin \beta} \left[(\cos \beta + f_k \sin \beta) \sin i_B + f_k \cos i_B \right] x^2 + T_{BOL} = 0$$

Set

$$a_1 = \frac{T_{BOL}}{m_x v_1}$$

$$a_2 = \left[\frac{\sigma \tan i_B}{m_x \sin \beta} (\cos \beta + f_k \sin \beta) \sin i_B + f_k \cos i_B \right]$$

$$a_3 = \frac{-T_{BOL}}{m_x}$$

$$\frac{d^2 x}{dt^2} + a_1 \frac{dx}{dt} + a_2 x^2 + a_3 = 0 \quad (B29)$$

Note that equation (B29) is independent of z and θ and can therefore be solved as an independent equation.

$$\text{At } t_c = 0 \quad x_1 = 0 \quad \left(\frac{dx}{dt}\right)_1 = v_1 \quad \left(\frac{d^2 x}{dt^2}\right)_1 = 0$$

Note that the solution of equation (B29) is a function of t. As a consequence the solutions to equations (B14) and (B15), F_{BZC} and V_{EXC} respectively are functions of t.

$$F_{BZC} = f(t) \quad F_{EXC} = f(t) \quad (B30)$$

It can be seen in Figure B-IX in the downward vertical direction (z-direction) that

$$F_z = m_z \frac{d^2 z}{dt^2}$$

$$\Delta - k_h \frac{dz}{dt} - (\Delta + T_f h) - T_{IB} \sin \theta - F_{BZC}$$

$$= m_z \frac{d^2 z}{dt^2}$$

$$\left[\frac{1}{2} \ln x^2 + \frac{1}{2} \ln (x^2 + 4) \right] = \frac{1}{2} \ln (x^2 + 4) + \frac{1}{2} \ln x^2$$

$$(20) \quad \frac{1}{2} \ln x^2 + \frac{1}{2} \ln (x^2 + 4) = \frac{1}{2} \ln (x^2 + 4) + \frac{1}{2} \ln x^2$$

Since the function (20) is independent of x and y and therefore its value is a constant expression.

$$C = \frac{1}{2} \ln \left(\frac{x^2 + 4}{x^2} \right) \quad C = \frac{1}{2} \ln \left(\frac{x^2 + 4}{x^2} \right) \quad C = \frac{1}{2} \ln \left(\frac{x^2 + 4}{x^2} \right) \quad C = \frac{1}{2} \ln \left(\frac{x^2 + 4}{x^2} \right)$$

Thus the solution of equation (20) is a function of x and y. The solution of equation (20) is a function of x and y. The solution of equation (20) is a function of x and y. The solution of equation (20) is a function of x and y.

$$(21) \quad \frac{1}{2} \ln x^2 + \frac{1}{2} \ln (x^2 + 4) = \frac{1}{2} \ln (x^2 + 4) + \frac{1}{2} \ln x^2$$

It can be seen from (21) that the solution of equation (21) is a function of x and y. The solution of equation (21) is a function of x and y. The solution of equation (21) is a function of x and y.

$$\frac{1}{2} \ln x^2 + \frac{1}{2} \ln (x^2 + 4) = \frac{1}{2} \ln (x^2 + 4) + \frac{1}{2} \ln x^2$$

$$\frac{1}{2} \ln x^2 + \frac{1}{2} \ln (x^2 + 4) = \frac{1}{2} \ln (x^2 + 4) + \frac{1}{2} \ln x^2$$

$$\frac{1}{2} \ln x^2 + \frac{1}{2} \ln (x^2 + 4) = \frac{1}{2} \ln (x^2 + 4) + \frac{1}{2} \ln x^2$$

Making appropriate substitutions from equations (E17), (E14), and (B28) this becomes

$$\begin{aligned}
 & -k_h \frac{dz}{dt} - t_{fz} - T_f (LCG-LCF)\theta - T_{BOL} \theta + \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt}\right) \theta \\
 & - \left[\frac{\sigma \tan i_B}{\sin \beta} \cos i_B (\cos \beta + f_k \sin \beta) - \frac{\sigma \tan i_B}{\sin \beta} f_k \sin i_B \right] x^2 \\
 & - m_z \frac{d^2 z}{dt^2} = 0 \\
 & \left[-m_z \right] \frac{d^2 z}{dt^2} + \left[-k_h \right] \frac{dz}{dt} + \left[-T_f \right] z \\
 & + \left[-T_f (LCG - LCF) - T_{BOL} + \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt}\right) \right] \theta \\
 & - \left[\cos i_B (\cos \beta + f_k \sin \beta) - f_k \sin i_B \right] \frac{\sigma \tan i_B}{\sin \beta} x^2 = 0 \quad (B31)
 \end{aligned}$$

It can be seen in Figure B-IX that the summation of moments about the center of gravity in the counterclockwise (+θ) direction may be expressed as follows:

$$\sum M = m_\theta k^2 \frac{d^2 \theta}{dt^2}$$

where k = radius of gyration.

$$F_{BZC}(GA)_x + F_{BXC}(GA)_z + T_{IB} \cos \theta (KG - d)$$

$$- (\Delta + T_{fh})GM_L \theta - k_p \frac{d\theta}{dt} - m_\theta k^2 \frac{d^2 \theta}{dt^2} = 0$$

Use (10) (11) and (12) to find the value of $\frac{d^2y}{dx^2}$ at $x=1$ (13)

$$\begin{aligned} \frac{d}{dx} \left(\frac{dy}{dx} \right) &= \frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{1}{x^2} \right) = -\frac{2}{x^3} = -\frac{2}{1^3} = -2 \end{aligned}$$

$$\frac{d^2y}{dx^2} = -\frac{2}{x^3}$$

$$\left. \frac{d^2y}{dx^2} \right|_{x=1} = -\frac{2}{1^3} = -2$$

Let $y = \frac{1}{x^2}$ then $\frac{dy}{dx} = -\frac{2}{x^3}$ and $\frac{d^2y}{dx^2} = \frac{6}{x^4}$. At $x=1$, $\frac{d^2y}{dx^2} = \frac{6}{1^4} = 6$.

$$\frac{d^2y}{dx^2} = \frac{6}{x^4}$$

where $x=1$, $\frac{d^2y}{dx^2} = 6$

$$(1 - 2x) = \frac{d}{dx} \left(\frac{1}{x^2} \right) = -\frac{2}{x^3}$$

$$0 = \frac{d}{dx} \left(\frac{1}{x^2} \right) = -\frac{2}{x^3} = -\frac{2}{1^3} = -2$$

Making appropriate substitutions from equations (B30), (E14), (B20), (E16), and (B28) this becomes

$$\begin{aligned}
 & F_{BZC} \left(\frac{L}{2} - LCG \right) \theta - F_{BZC} \left[(H-KG) + \frac{(L/2 - LCG)}{\tan i_B} \right] \theta \\
 & + F_{BZC} \left[\frac{1}{\tan i_B} \right] z + F_{BXC} [H-KG] + F_{BXC} [z] \\
 & + T_{BOL} (IG-d) - \left[\frac{T_{BOL}}{v_1} (KG-d) \right] \frac{dx}{dt} \\
 & - \left[\Delta GM_L \right] \theta - \left[T_f GM_L \right] \theta z - \left[T_f GM_L (LCG-LCF) \right] \theta^2 \\
 & - k_p \frac{d\theta}{dt} - m k^2 \frac{d^2\theta}{dt^2} = 0
 \end{aligned}$$

In this equation both terms which include T_f are multiplied by non-linear terms. As shown earlier, these particular non-linear terms are minute compared to the other terms in the equation and will be dropped. The equation may now be written as follows:

$$\begin{aligned}
 & \left[\frac{F_{BZC}}{\tan i_B} + F_{BXC} \right] z \\
 & + \left[-m k^2 \right] \frac{d^2\theta}{dt^2} \\
 & + \left[-k_p \right] \frac{d\theta}{dt} \\
 & + \left[-\Delta GM_L - F_{BZC} (H-KG) - F_{BZC} \frac{(L/2 - LCG)}{\tan i_B} \right] \theta
 \end{aligned}$$

(100) (101) (102) (103) (104) (105) (106) (107) (108) (109) (110)

(111) (112) (113) (114) (115) (116) (117) (118) (119) (120)

$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin(x) + C$$

$$\left[\frac{1}{2} x^2 - \left[\frac{1}{2} x^2 \right] \right] x^2 = \left[\frac{1}{2} x^2 \right] x^2 =$$

$$\frac{1}{2} \left[(1-x) \frac{dx}{x^2} \right] = (1-x) \frac{dx}{x^2}$$

$$E_0 \left[(1-x) \frac{dx}{x^2} \right] = \int \frac{1-x}{x^2} dx = \int \frac{1}{x^2} dx - \int \frac{x}{x^2} dx =$$

$$-\frac{1}{x} + \ln|x| + C$$

In this section we have seen how to integrate functions of the form $\frac{1}{\sqrt{1-x^2}}$. In the next section we will see how to integrate functions of the form $\frac{1}{\sqrt{a^2-x^2}}$. In the following section we will see how to integrate functions of the form $\frac{1}{\sqrt{x^2-a^2}}$. In the next section we will see how to integrate functions of the form $\frac{1}{\sqrt{x^2+a^2}}$. In the following section we will see how to integrate functions of the form $\frac{1}{\sqrt{1-x^2}}$.

$$\left[\frac{1}{2} x^2 - \left[\frac{1}{2} x^2 \right] \right] x^2 = \left[\frac{1}{2} x^2 \right] x^2 =$$

$$\frac{1}{2} \left[(1-x) \frac{dx}{x^2} \right] = (1-x) \frac{dx}{x^2}$$

$$\frac{1}{2} \left[\frac{1}{x^2} - \frac{x}{x^2} \right] dx = \frac{1}{2} \left[-\frac{1}{x} + \ln|x| \right] + C$$

$$= \left[\frac{(1-x) \ln|x|}{2} - \frac{1-x}{2x} \right] + C$$

$$+ \left[F_{BZC} \left(\frac{L}{2} - LCG \right) + F_{BXC} (H-KG) + T_{BOL} (KG-d) \right. \\ \left. - \frac{T_{BOL}}{v_1} (KG-d) \frac{dx}{dt} \right] = 0 \quad (B32)$$

Note that F_{BZC} , F_{BXC} , and $\frac{dx}{dt}$ are functions of t based on the solution of equation (B29).

Solution for x During Crushing

Many attempts, too many and too lengthy to be shown here, have been made to solve the non-linear summation equation (B29).

However, it would seem that an assumption concerning the second and third terms is in order. It is noted that these are

$$\frac{T_{BOL}}{m v_1} \frac{dx}{dt} \quad \text{and} \quad \frac{-T_{BOL}}{m}$$

and combined are

$$\frac{T_{BOL}}{m} \left(\frac{dx}{v_1} - 1 \right) .$$

This combined term is very small in the crushing range (See Figure B-XVI). In fact, it can readily be seen that the whole term is non-existent at initial contact.

Equation (B29) may now be written as

$$\begin{aligned} \sum F_x &= m \frac{d^2 x}{dt^2} \\ - F_{BXC} &= m \frac{d^2 x}{dt^2} \\ \text{Set } k_1 &= \left[\frac{\sigma \tan i_B}{\sin \beta} \sin i_B (\cos \beta + f_k \sin \beta) \right. \\ &\quad \left. + \frac{\sigma \tan i_B}{\sin \beta} f_k \cos i_B \right] \end{aligned} \tag{B33}$$

(See equation (B14))

QUESTION 1

Let $f(x) = x^2 + 2x + 1$ and $g(x) = x^2 - 2x + 1$.
 (a) Find $f(x) + g(x)$ and $f(x) - g(x)$.
 (b) Find the domain and range of $f(x)$ and $g(x)$.

$$f(x) + g(x) = (x^2 + 2x + 1) + (x^2 - 2x + 1) = 2x^2 + 2$$

and

$$f(x) - g(x) = (x^2 + 2x + 1) - (x^2 - 2x + 1) = 4x$$

(c) Find the domain and range of $f(x) + g(x)$ and $f(x) - g(x)$.
 (d) Find the domain and range of $f(x)$ and $g(x)$.

QUESTION 2

$$\frac{d}{dx} (x^2 + 2x + 1) = 2x + 2$$

$$\frac{d}{dx} (x^2 - 2x + 1) = 2x - 2$$

$$\left(\frac{d}{dx} (x^2 + 2x + 1) \right) - \left(\frac{d}{dx} (x^2 - 2x + 1) \right) = (2x + 2) - (2x - 2) = 4$$

(10)

$$\left[\frac{d}{dx} (x^2 + 2x + 1) - \frac{d}{dx} (x^2 - 2x + 1) \right] = 4$$

(10)

$$\text{Then } F_{\text{EXC}} = k_1 x^2 \quad (\text{B34})$$

$$-k_1 x^2 = m \frac{d^2 x}{dt^2}$$

$$\frac{d^2 x}{dt^2} = \frac{-k_1}{m} x^2 \quad (\text{B35})$$

Set $p = \dot{x} = \frac{dx}{dt}$ in accordance with reference (32)

$$\frac{d^2 x}{dt^2} = \dot{\dot{x}} = \frac{d}{dt} \left(\frac{dx}{dt} \right) = \frac{dp}{dt} = \frac{dp}{dx} \frac{dx}{dt} = p \frac{dp}{dx}$$

(Note that p = velocity of the center of gravity in the x -direction)

$$p \frac{dp}{dx} = \frac{-k_1}{m} x^2$$

$$\int_{v_1}^v p \, dp = \frac{-k_1}{m} \int_0^x x^2 \, dx$$

$$\left[\frac{p^2}{2} \right]_{v_1}^v = \frac{-k_1}{m} \left[\frac{x^3}{3} \right]_0^x$$

$$\frac{p^2}{2} - \frac{v_1^2}{2} = \frac{-k_1}{3m} x^3$$

$$\frac{p^2}{2} = \frac{v_1^2}{2} - \frac{k_1}{3m} x^3$$

$$p = \left(v_1^2 - \frac{2k_1}{3m} x^3 \right)^{1/2} = \frac{dx}{dt} \quad (\text{B36})$$

(103)

$$y'' + 2y' + 2y = 0$$

(104)

$$y'' + y = 0$$

(105) determine the constants of $y'' + 2y' + 2y = 0$

$$\frac{dy}{dx} = \frac{dy}{dx} + \frac{dy}{dx} = \left(\frac{dy}{dx}\right) \frac{1}{y} = \dots$$

(106) determine the constants of $y'' + y = 0$

$$y'' + y = 0$$

$$\int \left(\frac{dy}{dx} + y \right) dx$$

$$\int \left[\frac{dy}{dx} + y \right] dx = \int \frac{dy}{dx} dx + \int y dx$$

$$= y + \frac{y^2}{2} + C$$

$$= y + \frac{y^2}{2} + C$$

(107)

$$\frac{dy}{dx} = \dots$$

$$\int_0^x (v_1^2 - \frac{2k_1}{3m} x^3)^{-\frac{1}{2}} dx = \int_0^t dt$$

$$\int_0^x \frac{dx}{\sqrt{\frac{-2k_1}{3m} x^3 + v_1^2}} = t \tag{B37}$$

It is apparent that any exact solution to equation (B37) will be quite complex, unnecessarily complex.

Set $a = \frac{-2k_1}{3m}$ and $b = v_1^2$

temporarily.

Then the denominator of equation (B37) may be put into the numerator

as

$$f(x) = (a x^3 + b)^{-\frac{1}{2}}$$

This function may be expanded into a series using Maclaurin's Theorem (33).

$$f(x) = f(0) + f'(0) \frac{x}{1!} + f''(0) \frac{x^2}{2!} + f'''(0) \frac{x^3}{3!} + \dots$$

$$f(x) = (a x^3 + b)^{-\frac{1}{2}}$$

$$f'(x) = -\frac{1}{2}(a x^3 + b)^{-\frac{3}{2}} (3 a x^2)$$

$$f''(x) = -\frac{1}{2}(a x^3 + b)^{-\frac{3}{2}} (b a x) + \frac{3}{2} (a x^3 + b)^{-\frac{5}{2}} (3a x^2)(3 a x^2)$$

$$f(0) = b^{-1/2}$$

$$f'(0) = 0$$

$$f''(0) = 0$$

$$f'''(0) = 0 \text{ etc.}$$

(Note: Taylor's Theorem may be used with values where zero is not used but the result, in effect, is approximately the same but more cumbersome.)

$$\int_0^x \frac{1}{\sqrt{1-t^2}} dt = \arcsin t \Big|_0^x = \arcsin x$$

(20)

$$x = \frac{\sin y}{\sqrt{1-\sin^2 y}} = \frac{\sin y}{\cos y} = \tan y$$

If in equation (19) we make the substitution $y = \arcsin x$, then (20) will be true.

Therefore, we have

$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1-x^2}}$$

Q.E.D.

From the definition of equation (19) we get into the expression

$$\arcsin(x) = \arcsin(x) + 0$$

The function $f(x)$ is extended into a series using Taylor's Theorem (21)

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \dots$$

$$\arcsin(x) = \arcsin(0) + \frac{1}{\sqrt{1-0^2}}x + \dots$$

$$\arcsin(x) = 0 + x + \dots$$

$$\arcsin(x) = x + \frac{1}{6}x^3 + \dots$$

$$f(0) = 0$$

$$f'(0) = 1$$

$$f''(0) = 0$$

$$f'''(0) = 0 \text{ etc.}$$

(Note: Taylor's Theorem may be used with values other than 0 in order to use the result in other cases.)

Equation (B37) now becomes

$$\frac{1}{\sqrt{b}} \int_0^x dx = t$$

$$\frac{x}{v_1} = t \quad x = v_1 t \quad (B38)$$

In effect this states that velocity is approximated as constant. This value for x may now be substituted into equations (B35) and (B36) giving the following equations:

$$\frac{d^2 x}{dt^2} = \left(\frac{-k_1}{m_x} \right) x^2 = \frac{-k_1}{m_x} v_1^2 t^2 \quad (B39)$$

$$\frac{dx}{dt} = \left(v_1^2 - \frac{2k_1}{3 m_x} x^3 \right)^{1/2} = \left(v_1^2 - \frac{2k_1}{3 m_x} v_1^3 t^3 \right)^{1/2} \quad (B40)$$

$$x = v_1 t$$

It is advisable to check the validity of equation (B38) by substituting appropriate values into equation (B38) and (B39) and seeing if $\frac{dx}{dt}$ drops off excessively, too excessively to use equation (B38) for x .

Assume the following approximate values (14):

$$m = \frac{17.28 \times 10^6}{32.2} = 5 \times 10^5 \frac{\text{lb sec}^2}{\text{ft}}$$

$$v_1 \approx 10 \text{ ft/sec}$$

$$\sigma \approx 4 \times 10^4 \text{ psf (20 kg/cm}^2\text{)}$$

Equation (10) can be written

$$y'' + p(x)y' + q(x)y = r(x)$$

(11) $y'' + p(x)y' + q(x)y = r(x)$

Let y_1 and y_2 be two linearly independent solutions of the homogeneous equation (11). Then the general solution of (11) can be written as

(12) $y = C_1 y_1 + C_2 y_2 + y_p$

(13) $y'' + p(x)y' + q(x)y = r(x)$

$y_p = \dots$

It is not possible to find the values of C_1 and C_2 by substituting the values of y_1 and y_2 into equation (11) and solving for C_1 and C_2 . However, the values of C_1 and C_2 can be determined by using the initial conditions.

$$\frac{d}{dx} (C_1 y_1 + C_2 y_2 + y_p) = \dots$$

\dots

\dots

$$\begin{aligned} i_B = 30^\circ & \quad \tan i_B \approx 0.6 & \quad \sin i_B = 0.5 & \quad \cos i_B \approx 0.9 \\ \beta = 40^\circ & & \quad \sin \beta = 0.6 & \quad \cos \beta \approx 0.8 \\ \frac{c}{k} = 0.2 & \quad x \approx 10 \text{ ft} \end{aligned}$$

From equation (B33),

$$k_1 = \frac{(4 \times 10^4)(0.6)}{(0.6)} \left[(0.5)(0.8 + 0.12) + (0.18) \right]$$

$$k_1 = (4 \times 10^4) (0.46 + 0.18) = (4 \times 10^4) (0.64)$$

$$k_1 \approx 2.6 \times 10^4 \text{ lb/ft}^2$$

$$\frac{k_1}{m} \approx \frac{2.6 \times 10^4}{5 \times 10^5} \approx 5 \times 10^{-2}$$

$$-\frac{2}{3} \frac{k_1}{m} v_1^3 t^3 = -\left(\frac{2}{3}\right)(5 \times 10^{-2})(10^3)(1) = -33$$

$$\left(\frac{dx}{dt}\right)_{x=10'} = (100 - 33)^{1/2} = (66)^{1/2} = 8.1 \text{ ft/sec}$$

If the velocity decay had been linear this would imply that the distance travelled in 1 sec would have been

$$\left(\frac{10 + 8.1}{2}\right) = 9.05 \text{ ft/sec}$$

$$(9.05)(1) = 9.05 \text{ ft}$$

instead of 10 ft. (Since acceleration is increasingly negative, the velocity decay would not have been linear and the distance travelled in 1 sec would have been even closer to the 10 ft we started with.)

Using a series expansion it is possible to expand $f(x)$ of the integral of equation (B37) remembering that there exists the condition

$$\left(\frac{a}{b}\right)^2 x^6 < 1$$

$$f(x) = b^{-1/2} \left[1 - \frac{1}{2} \left(\frac{a}{b}\right) x^3 + \frac{1.3}{2.4} \left(\frac{a}{b}\right)^2 x^6 - \frac{1.3.5}{2.4.6} \left(\frac{a}{b}\right)^3 x^9 + \dots \right]$$

$$t = \int_0^x f(x) dx$$

$$t = \frac{1}{b^{1/2}} \int_0^x \left[1 - \frac{1}{2} \left(\frac{a}{b}\right) x^3 + \frac{3}{8} \left(\frac{a}{b}\right)^2 x^6 - \frac{5}{8} \left(\frac{a}{b}\right)^3 x^9 \right] dx$$

$$t = \frac{1}{b^{1/2}} \left[x - \frac{1}{8} \left(\frac{a}{b}\right) x^4 + \frac{3}{56} \left(\frac{a}{b}\right)^2 x^7 - \frac{1}{16} \left(\frac{a}{b}\right)^3 x^{10} \right]$$

$$a = \frac{-2k_1}{3 m_2} \quad b = v_1^2$$

$$a = \frac{-2(2.6 \times 10^4)}{3(5 \times 10^5)} = -3.47 \times 10^{-2}$$

$$b = 10^2$$

Assume $x = 4$ ft

$$\left(\frac{a}{b}\right) = -3.47 \times 10^{-4} \quad x^4 = 2.58 \times 10^2$$

$$\left(\frac{a}{b}\right)^2 = +1.205 \times 10^{-7} \quad x^7 = 1.65 \times 10^4$$

$$\left(\frac{a}{b}\right)^3 = -4.18 \times 10^{-11} \quad x^{10} = 1.02 \times 10^6$$

$$-\frac{1}{8} \left(\frac{a}{b}\right) x^4 = +1.12 \times 10^{-2}$$

Suppose that (1) is a group of order n and (2) is a group of order m .

Let G be the direct product of (1) and (2) . Then G is a group of order nm .

$$G \cong (1) \times (2)$$

$$\left[\begin{array}{l} \text{Let } (1) = \langle a \rangle \text{ and } (2) = \langle b \rangle. \text{ Then } G = \langle a, b \rangle. \\ \text{Since } a^m = 1 \text{ and } b^n = 1, \text{ we have } a^m b^n = 1. \\ \text{Thus } a^m = b^{-n}. \end{array} \right] \text{ (1)}$$

$$\text{Let } (1) = \langle a \rangle \text{ and } (2) = \langle b \rangle. \text{ Then } G = \langle a, b \rangle. \\ \text{Since } a^m = 1 \text{ and } b^n = 1, \text{ we have } a^m b^n = 1. \\ \text{Thus } a^m = b^{-n}.$$

$$\left[\begin{array}{l} \text{Let } (1) = \langle a \rangle \text{ and } (2) = \langle b \rangle. \text{ Then } G = \langle a, b \rangle. \\ \text{Since } a^m = 1 \text{ and } b^n = 1, \text{ we have } a^m b^n = 1. \\ \text{Thus } a^m = b^{-n}. \end{array} \right] \text{ (2)}$$

$$\text{Let } (1) = \langle a \rangle \text{ and } (2) = \langle b \rangle. \text{ Then } G = \langle a, b \rangle. \\ \text{Since } a^m = 1 \text{ and } b^n = 1, \text{ we have } a^m b^n = 1. \\ \text{Thus } a^m = b^{-n}.$$

$$\text{Let } (1) = \langle a \rangle \text{ and } (2) = \langle b \rangle. \text{ Then } G = \langle a, b \rangle.$$

Let $x = a^i b^j$.

$$\text{Let } (1) = \langle a \rangle \text{ and } (2) = \langle b \rangle. \text{ Then } G = \langle a, b \rangle. \\ \text{Since } a^m = 1 \text{ and } b^n = 1, \text{ we have } a^m b^n = 1. \\ \text{Thus } a^m = b^{-n}.$$

$$\text{Let } (1) = \langle a \rangle \text{ and } (2) = \langle b \rangle. \text{ Then } G = \langle a, b \rangle. \\ \text{Since } a^m = 1 \text{ and } b^n = 1, \text{ we have } a^m b^n = 1. \\ \text{Thus } a^m = b^{-n}.$$

$$\text{Let } (1) = \langle a \rangle \text{ and } (2) = \langle b \rangle. \text{ Then } G = \langle a, b \rangle. \\ \text{Since } a^m = 1 \text{ and } b^n = 1, \text{ we have } a^m b^n = 1. \\ \text{Thus } a^m = b^{-n}.$$

$$\text{Let } (1) = \langle a \rangle \text{ and } (2) = \langle b \rangle. \text{ Then } G = \langle a, b \rangle. \\ \text{Since } a^m = 1 \text{ and } b^n = 1, \text{ we have } a^m b^n = 1. \\ \text{Thus } a^m = b^{-n}.$$

$$+ \frac{3}{56} \left(\frac{a}{b}\right)^2 x^7 = -1.068 \times 10^{-4}$$

$$- \frac{1}{16} \left(\frac{a}{b}\right)^3 x^{10} = +2.56 \times 10^{-6}$$

$$t = \frac{1}{10} \left[4 + 0.0112 - 0.000107 + 0.0000026 \right]$$

$$t = \frac{4.0113096}{10} = 0.40113096 \text{ seconds}$$

If it had been assumed that

$$t = \frac{x}{v_1} \quad t = \frac{4}{10} = 0.40 \text{ seconds.}$$

Certainly it is satisfactory to use this relationship where the assumptions prove themselves correct later. This confirms equation (B38).

Using only the first and second terms,

$$t = \frac{1}{b^{1/2}} \left[x - \frac{1}{8} \left(\frac{a}{b}\right) x^4 \right]$$

$$b^{1/2} t = x - \frac{a}{8b} x^4 \quad \text{where } a = \frac{-2k_1}{3 m_x} \quad b = v_1^2$$

By using a reversion of the series (34)

$$(b^{1/2}t) = (1)x + (0)x^2 + (0)x^3 + \left(\frac{-a}{8b}\right) x^4$$

the equation becomes

$$x = A_1 (b^{1/2}t) + A_2 (b^{1/2}t)^2 + A_3 (b^{1/2}t)^3 + A_4 (b^{1/2}t)^4 + \dots$$

$$A_1 = \frac{1}{1} = 1 \quad A_2 = 0 \quad A_3 = 0$$

$$A_4 = \frac{1}{1} \left(+ \frac{a}{8b}\right) \quad A_5 = 0 \text{ etc.}$$

$$f(x) = \frac{1}{2}x^2 - \frac{1}{6}x^3$$

$$f'(x) = x - \frac{1}{2}x^2$$

$$f''(x) = 1 - x$$

$$f''(0) = 1 > 0$$

Therefore, $x=0$ is a local minimum.

$$f(0) = 0$$

Since $f(x) > 0$ for $x > 0$ and $f(x) < 0$ for $x < 0$, $x=0$ is a saddle point.

(iii) To find the local maxima and minima, we set $f'(x) = 0$.

Using the first derivative test,

$$f'(x) = x - \frac{1}{2}x^2 = 0$$

$$x = \frac{1}{2}x^2 \implies x = 0 \text{ or } x = 2$$

(iv) To find the local maxima and minima, we set $f''(x) = 0$.

$$f''(x) = 1 - x = 0 \implies x = 1$$

The equation becomes

$$f(x) = \frac{1}{2}x^2 - \frac{1}{6}x^3 = \frac{1}{6}x^2(3 - x)$$

$$f(0) = 0, f(2) = \frac{2}{3}$$

$$f(1) = \frac{1}{6}$$

$$x = b^{1/2} t + \frac{ab}{8b} t^{3/2}$$
$$x = v_1 t \frac{-2k_1 v_1^3}{24 m_x v_1^2} t^3 \quad x = v_1 \left(t - \frac{k_1 t^3}{12 m_x} \right) \quad (B41)$$

This equation, (B41), will be used.

Then

$$\dot{x} = \left(v_1^2 \frac{-2k_1 x^3}{3 m_x} \right)^{1/2} \quad (B42)$$

$$\ddot{x} = \left(\frac{-k_1}{m_x} \right) x^2 \quad (B43)$$

These three equations will be used to express x , \dot{x} , and \ddot{x} but since the last term of equation (B41) is almost always negligible it is dropped in calculations of other coordinates.

$$\frac{d}{dt} \left(\frac{1}{2} m v^2 \right) = \dots$$

(30) $\left(\frac{1}{2} m v^2 \right) = \dots$

... ..

(31) $\left(\frac{1}{2} m v^2 \right) = \dots$

(32) $\left(\frac{1}{2} m v^2 \right) = \dots$

... ..

Solution for θ During Crushing

The equation for the summation of the forces in the z-direction during crushing, (B31), may now be rewritten. The term for T_{IB} will be dropped for the reasons mentioned earlier.

Let

$$k_2 = \frac{\mu \tan i_B}{\sin \beta} \left[\cos i_B (\cos \beta + f_k \sin \beta) - f_k \sin i_B \right] \quad (B44)$$

$$\left[-m_x \right] \frac{d^2 z}{dt^2} + \left[-k_h \right] \frac{dz}{dt} + \left[-T_f \right] z$$

$$+ \left[-T_f (LCG-LCF) \right] \theta$$

$$\left[-k_2 \right] x^2 = 0 \quad (B45)$$

$$x^2 = v_1^2 t^2$$

Let

$$a_{12c} = +m_z \quad b_{12c} = +k_h \quad c_{12c} = +T_f$$

$$a_{13c} = 0 \quad b_{13c} = 0 \quad c_{13c} = +T_f (LCG-LCF)$$

(Note: These are constant coefficients)

Then

$$a_{12c} \ddot{z} + b_{12c} \dot{z} + c_{12c} z + a_{13c} \ddot{\theta} + b_{13c} \dot{\theta} + c_{13c} \theta = k_2 v_1^2 t^2 \quad (B46)$$

The equation for the summation of the moments about the center of gravity during crushing, (B32), may now be rewritten. The term for T_{IB}

Let us consider the orthogonal decomposition of the space $L^2(\mathbb{R})$ into the subspace \mathcal{H}_0 and its orthogonal complement \mathcal{H}_0^\perp . We have seen that \mathcal{H}_0 is the space of functions of the form $f(x) = \sum_{n \in \mathbb{Z}} c_n e^{inx}$ with $c_n = 0$ for $n < 0$. The orthogonal complement \mathcal{H}_0^\perp is the space of functions of the form $f(x) = \sum_{n \in \mathbb{Z}} c_n e^{inx}$ with $c_n = 0$ for $n > 0$.

$$(10.1) \quad \left(\sum_{n \in \mathbb{Z}} c_n e^{inx} \right) \left(\sum_{m \in \mathbb{Z}} d_m e^{imx} \right) = 0 \quad \text{for all } x \in \mathbb{R}$$

$$\Leftrightarrow \sum_{n, m \in \mathbb{Z}} c_n d_m e^{i(n+m)x} = 0 \quad \text{for all } x \in \mathbb{R}$$

$$\Leftrightarrow \sum_{k \in \mathbb{Z}} \left(\sum_{n+m=k} c_n d_m \right) e^{ikx} = 0 \quad \text{for all } x \in \mathbb{R}$$

$$\Leftrightarrow \sum_{k \in \mathbb{Z}} \left(\sum_{n+m=k} c_n d_m \right) = 0 \quad \text{for all } k \in \mathbb{Z}$$

$$(10.2) \quad c_n d_m = 0 \quad \text{for all } n, m \in \mathbb{Z}$$

$$\Leftrightarrow \left(\sum_{n \in \mathbb{Z}} c_n e^{inx} \right) \left(\sum_{m \in \mathbb{Z}} d_m e^{imx} \right) = 0 \quad \text{for all } x \in \mathbb{R}$$

$$(10.3) \quad \begin{aligned} c_n &= 0 & d_m &= 0 & \text{for all } n, m \in \mathbb{Z} \\ \text{or} & \quad c_n &= 0 & \text{for all } n < 0 & \text{and } d_m = 0 \text{ for all } m > 0 \\ \text{or} & \quad d_m &= 0 & \text{for all } m < 0 & \text{and } c_n = 0 \text{ for all } n > 0 \end{aligned}$$

(10.3) shows that the orthogonal complement of \mathcal{H}_0 is the space of functions of the form $f(x) = \sum_{n \in \mathbb{Z}} c_n e^{inx}$ with $c_n = 0$ for $n > 0$.

$$(10.4) \quad \begin{aligned} \mathcal{H}_0 &= \left\{ \sum_{n \in \mathbb{Z}} c_n e^{inx} \mid c_n = 0 \text{ for } n < 0 \right\} \\ \mathcal{H}_0^\perp &= \left\{ \sum_{n \in \mathbb{Z}} c_n e^{inx} \mid c_n = 0 \text{ for } n > 0 \right\} \end{aligned}$$

The orthogonal decomposition of the space $L^2(\mathbb{R})$ into the subspace \mathcal{H}_0 and its orthogonal complement \mathcal{H}_0^\perp is given by the orthogonal projection P_0 onto \mathcal{H}_0 . We have seen that P_0 is the operator $P_0 f(x) = \sum_{n \in \mathbb{Z}} c_n e^{inx}$ with $c_n = 0$ for $n < 0$.

will be dropped for reasons mentioned earlier.

In order to keep the coefficients of the equation constant, the first term,

$$\left[\frac{F_{BZC}}{\tan i_B} + F_{BXC} \right] z$$

will be dropped. This simplification should be valid since the magnitude of the term is negligible when compared to the term

$$\left[F_{BZC} \left(\frac{L}{2} - LCG \right) + F_{BXC} (H-KG) \right]$$

which will be retained. It is noted that this latter term is in the order of 200 times larger than the former.

$$\begin{aligned} & \left[-m k^2 \right] \frac{d^2 \theta}{dt^2} + \left[-k_p \right] \frac{d\theta}{dt} \\ & + \left[-\Delta GM_L - F_{BZC} (H-KG) - F_{BZC} \frac{\left(\frac{L}{2} - LCG \right)}{\tan i_B} \right] \theta \\ & + \left[F_{BZC} \left(\frac{L}{2} - LCG \right) + F_{BXC} (H-KG) \right] = 0 \end{aligned}$$

From equations (B14) and (B39) we find

$$F_{BZC} = k_2 x^2 \tag{B47}$$

From equations (B15) and (B33) we find

$$F_{BXC} = k_1 x^2 \tag{B48}$$

Let us suppose the initial conditions are

in order to keep the conditions of the system constant. The first

is

$$= \left[\text{initial} + \frac{\text{initial}}{e^{\lambda t}} \right]$$

will be constant. This differential equation should be solved for the constant

of the form of the solution. The solution is

$$\left[(10-1) \text{initial} + \left(\frac{1}{2} \right) \text{initial} \right]$$

which will be constant. It is noted that the solution is in the form

of the form of the solution.

$$\frac{d}{dt} \left[\frac{1}{e^{\lambda t}} \right] + \frac{d}{dt} \left[\frac{1}{e^{\lambda t}} \right]$$

$$= \left[\frac{(10-1) \text{initial}}{e^{\lambda t}} - (10-1) \text{initial} - \frac{1}{e^{\lambda t}} \right] =$$

$$= \left[(10-1) \text{initial} + \left(\frac{1}{2} \right) \text{initial} \right] +$$

From equation (10) and (11) we find

$$(10) \quad \frac{d}{dt} x = -\lambda x$$

From equation (10) and (11) we find

$$(11) \quad \frac{d}{dt} y = -\lambda y$$

Substitution of equation (B38) leads to

$$F_{BZC} = k_2 v_1^2 t^2 \quad (B49)$$

$$F_{BXC} = k_1 v_1^2 t^2 \quad (B50)$$

The summation-of-moments equation may now be written

$$\begin{aligned} & \left[+ mk^2 \right] \frac{d^2\theta}{dt^2} + \left[k_p \right] \frac{d\theta}{dt} \\ & + \left[+ \Delta GM_L + k_2 v_1^2 (H-KG)t^2 + \frac{k_2 v_1^2 (\frac{L}{2} - LCG)}{\tan i_B} t^2 \right] \theta \\ & = \left[k_2 v_1^2 (\frac{L}{2} - LCG) \right] t^2 + \left[k_1 v_1^2 (H-KG) \right] t^2 \end{aligned}$$

$$\text{Let } a_{23c} = \left[m_\theta k^2 \right] \quad b_{23c} = \left[k_p \right]$$

$$c_{23c} = \Delta GM_L + \left\{ k_2 v_1^2 \left[(H-KG) + \frac{(\frac{L}{2} - LCG)}{\tan i_B} \right] \right\} t^2$$

(Note that c_{23c} is a function of t .)

$$d_{23c} = v_1^2 \left[k_2 (\frac{L}{2} - LCG) + k_1 (H-KG) \right]$$

The summation of moments equation may now be written

$$a_{23c} \ddot{\theta} + b_{23c} \dot{\theta} + c_{23c} \theta = d_{23c} t^2 \quad (B51)$$

Let $c = \Delta GM_L$, the constant portion of the factor of θ in equation (B51).

Then this equation becomes

the second (3.1) reduces to equations

$$(2.11) \quad \frac{\partial}{\partial x} \left[\frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} \right] = 0$$

$$(2.12) \quad \frac{\partial}{\partial x} \left[\frac{\partial \psi}{\partial x} - \frac{\partial \psi}{\partial y} \right] = 0$$

where ψ is a function of x and y only. From (2.11) and (2.12) we get

$$\begin{aligned} & \frac{\partial}{\partial x} \left[\frac{\partial \psi}{\partial x} \right] = \frac{\partial}{\partial x} \left[\frac{\partial \psi}{\partial y} \right] \\ & \left(\frac{\partial}{\partial x} \left(\frac{\partial \psi}{\partial x} - \frac{\partial \psi}{\partial y} \right) + \frac{\partial}{\partial y} \left(\frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} \right) \right) = 0 \\ & \frac{\partial}{\partial x} \left[\left(\frac{\partial \psi}{\partial x} - \frac{\partial \psi}{\partial y} \right) \right] + \frac{\partial}{\partial y} \left[\left(\frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} \right) \right] = 0 \\ & \left[\frac{\partial \psi}{\partial x} \right] + \frac{\partial \psi}{\partial y} = \left[\frac{\partial \psi}{\partial x} \right] - \frac{\partial \psi}{\partial y} \\ & \frac{\partial \psi}{\partial y} = 0 \\ & \left(\frac{\partial \psi}{\partial x} - \frac{\partial \psi}{\partial y} \right) = \left(\frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} \right) \\ & \left(\frac{\partial \psi}{\partial x} - \frac{\partial \psi}{\partial y} \right) = \left(\frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} \right) \\ & \left(\frac{\partial \psi}{\partial x} - \frac{\partial \psi}{\partial y} \right) = \left(\frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} \right) \end{aligned}$$

the solution of which is $\psi = f(x) + g(y)$.

$$(2.13) \quad \frac{\partial \psi}{\partial x} = 0 \quad \frac{\partial \psi}{\partial y} = 0 \quad \frac{\partial \psi}{\partial z} = 0 \quad \frac{\partial \psi}{\partial t} = 0$$

Let $\psi = f(x) + g(y) + h(z) + k(t)$ be the general solution of (2.13).

Then this general solution

$$a \ddot{\theta} + b \dot{\theta} + c \theta = dt^2$$

$$\dot{\theta} + \frac{b}{a} \dot{\theta} + \frac{c}{a} \theta = \frac{d}{a} t^2 \quad (B52)$$

The related homogeneous equation is

$$\dot{\theta} + \frac{b}{a} \dot{\theta} + \frac{c}{a} \theta = 0$$

Roots

$$r = \frac{-\frac{b}{a} \pm \sqrt{\frac{b^2}{a^2} - 4\frac{c}{a}}}{2}$$

$$\theta = A_1 e^{(-\frac{b}{2a} + \frac{1}{2} \sqrt{\frac{b^2}{a^2} - 4\frac{c}{a}})t} + A_2 e^{(-\frac{b}{2a} - \frac{1}{2} \sqrt{\frac{b^2}{a^2} - 4\frac{c}{a}})t}$$

$$\theta = A_1 e^{r_1 t} + A_2 e^{r_2 t}$$

Note: $\frac{b^2}{a^2} > \frac{4c}{a}$
 $\frac{b^2}{a} > 4c$

As seen before, the partial solution

Assume the partial solution to be

$$\theta_p = A t^3 + B t^2 + C t + D$$

$$\dot{\theta}_p = 3 A t^2 + 2 B t + C$$

$$\ddot{\theta}_p = 6 A t + 2 B$$

Substitute these values in equation (B52).

$$6 A t + 2 B + \frac{b}{a} 3 A t^2 + \frac{b}{a} 2 B t + \frac{b}{a} C$$

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$$y = \frac{1}{2}x + \frac{1}{2} \quad (1)$$

$$\left(\frac{1}{2}x + \frac{1}{2}\right)^2 = \frac{1}{4} \quad (2)$$

if arbitrary real number x is

$$x = \frac{1}{2} \quad \frac{1}{2} = \frac{1}{2}$$

then

$$\frac{\left(\frac{1}{2}x + \frac{1}{2}\right)^2}{\left(\frac{1}{2}x + \frac{1}{2}\right)^2} = \frac{1}{4} = \frac{1}{4}$$

$$y \left(\frac{1}{2}x + \frac{1}{2} \right) \sqrt{\frac{1}{4} + \frac{1}{4}} \quad x \left(\frac{1}{2}x + \frac{1}{2} \right) \sqrt{\frac{1}{4} + \frac{1}{4}} \quad y \neq x$$

$$\frac{1}{2} < \frac{1}{2} \quad \text{no}$$

$$\frac{1}{2} < \frac{1}{2} \quad \text{no}$$

$$\frac{1}{2} < \frac{1}{2}$$

if some number x is not real number
 if it is real number x is not

$$x = \frac{1}{2} \quad \frac{1}{2} = \frac{1}{2} \quad \frac{1}{2} = \frac{1}{2}$$

$$x = \frac{1}{2} \quad \frac{1}{2} = \frac{1}{2} \quad \frac{1}{2} = \frac{1}{2}$$

$$x = \frac{1}{2} \quad \frac{1}{2} = \frac{1}{2} \quad \frac{1}{2} = \frac{1}{2}$$

(1) is the same as (2) if x is not real number

$$\frac{1}{2} = \frac{1}{2} \quad \frac{1}{2} = \frac{1}{2} \quad \frac{1}{2} = \frac{1}{2} \quad \frac{1}{2} = \frac{1}{2} \quad \frac{1}{2} = \frac{1}{2}$$

$$+ \frac{c}{a} A t^3 + \frac{c}{a} B t^2 + \frac{c}{a} C t + \frac{c}{a} D - \frac{d}{a} t^2 = 0$$

$$(3A \frac{b}{a} + \frac{c}{a} B - \frac{d}{a}) t^2 + (6A + 2\frac{b}{a} B + \frac{cC}{a}) t$$

$$+ (2B + \frac{b}{a} C + \frac{c}{a} D) + (\frac{c}{a} A) t^3 = 0$$

$$A = 0$$

$$B = \frac{d}{c}$$

$$C = (\frac{-2bB}{a}) (\frac{a}{c})$$

$$C = \frac{-2bB}{c} = \frac{-2bd}{c^2}$$

$$\frac{c}{a} D = -\frac{b}{a} C - 2B$$

$$D = \frac{-b}{c} C - \frac{2aB}{c}$$

$$D = (\frac{-b}{c}) (\frac{-2bd}{c^2}) - (\frac{2a}{c}) (\frac{d}{c})$$

$$D = \frac{2b^2d}{c^3} - \frac{2ad}{c^2}$$

$$\Theta_p = \frac{d}{c} t^2 + (\frac{-2bd}{c^2}) t + \frac{2b^2d}{c^3} - \frac{2ad}{c^2}$$

Find the partial solution, Θ_p , of equation (B2) $R = \frac{d}{a} t^2 = P(t)$

$$P'(t) = \frac{2d}{a} t \quad P''(t) = \frac{2d}{a}$$

$$\Theta_p = \frac{1}{c/a} \left[\frac{d}{a} t^2 - \frac{b/a}{c/a} (2) \frac{d}{a} t + \frac{\frac{b^2}{a^2} - \frac{c}{a}}{c/a^2} (2) \frac{d}{a} \right]$$

$$y = e^{-2x} \left(\frac{1}{2} + \frac{1}{2} e^{-2x} \right) = \frac{1}{2} e^{-2x} + \frac{1}{4} e^{-4x}$$

$$y = e^{-2x} \left(\frac{1}{2} + \frac{1}{4} e^{-2x} \right) = \frac{1}{2} e^{-2x} + \frac{1}{4} e^{-4x}$$

$$\left(\frac{1}{2} \right) \left(\frac{1}{2} \right) = \frac{1}{4} \quad \frac{1}{2} = 0 \quad \frac{1}{2} = 0$$

$$\frac{1}{2} = 0 \quad \frac{1}{2} = 0$$

$$\left(\frac{1}{2} \right) \left(\frac{1}{2} \right) = \left(\frac{1}{2} \right) \left(\frac{1}{2} \right) = 0$$

$$\frac{1}{2} = \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2} + \frac{1}{2} = 1$$

Find the general solution $y(x)$ of equation (2) $y'' + 4y' + 4y = 0$

$$y(x) = \frac{1}{2} e^{-2x} + \frac{1}{2} x e^{-2x}$$

$$\left[\frac{1}{2} e^{-2x} + \frac{1}{2} x e^{-2x} \right]$$

$$\theta_p = \left[\frac{a}{c} \frac{d}{a} t^2 - \frac{2bd}{ca} t + \frac{2d b^2 a^2}{a^3 c^2} - \frac{2cd a^2}{a^2 c^2} \right]$$

$$\theta_p = \frac{d}{c} t^2 - \frac{2bd}{c^2} t + \frac{2b^2 d}{c^3} - \frac{2ad}{c^2}$$

This confirms previous solution of θ_p .

$$\theta_p = \frac{-2d}{c^2} \left[\left(\frac{b}{r_1} - a + \frac{b^2}{c} \right) \left(\frac{r}{r_2 - r_1} \right) + \frac{b^2}{c} - a \right] e^{r_1 t}$$

$$+ \left[\frac{2d}{c^2} \left(\frac{b}{r_1} - a + \frac{b^2}{c} \right) \left(\frac{r}{r_2 - r_1} \right) \right] e^{r_2 t}$$

$$+ \frac{d}{c} t^2 - \frac{2bd}{c^2} t + \frac{2b^2 d}{c^3} - \frac{2ad}{c^2}$$

where $(k_p^2 > 4mk^2 \Delta GM_L)$

Before proceeding with $(b^2 < 4ac)$, look at solution where $b^2 > 4ac$ or equivalently where

$$k_p^2 < 4mk^2 \Delta GM_L .$$

$$\ddot{\theta} + \frac{b_1}{a_1} \dot{\theta} + \frac{c_1}{a_1} \theta = \frac{d_1}{a_1} t^2 \tag{B53}$$

$$r_1 = \frac{-b_1}{2a_1} + \frac{1}{2} \sqrt{\frac{b_1^2}{a_1^2} - \frac{4c_1}{a_1}} \quad r_2 = \frac{b_1}{2a_1} - \frac{1}{2} \sqrt{\frac{b_1^2}{a_1^2} - \frac{4c_1}{a_1}}$$

(for the homogeneous equation) (Both roots contain imaginary terms.)

$$\text{Let } \alpha_1 = \frac{-b_1}{2a_1} \quad \beta_1 = +\frac{1}{2}i \sqrt{\frac{b_1^2}{a_1^2} - \frac{4c_1}{a_1}}$$

$$\theta = e^{\alpha_1 t} (A_1 \cos \beta_1 t + A_2 \sin \beta_1 t)$$

(General solution to the homogeneous equation)

$$\text{As before, if } \theta_p = A_1 t^3 + B_1 t^2 + C_1 t + D_1$$

the general solution of the complete nonhomogeneous equation becomes

$$\theta = e^{\alpha_1 t} (A_1 \cos \beta_1 t + A_2 \sin \beta_1 t) + \frac{d_1}{c_1} t^2 - \frac{2 b_1 d_1}{c_1^2} t + \frac{2 b_1^2 d_1}{c_1^3} - \frac{2 a_1 d_1}{c_1^2}$$

(B54)

$$t = 0 \quad \theta = 0 = A_1 + \frac{2 b_1^2 d_1}{c_1^3} - \frac{2 a_1 d_1}{c_1^2}$$

$$A_1 = \frac{2 d_1}{c_1^2} \left(a_1 - \frac{b_1^2}{c_1} \right)$$

$$\dot{\theta} = \alpha_1 e^{\alpha_1 t} (A_1 \cos \beta_1 t + A_2 \sin \beta_1 t) + e^{\alpha_1 t} (-A_1 \beta_1 \sin \beta_1 t + A_2 \beta_1 \cos \beta_1 t) + \frac{2 d_1 t}{c_1} - \frac{2 b_1 d_1}{c_1^2}$$

(B55)

$$t = 0 \quad \dot{\theta} = 0 = \alpha_1 (A_1) + (A_2 \beta_1) - \frac{2 b_1 d_1}{c_1^2}$$

$$\frac{d^2x}{dt^2} = -\frac{GM}{r^3} \mathbf{r} \quad \text{with } \mathbf{r} = r \hat{r}$$

$$(\ddot{r} - r\dot{\theta}^2) \hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta}) \hat{\theta} = -\frac{GM}{r^3} r \hat{r}$$

(Equate components and set equal to zero)

$$\ddot{r} - r\dot{\theta}^2 = -\frac{GM}{r^2} \quad \text{and} \quad 2\dot{r}\dot{\theta} + r\ddot{\theta} = 0$$

The second equation is a differential equation for θ and can be separated as follows

$$\frac{2\dot{r}\dot{\theta}}{r} + \ddot{\theta} = 0 \quad \Rightarrow \quad \frac{d}{dt} \left(\frac{\dot{\theta}}{r} \right) = 0 \quad \Rightarrow \quad \frac{\dot{\theta}}{r} = \frac{h}{r^2}$$

(1)

$$\frac{d^2r}{dt^2} - r \left(\frac{h}{r^2} \right)^2 = -\frac{GM}{r^2}$$

$$\left(\frac{d^2r}{dt^2} - \frac{h^2}{r^3} \right) = -\frac{GM}{r^2}$$

$$\left(\frac{d^2r}{dt^2} - \frac{h^2}{r^3} \right) = -\frac{GM}{r^2} \quad \Rightarrow \quad \left(\frac{d^2r}{dt^2} + \frac{h^2}{r^3} \right) = -\frac{GM}{r^2}$$

(2)

$$\frac{d^2r}{dt^2} = -\frac{GM}{r^2} - \frac{h^2}{r^3}$$

$$\frac{d^2r}{dt^2} = -\frac{GM}{r^2} - \frac{h^2}{r^3} \quad \Rightarrow \quad \left(\frac{d^2r}{dt^2} + \frac{h^2}{r^3} \right) = -\frac{GM}{r^2}$$

$$A_2 = + \frac{2 b_1 d_1}{c_1^2 \beta_1} \frac{-\alpha_1 A_1}{\beta_1} = \frac{2 d_1}{c_1^2} \left[\frac{b_1}{\beta_1} - \frac{\alpha_1}{\beta_1} \left(a_1 - \frac{b_1^2}{c_1} \right) \right]$$

$$A_2 = \frac{2 d_1}{c_1^2} \left[\frac{b_1 - \alpha_1 \left(a_1 - \frac{b_1^2}{c_1} \right)}{\beta_1} \right]$$

The following terms are used concerning rotation during the crushing phase:

$$a_1 = n_{\theta} k^2 \tag{B56}$$

$$b_1 = k_p$$

$$c_1 = \Delta GM_L$$

$$k_3 = \left[k_2 \left(\frac{L}{2} - LCG \right) + k_1 (H-KG) \right]$$

$$d_1 = v_1^2 k_3$$

$$\alpha_1 = \frac{-b_1}{2a_1} \tag{B57}$$

$$\beta_1 = + \frac{1}{2} i \sqrt{\frac{b_1^2}{a_1^2} - \frac{4c_1}{a_1}} = + \frac{1}{2} \sqrt{\frac{4c_1}{a_1} - \frac{b_1^2}{a_1^2}}$$

$$A_1 = \frac{2a_1}{c_1^2} \left(a_1 - \frac{b_1^2}{c_1} \right)$$

$$A_2 = \frac{2a_1}{\beta_1 c_1^2} \left[b_1 - \alpha_1 \left(a_1 - \frac{b_1^2}{c_1} \right) \right]$$

$$\left[\frac{1}{\sqrt{2}} \rightarrow \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \right] \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0$$

$$\left[\frac{1}{\sqrt{2}} \rightarrow \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \right] \frac{1}{\sqrt{2}} = 0$$

Let's check the other two columns (columns 2 and 3) to see if they are also zero.

Column 2:

(1st)

$$\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$\left[\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \right] = \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2}$$

(2nd)

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2} \quad \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2} \quad \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$\left(\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \right) = 0$$

$$\left[\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \right] = 0$$

$$A_2 = \frac{2bd}{c^2\beta} - \frac{\alpha A_1}{\beta} = \frac{2bd}{c^2\beta} + \frac{b \frac{2d}{c^2}}{2a\beta} \left(a - \frac{b^2}{c} \right)$$

$$\frac{2bd}{c^2\beta} - \frac{\alpha}{\beta} \frac{2d}{c^2} \left(a - \frac{b^2}{c} \right) = \frac{2d}{c^2\beta} \left(b - \alpha \left(a - \frac{b^2}{c} \right) \right)$$

$$A_2 = \frac{2d}{c^2\beta} \left(b + \frac{b}{2a} \left(a - \frac{b^2}{c} \right) \right) = \frac{2d}{c^2\beta} \left(b + \frac{b}{2} - \frac{b^3}{2ac} \right)$$

$$A_2 = + \frac{2bd}{c^2\beta} \left(\frac{3}{2} - \frac{b^2}{2ac} \right) = \frac{bd}{c^2\beta} \left(3 - \frac{b^2}{ac} \right)$$

Equation (B54) may be written as

$$\theta = e^{\alpha_1 t} \left(A_1 \cos \beta_1 t + A_2 \sin \beta_1 t \right) + \frac{d_1}{c_1} t^2 - \frac{2 b_1 d_1}{c_1^2} t - A_1 \quad (B58)$$

Check values of A_1 and A_2

$$\alpha_1 A_1 + A_2 \beta_1 - \frac{2 b_1 d_1}{c_1^2} = 0$$

$$\frac{2 \alpha_1 d_1}{c_1} \left(a_1 - \frac{b_1^2}{c_1} \right) + \frac{2 b_1 d_1}{c_1^2} - \frac{2 \alpha_1 d_1}{c_1^2} \left(a_1 - \frac{b_1^2}{c_1} \right) - \frac{2 b_1 d_1}{c_1^2} = 0$$

A_1 and A_2 satisfy $\dot{\theta}$ and θ when $t = 0$.

$$\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) = \frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) = 0$$

$$\left(\frac{1}{2} - \frac{1}{2} \right) \left(\frac{1}{2} - \frac{1}{2} \right) = \left(\frac{1}{2} - \frac{1}{2} \right) \left(\frac{1}{2} - \frac{1}{2} \right) = 0$$

$$\left(\frac{1}{2} - \frac{1}{2} \right) \left(\frac{1}{2} - \frac{1}{2} \right) = \left(\frac{1}{2} - \frac{1}{2} \right) \left(\frac{1}{2} - \frac{1}{2} \right) = 0$$

$$\left(\frac{1}{2} - \frac{1}{2} \right) \left(\frac{1}{2} - \frac{1}{2} \right) = \left(\frac{1}{2} - \frac{1}{2} \right) \left(\frac{1}{2} - \frac{1}{2} \right) = 0$$

as shown in part (ii) above

(iii) $\frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) = \frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) = 0$

as shown in part (ii) above

$$0 = \frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) = \frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) = 0$$

$$0 = \frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) = \left(\frac{1}{2} - \frac{1}{2} \right) \left(\frac{1}{2} - \frac{1}{2} \right) = \frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) = \frac{1}{2} \left(\frac{1}{2} - \frac{1}{2} \right) = 0$$

as shown in part (ii) above

$$\ddot{\theta} = \alpha_1^2 e^{\alpha_1 t} (A_1 \cos \beta_1 t + A_2 \sin \beta_1 t) - \beta_1^2 e^{\alpha_1 t} (A_1 \cos \beta_1 t + A_2 \sin \beta_1 t) \\ + 2 \alpha_1 \beta_1 e^{\alpha_1 t} (-A_1 \sin \beta_1 t + A_2 \cos \beta_1 t) + \frac{2 d_1}{c_1}$$

$$\ddot{\theta} = (\alpha_1^2 - \beta_1^2) e^{\alpha_1 t} (A_1 \cos \beta_1 t + A_2 \sin \beta_1 t) \tag{B9}$$

$$+ 2 \alpha_1 \beta_1 e^{\alpha_1 t} (-A_1 \sin \beta_1 t + A_2 \cos \beta_1 t) + \frac{2 d_1}{c_1}$$

It is to be noted that the homogeneous equation expressed damped oscillatory motion, which is what the ship would have if there were no moment applied. Therefore, equations (B4) and (B5), and (B9) will be used. Furthermore it may be anticipated that the $e^{\alpha t}$ term should be very close to unity at the low values of t we expect.

Test to see that $\dot{\theta} = 0$ when $t = 0$.

$$\dot{\theta}_{t=0} = A_1 (\alpha^2 - \beta^2) + 2 \alpha \beta A_2 + \frac{2d}{c}$$

$$\dot{\theta}_{t=0} = \frac{2d}{c} \left(a - \frac{b^2}{c} \right) \left[\frac{b^2}{4a^2} - \frac{1}{4} \left(\frac{4c}{a} - \frac{b^2}{a^2} \right) \right]$$

$$\frac{-b}{a} \frac{bd}{c^2} \left(3 - \frac{b^2}{ac} \right) + \frac{2d}{c}$$

$$(1) \quad \frac{d}{dt} \left(\frac{1}{2} m v^2 + \frac{1}{2} k x^2 \right) = m v \frac{dv}{dt} + k x \frac{dx}{dt}$$

$$= (m v + k x) \frac{dx}{dt}$$

$$(2) \quad \frac{d}{dt} \left(\frac{1}{2} m v^2 + \frac{1}{2} k x^2 \right) = m v \frac{dv}{dt} + k x \frac{dx}{dt}$$

$$= (m v + k x) \frac{dx}{dt}$$

It is to be noted that the integrand in the above expression is not an exact differential, since it does not have the form $M dx + N dy$ where M and N are functions of x and y only. However, it can be recognized that the x^2 term can be written as $\frac{1}{2} \frac{d}{dt} (x^2)$ and the v^2 term as $\frac{1}{2} \frac{d}{dt} (v^2)$.

$$\frac{1}{2} \frac{d}{dt} (v^2 + x^2) = \frac{1}{2} \frac{d}{dt} (v^2 + x^2)$$

$$\frac{d}{dt} (v^2 + x^2) = \frac{d}{dt} (v^2 + x^2)$$

$$\left[\frac{d}{dt} \left(\frac{1}{2} m v^2 + \frac{1}{2} k x^2 \right) \right] = \frac{d}{dt} \left(\frac{1}{2} m v^2 + \frac{1}{2} k x^2 \right)$$

$$\frac{d}{dt} \left(\frac{1}{2} m v^2 + \frac{1}{2} k x^2 \right) = \frac{d}{dt} \left(\frac{1}{2} m v^2 + \frac{1}{2} k x^2 \right)$$

$$\ddot{\theta} = \frac{2 b^2 d}{4 a^2 c^2} \left(a - \frac{b^2}{c} \right) - \frac{8 c d}{4 a c^2} \left(a - \frac{b^2}{c} \right) + \frac{2 b^2 d}{4 a^2 c^2} \left(a - \frac{b^2}{c} \right)$$

$$= \frac{-3 b^2 d}{a c^2} + \frac{b^4 d}{a^2 c^3} + \frac{2 d}{c}$$

$$\ddot{\theta} = \frac{b^2 d}{2 a c^2} - \frac{b^4 d}{2 a^2 c^3} - \frac{2 d}{c^2} + \frac{2 b^2 d}{a c^2} + \frac{b^2 d}{2 a c^2}$$

$$= \frac{b^4 d}{2 a^2 c^3} - \frac{3 b^2 d}{a c^2} + \frac{b^4 d}{a^2 c^3} + \frac{2 d}{c}$$

$$\ddot{\theta} = 0$$

t=0

$$12) = \frac{12x}{x^2+1} = \frac{12x}{(x-i)(x+i)} = \frac{A}{x-i} + \frac{B}{x+i} \quad (1)$$

$$12 = \frac{A(x+i)}{x+i} + \frac{B(x-i)}{x-i}$$

$$\frac{12x}{x^2+1} = \frac{A(x+i)}{x-i} + \frac{B(x-i)}{x+i} \quad (2)$$

$$12 = \frac{A(x+i)}{x-i} + \frac{B(x-i)}{x+i}$$

Ans: 6

Solution for z During Crushing

In equation (B45) the factor of θ will be considered negligible; it is a small corrective term for the difference of draft between the center of gravity and the center of flotation. The equation becomes

$$(+m_z) \ddot{z} + (k_h) \dot{z} + (T_f)z = (-k_2 v_1^2) t^2$$

$$a_2 = +m_z \quad b_2 = k_h \quad c_2 = T_f \quad d_2 = (-k_2 v_1^2)$$

$$\ddot{z} + \frac{b_2}{a_2} \dot{z} + \frac{c_2}{a_2} z = \frac{d_2}{a_2} t^2 \quad (B60)$$

It is noted that equation (B60) is the same nonhomogeneous equation as (B53) and further that the motion represented by the homogeneous portion is the same type (damped oscillatory) that is represented by the solution to (B53). The initial conditions are the same.

We can therefore write the equations of motion using the following terms:

$$a_2 = +m_z \quad b_2 = k_h \quad c_2 = T_f \quad d_2 = (-k_2 v_1^2) \quad (B61)$$

m = total mass

k_h = leave damping coefficient

T_f = lb/ft immersion

$$\alpha_2 = \frac{-b_2}{2 a_2} \quad \beta_2 = +\frac{1}{2} t \sqrt{\frac{b_2^2}{a_2^2} - \frac{4 c_2}{a_2}} = \frac{1}{2} \sqrt{\frac{4 c_2}{a_2} - \frac{b_2^2}{a_2^2}}$$

$$B_1 = \frac{2 d_2}{c_2} \left(a_2 - \frac{b_2^2}{c_2} \right)$$

Problem 2: Linear Algebra

12. (a) Consider the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and the vector $\mathbf{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Compute $A\mathbf{v}$.

$$A\mathbf{v} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \cdot 1 + 2 \cdot 2 \\ 3 \cdot 1 + 4 \cdot 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 11 \end{pmatrix}$$

$$\begin{pmatrix} 5 \\ 11 \end{pmatrix} = \begin{pmatrix} 5 \\ 11 \end{pmatrix} \quad \mathbf{v}^T = \begin{pmatrix} 1 & 2 \end{pmatrix} \quad \mathbf{v}^T \mathbf{v} = \begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 5$$

(100)
$$\mathbf{v}^T \begin{pmatrix} 5 \\ 11 \end{pmatrix} = \begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 5 \\ 11 \end{pmatrix} = 1 \cdot 5 + 2 \cdot 11 = 27$$

(b) Let $\mathbf{u} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Compute the dot product $\mathbf{u} \cdot \mathbf{v}$.

(10) The dot product is $\mathbf{u} \cdot \mathbf{v} = \begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 5$.

(11) The magnitude of \mathbf{u} is $\|\mathbf{u}\| = \sqrt{1^2 + 2^2} = \sqrt{5}$.

(12) The magnitude of \mathbf{v} is $\|\mathbf{v}\| = \sqrt{1^2 + 2^2} = \sqrt{5}$.

(13)
$$\begin{pmatrix} 5 \\ 11 \end{pmatrix} = \begin{pmatrix} 5 \\ 11 \end{pmatrix} \quad \mathbf{v}^T = \begin{pmatrix} 1 & 2 \end{pmatrix} \quad \mathbf{v}^T \mathbf{v} = \begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 5$$

$\mathbf{v}^T \mathbf{v} = 5$ (14)	$\mathbf{v}^T \mathbf{v} = 5$ (15)	$\mathbf{v}^T \mathbf{v} = 5$ (16)
---------------------------------------	---------------------------------------	---------------------------------------

$$\frac{\mathbf{v}^T \mathbf{v}}{\mathbf{v}^T \mathbf{v}} = \frac{5}{5} = 1$$

$$\frac{\mathbf{v}^T \mathbf{v}}{\mathbf{v}^T \mathbf{v}} = \frac{5}{5} = 1$$

$$\frac{\mathbf{v}^T \mathbf{v}}{\mathbf{v}^T \mathbf{v}} = \frac{5}{5} = 1$$

$$B_2 = \frac{2 d_2}{c_2^2} \left[\frac{b_2 - \alpha_2 \left(a_2 \frac{-b_2^2}{c_2} \right)}{\beta_2} \right]$$

The equations for motion during crushing in the vertical direction are

$$z = e^{\alpha_2 t} (R_1 \cos \beta_2 t + B_2 \sin \beta_2 t) + \frac{d_2}{c_2} t^2$$

$$- \frac{2 b_2 d_2 t}{c_2^2} + \frac{2 b_2^2 d_2}{c_2^3} - \frac{2 a_2 d_2}{c_2^2} \quad (B62)$$

$$\dot{z} = \alpha_2 e^{\alpha_2 t} (R_1 \cos \beta_2 t + B_2 \sin \beta_2 t)$$

$$+ e^{\alpha_2 t} (-R_1 \beta_2 \sin \beta_2 t + B_2 \beta_2 \cos \beta_2 t)$$

$$+ \frac{2 d_2 t}{c_2} - \frac{2 b_2 d_2}{c_2^2} \quad (B63)$$

$$\ddot{z} = \alpha_2^2 e^{\alpha_2 t} (R_1 \cos \beta_2 t + B_2 \sin \beta_2 t)$$

$$+ 2 \alpha_2 e^{\alpha_2 t} (-R_1 \beta_2 \sin \beta_2 t + B_2 \beta_2 \cos \beta_2 t)$$

$$+ e^{\alpha_2 t} (-R_1 \beta_2^2 \cos \beta_2 t - B_2 \beta_2^2 \sin \beta_2 t)$$

$$+ \frac{2 d_2}{c_2}$$

$$\left[\frac{\sum_{i=1}^n x_i^2}{n} - \left(\frac{\sum_{i=1}^n x_i}{n} \right)^2 \right] \frac{1}{n} = s^2$$

The expression for the variance of the vertical direction is

$$s_y^2 = \frac{1}{n} \sum_{i=1}^n y_i^2 - \left(\frac{\sum_{i=1}^n y_i}{n} \right)^2$$

(20)

$$\frac{\sum_{i=1}^n x_i^2}{n} - \left(\frac{\sum_{i=1}^n x_i}{n} \right)^2 = s_x^2$$

$$\left(\sum_{i=1}^n x_i^2 - \frac{(\sum_{i=1}^n x_i)^2}{n} \right) \frac{1}{n} = s_x^2$$

$$\left(\sum_{i=1}^n x_i^2 - \frac{(\sum_{i=1}^n x_i)^2}{n} \right) \frac{1}{n} = s_x^2$$

(21)

$$\frac{\sum_{i=1}^n x_i^2}{n} - \left(\frac{\sum_{i=1}^n x_i}{n} \right)^2 = s_x^2$$

$$\left(\sum_{i=1}^n x_i^2 - \frac{(\sum_{i=1}^n x_i)^2}{n} \right) \frac{1}{n} = s_x^2$$

$$\left(\sum_{i=1}^n x_i^2 - \frac{(\sum_{i=1}^n x_i)^2}{n} \right) \frac{1}{n} = s_x^2$$

$$\left(\sum_{i=1}^n x_i^2 - \frac{(\sum_{i=1}^n x_i)^2}{n} \right) \frac{1}{n} = s_x^2$$

$$\frac{\sum_{i=1}^n x_i^2}{n}$$

$$\dot{z} = (\alpha_2^2 - \beta_2^2) e^{\alpha_2 t} \left[B_1 \cos \beta_2 t + B_2 \sin \beta_2 t \right] \quad (B64)$$
$$+ 2 \alpha_2 \beta_2 e^{\alpha_2 t} (-B_1 \sin \beta_2 t + B_2 \cos \beta_2 t) + \frac{2 d_2}{c_2}$$

Velocity at the Bow

It can be seen that the velocity at point A on the bow must equal v_1 , the approach velocity, at the time of initial contact. See Figure B-XVII.

As the criterion for the termination of the crushing phase, there must be no velocity at point A which is normal to the stem. Therefore the velocity of point A must be in the direction as indicated by $(i_B + \theta)$. See Figure B-XVIII.

Let the direction of the velocity of A at any time be defined by γ , as shown in Figure B-XIX. It follows that

$$\tan \gamma = \frac{(GA)_x \frac{d\theta}{dt} - \frac{dz}{dt}}{\frac{dx}{dt} - (GA)_z \left(\frac{d\theta}{dt}\right)} \quad (B65)$$

where $\frac{d\theta}{dt}$ is defined by equation (B55)

$\frac{dz}{dt}$ is defined by equation (B63)

and $\frac{dx}{dt}$ is defined by equation (B46)

We recall from equation (B16) that

$$(GA)_z = H - KG + z$$

and from equation (B20) that

$$(GA)_x = \left(\frac{L}{2} - LCG\right) - \left[(H - KG) + \frac{\left(\frac{L}{2} - LCG\right)}{\tan i_B} \right] \theta + \left(\frac{1}{\tan i_B}\right) z$$

Let $f(x) = \frac{1}{x^2} = x^{-2}$. Then $f'(x) = -2x^{-3} = -\frac{2}{x^3}$.
 Using the power rule, we have $f'(x) = -2x^{-3} = -\frac{2}{x^3}$.
 Now, we can find the derivative of $f(x)$ at $x = 2$.
 $f'(2) = -\frac{2}{2^3} = -\frac{2}{8} = -\frac{1}{4}$.
 Therefore, the derivative of $f(x)$ at $x = 2$ is $-\frac{1}{4}$.

Let $f(x) = \frac{1}{x^2}$. Then $f'(x) = -\frac{2}{x^3}$.
 Using the power rule, we have $f'(x) = -2x^{-3} = -\frac{2}{x^3}$.

(18)
$$\frac{\frac{1}{2} - \frac{1}{4}}{\left(\frac{1}{2}\right)^2 - \left(\frac{1}{4}\right)^2} = \frac{1}{4}$$

(19) average of function at $\frac{1}{2}$ and $\frac{1}{4}$

(20) average of function at $\frac{1}{2}$ and $\frac{1}{4}$

(21) average of function at $\frac{1}{2}$ and $\frac{1}{4}$

Let $f(x) = \frac{1}{x^2}$. Then $f'(x) = -\frac{2}{x^3}$.

$f'(2) = -\frac{2}{2^3} = -\frac{1}{4}$

Let $f(x) = \frac{1}{x^2}$. Then $f'(x) = -\frac{2}{x^3}$.

$$\frac{\left[\frac{1}{\left(\frac{1}{2}\right)^2} - \frac{1}{\left(\frac{1}{4}\right)^2} \right] - \left(\frac{1}{2} - \frac{1}{4} \right)}{\left(\frac{1}{2}\right)^2 - \left(\frac{1}{4}\right)^2} = \frac{1}{4}$$

$$\frac{\left[\frac{1}{\frac{1}{4}} - \frac{1}{\frac{1}{16}} \right] - \left(\frac{1}{2} - \frac{1}{4} \right)}{\frac{1}{4} - \frac{1}{16}} = \frac{1}{4}$$

Figure B-XVII

Point on Bow at First Contact with Ice. State 1

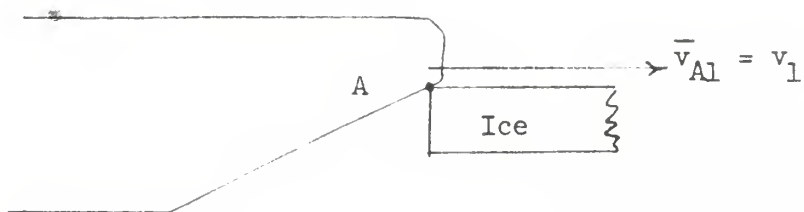


Figure B-XVIII

Point on Bow at End of Crushing Phase. State 2

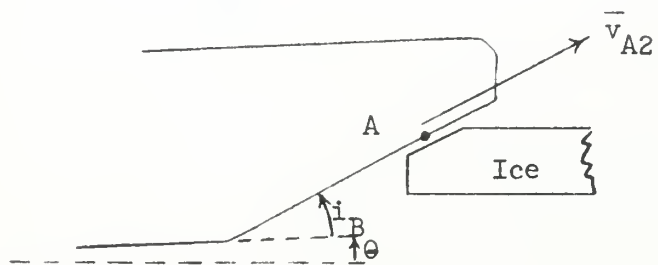
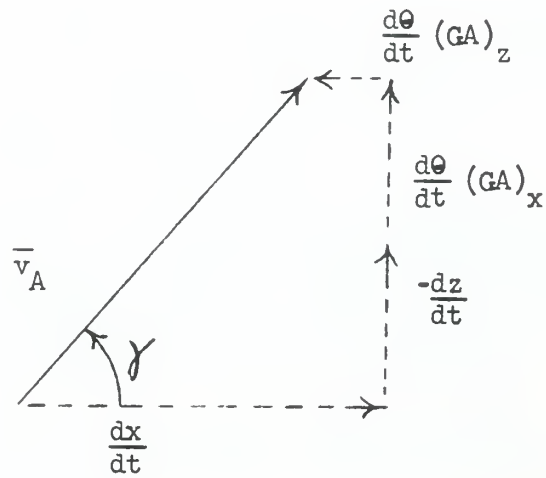


Figure B-XIX

Illustration of Vector Velocity of Point A on Bow

$$\bar{v}_A = \bar{v}_G + \bar{v}_{A/G}$$



Therefore, the equations on page 278 must also be solved on the basis of equations (B54) and (B62) before they may be substituted into equation (B65). If t is kept as the only unknown, then

$$\tan \gamma = f(t)$$

This may be solved for successive values of t until

$$\tan \gamma = \tan (i_B + \theta).$$

At this time, t_2 , State 2 is reached and the crushing has stopped.

Using t_2 , any crushing equation of motion may then be solved.

The following values must be known:

- f_k Dimensionless coefficient of kinetic friction
- β Angle between normal to plating and centerline plane.
- i_B Angle between stem line and base line
- σ Estimate of compressive failure stress of ice, lbs. per square foot.
- L Length between perpendiculars, ft.
- LCG Distance from midships to center of gravity, + if forward, - if aft, ft.
- H Draft, ft.
- KG Height of center of gravity above keel, ft.
- T_f Pounds per foot immersion
- k_p Coefficient of pitch damping, ft-lb-sec
- k_h Coefficient of heave damping, lb-sec/ft

... the ... of ... (1) ... (2) ...

$$f(x) = x^2$$

... the ... of ...

$$f(x) = x^2 + 1$$

... the ... of ...

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- m_x Mass of ship plus virtual mass in x-direction, $\frac{\text{lb-sec}^2}{t}$
- m_z Mass of ship plus virtual mass in z-direction, $\frac{\text{lb-sec}^2}{ft}$
- k Radius of gyration, ft
- m_θ Mass of ship plus virtual mass during rotation (pitch), $\frac{\text{lb-sec}^2}{ft}$
- LCF Distance from midships to center of flotation (+ if forward,
- if aft), ft.
- v_1 Velocity of ship immediately prior to initial contact, ft/sec
- GM_L Longitudinal metacentric height, ft.
- Δ Displacement in lb.
- LCG Distance from midships to center of gravity (+ if forward,
- if aft), ft.

$\frac{d}{dt} \left(\frac{1}{r} \right) = -\frac{1}{r^2} \frac{dr}{dt}$...

$\frac{d}{dt} \left(\frac{1}{r^2} \right) = -\frac{2}{r^3} \frac{dr}{dt}$...

... ..

$\frac{d}{dt} \left(\frac{1}{r^3} \right) = -\frac{3}{r^4} \frac{dr}{dt}$...

... ..

- 11 -

... ..

... ..

... ..

... ..

- 12 -

Mass and Mass Moment of Inertia

For the purpose of these calculations it may be assumed that the underwater shape of most icebreakers may be approximated as indicated in Table 62a on p.^s 423 of reference (24).

The following dimensions from a "Wind Class" Icebreaker will be used:

$$\begin{aligned} \text{LEP} = L &= 250 \text{ '} \\ H &= 25'9'' \\ B &= 62'0'' \\ \Delta &= 3500 \text{ tons} \end{aligned}$$

$$\frac{L}{H} = \frac{250}{25.75} = 0.97$$

$$\text{Fatness Ratio} = \frac{(3500)(35)}{(25)^3} = 7.75$$

D "Maximum Diameter"

$$\frac{L}{D} = \frac{250}{62} = 4.03$$

<u>Body No.</u>	<u>$\frac{L}{D}$</u>	<u>Fatness Ratio</u>	<u>CAMX</u>	<u>CAMZ</u>	<u>CAMθ</u>
1	4	31.91	0.087	0.854	0.598
5	8	7.98	0.031	0.942	0.835

where CAMX is the added mass coefficient for unsteady motion along the x-axis.

CAMZ is the added mass coefficient for unsteady motion along the z-axis.

Problem 2

The first part of the problem is to find the value of the function $f(x)$ at $x=1$ and $x=2$. The second part is to find the value of the function $f(x)$ at $x=3$ and $x=4$.

$$f(x) = \frac{1}{x^2} - \frac{1}{x}$$

The value of the function $f(x)$ at $x=1$ is $f(1) = \frac{1}{1^2} - \frac{1}{1} = 1 - 1 = 0$.

Similarly,

$$f(2) = \frac{1}{2^2} - \frac{1}{2} = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$$

$$f(3) = \frac{1}{3^2} - \frac{1}{3} = \frac{1}{9} - \frac{1}{3} = -\frac{2}{9}$$

$$f(4) = \frac{1}{4^2} - \frac{1}{4} = \frac{1}{16} - \frac{1}{4} = -\frac{3}{16}$$

$$f(5) = \frac{1}{5^2} - \frac{1}{5} = \frac{1}{25} - \frac{1}{5} = -\frac{4}{25}$$

$$f(x) = \frac{1}{x^2} - \frac{1}{x} = \frac{1-x}{x^2}$$

$$f(x) = \frac{1-x}{x^2} = \frac{1}{x^2} - \frac{1}{x}$$

The value of the function $f(x)$ at $x=1$ is $f(1) = \frac{1-1}{1^2} = 0$.

$$f(2) = \frac{1-2}{2^2} = -\frac{1}{4}$$

<u>x</u>	<u>f(x)</u>	<u>x</u>	<u>f(x)</u>	<u>x</u>	<u>f(x)</u>
1	0	2	-1/4	3	-2/9
2	-1/4	3	-2/9	4	-3/16
3	-2/9	4	-3/16	5	-4/25

The value of the function $f(x)$ at $x=3$ is $f(3) = \frac{1-3}{3^2} = -\frac{2}{9}$.

The value of the function $f(x)$ at $x=4$ is $f(4) = \frac{1-4}{4^2} = -\frac{3}{16}$.

CAM_θ is the added moment of inertia coefficient for pitch,

It is noted that in the reference the value for CAMZ is based on lateral translation but since these terms were developed for submerged shapes, the value is also valid for the z-direction.

Since none of the bodies in the table seem close enough to our typical shape, let us use the development of the prolate ellipsoid in Figure 62.B of the reference. This shape is not too far from the under-water shape of most icebreakers.

$$a = \frac{L}{2} = 125 \quad b = \frac{B}{2} = 31$$

$$a/b = 4.03$$

$$\text{For } \frac{a}{b} = 3.99 \quad k_1 = 0.860 \quad k_2 = 0.082 \quad k_3 = 0.608$$

where $k = \frac{\text{added mass (or mass moment of inertia)}}{\text{body mass (or mass moment of inertia)}}$

k_1 for the z-axis

k_2 for the x-axis

k_3 for pitching

Therefore, the mass (or mass moment of inertia) for ships of typical polar icebreaker form may be approximated as follows:

$$m_x = 1.08 \frac{\Delta}{g} = 0.0336 \Delta \quad (B66)$$

$$m_z = 1.86 \frac{\Delta}{g} = 0.0578 \Delta \quad (B67)$$

$$k^2 m_{\theta} = 1.61 k^2 \frac{\Delta}{g} = 0.050 k^2 \Delta \quad (B68)$$

... the ... of the ...
 It is ... in the ...
 ...
 ...
 ...
 ...
 ...

$$\begin{aligned}
 x &= \frac{1}{2} + \dots & y &= \frac{1}{2} - \dots \\
 x &= 1.02 & y &= 1.02 \\
 \text{for } \frac{1}{2} &= 2.00 & \text{for } \frac{1}{2} &= 2.00 \\
 800.0 &= x & 800.0 &= x & 800.0 &= x & 800.0 &= x
 \end{aligned}$$

... (of ... of ...)

...
 ...
 ...

... the ... of ...
 ... as follows:

$$\begin{aligned}
 (104) \quad & x^2 = 1.08 \frac{1}{2} - 0.015 \dots \\
 (105) \quad & x^2 = 1.06 \frac{1}{2} - 0.015 \dots \\
 (106) \quad & x^2 = 1.04 \frac{1}{2} - 0.015 \dots
 \end{aligned}$$

where Δ is displacement in lb and g is acceleration, 32.2 ft/sec^2 .

These factors correspond to length to beam ratios of 4 to 1 and are therefore representative. It is felt that it is not necessary to recalculate them for each proposed icebreaker for that reason plus the fact that solutions of the icebreaking equations are comparative and not, strictly speaking, absolute.

Damping Coefficients

It is necessary to use a convenient approximation for damping coefficients in heave (h_h) and pitch (k_p). It is to be remembered that these equations for icebreaking are to be used comparatively and do not warrant the precision and complexity of some methods of determining damping coefficients.

Vosser (25) uses the following dimensionless coefficients for damping:

$$\psi_{\psi}^{\circ} = \frac{N_{\psi\psi} \sqrt{gL}}{\rho g L^2 \nabla} \quad \text{for pitching}$$

and

$$\psi_z^{\circ} = \frac{N_{zz} \sqrt{gL}}{\rho g \nabla} \quad \text{for heaving}$$

For pitching

$$k_p = N_{\psi\psi} = \frac{\psi_{\psi}^{\circ} \rho g L^2 \nabla}{\sqrt{gL}} \quad \text{lb-ft-sec}$$

The first part of the paper is devoted to the study of the asymptotic behavior of the estimator $\hat{\beta}_n$ under the null hypothesis $H_0: \beta = \beta_0$. It is shown that $\hat{\beta}_n$ is asymptotically normal with mean β_0 and variance-covariance matrix $V(\beta_0)$. The second part of the paper is devoted to the study of the asymptotic behavior of the estimator $\hat{\beta}_n$ under the alternative hypothesis $H_1: \beta = \beta_1$. It is shown that $\hat{\beta}_n$ is asymptotically normal with mean β_1 and variance-covariance matrix $V(\beta_1)$.

Asymptotic Normality

To establish the asymptotic normality of the estimator $\hat{\beta}_n$, we first consider the case of the null hypothesis $H_0: \beta = \beta_0$. Let $\beta_0 = (\beta_{01}, \dots, \beta_{0k})'$ and let $\beta_1 = (\beta_{11}, \dots, \beta_{1k})'$ be a vector in R^k . Let $\beta = (\beta_1, \dots, \beta_k)'$ be a vector in R^k . Let $\beta_0 = (\beta_{01}, \dots, \beta_{0k})'$ and let $\beta_1 = (\beta_{11}, \dots, \beta_{1k})'$ be a vector in R^k . Let $\beta = (\beta_1, \dots, \beta_k)'$ be a vector in R^k .

Under H_0 , the following asymptotic normality result holds:

$$\sqrt{n}(\hat{\beta}_n - \beta_0) \xrightarrow{d} N(0, V(\beta_0))$$

and

$$\sqrt{n}(\hat{\beta}_n - \beta_1) \xrightarrow{d} N(0, V(\beta_1))$$

for all β_1 .

$$\sqrt{n}(\hat{\beta}_n - \beta) \xrightarrow{d} N(0, V(\beta))$$

where $V(\beta) = \sigma^2 (X'X)^{-1}$ is the variance-covariance matrix of the OLS estimator $\hat{\beta}_n$.

$$k_p = \frac{\psi_{\psi}^{\circ} \Delta L^{3/2}}{g^{1/2}}$$

An average value of ψ_{ψ}° for relatively low pitching frequency is selected from Gerritsma's work published by Vosser (25).

$$\text{Set } \psi_{\psi}^{\circ} = 0.10$$

Then

$$k_p = \frac{0.10}{\sqrt{g}} \Delta L^{3/2}$$

$$k_p = 0.0176 \Delta L^{3/2} \quad \text{lb-ft-sec} \quad (\text{B69})$$

For heaving

$$k_h = N_{zz} = \frac{\psi_z^{\circ} \rho g \nabla}{\sqrt{g L}} \quad \frac{\text{lb-sec}}{\text{ft}}$$

$$k_h = \frac{\psi_z^{\circ}}{\sqrt{g}} \frac{\Delta}{L^{1/2}}$$

An average value of ψ_z° based on Gerritsma's work is selected. (25)

$$\text{Set } \psi_z^{\circ} = 3.0$$

Then

$$k_h = \frac{3.0}{\sqrt{g}} \frac{\Delta}{L^{1/2}}$$

$$k_h = 0.529 \frac{\Delta}{L^{1/2}} \quad \frac{\text{lb-sec}}{\text{ft}} \quad (\text{B70})$$

$$\frac{d}{dx} \left(\frac{1}{\sqrt{x}} \right) = -\frac{1}{2} x^{-3/2}$$

is a constant multiple of a function whose derivative is known.

(2) Use the power rule for differentiation to find the derivative of $x^{-1/2}$.

$$\frac{d}{dx} x^{-1/2} = -\frac{1}{2} x^{-3/2}$$

$$\frac{d}{dx} \left(\frac{1}{\sqrt{x}} \right) = -\frac{1}{2} x^{-3/2}$$

(3)

power

$$\frac{d}{dx} x^{-1/2} = -\frac{1}{2} x^{-3/2}$$

power

$$\frac{d}{dx} \left(\frac{1}{\sqrt{x}} \right) = -\frac{1}{2} x^{-3/2}$$

$$\frac{d}{dx} \left(\frac{1}{\sqrt{x}} \right) = -\frac{1}{2} x^{-3/2}$$

(4) Use the power rule for differentiation to find the derivative of $x^{-1/2}$.

$$\frac{d}{dx} x^{-1/2} = -\frac{1}{2} x^{-3/2}$$

power

$$\frac{d}{dx} \left(\frac{1}{\sqrt{x}} \right) = -\frac{1}{2} x^{-3/2}$$

(5)

$$\frac{d}{dx} \left(\frac{1}{\sqrt{x}} \right) = -\frac{1}{2} x^{-3/2}$$

Radius of Gyration

It is necessary to have a suitable value for the mass moment of inertia about the center of gravity for a pitching motion (about the y-axis). A convenient way of finding this is to know (or approximate) the radius of gyration, k .

Vosser (25) indicates that the longitudinal radius of gyration of a fully loaded ship varies between $0.22L$ and $0.27L$. (A triangular weight distribution would have $k = 0.204L$).

Since an icebreaker is generally short, broad, and deep, much of its weight is toward amidships. For that reason, and as an approximation, set

$$k = 0.22 L \quad (B71)$$

Pounds per Foot Immersion

During initial design stages the area of the water plane may be known but the next step of calculating tons per foot immersion may not have been carried out. For that reason, T_f , will be expressed in the terms of water plane coefficient.

$$T_f = L \cdot B \text{ (Water plane coefficient)}$$

$$T_f = 64.2 L \alpha B \text{ lb/ft} \quad (B72)$$

for sea water.

Icebreakers constructed prior to 1962 have had various water plane coefficients from 0.650 to 0.761 with an average of 0.720.

The first part of the document discusses the importance of maintaining accurate records of all transactions. It emphasizes that proper record-keeping is essential for the integrity of the financial system and for the ability to detect and prevent fraud. The text also mentions the need for regular audits and the role of independent auditors in ensuring the accuracy of financial statements.

The second part of the document focuses on the specific requirements for the preparation and presentation of financial statements. It details the various components of the financial statements, including the balance sheet, income statement, and cash flow statement. The text also discusses the importance of providing clear and concise disclosures to users of the financial statements, particularly regarding any significant risks or uncertainties that may affect the company's financial position.

(18) 1980-1-1

Financial Reporting

The third part of the document discusses the various methods used to measure and evaluate the performance of a company. It compares different financial ratios and metrics, such as the profit margin, return on assets, and return on equity. The text also discusses the importance of benchmarking a company's performance against its peers in the industry and against its own historical performance. Finally, the text mentions the role of non-financial indicators, such as customer satisfaction and employee turnover, in providing a more comprehensive view of a company's performance.

(19) 1980-1-1

(20) 1980-1-1

1980-1-1

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Longitudinal Metacentric Height

This value may not be known during initial design stages. In that case it would be appropriate to use

$$GM_L \approx GM_L \approx L \quad (B73)$$

Bow Forces During Crushing

Reference to equations (B34) and (B14) will lead to values for the horizontal component of the bow force and the vertical components of the bow force respectively.

$$F_{BXC} = k_1 x^2 \quad (B39)$$

$$F_{BZC} = k_2 x^2 \quad (B14)$$

where k_1 and k_2 reflect the influence of σ , i_B , β , and f_k .

Sliding Phase, General

The sliding phase commences once local crushing has ceased. In other words, there is no component of velocity at the bow normal to the stem.

It is important to note that point A on the ice, the point of contact with the bow, is fixed relative to the coordinate system during the sliding phase.

Unlike the crushing phase, the friction force acts only parallel to the stem since that is the only direction of relative motion. See Figure B-XX.

If $\frac{N}{2}$ represents the force normal to the plating on each side, then the friction force can be represented by

$$\frac{F_s}{2} = f_k \frac{N}{2} \qquad F_s = f_k N \qquad (B74)$$

where f_k = coefficient of kinetic friction.

As may be seen in Figure B-XXI, the force normal to the stem, in the centerline plane, may be expressed as

$$P_s = N \cos \beta \qquad (B 75)$$

where β = angle between normal to plating and centerline plane

As may be seen in Figures B-XXII and B-XXIII the upward force under the bow is

$$F_{BZS} = P_s \cos (i_B + \theta) - F_s \sin (i_B + \theta)$$

RELATIONSHIP BETWEEN

The relative phase between the two waves is π when the path difference is an odd multiple of $\frac{\lambda}{2}$. In other words, there is a phase change of π when the path difference is $\frac{\lambda}{2}$.

It is important to note that when λ is very small compared to the distance between the slits, the path difference is approximately equal to the distance between the slits.

Using the condition for constructive interference, the intensity of the waves is maximum when the path difference is an even multiple of $\frac{\lambda}{2}$. In other words, the waves are in phase when the path difference is $n\lambda$. The intensity of the waves is minimum when the path difference is an odd multiple of $\frac{\lambda}{2}$.

(17)
$$n\lambda = d \sin \theta$$

$$\frac{\lambda}{d} \sin \theta = \sin \theta'$$

where θ' is the angle of diffraction. As can be seen in figure 17-11, the wave normal to the slits, in the direction of the slits, may be represented by

(18)
$$\lambda \sin \theta = d \sin \theta'$$

where θ' is the angle between the normal to the slits and the direction of the wave. As can be seen in figure 17-11, the wave normal to the slits, in the direction of the slits, may be represented by

$$(\theta - \theta') \sin \theta = (\theta + \theta') \cos \theta = \frac{\lambda}{d} \sin \theta'$$

Figure B-XX
Forces Acting on Bow During Sliding

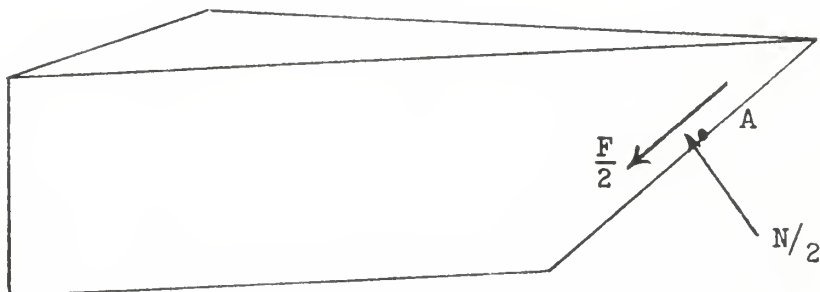


Figure B-XXI
Resolution of Friction and Normal Forces
During Sliding, Looking Down Stem

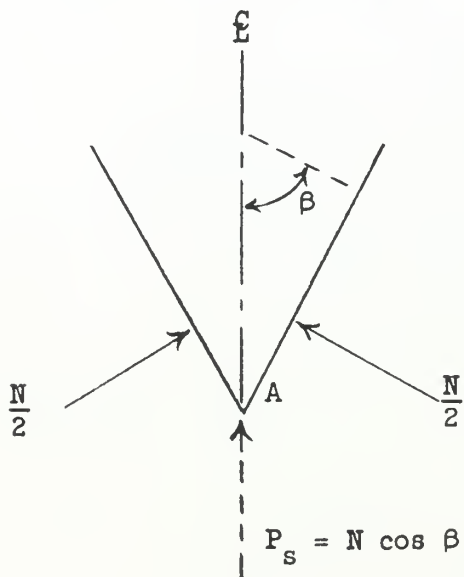


Figure B-XXII

Bow Forces During Sliding

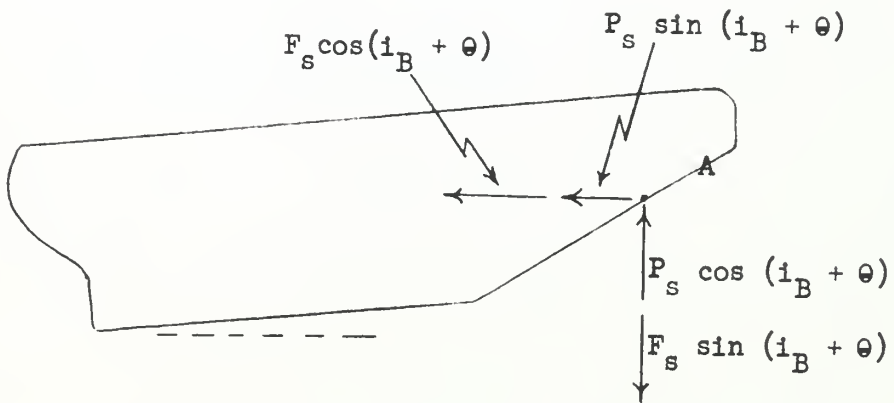
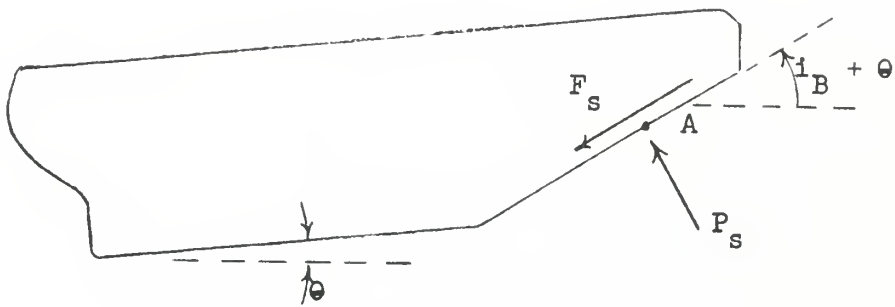
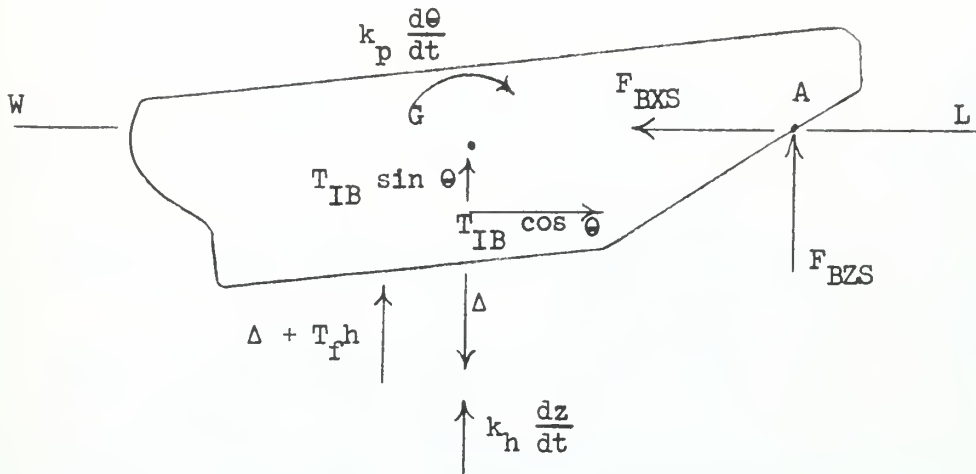


Figure B-XXIII
Free Body Diagram During Sliding Phase



$h \equiv$ increase of draft at LCF

$k_p \equiv$ coefficient of pitch damping

$k_h \equiv$ coefficient of Leave damping

$T_{IB} \equiv$ thrust available against ice

$T_f \equiv$ pounds per foot immersion

and the horizontal force to the left is

$$F_{BXS} = P_s \sin (i_B + \theta) + F_s \cos (i_B + \theta)$$

Substitution of equations (B74) and (B75) leads to

$$F_{BZS} = N \cos \beta \cos (i_B + \theta) - N f_k \sin (i_B + \theta)$$

$$F_{BZS} = N (\cos \beta \cos (i_B + \theta) - f_k \sin (i_B + \theta)) \quad (B76)$$

and

$$F_{BXS} = N \cos \beta \sin (i_B + \theta) + f_k N \cos (i_B + \theta)$$

$$F_{BXS} = N \left[\cos \beta \sin (i_B + \theta) + f_k \cos (i_B + \theta) \right] \quad (B77)$$

For reasons indicated and justified earlier, we shall use the following approximations:

$$\cos \theta = 1.0$$

$$\sin \theta = 0 \text{ radians}$$

$$\tan \theta = \theta \text{ radians}$$

Furthermore, the terms may also be rewritten using trigonometric substitutions.

Equation (B76) now becomes

$$F_{BZS} = N \left[\cos \beta (\cos i_B - \theta \sin i_B) - f_k (\sin i_B + \theta \cos i_B) \right]$$

$$F_{BZS} = N \left[\cos \beta \cos i_B - f_k \sin i_B - (\cos \beta \sin i_B + f_k \cos i_B) \theta \right] \quad (B78)$$

Equation (B77) now becomes

$$\left(\frac{1}{2} + \frac{1}{2} \right) \cos \frac{\pi}{2} + \left(\frac{1}{2} + \frac{1}{2} \right) \sin \frac{\pi}{2} = \cos \frac{\pi}{2}$$

or $\cos \frac{\pi}{2} = \cos \frac{\pi}{2}$ which is true

$$\left(\frac{1}{2} + \frac{1}{2} \right) \cos \frac{3\pi}{2} + \left(\frac{1}{2} + \frac{1}{2} \right) \sin \frac{3\pi}{2} = \cos \frac{3\pi}{2}$$

$$(iii) \left(\frac{1}{2} + \frac{1}{2} \right) \cos \frac{\pi}{2} - \left(\frac{1}{2} + \frac{1}{2} \right) \sin \frac{\pi}{2} = \cos \frac{\pi}{2}$$

$$\left(\frac{1}{2} + \frac{1}{2} \right) \cos \frac{3\pi}{2} - \left(\frac{1}{2} + \frac{1}{2} \right) \sin \frac{3\pi}{2} = \cos \frac{3\pi}{2}$$

$$(iv) \left[\left(\frac{1}{2} + \frac{1}{2} \right) \cos \frac{\pi}{2} + \left(\frac{1}{2} + \frac{1}{2} \right) \sin \frac{\pi}{2} \right] \cos \frac{\pi}{2} = \cos \frac{\pi}{2}$$

For values $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ and $\frac{\pi}{2}, \frac{3\pi}{2}$ we have $\cos \theta = \cos \theta$ and $\sin \theta = \sin \theta$

$$\cos \theta = \cos \theta$$

$$\sin \theta = \sin \theta$$

$$\cos \theta = \cos \theta$$

Therefore, the given equation is satisfied for all values of θ .

QED

Example (17) Prove that

$$\left[\cos \frac{\pi}{2} \cos \frac{\pi}{2} + \sin \frac{\pi}{2} \sin \frac{\pi}{2} \right] \cos \frac{\pi}{2} = \cos \frac{\pi}{2}$$

$$(18) \left[\cos \frac{\pi}{2} \cos \frac{\pi}{2} + \sin \frac{\pi}{2} \sin \frac{\pi}{2} \right] \sin \frac{\pi}{2} = \sin \frac{\pi}{2}$$

Example (17) Prove that

$$F_{\text{EKS}} = N \left[\cos \beta (\sin i_B + \theta) \cos i_B + f_k (\cos i_B - \theta \sin i_B) \right]$$

$$F_{\text{EKS}} = N \left[(\cos \beta \sin i_B + f_k \cos i_B) + (\cos \beta \cos i_B - f_k \sin i_B) \theta \right] \quad (\text{B79})$$

$$\text{Let } a_s = \cos \beta \sin i_B + f_k \cos i_B \quad (\text{B80})$$

$$\text{and } b_s = \cos \beta \cos i_B - f_k \sin i_B \quad (\text{B81})$$

$$\text{Then } F_{\text{BZS}} = N (b_s - a_s \theta) \quad (\text{B82})$$

$$\text{and } F_{\text{EKS}} = N (a_s + b_s \theta) \quad (\text{B83})$$

$$N = \frac{F_{\text{BZS}}}{(b_s - a_s \theta)} = \frac{F_{\text{EKS}}}{(a_s + b_s \theta)}$$

$$F_{\text{EKS}} = F_{\text{BZS}} \left(\frac{a_s + b_s \theta}{b_s - a_s \theta} \right) \quad (\text{B84})$$

Now F_{EKS} may be expressed in terms of the vertical force, F_{BZS} . The equation can be expanded and then the terms containing θ to a degree higher than the first may be dropped. This linearizing is valid since θ (in radians) will be relatively small.

$$F_{\text{EKS}} = F_{\text{BZS}} \left[\frac{a_s}{b_s} + \frac{a_s}{b_s} \left(\frac{a_s^2 + b_s^2}{a_s b_s} \right) \theta \right] \quad (\text{B85})$$

$$\left[\frac{1}{2} \left(\frac{1}{a^2} + \frac{1}{b^2} \right) x^2 - \frac{1}{a} x + \frac{1}{b} \right] x = \frac{1}{2} \ln^2 x$$

$$(197) \quad \left[\frac{1}{2} \left(\frac{1}{a^2} + \frac{1}{b^2} \right) x^2 - \frac{1}{a} x + \frac{1}{b} \right] x = \frac{1}{2} \ln^2 x$$

$$(198) \quad \frac{1}{2} \left(\frac{1}{a^2} + \frac{1}{b^2} \right) x^2 - \frac{1}{a} x + \frac{1}{b} = \frac{1}{2} \ln^2 x$$

$$(199) \quad \frac{1}{2} \left(\frac{1}{a^2} + \frac{1}{b^2} \right) x^2 - \frac{1}{a} x + \frac{1}{b} = \frac{1}{2} \ln^2 x$$

$$(200) \quad \left(\frac{1}{a^2} + \frac{1}{b^2} \right) x^2 - \frac{2}{a} x + \frac{2}{b} = \ln^2 x$$

$$(201) \quad \left(\frac{1}{a^2} + \frac{1}{b^2} \right) x^2 - \frac{2}{a} x + \frac{2}{b} = \ln^2 x$$

$$\frac{\frac{1}{2} \ln^2 x}{\left(\frac{1}{a^2} + \frac{1}{b^2} \right) x^2} = \frac{\frac{1}{2} \ln^2 x}{\left(\frac{1}{a^2} + \frac{1}{b^2} \right) x^2} = x$$

$$(202) \quad \left(\frac{1}{a^2} + \frac{1}{b^2} \right) x^2 - \frac{2}{a} x + \frac{2}{b} = \ln^2 x$$

Now we have to solve the equation in terms of the variable x . The

equation can be separated and then the terms containing x is a degree

higher than the first one to be dropped. This is similar to what we did

in the previous problem (another one) \square

$$(203) \quad \left[\frac{1}{2} \left(\frac{1}{a^2} + \frac{1}{b^2} \right) x^2 - \frac{1}{a} x + \frac{1}{b} \right] x = \frac{1}{2} \ln^2 x$$

Moment Arms

The free body diagram of the icebreaker during the sliding phase is shown in Figure B-XXIII.

The distance $(GA)_z$, the moment arm for the line of action of F_{BXS} may be expressed as

$$(GA)_z = H - KG + z \quad (B86)$$

where H is the initial draft and KG is the height of the center of gravity above the keel. It must be remembered that the origin of the coordinate system is at the position G had immediately prior to initial contact (State (1)). (See Figure B-XXIV).

At State(2), at the termination of crushing, the horizontal distance to point A is

$$(GA)_{x2} = \left(\frac{L}{2} - LCG\right) - \left[(H-KG) + \frac{\left(\frac{L}{2} - LCG\right)}{\tan i_B} \right] \theta_2 + \left[\frac{1}{\tan i_B} \right] z_2$$

See equation (B20).

Recall that, now that point A is fixed, any motion in the x-direction (beyond x_2) will reduce that value.

Therefore,

$$(GA)_x = \left(\frac{L}{2} - LCG\right) - (H-KG) + \frac{\left(\frac{L}{2} - LCG\right)}{\tan i_B} \theta_2 + \frac{1}{\tan i_B} z_2 - (x - x_2)$$

It is now possible to give a more detailed description of the
 system. The system is a linear system with a transfer function
 $G(s) = \frac{1}{s^2 + 2s + 1}$. The system is a second order system with a
 natural frequency of $\omega_n = 1$ and a damping ratio of $\zeta = 1$.

$$G(s) = \frac{1}{s^2 + 2s + 1} \quad (1)$$

The system is a linear system with a transfer function $G(s)$. The system is a second order system with a natural frequency of $\omega_n = 1$ and a damping ratio of $\zeta = 1$. The system is a linear system with a transfer function $G(s)$. The system is a second order system with a natural frequency of $\omega_n = 1$ and a damping ratio of $\zeta = 1$. The system is a linear system with a transfer function $G(s)$. The system is a second order system with a natural frequency of $\omega_n = 1$ and a damping ratio of $\zeta = 1$.

$$G(s) = \frac{1}{s^2 + 2s + 1} = \frac{1}{(s+1)^2}$$

$$G(s) = \frac{1}{(s+1)^2} = \frac{A}{s+1} + \frac{B}{(s+1)^2}$$

For $s = -1$

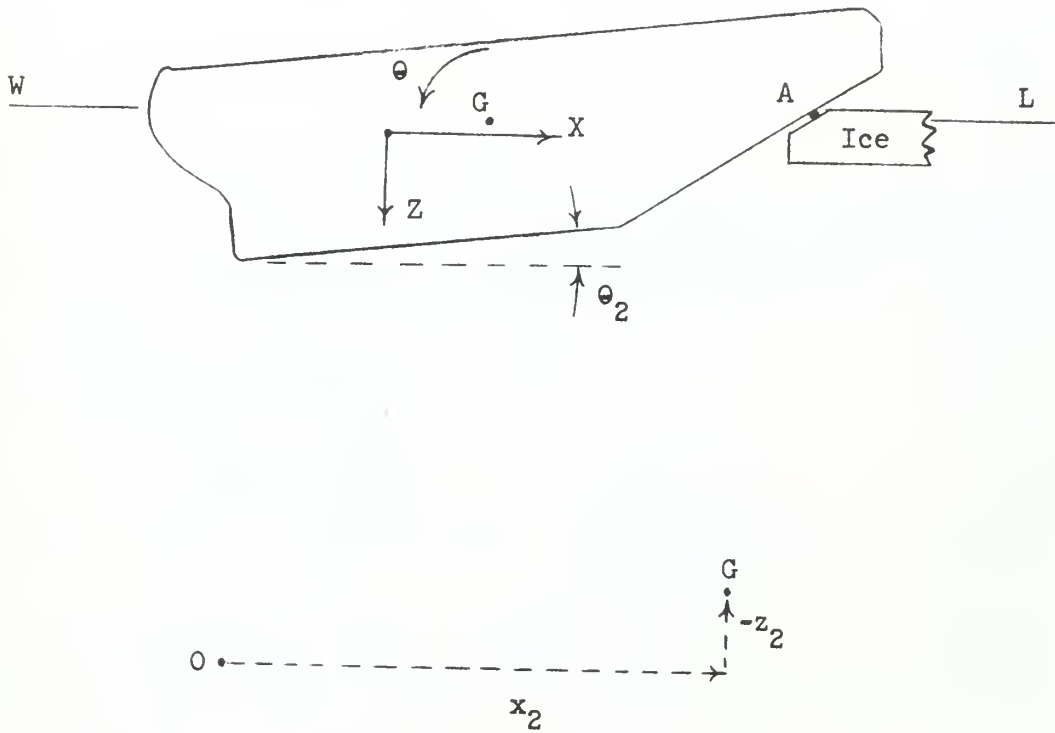
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$$G(s) = \frac{1}{s^2 + 2s + 1} = \frac{1}{(s+1)^2}$$

$$G(s) = \frac{1}{(s+1)^2} = \frac{A}{s+1} + \frac{B}{(s+1)^2}$$

Figure B-XXIV

Position of State 2, the Termination of the
Crushing Phase and the Commencement
of the Sliding Phase



$$\text{Let } k_4 = \left(\frac{L}{2} - LCG \right) - \left[(H-KG) + \frac{\left(\frac{L}{2} - LCG \right)}{\tan i_B} \right] \theta_2$$

$$+ \frac{z_2}{\tan i_B} + x_2 \quad (B37)$$

$$R_4 = (GA)_{x2} + x_2$$

$$\text{Then } (GA)_x = k_4 - x \quad (B38)$$

Newton's Laws of Motion During Sliding

With reference to Figure B-XXIII, Newton's Laws of motion may now be applied, for the sliding phase, rotationally about the center of gravity, in the x-direction, and in the z-direction.

In the horizontal direction

$$\sum F_x = m_x \frac{d^2 x}{dt^2}$$

$$T_{IB} \cos \theta - F_{BXS} = m_x \frac{d^2 x}{dt^2} \quad (B39)$$

From equation (B28)

$$T_{IB} = T_B \left(1 - \frac{dx}{v_1} \right)$$

From equation (B35)

$$F_{BXS} = F_{BZS} \frac{a_s}{b_s} + \frac{a_s}{b_s} \left(\frac{a_s^2 + b_s^2}{a_s b_s} \right) \theta$$

Equation (B39) may now be written as

$$\left[\frac{(1-x)^{-1}}{1-x} \right] = (1-x)^{-2} = \sum_{n=0}^{\infty} (n+1)x^n$$

(vii)

$$x^n = \frac{d^n}{dx^n} (1-x)^{-1}$$

$$x^n = \frac{d^n}{dx^n} (1-x)^{-1}$$

(viii)

$$x^n = \frac{d^n}{dx^n} (1-x)^{-1}$$

PROBLEM 10

Find the Taylor series for $f(x) = \ln(1+x)$ about $x=0$. Use the series to approximate $\ln(1.1)$ and $\ln(0.9)$. How many terms are needed to approximate $\ln(1.1)$ to within 10^{-4} ?

Solution:

$$\frac{d}{dx} \ln(1+x) = \frac{1}{1+x}$$

(ix)

$$\frac{d^n}{dx^n} \ln(1+x) = \frac{(-1)^{n-1} (n-2)!}{(1+x)^n}$$

(x)

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}$$

(xi)

$$\ln(1.1) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} (0.1)^n}{n}$$

as $n \rightarrow \infty$, $\frac{(-1)^{n-1} (0.1)^n}{n} \rightarrow 0$

$$T_{\text{BOL}} \left(1 - \frac{dx}{v_1 dt}\right) - F_{\text{BZS}} \left[\frac{a_s}{b_s} + \frac{a_s}{b_s} \left(\frac{a_s^2 + b_s^2}{a_s b_s} \right) \theta \right] - m_x \frac{d^2 x}{dt^2} = 0$$

$$T_{\text{BOL}} - \left(\frac{T_{\text{BOL}}}{v_1} \right) \frac{dx}{dt} - \frac{a_s}{b_s} F_{\text{BZS}} - \frac{a_s}{b_s} \left(\frac{a_s^2 + b_s^2}{a_s b_s} \right) F_{\text{BZS}} \theta - m_x \frac{d^2 x}{dt^2} = 0 \quad (\text{B90})$$

The summation of forces in the downward vertical direction (z-direction), as seen in Figure B-XXIII, may be expressed as

$$\sum F_z = m_z \frac{d^2 z}{dt^2}$$

$$- F_{\text{BZS}} - T_{\text{IB}} \theta - \Delta - T_f h - k_h \frac{dz}{dt} + \Delta - m_z \frac{d^2 z}{dt^2} = 0 \quad (\text{B91})$$

From equation (B17)

$$h = z + (\text{LCG} - \text{LCF}) \theta$$

Substitution of this and equation (B28) leads to

$$\begin{aligned} -F_{\text{BZS}} - T_{\text{BOL}} (\theta) + \frac{T_{\text{BOL}}}{v_1} \left(\frac{dx}{dt} \right) \theta - T_f z \\ F_{\text{BZS}} = - T_{\text{BOL}} \theta + \frac{T_{\text{BOL}}}{v_1} \left(\frac{dx}{dt} \right) \theta - T_f z \\ - T_f (\text{LCG} - \text{LCF}) \theta - k_h \frac{dz}{dt} - m_z \frac{d^2 z}{dt^2} \end{aligned} \quad (\text{B92})$$

Substitution of equation (B92) into equation (B90) leads to

$$\begin{aligned}
 T_{\text{BOL}} - \left(\frac{T_{\text{BOL}}}{v_1}\right) \frac{dx}{dt} + \frac{a_s}{b_s} T_{\text{BOL}} \theta - \frac{a_s}{b_s} \frac{T_{\text{BOL}}}{v_1} \left(\frac{dx}{dt}\right) \theta + \frac{a_s}{b_s} T_f z \\
 + \frac{a_s}{b_s} T_f (\text{LCG-LCF})\theta + k_h \frac{a_s}{b_s} \frac{dz}{dt} + \frac{a_s}{b_s} m_z \frac{d^2 z}{dt^2} \\
 + k_\gamma T_{\text{BOL}} \theta^2 - \frac{T_{\text{BOL}}}{v_1} k_\gamma \left(\frac{dx}{dt}\right) \theta^2 + k_\gamma T_f z \theta + k_\gamma T_f (\text{LCG-LCF})\theta^2 \\
 + k_\gamma k_h \frac{dz}{dt} \theta + k_\gamma m_z \theta \frac{d^2 z}{dt^2} - m_x \frac{d^2 x}{dt^2} = 0 \tag{B93}
 \end{aligned}$$

where $k_\gamma = \frac{a_s}{b_s} \left(\frac{a_s^2 + b_s^2}{a_s b_s} \right)$

$$k_\gamma = \frac{a_s^2 + b_s^2}{b_s^2} = 1 + \left(\frac{a_s}{b_s}\right)^2 \tag{B94}$$

The following products of variable terms appear in equation (B93):

$$\left(\frac{dx}{dt}\right)\theta \quad \theta^2$$

$$\left(\frac{dx}{dt}\right)\theta^2 \quad z \theta$$

$$\left(\frac{dz}{dt}\right) \theta$$

$$\left(\frac{d^2 z}{dt^2}\right) \theta$$

Maclaurin's Theorem may be used to put those terms in linear form.

Consider the equation (2) and (3) and let

$$z = \frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial x} \right) \frac{\partial \psi}{\partial y} + \frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial x} \right)^2 - \frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial y} \right)^2$$

$$\frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial x} \right)^2 - \frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial y} \right)^2 = \frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial x} \right)^2 - \frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial y} \right)^2$$

$$\frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial x} \right)^2 - \frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial y} \right)^2 = \frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial x} \right)^2 - \frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial y} \right)^2$$

(2)

$$\frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial x} \right)^2 - \frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial y} \right)^2 = \frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial x} \right)^2 - \frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial y} \right)^2$$

$$\left(\frac{\partial \psi}{\partial x} \right)^2 - \left(\frac{\partial \psi}{\partial y} \right)^2 = \frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial x} \right)^2 - \frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial y} \right)^2$$

(3)

$$\left(\frac{\partial \psi}{\partial x} \right)^2 - \left(\frac{\partial \psi}{\partial y} \right)^2 = \frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial x} \right)^2 - \frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial y} \right)^2$$

The following procedure of variable terms occurs in equation (2)

$$S = \frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial x} \right)^2 - \frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial y} \right)^2$$

$$S = \frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial x} \right)^2 - \frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial y} \right)^2$$

$$S = \frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial x} \right)^2 - \frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial y} \right)^2$$

$$S = \frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial x} \right)^2 - \frac{1}{\sqrt{a}} \left(\frac{\partial \psi}{\partial y} \right)^2$$

and the following procedure of variable terms occurs in equation (3)

$$\left(\frac{dx}{dt}\right) \Theta = \left(\frac{dx}{dt_2}\right) \Theta_2 + \left(\frac{dx}{dt}\right) \Theta - \left(\frac{dx}{dt_2}\right) \Theta_2 + \Theta_2 \frac{dx}{dt} - \left(\frac{dx}{dt_2}\right) \Theta_2$$

$$\left(\frac{dx}{dt}\right) \Theta = - \left(\frac{dx}{dt_2}\right) \Theta_2 + \left(\frac{dx}{dt_2}\right) \Theta + \Theta_2 \left(\frac{dx}{dt}\right)$$

$$\Theta^2 = -\Theta_2^2 + 2 \Theta_2 \Theta$$

$$\left(\frac{dx}{dt}\right) \Theta^2 = - \left(\frac{dx}{dt_2}\right) \Theta_2 \Theta + \left(\frac{dx}{dt_2}\right) \Theta^2 + \Theta_2 \left(\frac{dx}{dt}\right) \Theta$$

$$= - \left(\frac{dx}{dt_2}\right) \Theta_2 \Theta - \left(\frac{dx}{dt_2}\right) \Theta_2^2 + 2 \left(\frac{dx}{dt_2}\right) \Theta_2 \Theta$$

$$- \left(\frac{dx}{dt_2}\right) \Theta_2^2 + \left(\frac{dx}{dt_2}\right) \Theta_2 \Theta + \Theta_2^2 \left(\frac{dx}{dt}\right)$$

$$\left(\frac{dx}{dt}\right) \Theta^2 = - 2 \left(\frac{dx}{dt_2}\right) \Theta_2^2 + \left[2 \left(\frac{dx}{dt_2}\right) \Theta_2 \right] \Theta + \left[\Theta_2^2 \right] \left(\frac{dx}{dt}\right)$$

$$z \Theta = - z_2 \Theta_2 + z_2 \Theta + \Theta_2 z$$

$$\frac{dz}{dt} \Theta = - \left(\frac{dz}{dt_2}\right) \Theta_2 + \left(\frac{dz}{dt_2}\right) \Theta + \Theta_2 \left(\frac{dz}{dt}\right)$$

$$\left(\frac{d^2 z}{dt^2}\right) \Theta = - \left(\frac{d^2 z}{dt^2}\right) \Theta_2 + \left(\frac{d^2 z}{dt^2}\right) \Theta + \Theta_2 \left(\frac{d^2 z}{dt^2}\right)$$

Equation (B93) may now be written in linear form.

$$\begin{aligned} T_{\text{BOL}} - \left(\frac{T_{\text{BOL}}}{v_1}\right) \frac{dx}{dt} + \frac{a_s}{b_s} T_{\text{BOL}} \Theta + \frac{a_s}{b_s} \frac{T_{\text{BOL}}}{v_1} \left(\frac{dx}{dt_2}\right) \Theta_2 - \frac{a_s}{b_s} \frac{T_{\text{BOL}}}{v_1} \left(\frac{dx}{dt_2}\right) \Theta \\ - \frac{a_s}{b_s} \frac{T_{\text{BOL}}}{v_1} \Theta_2 \left(\frac{dx}{dt}\right) + \frac{a_s}{b_s} T_F z \end{aligned}$$

$$(1) \quad \frac{1}{x^2} = x^{-2} \Rightarrow \frac{d}{dx} x^{-2} = -2x^{-3} = -\frac{2}{x^3} \quad (1)$$

$$(2) \quad \frac{1}{x^3} = x^{-3} \Rightarrow \frac{d}{dx} x^{-3} = -3x^{-4} = -\frac{3}{x^4}$$

$$\dots \dots \dots$$

$$(n) \quad \frac{1}{x^n} = x^{-n} \Rightarrow \frac{d}{dx} x^{-n} = -nx^{-n-1} = -\frac{n}{x^{n+1}} \quad (2)$$

$$\therefore \frac{d}{dx} \left(\frac{1}{x^n} \right) = -\frac{n}{x^{n+1}} = -\frac{n}{x^n \cdot x} = -\frac{n}{x^{n+1}}$$

$$(3) \quad \frac{d}{dx} \left(\frac{1}{x} \right) = -\frac{1}{x^2}$$

$$(4) \quad \left[\frac{1}{x} \right]' = \left[x^{-1} \right]' = -x^{-2} = -\frac{1}{x^2} \quad (3)$$

$$\dots \dots \dots$$

$$(5) \quad \frac{d}{dx} \left(\frac{1}{x^2} \right) = -\frac{2}{x^3} = -\frac{2}{x^2 \cdot x} = -\frac{2}{x^3}$$

$$\left(\frac{1}{x^2} \right)' = -\frac{2}{x^3} = -\frac{2}{x^2 \cdot x} = -\frac{2}{x^3}$$

Example (1): Find the derivative of $\frac{1}{x^2}$ with respect to x .

$$\frac{d}{dx} \left(\frac{1}{x^2} \right) = \frac{d}{dx} x^{-2} = -2x^{-3} = -\frac{2}{x^3}$$

$$\therefore \frac{d}{dx} \left(\frac{1}{x^2} \right) = -\frac{2}{x^3}$$

$$+ \frac{a_s}{b_s} T_f (LCG-LCF)\theta + k_h \frac{a_s}{b_s} \frac{dz}{dt} + \frac{a_s}{b_s} m_z \frac{d^2 z}{dt^2} - k_5 T_{BOL} \theta_2^2 + 2 k_5 T_{BOL} \theta_2 \theta$$

$$+ \frac{2 T_{BOL} k_5}{v_1} \left(\frac{dx}{dt}\right)_2 \theta_2^2 - \frac{2 T_{BOL} k_5}{v_1} \left(\frac{dx}{dt}\right)_2 \theta_2 \theta - \frac{T_{BOL} k_5}{v_1} \theta_2^2 \frac{dx}{dt} - k_5 T_f z_2 \theta_2$$

$$+ k_5 T_f \theta_2 z$$

$$- k_5 T_f (LCG-LCF)\theta_2^2 + 2 k_5 T_f (LCG-LCF)\theta_2 \theta - k_5 k_h \left(\frac{dz}{dt}\right)_2 \theta + k_5 k_h \left(\frac{dz}{dt}\right)_2 \theta + k_5 k_h \theta_2 \frac{dz}{dt}$$

$$- k_5 m_z \left(\frac{d^2 z}{dt^2}\right)_2 \theta_2 + k_5 m_z \left(\frac{d^2 z}{dt^2}\right)_2 \theta + k_5 m_z \theta_2 \left(\frac{d^2 z}{dt^2}\right) - m_x \frac{d^2 x}{dt^2} = 0$$

$$a_{11} \dot{x} + b_{11} \dot{x} + c_{11} x + a_{12} \dot{z} + b_{12} \dot{z} + c_{12} z + a_{13} \dot{\theta} + b_{13} \dot{\theta} + c_{13} \theta = d_1$$

(B95)

where

$$a_{11} = -m_x$$

$$b_{11} = \frac{-T_{BOL}}{v_1} \frac{-a_s}{b_s} \frac{T_{BOL}}{v_1} \theta_2 - \frac{-T_{BOL} k_5}{v_1} \theta_2^2$$

$$c_{11} = 0$$

$$a_{12} = + \frac{a_s}{b_s} m_z + k_5 m_z \theta_2$$

$$b_{12} = + k_h \frac{a_s}{b_s} + k_5 k_h \theta_2$$

$$c_{12} = \frac{+a_s}{b_s} T_f + k_{\psi} T_f \theta_2$$

$$a_{13} = 0$$

$$b_{13} = 0$$

$$c_{13} = \frac{+a_s}{b_s} T_{BOL} - \frac{a_s}{b_s} \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt}\right)_2 + \frac{a_s}{b_s} T_f (LCG-LCF)$$

$$+ 2 k_{\psi} T_{BOL} \theta_2 - 2 \frac{T_{BOL}}{v_1} k_{\psi} \left(\frac{dx}{dt}\right)_2 \theta_2 + k_{\psi} T_f z_2$$

$$+ 2 k_{\psi} T_f (LCG-LCF) \theta_2 + k_{\psi} k_h \left(\frac{dz}{dt}\right)_2 + k_{\psi} m_z \left(\frac{d^2 z}{dt^2}\right)_2$$

$$d_1 = -T_{BOL} - \frac{a_s}{b_s} \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt}\right)_2 \theta_2 + k_{\psi} T_{BOL} \theta_2^2 - \frac{2 T_{BOL} k_{\psi}}{v_1} \left(\frac{dx}{dt}\right)_2 \theta_2^2$$

$$+ k_{\psi} T_f z_2 \theta_2 + k_{\psi} T_f (LCG-LCF) \theta_2^2 + k_{\psi} k_h \left(\frac{dz}{dt}\right)_2 \theta_2$$

$$+ k_{\psi} m_z \left(\frac{d^2 z}{dt^2}\right)_2 \theta_2$$

The summation of moments (counter clockwise) may be taken about the center of gravity. See Figure B-XXIII.

$$\sum M_{\curvearrowright} = I \frac{d^2 \theta}{dt^2}$$

$$r^2 = \frac{a^2}{2} = 2r^2$$

$$a = 2r^2$$

$$a = 2r^2$$

$$(1111-1111) = 2 \frac{a^2}{2} = \left(\frac{a^2}{2}\right) \frac{d^2}{1} = \frac{a^2}{2} \cdot d^2 = \frac{a^2}{2} \cdot \frac{a^2}{2} = 2r^2$$

$$2r^2 = 2r^2 \cdot \frac{a^2}{2} = \left(\frac{a^2}{2}\right) \left(\frac{d^2}{1}\right) = \frac{a^2 d^2}{2} = 2r^2$$

$$\left(\frac{a^2}{2}\right) \cdot \frac{d^2}{1} = \left(\frac{a^2}{2}\right) \cdot \frac{d^2}{1} + \left(\frac{a^2}{2}\right) \cdot \frac{d^2}{1} = 2r^2$$

$$\frac{1}{2} \left(\frac{a^2}{2}\right) \frac{d^2}{1} + \frac{1}{2} \left(\frac{a^2}{2}\right) \frac{d^2}{1} = \frac{a^2}{2} \cdot \frac{d^2}{1} = \frac{a^2}{2} \cdot \frac{a^2}{2} = 2r^2$$

$$2r^2 \left(\frac{a^2}{2}\right) \cdot \frac{d^2}{1} = \frac{1}{2} \left(\frac{a^2}{2}\right) \cdot \frac{d^2}{1} + \frac{1}{2} \left(\frac{a^2}{2}\right) \cdot \frac{d^2}{1} = 2r^2$$

$$\frac{1}{2} \left(\frac{a^2}{2}\right) \cdot \frac{d^2}{1} = 2r^2$$

The number of points is (1111-1111) times the number of points.

There are 1111 points in the plane.

$$\frac{a^2}{2} \cdot \frac{d^2}{1} = \frac{a^2}{2} \cdot \frac{d^2}{1}$$

$$F_{BZS} (GA)_x + F_{BXS} (GA)_z + T_{IB} \cos \theta (KG-d)$$

$$- (\Delta + T_f h) GM_L \theta - k_p \frac{d\theta}{dt} - I \frac{d^2\theta}{dt^2} = 0 \quad (B96)$$

From equation (B92)

$$F_{BZS} = - T_{BOL} \theta + \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt} \right) \theta - T_f z - T_f (LCG-LCF) \theta$$

$$- k_h \frac{dz}{dt} - m_z \frac{d^2 z}{dt^2}$$

From equations (B85) and (B94)

$$F_{BXS} = F_{BZS} \left[\frac{a_s}{b_s} + k_{\psi} \theta \right]$$

$$F_{BXS} = \frac{-a_s}{b_s} T_{BOL} \theta + \frac{a_s}{b_s} \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt} \right) \theta - \frac{a_s}{b_s} T_f z - \frac{a_s}{b_s} T_f (LCG-LCF) \theta$$

$$- \frac{a_s}{b_s} k_h \frac{dz}{dt} - \frac{a_s}{b_s} m_z \frac{d^2 z}{dt^2} - k_{\psi} T_{BOL} \theta^2 + k_{\psi} \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt} \right) \theta^2$$

$$- k_{\psi} T_f z \theta - k_{\psi} T_f (LCG-LCF) \theta^2 - k_{\psi} k_h \frac{dz}{dt} \theta - k_{\psi} m_z \left(\frac{d^2 z}{dt^2} \right) \theta$$

From equation (B23)

$$T_{IB} = T_{BOL} - \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt} \right)$$

From Equation (B17)

$$h = z + (LCG-LCF) \theta$$

From equation (B88)

$$(\ddot{GA})_x = k_4 \dot{x}$$

$$\text{where } k_4 = \left(\frac{L}{2} - LCG \right) - \left[(H-KG) \frac{+(\frac{L}{2} - LCG)}{\tan i_B} \right] \theta_2$$

$$\frac{+ z_2}{\tan i_B} + x_2$$

From equation (B86)

$$(GA)_z = (H-KG) + z$$

These equations must now be substituted into equation (B96).

$$-k_4 T_{BOL} \theta + k_4 \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt} \right) \theta - k_4 T_f z - k_4 T_f (LCG-LCF) \theta - k_4 k_h \frac{dz}{dt} - k_4 m_z \frac{d^2 z}{dt^2}$$

$$+ T_{BOL} \theta_x - \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt} \right) \theta_x + T_f z x + T_f (LCG-LCF) \theta_x + k_h \frac{dz}{dt} x + m_z \frac{d^2 z}{dt^2} x$$

$$- \frac{a_s}{b_s} T_{BOL} (H-KG) \theta + \frac{a_s}{b_s} \frac{T_{BOL}}{v_1} (H-KG) \frac{dx}{dt} \theta - \frac{a_s}{b_s} T_f (H-KG) z - \frac{a_s}{b_s} T_f (LCG-LCF) (H-KG) \theta$$

$$- \frac{a_s}{b_s} k_h (H-KG) \frac{dz}{dt} - \frac{a_s}{b_s} m_z (H-KG) \frac{d^2 z}{dt^2} - k_5 T_{BOL} (H-KG) \theta^2 + k_5 \frac{T_{BOL}}{v_1} (H-KG) \left(\frac{dx}{dt} \right) \theta^2$$

(11) $\frac{d}{dx} \left(\frac{1}{x^2} \right)$

$$= \frac{d}{dx} (x^{-2}) = -2x^{-3} = -\frac{2}{x^3}$$

(12) $\frac{d}{dx} \left(\frac{1}{x^3} \right)$

$$= \frac{d}{dx} (x^{-3}) = -3x^{-4} = -\frac{3}{x^4}$$

$$\frac{d}{dx} \left[\frac{(x^2 - 3)^2}{x^2 + 2x} \right] = \frac{2(x^2 - 3)(2x)}{(x^2 + 2x)^2} - \frac{2x(x^2 - 3)^2}{(x^2 + 2x)^3}$$

$$= \frac{2x^2 - 6}{(x^2 + 2x)^2} - \frac{2x^3 - 6x^2}{(x^2 + 2x)^3}$$

(13) $\frac{d}{dx} \left(\frac{1}{x^4} \right)$

$$= \frac{d}{dx} (x^{-4}) = -4x^{-5} = -\frac{4}{x^5}$$

(14) $\frac{d}{dx} \left(\frac{1}{x^5} \right)$

$$\frac{d}{dx} x^{-5} = -\frac{5}{x^6} = -\frac{5}{x^2} \cdot \frac{1}{x^4} = -\frac{5}{x^2} \cdot x^{-4} = -\frac{5}{x^2} \cdot \frac{1}{x^4} = -\frac{5}{x^6}$$

$$= \frac{d}{dx} x^{-5} = -\frac{5}{x^6} = -\frac{5}{x^2} \cdot \frac{1}{x^4} = -\frac{5}{x^2} \cdot x^{-4} = -\frac{5}{x^6}$$

$$\frac{d}{dx} \left(\frac{1}{x^6} \right) = \frac{d}{dx} (x^{-6}) = -6x^{-7} = -\frac{6}{x^7} = -\frac{6}{x^3} \cdot \frac{1}{x^4} = -\frac{6}{x^3} \cdot x^{-4} = -\frac{6}{x^7}$$

$$\frac{d}{dx} \left(\frac{1}{x^7} \right) = \frac{d}{dx} (x^{-7}) = -7x^{-8} = -\frac{7}{x^8} = -\frac{7}{x^4} \cdot \frac{1}{x^4} = -\frac{7}{x^4} \cdot x^{-4} = -\frac{7}{x^8}$$

$$- k_{\bar{y}} T_f (H-KG) z \theta - k_{\bar{y}} T_f (LCG-LCF)(H-KG) \theta^2 - k_{\bar{y}} k_h (H-KG) \frac{dz}{dt} \theta - k_{\bar{y}} m_z \left(\frac{d^2 z}{dt^2} \right) \theta (H-KG)$$

$$\frac{-a_s}{b_s} T_{BOL} \theta z + \frac{a_s z}{b_s} \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt} \right) \theta z - \frac{a_s}{b_s} T_f z^2 - \frac{a_s}{b_s} T_f (LCG-LCF) \theta z - \frac{a_s}{b_s} k_h \frac{dz}{dt} z$$

$$- \frac{a_s}{b_s} m_z \frac{d^2 z}{dt^2} z$$

$$- k_{\bar{y}} T_{BOL} \theta^2 z + k_{\bar{y}} \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt} \right) \theta^2 z - k_{\bar{y}} T_f \theta z^2 - k_{\bar{y}} T_f (LCG-LCF) \theta^2 z - k_{\bar{y}} k_h \frac{dz}{dt} \theta z$$

$$- k_{\bar{y}} m_z \left(\frac{d^2 z}{dt^2} \right) \theta z$$

$$+ T_{BOL} (KG-d) - \frac{T_{BOL}}{v_1} (KG-d) \frac{dx}{dt} - \Delta GM_L \theta - T_f GM_L \theta z - T_f GM_L (LCG-LCF) \theta^2 - k_p \frac{d\theta}{dt}$$

$$- I \frac{d^2 \theta}{dt^2} = 0 \quad (B98)$$

Non-linear terms of equation (B98) must be put into linear form, as was done earlier.

$$\left(\frac{dx}{dt} \right) \theta = - \left(\frac{dx}{dt} \right)_2 \theta_2 + \left(\frac{dx}{dt} \right)_2 \theta + \theta_2 \left(\frac{dx}{dt} \right)$$

$$\theta x = - \theta_2 x_2 + \theta_2 x + x_2 \theta$$

$$z s = - z_2 x_2 + z_2 s + x_2 z$$

$$10000 \left(\frac{1}{1.05} \right)^1 + 10000 \left(\frac{1}{1.05} \right)^2 + \dots + 10000 \left(\frac{1}{1.05} \right)^n = 10000 \left(\frac{1}{1.05} \right) \left[1 - \left(\frac{1}{1.05} \right)^n \right] / \left(\frac{1}{1.05} - 1 \right)$$

$$= \frac{10000}{0.05} \left[1 - \left(\frac{1}{1.05} \right)^n \right] = \frac{10000}{0.05} \left[1 - \frac{1}{1.05^n} \right] = 200000 \left[1 - \frac{1}{1.05^n} \right]$$

$$10000 \left(\frac{1}{1.05} \right)^1 + 10000 \left(\frac{1}{1.05} \right)^2 + \dots + 10000 \left(\frac{1}{1.05} \right)^n = 200000 \left[1 - \frac{1}{1.05^n} \right]$$

$$\frac{10000}{0.05} \left[1 - \left(\frac{1}{1.05} \right)^n \right] = 200000 \left[1 - \frac{1}{1.05^n} \right]$$

Therefore, the value of the annuity is 200,000 (1 - 1/1.05^n) / 0.05

$$\left(\frac{10000}{0.05} \right) \left[1 - \left(\frac{1}{1.05} \right)^n \right] = 200000 \left[1 - \frac{1}{1.05^n} \right]$$

$$200000 \left[1 - \frac{1}{1.05^n} \right] = 200000 \left[1 - \frac{1}{1.05^n} \right]$$

$$1 - \frac{1}{1.05^n} = 1 - \frac{1}{1.05^n}$$

$$\Theta^2 = -\Theta_2^2 + 2\Theta_2\Theta$$

$$z\Theta = -z_2\Theta_2 + z_2\Theta + \Theta_2 z$$

$$\left(\frac{dz}{dt}\right)\Theta = -\left(\frac{dz}{dt}\right)_2\Theta_2 + \left(\frac{dz}{dt}\right)_2\Theta + \Theta_2\left(\frac{dz}{dt}\right)$$

$$\left(\frac{d^2z}{dt^2}\right)\Theta = -\left(\frac{d^2z}{dt^2}\right)_2\Theta_2 + \left(\frac{d^2z}{dt^2}\right)_2\Theta + \Theta_2\left(\frac{d^2z}{dt^2}\right)$$

$$z^2 = -z_2^2 + 2z_2 z$$

$$\left(\frac{d}{dt}\right)z = -\left(\frac{dz}{dt}\right)_2 z_2 + \left(\frac{dz}{dt}\right)_2 z + z_2\left(\frac{dz}{dt}\right)$$

$$\left(\frac{d^2z}{dt^2}\right)z = -\left(\frac{d^2z}{dt^2}\right)_2 z_2 + \left(\frac{d^2z}{dt^2}\right)_2 z + z_2\left(\frac{d^2z}{dt^2}\right)$$

$$\left(\frac{dx}{dt}\right)\Theta x = -\left(\frac{dx}{dt}\right)_2\Theta_2 x + \left(\frac{dx}{dt}\right)_2\Theta x + \Theta_2\left(\frac{dx}{dt}\right)x$$

$$\begin{aligned} \left(\frac{dx}{dt}\right)\Theta x &= -\left(\frac{dx}{dt}\right)_2\Theta_2 x - \left(\frac{dx}{dt}\right)_2\Theta_2 x_2 + \left(\frac{dx}{dt}\right)_2\Theta_2 x \\ &+ \left(\frac{dx}{dt}\right)_2 x_2\Theta - \Theta_2\left(\frac{dx}{dt}\right)_2 x_2 + \Theta_2\left(\frac{dx}{dt}\right)_2 x \\ &+ \Theta_2 x_2\left(\frac{dx}{dt}\right) \end{aligned}$$

$$x^2 + 2x + 1 = (x+1)^2$$

$$x^2 + 2x + 1 = (x+1)^2$$

$$\frac{1}{x^2 + 2x + 1} = \frac{1}{(x+1)^2}$$

$$\frac{1}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2}$$

$$1 = A(x+1) + B$$

$$\frac{1}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2}$$

$$\frac{1}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2}$$

$$\frac{1}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2}$$

$$\frac{1}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2}$$

$$\frac{1}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2}$$

$$\frac{1}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2}$$

$$\left(\frac{dx}{dt}\right) \theta x = -2 \left(\frac{dx}{dt}\right)_2 \theta_2 x_2 + \left(\frac{dx}{dt}\right) x_2 \theta + \left(\frac{dx}{dt}\right)_2 \theta_2 x$$

$$+ \theta_2 x_2 \left(\frac{dx}{dt}\right)$$

$$\left(\frac{dx}{dt}\right) \theta^2 = -2 \left(\frac{dx}{dt}\right)_2 \theta_2^2 + 2 \left(\frac{dx}{dt}\right)_2 \theta_2 \theta + \theta_2^2 \left(\frac{dx}{dt}\right)$$

$$\left(\frac{dx}{dt}\right) \theta z = -2 \left(\frac{dx}{dt}\right)_2 \theta_2 z_2 + \left(\frac{dx}{dt}\right)_2 z_2 \theta + \left(\frac{dx}{dt}\right)_2 \theta_2 z$$

$$+ \theta_2 z_2 \left(\frac{dx}{dt}\right)$$

$$\theta^2 z = -2 z_2 \theta_2^2 + 2 z_2 \theta_2 \theta + \theta_2^2 z$$

$$\theta z^2 = -2 \theta_2 z_2^2 + 2 z_2 \theta_2 z + z_2^2 \theta$$

$$\left(\frac{dz}{dt}\right) \theta z = -2 \left(\frac{dz}{dt}\right)_2 \theta_2 z_2 + \left(\frac{dz}{dt}\right)_2 z_2 \theta + \left(\frac{dz}{dt}\right)_2 \theta_2 z$$

$$+ \theta_2 z_2 \left(\frac{dz}{dt}\right)$$

$$\left(\frac{d^2z}{dt^2}\right) \theta z = -2 \left(\frac{d^2z}{dt^2}\right)_2 \theta_2 z_2 + \left(\frac{d^2z}{dt^2}\right)_2 z_2 \theta + \left(\frac{d^2z}{dt^2}\right)_2 \theta_2 z$$

$$+ \theta_2 z_2 \left(\frac{d^2z}{dt^2}\right)$$

$$-2x \left(\frac{1}{x^2}\right) + 2 \left(\frac{1}{x^2}\right) = -\frac{2}{x} + \frac{2}{x^2} \quad (1)$$

$$(1) \quad x^2$$

$$\frac{1}{x^2} \cdot x^2 + 2 \left(\frac{1}{x^2}\right) \cdot x^2 = \frac{1}{x^2} \cdot x^2 + 2 \cdot x^2 \quad (2)$$

$$-2x \left(\frac{1}{x^2}\right) + 2 \left(\frac{1}{x^2}\right) = -2x \left(\frac{1}{x^2}\right) + 2 \left(\frac{1}{x^2}\right) \quad (3)$$

$$(3) \quad x^2$$

$$-2x \left(\frac{1}{x^2}\right) + 2 \left(\frac{1}{x^2}\right) = -2x \left(\frac{1}{x^2}\right) + 2 \left(\frac{1}{x^2}\right) \quad (4)$$

$$-2x \left(\frac{1}{x^2}\right) + 2 \left(\frac{1}{x^2}\right) = -2x \left(\frac{1}{x^2}\right) + 2 \left(\frac{1}{x^2}\right) \quad (5)$$

$$-2x \left(\frac{1}{x^2}\right) + 2 \left(\frac{1}{x^2}\right) = -2x \left(\frac{1}{x^2}\right) + 2 \left(\frac{1}{x^2}\right) \quad (6)$$

$$(6) \quad x^2$$

$$-2x \left(\frac{1}{x^2}\right) + 2 \left(\frac{1}{x^2}\right) = -2x \left(\frac{1}{x^2}\right) + 2 \left(\frac{1}{x^2}\right) \quad (7)$$

$$\left(\frac{1}{x^2}\right) \cdot x^2 + 2 \cdot x^2 =$$

$$\left(\frac{dx}{dt}\right) \theta^2 z = -2 \left(\frac{dx}{dt}\right)_2 \theta_2 z_2 \theta + \left(\frac{dx}{dt}\right)_2 z_2 \theta^2 + \left(\frac{dx}{dt}\right)_2 \theta_2 z \theta$$

$$+ \theta_2 z_2 \left(\frac{dx}{dt}\right) \theta$$

$$\left(\frac{dx}{dt}\right) \theta^2 z = -2 \left(\frac{dx}{dt}\right)_2 \theta_2 z_2 \theta - \left(\frac{dx}{dt}\right)_2 z_2 \theta_2^2 + 2 \left(\frac{dx}{dt}\right)_2 z_2 \theta_2 \theta$$

$$- \left(\frac{dx}{dt}\right)_2 \theta_2^2 z_2 + \left(\frac{dx}{dt}\right)_2 \theta_2 z_2 \theta + \left(\frac{dx}{dt}\right)_2 \theta_2^2 z$$

$$- \theta_2 z_2 \left(\frac{dx}{dt}\right)_2 \theta_2 + \theta_2 z_2 \left(\frac{dx}{dt}\right)_2 \theta$$

$$+ \theta_2 z_2 \theta_2 \left(\frac{dx}{dt}\right)$$

$$\left(\frac{dx}{dt}\right) \theta^2 z = -3 \left(\frac{dx}{dt}\right)_2 z_2 \theta_2^2 + 2 \left(\frac{dx}{dt}\right)_2 \theta_2 z_2 \theta + \left(\frac{dx}{dt}\right)_2 \theta_2^2 z$$

$$+ z_2 \theta_2^2 \left(\frac{dx}{dt}\right)$$

Equation (B98) may now be written in linear form.

$$-k_4 T_{BOL} \theta - k_4 \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt}\right)_2 \theta_2 + k_4 \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt}\right)_2 \theta + k_4 \frac{T_{BOL}}{v_1} \theta_2 \left(\frac{dx}{dt}\right) - k_4 T_f z$$

$$- k_4 T_f (LCG - LCF) \theta$$

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \quad (1)$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \quad (2)$$

$$a_{31}x_1 + a_{32}x_2 + \dots + a_{3n}x_n = b_3 \quad (3)$$

$$a_{41}x_1 + a_{42}x_2 + \dots + a_{4n}x_n = b_4 \quad (4)$$

$$a_{51}x_1 + a_{52}x_2 + \dots + a_{5n}x_n = b_5 \quad (5)$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \quad (m)$$

$$\vdots$$

where a_{ij} and b_i are constants and x_1, x_2, \dots, x_n are variables.

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

$$a_{31}x_1 + a_{32}x_2 + \dots + a_{3n}x_n = b_3$$

$$\vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m$$

$$a_{ij}x_j = b_i \quad (i=1, 2, \dots, m)$$

$$\begin{aligned}
 & -k_h k_h \left(\frac{dz}{dt}\right) - k_h m_z \left(\frac{d^2 z}{dt^2}\right) - T_{BOL} \theta_2 x_2 + T_{BOL} \theta_2 x + T_{BOL} x_2 \theta - \frac{+2T_{BOL}}{v_1} \left(\frac{dx}{dt}\right)_2 \theta_2 x_2 \\
 & \frac{-T_{BOL}}{v_1} \left(\frac{dx}{dt}\right)_2 x_2 \theta \\
 & \frac{-T_{BOL}}{v_1} \left(\frac{dx}{dt}\right)_2 \theta_2 x - \frac{-T_{BOL}}{v_1} \theta_2 x_2 \left(\frac{dx}{dt}\right) - T_f z_2 x_2 + T_f z_2 x + T_f x_2 z - T_f (LCG-LCF) \theta_2 x_2 \\
 & + T_f (LCG-LCF) \theta_2 x + T_f (LCG-LCF) x_2 \theta - k_h \left(\frac{dz}{dt}\right)_2 x_2 + k_h \left(\frac{dz}{dt}\right)_2 x + k_h x_2 \left(\frac{dz}{dt}\right) \\
 & - m_z \left(\frac{d^2 z}{dt^2}\right)_2 x_2 + m_z \left(\frac{d^2 z}{dt^2}\right)_2 x + m_z x_2 \left(\frac{d^2 z}{dt^2}\right) - \frac{a_s}{b_s} T_{BOL} (H-KG) \theta - \frac{a_s}{b_s} \frac{T_{BOL}}{v_1} (H-KG) \left(\frac{dx}{dt}\right)_2 \theta_2 \\
 & + \frac{a_s}{b_s} \frac{T_{BOL}}{v_1} (H-KG) \left(\frac{dx}{dt}\right)_2 \theta + \frac{a_s}{b_s} \frac{T_{BOL}}{v_1} (H-KG) \theta_2 \left(\frac{dx}{dt}\right) - \frac{a_s}{b_s} T_f (H-KG) z \\
 & - \frac{a_s}{b_s} T_f (LCG - LCF) (H-KG) \theta \\
 & - \frac{a_s}{b_s} k_h (H-KG) \frac{dz}{dt} - \frac{a_s}{b_s} m_z (H-KG) \frac{d^2 z}{dt^2} + k_5 T_{BOL} (H-KG) \theta_2^2 - 2 k_5 T_{BOL} (H-KG) \theta_2 \theta \\
 & - 2 k_5 \frac{T_{BOL}}{v_1} (H-KG) \left(\frac{dx}{dt}\right)_2 \theta_2^2 + 2 k_5 \frac{T_{BOL}}{v_1} (H-KG) \left(\frac{dx}{dt}\right)_2 \theta_2 \theta + k_5 \frac{T_{BOL}}{v_1} (H-KG) \theta_2^2 \left(\frac{dx}{dt}\right) \\
 & + k_5 T_f (H-KG) z_2 \theta_2
 \end{aligned}$$

$$x^2 \left[\frac{\partial}{\partial x} \left(\frac{\partial \psi}{\partial x} \right) - \frac{\partial \psi}{\partial x} \right] + \dots = \left(\frac{\partial \psi}{\partial x} \right) \dots - \left(\frac{\partial \psi}{\partial x} \right) \dots$$

$$\frac{\partial}{\partial x} \left(\frac{\partial \psi}{\partial x} \right) \dots = \dots \left(\frac{\partial \psi}{\partial x} \right) \dots$$

$$\left(\frac{\partial \psi}{\partial x} \right) \dots + \dots \left(\frac{\partial \psi}{\partial x} \right) \dots = \dots \left(\frac{\partial \psi}{\partial x} \right) \dots + \dots \left(\frac{\partial \psi}{\partial x} \right) \dots$$

$$\frac{\partial}{\partial x} \left(\frac{\partial \psi}{\partial x} \right) \dots = \dots \left(\frac{\partial \psi}{\partial x} \right) \dots + \dots \left(\frac{\partial \psi}{\partial x} \right) \dots$$

$$= \dots \left(\frac{\partial \psi}{\partial x} \right) \dots - \dots \left(\frac{\partial \psi}{\partial x} \right) \dots + \dots \left(\frac{\partial \psi}{\partial x} \right) \dots - \dots \left(\frac{\partial \psi}{\partial x} \right) \dots$$

$$\frac{\partial}{\partial x} \left(\frac{\partial \psi}{\partial x} \right) \dots = \dots \left(\frac{\partial \psi}{\partial x} \right) \dots$$

$$\frac{\partial}{\partial x} \left(\frac{\partial \psi}{\partial x} \right) \dots = \dots \left(\frac{\partial \psi}{\partial x} \right) \dots + \dots \left(\frac{\partial \psi}{\partial x} \right) \dots - \dots \left(\frac{\partial \psi}{\partial x} \right) \dots$$

$$\left(\frac{\partial \psi}{\partial x} \right) \dots + \dots \left(\frac{\partial \psi}{\partial x} \right) \dots = \dots \left(\frac{\partial \psi}{\partial x} \right) \dots + \dots \left(\frac{\partial \psi}{\partial x} \right) \dots$$

$$\frac{\partial}{\partial x} \left(\frac{\partial \psi}{\partial x} \right) \dots = \dots \left(\frac{\partial \psi}{\partial x} \right) \dots$$

$$-k_{\gamma} T_f (H-KG) z_2 \theta - k_{\gamma} T_f (H-KG) \theta_2 z + k_{\gamma} T_f (LCG-LCF)(H-KG) \theta_2^2 - 2k_{\gamma} T_f (LCG-LCF)(H-KG) \theta_2 \theta$$

$$+k_{\gamma} k_h (H-KG) \left(\frac{dz}{dt}\right)_2 \theta_2 - k_{\gamma} k_h (H-KG) \left(\frac{dz}{dt}\right)_2 \theta - k_{\gamma} k_h (H-KG) \theta_2 \left(\frac{dz}{dt}\right) + k_{\gamma} m_z (H-KG) \left(\frac{d^2 z}{dt^2}\right)_2 \theta_2$$

$$-k_{\gamma} m_z (H-KG) \left(\frac{d^2 z}{dt^2}\right)_2 \theta - k_{\gamma} m_z (H-KG) \theta_2 \left(\frac{d^2 z}{dt^2}\right) + \frac{a_s}{b_s} T_{BOL} \theta_2 z_2 - \frac{a_s}{b_s} T_{BOL} \theta_2 z - \frac{a_s}{b_s} T_{BOL} z_2 \theta$$

$$- \frac{2 a_s}{b_s} \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt}\right)_2 \theta_2 z_2 + \frac{a_s}{b_s} \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt}\right)_2 z_2 \theta + \frac{a_s}{b_s} \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt}\right)_2 \theta_2 z$$

$$+ \frac{a_s}{b_s} \frac{T_{BOL}}{v_1} \theta_2 z_2 \left(\frac{dx}{dt}\right)$$

$$+ \frac{a_s}{b_s} T_f z_2^2 - \frac{2 a_s}{b_s} T_f z_2 z + \frac{a_s}{b_s} T_f (LCG-LCF) \theta_2 z_2 - \frac{a_s}{b_s} T_f (LCG-LCF) \theta_2 z$$

$$- \frac{a_s}{b_s} T_f (LCG-LCF) z_2 \theta$$

$$+ \frac{a_s}{b_s} k_h \left(\frac{dz}{dt}\right)_2 z_2 - \frac{a_s}{b_s} k_h \left(\frac{dz}{dt}\right)_2 z - \frac{a_s}{b_s} k_h z_2 \left(\frac{dz}{dt}\right) + \frac{a_s}{b_s} m_z \left(\frac{d^2 z}{dt^2}\right)_2 z_2$$

$$- \frac{a_s}{b_s} m_z \left(\frac{d^2 z}{dt^2}\right)_2 z$$

مجموعه اول: $(1, 2, 3, \dots, n)$ و مجموعه دوم: $(1, 2, 3, \dots, n)$

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$$n \cdot \left(\frac{n-1}{2}\right) \cdot \frac{n-1}{2} = n \cdot \left(\frac{n-1}{2}\right) \cdot \frac{n-1}{2}$$

$$\left(\frac{n-1}{2}\right) \cdot \frac{n-1}{2}$$

$$n \cdot \left(\frac{n-1}{2}\right) \cdot \frac{n-1}{2} = n \cdot \left(\frac{n-1}{2}\right) \cdot \frac{n-1}{2}$$

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$$n \cdot \left(\frac{n-1}{2}\right) \cdot \frac{n-1}{2} = n \cdot \left(\frac{n-1}{2}\right) \cdot \frac{n-1}{2}$$

$$\left(\frac{n-1}{2}\right) \cdot \frac{n-1}{2}$$

$$\begin{aligned}
 & -\frac{a}{b} m_z z_2 \left(\frac{d^2 z}{dt^2} \right) + 2 k_5 T_{BOL} \theta_2^2 z_2 - 2 k_5 T_{BOL} z_2 \theta_2 \theta - k_5 T_{BOL} \theta_2^2 z \\
 & - 3 k_5 \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt} \right)_2 z_2 \theta_2^2 \\
 & + 2 k_5 \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt} \right)_2 \theta_2 z_2 \theta + k_5 \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt} \right)_2 \theta_2^2 z + k_5 \frac{T_{BOL}}{v_1} z_2 \theta_2^2 \left(\frac{dx}{dt} \right) + 2 k_5 T_f \theta_2 z_2^2 \\
 & - 2 k_5 T_f z_2 \theta_2 z - k_5 T_f z_2^2 \theta + 2 k_5 T_f (LCG-LCF) z_2 \theta_2^2 - 2 k_5 T_f (LCG-LCF) \theta_2 z_2 \theta \\
 & - k_5 T_f (LCG-LCF) \theta_2^2 z + 2 k_5 k_h \left(\frac{dz}{dt} \right)_2 \theta_2 z_2 - k_5 k_h \left(\frac{dz}{dt} \right)_2 z_2 \theta - k_5 k_h \left(\frac{dz}{dt} \right)_2 \theta_2 z \\
 & - k_5 k_h \theta_2 z_2 \left(\frac{dz}{dt} \right) \\
 & + 2 k_5 m_z \left(\frac{d^2 z}{dt^2} \right)_2 \theta_2 z_2 - k_5 m_z \left(\frac{d^2 z}{dt^2} \right)_2 \theta_2 z - k_5 m_z \left(\frac{d^2 z}{dt^2} \right)_2 z_2 \theta - k_5 m_z \theta_2 z_2 \left(\frac{d^2 z}{dt^2} \right) + T_{BOL} (KG-d) \\
 & - \frac{T_{BOL}}{v_1} (KG-d) \left(\frac{dx}{dt} - \Delta GM_L \theta + T_f GM_L \theta_2 z_2 - T_f GM_L z_2 \theta - T_f GM_L \theta_2 z \right. \\
 & \quad \left. + T_f GM_L (LCG-LCF) \theta_2^2 \right) \\
 & - 2 T_f GM_L (LCG-LCF) \theta_2 \theta - k_p \frac{d\theta}{dt} - I \frac{d^2 \theta}{dt^2} + 0 \tag{B99}
 \end{aligned}$$

دو طرفوں سے ضرب کر کے اور $\frac{1}{2}$ سے ضرب کر کے

$$\frac{1}{2} \left(\frac{1}{2} \right) = \frac{1}{4}$$

اس لیے $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ اور $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

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$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

(100) $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ اور $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$$a_{21} \dot{x} + b_{21} \ddot{x} + c_{21} x + a_{22} \dot{z} + b_{22} \ddot{z} + c_{22} z + a_2 \dot{\theta} + b_{23} \ddot{\theta} + c_{23} \theta = d_2 \quad (E100)$$

where

$$a_{21} = 0$$

$$b_{21} = +k_4 \frac{T_{BOL}}{v_1} \theta_2 - \frac{T_{BOL}}{v_1} \theta_2 x_2 + \frac{a_s}{b_s} \frac{T_{BOL}}{v_1} (H-KG) \theta_2$$

$$+ k_5 \frac{T_{BOL}}{v_1} (H-KG) \theta_2^2 + \frac{a_s}{b_s} \frac{T_{BOL}}{v_1} \theta_2 z_2 + k_5 \frac{T_{BOL}}{v_1} z_2 \theta_2^2$$

$$- \frac{T_{BOL}}{v_1} (KG-d)$$

$$c_{21} = +T_{BOL} \theta_2 \frac{-T_{BOL}}{v_1} \left(\frac{dx}{dt}\right)_2 \theta_2 + T_f z_2 + T_f (LCG-LCF) \theta_2$$

$$+ k_h \left(\frac{dz}{dt}\right)_2 + m_z \left(\frac{d^2 z}{dt^2}\right)_2$$

$$a_{22} = -k_4 m_z + m_z x_2 - \frac{a_s}{b_s} m_z (H-KG) - k_5 m_z (H-KG) \theta_2$$

$$\frac{-a_s}{b_s} m_z z_2 - k_5 m_z \theta_2 z_2$$

$$b_{22} = -k_4 k_h + k_h x_2 \frac{-a_s}{b_s} k_h (H-KG) - k_5 k_h (H-KG) \theta_2$$

$$- \frac{a_s}{b_s} k_h z_2 - k_5 k_h \theta_2 z_2$$

$$c_{22} = T_f x_2 - \frac{a_s}{b_s} T_f (H-KG) - k_5 T_f (H-KG) \theta_2 - k_4 T_f$$

$$- \frac{a_s}{b_s} T_{BOL} \theta_2 + \frac{a_s}{b_s} \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt} \right)_2 \theta_2 - \frac{2 a_s}{b_s} T_f z_2$$

$$- \frac{a_s}{b_s} T_f (LCG-LCF) \theta_2 - \frac{a_s}{b_s} k_h \left(\frac{dz}{dt} \right)_2 - \frac{a_s}{b_s} m_z \left(\frac{d^2 z}{dt^2} \right)_2$$

$$- k_5 T_{BOL} \theta_2^2 + k_5 \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt} \right)_2 \theta_2^2 - 2 k_5 T_f z_2 \theta_2$$

$$- k_5 T_f (LCG-LCF) \theta_2^2 - k_5 k_h \left(\frac{dz}{dt} \right)_2 \theta_2$$

$$- k_5 m_z \left(\frac{d^2 z}{dt^2} \right)_2 \theta_2 - T_f GM_L \theta_2$$

$$a_{23} = - I_B = - m \theta k^2$$

$$b_{23} = - k_p$$

$$c_{23} = -k_4 T_{BOL} + k_4 \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt}\right)_2 - k_4 T_f (LCG-LCF) + T_{BOL} x_2$$

$$- \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt}\right)_2 x_2 + T_f (LCG-LCF)x_2 - \frac{a_s}{b_s} T_{BOL} (H-KG)$$

$$+ \frac{a_s}{b_s} \frac{T_{BOL}}{v_1} (H-KG) \left(\frac{dx}{dt}\right)_2 - \frac{a_s}{b_s} T_f (LCG-LCF)(H-KG)$$

$$-2 k_5 T_{BOL} (H-KG)\theta_2 + 2 k_5 \frac{T_{BOL}}{v_1} (H-KG) \left(\frac{dx}{dt}\right)_2 \theta_2$$

$$-k_5 T_f (H-KG) z_2 - 2 k_5 T_f (LCG-LCF)(H-KG)\theta_2$$

$$-k_5 k_h (H-KG) \left(\frac{dz}{dt}\right)_2 - k_5 m_z (H-KG) \left(\frac{d^2 z}{dt^2}\right)_2$$

$$- \frac{a_s}{b_s} T_{BOL} z_2 + \frac{a_s}{b_s} \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt}\right)_2 z_2 - \frac{a_s}{b_s} T_f (LCG-LCF) z_2$$

$$- 2 k_5 T_{BOL} z_2 \theta_2 + 2 k_5 \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt}\right)_2 \theta_2 z_2$$

$$- k_5 T_f z_2^2 - 2 k_5 T_f (LCG-LCF)\theta_2 z_2$$

$$2 = 2x^2 + (20-2x) \cdot x + \frac{1}{2} \cdot \frac{2x^2}{x} + \frac{1}{2} \cdot \frac{2x^2}{x} \quad (2)$$

$$(20-2x) \cdot x = \frac{2x^2}{x} - 2x^2 \quad \text{من (1) } \frac{2x^2}{x} = 2x$$

$$(20-2x)(2x) = \frac{2x^2}{x} + \frac{2x^2}{x}$$

$$\text{من (2) } \frac{2x^2}{x} = 2x \quad \text{من (3) } 2x = 2x$$

$$(20-2x)(2x) = 2x + 2x$$

$$\frac{2x^2}{x} = 2x \quad \text{من (4) } 2x = 2x$$

$$(20-2x) \cdot x = \frac{2x^2}{x} - 2x^2 \quad \text{من (5) } \frac{2x^2}{x} = 2x$$

$$\frac{2x^2}{x} = 2x \quad \text{من (6) } 2x = 2x$$

$$\frac{2x^2}{x} = 2x \quad \text{من (7) } 2x = 2x$$

$$- k_{\gamma} k_h \left(\frac{dz}{dt} \right)_2 z_2 - k_{\gamma} m_z \left(\frac{d^2 z}{dt^2} \right)_2 z_2$$

$$- \Delta GM_L - T_f GM_L z_2 - 2 T_f GM_L (LCG-LCF) \theta_2$$

$$d_2 = + k_4 \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt} \right)_2 \theta_2 + T_{BOL} \theta_2 x_2 - 2 \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt} \right)_2 \theta_2 x_2$$

$$+ T_f z_2 x_2 + T_f (LCG-LCF) \theta_2 x_2 + k_h \left(\frac{dz}{dt} \right)_2 x_2$$

$$+ m_z \left(\frac{d^2 z}{dt^2} \right)_2 x_2 + \frac{a_s}{b_s} \frac{T_{BOL}}{v_1} (H-KG) \left(\frac{dx}{dt} \right)_2 \theta_2$$

$$- k_{\gamma} T_{BOL} (H-KG) \theta_2^2 + 2 k_{\gamma} \frac{T_{BOL}}{v_1} (H-KG) \left(\frac{dx}{dt} \right)_2 \theta_2^2$$

$$- k_{\gamma} T_f (H-KG) z_2 \theta_2 \omega - k_{\gamma} T_f (LCG-LCF) (H-KG) \theta_2^2$$

$$- k_{\gamma} k_h (H-KG) \left(\frac{dz}{dt} \right)_2 \theta_2^2 - k_{\gamma} m_z (H-KG) \left(\frac{d^2 z}{dt^2} \right)_2 \theta_2^2$$

$$- \frac{a_s}{b_s} T_{BOL} \theta_2 z_2 + \frac{2 a_s}{b_s} \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt} \right)_2 \theta_2 z_2$$

$$f_2\left(\frac{1}{2}\right) = f_2\left(\frac{1}{2}\right) = -$$

$$f_2(1) = f_2(1) = -$$

$$f_2\left(\frac{1}{2}\right) = \frac{1}{2} = - \frac{1}{2} \cdot \frac{1}{2} = - \frac{1}{4}$$

$$f_2\left(\frac{1}{4}\right) = \frac{1}{4} = - \frac{1}{4} \cdot \frac{1}{4} = - \frac{1}{16}$$

$$f_2\left(\frac{1}{8}\right) = \frac{1}{8} = - \frac{1}{8} \cdot \frac{1}{8} = - \frac{1}{64}$$

$$f_2\left(\frac{1}{16}\right) = \frac{1}{16} = - \frac{1}{16} \cdot \frac{1}{16} = - \frac{1}{256}$$

$$f_2\left(\frac{1}{32}\right) = \frac{1}{32} = - \frac{1}{32} \cdot \frac{1}{32} = - \frac{1}{1024}$$

$$f_2\left(\frac{1}{64}\right) = \frac{1}{64} = - \frac{1}{64} \cdot \frac{1}{64} = - \frac{1}{4096}$$

$$f_2\left(\frac{1}{128}\right) = \frac{1}{128} = - \frac{1}{128} \cdot \frac{1}{128} = - \frac{1}{16384}$$

$$-\frac{a_s}{b_s} T_f z_2^2 - \frac{a_s}{b_s} T_f (\text{LCG-LCF}) \theta_2 z_2$$

$$-\frac{a_s}{b_s} k_h \left(\frac{dz}{dt}\right)_2 z_2 - \frac{a_s}{b_s} m_z \left(\frac{d^2 z}{dt^2}\right)_2 z_2$$

$$-2 k_\psi T_{\text{BOL}} \theta_2^2 z_2 + 3 k_\psi \frac{T_{\text{BOL}}}{v_1} \left(\frac{dx}{dt}\right)_2 z_2 \theta_2^2$$

$$-2 k_\psi T_f \theta_2 z_2^2 - 2 k_\psi T_f (\text{LCG-LCF}) z_2 \theta_2^2$$

$$-2 k_\psi k_h \left(\frac{dz}{dt}\right)_2 \theta_2 z_2 - 2 k_\psi m_z \left(\frac{d^2 z}{dt^2}\right)_2 \theta_2 z_2$$

$$-T_{\text{BOL}} (\text{KG-d}) - T_f GM_L \theta_2 z_2 - T_f GM_L (\text{LCG-LCF}) \theta_2^2$$

1. $\frac{1}{x^2} = x^{-2}$ - $\frac{d}{dx} x^{-2} = -2x^{-3} = -\frac{2}{x^3}$

2. $\frac{1}{x^3} = x^{-3}$ - $\frac{d}{dx} x^{-3} = -3x^{-4} = -\frac{3}{x^4}$

3. $\frac{1}{x^4} = x^{-4}$ - $\frac{d}{dx} x^{-4} = -4x^{-5} = -\frac{4}{x^5}$

4. $\frac{1}{x^5} = x^{-5}$ - $\frac{d}{dx} x^{-5} = -5x^{-6} = -\frac{5}{x^6}$

5. $\frac{1}{x^6} = x^{-6}$ - $\frac{d}{dx} x^{-6} = -6x^{-7} = -\frac{6}{x^7}$

6. $\frac{1}{x^7} = x^{-7}$ - $\frac{d}{dx} x^{-7} = -7x^{-8} = -\frac{7}{x^8}$

Location Geometry

During the sliding phase point A, on the ice, is fixed relative to our coordinate system. Since the bow maintains contact with this point there must exist a definite relationship among $(\theta - \theta_2)$, $(z - z_2)$, and $(x - x_2)$. These relationships are illustrated in Figure B-XXV.

If the ship is rotated $(\theta - \theta_2)$ counterclockwise and raised $-(z - z_2)$, the ship must be advanced $(x - x_2)$ in order to maintain contact.

It may be seen that

$$\tan (i_B + \theta) = \frac{(GA)_{x2} (\theta - \theta_2) - (z - z_2)}{(x - x_2) - (GA)_{z2} (\theta - \theta_2)}$$

$$(x - x_2) = \frac{(GA)_{x2} (\theta - \theta_2) - (z - z_2)}{\tan (i_B + \theta)} + (GA)_{z2} (\theta - \theta_2)$$

$$(x - x_2) - \frac{(GA)_{x2} (\theta - \theta_2)}{\tan (i_B + \theta)} + \frac{(z - z_2)}{\tan (i_B + \theta)} + (GA)_{z2} (\theta - \theta_2) = 0 \tag{B101}$$

$$\text{Set } (GA)_{x2} = k_6 = \left(\frac{L}{2} - LCG \right) - \left[(H-KG) + \frac{\left(\frac{L}{2} - LCG \right)}{\tan i_B} \right] \theta_2$$

$$+ \frac{z_2}{\tan i_B}$$

$$\text{Set } (GA)_{z2} = k_7 = (H-KG) + z_2$$

Problem 10

Let $f(x) = \frac{1}{x^2} = x^{-2}$. Then $f'(x) = -2x^{-3} = -\frac{2}{x^3}$. We want to find the value of $f'(2)$.

Using the power rule, we have $f'(x) = -2x^{-3}$. Substituting $x = 2$, we get $f'(2) = -\frac{2}{2^3} = -\frac{2}{8} = -\frac{1}{4}$.

Alternatively, we can use the definition of the derivative: $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$. For $x = 2$, we have $f'(2) = \lim_{h \rightarrow 0} \frac{\frac{1}{(2+h)^2} - \frac{1}{2^2}}{h}$.

Combining the fractions in the numerator, we get $f'(2) = \lim_{h \rightarrow 0} \frac{\frac{1}{(2+h)^2} - \frac{1}{4}}{h} = \lim_{h \rightarrow 0} \frac{\frac{4 - (2+h)^2}{4(2+h)^2}}{h}$.

$$f'(2) = \lim_{h \rightarrow 0} \frac{4 - (2+h)^2}{4h(2+h)^2} = \lim_{h \rightarrow 0} \frac{4 - (4 + 4h + h^2)}{4h(2+h)^2} = \lim_{h \rightarrow 0} \frac{-4h - h^2}{4h(2+h)^2}$$

$$f'(2) = \lim_{h \rightarrow 0} \frac{-4h - h^2}{4h(2+h)^2} = \lim_{h \rightarrow 0} \frac{-4 - h}{4(2+h)^2} = \frac{-4 - 0}{4(2+0)^2} = \frac{-4}{4 \cdot 4} = -\frac{1}{4}$$

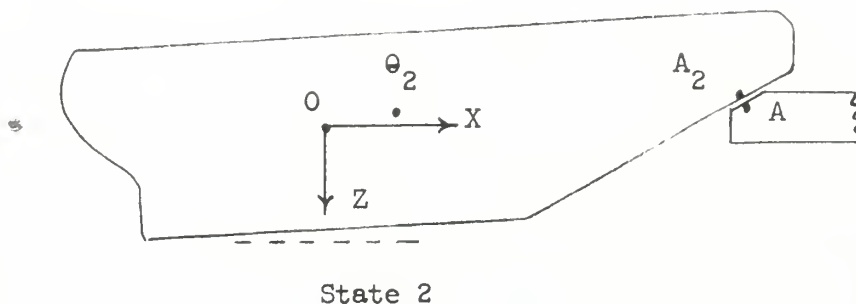
Thus, the derivative of $f(x) = \frac{1}{x^2}$ at $x = 2$ is $-\frac{1}{4}$.

$$f'(2) = -\frac{1}{4}$$

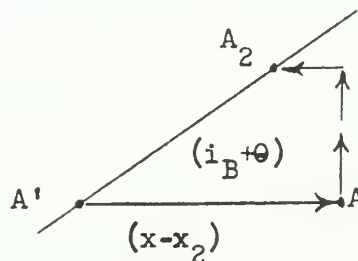
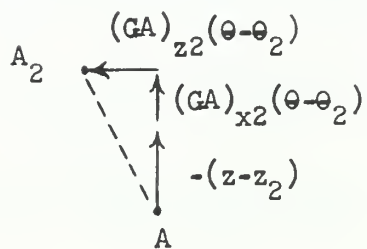
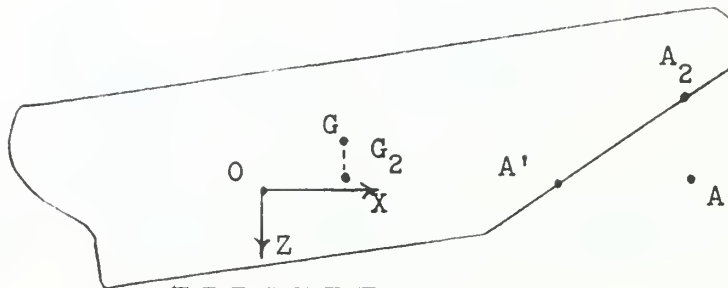
$$\frac{d}{dx} x^{-2} = -2x^{-3}$$

$$f'(2) = -\frac{2}{2^3} = -\frac{1}{4}$$

Figure B-XXV
Illustration of Position Geometry



Rotate $(\theta - \theta_2)$ about the center of gravity and then raise it $-(z-z_2)$.



From equation (B19)

$$\frac{1}{\tan(i_B + \theta)} = \frac{1}{\tan i_B} - \frac{\theta}{\sin^2 i_B}$$

Then equation (B101) becomes

$$x - x_2 - \frac{k_6(\theta - \theta_2)}{\tan i_B} + \frac{k_6(\theta - \theta_2)\theta}{\sin^2 i_B} + \frac{(z - z_2)}{\tan i_B}$$

$$- \frac{(z - z_2)\theta}{\sin^2 i_B} + k_7(\theta - \theta_2) = 0$$

$$x - x_2 - \frac{k_6}{\tan i_B} \theta + \frac{k_6 \theta_2}{\tan i_B} + \frac{k_6}{\sin^2 i_B} \theta^2 - \frac{k_6 \theta_2}{\sin^2 i_B} \theta$$

$$+ \frac{z}{\tan i_B} - \frac{z_2}{\tan i_B} - \frac{z\theta}{\sin^2 i_B} + \frac{z_2\theta}{\sin^2 i_B} + k_7\theta - k_7\theta_2 = 0$$

As was done previously, the non-linear terms must be linearized.

$$\theta^2 = -\theta_2^2 + 2\theta_2\theta$$

$$z\theta = -z_2\theta_2 + z_2\theta + \theta_2^2$$

$$x - x_2 - \frac{k_6}{\tan i_B} \theta + \frac{k_6}{\tan i_B} \theta_2 - \frac{k_6}{\sin^2 i_B} \theta_2^2$$

Two members (D) and (E)

$$\frac{1}{2} \frac{d^2 y}{dx^2} - \frac{1}{2} \frac{d^2 y}{dx^2} = \frac{1}{2} \frac{d^2 y}{dx^2}$$

Two members (D) and (E)

$$\frac{(y'' - y)}{2} = \frac{y'' - y}{2} + \frac{y'' - y}{2} = y'' - y$$

$$0 = (y'' - y) \Rightarrow \frac{d^2 y}{dx^2} - y = 0$$

$$y'' - y = 0 \Rightarrow y = C_1 e^x + C_2 e^{-x}$$

$$y = C_1 e^x + C_2 e^{-x} \Rightarrow y' = C_1 e^x - C_2 e^{-x}$$

As we have previously, the non-linear terms can be linearized.

$$y'' - y = 0$$

$$y'' - y = 0$$

$$y'' - y = 0 \Rightarrow y = C_1 e^x + C_2 e^{-x}$$

$$\begin{aligned}
 & + \frac{2 k_6 \theta_2}{\sin^2 i_B} \theta - \frac{k_6 \theta_2}{\sin^2 i_B} \theta + \frac{z}{\tan i_B} - \frac{z_2}{\tan i_B} \\
 & + \frac{z_2 \theta_2}{\sin^2 i_B} - \frac{z_2}{\sin^2 i_B} \theta - \frac{\theta_2}{\sin^2 i_B} z \\
 & + \frac{z_2 \theta}{\sin^2 i_B} + k_7 \theta - k_7 \theta_2 = 0 \\
 x - x_2 & - \frac{k_6}{\tan i_B} \theta + \frac{k_6}{\tan i_B} \theta_2 - \frac{k_6 \theta_2^2}{\sin^2 i_B} + \frac{k_6 \theta_2}{\sin^2 i_B} \theta \\
 & + \frac{z}{\tan i_B} - \frac{z_2}{\tan i_B} + \frac{z_2 \theta_2}{\sin^2 i_B} - \frac{\theta_2}{\sin^2 i_B} z \\
 & + k_7 \theta - k_7 \theta_2 = 0 \tag{E102}
 \end{aligned}$$

Equation (E102) may be written as

$$\begin{aligned}
 a_{31} \dot{x} + b_{31} \dot{x} + c_{31} x + a_{32} \dot{z} + b_{32} \dot{z} + c_{32} z \\
 + a_{33} \dot{\theta} + b_{33} \dot{\theta} + c_{33} \theta = d_3 \tag{E103}
 \end{aligned}$$

where

$$\frac{1}{x^2} = x^{-2} \Rightarrow \frac{d}{dx} x^{-2} = -2x^{-3} = -\frac{2}{x^3}$$

$$\frac{d}{dx} \frac{1}{x^3} = \frac{d}{dx} x^{-3} = -3x^{-4} = -\frac{3}{x^4}$$

$$\frac{d}{dx} \frac{1}{x^4} = \frac{d}{dx} x^{-4} = -4x^{-5} = -\frac{4}{x^5}$$

$$\frac{d}{dx} \frac{1}{x^5} = \frac{d}{dx} x^{-5} = -5x^{-6} = -\frac{5}{x^6}$$

$$\frac{d}{dx} \frac{1}{x^6} = \frac{d}{dx} x^{-6} = -6x^{-7} = -\frac{6}{x^7}$$

(2013) $\frac{d}{dx} \frac{1}{x^7} = \frac{d}{dx} x^{-7} = -7x^{-8} = -\frac{7}{x^8}$

no need to use the rule (2013) and (2014)

$$\frac{d}{dx} \frac{1}{x^8} = \frac{d}{dx} x^{-8} = -8x^{-9} = -\frac{8}{x^9}$$

(2014) $\frac{d}{dx} \frac{1}{x^9} = \frac{d}{dx} x^{-9} = -9x^{-10} = -\frac{9}{x^{10}}$

answer

$$a_{31} = 0$$

$$b_{31} = 0$$

$$c_{31} = +1$$

$$a_{32} = 0$$

$$b_{32} = 0$$

$$c_{32} = \frac{1}{\tan i_B} - \frac{\theta_2}{\sin^2 i_B}$$

$$a_{33} = 0$$

$$b_{33} = 0$$

$$c_{33} = \frac{-k_6}{\tan i_B} + \frac{k_6 \theta_2}{\sin^2 i_B} + k_7$$

$$d_3 = +x_2 \frac{-k_6 \theta_2}{\tan i_B} + \frac{k_6 \theta_2^2}{\sin^2 i_B} + \frac{z_2}{\tan i_B}$$

$$\frac{-z_2 \theta_2}{\sin^2 i_B} + k_7 \theta_2$$

where $k_6 = (GA)_{x2}$ $k_7 = (GA)_{z2}$

$$x = 2^t$$

$$y = 2^{2t}$$

$$z = 2^{4t}$$

$$w = 2^{8t}$$

$$v = 2^{16t}$$

$$\frac{d^2 x}{dt^2} = \frac{1}{2^t} = 2^{-t}$$

$$x = 2^{2t}$$

$$y = 2^{4t}$$

$$z = \frac{2^{6t} \cdot 2^t}{2^t \cdot 2^{2t}} = \frac{2^{6t}}{2^t} = 2^{5t}$$

$$\frac{d^2 y}{dt^2} = \frac{2^{2t} \cdot 2^t}{2^t \cdot 2^{2t}} = \frac{2^{2t}}{2^t} = 2^t$$

$$x^2 = \frac{2^{4t} \cdot 2^t}{2^t \cdot 2^{2t}} = \frac{2^{4t}}{2^t} = 2^{3t}$$

$$S_x(40) = 2^{40}$$

$$S_x(40) = 2^{40} \text{ decoupled}$$

Simultaneous Equations of Sliding

Grouping the three equations,

Equation (B95)

$$a_{11}\dot{x} + b_{11}\dot{x} + (0)x + a_{12}\dot{z} + b_{12}\dot{z} + c_{12}z + (0)\dot{\theta} + (0)\dot{\theta} + c_{13}\theta = d_1$$

Equation (B100)

$$(0)\dot{x} + b_{21}\dot{x} + c_{21}x + a_{22}\dot{z} + b_{22}\dot{z} + c_{22}z + a_{23}\dot{\theta} + b_{23}\dot{\theta} + c_{23}\theta = d_2$$

Equation (B103)

$$(0)\dot{x} + (0)\dot{x} + c_{31}x + (0)\dot{z} + (0)\dot{z} + c_{32}z + (0)\dot{\theta} + (0)\dot{\theta} + c_{33}\theta = d_3$$

Rewriting in operational form.

$$(a_{11}D^2 + b_{11}D + 0)x + (a_{12}D^2 + b_{12}D + c_{12})z + (0D^2 + 0D + c_{13})\theta = d_1$$

$$(0D^2 + b_{21}D + c_{21})x + (a_{22}D^2 + b_{22}D + c_{22})z + (a_{23}D^2 + b_{23}D + c_{23})\theta = d_2$$

$$(c_{31})x + (c_{32})z + (c_{33})\theta = d_3$$

Method of Variation of Parameters

and find the particular solution

(199) method

$$y'' - 0.2y' + 0.1y = 0 \quad y(0) = 1 \quad y(\infty) = 0 \quad y_1 = e^{-0.1x} \quad y_2 = e^{-0.1x}$$

(200) method

$$y'' + 0.2y' + 0.1y = 0 \quad y(0) = 1 \quad y(\infty) = 0 \quad y_1 = e^{-0.1x} \quad y_2 = e^{-0.1x}$$

(201) method

$$y'' + 0.2y' + 0.1y = 0 \quad y(0) = 1 \quad y(\infty) = 0 \quad y_1 = e^{-0.1x} \quad y_2 = e^{-0.1x}$$

Method of Variation of Parameters

$$y'' + 0.2y' + 0.1y = 0 \quad y(0) = 1 \quad y(\infty) = 0 \quad y_1 = e^{-0.1x} \quad y_2 = e^{-0.1x}$$

$$y'' + 0.2y' + 0.1y = 0 \quad y(0) = 1 \quad y(\infty) = 0 \quad y_1 = e^{-0.1x} \quad y_2 = e^{-0.1x}$$

$$y'' + 0.2y' + 0.1y = 0 \quad y(0) = 1 \quad y(\infty) = 0 \quad y_1 = e^{-0.1x} \quad y_2 = e^{-0.1x}$$

Laplace Transforms

$$\underline{f(t)} \qquad \qquad \underline{F(s)} = \underline{[f(t)]}$$

$$\frac{d}{dt} [f(t)] \qquad \qquad s \underline{[f(t)]} - f(0^+)$$

$$\frac{d^2}{dt^2} [f(t)] \qquad \qquad s^2 \underline{[f(t)]} - s f(0^+) - \frac{df}{dt}(0^+)$$

For example, if $f(t) = x$

$$\underline{[a D^2 x]} = a s^2 \underline{(x)} - a s x_0 - a \dot{x}_0$$

$$\underline{[D x]} = b s \underline{(x)} - b x_0$$

$$\underline{[c x]} = c \underline{(x)}$$

For further example, if $x = x_2$ and $\frac{dx}{dt} = (\frac{dx}{dt})_2$ at $t = 0$,

$$\underline{[a D^2 + b D + c] x} = a s^2 \underline{(x)} - a s x_2 - a \dot{x}_2$$

$$+ b s \underline{(x)} - b x_2$$

$$+ c \underline{(x)}$$

$$\underline{[a D^2 + b D + c] x} = (a s^2 + b s + c) \underline{[x]} - a s x_2 - a \dot{x}_2 - b x_2$$

10/10/2021

$$x^2 - 1 = (x-1)(x+1)$$

$$x^2 - 1 = (x-1)(x+1)$$

$$x^2 - 1 = (x-1)(x+1)$$

$$x^2 - 1 = (x-1)(x+1)$$

$$(x^2 - 1) - (x-1)(x+1) = 0$$

$$(x^2 - 1) - (x-1)(x+1) = 0$$

For example, let $f(x) = x^2 - 1$

$$f(x) = x^2 - 1 = (x-1)(x+1)$$

$$f(x) = x^2 - 1 = (x-1)(x+1)$$

$$f(x) = x^2 - 1 = (x-1)(x+1)$$

The function $f(x) = x^2 - 1$ is a polynomial function of degree 2.

$$f(x) = x^2 - 1 = (x-1)(x+1)$$

$$f(x) = x^2 - 1 = (x-1)(x+1)$$

$$f(x) = x^2 - 1 = (x-1)(x+1)$$

$$f(x) = x^2 - 1 = (x-1)(x+1)$$

The three simultaneous equations (B95) (B100), and (B103) may be written as follows using Laplace Transforms.

$$(a_{11}s^2 + b_{11}s) \mathcal{L}[x] + (a_{12}s^2 + b_{12}s + c_{12}) \mathcal{L}[z] + (c_{13}) \mathcal{L}[\theta] =$$

$$a_{11}s x_2 + a_{11}\dot{x}_2 + b_{11}x_2 + a_{12}s z_2 + a_{12}\dot{z}_2 + b_{12}z_2 + \frac{d_1}{s}$$

$$(b_{21}s + c_{21}) \mathcal{L}[x] + (a_{22}s^2 + b_{22}s + c_{22}) \mathcal{L}[z] + (a_{23}s^2 + b_{23}s + c_{23}) \mathcal{L}[\theta] =$$

$$b_{21}x_2 + a_{22}s z_2 + a_{22}\dot{z}_2 + b_{22}z_2 + a_{23}s \theta_2 + a_{23}\dot{\theta}_2 + b_{23}\theta_2 + \frac{d_2}{s}$$

$$(c_{21}) \mathcal{L}[x] + (c_{32}) \mathcal{L}[z] + (c_{33}) \mathcal{L}[\theta] = \frac{d_3}{s}$$

Let $d_{11} = a_{11}\dot{x}_2 + b_{11}x_2 + a_{12}\dot{z}_2 + b_{12}z_2$

$$d_{12} = a_{11}x_2 + a_{12}z_2 \tag{B104}$$

$$d_{13} = d_1$$

$$d_{21} = b_{21}x_2 + a_{22}\dot{z}_2 + b_{22}z_2 + a_{23}\dot{\theta}_2 + b_{23}\theta_2$$

$$d_{22} = a_{22}z_2 + a_{23}\theta_2$$

$$d_{23} = d_2$$

$$d_{33} = d_3$$

Let \$f(x) = x^2 + 2x + 1\$ and \$g(x) = x^2 + 1\$. Find \$f(x) + g(x)\$ and \$f(x) - g(x)\$.

$$f(x) + g(x) = (x^2 + 2x + 1) + (x^2 + 1)$$

$$= x^2 + 2x + 1 + x^2 + 1$$

$$= (x^2 + x^2) + (2x) + (1 + 1)$$

$$= 2x^2 + 2x + 2$$

$$f(x) - g(x) = (x^2 + 2x + 1) - (x^2 + 1)$$

$$= x^2 + 2x + 1 - x^2 - 1$$

$$(2007) \quad = x^2 - x^2 + 2x + 1 - 1$$

$$= 2x$$

$$= 2x^2 + 2x + 2 - 2x^2 - 2$$

$$= 2x^2 - 2x^2 + 2x + 2 - 2$$

$$= 2x$$

$$= 2x$$

Then the simultaneous equations may be written in their shortened form.

$$(a_{11}s^2 + b_{11}s) \mathcal{L}[x] + (a_{12}s^2 + b_{12}s + c_{12}) \mathcal{L}[z] + (c_{13}) \mathcal{L}[\theta] = d_{11} + d_{12}s + \frac{d_{13}}{s} \quad (E105)$$

$$(b_{21}s + c_{21}) \mathcal{L}[x] + (a_{22}s^2 + b_{22}s + c_{22}) \mathcal{L}[z] + (a_{23}s^2 + b_{23}s + c_{23}) \mathcal{L}[\theta] = d_{21} + d_{22}s + \frac{d_{23}}{s} \quad (E106)$$

$$(c_{31}) \mathcal{L}[x] + (c_{32}) \mathcal{L}[z] + (c_{33}) \mathcal{L}[\theta] = \frac{d_{33}}{s}$$

Using determinant form, the simultaneous Laplace equations may be solved for $\mathcal{L}[x]$, $\mathcal{L}[z]$, and $\mathcal{L}[\theta]$.

Find the value of the expression...

$$\frac{1}{2} \int_{-1}^1 (x^2 + 1) dx = \frac{1}{2} \left[\frac{x^3}{3} + x \right]_{-1}^1 = \frac{1}{2} \left(\frac{1}{3} + 1 - \left(-\frac{1}{3} - 1 \right) \right) = \frac{1}{2} \left(\frac{1}{3} + 1 + \frac{1}{3} + 1 \right) = \frac{1}{2} \left(\frac{2}{3} + 2 \right) = \frac{1}{2} \left(\frac{8}{3} \right) = \frac{4}{3}$$

(2018)

$$\int_{-1}^1 (x^2 + 1) dx = \left[\frac{x^3}{3} + x \right]_{-1}^1 = \left(\frac{1}{3} + 1 \right) - \left(-\frac{1}{3} - 1 \right) = \frac{1}{3} + 1 + \frac{1}{3} + 1 = \frac{2}{3} + 2 = \frac{8}{3}$$

(2018)

$$\frac{1}{2} \int_{-1}^1 (x^2 + 1) dx = \frac{4}{3}$$

$$\frac{1}{2} \int_{-1}^1 (x^2 + 1) dx = \frac{1}{2} \left[\frac{x^3}{3} + x \right]_{-1}^1 = \frac{1}{2} \left(\frac{1}{3} + 1 - \left(-\frac{1}{3} - 1 \right) \right) = \frac{1}{2} \left(\frac{1}{3} + 1 + \frac{1}{3} + 1 \right) = \frac{1}{2} \left(\frac{2}{3} + 2 \right) = \frac{1}{2} \left(\frac{8}{3} \right) = \frac{4}{3}$$

Find the value of the expression...

$$\int_{-1}^1 (x^2 + 1) dx = \frac{8}{3}$$

$$(d_{11} + d_{12}s + \frac{d_{13}}{s}) (a_{12}s^2 + b_{12}s + c_{12}) \quad (c_{13})$$

$$(d_{21} + d_{22}s + \frac{d_{23}}{s}) (a_{22}s^2 + b_{22}s + c_{22}) (a_{23}s^2 + b_{23}s + c_{23})$$

$$L \{x\} = N_1 = \frac{\begin{matrix} (\frac{d_{33}}{s}) & (c_{32}) & (c_{33}) \end{matrix}}{\begin{matrix} (a_{11}s^2 + b_{11}s) & (a_{12}s^2 + b_{12}s + c_{12}) & (c_{13}) \\ (b_{21}s + c_{21}) & (a_{22}s^2 + b_{22}s + c_{22}) & (a_{23}s^2 + b_{23}s + c_{23}) \\ (c_{31}) & (c_{32}) & (c_{33}) \end{matrix}}$$

$$N_1 = d_{11}a_{22}c_{33}s^2 + d_{11}b_{22}c_{33}s + d_{11}c_{22}c_{33} + d_{12}a_{22}c_{33}s^3 + d_{12}b_{22}c_{33}s^2 + d_{12}c_{22}c_{33}s$$

$$+ d_{13}a_{22}c_{33}s + d_{13}b_{22}c_{33} + d_{13}c_{22}c_{33} \frac{1}{s} + a_{12}a_{23}d_{33}s^3 + a_{12}b_{23}d_{33}s^2 + a_{12}c_{23}d_{33}s$$

$$+ b_{12}a_{23}d_{33}s^2 + b_{12}b_{23}d_{33}s + b_{12}c_{23}d_{33} + c_{12}a_{23}d_{33}s + c_{12}b_{23}d_{33} + c_{12}c_{23}d_{33} \frac{1}{s}$$

$$+ c_{13}d_{21}c_{32} + c_{13}d_{22}c_{32}s + c_{13}d_{23}c_{32} \frac{1}{s} - d_{11}a_{23}c_{32}s^2 - d_{11}b_{23}c_{32}s - d_{11}c_{23}c_{32}$$

$$- d_{12}a_{23}c_{32}s^3 - d_{12}b_{23}c_{32}s^2 - d_{12}c_{23}c_{32}s - d_{13}a_{23}c_{32}s - d_{13}b_{23}c_{32} - d_{13}c_{23}c_{32} \frac{1}{s}$$

1. $\int \frac{1}{x^2} dx = -\frac{1}{x} + C$

2. $\int \frac{1}{x^3} dx = -\frac{1}{2x^2} + C$

$$\int \frac{1}{x^2} dx = -\frac{1}{x} + C \quad \int \frac{1}{x^3} dx = -\frac{1}{2x^2} + C$$

3. $\int \frac{1}{x^4} dx = -\frac{1}{3x^3} + C$

4. $\int \frac{1}{x^5} dx = -\frac{1}{4x^4} + C$

5. $\int \frac{1}{x^6} dx = -\frac{1}{5x^5} + C$

6. $\int \frac{1}{x^7} dx = -\frac{1}{6x^6} + C$

7. $\int \frac{1}{x^8} dx = -\frac{1}{7x^7} + C$

8. $\int \frac{1}{x^9} dx = -\frac{1}{8x^8} + C$

9. $\int \frac{1}{x^{10}} dx = -\frac{1}{9x^9} + C$

$$\begin{aligned}
 & -a_{12}d_{21}c_{33}s^2 - a_{12}d_{22}c_{33}s^3 - a_{12}d_{23}c_{33}s - b_{12}d_{21}c_{33}s - b_{12}d_{22}c_{33}s^2 - b_{12}d_{23}c_{33} \\
 & -c_{12}d_{21}c_{33} - c_{12}d_{22}c_{33}s - c_{12}d_{23}c_{33} \frac{1}{2} - c_{13}a_{22}d_{33}s - c_{13}b_{22}d_{33} - c_{13}d_{22}d_{33} \frac{1}{s}
 \end{aligned}$$

$$N_1 = N_{13}s^3 + N_{12}s^2 + N_{11}s + N_{10} + N_{09} \frac{1}{2} \quad (E108)$$

$$N_{13} = (d_{12}a_{22}c_{33} + a_{12}a_{23}d_{33} - d_{12}a_{23}c_{32} - a_{12}d_{22}c_{33})$$

$$\begin{aligned}
 N_{12} = & (d_{11}a_{22}c_{33} + d_{12}b_{22}c_{33} + a_{12}b_{23}d_{33} + b_{12}a_{23}d_{33} - d_{11}a_{23}c_{32} - d_{12}b_{23}c_{32} \\
 & - a_{12}d_{21}c_{33} - b_{12}d_{22}c_{33})
 \end{aligned}$$

$$\begin{aligned}
 N_{11} = & (d_{11}b_{22}c_{33} + d_{12}c_{22}c_{33} + d_{13}a_{22}c_{33} + a_{12}c_{23}d_{33} + b_{12}b_{23}d_{33} + c_{12}a_{23}d_{33}) \\
 & + c_{13}d_{22}c_{32} - d_{11}b_{23}c_{32} - d_{12}c_{23}c_{32} - d_{13}a_{23}c_{32} - a_{12}d_{23}c_{33} \\
 & - b_{12}d_{21}c_{33} - c_{12}d_{22}c_{33} - c_{13}a_{22}d_{33})
 \end{aligned}$$

$$\begin{aligned}
 N_{10} = & (d_{11}c_{22}c_{33} + d_{13}b_{22}c_{33} + b_{12}c_{23}d_{33} + c_{12}b_{23}d_{33} + c_{13}d_{21}c_{32} - d_{11}c_{23}c_{32} \\
 & - d_{13}b_{23}c_{32} - b_{12}d_{23}c_{33} - c_{12}d_{21}c_{33} - c_{13}b_{22}d_{33})
 \end{aligned}$$

$$N_{09} = (d_{13}c_{22}c_{33} + c_{12}c_{23}d_{33} + c_{13}d_{23}c_{32} - d_{13}c_{23}c_{32} - c_{12}d_{23}c_{33} - c_{13}c_{22}d_{33})$$

1. $\frac{1}{x^2} = x^{-2}$

2. $\frac{1}{x^3} = x^{-3}$

3. $\frac{1}{x^4} = x^{-4}$

4. $\frac{1}{x^5} = x^{-5}$

5. $\frac{1}{x^6} = x^{-6}$

6. $\frac{1}{x^7} = x^{-7}$

7. $\frac{1}{x^8} = x^{-8}$

8. $\frac{1}{x^9} = x^{-9}$

9. $\frac{1}{x^{10}} = x^{-10}$

10. $\frac{1}{x^{11}} = x^{-11}$

11. $\frac{1}{x^{12}} = x^{-12}$

12. $\frac{1}{x^{13}} = x^{-13}$

$$\begin{aligned}
 \text{Den.} = & a_{11}a_{22}c_{33}s^4 + a_{11}b_{22}c_{33}s^3 + a_{11}c_{22}c_{33}s^2 + b_{11}a_{22}c_{33}s^3 + b_{11}b_{22}c_{33}s^2 \\
 & + b_{11}c_{22}c_{33}s \\
 & + a_{12}a_{23}c_{31}s^4 + a_{12}b_{23}c_{31}s^3 + a_{12}c_{23}c_{31}s^2 + b_{12}a_{23}c_{31}s^3 + b_{12}b_{23}c_{31}s^2 \\
 & + b_{12}c_{23}c_{31}s \\
 & + c_{12}a_{23}c_{31}s^2 + c_{12}b_{23}c_{31}s + c_{12}c_{23}c_{31} + c_{13}b_{21}c_{32}s + c_{13}c_{21}c_{32} \\
 & - a_{11}a_{23}c_{32}s^4 - a_{11}b_{23}c_{32}s^3 - a_{11}c_{23}c_{32}s^2 - b_{11}a_{23}c_{32}s^3 - b_{11}b_{23}c_{32}s^2 \\
 & - b_{11}c_{23}c_{32}s \\
 & - a_{12}b_{21}c_{33}s^3 - a_{12}c_{21}c_{33}s^2 - b_{12}b_{21}c_{33}s^2 - b_{12}c_{21}c_{33}s - c_{12}b_{21}c_{33}s - c_{12}c_{21}c_{33} \\
 & - c_{13}a_{22}c_{31}s^2 - c_{13}b_{22}c_{31}s - c_{13}c_{22}c_{31}
 \end{aligned}$$

$$\text{Den} = D_4 s^4 + D_3 s^3 + D_2 s^2 + D_1 s + D_0 \quad (\text{B109})$$

$$D_4 = (a_{11}a_{22}c_{33} + a_{12}a_{23}c_{31} - a_{11}a_{23}c_{32})$$

$$\begin{aligned}
 D_3 = & (a_{11}b_{22}c_{33} + b_{11}a_{22}c_{33} + a_{12}b_{23}c_{31} + b_{12}a_{23}c_{31} - a_{11}b_{23}c_{32} - b_{11}a_{23}c_{32} \\
 & - a_{12}b_{21}c_{33})
 \end{aligned}$$

$$S_1(x) = S_2(x) = S_3(x) = S_4(x) = S_5(x) = \dots$$

$$S_1(x) = S_2(x) = S_3(x) = S_4(x) = S_5(x) = \dots$$

$$S_1(x) = S_2(x) = S_3(x) = S_4(x) = S_5(x) = \dots$$

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$$S_1(x) = S_2(x) = S_3(x) = S_4(x) = S_5(x) = \dots$$

(1000)

$$S_1(x) = S_2(x) = S_3(x) = S_4(x) = S_5(x) = \dots$$

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$$S_1(x) = S_2(x) = S_3(x) = S_4(x) = S_5(x) = \dots$$

$$D_2 = (a_{11}c_{22}c_{33} + b_{11}b_{22}c_{33} + a_{12}c_{23}c_{31} + b_{12}b_{23}c_{31} + c_{12}a_{23}c_{31} - a_{11}c_{23}c_{32} \\ - b_{11}b_{23}c_{32} - a_{12}c_{21}c_{33} - b_{12}b_{21}c_{33} - c_{13}a_{22}c_{31})$$

$$D_1 = (b_{11}c_{22}c_{33} + b_{12}c_{23}c_{31} + c_{12}b_{23}c_{31} + c_{13}b_{21}c_{32} - b_{11}c_{23}c_{32} - b_{12}c_{21}c_{33} \\ - c_{12}b_{21}c_{33} - c_{13}b_{22}c_{31})$$

$$D_0 = (c_{12}c_{23}c_{31} + c_{13}c_{21}c_{32} - c_{12}c_{21}c_{33} - c_{13}c_{22}c_{31})$$

$$L[x] = \frac{N_1}{\text{Den}} = \frac{N_{13}s^3 + N_{12}s^2 + N_{11}s + N_{10} + N_{09}\frac{1}{s}}{D_4s^4 + D_3s^3 + D_2s^2 + D_1s + D_0}$$

$$L[x] = \frac{N_{13}s^4 + N_{12}s^3 + N_{11}s^2 + N_{10}s + N_{09}}{s D_4s^4 + D_5s^3 + D_2s^2 + D_1s + D_0} \quad (\text{E110})$$

$$2x^2 + 3x - 4 = 0 \Rightarrow x = \frac{-3 \pm \sqrt{9 + 32}}{4} = \frac{-3 \pm \sqrt{41}}{4}$$

$$x^2 - 5x + 6 = 0 \Rightarrow (x-2)(x-3) = 0 \Rightarrow x = 2, 3$$

$$x^2 + 2x - 3 = 0 \Rightarrow x = -3, 1$$

$$x^2 - 4x + 4 = 0 \Rightarrow (x-2)^2 = 0 \Rightarrow x = 2$$

$$(x^2 + 3x - 4) - (x^2 - 5x + 6) = 8x - 10 = 0 \Rightarrow x = \frac{5}{4}$$

$$\frac{\frac{1}{2}x^2 + \frac{3}{4}x - \frac{1}{2} - (\frac{1}{2}x^2 - \frac{5}{4}x + \frac{3}{2})}{\frac{1}{2}x^2 + \frac{3}{4}x - \frac{1}{2} - (\frac{1}{2}x^2 - \frac{5}{4}x + \frac{3}{2})} = \frac{8x - 10}{8x - 10} = [1]$$

(02m)

$$\frac{x^2 + 2x - 3 - (x^2 - 4x + 4)}{x^2 + 2x - 3 - (x^2 - 4x + 4)} = [1]$$

Solution of the LaPlace Transforms

$$\begin{aligned}
 \mathcal{L}[x] &= \frac{N_{13}}{D_4} s^4 + \frac{N_{12}}{D_4} s^3 + \frac{N_{11}}{D_4} s^2 + \frac{N_{10}}{D_4} s + \frac{N_{09}}{D_4} \\
 &\frac{D_4}{D_4} s^5 + \frac{D_3}{D_4} s^4 + \frac{D_2}{D_4} s^3 + \frac{D_1}{D_4} s^2 + \frac{D_0}{D_4} s
 \end{aligned}
 \tag{B11}$$

$$\text{Let } a_{4x} = \frac{N_{13}}{D_4} \quad a_{3x} = \frac{N_{12}}{D_4} \quad a_{2x} = \frac{N_{11}}{D_4} \quad a_{1x} = \frac{N_{10}}{D_4} \quad a_{0x} = \frac{N_{09}}{D_4}$$

(B12)

$$b_4 = \frac{D_3}{D_4} \quad b_3 = \frac{D_2}{D_4} \quad b_2 = \frac{D_1}{D_4} \quad b_1 = \frac{D_0}{D_4} \quad b_0 = 0$$

Then,

$$\mathcal{L}[x] = \frac{a_{4x} s^4 + a_{3x} s^3 + a_{2x} s^2 + a_{1x} s + a_{0x}}{s^5 + b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0}$$

$$\mathcal{L}[x] = \frac{a_p s^p + a_{p-1} s^{p-1} + \dots}{s^q + b_{q-1} s^{q-1} + \dots} = \frac{A(s)}{B(s)}$$

$$\mathcal{L}[x] = \frac{A(s)}{(s-s_1)(s-s_2)(s-s_3)(s-s_4)(s-s_5)}$$

(B14)

Since $b_0 = 0$ $s_1 = 0$

Biquadratic Solution

Transform

$$s^4 + b_4 s^3 + b_3 s^2 + b_2 s + b_1$$

$$\text{to } \left[(s + B_1)^2 + A_1^2 \right] \left[(s + B_2)^2 + A_2^2 \right] \quad (\text{B115})$$

$$\text{or } \left[(s + \alpha_3)^2 + \beta_3^2 \right] \left[(s + \alpha_4)^2 + \beta_4^2 \right]$$

$$(s + B_1)^2 + A_1^2 = s^2 + 2 B_1 s + (A_1^2 + B_1^2) \quad (\text{B116})$$

$$s^2 + 2 B_1 s + (A_1^2 + B_1^2) \frac{s^2 + (b_4 - 2B_1)s + [b_3 - (A_1^2 + B_1^2) - 2 B_1 (b_4 - 2B_1)]}{s^4 + b_4 s^3 + b_3 s^2 + b_2 s + b_1}$$

$$\frac{s^4 + 2 B_1 s^3 + (A_1^2 + B_1^2) s^2}{(b_4 - 2 B_1) s^3 + [b_3 - (A_1^2 + B_1^2)] s^2 + b_2 s + b_1}$$

$$\frac{(b_4 - 2 B_1) s^3 + 2 B_1 (b_4 - 2 B_1) s^2 + (b_4 - 2 B_1) (A_1^2 + B_1^2) s}{[b_3 - (A_1^2 + B_1^2) - 2 B_1 (b_4 - 2 B_1)] s^2 + [b_2 - (b_4 - 2 B_1) (A_1^2 + B_1^2)] s + b_1}$$

$$\begin{aligned} & [b_3 - (A_1^2 + B_1^2) - 2 B_1 (b_4 - 2 B_1)] s^2 + [2 B_1 [b_3 - (A_1^2 + B_1^2) - 2 B_1 (b_4 - 2 B_1)] \\ & + [b_2 - \dots] [A_1^2 + B_1^2] \end{aligned}$$

$$2 B_2 = (b_4 - 2 B_1)$$

$$(A_2^2 + B_2^2) = b_3 - (A_1^2 + B_1^2) - 2 B_1 (b_4 - 2 B_1)$$

$$A_2 = \sqrt{b_3 - (A_1^2 + B_1^2) - 2 B_1 (b_4 - 2 B_1) - B_2^2} \quad (\text{B117})$$

PROBLEM 11

PROBLEM

$$x^2 + 2x + 1 = (x+1)^2$$

(1) $\left[\frac{x^2 + 2x + 1}{x^2 + 2x + 1} \right] \left[\frac{x^2 + 2x + 1}{x^2 + 2x + 1} \right] = 1$

$$\left[\frac{x^2 + 2x + 1}{x^2 + 2x + 1} \right] \left[\frac{x^2 + 2x + 1}{x^2 + 2x + 1} \right] = 1$$

(2) $\left(\frac{x^2 + 2x + 1}{x^2 + 2x + 1} \right) = \frac{x^2 + 2x + 1}{x^2 + 2x + 1}$

$$\frac{(x^2 + 2x + 1) \cdot 1 - (x^2 + 2x + 1) \cdot 1}{x^2 + 2x + 1} = \frac{x^2 + 2x + 1 - x^2 - 2x - 1}{x^2 + 2x + 1} = \frac{0}{x^2 + 2x + 1} = 0$$

$$\frac{x^2 + 2x + 1}{x^2 + 2x + 1} = \frac{x^2 + 2x + 1}{x^2 + 2x + 1}$$

$$\frac{(x^2 + 2x + 1)(x^2 + 2x + 1) - (x^2 + 2x + 1)(x^2 + 2x + 1)}{(x^2 + 2x + 1)(x^2 + 2x + 1) - (x^2 + 2x + 1)(x^2 + 2x + 1)}$$

$$\left[\frac{(x^2 + 2x + 1) \cdot 1 - (x^2 + 2x + 1) \cdot 1}{x^2 + 2x + 1} \right] \left[\frac{(x^2 + 2x + 1) \cdot 1 - (x^2 + 2x + 1) \cdot 1}{x^2 + 2x + 1} \right]$$

$$\frac{(x^2 + 2x + 1) \cdot 1 - (x^2 + 2x + 1) \cdot 1}{x^2 + 2x + 1} = \frac{0}{x^2 + 2x + 1} = 0$$

(3)

$$\frac{x^2 + 2x + 1}{x^2 + 2x + 1} = \frac{x^2 + 2x + 1}{x^2 + 2x + 1}$$

The remainder must equal zero.

Therefore,

$$\left[b_2 - (b_4 - 2B_1)(A_1^2 + B_1^2) \right] - \left[b_3 - (A_1^2 + B_1^2) - 2B_1(b_4 - 2B_1) \right] \left[2B_1 \right] = 0 \quad (E118)$$

$B_1 \neq 0$ B_1 must be pos. to get $e^{-B_1 t}$, B_2 must be pos. too so $(b_4 - 2B_1) > 0$

$$\text{or } \frac{b_4}{2} > B_1$$

$$\text{So } 0 < B_1 < \frac{b_4}{2}$$

$$b_2 - b_4 A_1^2 + 2A_1^2 B_1 - b_4 B_1^2 + 2B_1^3 - 2b_3 B_1 + 2A_1^2 B_1 + 2B_1^3 + 4b_4 B_1^2 - 8B_1^3 = 0$$

$$b_2 - b_4 A_1^2 + 4A_1^2 B_1 + 3b_4 B_1^2 - 4B_1^3 - 2b_3 B_1 = 0 \quad (E119)$$

$$b_1 - \left[b_3 - (A_1^2 + B_1^2) - 2B_1(b_4 - 2B_1) \right] \left[A_1^2 + B_1^2 \right] = 0$$

$$b_1 - \left[b_3 - A_1^2 - B_1^2 - 2b_4 B_1 + 4B_1^2 \right] \left[A_1^2 + B_1^2 \right] = 0$$

$$b_1 - b_3 A_1^2 + A_1^4 + B_1^2 A_1^2 + 2b_4 A_1^2 B_1 - 4A_1^2 B_1^2 - b_3 B_1^2 + A_1^2 B_1^2 + B_1^4 + 2b_4 B_1^3 - 4B_1^4 = 0$$

(E120)

$$b_1 - b_3 A_1^2 + A_1^4 - 2A_1^2 B_1^2 + 2b_4 A_1^2 B_1 - b_3 B_1^2 - 3B_1^4 + 2b_4 B_1^3 = 0$$

$$(111) \quad 0 = \left[\frac{m}{r} \right] \left[(r^2 - a^2) \frac{d^2 r}{dt^2} - (r^2 - a^2) \dot{r}^2 - (r^2 - a^2) \left(\frac{d\theta}{dt} \right)^2 \right] - \left[\left(\frac{d^2 r}{dt^2} - \frac{d^2 a}{dt^2} \right) (r^2 - a^2) - 2r \dot{r} \dot{a} \right]$$

Since $(r^2 - a^2)$ is not zero at any point, $\frac{d^2 r}{dt^2} = \frac{d^2 a}{dt^2} + \frac{2r \dot{r} \dot{a}}{r^2 - a^2}$

$$r < \frac{d^2 r}{dt^2} + a$$

$$\frac{d^2 r}{dt^2} > r > 0$$

$$0 = \frac{d^2 r}{dt^2} + \frac{2r \dot{r} \dot{a}}{r^2 - a^2} - \frac{d^2 a}{dt^2} - \frac{2r \dot{r} \dot{a}}{r^2 - a^2} - \frac{d^2 r}{dt^2} - \frac{d^2 a}{dt^2} - \frac{2r \dot{r} \dot{a}}{r^2 - a^2} - \frac{d^2 r}{dt^2} - \frac{d^2 a}{dt^2} - \frac{2r \dot{r} \dot{a}}{r^2 - a^2}$$

$$(112) \quad 0 = \frac{d^2 r}{dt^2} + \frac{2r \dot{r} \dot{a}}{r^2 - a^2} - \frac{d^2 a}{dt^2} - \frac{2r \dot{r} \dot{a}}{r^2 - a^2} - \frac{d^2 r}{dt^2} - \frac{d^2 a}{dt^2} - \frac{2r \dot{r} \dot{a}}{r^2 - a^2} - \frac{d^2 r}{dt^2} - \frac{d^2 a}{dt^2} - \frac{2r \dot{r} \dot{a}}{r^2 - a^2}$$

$$0 = \left[\frac{d^2 r}{dt^2} - \frac{d^2 a}{dt^2} \right] \left[(r^2 - a^2) \frac{d^2 r}{dt^2} - (r^2 - a^2) \dot{r}^2 - (r^2 - a^2) \left(\frac{d\theta}{dt} \right)^2 \right] - \frac{d^2 r}{dt^2}$$

$$0 = \left[\frac{d^2 r}{dt^2} - \frac{d^2 a}{dt^2} \right] \left[\frac{d^2 r}{dt^2} + \frac{2r \dot{r} \dot{a}}{r^2 - a^2} - \frac{d^2 a}{dt^2} - \frac{d^2 r}{dt^2} - \frac{d^2 a}{dt^2} - \frac{2r \dot{r} \dot{a}}{r^2 - a^2} \right] - \frac{d^2 r}{dt^2}$$

$$0 = \frac{d^2 r}{dt^2} - \frac{d^2 a}{dt^2} + \frac{d^2 r}{dt^2} + \frac{2r \dot{r} \dot{a}}{r^2 - a^2} - \frac{d^2 a}{dt^2} - \frac{d^2 r}{dt^2} - \frac{d^2 a}{dt^2} - \frac{2r \dot{r} \dot{a}}{r^2 - a^2} + \frac{d^2 r}{dt^2} + \frac{d^2 a}{dt^2} + \frac{2r \dot{r} \dot{a}}{r^2 - a^2} - \frac{d^2 r}{dt^2}$$

(113)

$$0 = \frac{d^2 r}{dt^2} + \frac{d^2 r}{dt^2} - \frac{d^2 a}{dt^2} - \frac{d^2 r}{dt^2} + \frac{d^2 r}{dt^2} - \frac{d^2 a}{dt^2} + \frac{d^2 r}{dt^2} - \frac{d^2 a}{dt^2} - \frac{d^2 r}{dt^2}$$

From (RL9), $A_1^2 (4B_1 - b_4) = -b_2 - 3b_4 B_1^2 + 4B_1^3 + 2b_3 B_1$

$$A_1^2 = \frac{-b_2 - 3b_4 B_1^2 + 4B_1^3 + 2b_3 B_1}{(4B_1 - b_4)} \quad (\text{Note that } B_1 \neq \frac{b_4}{4} \text{ or } A_1 = \infty)$$

$$A_1^4 = \frac{1}{(16B_1^2 - 8b_4 B_1 + b_4^2)} (+b_2^2 + 3b_2 b_4 B_1^2 - 4b_2 B_1^3 - 2b_2 b_3 B_1$$

$$+ 3b_2 b_4 B_1^2 + 9b_4^2 B_1^4 - 12b_4 B_1^5 - 6b_3 b_4 B_1^3$$

$$+ 8b_3 B_1^4 - 4b_2 B_1^3 - 12b_4 B_1^5 + 16B_1^6$$

$$+ 4b_3^2 B_1^2 + 8b_3 B_1^4 - 2b_2 b_3 B_1 - 6b_3 b_4 B_1^3$$

$$A_1^4 = \frac{(+b_2^2 + 9b_4^2 B_1^4 + 16B_1^6 + 4b_3^2 B_1^2 + 6b_2 b_4 B_1^2 - 8b_2 B_1^3 - 4b_2 b_3 B_1 + 16b_3 B_1^4 - 24b_4 B_1^5 - 12b_3 b_4 B_1^3)}{(16B_1^2 - 8b_4 B_1 + b_4^2)}$$

$$b_1 + A_1^2 (-b_3 - 2B_1^2 + 2b_4 B_1) + A_1^4 (-b_3 B_1^2 - 3B_1^4 + 2b_4 B_1^3) = 0$$

$$b_1 (16B_1^2 - 8b_4 B_1 + b_4^2) + (4B_1 - b_4) (-b_2 - 3b_4 B_1^2 + 4B_1^3 + 2b_3 B_1) (-b_3 - 2B_1^2 + 2b_4 B_1)$$

$$+ (-b_3 B_1^2 - 3B_1^4 + 2b_4 B_1^3) (16B_1^2 - 8b_4 B_1 + b_4^2)$$

$$+ b_2^2 + 9b_4^2 B_1^4 + 16B_1^6 + 4b_3^2 B_1^2 + 6b_2 b_4 B_1^2 - 8b_2 B_1^3 - 4b_2 b_3 B_1 + 16b_3 B_1^4 - 24b_4 B_1^5$$

$$- 12 b_3 b_4 B_1 = 0$$

$$p_1 = (p_1 - \sum_{i=1}^n p_i) - (p_1 - \sum_{i=1}^n p_i) \frac{p_1}{(p_1 - \sum_{i=1}^n p_i)}$$

$$\text{for } p_1 = \frac{p_1}{p_1 - \sum_{i=1}^n p_i} \text{ then } \frac{p_1}{p_1 - \sum_{i=1}^n p_i} = \frac{p_1 + \sum_{i=1}^n p_i}{p_1 - \sum_{i=1}^n p_i} = \frac{p_1}{p_1 - \sum_{i=1}^n p_i}$$

$$\frac{p_1 p_2 + \dots + p_1 p_n + \sum_{i=1}^n p_i^2}{(p_1 - \sum_{i=1}^n p_i) - \sum_{i=1}^n p_i} = \frac{p_1}{p_1 - \sum_{i=1}^n p_i}$$

$$\frac{p_1 p_2 + \dots + p_1 p_n + \sum_{i=1}^n p_i^2}{p_1 - \sum_{i=1}^n p_i} = \frac{p_1}{p_1 - \sum_{i=1}^n p_i}$$

$$\frac{p_1 p_2 + \dots + p_1 p_n + \sum_{i=1}^n p_i^2}{p_1 - \sum_{i=1}^n p_i} = \frac{p_1}{p_1 - \sum_{i=1}^n p_i}$$

$$\frac{p_1 p_2 + \dots + p_1 p_n + \sum_{i=1}^n p_i^2}{p_1 - \sum_{i=1}^n p_i} = \frac{p_1}{p_1 - \sum_{i=1}^n p_i}$$

$$\frac{(p_1 p_2 + \dots + p_1 p_n + \sum_{i=1}^n p_i^2) - (p_1 p_2 + \dots + p_1 p_n + \sum_{i=1}^n p_i^2) \frac{p_1}{p_1 - \sum_{i=1}^n p_i}}{(p_1 - \sum_{i=1}^n p_i) - \sum_{i=1}^n p_i} = \frac{p_1}{p_1 - \sum_{i=1}^n p_i}$$

$$0 = (p_1 p_2 + \dots + p_1 p_n + \sum_{i=1}^n p_i^2) - (p_1 p_2 + \dots + p_1 p_n + \sum_{i=1}^n p_i^2) \frac{p_1}{p_1 - \sum_{i=1}^n p_i} = 0$$

$$(p_1 p_2 + \dots + p_1 p_n + \sum_{i=1}^n p_i^2) (p_1 - \sum_{i=1}^n p_i) - (p_1 p_2 + \dots + p_1 p_n + \sum_{i=1}^n p_i^2) p_1 = 0$$

$$(p_1 p_2 + \dots + p_1 p_n + \sum_{i=1}^n p_i^2) (p_1 - \sum_{i=1}^n p_i - p_1) = 0$$

$$\frac{p_1 p_2 + \dots + p_1 p_n + \sum_{i=1}^n p_i^2}{p_1 - \sum_{i=1}^n p_i - p_1} = \frac{p_1 p_2 + \dots + p_1 p_n + \sum_{i=1}^n p_i^2}{- \sum_{i=1}^n p_i} = \frac{p_1 p_2 + \dots + p_1 p_n + \sum_{i=1}^n p_i^2}{- \sum_{i=1}^n p_i}$$

$$0 = \frac{p_1 p_2 + \dots + p_1 p_n + \sum_{i=1}^n p_i^2}{- \sum_{i=1}^n p_i}$$

$$16 b_1 R_1^2 - 8b_1 b_4 R_1 + b_1 b_4^2 + (4R_1 - b_4)(b_2 b_3 + 3b_3 b_4 R_1^2 - 4b_3 R_1^3 - 2b_3^2 R_1 + 2b_2 R_1^2$$

$$+ 6 b_4 R_1^4 - 8R_1^5 - 4 b_3 R_1^3$$

$$- 2b_2 b_4 R_1 - 6b_4^2 R_1^3 + 8b_4 R_1^4 + 4b_3 b_4 R_1^2) - 16b_3 R_1^4 + 8b_3 b_4 R_1^3 - b_3^2 b_4 R_1^2 - 48R_1^6$$

$$+ 24 b_4 R_1^5 - 3b_4^2 R_1^4$$

$$+ 32 b_4 R_1^5 - 16 b_4^2 R_1^4 + 2b_4^3 R_1^3 + b_2^2 + 9b_4^2 R_1^4 + 16R_1^6 + 4b_3^2 R_1^2 + 6b_2 b_4 R_1^2$$

$$- 8 b_2 R_1^3 - 4b_2 b_3 R_1 + 16 b_3 R_1^4$$

$$- 24 b_4 R_1^5 - 12 b_3 b_4 R_1^3 = 0$$

$$16 b_1 R_1^2 - 8b_1 b_4 R_1 + b_1 b_4^2 + 4 b_2 b_3 R_1 + 12b_3 b_4 R_1^3 - 16b_3 R_1^4 - 8b_3^2 R_1^2 + 8b_2 R_1^3$$

$$+ 24b_4 R_1^5 - 32 R_1^6 - 16 b_3 R_1^4$$

$$- 8 b_2 b_4 R_1^2 - 24 b_4^2 R_1^4 + 32b_4 R_1^5 + 16b_3 b_4 R_1^3 - b_2 b_3 b_4 - 3b_3 b_4^2 R_1^2 + 4b_3 b_4 R_1^3$$

$$+ 2b_3^2 b_4 R_1 - 2b_2 b_4 R_1^2 - 6b_4^2 R_1^4$$

$$+ 8 b_4 R_1^5 + 4b_3 b_4 R_1^3 + 2b_2 b_4 R_1^2 + 6b_4^3 R_1^3 - 8b_4^2 R_1^4 - 4b_3 b_4^2 R_1^2 - 16b_3 R_1^4$$

$$+ 8 b_3 b_4 R_1^3 - b_3 b_4^2 R_1^2$$

$$a^2 + b^2 - (a+b)^2 = a^2 + b^2 - (a^2 + 2ab + b^2) = -2ab$$

$$\frac{1}{2}(a+b)^2 - \frac{1}{2}(a-b)^2 = ab$$

$$a^2 - b^2 = (a+b)(a-b) \implies \frac{a^2 - b^2}{a+b} = a-b$$

$$\frac{1}{2}(a+b)^2 - \frac{1}{2}(a-b)^2 = ab$$

$$a^2 + b^2 + 2ab - (a^2 + b^2 - 2ab) = 4ab$$

$$\frac{1}{2}(a+b)^2 + \frac{1}{2}(a-b)^2 = a^2 + b^2$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$a^2 + b^2 - (a+b)^2 = -2ab$$

$$\frac{1}{2}(a+b)^2 - \frac{1}{2}(a-b)^2 = ab$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$\frac{1}{2}(a+b)^2 + \frac{1}{2}(a-b)^2 = a^2 + b^2$$

$$a^2 + b^2 - (a+b)^2 = -2ab$$

$$\frac{1}{2}(a+b)^2 - \frac{1}{2}(a-b)^2 = ab$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$- 48B_1^6 + 24b_4B_1^5 - 3b_4^2B_1^4 + 32b_4B_1^5 - 16b_4^2B_1^4 + 2b_4^3B_1^3 + b_2^2 + 9b_4^2B_1^4 + 16B_1^6$$

$$+ 4b_3^2B_1^2 + 6b_3b_4B_1^2$$

$$- 8b_2B_1^3 - 4b_2b_3B_1 + 16b_3B_1^4 - 24b_4B_1^5 - 12b_3b_4B_1^3 = 0$$

$$B_1^6 (-32 - 48 + 16) + B_1^5 (+ 24b_4 + 32b_4 + 8b_4 + 24b_4 + 32b_4 - 24b_4)$$

$$+ B_1^4 (-16b_3 - 16b_3 - 6b_4^2 - 8b_4^2 - 16b_3 - 3b_4^2 - 16b_4^2 + 9b_4^2 + 16b_3 - 24b_4^2)$$

$$+ B_1^3 (+12b_3b_4 + 8b_2 + 16b_3b_4 + 4b_3b_4 + 4b_3b_4 + 6b_4^3 + 8b_3b_4 - 8b_2 - 12b_3b_4 + 2b_4^3)$$

$$+ B_1^2 (+16b_1 - 8b_3^2 - 8b_2b_4 - 3b_3b_4^2 - 2b_2b_4 - 4b_3b_4^2 - b_3b_4^2 + 4b_3^2 + 6b_2b_4)$$

$$+ B_1 (-8b_1b_4 + 4b_2b_3 + 2b_3^2b_4 + 2b_2b_4^2 - 4b_2b_3)$$

$$+ (+b_1b_4^2 - b_2b_3b_4 + b_2^2) = 0$$

$$B_1^6 (-64) + B_1^5 (96b_4) + B_1^4 (-48b_4^2 - 32b_3) + B_1^3 (+32b_3b_4 + 8b_4^3)$$

$$B_1^2 (+16b_1 - 4b_3^2 - 4b_2b_4 - 8b_3b_4^2) + B_1 (-8b_1b_4 + 2b_3^2b_4 + 2b_2b_4^2)$$

$$+ (b_1b_4^2 - b_2b_3b_4 + b_2^2) = 0$$

$$\frac{d}{dt} \left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 \right) = m \dot{x} \ddot{x} + k x \dot{x} = \dot{x} (m \ddot{x} + k x) = \dot{x} F = \dot{x} \cdot 0 = 0$$

$$\frac{d}{dt} \left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 \right) = 0$$

$$0 = \frac{d}{dt} \left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 \right) = m \dot{x} \ddot{x} + k x \dot{x} = \dot{x} (m \ddot{x} + k x) = \dot{x} \cdot 0 = 0$$

$$\left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 \right) = \text{const} = \frac{1}{2} m \dot{x}_0^2 + \frac{1}{2} k x_0^2 = \text{const}$$

$$\left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 \right) = \frac{1}{2} m \dot{x}_0^2 + \frac{1}{2} k x_0^2 = \text{const}$$

$$\left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 \right) = \frac{1}{2} m \dot{x}_0^2 + \frac{1}{2} k x_0^2 = \text{const}$$

$$\left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 \right) = \frac{1}{2} m \dot{x}_0^2 + \frac{1}{2} k x_0^2 = \text{const}$$

$$\left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 \right) = \frac{1}{2} m \dot{x}_0^2 + \frac{1}{2} k x_0^2 = \text{const}$$

$$0 = \left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 \right) = \text{const}$$

$$\left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 \right) = \frac{1}{2} m \dot{x}_0^2 + \frac{1}{2} k x_0^2 = \text{const}$$

$$\left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 \right) = \frac{1}{2} m \dot{x}_0^2 + \frac{1}{2} k x_0^2 = \text{const}$$

$$0 = \left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 \right) = \text{const}$$

Since $0 < R_1 < \frac{b_4}{2}$ and $R_1 \neq \frac{b_4}{4}$

let $R_1 = c b_4$ where $0 < c < \frac{1}{4}$ and/or $\frac{1}{4} < c < \frac{1}{2}$ (B121)

$$R_1^2 = c^2 b_4^2 \quad R_1^3 = c^3 b_4^3 \quad R_1^4 = c^4 b_4^4 \quad R_1^5 = c^5 b_4^5 \quad R_1^6 = c^6 b_4^6$$

$$\begin{aligned} & c^6 (-64b_4^6) + c^5 (+96 b_4^6) + c^4 \left[b_4^4 (-48 b_4^2 - 32b_3) \right] + c^3 \left[b_4^3 (+32b_3 b_4 + 8b_4^3) \right] \\ & + c^2 \left[b_4^2 (+16 b_1 - 4b_3^2 - 4b_2 b_4 - 8b_3 b_4^2) \right] + c \left[b_4 (-8b_1 b_4 + 2b_3^2 b_4 + 2b_2 b_4^2) \right] \\ & + (b_1 b_4^2 - b_2 b_3 b_4 + b_2^2) = 0 \end{aligned}$$

Try various values of c within both sets of limits indicated.

Let $w_6 = -64 b_4^6$

$w_5 = +96 b_4^6$ (B122)

$$w_4 = b_4^4 (-48 b_4^2 - 32 b_3)$$

$$w_3 = b_4^3 (+32 b_3 b_4 + 8b_4^3)$$

$$w_2 = b_4^2 (+16 b_1 - 4b_3^2 - 4b_2 b_4 - 8b_3 b_4^2)$$

$$w_1 = b_4 (-8b_1 b_4 + 2b_3^2 b_4 + 2b_2 b_4^2)$$

$$w_0 = (b_1 b_4^2 - b_2 b_3 b_4 + b_2^2) \quad \text{(B123)}$$

$$\frac{d^2}{dt^2} + \dots = 0$$

(1.10) $\frac{d^2}{dt^2} + \dots = 0$

$$\frac{d^2}{dt^2} + \dots = 0$$

$$\left[\left(\frac{d^2}{dt^2} + \dots \right) \dots \right] = \dots$$

$$\left[\left(\frac{d^2}{dt^2} + \dots \right) \dots \right] = \dots$$

$$\dots = \dots$$

The various terms on the right hand side of (1.10) are:

$$\frac{d^2}{dt^2} + \dots$$

(1.11) $\frac{d^2}{dt^2} + \dots = \dots$

$$\left(\frac{d^2}{dt^2} + \dots \right) \dots = \dots$$

$$\left(\frac{d^2}{dt^2} + \dots \right) \dots = \dots$$

$$\left(\frac{d^2}{dt^2} + \dots \right) \dots = \dots$$

$$\left(\frac{d^2}{dt^2} + \dots \right) \dots = \dots$$

(1.12) $\left(\frac{d^2}{dt^2} + \dots \right) \dots = \dots$

Then

$$w_6 c^6 + w_5 c^5 + w_4 c^4 + w_3 c^3 + w_2 c^2 + w_1 c + w_0 = 0$$

$$0 < c < \frac{1}{4} \quad \text{and} \quad \frac{1}{4} < c < \frac{1}{2}$$

Let $\alpha_3 = B_1$

$$\beta_3 = A_1$$

$$\alpha_4 = B_2$$

$$\beta_4 = A_2$$

Then

$$\alpha_3 = c b_4 \tag{H24}$$

$$\beta_3 = \sqrt{\frac{-b_2 - 3b_4 \alpha_3^2 + 4\alpha_3^3 + 2b_3 \alpha_3}{4 \alpha_3 - b_4}} \tag{H25}$$

$$\alpha_4 = \frac{1}{2} (b_4 - 2\alpha_3) = \frac{1}{2} (b_4 - 2cb_4) = \frac{b_4}{2} (1 - 2c) \tag{H26}$$

$$\beta_4 = \sqrt{b_3 - (\beta_3^2 + \alpha_3^2) - 4\alpha_3 \alpha_4 - \alpha_4^2} \tag{H27}$$

(1) $\frac{1}{x^2} = x^{-2}$

$\frac{d}{dx} x^{-2} = -2x^{-3}$

$= -\frac{2}{x^3}$

$= -\frac{2}{x^3}$

$= -\frac{2}{x^3}$

$= -\frac{2}{x^3}$

(2)

$\frac{d}{dx} x^{-2} = -\frac{2}{x^3}$

(3)

$\frac{d}{dx} \left(\frac{x^2 + 3x + 5}{x^2} \right) = \frac{2x + 3}{x^2} - \frac{2x(x^2 + 3x + 5)}{x^4}$

(4)

$\frac{d}{dx} \left(\frac{x^2 + 3x + 5}{x^2} \right) = \frac{2x + 3}{x^2} - \frac{2x(x^2 + 3x + 5)}{x^4}$

(5)

$\frac{d}{dx} \left(\frac{x^2 + 3x + 5}{x^2} \right) = \frac{2x + 3}{x^2} - \frac{2x(x^2 + 3x + 5)}{x^4}$

Partial Fraction Form

It has been assumed that the denominator of equation (E113) has the following form:

$$s \left[(s + \alpha_3)^2 + \beta_3^2 \right] \left[(s + \alpha_4)^2 + \beta_4^2 \right]$$

A solution of the roots of the denominator has been solved accordingly. See equations (E114), (E124), (E125), (E126), and (E127).

Therefore, from equation (E113), we must go to partial fractions.

$$\mathcal{L} [x] = \frac{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}{s^5 + b_4 s^4 + b_3 s^3 + b_2 s^2 + b_1 s} =$$

$$\frac{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}{s \left[(s + \alpha_3)^2 + \beta_3^2 \right] \left[(s + \alpha_4)^2 + \beta_4^2 \right]} =$$

$$\frac{A_1}{s} + \frac{A_3 s + B_3}{\left[(s + \alpha_3)^2 + \beta_3^2 \right]} + \frac{A_4 s + B_4}{\left[(s + \alpha_4)^2 + \beta_4^2 \right]} =$$

$$\frac{A_1}{s} + \frac{A_3 s + B_3}{\left[s^2 + 2\alpha_3 s + (\alpha_3^2 + \beta_3^2) \right]} + \frac{A_4 s + B_4}{\left[s^2 + 2\alpha_4 s + (\alpha_4^2 + \beta_4^2) \right]}$$

Put the last term in the form of a polynomial with a common denominator; it then becomes

Let $f(x) = x^2 + 2x + 1$ and $g(x) = x^2 + 3x + 2$

Find $\frac{f(x)}{g(x)}$

$$\frac{x^2 + 2x + 1}{x^2 + 3x + 2}$$

Perform polynomial long division

$x^2 + 2x + 1 \div x^2 + 3x + 2$

Step 1: Divide x^2 by x^2 to get 1

$$1 \cdot (x^2 + 3x + 2) = x^2 + 3x + 2$$

$$\frac{x^2 + 2x + 1}{x^2 + 3x + 2} = 1 + \frac{-x - 1}{x^2 + 3x + 2}$$

$$= 1 + \frac{-x - 1}{(x+1)(x+2)}$$

$$= 1 + \frac{A}{x+1} + \frac{B}{x+2}$$

For the first term in the form of a polynomial with a common denominator

$$\left\{ A_1 \left[s^2 + 2\alpha_3 s + (\alpha_3^2 + \beta_3^2) \right] \left[s^2 + 2\alpha_4 s + (\alpha_4^2 + \beta_4^2) \right] + A_3 s^2 \left[s^2 + 2\alpha_4 s + (\alpha_4^2 + \beta_4^2) \right] \right. \\ \left. + B_3 s \left[s^2 + 2\alpha_4 s + (\alpha_4^2 + \beta_4^2) \right] + A_4 s^2 \left[s^2 + 2\alpha_3 s + (\alpha_3^2 + \beta_3^2) \right] \right. \\ \left. + B_4 s \left[s^2 + 2\alpha_3 s + (\alpha_3^2 + \beta_3^2) \right] \right\} \div \left\{ \left[s^2 + 2\alpha_3 s + (\alpha_3^2 + \beta_3^2) \right] \right. \\ \left. \left[s^2 + 2\alpha_4 s + (\alpha_4^2 + \beta_4^2) \right] \right\}$$

The numerator becomes

$$A_1 \left[s^4 + 2\alpha_4 s^3 + s^2(\alpha_4^2 + \beta_4^2) + 2\alpha_3 s^3 + 4\alpha_3 \alpha_4 s^2 + 2\alpha_3 s(\alpha_4^2 + \beta_4^2) + s^2(\alpha_3^2 + \beta_3^2) \right. \\ \left. + 2\alpha_4(\alpha_3^2 + \beta_3^2)s \right. \\ \left. + (\alpha_3^2 + \beta_3^2)(\alpha_4^2 + \beta_4^2) \right] + A_3 s^4 + 2\alpha_4 A_3 s^3 + A_3 (\alpha_4^2 + \beta_4^2) s^2 + B_3 s^3 + 2\alpha_4 B_3 s^2 \\ + B_3 (\alpha_4^2 + \beta_4^2) s + A_4 s^4 + 2\alpha_3 A_4 s^3 + A_4 (\alpha_3^2 + \beta_3^2) s^2 + B_4 s^3 + 2\alpha_4 B_4 s^2 \\ + B_4 (\alpha_3^2 + \beta_3^2) s$$

If coefficients of like terms are grouped, the numerator becomes

$$\begin{aligned}
 & s^4 \left[A_1 + A_3 + A_4 \right] + s^3 \left[2 \alpha_4 A_1 + 2 \alpha_3 A_1 + 2 \alpha_4 A_3 + B_3 + 2 \alpha_3 A_4 + B_4 \right] \\
 & + s^2 \left[(\alpha_4^2 + \beta_4^2) A_1 + 4 \alpha_3 \alpha_4 A_1 + (\alpha_3^2 + \beta_3^2) A_1 + (\alpha_4^2 + \beta_4^2) A_3 + 2 \alpha_4 B_3 + (\alpha_3^2 + \beta_3^2) A_4 + 2 B_4 \alpha_3 \right] \\
 & + s \left[2 \alpha_3 (\alpha_4^2 + \beta_4^2) A_1 + 2 \alpha_4 (\alpha_3^2 + \beta_3^2) A_1 + (\alpha_4^2 + \beta_4^2) B_3 + (\alpha_3^2 + \beta_3^2) B_4 \right] \\
 & + \left[A_1 (\alpha_3^2 + \beta_3^2) (\alpha_4^2 + \beta_4^2) \right]
 \end{aligned}$$

This numerator must equal

$$a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0$$

Therefore, coefficients of like terms may be equated.

$$a_4 = A_1 + A_3 + A_4 \quad (E128)$$

$$a_3 = 2 \alpha_4 A_1 + 2 \alpha_3 A_1 + 2 \alpha_4 A_3 + B_3 + 2 \alpha_3 A_4 + B_4 \quad (E129)$$

$$\begin{aligned}
 a_2 = & (\alpha_4^2 + \beta_4^2) A_1 + 4 \alpha_3 \alpha_4 A_1 + (\alpha_3^2 + \beta_3^2) A_1 + (\alpha_4^2 + \beta_4^2) A_3 + 2 \alpha_4 B_3 \\
 & + (\alpha_3^2 + \beta_3^2) A_4 + 2 B_4 \alpha_3 \quad (E130)
 \end{aligned}$$

$$a_1 = 2 \alpha_3 (\alpha_4^2 + \beta_4^2) A_1 + 2 \alpha_4 (\alpha_3^2 + \beta_3^2) A_1 + (\alpha_4^2 + \beta_4^2) B_3 + (\alpha_3^2 + \beta_3^2) B_4 \quad (E131)$$

$$a_0 = (\alpha_3^2 + \beta_3^2) (\alpha_4^2 + \beta_4^2) A_1 \quad (E132)$$

$$[x^2 - 2x + 1] \cdot [x^2 + 2x + 1] = [x^2 + 1]^2 - [2x]^2 = x^4 + 2x^2 + 1 - 4x^2 = x^4 - 2x^2 + 1$$

$$= x^4 - 2x^2 + 1$$

$$= [x^2 - 2x + 1] \cdot [x^2 + 2x + 1] = x^4 - 2x^2 + 1$$

$$= [x^2 - 2x + 1] \cdot [x^2 + 2x + 1]$$

Using the binomial formula

$$(a+b)^2 = a^2 + 2ab + b^2$$

Therefore, we can write the binomial formula as follows:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$= a^2 + 2ab + b^2$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

From equation (H132)

$$A_1 = \frac{a_0}{(\alpha_3^2 + \beta_3^2)(\alpha_4^2 + \beta_4^2)} \quad (\text{H133})$$

$$\text{Let } g_3 = (\alpha_3^2 + \beta_3^2) \quad g_4 = (\alpha_4^2 + \beta_4^2) \quad (\text{H134})$$

$$\text{Then } A_1 = \frac{a_0}{(g_3)(g_4)}$$

Equation (H128) becomes

$$A_3 + A_4 = a_4 - A_1 = d_1$$

Equation (H129) becomes

$$2 \alpha_4 A_3 + 2 \alpha_3 A_4 + B_3 + B_4 = a_3 - 2 A_1 (\alpha_3 + \alpha_4) = d_2$$

Equation (H130) becomes

$$g_4 A_3 + g_3 A_4 + 2 \alpha_4 B_3 + 2 \alpha_3 B_4 = a_2 - A_1 (g_4 + g_3 + 4 \alpha_3 \alpha_4) = d_3$$

Equation (H131) becomes

$$g_4 B_3 + g_3 B_4 = a_1 - A_1 (2 \alpha_3 g_4 + 2 \alpha_4 g_3) = d_4$$

In equation (H135),

$$\text{let } c_{1x} = A_1 = \frac{a_0 x}{(g_3)(g_4)} \quad (\text{H136})$$

Then

Equation (1.10) becomes

(1.11) $\frac{d^2x}{dt^2} + \omega^2 x = 0$

(1.12) $x(0) = x_0, \quad \dot{x}(0) = v_0$

Equation (1.11) is a homogeneous second-order linear differential equation with constant coefficients.

The characteristic equation is

$$r^2 + \omega^2 = 0$$

which has roots

$$r = \pm i\omega$$

Therefore, the general solution is

$$x(t) = A \cos(\omega t) + B \sin(\omega t)$$

where A and B are constants to be determined from the initial conditions.

$$x(0) = A = x_0$$

and

(1.13) $\dot{x}(0) = -A\omega \sin(0) + B\omega \cos(0) = B\omega = v_0$

Thus

$$d_{1x} = a_{4x} - c_{1x} \quad (\text{E137})$$

$$d_{2x} = a_{3x} - c_{1x} b_4 \quad (\text{E138})$$

$$d_{3x} = a_{2x} - c_{1x} b_3 \quad (\text{E139})$$

$$d_{4x} = a_{1x} - c_{1x} b_2 \quad (\text{E140})$$

Solve these four simultaneous equations using matrices

$$\begin{bmatrix} (1) & (1) & (0) & (0) & d_1 \\ (2\alpha_4) & 2\alpha_3 & 1 & 1 & d_2 \\ \varepsilon_4 & \varepsilon_3 & 2\alpha_4 & 2\alpha_3 & d_3 \\ 0 & 0 & \varepsilon_4 & \varepsilon_3 & d_4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & d_1 \\ (2\alpha_4 - 2\alpha_4) & (2\alpha_3 - 2\alpha_4) & 1 & 1 & (d_2 - 2\alpha_4 d_1) \\ (\varepsilon_4 - \varepsilon_4) & (\varepsilon_3 - \varepsilon_4) & 2\alpha_4 & 2\alpha_3 & (d_3 - \varepsilon_4 d_1) \\ 0 & 0 & \varepsilon_4 & \varepsilon_3 & d_4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & d_1 \\ 0 & 1 & \frac{1}{(2\alpha_3 - 2\alpha_4)} & \frac{1}{(2\alpha_3 - 2\alpha_4)} & \frac{(d_2 - 2\alpha_4 d_1)}{(2\alpha_3 - 2\alpha_4)} \\ 0 & (\varepsilon_3 - \varepsilon_4) & 2\alpha_4 & 2\alpha_3 & (d_3 - \varepsilon_4 d_1) \\ 0 & 0 & \varepsilon_4 & \varepsilon_3 & d_4 \end{bmatrix}$$

$\frac{d^2}{dt^2}$

$\frac{d}{dt}$

$$\begin{bmatrix}
 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 1 \\
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0
 \end{bmatrix}
 \begin{bmatrix}
 x \\
 y \\
 z \\
 w
 \end{bmatrix}
 =
 \begin{bmatrix}
 0 \\
 0 \\
 0 \\
 0
 \end{bmatrix}$$

$$\begin{bmatrix}
 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 1 \\
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0
 \end{bmatrix}
 \begin{bmatrix}
 x \\
 y \\
 z \\
 w
 \end{bmatrix}
 =
 \begin{bmatrix}
 0 \\
 0 \\
 0 \\
 0
 \end{bmatrix}$$

$$\begin{bmatrix}
 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 1 \\
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0
 \end{bmatrix}
 \begin{bmatrix}
 x \\
 y \\
 z \\
 w
 \end{bmatrix}
 =
 \begin{bmatrix}
 0 \\
 0 \\
 0 \\
 0
 \end{bmatrix}$$

$$\begin{bmatrix}
 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 1 \\
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0
 \end{bmatrix}
 \begin{bmatrix}
 x \\
 y \\
 z \\
 w
 \end{bmatrix}
 =
 \begin{bmatrix}
 0 \\
 0 \\
 0 \\
 0
 \end{bmatrix}$$

$$d \cdot \frac{(a^2 - ab)(a^2 - b^2) - a^2(a^2 - ab) - a^2(a^2 - ab)(a^2 - ab) + a^2(a^2 - ab)}{(a^2 - ab)(a^2 - ab) - a^2(a^2 - ab) - a^2(a^2 - ab)(a^2 - ab) + a^2(a^2 - ab)}$$

$$\left[\frac{a^2(a^2 - ab)}{(a^2 - ab)} - \frac{a^2(a^2 - ab)}{a^2} + \frac{a^2(a^2 - ab)}{(a^2 - ab)} \right] \cdot a^2$$

$$d \cdot \left[\frac{(a^2 - ab)}{(a^2 - ab)(a^2 - ab)} - \frac{a^2(a^2 - ab)}{a^2} + \frac{a^2(a^2 - ab)}{(a^2 - ab)} \right]$$

$$0 \quad 0 \quad 0 \quad 0 \quad \left[\frac{a^2}{a^2} - \frac{a^2}{a^2} \right]$$

$$0 \quad 0 \quad 0 \quad 0 \quad a^2$$

$$0 \quad 1 \quad 0 \quad 0 \quad 0 \quad \left[\frac{(a^2 - ab)}{(a^2 - ab)} - \frac{a^2(a^2 - ab)}{a^2} \right] \cdot a^2 \left[\frac{(a^2 - ab)}{(a^2 - ab)} - \frac{a^2(a^2 - ab)}{a^2} \right]$$

$$1 \quad 0 \quad 0 \quad 0 \quad 0 \quad \left[\frac{(a^2 - ab)}{(a^2 - ab)} - \frac{a^2(a^2 - ab)}{a^2} \right] \cdot a^2 \left[\frac{(a^2 - ab)}{(a^2 - ab)} - \frac{a^2(a^2 - ab)}{a^2} \right]$$

$$a^2 \quad a^2 \quad 0 \quad 0 \quad 0$$

$$c_{6x} = B_{4x} = \frac{2(\alpha_3 - \alpha_4) [\varepsilon_4 d_{3x} - \varepsilon_4^2 d_{1x} - 2\alpha_4 d_{4x}] + (\varepsilon_3 - \varepsilon_4) [-\varepsilon_4 d_{2x} + 2\alpha_4 \varepsilon_4 d_{1x} + d_{4x}]}{2(\alpha_3 - \alpha_4) [2\alpha_3 \varepsilon_4 - 2\alpha_4 \varepsilon_3] + (\varepsilon_3 - \varepsilon_4) [\varepsilon_3 - \varepsilon_4]} \quad (\text{Bl 41})$$

$$c_{5x} = B_{3x} = \frac{d_{4x} - \varepsilon_3 B_{4x}}{\varepsilon_4} = \frac{d_{4x} - \varepsilon_3 c_{6x}}{\varepsilon_4}$$

$$c_{4x} = A_{4x} = \left[\frac{\varepsilon_4 (d_{2x} - 2\alpha_4 d_{1x})}{2 \varepsilon_4 (\alpha_3 - \alpha_4)} \right] - B_{4x} \left[\frac{\varepsilon_4 - \varepsilon_3}{2 \varepsilon_4 (\alpha_3 - \alpha_4)} \right] = \frac{\varepsilon_4 (d_{2x} - 2\alpha_4 d_{1x}) - d_{4x} + B_{4x} (\varepsilon_3 - \varepsilon_4)}{2 \varepsilon_4 (\alpha_3 - \alpha_4)} \quad (\text{Bl 43})$$

$$c_{3x} = A_{3x} = d_{1x} - A_{4x} = d_{1x} - c_{4x} \quad (\text{Bl 44})$$

$$[x] = \frac{A_1}{s} + \frac{A_3 s + B_3}{[(s + \alpha_3)^2 + \beta_3^2]} + \frac{A_4 s + B_4}{[(s + \alpha_4)^2 + \beta_4^2]} =$$

$$\frac{A_1}{s} + \frac{(s + \alpha_3) \phi_{23} + \beta_3 \phi_{13}}{\beta_3 [(s + \alpha_3)^2 + \beta_3^2]} + \frac{(s + \alpha_4) \phi_{24} + \beta_4 \phi_{14}}{\beta_4 [(s + \alpha_4)^2 + \beta_4^2]} \quad (\text{Bl 45})$$

where, according to (36),

$$\phi_{23} = \beta_3 A_3 \quad \phi_{24} = \beta_4 A_4 \quad (\text{Bl 46})$$

$$\phi_{13} = B_3 - \alpha_3 A_3 \quad \phi_{14} = B_4 - \alpha_4 A_4$$

$$\frac{[x^2 - 2x + 1] - [x^2 - 2x + 1]}{(x+1)(x-1)} = \frac{0}{(x+1)(x-1)}$$

(21)

$$\frac{x^2 - 2x + 1}{(x+1)(x-1)} = \frac{x^2 - 2x + 1}{x^2 - 1}$$

$$\frac{(x^2 - 2x + 1) - (x^2 - 2x + 1)}{(x+1)(x-1)} = \frac{0}{(x+1)(x-1)}$$

(22)

$$(23) \quad x^2 - 2x + 1 = x^2 - 2x + 1$$

$$= \frac{x^2 - 2x + 1}{x^2 - 1} = \frac{x^2 - 2x + 1}{(x+1)(x-1)}$$

(24)

$$\frac{x^2 - 2x + 1}{x^2 - 1} = \frac{x^2 - 2x + 1}{(x+1)(x-1)}$$

(25) of (24) is

$$x^2 - 2x + 1 = x^2 - 2x + 1$$

(26)

$$x^2 - 2x + 1 = x^2 - 2x + 1$$

$$x = A_1 + \frac{1}{\beta_3} e^{-\alpha_3 t} (\phi_{23} \cos \beta_3 t + \phi_{13} \sin \beta_3 t) + \frac{1}{\beta_4} e^{-\alpha_4 t} (\phi_{24} \cos \beta_4 t + \phi_{14} \sin \beta_4 t) \quad (E147)$$

$$P_{23x} = (\beta_3)(c_{3x}) \quad P_{24x} = (\beta_4)(c_{4x})$$

$$P_{13x} = (c_{5x}) - (\alpha_3)(c_{3x}) \quad P_{14x} = (c_{6x}) - (\alpha_4)(c_{4x}) \quad (E148)$$

$$x = c_{1x} + \frac{1}{\beta_3} e^{-\alpha_3 t} (P_{23x} \cos \beta_3 t + P_{13x} \sin \beta_3 t) + \frac{1}{\beta_4} e^{-\alpha_4 t} (P_{24x} \cos \beta_4 t + P_{14x} \sin \beta_4 t) \quad (E149)$$

$$\dot{x} = \frac{-\alpha_3}{\beta_3} e^{-\alpha_3 t} (P_{23x} \cos \beta_3 t + P_{13x} \sin \beta_3 t)$$

$$+ e^{-\alpha_3 t} (-P_{23x} \sin \beta_3 t + P_{13x} \cos \beta_3 t)$$

$$\frac{-\alpha_4}{\beta_4} e^{-\alpha_4 t} (P_{24x} \cos \beta_4 t + P_{14x} \sin \beta_4 t)$$

$$+ e^{-\alpha_4 t} (-P_{24x} \sin \beta_4 t + P_{14x} \cos \beta_4 t) \quad (E150)$$

$$\ddot{x} = \frac{+\alpha_3^2}{\beta_3} e^{-\alpha_3 t} (P_{23x} \cos \beta_3 t + P_{13x} \sin \beta_3 t)$$

$$- 2\alpha_3 e^{-\alpha_3 t} (-P_{23x} \sin \beta_3 t + P_{13x} \cos \beta_3 t)$$

17) $\int_0^1 (x^2 + 1) dx = \left[\frac{x^3}{3} + x \right]_0^1 = \frac{1}{3} + 1 = \frac{4}{3}$

$$\int_0^1 (x^2 + 1) dx = \frac{4}{3}$$

$$(17) \int_0^1 (x^2 + 1) dx = \left[\frac{x^3}{3} + x \right]_0^1 = \frac{1}{3} + 1 = \frac{4}{3}$$

18) $\int_0^1 (x^2 + 1) dx = \left[\frac{x^3}{3} + x \right]_0^1 = \frac{1}{3} + 1 = \frac{4}{3}$

$$\int_0^1 (x^2 + 1) dx = \left[\frac{x^3}{3} + x \right]_0^1 = \frac{1}{3} + 1 = \frac{4}{3}$$

$$\int_0^1 (x^2 + 1) dx = \left[\frac{x^3}{3} + x \right]_0^1 = \frac{1}{3} + 1 = \frac{4}{3}$$

$$\int_0^1 (x^2 + 1) dx = \left[\frac{x^3}{3} + x \right]_0^1 = \frac{1}{3} + 1 = \frac{4}{3}$$

$$(18) \int_0^1 (x^2 + 1) dx = \left[\frac{x^3}{3} + x \right]_0^1 = \frac{1}{3} + 1 = \frac{4}{3}$$

$$\int_0^1 (x^2 + 1) dx = \left[\frac{x^3}{3} + x \right]_0^1 = \frac{1}{3} + 1 = \frac{4}{3}$$

$$\int_0^1 (x^2 + 1) dx = \left[\frac{x^3}{3} + x \right]_0^1 = \frac{1}{3} + 1 = \frac{4}{3}$$

$$\begin{aligned}
 & + \beta_3 e^{-\alpha_3 t} (-P_{23x} \cos \beta_3 t - P_{13x} \sin \beta_3 t) \\
 & + \frac{(\alpha_4^2 - \beta_4^2)}{\beta_4} e^{-\alpha_4 t} (P_{24x} \cos \beta_4 t + P_{14x} \sin \beta_4 t) \\
 & - 2\alpha_4 e^{-\alpha_4 t} (-P_{24x} \sin \beta_4 t + P_{14x} \cos \beta_4 t) \\
 \ddot{x} = & \frac{(\alpha_3^2 - \beta_3^2)}{\beta_3} e^{-\alpha_3 t} (P_{23x} \cos \beta_3 t + P_{13x} \sin \beta_3 t) \\
 & - 2\alpha_3 e^{-\alpha_3 t} (-P_{23x} \sin \beta_3 t + P_{13x} \cos \beta_3 t) \\
 & + \frac{(\alpha_4^2 - \beta_4^2)}{\beta_4} e^{-\alpha_4 t} (P_{24x} \cos \beta_4 t + P_{14x} \sin \beta_4 t) \\
 & - 2\alpha_4 e^{-\alpha_4 t} (-P_{24x} \sin \beta_4 t + P_{14x} \cos \beta_4 t) \tag{E151}
 \end{aligned}$$

The solution for z is as follows:

$$\begin{aligned}
 [z] = \frac{N_2}{\text{Den.}} = & \begin{pmatrix} (a_{11}s^2 + b_{11}s) & (d_{11} + d_{12}s + \frac{d_{13}}{s}) & (c_{13}) \\ (b_{21}s + c_{21}) & (d_{21} + d_{22}s + \frac{d_{23}}{s}) & (a_{23}s^2 + b_{23}s + c_{23}) \\ (c_{31}) & (\frac{d_{33}}{s}) & (c_{33}) \end{pmatrix}
 \end{aligned}$$

$$D_4 s^4 + D_3 s^3 + D_2 s^2 + D_1 s + D_0$$

$$(x_1^2 + x_2^2) \cos \theta = x_1 \cos \theta + x_2 \sin \theta$$

$$(x_1^2 + x_2^2) \sin \theta = x_1 \sin \theta + x_2 \cos \theta$$

$$(x_1^2 + x_2^2) \cos 2\theta = x_1 \cos 2\theta + x_2 \sin 2\theta$$

$$(x_1^2 + x_2^2) \sin 2\theta = x_1 \sin 2\theta + x_2 \cos 2\theta$$

$$(x_1^2 + x_2^2) \cos 3\theta = x_1 \cos 3\theta + x_2 \sin 3\theta$$

$$(x_1^2 + x_2^2) \sin 3\theta = x_1 \sin 3\theta + x_2 \cos 3\theta$$

(100) $(x_1^2 + x_2^2) \cos 4\theta = x_1 \cos 4\theta + x_2 \sin 4\theta$

The solution for θ is as follows:

$$(101) \quad \left(\frac{x_1}{x_2} \right) \cos \theta = \frac{x_1 \cos \theta + x_2 \sin \theta}{x_2 \sin \theta}$$

$$(102) \quad \left(\frac{x_1}{x_2} \right) \sin \theta = \frac{x_1 \sin \theta + x_2 \cos \theta}{x_2 \cos \theta} \quad \left[\frac{x_1}{x_2} \right] = \frac{x_1 \cos \theta + x_2 \sin \theta}{x_2 \sin \theta}$$

$$(103) \quad \left(\frac{x_1}{x_2} \right) \cos 2\theta = \frac{x_1 \cos 2\theta + x_2 \sin 2\theta}{x_2 \sin 2\theta}$$

$$(104) \quad \left(\frac{x_1}{x_2} \right) \sin 2\theta = \frac{x_1 \sin 2\theta + x_2 \cos 2\theta}{x_2 \cos 2\theta}$$

$$N_2 = a_{11}d_{21}c_{33}^2 + a_{11}d_{22}c_{33}^3 + a_{11}d_{23}c_{33}^3 + b_{11}d_{21}c_{33}^3 + b_{11}d_{22}c_{33}^3 + b_{11}d_{23}c_{33}^3$$

$$+ d_{11}a_{23}c_{31}^2 + d_{11}b_{23}c_{31} + d_{11}c_{23}c_{31} + d_{12}a_{23}c_{31}^3 + d_{12}b_{23}c_{31}^3$$

$$+ d_{13}a_{23}c_{31}^3 + d_{13}b_{23}c_{31} + d_{13}c_{23}c_{31} \left(\frac{1}{s}\right) + c_{13}b_{21}d_{33} + c_{13}c_{21}d_{33} \left(\frac{1}{2}\right)$$

$$- a_{11}a_{23}d_{33}^3 - a_{11}b_{23}d_{33}^2 - a_{11}c_{23}d_{33}^3 - b_{11}a_{23}d_{33}^2 - b_{11}d_{23}d_{33} - b_{11}c_{23}d_{33}$$

$$- d_{11}b_{21}c_{33}^3 - d_{11}c_{21}c_{33} - d_{12}b_{21}c_{33}^2 - d_{12}c_{21}c_{33} - d_{12}b_{21}c_{33} - d_{13}c_{21}c_{33} \left(\frac{1}{s}\right)$$

$$- c_{13}d_{21}c_{31} - c_{13}d_{22}c_{31} - c_{13}d_{23}c_{31} \left(\frac{1}{s}\right)$$

$$N_2 = N_{23}^3 + N_{22}^2 + N_{21}^3 + N_{20} + N_{19} \left(\frac{1}{s}\right)$$

$$N_{23} = (a_{11}d_{22}c_{33} + d_{12}a_{23}c_{31} - a_{11}a_{23}d_{33})$$

(H52)

$$x^2 = (x^2 + y^2 + z^2) - (y^2 + z^2) = r^2 - (y^2 + z^2)$$

$$y^2 = (x^2 + y^2 + z^2) - (x^2 + z^2) = r^2 - (x^2 + z^2)$$

$$z^2 = (x^2 + y^2 + z^2) - (x^2 + y^2) = r^2 - (x^2 + y^2)$$

$$x^2 + y^2 = (r^2 - (y^2 + z^2)) + (r^2 - (x^2 + z^2)) = 2r^2 - x^2 - y^2 - 2z^2$$

$$x^2 + z^2 = (r^2 - (y^2 + z^2)) + (r^2 - (x^2 + z^2)) = 2r^2 - x^2 - y^2 - z^2$$

$$y^2 + z^2 = (r^2 - (x^2 + z^2)) + (r^2 - (x^2 + y^2)) = 2r^2 - x^2 - y^2 - z^2$$

$$x^2 + y^2 + z^2 = r^2$$

$$r^2 = x^2 + y^2 + z^2$$

$$N_{22} = (a_{11}d_{21}c_{33} + b_{11}d_{22}c_{33} + d_{11}a_{23}c_{31} + d_{12}b_{23}c_{31} - a_{11}b_{23}d_{33} - b_{11}a_{23}d_{33} - d_{12}b_{21}c_{33})$$

$$N_{21} = (a_{11}d_{23}c_{33} + b_{11}d_{21}c_{33} + d_{11}b_{23}c_{31} + d_{12}c_{23}c_{31} + d_{13}a_{23}c_{31} - a_{11}c_{23}d_{33} - b_{11}b_{23}d_{33})$$

$$-d_{11}b_{21}c_{33} - d_{12}c_{21}c_{33} - c_{13}d_{22}c_{31})$$

$$N_{20} = (b_{11}d_{23}c_{33} + d_{11}c_{23}c_{31} + d_{13}b_{23}c_{31} + c_{13}b_{21}d_{33} - b_{11}c_{23}d_{33} - d_{11}c_{21}c_{33} - d_{13}b_{21}c_{33})$$

$$-c_{13}d_{21}c_{31})$$

$$N_{19} = (d_{13}c_{23}c_{31} + c_{13}c_{21}d_{33} - d_{13}c_{21}c_{33} - c_{13}d_{23}c_{31})$$

$$L[z] = \frac{N_{19} + N_{20}s + N_{21}s^2 + N_{22}s^3 + N_{23}s^4}{s(D_0 + D_1s + D_2s^2 + D_3s^3 + D_4s^4)} \quad (B153)$$

Let $a_{4z} = \frac{N_{23}}{D_4}$ $a_{3z} = \frac{N_{22}}{D_4}$ $a_{2z} = \frac{N_{21}}{D_4}$ $a_{1z} = \frac{N_{20}}{D_4}$ $a_{0z} = \frac{N_{19}}{D_4}$

$$\frac{d}{dt} \left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{y}^2 + \frac{1}{2} m \dot{z}^2 \right) = \frac{d}{dt} \left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{y}^2 + \frac{1}{2} m \dot{z}^2 \right)$$

$$\Gamma \left[\frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{y}^2 + \frac{1}{2} m \dot{z}^2 \right]$$

$$\frac{d}{dt} \left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{y}^2 + \frac{1}{2} m \dot{z}^2 \right) = \frac{d}{dt} \left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{y}^2 + \frac{1}{2} m \dot{z}^2 \right)$$

$$\frac{d}{dt} \left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{y}^2 + \frac{1}{2} m \dot{z}^2 \right)$$

$$\frac{d}{dt} \left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{y}^2 + \frac{1}{2} m \dot{z}^2 \right) = \frac{d}{dt} \left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{y}^2 + \frac{1}{2} m \dot{z}^2 \right)$$

$$\frac{d}{dt} \left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{y}^2 + \frac{1}{2} m \dot{z}^2 \right)$$

$$\frac{d}{dt} \left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{y}^2 + \frac{1}{2} m \dot{z}^2 \right) = \frac{d}{dt} \left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{y}^2 + \frac{1}{2} m \dot{z}^2 \right)$$

$$\frac{d}{dt} \left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{y}^2 + \frac{1}{2} m \dot{z}^2 \right) = \frac{d}{dt} \left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{y}^2 + \frac{1}{2} m \dot{z}^2 \right)$$

From (B112),

$$b_4 = \frac{D_3}{D_4} \quad b_3 = \frac{D_2}{D_4} \quad b_2 = \frac{D_1}{D_4} \quad b_1 = \frac{D_0}{D_4} \quad b_0 = 0$$

Then,

$$\begin{bmatrix} z \end{bmatrix} = \frac{a_{4z}s^4 + a_{3z}s^3 + a_{2z}s^2 + a_{1z}s + a_{0z}}{s^5 + b_4s^4 + b_3s^3 + b_2s^2 + b_1s}$$

It is noted that this is the same as equation (B113) except the constant coefficients (a_{4z} , a_{3z} , etc.) of the numerator have different values. The solution for z is the same as for x except for that change.

$$c_{1z} = \frac{a_{0z}}{(g_3)(g_4)} \tag{B156}$$

$$d_{1z} = a_{4z} - c_{1z} \tag{B157}$$

$$d_{2z} = a_{3z} - (c_{1z})(b_4)$$

$$d_{3z} = a_{2z} - (c_{1z})(b_3)$$

$$d_{4z} = a_{1z} - (c_{1z})(b_2) \tag{B158}$$

$$c_{6z} = B_{4z} = \frac{2(\alpha_3 - \alpha_4) \left[g_4 d_{3z} - g_4^2 d_{1z} - 2\alpha_4 d_{4z} \right] + (g_3 - g_4) \left[-g_4 d_{2z} + 2\alpha_4 g_4 d_{1z} + d_{4z} \right]}{2(\alpha_3 - \alpha_4) \left[2\alpha_3 g_4 - 2\alpha_4 g_3 \right] + (g_3 - g_4)^2}$$

$$c_{5z} = B_{3z} = \frac{d_{4z} - g_3 B_{4z}}{g_4} = \frac{d_{4z} - g_3 c_{6z}}{g_4} \quad (E159)$$

$$c_{4z} = A_{4z} = \frac{g_4(d_{2z} - 2\alpha_4 d_{1z}) - d_{4z} + c_{6z}(g_3 - g_4)}{2g_4(\alpha_3 - \alpha_4)} \quad (E160)$$

$$c_{3z} = A_{3z} = d_{1z} - A_{4z} = d_{1z} - c_{4z} \quad (E161)$$

$$P_{23z} = (\beta_3)(c_{3z}) \quad (E162)$$

$$P_{13z} = (c_{5z}) - (\alpha_3)(c_{3z})$$

$$P_{24z} = (\beta_4)(c_{4z})$$

$$P_{14z} = (c_{6z}) - (\alpha_4)(c_{4z})$$

$$z = c_{1z} + \frac{1}{\beta_3} e^{-\alpha_3 t} (P_{23z} \cos \beta_3 t + P_{13z} \sin \beta_3 t) + \frac{1}{\beta_4} e^{-\alpha_4 t} (P_{24z} \cos \beta_4 t + P_{14z} \sin \beta_4 t) \quad (E163)$$

$$\dot{z} = \frac{-\alpha_3}{\beta_3} e^{-\alpha_3 t} (P_{23z} \cos \beta_3 t + P_{13z} \sin \beta_3 t)$$

$$+ e^{-\alpha_3 t} (-P_{23z} \sin \beta_3 t + P_{13z} \cos \beta_3 t)$$

(1000) $\frac{200000 - 100000}{100000} = \frac{100000 - 50000}{50000} \Rightarrow 100\% = 200\%$

(1010) $\frac{(100 - 50) \times 10^6 - (100 \times 10^6 - 50 \times 10^6)}{100 \times 10^6} = 100\% = 50\%$

(1020) $100\% = 100\% = 100\% \quad 100\% = 100\% = 100\%$

(1030) $(100\%)(100\%) = 100\%$

$(100\%)(100\%) = 100\%$

$(100\%)(100\%) = 100\%$

$(100\%)(100\%) = 100\%$

$(100\% \times 100\% + 100\% \times 100\%) \times 100\% = \frac{1}{2} + 100\% = 2$

(1040) $(100\% \times 100\% + 100\% \times 100\%) \times 100\% = \frac{1}{2} + 1$

$(100\% \times 100\% + 100\% \times 100\%) \times 100\% = \frac{1}{2} + 1 = 1.5$

$(100\% \times 100\% + 100\% \times 100\%) \times 100\% = 2$

$$\begin{aligned} & \frac{-\alpha_4}{\beta_4} e^{-\alpha_4 t} (P_{24z} \cos \beta_4 t + P_{14z} \sin \beta_4 t) \\ & + e^{-\alpha_4 t} (-P_{24z} \sin \beta_4 t + P_{14z} \cos \beta_4 t) \end{aligned} \quad (B164)$$

$$\ddot{z} = \frac{(\alpha_3^2 - \beta_3^2)}{\beta_3} e^{-\alpha_3 t} (P_{23z} \cos \beta_3 t + P_{13z} \sin \beta_3 t)$$

$$-2 \alpha_3 e^{-\alpha_3 t} (-P_{23z} \sin \beta_3 t + P_{13z} \cos \beta_3 t)$$

$$+ \frac{(\alpha_4^2 - \beta_4^2)}{\beta_4} e^{-\alpha_4 t} (P_{24z} \cos \beta_4 t + P_{14z} \sin \beta_4 t)$$

$$-2 \alpha_4 e^{-\alpha_4 t} (-P_{24z} \sin \beta_4 t + P_{14z} \cos \beta_4 t) \quad (B165)$$

The solution for θ is as follows:

$$L[\theta] \frac{N_3}{\text{Den.}} = \begin{array}{|c|c|c|} \hline (a_{11}s^2 + b_{11}s) & (a_{12}s^2 + b_{12}s + c_{12}) & (d_{11} + d_{12}s + \frac{d_{13}}{s}) \\ \hline (b_{21}s + c_{21}) & (a_{22}s^2 + b_{22}s + c_{22}) & (d_{21} + d_{22}s + \frac{d_{23}}{s}) \\ \hline (c_{31}) & (c_{32}) & (\frac{d_{33}}{s}) \\ \hline \end{array}$$

$$D_4 s^4 + D_3 s^3 + D_2 s^2 + D_1 s + D_0$$

$$(r_1^2 \cos^2 \theta + r_2^2 \sin^2 \theta)^{1/2} = \frac{r_1 r_2}{r}$$

(10.12) $(r_1^2 \cos^2 \theta + r_2^2 \sin^2 \theta)^{1/2} = r$

$$(r_1^2 \cos^2 \theta + r_2^2 \sin^2 \theta)^{1/2} = \frac{r_1^2 - r_2^2}{r} = \frac{r_1^2 - r_2^2}{r}$$

$$(r_1^2 \cos^2 \theta + r_2^2 \sin^2 \theta)^{1/2} = \frac{r_1^2 - r_2^2}{r}$$

$$(r_1^2 \cos^2 \theta + r_2^2 \sin^2 \theta)^{1/2} = \frac{(r_1^2 - r_2^2)}{r}$$

(10.13) $(r_1^2 \cos^2 \theta + r_2^2 \sin^2 \theta)^{1/2} = \frac{r_1^2 - r_2^2}{r}$

The equation for r is

$\left(\frac{r_1^2}{r} = r_1^2 \cos^2 \theta + r_2^2 \sin^2 \theta\right)$	$(r_1^2 \cos^2 \theta + r_2^2 \sin^2 \theta)$	$(r_1^2 \cos^2 \theta + r_2^2 \sin^2 \theta)$	$= \frac{r_1^2 - r_2^2}{r}$
$\left(\frac{r_2^2}{r} = r_1^2 \cos^2 \theta + r_2^2 \sin^2 \theta\right)$	$(r_1^2 \cos^2 \theta + r_2^2 \sin^2 \theta)$	$(r_1^2 \cos^2 \theta + r_2^2 \sin^2 \theta)$	
$\left(\frac{r_1^2}{r}\right)$	$(r_1^2 \cos^2 \theta)$	$(r_2^2 \sin^2 \theta)$	

$$r = r_1^2 \cos^2 \theta + r_2^2 \sin^2 \theta + \frac{r_1^2 - r_2^2}{r}$$

$$\begin{aligned}
 N_3 = & a_{11}^a d_{22}^d s^3 + a_{11}^b d_{22}^d s^2 + a_{11}^c d_{22}^d s + b_{11}^a d_{22}^d s^2 + b_{11}^b d_{22}^d s + b_{11}^c d_{22}^d \\
 & + a_{12}^d d_{21}^c s^2 + a_{12}^d d_{22}^c s^3 + a_{12}^d d_{23}^c s + b_{12}^d d_{21}^c s + b_{12}^d d_{22}^c s^2 + b_{12}^d d_{23}^c s \\
 & + c_{12}^d d_{21}^c s + c_{12}^d d_{22}^c s + c_{12}^d d_{23}^c \left(\frac{1}{s}\right) + d_{11}^b d_{21}^c s + d_{11}^c d_{21}^c s + d_{12}^b d_{21}^c s^2 \\
 & + d_{12}^c d_{21}^c s + d_{13}^c d_{21}^c s + d_{13}^c d_{21}^c \left(\frac{1}{s}\right) - a_{11}^d d_{21}^c s^2 - a_{11}^d d_{22}^c s^3 - a_{11}^d d_{23}^c s^2 \\
 & - b_{11}^d d_{21}^c s - b_{11}^d d_{22}^c s^2 - b_{11}^d d_{23}^c s - a_{12}^b d_{21}^d s^3 - b_{12}^b d_{21}^d s^3 \\
 & - b_{12}^c d_{21}^d s - c_{12}^b d_{21}^d s - c_{12}^c d_{21}^d \left(\frac{1}{s}\right) - d_{11}^a d_{22}^c s^2 - d_{11}^b d_{22}^c s - d_{11}^c d_{22}^c s \\
 & - d_{12}^a d_{22}^c s^3 - d_{12}^b d_{22}^c s^2 - d_{12}^c d_{22}^c s - d_{13}^a d_{22}^c s - d_{13}^b d_{22}^c s - d_{13}^c d_{22}^c \left(\frac{1}{s}\right) \\
 N_3 = & N_{33} s^3 + N_{32} s^2 + N_{31} s + N_{30} + N_{29} \left(\frac{1}{s}\right)
 \end{aligned}$$

(B166)

$$N_{33} = (a_{11} a_{22} d_{33} + a_{12} d_{22} c_{31} - a_{11} d_{22} c_{32} - d_{12} a_{22} c_{31})$$

$$N_{32} = (a_{11} b_{22} d_{33} + b_{11} a_{22} d_{33} + a_{12} d_{21} c_{31} + b_{12} d_{22} c_{31} + d_{12} b_{21} c_{32} - a_{11} d_{21} c_{32} - b_{11} d_{22} c_{32}$$

$$- a_{12} b_{21} d_{33} - d_{11} a_{22} c_{31} - d_{12} b_{22} c_{31})$$

$$N_{31} = (a_{11} c_{22} d_{33} + b_{11} b_{22} d_{33} + a_{12} d_{23} c_{31} + b_{12} d_{21} c_{31} + c_{12} d_{22} c_{31} + d_{11} b_{21} c_{32} + d_{12} c_{21} c_{32}$$

$$- a_{11} d_{23} c_{32} - b_{11} d_{21} c_{32} - a_{12} c_{21} d_{33} - b_{12} b_{21} d_{33} - d_{11} b_{22} c_{31} - d_{12} c_{22} c_{31} - d_{13} a_{22} c_{31})$$

$$N_{30} = (b_{11} c_{22} d_{33} + b_{12} d_{23} c_{31} + c_{12} d_{21} c_{31} + d_{11} c_{21} c_{32} + d_{13} b_{21} c_{32} - b_{11} d_{23} c_{32} - b_{12} c_{21} d_{33}$$

$$- c_{12} b_{21} d_{33} - d_{11} c_{22} c_{31} - d_{13} b_{22} c_{31})$$

$$N_{29} = (c_{12} d_{23} c_{31} + d_{13} c_{21} c_{32} - c_{12} c_{21} d_{33} - d_{13} c_{22} c_{31})$$

$$200 = 100 + 100 \Rightarrow 200 = 100 + 100$$

$$100 = 100 + 0 \Rightarrow 100 = 100 + 0$$

$$100 = 100 + 0 \Rightarrow 100 = 100 + 0$$

$$100 = 100 + 0 \Rightarrow 100 = 100 + 0$$

$$100 = 100 + 0 \Rightarrow 100 = 100 + 0$$

$$100 = 100 + 0 \Rightarrow 100 = 100 + 0$$

$$100 = 100 + 0 \Rightarrow 100 = 100 + 0$$

$$100 = 100 + 0 \Rightarrow 100 = 100 + 0$$

$$L[\theta] = \frac{N_{29} + N_{30}s + N_{31}s^2 + N_{32}s^3 + N_{33}s^4}{s(D_0 + D_1s + D_2s^2 + D_3s^3 + D_4s^4)} \quad (E167)$$

$$\text{Let } a_{40} = \frac{N_{33}}{D_4} \quad a_{30} = \frac{N_{32}}{D_4} \quad a_{20} = \frac{N_{31}}{D_4} \quad a_{10} = \frac{N_{30}}{D_4} = a_{00} = \frac{N_{29}}{D_4}$$

Then,

$$L[\theta] = \frac{a_{40}s^4 + a_{30}s^3 + a_{20}s^2 + a_{10}s + a_{00}}{s^5 + b_4s^4 + b_3s^3 + b_2s^2 + b_1s} \quad (E169)$$

It is noted that this is the same as equations (E113) and (E155) except for the constants in the numerator. Therefore the solution is as follows:

$$c_{10} = \frac{a_{00}}{(g_3)(g_4)} \quad (E170)$$

$$d_{10} = a_{40} - c_{10} \quad (E171)$$

$$d_{20} = a_{30} - (c_{10})(b_4)$$

$$d_{30} = a_{20} - (c_{10})(b_3)$$

$$d_{40} = a_{10} - (c_{10})(b_2)$$

(123)
$$\frac{1000 - 1000(0.05)^n}{0.05} = 1000 \left[\frac{1 - (0.95)^n}{0.05} \right] \quad (1)$$

$$\frac{1000}{0.05} - \frac{1000(0.05)^n}{0.05} = 1000 \left[\frac{1 - (0.95)^n}{0.05} \right] \quad \text{or} \quad \frac{1000}{0.05} - \frac{1000(0.05)^n}{0.05} = 1000 \left[\frac{1 - (0.95)^n}{0.05} \right]$$

(124)
$$\frac{1000 - 1000(0.05)^n}{0.05} = 1000 \left[\frac{1 - (0.95)^n}{0.05} \right] \quad (2)$$

It is noted that this is the same as equation (1) and (2) are identical. Therefore the solution is the same for both equations. (125)

(126)
$$\frac{1000}{0.05} - \frac{1000(0.05)^n}{0.05} = 1000 \left[\frac{1 - (0.95)^n}{0.05} \right]$$

(127)
$$1000 - 1000(0.05)^n = 1000 \left[1 - (0.95)^n \right]$$

$$(1000)(0.05)^n = 1000 - 1000(0.95)^n$$

$$(1000)(0.05)^n = 1000 - 1000(0.95)^n$$

$$(1000)(0.05)^n = 1000 - 1000(0.95)^n$$

$$c_{60} = \frac{2(\alpha_3 - \alpha_4) \left[\varepsilon_4 d_{30} - \varepsilon_4^2 d_{10} - 2\alpha_4 d_{40} \right] + (\varepsilon_3 - \varepsilon_4) \left[-\varepsilon_4 d_{20} + 2\alpha_4 \varepsilon_4 d_{10} + d_{40} \right]}{2(\alpha_3 - \alpha_4) \left[2\alpha_3 \varepsilon_4 - 2\alpha_4 \varepsilon_3 \right] + (\varepsilon_3 - \varepsilon_4)^2}$$

$$c_{50} = \frac{d_{40} - \varepsilon_3 c_{60}}{\varepsilon_4} \quad (B173)$$

$$c_{40} = \frac{\varepsilon_4 (d_{20} - 2\alpha_4 d_{10}) - d_{40} + c_{60} (\varepsilon_3 - \varepsilon_4)}{2 \varepsilon_4 (\alpha_3 - \alpha_4)} \quad (B174)$$

$$c_{30} = d_{10} - c_{40} \quad (B175)$$

$$P_{230} = (\beta_3)(c_{30}) \quad (B176)$$

$$P_{130} = (c_{50}) - (\alpha_3)(c_{30})$$

$$P_{240} = (\beta_4)(c_{40})$$

$$P_{140} = (c_{60}) - (\alpha_4)(c_{40})$$

$$\theta = c_{10} + \frac{1}{\beta_3} e^{-\alpha_3 t} (P_{230} \cos \beta_3 t + P_{130} \sin \beta_3 t) + \frac{1}{\beta_4} e^{-\alpha_4 t} (P_{240} \cos \beta_4 t + P_{140} \sin \beta_4 t) \quad (B177)$$

$$\frac{[a^2 \cdot \frac{1}{a^2} + \frac{1}{a^2} \cdot a^2] (a^2 - a) + [a^2(a^2 - a) - a^2(a^2 - a)] (a^2 - a)^{-1} = 0}{(a^2 - a) = [a^2(a^2 - a) - a^2(a^2 - a)] (a^2 - a)^{-1}}$$

(17) $\frac{a^2(a^2 - a) - a^2(a^2 - a)}{a^2 - a} = 0$

(18) $\frac{(a^2 - a) a^2 + (a^2 - a) a^2 - a^2(a^2 - a)}{(a^2 - a)^2} = 0$

(19) $a^2 - a^2 = 0$

(20) $(a^2)(a^2) = a^2 a^2$

$(a^2)(a^2) - (a^2) = a^2 a^2 - a^2$

$(a^2)(a^2) = a^2 a^2$

$(a^2)(a^2) - (a^2) = a^2 a^2 - a^2$

$(a^2 a^2 + a^2 a^2) \frac{1}{a^2} + a^2 - a^2 = 0$

(21) $(a^2 a^2 + a^2 a^2) \frac{1}{a^2} = 0$

$$\begin{aligned} \dot{\theta} &= \frac{-\alpha_3}{\beta_3} e^{-\alpha_3 t} (P_{230} \cos \beta_3 t + P_{130} \sin \beta_3 t) \\ &+ e^{-\alpha_3 t} (-P_{230} \sin \beta_3 t + P_{130} \cos \beta_3 t) \\ &+ \frac{-\alpha_4}{\beta_4} e^{-\alpha_4 t} (P_{240} \cos \beta_4 t + P_{140} \sin \beta_4 t) \\ &+ e^{-\alpha_4 t} (-P_{240} \sin \beta_4 t + P_{140} \cos \beta_4 t) \end{aligned} \quad (E178)$$

$$\begin{aligned} \ddot{\theta} &= \frac{(\alpha_3^2 - \beta_3^2)}{\beta_3} e^{-\alpha_3 t} (P_{230} \cos \beta_3 t + P_{130} \sin \beta_3 t) \\ &- 2 \alpha_3 e^{-\alpha_3 t} (-P_{230} \sin \beta_3 t + P_{130} \cos \beta_3 t) \\ &+ \frac{(\alpha_4^2 - \beta_4^2)}{\beta_4} e^{-\alpha_4 t} (P_{240} \cos \beta_4 t + P_{140} \sin \beta_4 t) \\ &- 2 \alpha_4 e^{-\alpha_4 t} (-P_{240} \sin \beta_4 t + P_{140} \cos \beta_4 t) \end{aligned} \quad (E179)$$

Now there is a complete description of the motion during the sliding phase.

It is noted that x_2 , \dot{x}_2 , z_2 , \dot{z}_2 , θ_2 and $\dot{\theta}_2$ are used as initial conditions for the LaPlace equations for sliding. This is in addition to their use in linearizing.

However, \dot{x}_2 , \dot{z}_2 and $\dot{\theta}_2$ are used only for linearization (and not for initial conditions). It is not appropriate to use the State 2 conditions of accelerations for initial conditions for sliding. For that reason it is considered, for use in the sliding equations, to consider

$$\dot{x}_2 = -1.0 \quad (B180)$$

$$\dot{z}_2 = 0.0$$

$$\dot{\theta}_2 = 0.0$$

(1) The first condition is that the function $f(x)$ must be continuous on the interval $[a, b]$.
 (2) The second condition is that the function $f(x)$ must be bounded on the interval $[a, b]$.
 (3) The third condition is that the function $f(x)$ must have a finite number of discontinuities on the interval $[a, b]$.

(182)

$$f(x) = \frac{1}{x}$$

$$f'(x) = -\frac{1}{x^2}$$

$$f''(x) = \frac{2}{x^3}$$

Vertical Force on Bow During Sliding

It is important to note that the downward force, F_{BZS} , may be solved for each time t during the sliding phase. (This value should not exceed $(F_{BZC})_2$ since this would imply local crushing should have started again.)

The value of F_{BZS} may be determined from equation (B92).

$$F_{BZS} = -T_{BOL}\theta + \frac{T_{BOL}}{v_1} \left(\frac{dx}{dt}\right)\theta - T_f z$$

$$- T_f (LCG-LCF)\theta - k_h \left(\frac{dz}{dt}\right) - m_z \frac{d^2 z}{dt^2}$$

where θ may be obtained from equation (B177)

$$\frac{dx}{dt} \quad (B150)$$

$$z \quad (B163)$$

$$\frac{dz}{dt} \quad (B164)$$

$$\frac{d^2 z}{dt^2} \quad (B165)$$

PROBLEM 1

Let $f(x) = x^3 - 3x^2 + 2x - 1$. Find the local extrema of $f(x)$ and determine their nature. Also, find the intervals where $f(x)$ is increasing or decreasing.

$$f'(x) = 3x^2 - 6x + 2 = 0 \implies x = \frac{6 \pm \sqrt{36 - 24}}{6} = \frac{6 \pm \sqrt{12}}{6} = 1 \pm \frac{\sqrt{3}}{3}$$

$$f''(x) = 6x - 6 \implies f''\left(1 + \frac{\sqrt{3}}{3}\right) = 2\sqrt{3} > 0 \implies \text{local minimum}$$

$$f''\left(1 - \frac{\sqrt{3}}{3}\right) = -2\sqrt{3} < 0 \implies \text{local maximum}$$

Since $f'(x) > 0$ for $x < 1 - \frac{\sqrt{3}}{3}$ and $f'(x) < 0$ for $1 - \frac{\sqrt{3}}{3} < x < 1 + \frac{\sqrt{3}}{3}$, $f(x)$ is increasing on $(-\infty, 1 - \frac{\sqrt{3}}{3})$ and decreasing on $(1 - \frac{\sqrt{3}}{3}, 1 + \frac{\sqrt{3}}{3})$.

- (100) $\frac{1}{2}$
- (101) $\frac{1}{3}$
- (102) $\frac{1}{4}$
- (103) $\frac{1}{5}$
- (104) $\frac{1}{6}$

Termination of Sliding Phase, State 3

If the equations of velocity (E150), (E164), and (E178) combined, the velocity of a point on the bow in contact with the ice may be determined. When this velocity becomes zero the sliding phase has terminated, State 3. Using this time, t_3 , x_3 , z_3 , and θ_3 can be determined from (E149), (E163) and (E177) and this will give us the location on the ship of the point of ice support. Using this point the static equilibrium problem may be solved (presuming slipping does not start immediately) and the downward force under the bow may be determined.

From Figure B-XIX it may be seen that the velocity of point A on the bow may be expressed in terms of the z-component

$$v_{Az} = \frac{dz}{dt} - \frac{d\theta}{dt} (GA)_x \quad (E181)$$

and the x-component

$$v_{Ax} = \frac{dx}{dt} - \frac{d\theta}{dt} (GA)_z \quad (E182)$$

Since these two components are related as shown in Figure B-XIX, the value of one may be used to determine the value of the other. However, when $v_{Az} = 0$ then $v_{Ax} = 0$. For that reason either equation (E181) or (E182) may be used to find the time, t_3 , for $v_A = 0$.

Equation (E182) for the velocity component of A in the x-direction will be used.

Verification of Euler's Theorem

In the previous section we saw that if x, y, z are positive integers such that $x^2 + y^2 = z^2$, then x, y, z can be written in the form $x = k(m^2 - n^2), y = k(2mn), z = k(m^2 + n^2)$ where m, n are coprime integers, $m > n$, and m, n are of opposite parity. We now verify that these expressions satisfy Euler's equation. Substituting these expressions into Euler's equation, we get $k^2(m^2 - n^2)^2 + k^2(2mn)^2 = k^2(m^2 + n^2)^2$. Dividing both sides by k^2 , we get $(m^2 - n^2)^2 + (2mn)^2 = (m^2 + n^2)^2$. Expanding the squares, we get $m^4 - 2m^2n^2 + n^4 + 4m^2n^2 = m^4 + 2m^2n^2 + n^4$. Simplifying, we get $2m^2n^2 = 2m^2n^2$, which is true. Thus, the expressions for x, y, z satisfy Euler's equation.

(187)
$$x^2 + y^2 = z^2$$

and the converse.

(188)
$$x^2 + y^2 = z^2$$

Since these two equations are related as in (187) and (188), we may use the same method to describe the solutions of the other equation. Let x, y, z be a primitive solution of (188). Then $x^2 + y^2 = z^2$ and x, y, z are coprime. We may assume that x and y are of opposite parity. Then $x^2 + y^2 \equiv 1 \pmod{4}$, so $z^2 \equiv 1 \pmod{4}$ and z is odd. Let $z = 2k + 1$. Then $x^2 + y^2 = (2k + 1)^2 = 4k^2 + 4k + 1$. Thus $x^2 + y^2 - 1 = 4k^2 + 4k$. Dividing both sides by 4, we get $\frac{x^2 + y^2 - 1}{4} = k^2 + k$. This is a quadratic equation in k . Solving for k , we get $k = \frac{-1 \pm \sqrt{1 + x^2 + y^2}}{2}$. Since k is an integer, $1 + x^2 + y^2$ must be a perfect square. Let $1 + x^2 + y^2 = w^2$. Then $w^2 - x^2 - y^2 = 1$. This is a Diophantine equation. We can solve this equation by the method of descent. Let $w = m^2 - n^2, x = 2mn, y = m^2 + n^2$. Then $(m^2 - n^2)^2 - (2mn)^2 - (m^2 + n^2)^2 = 1$. Simplifying, we get $-4m^2n^2 - 4m^2n^2 = 1$, which is impossible. Thus, there are no primitive solutions of (188).

$$v_{Ax} = \frac{dx}{dt} - \frac{d\theta}{dt} (GA)_z$$

From equation (B86)

$$(GA)_z = (H-KG) + z$$

$$v_{Ax} = \frac{dx}{dt} - \frac{d\theta}{dt} [(H-KG) + z] \quad (E183)$$

The values to be used come from equations (E150), (E178), and (E163) respectively. For each substitution of t into these equations and then the substitution of these values into equation, a velocity v_{Ax} results. When v_{Ax} becomes zero the sliding phase has terminated and State 3 has been reached.

$$(16) \quad \frac{d^2 u}{dx^2} = -u^2$$

(17) $u(0) = 1$

$$u(1) = 0$$

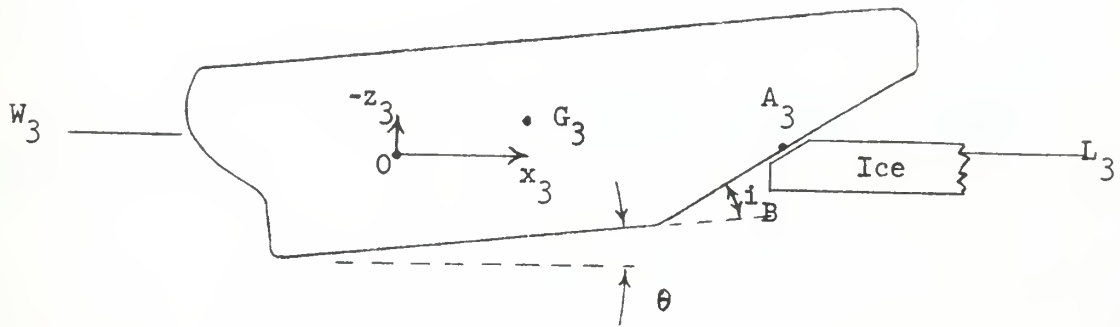
(18)

$$\left[\frac{d}{dx} + (2x-1) \right] \frac{d^2 u}{dx^2} = -\frac{d^2 u}{dx^2} - u^2$$

The values to be used are: (16) , (17) , (18) , (19) , (20) , (21) , (22) , (23) , (24) , (25) , (26) , (27) , (28) , (29) , (30) , (31) , (32) , (33) , (34) , (35) , (36) , (37) , (38) , (39) , (40) , (41) , (42) , (43) , (44) , (45) , (46) , (47) , (48) , (49) , (50) , (51) , (52) , (53) , (54) , (55) , (56) , (57) , (58) , (59) , (60) , (61) , (62) , (63) , (64) , (65) , (66) , (67) , (68) , (69) , (70) , (71) , (72) , (73) , (74) , (75) , (76) , (77) , (78) , (79) , (80) , (81) , (82) , (83) , (84) , (85) , (86) , (87) , (88) , (89) , (90) , (91) , (92) , (93) , (94) , (95) , (96) , (97) , (98) , (99) , (100) .

Figure B-XXVI

Illustration of Position at State 3



From equation (B88),

$$(GA_3)_x = k_4 - x_3$$

From equation (B86)

$$(GA_3)_z = (H-KG) + z_3$$

State 4

It seems quite probable that when there is no velocity of the bow relative to the ice that all velocities will be zero, or negligible. However, the static equilibrium problem should be solved regardless based on support of the bow at point A_3 for time t_3 .

It should also be noted that the friction force due to sliding has now disappeared.

1911

The first thing I noticed when I stepped out of the train
was the smell of the sea. It was a strange, salty
smell that I had never before. The air was cool and
fresh, a relief from the hot, stifling air of the
city. I looked out at the vast expanse of the ocean,
the waves crashing against the shore. It was a sight
that I had never before. The sun was shining
brightly, and the sky was a clear, deep blue.
I felt a sense of peace and tranquility that I
had never before. It was a feeling that I had
never before. I had found a new world, a new
place to call home.

Based on Figure B-XXVII Newton's equations may be applied for static equilibrium.

$$\sum F_x = 0$$

$$T_{BOL} \cos \theta_4 - F_{BX4} = 0 \quad (E184)$$

$$\sum F_z = 0$$

$$-F_{BZ4} - T_{BOL} \sin \theta_4 - \Delta - T_f h_4 + \Delta = 0$$

$$-F_{BZ4} - T_{BOL} \sin \theta_4 - T_f h_4 = 0 \quad (E185)$$

It is to be noted that F_{BZ4} will be of greater magnitude if the bollard thrust is eliminated (stopping the screws) as long as static equilibrium can be maintained by static friction at the bow.

It may be seen in Figure B-XXVIII that the change in draft at the center of gravity from State 3 to State 4 is equal to

$$z_4 - z_3 = (GA_3)_x (\theta_4 - \theta_3)$$

Therefore

$$z_4 = z_3 + (GA_3)_x (\theta_4 - \theta_3) \quad (E186)$$

From equation (E17)

Let $f(x) = \sum_{k=0}^n a_k x^k$ and $g(x) = \sum_{k=0}^m b_k x^k$ be two polynomials.

Then their product is

$$f(x)g(x) = \sum_{k=0}^{n+m} c_k x^k$$

(1)

$$c_k = \sum_{i+j=k} a_i b_j$$

$$c_k = \sum_{i+j=k} a_i b_j$$

$$c_k = \sum_{i+j=k} a_i b_j = \sum_{i=0}^k a_i b_{k-i}$$

(2)

$$c_k = \sum_{i+j=k} a_i b_j = \sum_{i=0}^k a_i b_{k-i}$$

It is clear that c_k is the coefficient of x^k in the product of $f(x)$ and $g(x)$.
If we compare the coefficient of x^k in $f(x)g(x)$ with the coefficient of x^k in $\sum_{i=0}^k a_i b_{k-i} x^k$, we find that they are equal.
Hence $c_k = \sum_{i=0}^k a_i b_{k-i}$ for all k .

$$c_k = \sum_{i=0}^k a_i b_{k-i}$$

Therefore

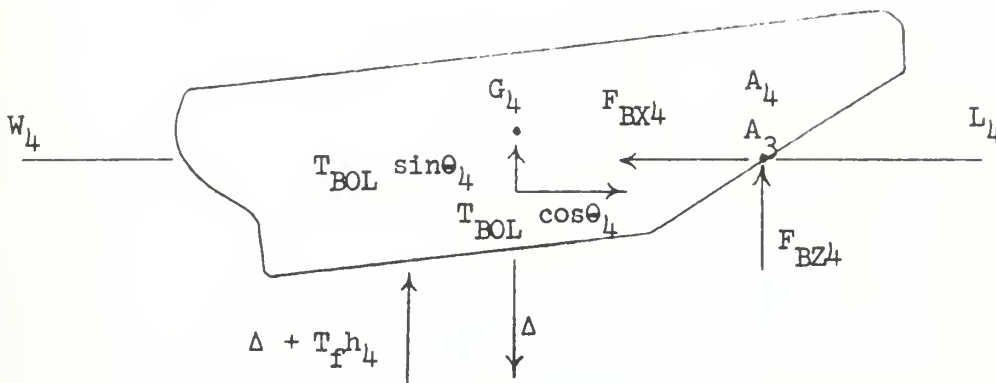
(3)

$$c_k = \sum_{i=0}^k a_i b_{k-i}$$

From equation (3)

Figure B-XXVII

Free Body Diagram for Static Equilibrium,
State 4



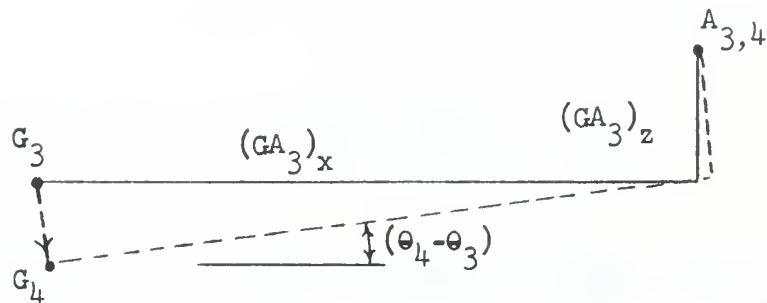
$h \equiv$ increase in draft at LCF

$T_f \equiv$ pounds per foot immersion

$T_{BOL} \equiv$ bollard thrust, pounds

Figure B-XXVIII

Illustration of Moment Arms $(GA_4)_x$ and $(GA_4)_z$



$$(G_4A_4)_x = (GA_3)_x \cos (\theta_4 - \theta_3) - (GA_3)_z \sin (\theta_4 - \theta_3)$$

and for small angle $(\theta_4 - \theta_3)$

$$(G_4A_4)_x = (GA_3)_x - (GA_3)_z (\theta_4 - \theta_3)$$

and

$$(G_4A_4)_z = (GA_3)_z + (GA_3)_x (\theta_4 - \theta_3)$$

$$h = z + (LCG - LCF) \theta$$

$$h_4 = z_4 + (LCG - LCF) \theta_4$$

From equation (E186)

$$h_4 = z_3 + (LCG - LCF) \theta_4 + (GA_3)_x (\theta_4 - \theta_3)$$

$$\sum M_A = 0$$

$$F_{BZ4} (G_{4A4})_x + F_{BX4} (G_{4A4})_z + T_{BOL} \quad (KG-d)$$

$$- (\Delta + T_F h_4) GM_L \theta_4 = 0 \quad (E187)$$

$$F_{BZ4} \left[(GA_3)_x - (GA_3)_z (\theta_4 - \theta_3) \right] + F_{BX4} \left[(GA_3)_z + (GA_3)_x (\theta_4 - \theta_3) \right]$$

$$+ T_{BOL} (KG-d) - \Delta GM_L \theta_4 - T_F GM_L \theta_4 \left[z_3 + (LCG - LCF) \theta_4 + (GA_3)_x (\theta_4 - \theta_3) \right]$$

$$= 0 \quad (E188)$$

It may be seen that three unknowns (θ_4 , F_{BX4} , and F_{BZ4}) appear in the three equations (E184), (E185), and (E188).

As mentioned before, the maximum static vertical force F_{BZ4} can be attained when the bollard thrust is eliminated. (Under this condition it

$$m(\ddot{x} - a) = -kx$$

$$m(\ddot{x} - a) + kx = 0$$

From equation (2.5)

$$(\ddot{x} - a) + \frac{k}{m}x = 0$$

$$\sum \ddot{x} = 0$$

$$(2.6) \quad \ddot{x} + \frac{k}{m}x = a$$

$$(2.7) \quad \ddot{x} + \frac{k}{m}x = a$$

$$\left[\frac{1}{2} \frac{d}{dt} \left(\dot{x}^2 + \frac{k}{m} x^2 \right) - \dot{x} a \right]_{t_1}^{t_2} = \left[\frac{1}{2} \frac{d}{dt} \left(\dot{x}^2 + \frac{k}{m} x^2 \right) - \dot{x} a \right]_{t_1}^{t_2}$$

$$\left[\frac{1}{2} \frac{d}{dt} \left(\dot{x}^2 + \frac{k}{m} x^2 \right) - \dot{x} a \right]_{t_1}^{t_2} = \left[\frac{1}{2} \frac{d}{dt} \left(\dot{x}^2 + \frac{k}{m} x^2 \right) - \dot{x} a \right]_{t_1}^{t_2}$$

$$\left[\frac{1}{2} \frac{d}{dt} \left(\dot{x}^2 + \frac{k}{m} x^2 \right) - \dot{x} a \right]_{t_1}^{t_2}$$

(2.8)

It may be seen that three constants \dot{x}_1 , \dot{x}_2 and \dot{x}_3 appear in the three equations (2.5), (2.6), and (2.7).

As mentioned before, the maximum kinetic energy T_{max} can be obtained when the potential energy is minimum. (Under this condition it

is assumed that static friction at the bow is sufficient to maintain equilibrium or at least the acceleration sliding back down the ice is negligible.)

Set $T_{BOL} = 0$ by stopping thrust.

Then $F_{EX4} = 0$

Equations (B185) and (B188) may now be written as follows:

$$-F_{BZ4} - T_f z_3 - T_f (LCG-LCF)\theta_4 - T_f (GA_3)_x (\theta_4 - \theta_3) = 0 \quad (B189)$$

$$F_{BZ4} \left[(GA_3)_x - (GA_3)_z (\theta_4 - \theta_3) \right] - \Delta GM_L \theta_4 \quad (B190)$$

$$- T_f GM_L \theta_4 \left[z_3 + (LCG - LCF)\theta_4 + (GA_3)_x (\theta_4 - \theta_3) \right] = 0 \quad (B190)$$

Expand equation (B189).

$$-F_{BZ4} - T_f z_3 - T_f (LCG-LCF)\theta_4 - T_f (GA_3)_x \theta_4$$

$$+ T_f (GA_3)_x \theta_3 = 0$$

$$-F_{BZ4} - T_f z_3 + T_f (GA_3)_x \theta_3 = T_f \left[(LCG - LCF) + (GA_3)_x \right] \theta_4$$

... (faint text) ...

$$\dots$$

... (faint text) ...

(14a) \dots

(14b) \dots

(14c) \dots

(14d) ...

$$\dots$$

$$\dots$$

$$\dots$$

$$\theta_4 = \frac{-F_{BZ4} - T_f z_3 + T_f (GA_3)_x \theta_3}{T_f [(LCG - LCF) + (GA_3)_x]}$$

$$\theta_4 = \frac{\frac{-F_{BZ4}}{T_f} - z_3 + (GA_3)_x \theta_3}{(LCG - LCF) + (GA_3)_x} \quad (E191)$$

Set $d_1 = (LCG - LCF) + (GA_3)_x$ (E192)

$$\theta_4 = \frac{-F_{BZ4}}{d_1 T_f} - \frac{z_3}{d_1} + \frac{(GA_3)_x \theta_3}{d_1} \quad (E193)$$

Substitute equation (E193) into equation (E190) and solve for F_{BZ4} .

$$F_{BZ4} (GA_3)_x + F_{BZ4} (GA_3)_z \theta_3$$

$$- F_{BZ4} (GA_3)_z \left[\frac{-F_{BZ4}}{d_1 T_f} - \frac{z_3}{d_1} + \frac{(GA_3)_x \theta_3}{d_1} \right]$$

$$- \Delta GM_L \left[\frac{-F_{BZ4}}{d_1 T_f} - \frac{z_3}{d_1} + \frac{(GA_3)_x \theta_3}{d_1} \right]$$

$$T_f GM_L z_3 \left[\frac{-F_{BZ4}}{d_1 T_f} - \frac{z_3}{d_1} + \frac{(GA_3)_x \theta_3}{d_1} \right]$$

$$\frac{f(x) + g(x)}{h(x)} = \frac{f(x)}{h(x)} + \frac{g(x)}{h(x)}$$

(100)

$$\frac{f(x) + g(x)}{h(x)} = \frac{f(x)}{h(x)} + \frac{g(x)}{h(x)}$$

(101)

$$f(x) + g(x) = h(x)$$

(102)

$$\frac{f(x)}{g(x)} = \frac{h(x)}{k(x)}$$

Indicate the equation (100) has solution (101) and solve for f(x)

$$f(x) + g(x) = h(x)$$

$$\left[\frac{f(x)}{g(x)} + \frac{h(x)}{k(x)} \right] = \frac{f(x)}{g(x)} + \frac{h(x)}{k(x)}$$

$$\left[\frac{f(x)}{g(x)} + \frac{h(x)}{k(x)} \right] = \frac{f(x)}{g(x)} + \frac{h(x)}{k(x)}$$

$$\left[\frac{f(x)}{g(x)} + \frac{h(x)}{k(x)} \right] = \frac{f(x)}{g(x)} + \frac{h(x)}{k(x)}$$

$$- T_f GM_L (LCG - LCF) \left[\frac{-F_{BZ4}}{d_1 T_f} - \frac{z_3}{d_1} + \frac{(GA_3)_x \theta_3}{d_1} \right]^2$$

$$+ T_f GM_L (GA_3)_x \theta_3 \left[\frac{-F_{BZ4}}{d_1 T_f} - \frac{z_3}{d_1} + \frac{(GA_3)_x \theta_3}{d_1} \right]$$

$$- T_f GM_L (GA_3)_x \left[\frac{-F_{BZ4}}{d_1 T_f} - \frac{z_3}{d_1} + \frac{(GA_3)_x \theta_3}{d_1} \right]^2$$

= 0

$$\theta_4^2 = \left[\frac{-F_{BZ4}}{d_1 T_f} - \frac{z_3}{d_1} + \frac{(GA_3)_x \theta_3}{d_1} \right]^2$$

$$\theta_4^2 = \frac{F_{BZ4}^2}{d_1^2 T_f^2} + \frac{F_{BZ4} z_3}{d_1^2 T_f} - \frac{F_{BZ4} (GA_3)_x \theta_3}{d_1^2 T_f}$$

$$+ \frac{z_3^2}{d_1^2} + \frac{F_{BZ4} z_3}{d_1^2 T_f} - \frac{(GA_3)_x z_3 \theta_3}{d_1^2}$$

$$+ \frac{(GA_3)_x^2 \theta_3^2}{d_1^2} - \frac{(GA_3)_x z_3 \theta_3}{d_1^2} - \frac{F_{BZ4} (GA_3)_x \theta_3}{d_1^2 T_f}$$

$$\left[\frac{a^2 - 1}{2} + 5 \frac{a^2}{2} \right] \text{ (for } a > 1)$$

$$\left[\frac{a^2 - 1}{2} - 5 \frac{a^2}{2} \right] \text{ (for } a < 1)$$

$$\left[\frac{a^2 - 1}{2} + 5 \frac{a^2}{2} \right] \text{ (for } a > 1)$$

0 -

$$\left[\frac{a^2 - 1}{2} - 5 \frac{a^2}{2} \right] = 0$$

$$\frac{a^2 - 1}{2} - 5 \frac{a^2}{2} = 0 \Rightarrow \frac{a^2 - 1 - 10a^2}{2} = 0$$

$$\frac{a^2 - 1 - 10a^2}{2} = 0 \Rightarrow \frac{-9a^2 - 1}{2} = 0$$

$$\frac{a^2 - 1 - 10a^2}{2} = 0 \Rightarrow \frac{-9a^2 - 1}{2} = 0$$

$$\theta_4^2 = \frac{F_{BZ4}^2}{d_1^2 T_f^2} + \frac{z_3^2}{d_1^2} + \frac{(GA_3)_x^2 \theta_3^2}{d_1^2} + \frac{2 F_{BZ4} z_3}{d_1^2 T_f}$$

$$- \frac{2 F_{BZ4} (GA_3)_x \theta_3}{d_1^2 T_f} - \frac{2 (GA_3)_x z_3 \theta_3}{d_1^2}$$

$$F_{BZ4} (GA)_3^x + F_{BZ4} (GA)_3^z \ominus_3 + T_{BZ4}^2 \frac{(GA)_3^z}{d_1 T_f} + F_{BZ4} \frac{(GA)_3^z z_3}{d_1} - F_{BZ4} \frac{(GA)_3^z (GA)_3^x \ominus_3}{d_1}$$

$$+ F_{BZ4} \frac{\Delta GM_L}{d_1} \frac{T_f}{d_1} + \frac{\Delta GM_L z_3}{d_1} - \frac{\Delta GM_L (GA)_3^x \ominus_3}{d_1} + \frac{F_{BZ4} GM_L z_3}{d_1} + \frac{T_f GM_L^2 z_3}{d_1}$$

$$- \frac{T_f GM_L z_3 (GA)_3^x \ominus_3}{d_1} - \frac{F_{BZ4} GM_L (GA)_3^x \ominus_3 z_3}{d_1} + \frac{(GA)_3^z \ominus_3^2}{d_1}$$

$$- \frac{2}{F_{BZ4}} \frac{GM_L (LCG-LCF)}{d_1^2} \frac{T_f}{d_1} - \frac{T_f GM_L (LCG-LCF) z_3^2}{d_1^2} - \frac{T_f GM_L (LCG-LCF) (GA)_3^z \ominus_3^2}{d_1^2}$$

$$- \frac{F_{BZ4}}{d_1^2} \frac{GM_L (LCG-LCF) z_3}{d_1} + F_{BZ4} \frac{(2) GM_L (LCG-LCF) (GA)_3^z \ominus_3}{d_1^2} + \frac{2 T_f GM_L (LCG-LCF) (GA)_3^z \ominus_3^3}{d_1^2}$$

$$= \frac{2^{2004} \cdot 2^2}{(5) 2^2 (1000 - 1000) 2^2} = \frac{2^{2006}}{(5) 2^2 (1000 - 1000) 2^2} = \frac{2^{2006}}{(5) 2^2 (1000 - 1000) 2^2}$$

$$= \frac{2^{2004} \cdot 2^2}{(5) 2^2 (1000 - 1000) 2^2} = \frac{2^{2006}}{(5) 2^2 (1000 - 1000) 2^2} = \frac{2^{2006}}{(5) 2^2 (1000 - 1000) 2^2}$$

$$= \frac{2^{2004} \cdot 2^2}{(5) 2^2 (1000 - 1000) 2^2} = \frac{2^{2006}}{(5) 2^2 (1000 - 1000) 2^2} = \frac{2^{2006}}{(5) 2^2 (1000 - 1000) 2^2}$$

$$= \frac{2^{2004} \cdot 2^2}{(5) 2^2 (1000 - 1000) 2^2} = \frac{2^{2006}}{(5) 2^2 (1000 - 1000) 2^2} = \frac{2^{2006}}{(5) 2^2 (1000 - 1000) 2^2}$$

$$= \frac{2^{2004} \cdot 2^2}{(5) 2^2 (1000 - 1000) 2^2} = \frac{2^{2006}}{(5) 2^2 (1000 - 1000) 2^2} = \frac{2^{2006}}{(5) 2^2 (1000 - 1000) 2^2}$$

$$- \frac{2}{F_{BZ4}} \left[\frac{GM_L (GA_3)_x}{d_1^2 T_f} - \frac{T_f GM_L (GA_3)_{z3}^2}{d_1^2} - \frac{T_f GM_L (GA_3)_{x3}^3 \theta_3^2}{d_1^2} - F_{BZ4} \right] - \frac{(2) GM_L (GA_3)_{x3}^2}{d_1^2}$$

$$+ \frac{2}{F_{BZ4}} \left[\frac{GM_L (GA_3)_{z3}^2 \theta_3}{d_1^2} + \frac{2 T_f GM_L (GA_3)_{z3}^2 \theta_3}{d_1^2} \right] = 0$$

$$+ \frac{2}{F_{BZ4}} \left[\frac{(GA_3)_z}{d_1 T_f} - \frac{GM_L (LCG - LCF)}{d_1^2 T_f} - \frac{GM_L (GA_3)_x}{d_1^2 T_f} \right]$$

$$+ \frac{2}{F_{BZ4}} \left[(GA_3)_x + (GA_3)_{z3}^2 \theta_3 + \frac{(GA_3)_{z3}^2}{d_1} - \frac{(GA_3)_z (GA_3)_{x3} \theta_3}{d_1} + \frac{\Delta GM_L}{d_1 T_f} + \frac{GM_L z_3}{d_1} \right]$$

$$- \frac{GM_L (GA_3)_{x3} \theta_3}{d_1} - \frac{2 GM_L (LCG - LCF) z_3}{d_1^2} + \frac{2 GM_L (LCG - LCF) (GA_3)_{x3} \theta_3}{d_1^2}$$

$$- \frac{2 GM_L (GA_3)_{x3}^2}{d_1^2} + \frac{2 GM_L (GA_3)_{x3}^2 \theta_3}{d_1^2}$$

$$+ \left[\frac{\Delta GM_L z_3}{d_1} - \frac{\Delta GM_L (GA_3)_{x3} \theta_3}{d_1} + \frac{T_F GM_L z_3^2}{d_1} - \frac{T_F GM_L z_3 (GA_3)_{z3} \theta_3}{d_1} - \frac{T_F GM_L (GA_3)_{x3} z_3 \theta_3}{d_1} \right]$$

$$+ \frac{T_F GM_L (GA_3)_{x3}^2 \theta_3^2}{d_1} - \frac{T_F GM_L (LCG-LCF) z_3^2}{d_1^2} - \frac{T_F GM_L (LCG-LCF) (GA_3)_{x3}^2 \theta_3^2}{d_1^2}$$

$$+ \frac{2 T_F GM_L (LCG-LCF)(GA_3)_{x3} z_3 \theta_3}{d_1^2} - \frac{T_F GM_L (GA_3)_{x3}^2}{d_1^2} - \frac{T_F GM_L (GA_3)_{x3}^3 \theta_3^2}{d_1^2} + \frac{2 T_F GM_L (GA_3)_{x3} z_3 \theta_3}{d_1^2}$$

= 0

Therefore

$$a_4 F_{BZ4}^2 + b_4 F_{BZ4} + c_4 = 0$$

(B194)

$$a^2 \sqrt{2a^2} + a^2 \sqrt{2a^2} + a^2 = 0$$

(or)

2a^2 \sqrt{2a^2} + a^2 = 0

-a^2

$$\frac{a^2}{2 \sqrt{2a^2} (2a^2 - 2a^2) + a^2} = \frac{a^2}{2 \sqrt{2a^2} (2a^2 - 2a^2) + a^2} = \frac{a^2}{a^2}$$

$$\frac{a^2}{2 \sqrt{2a^2} (2a^2 - 2a^2) + a^2} = \frac{a^2}{2 \sqrt{2a^2} (2a^2 - 2a^2) + a^2}$$

$$\frac{a^2}{2 \sqrt{2a^2} (2a^2 - 2a^2) + a^2} = \frac{a^2}{2 \sqrt{2a^2} (2a^2 - 2a^2) + a^2}$$

$$\frac{a^2}{2 \sqrt{2a^2} (2a^2 - 2a^2) + a^2} = \frac{a^2}{2 \sqrt{2a^2} (2a^2 - 2a^2) + a^2}$$

where

$$a_4 = \frac{+(GA_3)_z}{d_1 T_f} - \frac{GM_L (LOG - LCF)}{d_1^2 T_f} - \frac{GM_L (GA_3)_x}{d_1^2 T_f}$$

$$b_4 = (GA_3)_x + (GA_3)_{z^2} + \frac{(GA_3)_z (GA_3)_x}{d_1} + \frac{\Delta GM_L}{d_1 T_f} + \frac{GM_L z_3}{d_1} - \frac{GM_L (GA_3)_{x^2}}{d_1}$$

$$- 2 \frac{GM_L (LOG - LCF) z_3}{d_1^2} + \frac{2 GM_L (LOG - LCF) (GA_3)_x}{d_1^2} - 2 \frac{GM_L (GA_3)_{z^2}}{d_1^2} + \frac{2 GM_L (GA_3)_{x^2}}{d_1^2}$$

$$c_4 = \frac{\Delta GM_L z_3}{d_1} - \frac{\Delta GM_L (GA_3)_x}{d_1} + \frac{T_f GM_L z_3^2}{d_1} + \frac{T_f GM_L z_3 (GA_3)_x}{d_1} - \frac{T_f GM_L (GA_3)_{z^2}}{d_1}$$

$$+ \frac{T_f GM_L (GA_3)_{x^2}}{d_1} - \frac{T_f GM_L}{d_1^2} \left[(LOG - LCF) z_3^2 + (LOG - LCF) (GA_3)_x^2 - 2 (LOG - LCF) (GA_3)_{x^2} \right]$$

$$+ (GA_3)_{x^2} z_3^2 + (GA_3)_{z^2} - 2 (GA_3)_{z_3 x} \left. \right]$$

$$+ (\omega^2)^3 e_3^3 + (\omega^2)^2 e_2^3 - 0 (\omega^2)^1 e_1^3 + 0$$

$$\frac{\sigma^T}{\omega^2} = \frac{\sigma^T}{\omega^2} \left[\frac{1}{\omega^2} (100) e_1^2 + \frac{1}{\omega^2} (100) e_2^2 + \frac{1}{\omega^2} (100) e_3^2 \right]$$

$$\sigma^T = \frac{\sigma^T}{\omega^2} \left[\frac{1}{\omega^2} (100) e_1^2 + \frac{1}{\omega^2} (100) e_2^2 + \frac{1}{\omega^2} (100) e_3^2 \right]$$

$$= \frac{\sigma^T}{\omega^2} \left[\frac{1}{\omega^2} (100) e_1^2 + \frac{1}{\omega^2} (100) e_2^2 + \frac{1}{\omega^2} (100) e_3^2 \right]$$

$$\sigma^T = (\omega^2)^3 e_3^3 + (\omega^2)^2 e_2^3 + (\omega^2)^1 e_1^3 + 0$$

$$\sigma^T = \frac{\sigma^T}{\omega^2} \left[\frac{1}{\omega^2} (100) e_1^2 + \frac{1}{\omega^2} (100) e_2^2 + \frac{1}{\omega^2} (100) e_3^2 \right]$$

Q.E.D.

$$F_{BZ4} = \frac{-b_4 + \sqrt{b_4^2 - 4a_4c_4}}{2a_4}$$

It is presumed the values are real and unequal (i.e. $b_4^2 - 4a_4c_4 > 0$) and that the larger of these values is the significant one. Therefore the static force under the bow is

$$F_{BZ4} = \frac{-b_4 + \sqrt{b_4^2 - 4a_4c_4}}{2a_4} \quad (B195)$$

The final trim (θ_4), may be obtained from equation (B193),

$$\theta_4 = \frac{-F_{BZ4}}{d_1 T_f} - \frac{z_3}{d_1} + \frac{(GA)_{3x} \theta_3}{d_1}$$

The change of position in the x-direction, on settling, is negligible.

The final value for z (z_4) may be obtained from equation (B186),

$$z_4 = z_3 + (GA_3)_x (\theta_4 - \theta_3)$$

$$\frac{d}{dx} \left(\frac{1}{x^2} \right) = -\frac{2}{x^3} = -2x^{-3}$$

(1) $y = x^{-2}$ is a function of x . We can find its derivative by using the power rule. The derivative of x^n is nx^{n-1} . Here, $n = -2$, so the derivative is $-2x^{-3}$.

(2) $y = x^{-2}$

$$\frac{d}{dx} (x^{-2}) = -2x^{-3} = -\frac{2}{x^3}$$

(3) $y = x^{-2}$ is a function of x . We can find its derivative by using the power rule. The derivative of x^n is nx^{n-1} . Here, $n = -2$, so the derivative is $-2x^{-3}$.

$$\frac{d}{dx} (x^{-2}) = -2x^{-3} = -\frac{2}{x^3}$$

The derivative of $y = x^{-2}$ is $-2x^{-3}$.

(4) $y = x^{-2}$ is a function of x . We can find its derivative by using the power rule. The derivative of x^n is nx^{n-1} . Here, $n = -2$, so the derivative is $-2x^{-3}$.

$$(x^{-2})' = -2x^{-3} = -\frac{2}{x^3}$$

Extracting Thrust

Once the icebreaker has completely ceased moving (State 4) it frequently becomes stuck in that position due to static friction which can be significantly greater than kinetic friction.

It is possible, indeed probable, that an icebreaker designed to attain a high downward sustained force may also, unfortunately, require a very large backing thrust to remove itself. For that reason it is important to know what backing thrust will be required to back down. This will be called "extracting thrust",

It is necessary to create enough extracting thrust to overcome the friction between the bow plating and the ice. The direction of this friction force is parallel to the stem since that is the direction of impending motion at the bow.

The values of x_4 , z_4 , $(GA)_{z4}$, $(GA)_{x4}$, and θ_4 are known and valid for this condition since changing them would imply the icebreaker is not held by static friction.

Refer to Figure B-XXIX. The force normal to the bow plating on each side is $N/2$. The friction force is then $f_g N/2$.

As may be seen in Figure B-XXX the force normal to the stem, in the centerline plane, may be expressed as

$$N \cos \beta$$

where β = angle between normal to plating and centerline plane.

These forces may be resolved into a vertical component and a horizontal component respectively. See Figure B-XXX.

The first part of the paper is devoted to a study of the conditions under which the reaction between the metal and the gas is controlled by the rate of diffusion of the gas through the boundary layer.

It is assumed that the reaction is first order with respect to the gas concentration. The rate of diffusion of the gas through the boundary layer is given by Fick's law, and the rate of reaction is given by the Arrhenius equation. The overall rate of reaction is then given by the sum of the two rates.

It is necessary to consider the effect of the rate of diffusion of the gas through the boundary layer on the overall rate of reaction. The rate of diffusion of the gas through the boundary layer is given by Fick's law, and the rate of reaction is given by the Arrhenius equation. The overall rate of reaction is then given by the sum of the two rates.

The value of the rate constant k is determined by the temperature and the activation energy of the reaction. The rate constant k is given by the Arrhenius equation, and the activation energy E_a is determined by the slope of the Arrhenius plot.

From Figure 1 it is seen that the rate of reaction increases with increasing temperature. The rate of reaction is given by the Arrhenius equation, and the activation energy E_a is determined by the slope of the Arrhenius plot.

It can be seen from Figure 1 that the rate of reaction increases with increasing temperature. The rate of reaction is given by the Arrhenius equation, and the activation energy E_a is determined by the slope of the Arrhenius plot.

Figure B-XXIX

Forces on Bow When Backing is Impending

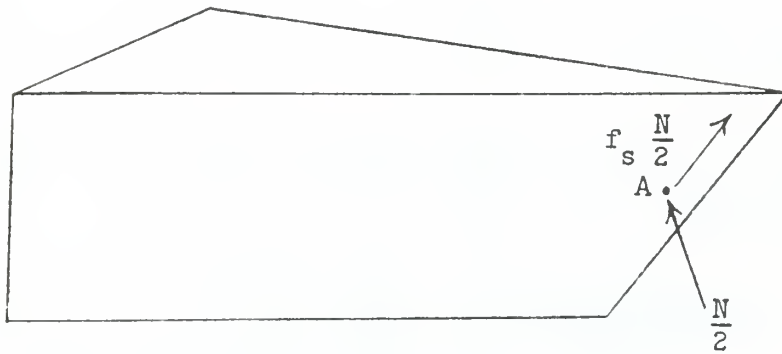
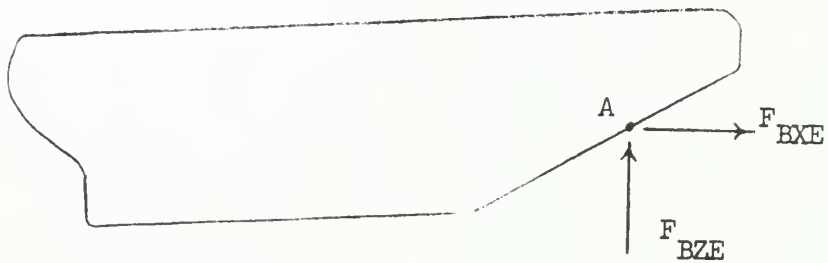


Figure B-XXX

Component Bow Forces When Backing is Impending



$$F_{BZE} = (N \cos \beta) \cos (i_B + \theta_4) + f_s N \sin (i_B + \theta_4)$$

$$F_{BZE} = N \left[(\cos \beta) \cos (i_B + \theta_4) + f_s \sin (i_B + \theta_4) \right] \quad (E196)$$

$$F_{BXE} = - (N \cos \beta) \sin (i_B + \theta_4) + f_s N \cos (i_B + \theta_4)$$

$$F_{BXE} = N \left[- (\cos \beta) \sin (i_B + \theta_4) + f_s \cos (i_B + \theta_4) \right] \quad (E197)$$

Summing forces in the z-direction, (See Figure B-XXXI)

$$\Delta - (\Delta + T_f h_4) + E_t \sin \theta_4 - F_{BZE} = 0$$

As may be realized from equation (E189),

$$\Delta - (\Delta + T_f h_4) = + F_{BZ4}$$

Therefore,

$$+ F_{BZ4} + E_t \sin \theta_4 - F_{BZE} = 0$$

$$F_{BZE} = F_{BZ4} + E_t \sin \theta_4 \quad (E198)$$

Summing forces in the x-direction,

$$F_{BXE} - E_t \cos \theta_4 = 0 \quad (E199)$$

$$(120) \quad \left[\cos(\theta) + i \sin(\theta) \right]^n = \cos(n\theta) + i \sin(n\theta)$$

$$(121) \quad \left[\cos(\theta) + i \sin(\theta) \right]^n = \cos(n\theta) + i \sin(n\theta)$$

$$(122) \quad \left[\cos(\theta) + i \sin(\theta) \right]^n = \cos(n\theta) + i \sin(n\theta)$$

$$(123) \quad \left[\cos(\theta) + i \sin(\theta) \right]^n = \cos(n\theta) + i \sin(n\theta)$$

Using De Moivre's theorem, we have:

$$0 = \cos^2 \theta - \sin^2 \theta + i(2 \sin \theta \cos \theta) = z$$

Let $z = \cos \theta + i \sin \theta$, then $z^2 = \cos 2\theta + i \sin 2\theta$.

$$\cos^2 \theta - \sin^2 \theta + i(2 \sin \theta \cos \theta) = \cos 2\theta + i \sin 2\theta$$

Therefore,

$$0 = \cos^2 \theta - \sin^2 \theta + i(2 \sin \theta \cos \theta) = z$$

(124)

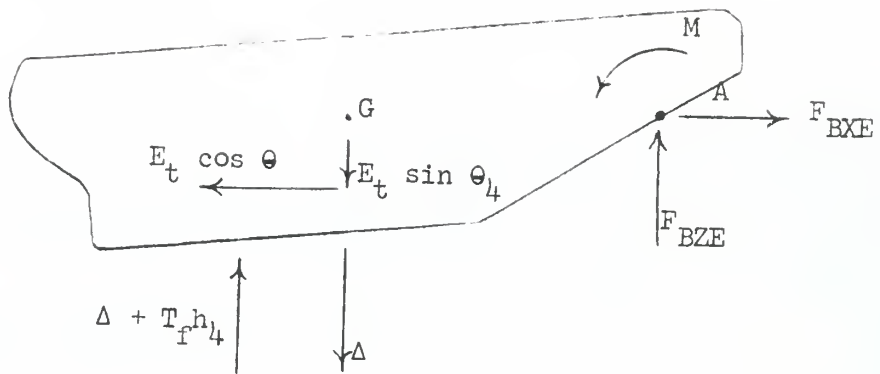
$$\cos^2 \theta - \sin^2 \theta + i(2 \sin \theta \cos \theta) = \cos 2\theta + i \sin 2\theta$$

Equating terms in the equation,

(125)

$$0 = \cos^2 \theta - \sin^2 \theta + i(2 \sin \theta \cos \theta) = z$$

Figure B-XXXI
Free Body Diagram for Extraction



$$\text{Let } a_7 = (\cos \beta) \cos (i_B + \theta_4) + f_s \sin (i_B + \theta_4) \quad (\text{B200})$$

$$\text{and } b_7 = -(\cos \beta) \sin (i_B + \theta_4) + f_s \cos (i_B + \theta_4) \quad (\text{B201})$$

Then equation (H198) becomes

$$a_7 N = F_{BZ4} + E_t \sin \theta_4$$

$$N = \frac{1}{a_7} (F_{BZ4} + E_t \sin \theta_4)$$

Equation (H199) becomes

$$b_7 N = E_t \cos \theta_4$$

$$N = E_t \frac{\cos \theta_4}{b_7}$$

Equating these equations

$$\frac{F_{BZ4}}{a_7} + E_t \frac{\sin \theta_4}{a_7} = N = E_t \frac{\cos \theta_4}{b_7}$$

$$\frac{F_{BZ4}}{a_7} = E_t \frac{\cos \theta_4}{b_7} - E_t \frac{\sin \theta_4}{a_7}$$

$$\frac{F_{BZ4}}{a_7} = E_t \left(\frac{\cos \theta_4}{b_7} - \frac{\sin \theta_4}{a_7} \right)$$

$$E_t = \frac{F_{BZ4}}{a_7 \left(\frac{\cos \theta_4}{b_7} - \frac{\sin \theta_4}{a_7} \right)}$$

(2022) $(\mu + \sigma^2) \ln \frac{1}{\mu} + \sigma^2 \ln \frac{1}{\sigma^2} = (\mu + \sigma^2) \ln \frac{1}{\mu} + \sigma^2 \ln \frac{1}{\sigma^2}$

(2023) $(\mu + \sigma^2) \ln \frac{1}{\mu} + \sigma^2 \ln \frac{1}{\sigma^2} = (\mu + \sigma^2) \ln \frac{1}{\mu} + \sigma^2 \ln \frac{1}{\sigma^2}$

Derivative with respect to μ

$$\ln \frac{1}{\mu} - \frac{1}{\mu} + \frac{2\sigma^2}{\mu^2} = 0$$

$$\ln \frac{1}{\mu} - \frac{1}{\mu} + \frac{2\sigma^2}{\mu^2} = 0$$

Derivative with respect to σ^2

$$\frac{2\sigma^2}{\mu^2} - \frac{1}{\sigma^2} = 0$$

$$\frac{2\sigma^2}{\mu^2} - \frac{1}{\sigma^2} = 0$$

Derivative with respect to σ^2

$$\frac{2\sigma^2}{\mu^2} - \frac{1}{\sigma^2} = 0 \Rightarrow \frac{2\sigma^2}{\mu^2} = \frac{1}{\sigma^2} \Rightarrow \frac{2\sigma^4}{\mu^2} = 1$$

$$\frac{2\sigma^4}{\mu^2} = 1 \Rightarrow \frac{2\sigma^4}{\mu^2} = 1 \Rightarrow \frac{2\sigma^4}{\mu^2} = 1$$

$$\left(\frac{2\sigma^4}{\mu^2} - \frac{1}{\sigma^2} \right) \sigma^2 = \frac{2\sigma^6}{\mu^2} - \frac{1}{\sigma^2}$$

$$\frac{2\sigma^6}{\mu^2} - \frac{1}{\sigma^2} = 0 \Rightarrow \frac{2\sigma^6}{\mu^2} = \frac{1}{\sigma^2} \Rightarrow \frac{2\sigma^8}{\mu^2} = 1$$

$$E_t = \frac{F_{BZ4}}{\left(\frac{a_7}{h_7} \cos \theta_4 - \sin \theta_4\right)} \quad (B202)$$

It is noted that all values needed to solve E_t from equation (B202) are known.

There is a small moment created which will help free the icebreaker. So neglecting this is on the safe side. Furthermore, if the line of action of the thrust passes through the point of contact, this moment vanishes.

(1951)

$$\frac{100^2}{(100 \times 100 - 100 \times 100)}$$

It is noted that all values listed in Table 1 are

values are (1951) values

There is a well known trend that will help you to understand

the way in which the values are related to the data on

values of the other values through the points of contact. This means

values

Table 1

Values of the other values through the points of contact.

Table 2

Values of the other values through the points of contact.

Table 3

Values of the other values through the points of contact.

Table 4

Values of the other values through the points of contact.

Computer Program

Naturally the solution of all the preceding equations would be quite tedious and there would be a high probability of error. This is compounded by the fact that there are three iterative solutions involved. Furthermore, one solution by itself would be of little value; comparisons are needed.

For the reasons mentioned the solution has been programmed in Fortran and carried out on an I.B.M. 7094 computer.*

The following is a listing of the input data which must be supplied:

- BP Length between perpendiculars, ft.
- B Beam at waterline, ft.
- H Mean draft, ft.
- DIS Displacement, lb.
- BA Bow angle (from base line to stem), radians
- SA "Spread angle complement (normal to bow plating with respect to centerline plane), radians
- VI Impact velocity, ft./sec.
- AL α , Waterplane coefficient, dimensionless.
- CF LCF, Longitudinal position of the center of flotation (- if aft of amidships, + if fwd), ft.
- CG LCG, Longitudinal position of the center of gravity (- if aft of amidships, + if fwd), ft.
- GK KG, Height of center of gravity above base line, ft.

* This work was done in part at the Computation Center at M.I.T., Cambridge, Massachusetts.

- D Height of thrust line above base line near center of gravity, ft.
- TB Bollard thrust which would be attained for rpm used during crushing and skidding, lbs.
- GM GM_L , Longitudinal metacentric height, ft.
- FK Ice/ship kinetic friction coeff., dimensionless.
- FS Ice/ship static friction coefficient, dimensionless.
- S/G Compressive failure stress of ice, lb/ft^2

M4045-3564,FMS,TEST,5,5,5000,0 DYNAMIC ICEBREAKING R.M.WHITE
XEQ

DYNAMIC ICEBREAKING R. M. WHITE

36 READ 5,BP,B,H,DIS,BA,SA,V1,AL,CF,CG,GK,D,TB,GM,FK,FS,SIG

5 FORMAT (4F15.3/4F15.3/4F15.3/4F15.3/F15.3)

PRINT 41, BP,B,H,DIS,BA,SA,V1,AL,CF,CG,GK,D,TB,GM,FK,FS,SIG

41 FORMAT (6H BP=,F15.3,5H B=,F15.3,5H H=,F15.3,7H DIS=,F15.3

1/6H BA=,F15.3,6H SA=,F15.3,6H V1=,F15.3,6H AL=,F15.3/

26H CF=,F15.3,6H CG=,F15.3,6H GK=,F15.3,5H D=,F15.3/6H TB

3=,F15.3,6H GM=,F15.3,6H FK=,F15.3,6H FS=,F15.3/7H SIG=,

4F15.3//)

XM = (3.36E-2)*DIS (B66)

ZM = (5.78E-2)*DIS (B67)

RG = 0.22*BP (B71)

THM = (5.0E-2)*(RG**2)*DIS (B68)

DP = (1.76E-2)*DIS*BP**1.5 (B69)

DH = (5.29E-1)*DIS/BP**0.5 (B70)

TF = (64.2)*BP*B*AL

SIBA = SIN(BA)

COBA = COS(BA)

TABA = SIBA/COBA

SISA = SIN(SA)

COSA = COS(SA)

P1 = (SIG*TABA/SISA)*(SIBA*(COSA+FK*SISA)+FK*COBA) (B33)

P2 = (SIG*TABA/SISA)*(COBA*(COSA+FK*SISA)-FK*SIBA) (B39)

P3 = P2*(BP/2.-CG)+P1*(H-GK) (B51)

A1 = THM

B1 = DP

C1 = DIS*GM

D1 = V1**2*P3

AL1 = -B1/(2.*A1)

DISC1 = 4.*C1/A1-(B1**2)/(A1**2)

IF (DISC1) 11,2,2

2 BE1 = 0.5*SQRTF(DISC1) (B57)

AA1 = (2.*D1/C1**2)*(A1-B1**2/C1)

AA2 = (2.*D1/((C1**2)*BE1))*(B1-AL1*(A1-(B1**2)/C1))

A2 = ZM

B2 = DH

C2 = TF

D2 = -P2*V1**2

AL2 = -B2/(2.*A2)

DISC2 = 4.*C2/A2-(B2**2)/(A2**2)

IF (DISC2) 12,3,3

3 BE2 = 0.5*SQRTF(DISC2) (B61)

BB1 = (2.*D2/C2**2)*(A2-B2**2/C2)

BB2 = (2.*D2/((C2**2)*BE2))*(B2-AL2*(A2-(B2**2)/C2))

PRINT 4, XM,ZM,RG,THM,DP,DH,TF,P1,P2,A1,B1,C1,P3,D1,AL1,BE1,AA1,

1AA2,A2,B2,C2,D2,AL2,BE2,BB1,BB2

4 FORMAT (4E12.4/5E12.4/5E12.4/4E12.4/4E12.4/4E12.4//)

T = -0.05

1 T = T+0.05

EAL1T = EXPF(AL1*T)

COB1T = COSF(BE1*T)

SIB1T = SIN(BE1*T)

TH = EAL1T*(AA1*COB1T+AA2*SIB1T)+D1*(T**2)/C1-2.*B1*D1*T/(C1**2)

(B58)



```
1-AA1
  THD = AL1*EAL1T*(AA1*COB1T+AA2*SIB1T)+EAL1T*(-AA1*BE1*SIB1T+AA2 (B55)
1*BE1*COB1T)+2.*D1*T/C1-2.*B1*D1/C1**2
  THDD = (AL1**2-BE1**2)*EAL1T*(AA1*COB1T+AA2*SIB1T)+2.*AL1*BE1* (B59)
1EAL1T*(-AA1*SIB1T+AA2*COB1T)+2.*D1/C1
  EAL2T = EXPF(AL2*T)
  COB2T = COSF(BE2*T)
  SIB2T = SINF(BE2*T)
  Z =EAL2T*(BB1*COB2T+BB2*SIB2T)+D2*(T**2)/C2-2.*B2*D2*T/(C2**2)-BB1(B62)
  ZD = AL2*EAL2T*(BB1*COB2T+BB2*SIB2T)+EAL2T*(-BB1*BE2*SIB2T+BB2 (B63)
1*BE2*COB2T)+2.*D2*T/C2-2.*B2*D2/C2**2
  ZDD = (AL2**2-BE2**2)*EAL2T*(BB1*COB2T+BB2*SIB2T)+2.*AL2*BE2* (B64)
1EAL2T*(-BB1*SIB2T+BB2*COB2T)+2.*D2/C2
  X = V1*(T-P1*T**3/(12.*XM)) (B41)
  XD = SQRTF(V1**2-2.*P1*X**3/(3.*XM)) (B40)
  XDD = -P1*X**2/XM (B39)
  FXC = P1*X**2 (E47)
  FZC = P2*X**2 (E48)
  GAX = (BP/2.-CG)-((H-GK)+(BP/2.-CG)/TABA)*TH+Z/TABA (B20)
  GAZ = H-GK+Z (B16)
  TAGA = (GAX*THD-ZD)/(XD-GAZ*THD) (B65)
  DIF = SINF(BA+TH)/COSF(BA+TH)-TAGA
  PRINT 6,T,TH,THD,THDD,Z,ZD,ZDD,X,XD,XDD,FXC,FZC,TAGA,GAX,GAZ,DIF
6 FORMAT (F11.5/3F11.5/3F11.5/3F11.5/2E12.5/4F11.5//)
  IF (XD) 38,38,37
38 PRINT 39, FZC
  GO TO 36
39 FORMAT (44H SHIP STOPPED DURING CRUSHING PHASE, FZC2=,E12.5//)
37 IF (DIF) 14,14,7
7 TL = T
  THL = TH
  THDL = THD
  THDDL = THDD
  ZL = Z
  ZDL = ZD
  ZDDL = ZDD
  XL = X
  XDL = XD
  XDDL = XDD
  FXCL = FXC
  FZCL = FZC
  TAGAL = TAGA
  GAXL = GAX
  GAZL = GAZ
  DIFL = DIF
  GO TO 1
14 TERP = DIFL/(DIFL-DIF)
  T2 = TL+TERP*(T-TL)
  TH2 = THL+TERP*(TH-THL)
  THD2 = THDL+TERP*(THD-THDL)
  THDD2 = THDDL+TERP*(THDD-THDDL)
  Z2 = ZL+TERP*(Z-ZL)
  ZD2 = ZDL+TERP*(ZD-ZDL)
  ZDD2 = ZDDL+TERP*(ZDD-ZDDL)
  X2 = XL+TERP*(X-XL)
```



```

XD2 = XDL+TERP*(XD-XDL)
XDD2 = XDDL+TERP*(XDD-XDDL)
FXC2 = FXCL+TERP*(FXC-FXCL)
FZC2 = FZCL+TERP*(FZC-FZCL)
TAGA2 = TAGAL+TERP*(TAGA-TAGAL)
GAX2 = GAXL+TERP*(GAX-GAXL)
GAZ2 = GAZL+TERP*(GAZ-GAZL)
DIF2 = DIFL+TERP*(DIF-DIFL)
10 PRINT 15, T2, TH2, THD2, THDD2, Z2, ZD2, ZDD2, X2, XD2, XDD2, FXC2, FZC2,
1TAGA2, GAX2, GAZ2, DIF2
15 FORMAT (6H T2=,F11.5/7H TH2=,F11.5,8H THD2=,F11.5,
19H THDD2=,F11.5/6H Z2=,F11.5,7H ZD2=,F11.5,8H ZDD2=,F11.5/
26H X2=,F11.5,7H XD2=,F11.5,8H XDD2=,F11.5/8H FXC2=,E12.5,
38H FZC2=,E12.5/9H TAGA2=,F11.5,6H GAX2=,F11.5,8H GAZ2=,
4F11.5,8H DIF2=,F11.5//)
GO TO 16
11 PRINT 13, DISC1
13 FORMAT (E12.4)
GO TO 36
12 PRINT 13, DISC2
GO TO 36
C ICEBREAKER SLIDING PHASE SOLUTION R. M. WHITE
16 AS = COSA*SIBA+FK*COBA
BS = COSA*COBA-FK*SIBA
XDD2 = -1.0
ZDD2 = 0.0
THDD2 = 0.0
P4 = GAX2+X2
P5 = 1.+(AS/BS)**2
HGK = H-GK
CGCF = CG-CF
GKD = GK-D
A11 = -XM
B11 = -(TB/V1)*(1.+AS*TH2/BS+P5*TH2**2)
C11 = 0.0
A12 = ZM*(AS/BS+P5*TH2)
B12 = DH*(AS/BS+P5*TH2)
C12 = TF*(AS/BS+P5*TH2)
A13 = 0.0
B13 = 0.0
C13 = TB*(AS/BS-AS*XD2/(BS*V1)+2.*P5*TH2-2.*P5*XD2*TH2/V1)
1+TF*(AS*CGCF/BS+P5*Z2+2.*P5*CGCF*TH2)+P5*DH*ZD2+P5*ZM*ZDD2
D1 = -TB*(1.+AS*XD2*TH2/(BS*V1)-P5*TH2**2+2.*P5*XD2*TH2**2/V1)
1+TF*(P5*Z2*TH2+P5*CGCF*TH2**2)+P5*DH*ZD2*TH2+P5*ZM*ZDD2*TH2
A21 = 0.0
B21 = TB*(P4*TH2/V1-TH2*X2/V1+AS*HGK*TH2/(BS*V1)+P5*HGK*TH2**2/V1
1+AS*TH2*Z2/(BS*V1)+P5*Z2*TH2**2/V1-GKD/V1)
C21 = TB*(TH2-XD2*TH2/V1)+TF*(Z2+CGCF*TH2)+DH*ZD2+ZM*ZDD2
A22 = ZM*(-P4+X2-AS*HGK/BS-P5*HGK*TH2-AS*Z2/BS-P5*TH2*Z2)
B22 = DH*(-P4+X2-AS*HGK/BS-P5*HGK*TH2-AS*Z2/BS-P5*TH2*Z2)
C22 = TF*(X2-AS*HGK/BS-P5*HGK*TH2-P4-2.*AS*Z2/BS-AS*CGCF*TH2/BS
1-2.*P5*Z2*TH2-P5*CGCF*TH2**2-GM*TH2)+TB*(-AS*TH2/BS+AS*XD2*TH2/
2(BS*V1)-P5*TH2**2+P5*XD2*TH2**2/V1)+DH*(-AS*ZD2/BS-P5*ZD2*TH2)
3+ZM*(-AS*ZDD2/BS-P5*ZDD2*TH2)
A23 = -THM

```

(B80)
(B81)

} (B180)

(B87)
(B94)

(B95)

(B100)

R23 = -DP

C23 = TB*(-P4+P4*XD2/V1+X2-XD2*X2/V1-AS*HG'/BS+AS*HGK*XD2/(BS*V1)
1-2.*P5*HGK*TH2+2.*P5*HGK*XD2*TH2/V1-AS*Z2/BS+AS*XD2*Z2/(BS*V1)
2-2.*P5*Z2*TH2+2.*P5*XD2*TH2*Z2/V1)+TF*(-P4*CGCF+CGCF*X2-AS*CGCF*
3HGK/BS-P5*HGK*Z2-2.*P5*CGCF*HGK*TH2-AS*CGCF*Z2/BS-P5*Z2**2-2.*P5
4*CGCF*TH2*Z2-GM*Z2-2.*GM*CGCF*TH2)+DH*(-P5*HGK*ZD2-P5*ZD2*Z2)+ZM*
5(-P5*HGK*ZDD2-P5*ZDD2*Z2)-DIS*GM

D2 = TB*(P4*XD2*TH2/V1+TH2*X2-2.*XD2*TH2*X2/V1+AS*HGK*XD2*TH2/(BS*
1V1)-P5*HGK*TH2**2+2.*P5*HGK*XD2*TH2**2/V1-AS*TH2*Z2/BS+2.*AS*XD2
2*TH2*Z2/(BS*V1)-2.*P5*TH2**2*Z2+3.*P5*XD2*Z2*TH2**2/V1-GKD)+TF*(
3Z2*X2+CGCF*TH2*X2-P5*HGK*Z2*TH2-P5*CGCF*HGK*TH2**2-AS*Z2**2/BS-AS
4*CGCF*TH2*Z2/BS-2.*P5*TH2*Z2**2-2.*P5*CGCF*Z2*TH2**2-GM*TH2*Z2-GM*
5CGCF*TH2**2)+DH*(ZD2*X2-P5*HGK*ZD2*TH2-AS*ZD2*Z2/BS-2.*P5*ZD2*TH2*
6Z2)+ZM*(ZDD2*X2-P5*HGK*ZDD2*TH2-AS*ZDD2*Z2/BS-2.*P5*ZDD2*TH2*Z2)

(B100)

A31 = 0.0

B31 = 0.0

C31 = 1.0

A32 = 0.0

B32 = 0.0

C32 = 1./TABA-TH2/SIBA**2

A33 = 0.0

B33 = 0.0

C33 = -GAX2/TABA+GAX2*TH2/SIBA**2+GAZ2

D3 = X2-GAX2*TH2/TABA+GAX2*TH2**2/SIBA**2+Z2/TABA-Z2*TH2/SIBA**2

(B103)

1+GAZ2*TH2

PRINT 17,AS,BS,P4,P5,A11,B11,C11,A12,B12,C12,A13,B13,C13,D1,
1A21,B21,C21,A22,B22,C22,A23,B23,C23,D2,A31,B31,C31,A32,B32,C32,
2A33,B33,C33,D3

17 FORMAT (4E14.6//3E14.6/3E14.6/3E14.6/E14.6//3E14.6/3E14.6/3E14.6/
1F14.6//3E14.6/3F14.6/3E14.6/E14.6//)

D11 = A11*XD2+B11*X2+A12*ZD2+B12*Z2

D12 = A11*X2+A12*Z2

D13 = D1

D21 = B21*X2+A22*ZD2+B22*Z2+A23*THD2+B23*TH2

D22 = A22*Z2+A23*TH2

D23 = D2

D33 = D3

(B104)

DD4 = A11*A22*C33+A12*A23*C31-A11*A23*C32

DD3 = A11*B22*C33+B11*A22*C33+A12*B23*C31+B12*A23*C31-A11*B23*C32

1-B11*A23*C32-A12*B21*C33

DD2 = A11*C22*C33+B11*B22*C33+A12*C23*C31+B12*B23*C31+C12*A23*C31

1-A11*C23*C32-B11*B23*C32-A12*C21*C33-B12*B21*C33-C13*A22*C31

DD1 = B11*C22*C33+B12*C23*C31+C12*B23*C31+C13*B21*C32-B11*C23*C32

1-B12*C21*C33-C12*B21*C33-C13*B22*C31

DD0 = C12*C23*C31+C13*C21*C32-C12*C21*C33-C13*C22*C31

U13 = D12*A22*C33+A12*A23*D33-D12*A23*C32-A12*D22*C33

U12 = D11*A22*C33+D12*B22*C33+A12*B23*D33+B12*A23*D33-D11*A23*C32

1-D12*B23*C32-A12*D21*C33-B12*D22*C33

U11 = D11*B22*C33+D12*C22*C33+D13*A22*C33+A12*C23*D33+B12*B23*D33

1+C12*A23*D33+C13*D22*C32-D11*B23*C32-D12*C23*C32-D13*A23*C32

2-A12*A23*C33-B12*D21*C33-C12*D22*C33-C13*A22*D33

U10 = D11*C22*C33+D13*B22*C33+B12*C23*D33+C12*B23*D33+C13*D21*C32

1-D11*C23*C32-D13*B23*C32-B12*D23*C33-C12*D21*C33-C13*B22*D33

U09 = D13*C22*C33+C12*C23*D33+C13*D23*C32-D13*C23*C32-C12*D23*C33

1-C13*C22*D33

(B109)

(B108)


```

U23 = A11*D22*C33+D12*A23*C31-A11*A23*D33
U22 = A11*D21*C33+B11*D22*C33+D11*A23*C31+D12*B23*C31-A11*B23*D33
1-B11*A23*D33-D12*B21*C33
U21 = A11*D23*C33+B11*D21*C33+D11*B23*C31+D12*C23*C31+D13*A23*C31
1-A11*C23*D33-B11*B23*D33-D11*B21*C33-D12*C21*C33-C13*D22*C31
U20 = B11*D23*C33+D11*C23*C31+D13*B23*C31+C13*B21*D33-B11*C23*D33
1-D11*C21*C33-D13*B21*C33-C13*D21*C31
U19 = D13*C23*C31+C13*C21*D33-D13*C21*C33-C13*D23*C31
U33 = A11*A22*D33+A12*D22*C31-A11*D22*C32-D12*A22*C31
U32 = A11*B22*D33+B11*A22*D33+A12*D21*C31+B12*D22*C31+D12*B21*C32
1-A11*D21*C32-B11*D22*C32-A12*B21*D33-D11*A22*C31-D12*B22*C31
U31 = A11*C22*D33+B11*B22*D33+A12*D23*C31+B12*D21*C31+C12*D22*C31
1+D11*B21*C32+D12*C21*C32-A11*D23*C32-B11*D21*C32-A12*C21*D33
2-B12*B21*D33-D11*B22*C31-D12*C22*C31-D13*A22*C31
U30 = B11*C22*D33+B12*D23*C31+C12*D21*C31+D11*C21*C32+D13*B21*C32
1-B11*D23*C32-B12*C21*D33-C12*B21*D33-D11*C22*C31-D13*B22*C31
U29 = C12*D23*C31+D13*C21*C32-C12*C21*D33-D13*C22*C31
PRINT 13,D11,D12,D13,D21,D22,D23,D33,DD4,DD3,DD2,DD1,DD0,U13,U12,
1U11,U10,U09,U23,U22,U21,U20,U19,U33,U32,U31,U30,U29

```

(B152)

(B53)

```

WB4 = DD3/DD4
WB3 = DD2/DD4
WB2 = DD1/DD4
WB1 = DD0/DD4

```

(B112)

```

W6 = -64.*WB4**6
W5 = 96.*WB4**6
W4 = (WB4**4)*(-48.*WB4**2-32.*WB3)
W3 = (WB4**3)*(32.*WB3*WB4+8.*WB4**3)
W2 = (WB4**2)*(16.*WB1-4.*WB3**2-4.*WB2*WB4-8.*WB3*WB4**2)
W1 = (WB4)*(-8.*WB1*WB4+2.*WB3**2*WB4+2.*WB2*WB4**2)
W0 = WB1*WB4**2-WB2*WB3*WB4+WB2**2

```

(B122)

```

C = 0
19 CL = C
TOTL = TOT
C = C+0.001
TOT = W6*C**6+W5*C**5+W4*C**4+W3*C**3+W2*C**2+W1*C+W0
PRINT 13, C, TOT
IF (TOT) 19,20,20

```

(B123)

```

20 C = CL-TOTL*0.001/(TOT-TOTL)
PRINT 13, C
AL3 = C*WB4
DISC3 = (-WB2-3.*WB4*(AL3**2)+4.*(AL3**3)+2.*WB3*AL3)/(4.*AL3-WB4)
IF (DISC3) 21,22,22

```

(B124)

(B125)

```

21 PRINT 13, DISC3
GO TO 36
22 BE3 = SQRTF (DISC3)
AL4 = (1.-2.*C)*WB4/2.
DISC4 = WB3-(BE3**2)-(AL3**2)-4.*AL3*AL4-(AL4**2)
IF (DISC4) 23,24,24

```

(B125)

(B126)

```

23 PRINT 13, DISC4
GO TO 36
24 BE4 = SQRTF (DISC4)
G3 = AL3**2+BE3**2
G4 = AL4**2+BE4**2
PRINT 13, AL3,BE3,AL4,BE4,G3,G4
A4X = U13/DD4

```

(B127)

(B134)

(B111)

A3X = U12/DD4
A2X = U11/DD4
A1X = U10/DD4
A0X = U09/DD4
PRINT 13, A4X, A3X, A2X, A1X, A0X
C1X = A0X/(G3*G4) (E136)
D1X = A4X-C1X (E137)
D2X = A3X-C1X*WB4 (E138)
D3X = A2X-C1X*WB3 (E139)
D4X = A1X-C1X*WB2 (E140)
C6X = (2.*(AL3-AL4)*(G4*D3X-D1X*G4**2-2.*AL4*D4X)+(G3-G4)*(-G4*D2X
1+2.*AL4*G4*D1X+D4X))/(2.*(AL3-AL4)*(2.*AL3*G4-2.*AL4*G3)+(G3-G4) (E141)
2**2)

C5X = (D4X-G3*C6X)/G4 (E142)
C4X = (G4*(D2X-2.*AL4*D1X)-D4X+C6X*(G3-G4))/(2.*G4*(AL3-AL4)) (E143)
C3X = D1X-C4X (E144)

P23X = BE3*C3X
P13X = C5X-AL3*C3X
P24X = BE4*C4X
P14X = C6X-AL4*C4X

PRINT 13, C1X, D1X, D2X, D3X, D4X, C6X, C5X, C4X, C3X, P23X, P13X, P24X, P14X
A4Z = U23/DD4
A3Z = U22/DD4
A2Z = U21/DD4
A1Z = U20/DD4
A0Z = U19/DD4

PRINT 13, A4Z, A3Z, A2Z, A1Z, A0Z
C1Z = A0Z/(G3*G4) (E156)
D1Z = A4Z-C1Z
D2Z = A3Z-C1Z*WB4 (E157)
D3Z = A2Z-C1Z*WB3
D4Z = A1Z-C1Z*WB2
C6Z = (2.*(AL3-AL4)*(G4*D3Z-D1Z*G4**2-2.*AL4*D4Z)+(G3-G4)*(-G4*D2Z
1+2.*AL4*G4*D1Z+D4Z))/(2.*(AL3-AL4)*(2.*AL3*G4-2.*AL4*G3)+(G3-G4) (E163)
2**2)

C5Z = (D4Z-G3*C6Z)/G4 (E159)
C4Z = (G4*(D2Z-2.*AL4*D1Z)-D4Z+C6Z*(G3-G4))/(2.*G4*(AL3-AL4)) (E160)
C3Z = D1Z-C4Z (E161)

P23Z = BE3*C3Z
P13Z = C5Z-AL3*C3Z
P24Z = BE4*C4Z
P14Z = C6Z-AL4*C4Z

PRINT 13, C1Z, D1Z, D2Z, D3Z, D4Z, C6Z, C5Z, C4Z, C3Z, P23Z, P13Z, P24Z, P14Z
A4T = U33/DD4
A3T = U32/DD4
A2T = U31/DD4
A1T = U30/DD4
A0T = U29/DD4

PRINT 13, A4T, A3T, A2T, A1T, A0T
C1T = A0T/(G3*G4) (E170)
D1T = A4T-C1T
D2T = A3T-C1T*WB4 (E171)
D3T = A2T-C1T*WB3
D4T = A1T-C1T*WB2
C6T = (2.*(AL3-AL4)*(G4*D3T-D1T*G4**2-2.*AL4*D4T)+(G3-G4)*(-G4*D2T (E172)

1+2.*AL4*G4*D1T+D4T))/(2.*(AL3-AL4)*(2.*AL3*G4-2.*AL4*G3)+(G3-G4)
2**2) (E172)

C5T = (D4T-G3*C6T)/G4 (E173)

C4T = (G4*(D2T-2.*AL4*D1T)-D4T+C6T*(G3-G4))/(2.*G4*(AL3-AL4)) (E174)

C3T = D1T-C4T (E175)

P23T = BE3*C3T

P13T = C5T-AL3*C3T

P24T = BE4*C4T

P14T = C6T-AL4*C4T

} (E176)

PRINT 13, C1T,D1T,D2T,D3T,D4T,C6T,C5T,C4T,C3T,P23T,P13T,P24T,P14T

T = -0.100

25 T = T+0.100

27 EAL3T = EXPF (AL3*T)

COB3T = COSF (BE3*T)

SIB3T = SIN F (BE3*T)

EAL4T = EXPF (AL4*T)

COB4T = COSF (BE4*T)

SIB4T = SIN F (BE4*T)

X = C1X+(1./(BE3*EAL3T))*(P23X*COB3T+P13X*SIB3T)+(1./(BE4*EAL4T))*
1(P24X*COB4T+P14X*SIB4T) (E149)

XD = (-AL3/(BE3*EAL3T))*(P23X*COB3T+P13X*SIB3T)+(1./EAL3T)*(-P23X
1*SIB3T+P13X*COB3T)-(AL4/(BE4*EAL4T))*(P24X*COB4T+P14X*SIB4T) (E150)

2+(1./EAL4T)*(-P24X*SIB4T+P14X*COB4T);
XDD = ((AL3**2-BE3**2)/(BE3*EAL3T))*(P23X*COB3T+P13X*SIB3T)-(2.* (E151)

1AL3/EAL3T)*(-P23X*SIB3T+P13X*COB3T)+((AL4**2-BE4**2)/(BE4*EAL4T))*
2(P24X*COB4T+P14X*SIB4T)-(2.*AL4/EAL4T)*(-P24X*SIB4T+P14X*COB4T)

Z = C1Z+(1./(BE3*EAL3T))*(P23Z*COB3T+P13Z*SIB3T)+(1./(BE4*EAL4T))*
1(P24Z*COB4T+P14Z*SIB4T) (E163)

ZD = (-AL3/(BE3*EAL3T))*(P23Z*COB3T+P13Z*SIB3T)+(1./EAL3T)*(-P23Z
1*SIB3T+P13Z*COB3T)-(AL4/(BE4*EAL4T))*(P24Z*COB4T+P14Z*SIB4T) (E164)

2+(1./EAL4T)*(-P24Z*SIB4T+P14Z*COB4T)
ZDD = ((AL3**2-BE3**2)/(BE3*EAL3T))*(P23Z*COB3T+P13Z*SIB3T)-(2.* (E165)

1AL3/EAL3T)*(-P23Z*SIB3T+P13Z*COB3T)+((AL4**2-BE4**2)/(BE4*EAL4T))*
2(P24Z*COB4T+P14Z*SIB4T)-(2.*AL4/EAL4T)*(-P24Z*SIB4T+P14Z*COB4T)

TH= C1T+(1./(BE3*EAL3T))*(P23T*COB3T+P13T*SIB3T)+(1./(BE4*EAL4T))*
1(P24T*COB4T+P14T*SIB4T) (E177)

THD= (-AL3/(BE3*EAL3T))*(P23T*COB3T+P13T*SIB3T)+(1./EAL3T)*(-P23T
1*SIB3T+P13T*COB3T)-(AL4/(BE4*EAL4T))*(P24T*COB4T+P14T*SIB4T) (E178)

2+(1./EAL4T)*(-P24T*SIB4T+P14T*COB4T)
THDD= ((AL3**2-BE3**2)/(BE3*EAL3T))*(P23T*COB3T+P13T*SIB3T)-(2.* (E179)

1AL3/EAL3T)*(-P23T*SIB3T+P13T*COB3T)+((AL4**2-BE4**2)/(BE4*EAL4T))*
2(P24T*COB4T+P14T*SIB4T)-(2.*AL4/EAL4T)*(-P24T*SIB4T+P14T*COB4T)

FBZS = -TB*TH+TB*XD*TH/V1-TF*Z-TF*CGCF*TH-DH*ZD-XM*ZDD (E192)

WRAT = FBZS/(V1*DIS) (E214)

VAX = XD-(HGK+Z)*THD (E163)

TT = T+T2

PRINT 26, TT,T,X,XD,XDD,Z,ZD,ZDD,TH,THD,THDD,FBZS,WRAT,VAX

26 FORMAT (14H TOTAL TIME=,F11.5,5H T=,F11.5/5H X=,F11.5,

16H XD=,F11.5,7H XDD=,F11.5/5H Z=,F11.5,6H ZD=,F11.5,

27H ZDD=,F11.5/6H TH=,F11.5,7H THD=,F11.5,8H THDD=,F11.5/

38H FBZS=,E12.5,8H WRAT=,F10.6,7H VAX=,F11.5//)

TEST1 = A11*XDD+B11*XD+A12*ZDD+B12*ZD+C12*Z+C13*TH-D13 (E195)

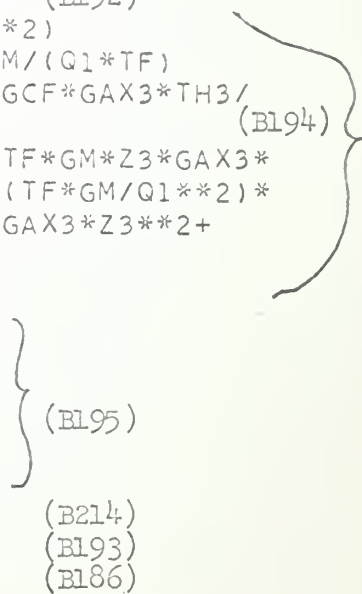
TEST2 = B21*XD+C21*X+A22*ZDD+B22*ZD+C22*Z+A 23*THDD+B23*THD+C23*TH- (E100)

1D23
TEST3 = C31*X+C32*Z+C33*TH-D33 (E103)


```

PRINT 13, TEST1,TEST2,TEST3
IF (VAX) 30,30,31
30 IF (VAX+0.02) 29,28,28
29 T = T-0.005
GO TO 27
31 IF (VAX-0.02) 28,28,25
28 TT3 = TT
T3 = T
X3 = X
XD3 = XD
XDD3 = XDD
Z3 = Z
ZD3 = ZD
ZDD3 = ZDD
TH3 = TH
THD3 = THD
THDD3 = THDD
FBZ3 = FBZS
WRAT3 = WRAT
VAX3 = VAX
PRINT 32, TT3,T3,X3,XD3,XDD3,Z3,ZD3,ZDD3,TH3,THD3,THDD3,FBZ3,
1WRAT3,VAX3
32 FORMAT (17H STATE 3 VALUES/14H TOTAL TIME=,F11.5,6H T3=,
1F11.5/6H X3=,F11.5,7H XD3=,F11.5,8H XDD3=,F11.5/6H Z3=,
2F11.5,7H ZD3=,F11.5,8H ZDD3=,F11.5/7H TH3=,F11.5,8H THD3=,
3F11.5,9H THDD3=,F11.5/8H FBZ3=,E12.5,9H WRAT3=,F10.6,
48H VAX3=,F11.5//)
PRAT = FZC2/FBZ3
IF (PRAT-1.0) 42,44,44
42 PRINT 43, PRAT
43 FORMAT (46H CAUTION, CRUSHING FORCE / SLIDING FORCE IS ,F8.5//)
44 GAX3 = P4-X3 (B87)
GAZ3 = HGK+Z3 (B86)
Q1 = CGCF+GAX3 (B192)
A4 = GAZ3/(Q1*TF)-GM*CGCF/(TF*Q1**2)-GM*GAX3/(TF*Q1**2)
B4 = GAX3+GAZ3*TH3+GAZ3*Z3/Q1-GAZ3*GAX3*TH3/Q1+DIS*GM/(Q1*TF)
1+GM*Z3/Q1-GM*GAX3*TH3/Q1-2.*GM*CGCF*Z3/Q1**2+2.*GM*CGCF*GAX3*TH3/
2Q1**2-2.*GM*GAX3*Z3/Q1**2+2.*GM*GAX3**2*TH3/Q1**2 (B194)
C4 = DIS*GM*Z3/Q1-DIS*GM*GAX3*TH3/Q1+TF*GM*Z3**2/Q1-TF*GM*Z3*GAX3*
1TH3/Q1-TF*GM*GAX3*TH3*Z3/Q1+TF*GM*GAX3**2*TH3**2/Q1-(TF*GM/Q1**2)*
2(CGCF*Z3**2+CGCF*GAX3**2*TH3**2-2.*CGCF*GAX3*Z3*TH3+GAX3*Z3**2+
3GAX3**3*TH3**2-2.*GAX3**2*Z3*TH3)
PRINT 13,GAX3,GAZ3,Q1,A4,B4,C4
DISC5 = (B4**2)-4.*A4*C4
IF (DISC5) 34,33,33
34 PRINT 13, DISC5
GO TO 36
33 RAD = SQRTF (DISC5)
FBZ4 = (-B4+RAD)/(2.*A4)
WRAT4 = FBZ4/(V1*DIS)
TH4 = -FBZ4/(Q1*TF)-Z3/Q1+GAX3*TH3/Q1
Z4 = Z3+GAX3*(TH4-TH3)
X4 = X3
PRINT 35, X4,Z4,TH4,FBZ4,WRAT4
35 FORMAT (17H STATE 4 VALUES/6H X4=,F11.5,6H Z4=,F11.5,

```





```
17H TH4=,F11.5/26H VERTICAL FORCE AT BOW =,E12.5/16H WHITE RA  
2TIO =,F10.6//)  
COBA5 = COSF(BA+TH4)  
SIBA5 = SINF(BA+TH4)  
A7 = COSA*COBA5+FS*SIBA5 (B200)  
B7 = -COSA*SIBA5+FS*COBA5 (B201)  
ET = FBZ4/((A7/B7)*COSF(TH4)-SINF(TH4)) (B202)  
RAT = ET/TB  
PRINT 40,ET,RAT
```

```
40 FORMAT (22H EXTRACTING THRUST =,E12.5/50H RATIO OF EXTRACTING  
1THRUST TO DOLLARD THRUST IS ,F15.3//)  
GO TO 36  
END
```

DATA

250.000	62.000	25.750	11880000.000
0.523	0.886	10.000	0.724
-4.300	-3.300	22.750	6.750
50000.000	240.000	0.200	0.800
43200.000			
250.000	62.000	25.750	11880000.000
0.523	0.886	10.000	0.724
-4.300	-3.300	22.750	6.750
50000.000	240.000	0.200	0.800
50000.000			
250.000	62.000	25.750	11880000.000
0.523	0.886	6.000	0.724
-4.300	-3.300	22.750	6.750
50000.000	240.000	0.200	0.800
43200.000			
250.000	62.000	25.750	11880000.000
0.523	0.886	6.000	0.724
-4.300	-3.300	22.750	6.750
50000.000	240.000	0.200	0.800
50000.000			

The most important output of the program is the relatively sustained downward force under the bow during State 4.

$$F_{BZ4} = \text{Vertical Force at Bow, lbs.}$$

In addition other output is available as follows:

$X4$ = Forward motion from initial point of contact, ft.

$Z4$ = Vertical position of the center of gravity relative to the original position at the time of contact, ft.

$TH4$ = θ_4 , Final trim, radians

$$WRAT = \text{"White Ratio"} = \frac{F_{BZ4}}{(\text{Displacement})(\text{Impact velocity})} \text{ sec/ft}$$

ET = Extracting thrust, lbs.

RAT = Extracting thrust/Bollard thrust, dimensionless.

Other information is readily available (if desired) as a function of time.

Forward position and its derivatives $X, XD, XDD = x, \dot{x}, \ddot{x}$
(ft, ft/sec, and ft/sec²)

Vertical position of the center of $Z, ZD, ZDD = z, \dot{z}, \ddot{z}$
gravity and its derivatives (ft, ft/sec, and ft/sec²)

$TH, THD, THDD = \theta, \dot{\theta}, \ddot{\theta}$ Pitch angle and its derivatives
(radians, rad/sec, and rad/sec²)

F_{BZ} Downward force under bow during all phases as a function of time. lbs.

The first important result of the theory is the relationship between

the various quantities which appear in the equations of motion.

$$\frac{d^2x}{dt^2} = -\frac{GM}{r^2}$$

The general solution of this equation is given by

$$r = \frac{GM}{v^2} (1 + e \cos \theta)$$

$$t = \frac{GM}{v^3} (2\theta - e \sin \theta)$$

where r is the radial distance, t is the time, θ is the true anomaly,

$$e$$
 is the eccentricity, and v is the orbital velocity.

$$\frac{GM}{v^2} = \frac{GM}{v^2} \left(\frac{v}{v} \right)^2 = \frac{GM}{v^2}$$

$$t = \frac{GM}{v^3} (2\theta - e \sin \theta)$$

$$e = \frac{GM}{v^2} \left(\frac{v}{v} \right)^2 = \frac{GM}{v^2}$$

The general solution of this equation is given by

where

$$r = \frac{GM}{v^2} (1 + e \cos \theta)$$

$$t = \frac{GM}{v^3} (2\theta - e \sin \theta)$$

$$e = \frac{GM}{v^2} \left(\frac{v}{v} \right)^2 = \frac{GM}{v^2}$$

$$e = \frac{GM}{v^2} \left(\frac{v}{v} \right)^2 = \frac{GM}{v^2}$$

$$e = \frac{GM}{v^2} \left(\frac{v}{v} \right)^2 = \frac{GM}{v^2}$$

$$e = \frac{GM}{v^2} \left(\frac{v}{v} \right)^2 = \frac{GM}{v^2}$$

The general solution of this equation is given by

of time. The

Other output is available directly but is only incidental to the solution of the basic problem. This includes total mass, including virtual (in each sense, x , z , θ), radius of gyration, pounds per foot immersion, pitch damping coefficient, heave damping coefficient, and scores of coefficients used in the solution.

Suitable Simplifications

From Milano's work (18) it may be seen that the longitudinal inertia coefficient of the waterplane is approximately linear as a function of α , the waterplane coefficient.

This may be expressed as

$$C_{1l} = 0.030 + \frac{(0.060 - 0.030)(\alpha - 0.65)}{(0.88 - 0.65)}$$

$$C_{1l} = 0.030 + \frac{0.030}{0.23} \alpha$$

$$C_{1l} = 0.030 + 0.1304 (\alpha - 0.65) \quad (B203)$$

$$BM_L = \frac{I}{W_{vol}} = \frac{C_{1l} HL^3}{35 (DIS)}$$

$$BM_L = \frac{I}{vol} = \frac{C_{1l} HL^3}{(DIS)/64.2} = \frac{64.2 C_{1l} HL^3}{DIS} \quad (B204)$$

$$C_b = \frac{DIS}{64.2 LBH} \quad (B205)$$

Then,

$$BM_L = \frac{C_{1l} L^2}{C_b H} \quad (B206)$$

$$KB = \frac{\alpha}{C_b + \alpha} H \quad (B207)$$

QUESTION 1

Let $f(x) = \frac{1}{x^2}$. Find $f'(x)$ using the definition of the derivative.

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \frac{x^2 - (x+h)^2}{h(x+h)^2 x^2}$$

$$= \frac{x^2 - (x^2 + 2xh + h^2)}{h(x+h)^2 x^2} = \frac{-2xh - h^2}{h(x+h)^2 x^2}$$

(cont)

$$= \frac{-2x - h}{(x+h)^2 x^2}$$

$$\lim_{h \rightarrow 0} \frac{-2x - h}{(x+h)^2 x^2} = \frac{-2x}{x^2 x^2} = -\frac{2}{x^3}$$

(cont)

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = -\frac{2}{x^3}$$

(cont)

$$f'(x) = -\frac{2}{x^3}$$

(cont)

$$f'(x) = -\frac{2}{x^3}$$

(cont)

$$f'(x) = -\frac{2}{x^3}$$

It is noted from the "Wind Class" inclining experiment (29) that the height of the center of gravity above the keel is 23.4 ft at a draft of 26.25 ft, or

$$KG = 0.89 H \quad (B208)$$

Milano indicates the center of gravity may be expressed as follows:

$$0.95 H = KG = 1.20 H \quad (B209)$$

Let it be assumed, as an approximation that

$$KG = 0.95 H \quad (B210)$$

The longitudinal metacentric height, GM_L , may be determined by using equations (B205), (B203), (B207), (B206, and (B210).

$$GM_L = KG + EM_L - KG \quad (B211)$$

Bollard thrust may be approximated by using a propeller loading factor, $T_B / (\text{Prop. diam.})^2$

It has been shown (for twin screw icebreakers) that the ratio of propeller diameter to draft varies linearly with design draft (18).

$$PD/H = 0.82 - \frac{(0.22)}{(22)} (H-10) = 0.82 - \frac{(H-10)}{100}$$

It follows that

$$\text{Prop. Diam.} = H \left[0.82 - \frac{1}{100} (H - 10) \right] \quad (B212)$$

It is noted that the "1984" label is missing from the
 label of the graph of the function $f(x) = 2x^2 - 12x + 10$
 at $x = 10$.

(1000) $f(x) = 2x^2 - 12x + 10$

When the graph is plotted, the vertex is at $(3, -8)$ and the x-axis is at $y = 0$.

(1010) $0 = 2x^2 - 12x + 10$

Let it be assumed that the graph is a parabola.

(1020) $0 = 2x^2 - 12x + 10$

The horizontal asymptote is $y = 0$, and the vertical axis is

using equations (1000), (1010), (1020), (1030), and (1040).

(1030) $0 = 2x^2 - 12x + 10$

Let the graph be approximated by using a projection method.

$f(x) = 2x^2 - 12x + 10$

It has been shown that the graph is a parabola.

Projection method is used to find the vertex of the parabola.

$$\frac{f(10)}{f'(10)} = \frac{2(10)^2 - 12(10) + 10}{4(10) - 12} = \frac{200 - 120 + 10}{40 - 12} = \frac{90}{28} = \frac{45}{14}$$

It follows that

(1040) $\left[\frac{45}{14} - 10 \right] = \frac{45}{14} - 10 = \frac{45 - 140}{14} = \frac{-95}{14}$

For twin screw icebreakers the propeller diameter can be based on equation (B212). From this the bollard thrust can be approximated (for icebreakers over 300 ft) by (18),

$$\bar{T}_B = (0.38)(2240)(\text{Prop Diam})^2 \quad (\text{B213})$$

Parametric Study, General

It is important to determine how the variation of a parameter effects the sustained downward force. For example, it would seem obvious that an increase in displacement would yield a greater downward force. It is to be noted, however, that there are sixteen variables as parameters and only a few of these can be considered as approximately independent (i.e., the bow angle).

As a first basis, assume an icebreaker the size of a "Wind Class" and see what effect there would be from shifting some parameters independently within reasonable limits.

"Wind Class" Parameters

BP	(length between perpendiculars)	=	250.0 ft.
B	(beam at waterline)	=	64.0 ft.
H	(draft mean)	=	25.75 ft.
DIS	(displacement)	=	12,100,000.0 lb.
BA	(i_B , bow angle)	=	0.523 rad.
SA	(β , spread angle complement)	=	0.886 rad.
VI	(impact velocity)	varies from 0 to	25.0 ft/sec.

The following information is for your information only. It is not intended to constitute an offer of insurance or any other financial product. Please contact your agent for more information.

Policy No. 123456789

Insurance Policy Details

This policy provides coverage for the insured against the risk of fire and theft. The policy is subject to the terms, conditions, and exclusions set forth in the policy document. The insured is required to maintain the property in a safe and sound condition and to take all reasonable precautions to prevent loss. The policy is not intended to cover losses caused by war, terrorism, or nuclear energy.

The policy is issued to the insured for a term of one year, commencing on the date of issue. The policy is subject to renewal at the discretion of the insurer. The insured is required to pay the premium for this policy on the date specified in the policy document.

Policy Schedule

Item	Description	Amount
1	Fire and Theft Insurance	\$100,000
2	Business Interruption Insurance	\$50,000
3	Legal Defense Costs	\$25,000
4	Professional Fees	\$10,000
5	Administrative Expenses	\$5,000
6	Contingent Liability	\$10,000
7	Other Expenses	\$5,000
8	Total	\$210,000

AL	(α , waterplane coeff.)	=	0.724
CF	(LCF, center of flotation)	=	-1.25 ft
CG	(LCG, center of gravity)	=	-2.40 ft
GK	(KG, height of center of gravity)	=	23.4 ft.
D	(height of thrust line at c.g.)	=	16.0 ft.
TB	(bollard thrust)	=	270,000 max.
GM	(GM_L , long. metacentric height)	=	195.6 ft.
FK	(ice/ship kinetic friction)	=	0.2
FS*	(ice/ship static friction)	=	0.8
SIG	(failure stress of ice)	=	144,000.0 lb/ft ²

Some of these properties may be varied independently (i.e. VL, FK, SIG). Other parameters may be varied within reasonable limits and under that condition it may be assumed that they are independent (i.e., TB depends on shaft r.p.m., used during the crushing and sliding, D, GK, CG, SA, BA). The remaining parameters (GM, CF, AL, DIS, H, B, HP) may not be varied independently.

The impact velocity, VL, will be varied on subsequent solutions (from 0 to 25.0 ft/sec, 0 to 14.8 knots) along with one other parameter.

The ice/ship kinetic friction, FK, will be varied from 0.1 to 0.3. These are reasonable limits (14), (15).

* FS is not necessary for the solution of the downward force.

The compressive failure stress of the ice, SIG, will be varied from 25,000 lb/ft² to 200,000 lb/ft². (30) (Strengths below that would probably not give a "Wind Class" icebreaker any difficulty at all (37).) It should be noted this parameter cannot be controlled.

As the solution was worked out, it was presumed the bollard thrust (TB) would be based on the rpm of the shaft necessary to maintain impact velocity in open water. The variation in practice, depending on the choice of the Commanding Officer, is from zero-thrust (stopping ships screws at the time of initial contact) to maximum thrust (by applying full power at the time of initial contact, as was done during the 1963 D.T.M.B. - Westwind tests (37)). In any event, the solution considers that only a partial thrust is used against the ice until the ship stops. At that point only, bollard thrust is completely against the ice. For that reason, the bollard thrust may be considered independent and will be varied from 0 to 270,000 lbs. for the "Wind" class, the maximum available. (Other classes will have different limits.)

The height of the thrust line of action, D, measured near the center of gravity could reasonably be varied from 10.0 ft to 18.0 feet for the "Wind" class. It is noted that this is merely an extrapolation of the shafting line and may not in fact truly represent the line of action of the thrust. The solution disregards any vertical component of thrust when the ship is in trim. The solution does take into account a vertical component as the ship has it's bow raised by the ice.

The Commission has received information from the State Department that the Soviet Government has agreed to the proposed terms of the armistice. The Commission has also received information from the State Department that the Soviet Government has agreed to the proposed terms of the armistice.

(a) The Commission has received information from the State Department that the Soviet Government has agreed to the proposed terms of the armistice. The Commission has also received information from the State Department that the Soviet Government has agreed to the proposed terms of the armistice.

[Section Header]

The Commission has received information from the State Department that the Soviet Government has agreed to the proposed terms of the armistice. The Commission has also received information from the State Department that the Soviet Government has agreed to the proposed terms of the armistice.

The height of the center of gravity above the keel, GK, is 0.95 to 1.2 times the mean draft for most icebreakers. In normal load the "Wind Class" GK equals 0.91 times the mean draft. Therefore, GK will be varied from 22.0 ft. to 31.0 ft. for the "Wind" class. It must be noted that GM_L must be varied accordingly to keep GM_L constant.

The longitudinal position of the center of gravity, CG, may be varied but this implies there is an initial trim which effects, among other things, the effective bow angle. The secondary effects will be disregarded. (For example, if the icebreaker is up 30" by the bow as the result of shifting the center of gravity back two feet, the effective bow angle is increased about 2 o/o. CG will be varied from -4.4 ft to -1.4 ft. for the "Wind" Class.)

The spread angle complement (the angle from a normal to the hull plating to the center line plane, β), SA, may be considered as quite independent. A "sharp" bow may have $\beta = 70^\circ$ while a "blunt" bow may have $\beta = 20^\circ$. Therefore, SA will be varied from 1.2 radians (sharp) to 0.35 radians (blunt).

Probably the most often discussed variable of icebreaker design is the bow angle, BA, (i_B , the angle from the base line to the stem). Assuming that the stem is a straight line from the forward perpendicular back down to the keel, as this solution does, the lower limit must be of the magnitude of 15° . (At about 6° the stem becomes the keel of an icebreaker with a large designed drag.) In fact, at this low angle the bow

The value of the vector of gravity \vec{g} is 9.8 m/s^2 and it is directed downwards. The vector of the velocity \vec{v} is 10 m/s and it is directed upwards. The vector of the acceleration \vec{a} is 9.8 m/s^2 and it is directed downwards. The vector of the displacement \vec{s} is 10 m and it is directed upwards.

Let us consider the motion of a body in a uniform gravitational field.

The trajectory of the motion of the body in a uniform gravitational field is a parabola. The vector of the velocity \vec{v} is 10 m/s and it is directed upwards. The vector of the acceleration \vec{a} is 9.8 m/s^2 and it is directed downwards. The vector of the displacement \vec{s} is 10 m and it is directed upwards. The vector of the velocity \vec{v} is 10 m/s and it is directed upwards. The vector of the acceleration \vec{a} is 9.8 m/s^2 and it is directed downwards. The vector of the displacement \vec{s} is 10 m and it is directed upwards.

For the "ball" case:

The speed of the ball is 10 m/s and it is directed upwards.

Let us consider the motion of a body in a uniform gravitational field. The vector of the velocity \vec{v} is 10 m/s and it is directed upwards. The vector of the acceleration \vec{a} is 9.8 m/s^2 and it is directed downwards. The vector of the displacement \vec{s} is 10 m and it is directed upwards. The vector of the velocity \vec{v} is 10 m/s and it is directed upwards. The vector of the acceleration \vec{a} is 9.8 m/s^2 and it is directed downwards. The vector of the displacement \vec{s} is 10 m and it is directed upwards.

(10)

Let us consider the motion of a body in a uniform gravitational field.

The vector of the velocity \vec{v} is 10 m/s and it is directed upwards. The vector of the acceleration \vec{a} is 9.8 m/s^2 and it is directed downwards. The vector of the displacement \vec{s} is 10 m and it is directed upwards. The vector of the velocity \vec{v} is 10 m/s and it is directed upwards. The vector of the acceleration \vec{a} is 9.8 m/s^2 and it is directed downwards. The vector of the displacement \vec{s} is 10 m and it is directed upwards.

angle cannot be considered completely independent. However, the bow angle will be varied from 0.262 radians (15°) to 0.80 (about 45°).

"White Ratio"

It is anticipated that the downward force under the bow in the static condition (State 4) following ramming will be effected approximately linearly by displacement and impact velocity. For that reason, the following coefficient may be of use in comparison of parameter effects.

$$\text{WRAT} = \frac{F_{BZ4}}{(MS)(VI)} \quad \text{sec/ft} \quad (\text{B214})$$

... (faint text) ...

... (faint text) ...

... (faint text) ...

(1947) $\frac{1}{(1+r)^t}$...

... (faint text) ...

"Glacier" Class Parameters

BP = 290.0 ft
B = 72.5 ft
H = 28.0 ft
DIS = 8640 tons = 19,350,000.0 lb
BA = $30^{\circ} = 0.523$ radians
SA = $50.8^{\circ} = 0.886$ radians
VI = variable
AL = 0.8
CF = -1.45 ft (scaled from Wind Class length)
CG = -2.78 ft (scaled from Wind Class length)
GK = 24.5 ft (scaled from Wind Class draft)
D = 16.8 ft (scaled from Wind Class draft)
TB = 455,000.0 lb (4)
 GM_L = 275.0 ft
FK = 0.2
FS = 0.8
SIG = 144,000 lb/ft² (30)

Appendix 1 - 1992

	1992	1991
	1990	1989
	1988	1987
	1986	1985
	1984	1983
	1982	1981
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	1978	1977
	1976	1975
	1974	1973
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	1108	1107
	1106	1105
	1104	1103
	1102	1101
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"Lenin" Class Parameters (4)

BP = 420.0 ft
B = 90.0 ft
H = 39.25 ft
DIS = 16,000 tons = 35,800,000.0 lbs.
BA = $30^\circ = 0.523$ radians
SA = $50.8^\circ = 0.886$ (est. equal to Glacier, Wind Class)
V1 = variable
AL = 0.80
CF = -2.10 (scaled from Wind Class length)
CG = -4.04 (scaled from Wind Class length)
GK = 27.5 (scaled from Wind Class draft)
D = 18.8 (scaled from Wind Class draft)
TB = 730,000.0 lb
GM_L = 545.0
FK = 0.2
FS = 0.8
SIG = 144,000 lb/ft² (30)

As noted previously, some of the variables may not be varied independently. For example, a change in α , the waterplane coefficient will cause a change in pounds-per-foot-immersion (TF), the height of the center of buoyancy (KB), and the distance from the center of buoyancy to the longitudinal metacenter (EM_L). There could be a change in the height of the center of gravity but it will be assumed this is unchanged. For the sake of comparison it shall be assumed that the displacement does not change (the block coefficient remains constant). This implies that if the higher waterplanes have an increased coefficient, the lower waterplanes must have a decreased coefficient.

Assume that α is changed over a range from 0.70 to 0.85, then,

$$C_{iL} = 0.030 + 0.1304 (\alpha - 0.65) \quad (B203)$$

$$C_b = \frac{DIS}{64.2 LBH} \quad (B205)$$

$$GM_L = \frac{C_{iL} L^2}{C_b H} \quad (B206)$$

$$KB = \frac{\alpha H}{C_b + \alpha} \quad (B207)$$

$$GM_L = KB + EM_L - KG \quad (B211)$$

is not possible, one of the following is to be done:

1. The following conditions are to be satisfied:

(a) The value of λ is to be determined such that the characteristic equation of the matrix $(A - \lambda I)$ has a zero root. This is done by setting the determinant of the matrix $(A - \lambda I)$ equal to zero.

(b) The value of λ is to be determined such that the characteristic equation of the matrix $(A - \lambda I)$ has a zero root. This is done by setting the determinant of the matrix $(A - \lambda I)$ equal to zero.

(c) The value of λ is to be determined such that the characteristic equation of the matrix $(A - \lambda I)$ has a zero root. This is done by setting the determinant of the matrix $(A - \lambda I)$ equal to zero.

(d) The value of λ is to be determined such that the characteristic equation of the matrix $(A - \lambda I)$ has a zero root. This is done by setting the determinant of the matrix $(A - \lambda I)$ equal to zero.

where λ is the eigenvalue and \mathbf{x} is the eigenvector.

(1001)
$$(A - \lambda I) \mathbf{x} = \mathbf{0}$$

(1002)
$$\begin{bmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(1003)
$$\begin{bmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(1004)
$$\begin{bmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(1005)
$$a_{11}x_1 + a_{12}x_2 = 0$$

Utilizing the equations listed above the computer program may then give results based on a change of α and consequently a change of GM_L and pounds-per-foot immersion (the latter is already a calculation contained in the program).

It is possible to assume that the longitudinal position of the center of flotation varies slightly. Within reasonable limits other terms may be held constant even though a change in ship form would be necessary so as not to introduce trim. (However, displacement, length, draft, and the coefficients could remain constant.) It will be assumed that $\frac{LCF}{LBP}$ varies from 0 to -0.010. (For the Wind Class $\frac{LLF}{LBP} = -0.005$.)

In order to find the effect caused by changing draft, H, the beam to draft ratio may be varied (B/H). Most other values will be held constant (i.e. displacement and block coefficient, length). This implies that the product B x H remains constant. Let the beam to draft ratio vary from 2.0 to 4.0.

First determine the product of the parent ship.

$$B \times H = C_{BH} \quad (B215)$$

$$B = (B - H \text{ Ratio}) H \quad (B216)$$

$$H = \sqrt{\frac{C_{BH}}{(B-H \text{ ratio})}} \quad (B217)$$

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$$(-1)^{n-1} \frac{1}{n!} \dots$$

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0.4 or 0.5

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(122) \dots

(123) \dots

(124) $\sqrt{\frac{\dots}{(2-2) \dots}}$

Using the value of C_{BH} determined in equation (B215), find the new draft H from equation (B217) and then find the new beam, B, from equation (B216).

Assume, for the sake of comparison, that

$$KG = 0.95 H$$

Then utilize equations (B203), (B205), (B206, (B207), and (B211) to determine the corresponding longitudinal metacentric height, GM_L . By entering these changes into the program and by varying beam-to-draft-ratio as indicated, the corresponding effect may be obtained.

In most polar icebreakers the length-to-beam-ratio is approximately 4 to 1. It is possible to determine the effect of varying this ratio, BP/B , by modifying the solution but holding displacement, draft, and the block and waterplane coefficients constant. This implies $BP \times B$ remains constant.

$$BP \times B = C_{EPB} \quad (B218)$$

$$BP = (BP-B \text{ ratio})B \quad (B219)$$

$$B (BP-B \text{ ratio})B = C_{EPB} \quad (B220)$$

$$B = \sqrt{\frac{C_{EPB}}{(BP-B \text{ ratio})}} \quad (B221)$$

Using the value of C_{EPB} determined in equation (B218), find the new beam, B, from equation (B221) and then find the new length, BP, from

Let $\mathcal{L} = \{L_1, L_2, \dots, L_n\}$ be a set of linear forms in n variables over \mathbb{C} .
 Let $\mathcal{L}^* = \{L_1^*, L_2^*, \dots, L_n^*\}$ be the dual set of linear forms.
 Let $\mathcal{L}^{\otimes k} = \{L_1^{\otimes k}, L_2^{\otimes k}, \dots, L_n^{\otimes k}\}$ be the set of k -th powers of the linear forms.

1.3.1. The

Let $\mathcal{L} = \{L_1, L_2, \dots, L_n\}$ be a set of linear forms in n variables over \mathbb{C} .
 Let $\mathcal{L}^* = \{L_1^*, L_2^*, \dots, L_n^*\}$ be the dual set of linear forms.
 Let $\mathcal{L}^{\otimes k} = \{L_1^{\otimes k}, L_2^{\otimes k}, \dots, L_n^{\otimes k}\}$ be the set of k -th powers of the linear forms.
 Let $\mathcal{L}^{\otimes k, \otimes l} = \{L_1^{\otimes k}, L_2^{\otimes k}, \dots, L_n^{\otimes k}, L_1^{\otimes l}, L_2^{\otimes l}, \dots, L_n^{\otimes l}\}$ be the set of k -th and l -th powers of the linear forms.
 Let $\mathcal{L}^{\otimes k, \otimes l, \otimes m} = \{L_1^{\otimes k}, L_2^{\otimes k}, \dots, L_n^{\otimes k}, L_1^{\otimes l}, L_2^{\otimes l}, \dots, L_n^{\otimes l}, L_1^{\otimes m}, L_2^{\otimes m}, \dots, L_n^{\otimes m}\}$ be the set of k -th, l -th, and m -th powers of the linear forms.

(1.3.1)

$$L_1^{\otimes k} + L_2^{\otimes k} + \dots + L_n^{\otimes k} = 0$$

(1.3.2)

$$L_1^{\otimes k} + L_2^{\otimes k} + \dots + L_n^{\otimes k} = 0$$

(1.3.3)

$$L_1^{\otimes k} + L_2^{\otimes k} + \dots + L_n^{\otimes k} = 0$$

(1.3.4)

$$\sqrt{\frac{L_1^{\otimes k} + L_2^{\otimes k} + \dots + L_n^{\otimes k}}{L_1^{\otimes l} + L_2^{\otimes l} + \dots + L_n^{\otimes l}}} = 0$$

Let $\mathcal{L} = \{L_1, L_2, \dots, L_n\}$ be a set of linear forms in n variables over \mathbb{C} .
 Let $\mathcal{L}^* = \{L_1^*, L_2^*, \dots, L_n^*\}$ be the dual set of linear forms.
 Let $\mathcal{L}^{\otimes k} = \{L_1^{\otimes k}, L_2^{\otimes k}, \dots, L_n^{\otimes k}\}$ be the set of k -th powers of the linear forms.
 Let $\mathcal{L}^{\otimes k, \otimes l} = \{L_1^{\otimes k}, L_2^{\otimes k}, \dots, L_n^{\otimes k}, L_1^{\otimes l}, L_2^{\otimes l}, \dots, L_n^{\otimes l}\}$ be the set of k -th and l -th powers of the linear forms.

equation (B219).

Then utilize equations (B203), (B205), (B206), (B207), and (B211) to determine the corresponding longitudinal metacentric height, GM_L . By entering these changes into the program and by varying the length-to-beam ratio from 3.5 to 5.0, the corresponding effect may be determined.

One method of variation of displacement is to vary the block coefficient while holding length, EP, draft, H, and beam, B, constant. (Assume that the waterplane coefficient of the waterplane of the effective draft remains constant.)

$$DIS = C_b (69.2) LBH$$

Then utilize equations (B203), (B206), (B207), and (B211) to determine the corresponding longitudinal metacentric height. By entering these changes into the program and by varying the block coefficient, C_b , from 0.4 to 0.7, the corresponding effect may be determined.

The effect brought about by changing displacement may be examined by increasing the size of the ship such that the new ship is geometrically similar. To do this, multiply TB (assuming a constant thrust-to-displacement-ratio), and DIS by (Scale ratio)³. Multiply the following length dimensions by the (Scale ratio):

EP, B, H, CF, CG, GK, D, and GM_L .

By utilizing these changes the effect of varying the scale ratio from 0.8 to 1.6 may be determined for geometrically similar ships.

Model Parameters

Let the length of the ship divided by the length of the model equal lambda.

$$\frac{L_s}{L_m} = \lambda \quad (B222)$$

It then follows that the linear dimensions must be multiplied by $1/\lambda$.

<u>Model</u>	=	<u>Ship</u>	
BP_m	=	BP_s/λ	(B223)
B_m	=	B_s/λ	
H_m	=	H_s/λ	
CF_m	=	CF_s/λ	
CG_m	=	CG_s/λ	
GK_m	=	GK_s/λ	
D_m	=	D_s/λ	
GM_{Lm}	=	GM_{Ls}/λ	

Assuming a constant density fluid (fresh water or sea water),

$$DIS_m = DIS / \lambda^3 \quad (B224)$$

Write down the steps of the following process in a flow chart.

Answer

(2021)

1.5

of the following process, write down the steps in a flow chart.

1/c

(2021)

Step	Order
1.6	1
1.7	2
1.8	3
1.9	4
1.10	5
1.11	6
1.12	7

(Write down the steps of the following process in a flow chart.)

(2021)

5/1

Assuming a constant thrust-to-displacement ratio,

$$TB_m = TB_s / \lambda^3 \quad (B225)$$

The coefficients are dimensionless and are not changed.

$$AL_m = AL_s \quad FK_m = FK_s \quad FS_m = FS_s \quad (B226)$$

Angles are not changed.

$$BA_m = BA_s \quad SA_m = SA_s \quad (B227)$$

The value for compressive failure stress of ice (or the model-supporting-medium) must be changed.

$$\text{Pounds force} = \rho g L^3$$

$$\text{where } \rho_m = \rho_s \quad \text{and } g_m = g_s$$

$$SIG = \frac{\text{Pounds force}}{L^2} = \frac{\rho g L^3}{L^2} = \rho g L$$

It follows that

$$SIG_m = \rho_m g_m L_m \quad SIG_s = \rho_s g_s L_s$$

$$\frac{SIG_m}{SIG_s} = \frac{\rho_m g_m L_m}{\rho_s g_s L_s} = \frac{L_m}{L_s} = \frac{1}{\lambda}$$

Therefore

where μ is the mean and σ^2 is the variance.

(100)
$$\int_{-\infty}^{\infty} f(x) \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \int_{-\infty}^{\infty} f(x) dx$$

The equality holds for all functions $f(x)$ for which the integral exists.

(101)
$$\int_{-\infty}^{\infty} x^n \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \begin{cases} \mu^n & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases}$$

where n is any integer.

(102)
$$\int_{-\infty}^{\infty} x^n \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \mu^n + \frac{n(n-1)}{2\sigma^2} \int_{-\infty}^{\infty} x^{n-2} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

The value for $n=0$ is 1 and for $n=1$ is μ .

(The value for $n=2$ is $\mu^2 + \sigma^2$.)

where n is any integer.

where n is any integer.

$$\int_{-\infty}^{\infty} x^n \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \mu^n + \frac{n(n-1)}{2\sigma^2} \int_{-\infty}^{\infty} x^{n-2} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

It follows that

$$\int_{-\infty}^{\infty} x^n \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \mu^n + \frac{n(n-1)}{2\sigma^2} \int_{-\infty}^{\infty} x^{n-2} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

$$\int_{-\infty}^{\infty} x^n \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = \mu^n + \frac{n(n-1)}{2\sigma^2} \int_{-\infty}^{\infty} x^{n-2} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

Therefore

$$\text{SIG}_m = \text{SIG}_s / \lambda \quad (\text{B228})$$

Since gravity and dynamics are involved it follows that

$$\frac{(V_L)_m^2}{g_m L_m} = \frac{(V_L)_s^2}{g_s L_s}$$

$$\frac{V_L}_m = \frac{V_L}_s \sqrt{\frac{L_s}{L_m}}$$

It is apparent that the respective Froude Numbers must be equal and

$$V_L}_m = V_L}_s / \sqrt{\lambda} \quad (\text{B229})$$

If these model parameters are used, the values of x_m and z_m for the model in its final position should equal x_s / λ and z_s / λ respectively. θ_m in its final position should be equal to θ_s . The values for force (both crushing peak and final sustained value) should be related by λ^3 .

$$F_m = F_s / \lambda^3 \quad (\text{B230})$$

Equation (B33) is used in the program in place of equation (B41) because it is more suitable for both model and ship.

The relationship of time of events for the ship to the time of events for the model may be developed as follows:

(1981)

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$$\frac{1000}{100} = \frac{100}{10}$$

$$\frac{1000}{100} = \frac{100}{10}$$

... ..

(1982)

$$\frac{1000}{100} = \frac{100}{10}$$

... ..

(1983)

$$\frac{1000}{100} = \frac{100}{10}$$

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Since $\frac{V_m^2}{L_m} = \frac{V_s^2}{L_s}$

$$\frac{L_m^2/T_m^2}{L_m} = \frac{L_s^2/T_s^2}{L_s}$$

where T = time

$$\frac{L_m}{T_m^2} = \frac{L_s}{T_s^2}$$

$$\frac{T_s}{T_m} = \sqrt{\frac{L_s}{L_m}} = \sqrt{\Lambda}$$

(B231)

1

$$\frac{1}{x^2} = x^{-2}$$

1

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$$\frac{d}{dx} x^{-2} = -2x^{-3}$$

$$= -\frac{2}{x^3}$$

2021

$$= -\frac{2}{x^3}$$

C. SYMBOLS AND THEIR TITLES

This appendix list the symbols generally used in this research.*
Symbols of special or limited use are defined separately as used in this research.

- a - Acceleration, linear
- A - Area in general
- B - Beam at the designed waterline
- C_B - Coefficient, block = $V/(LB_x H_x)$
- e - Base of Napierian or natural logarithms
- e - Coefficient of restitution
- f_k = Coefficient of kinetic friction of ice and hull
- f_s = Coefficient of static friction of ice and hull

- F = Force in general
- F_x = Component of force in x direction in lb.
- F_z = Component of force in Z direction in lb.
- F_B = Force against bow perpendicular to stem in \hat{E} plane, in lb.
- F_{BZ} = Component of force against bow in Z-direction in lb.
- F_{BX} = Component of force against bow in X-direction in lb.

* These are, for the most part, in agreement with recommendations of the Society of Naval Architects and Marine Engineers

The opposite of the force exerted by the spring is the force exerted by the mass on the spring. This force is called the reaction force.

$F = -kx$ - Hooke's Law

$F = ma$ - Newton's Second Law

$F = \frac{dW}{dt}$ - Power

$W = \int F \cdot dx$ - Work

$U = \frac{1}{2}kx^2$ - Spring Potential Energy

$U = mgh$ - Gravitational Potential Energy

$E = \frac{1}{2}mv^2$ - Kinetic Energy

$E = \frac{1}{2}kx^2$ - Spring Potential Energy

$F = \frac{dW}{dt}$ - Power

$F_x = F \cos \theta$ - Component of force in x direction

$F_y = F \sin \theta$ - Component of force in y direction

$F = \frac{dW}{dt}$ - Power

$F_x = F \cos \theta$ - Component of force in x direction

$F_y = F \sin \theta$ - Component of force in y direction

*These are for the most part, in agreement with the conventions of the Society of American Scientists and the American Physical Society.

- g - Acceleration due to gravity
- H - Draft of a floating body or ship
- i_B - Slope, bowline or buttock with reference to baseplane, stem angle.
- L - Length, the principal longitudinal dimension of a ship, generally length between perpendiculars
- \bar{Q} - Amidships in general
- T_{IB} = Thrust, available for breaking ice, lb, (Total thrust - thrust used to overcome non-ice resistance)
- t - Thrust-deduction fraction, = $(T - R_T)/T$
- t - Time in general
- T - Thrust; usually ahead thrust; specifically, thrust developed by a propulsion device, lb.
- v - Velocity, linear
- w - Wake fraction of Taylor
- β = Angle with respect to the \bar{E} plane of a normal to the shell at the bow. (Note that this is the complement of half the angle of "spread" as one looks down the stem line), deg.
- x - Longitudinal body axis, positive forward
- z - Vertical body axis, positive from deck to keel
- α - Designed waterplane coefficient
- Δ - (delta, large capital) - Displacement weight in lbs of salt water
- η - Efficiency, general

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$$T \left(\frac{1}{2} \pi - \theta \right) = \dots$$

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θ - Angle of pitch or trim in a ship, with reference to the designed or normal attitude in the fore-and-aft plane. Its natural tangent is the algebraic difference of the changes in elevation of the designed waterline at the end perpendiculars, divided by the length L .

λ - Ratio, linear or scale, full-size body or ship to model, generally expressed as a number greater than unity; for example, 20th scale or 1:20 model.

CB - Center of buoyance of a body or ship

CF - Center of flotation; geometric or moment center of the surface waterplane area A_w

CG - Center of gravity or center of mass of a body or ship

CM_L, M_L - Meacenter, for longitudinal inclination

BM_L - Metacentric radius for longitudinal inclination

GM_L - Metacentric height, longitudinal, from CG to CM_L , for longitudinal inclination

LCF - Longitudinal center of flotation abaft X

LCG - Longitudinal center of gravity abaft X

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Abbreviations for Units of Measurement

- ft - foot
- ft² - square foot
- ft³ - cubic foot
- lb - pound
- lb ft² - pounds per square foot
- ft-lb - foot-pound
- hp - horsepower
- ehp - effective power, in English horses
- ihp - indicated power, in English horses
- shp - shaft power, in English horses
- sec - second
- rpm - revolutions per minute
- kt - knot, one nautical mile per hour

Investment by firms for advertising

1961 - 21

1962 - 22

1963 - 23

1964 - 24

1965 - 25

1966 - 26

1967 - 27

1968 - 28

1969 - 29

1970 - 30

1971 - 31

1972 - 32

1973 - 33

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E. SAMPLE CALCULATIONS

Sample Calculation using Raneberg's Equation

Wanted: F_{BZ} for U.S.S. Glacier

Given (14): $T_{IB} = 160$ tons

$$T_{IB} = (2240)(160) = 358,400 \text{ lb.}$$

$$i_B = 30.0^\circ$$

$$\beta = 40.3^\circ$$

$$f_k = 0.05$$

Solution: $\cos i_B = 0.866$

$$\sin i_B = 0.500$$

$$\cos \beta = 0.763$$

$$F_{BZ} = \frac{T_{IB} (\cos i_B \cos \beta - f_k \sin i_B)}{(\sin i_B \cos \beta + f_k \cos i_B)} \quad (A12)$$

$$F_{BZ} = \frac{T_{IB} [(0.866)(0.763) - (0.05)(0.500)]}{[(0.500)(0.763) + (0.05)(0.866)]}$$

$$F_{BZ} = T_{IB} \frac{(0.660) - (0.025)}{(0.381) + (0.043)}$$

DIFFERENTIAL EQUATIONS

Problem 1. Find the general solution of the differential equation

$$y'' + 2y' + 2y = 0$$

$$\text{Let } y = e^{rt} \text{ then } r^2 + 2r + 2 = 0$$

$$(r + 1)^2 + 1 = 0 \Rightarrow r^2 + 2r + 2 = 0$$

$$r^2 + 2r + 1 + 1 = 0$$

$$(r + 1)^2 = -1$$

$$r + 1 = \pm i$$

$$r_1 = -1 + i \quad \text{and} \quad r_2 = -1 - i$$

$$y_1 = e^{(-1+i)t} = e^{-t} e^{it}$$

$$y_2 = e^{(-1-i)t} = e^{-t} e^{-it}$$

$$(11A) \quad \frac{\frac{1}{2} e^{it} + \frac{1}{2} e^{-it}}{\frac{1}{2} e^{it} + \frac{1}{2} e^{-it}} = \frac{e^{it} + e^{-it}}{e^{it} + e^{-it}} = 1$$

$$\frac{(e^{it} + e^{-it}) - (e^{it} - e^{-it})}{(e^{it} + e^{-it}) + (e^{it} - e^{-it})} = \frac{2e^{-it}}{2e^{it}} = e^{-2it}$$

$$\frac{(e^{it}) - (e^{-it})}{(e^{it}) + (e^{-it})} = \frac{e^{it} - e^{-it}}{e^{it} + e^{-it}}$$

$$F_{BZ} = T_{IB} \left(\frac{0.635}{0.424} \right) = 1.50 T_{IB}$$

Glacier

$$F_{BZ} = 537,600 \text{ lb}$$

Wanted: F_{BZ} for Stalin Class U.S.S.R.

Given (3): $i_B = 25.0^\circ$

$i_E = \text{Half Entrance angle in } \omega L \text{ plane} = 21^\circ$

$$f_k = 0.05$$

Solution: $\tan \beta = \frac{\sin i_B}{\tan i_E}$

$$\tan i_E = \tan 21^\circ = 0.384$$

$$\sin i_B = \sin 25^\circ = 0.423$$

$$\tan \beta = \frac{0.423}{0.384} = 1.100$$

$$\beta = 47.8^\circ \quad \cos \beta = 0.672$$

$$\cos i_B = \cos 25^\circ = 0.906$$

$$F_{BZ} = \frac{T_{IB} [(0.906)(0.672) - (0.05)(0.423)]}{[(0.423)(0.672) + (0.05)(0.906)]}$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2} \quad \sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

where

$$\cos 2\theta = 2\cos^2 \theta - 1$$

Substituting in the above equation

$$\sin^2 \theta = \frac{1 - (2\cos^2 \theta - 1)}{2} \quad \text{Given (1)}$$

$\sin^2 \theta = \frac{1 - 2\cos^2 \theta + 1}{2} \Rightarrow \sin^2 \theta = \frac{2 - 2\cos^2 \theta}{2}$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$\frac{1 - \cos 2\theta}{2} = 1 - \cos^2 \theta \quad \text{Substituting}$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2} = 1 - \cos^2 \theta$$

$$\sin^2 \theta = 1 - \cos^2 \theta \quad \sin^2 \theta = 1 - \cos^2 \theta$$

$$\sin^2 \theta = 1 - \cos^2 \theta = \frac{1 - \cos 2\theta}{2}$$

$$\frac{[\cos^2 \theta (1 - \cos 2\theta) - (1 - \cos^2 \theta) \cos 2\theta]}{[\cos^2 \theta (1 - \cos 2\theta) + (1 - \cos^2 \theta) \cos 2\theta]} = \sin^2 \theta$$

$$F_{BZ} = T_{IB} \left[\frac{(0.608) - (0.021)}{(0.284) + (0.045)} \right]$$

$$F_{BZ} = T_{IB} \left(\frac{0.587}{0.329} \right) = \boxed{1.78 T_{IB}}$$

Stalin

1000

$$\left(\frac{1000 \cdot 0}{1000 \cdot 1} - \frac{1000 \cdot 0}{1000 \cdot 1} \right) 1000 = 1000$$

$$\boxed{1000 \cdot 1} = \left(\frac{1000 \cdot 0}{1000 \cdot 1} \right) 1000 = 1000$$

1000

Sample Calculation using Kari's Equation

Wanted: F_{BZ} for U.S.S. Glacier

Given (14): $\Delta = 8640$ tons

$\theta =$ Variable

$L = 290$ ft.

$H = 28$ ft.

$C = 0.07$

Solution: $F_{BZ} = \frac{4480 \Delta C L \sin \theta}{H}$ (A26)

$$F_{BZ} = \frac{(4480)(8640)(0.07)(290)}{(28)}$$

$F_{BZ} = 28,100,000 \sin \theta$

Glacier

θ	$\sin \theta$	F_{BZ}
0	0	0
1°	0.01745	491,000 lb.
2°	0.03490	981,000
3°	0.05234	1,470,000
4°	0.06976	1,960,000
5°	0.08716	2,450,000

QUESTION 1: (10 marks)

Consider the following system of linear equations:

$$\begin{cases} x + 2y + 3z = 1 \\ 2x + 3y + 4z = 2 \\ 3x + 4y + 5z = 3 \end{cases}$$

(a) Write down the augmented matrix for the system above.

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 2 & 3 & 4 & 2 \\ 3 & 4 & 5 & 3 \end{array} \right]$$

$$R_2 \leftarrow R_2 - 2R_1, R_3 \leftarrow R_3 - 3R_1$$

Therefore

R_1	R_2	R_3
$x + 2y + 3z = 1$	$y + z = 0$	$0 = 0$
$x + 2y + 3z = 1$	$y + z = 0$	$0 = 0$
$x + 2y + 3z = 1$	$y + z = 0$	$0 = 0$
$x + 2y + 3z = 1$	$y + z = 0$	$0 = 0$
$x + 2y + 3z = 1$	$y + z = 0$	$0 = 0$

Wanted: F_{BZ} for Stalin Class U.S.S.R.

Given (3): $\Delta = 11,000$ tons

$\theta =$ Variable

$L = 335$ ft.

$H = 29.5$ ft.

$C = 0.07$ ft.

Solution:
$$F_{BZ} = \frac{4480 \Delta C L \sin \theta}{H} \quad (A26)$$

$$F_{BZ} = \frac{(4480)(11,000)(0.07)(335)}{(29.5)} \sin \theta$$

$F_{BZ} = 39,200,000 \sin \theta$

Stalin

θ	<u>$\sin \theta$</u>	<u>F_{BZ}</u>
0	0	0
1°	0.01745	684,000 lb
2°	0.03490	1,365,000
3°	0.05234	2,045,000
4°	0.06976	2,730,000
5°	0.08716	3,415,000

$1000000 \times 0.05 = 50000$
 $1000000 \times 0.05 = 50000$
 $1000000 \times 0.05 = 50000$
 $1000000 \times 0.05 = 50000$
 $1000000 \times 0.05 = 50000$

1000000
 (c) 1000000

(1000)

$$\frac{1000000 \times 0.05}{1000000} = 0.05$$

1000000

$$1000000 \times \frac{(0.05)(1000000)(0.05)(1000000)}{(1000000)} = 1000000$$

$$1000000 \times 0.05 = 50000$$

1000000

$\frac{1000000}{1000000}$	$\frac{1000000}{1000000}$	0
0	0	0
0.05	0.05	0.05
0.05	0.05	0.05
0.05	0.05	0.05
0.05	0.05	0.05
0.05	0.05	0.05

Sample Calculation using Simonson's Equation

Wanted: F_{BZ} for U.S.S. Glacier

Given (14): $T_{IB} = 160$ tons

$$T_{IB} = 358,400 \text{ lb}$$

$$i_B = 30.0^\circ$$

Solution:
$$F_{BZ} = \frac{T_{IB}}{\tan(i_B + \theta)} \quad (A43)$$

$$F_{BZ} = \frac{T_{IB}}{\tan(30^\circ + \theta)}$$

Glacier

$$\theta = 0 \quad F_{BZ} = \frac{T_{IB}}{0.577} = 1.73 T_{IB}$$

$$\theta = 2^\circ \quad F_{BZ} = \frac{T_{IB}}{0.625} = 1.60 T_{IB}$$

Try $\theta = 0$ $T_{IB} = 358,400$ lb

$$F_{BZ} = (1.73)(358,400) = 620,000 \text{ lb}$$

A check against Figure II, Kari's Equation, shows that the trim would be about 1° .

Try $\theta = 1^\circ$ $T_{IB} = 358,400$ lb

$$F_{BZ} = (1.66)(358,400) = 595,000 \text{ lb}$$

PROBLEM 1: (10 points)

Consider the function $f(x, y, z) = x^2 + y^2 + z^2$.

Find the gradient of f at the point $(1, 1, 1)$.

$\nabla f(1, 1, 1) = \nabla f$

$\nabla f = 2x\mathbf{i} + 2y\mathbf{j} + 2z\mathbf{k}$

$\nabla f(1, 1, 1) = 2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$

$$\nabla f(1, 1, 1) = 2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$$

Answer:

$\nabla f(1, 1, 1) = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k} = 2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$

$\nabla f(1, 1, 1) = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k} = 2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$

$\nabla f(1, 1, 1) = 2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$

$\nabla f(1, 1, 1) = (2, 2, 2)$

A space curve is given by the vector equation $\mathbf{r}(t) = (t^2)\mathbf{i} + (t^3)\mathbf{j} + (t^4)\mathbf{k}$. Show that the curve is concave up at the point $(1, 1, 1)$.

Answer:

$\mathbf{r}(t) = t^2\mathbf{i} + t^3\mathbf{j} + t^4\mathbf{k}$

$$\mathbf{r}(1) = \mathbf{i} + \mathbf{j} + \mathbf{k}$$

This is in approximate agreement with the trim indicated by Kari.

Wanted: F_{BZ} for Stalin Class U.S.S.R.

Given (3): $i_B = 25.0^\circ$

$$F_{BZ} = \frac{T_{IB}}{\tan (25^\circ + \theta)}$$

$$\theta = 0^\circ \quad F_{BZ} = 2.14 T_{IB}$$

$$\theta = 2^\circ \quad F_{BZ} = 1.96 T_{IB}$$

Let $f(x) = x^2 + 2x + 1$ and $g(x) = x^2 - 2x + 1$

$f(x) + g(x) = (x^2 + 2x + 1) + (x^2 - 2x + 1) = 2x^2 + 2$ (1)

$f(x) - g(x) = (x^2 + 2x + 1) - (x^2 - 2x + 1) = 4x$ (2)

$$\frac{f(x) + g(x)}{f(x) - g(x)} = \frac{2x^2 + 2}{4x} = \frac{x^2 + 1}{2x}$$

$\frac{f(x) + g(x)}{f(x) - g(x)} = \frac{x^2 + 1}{2x}$ (3)

$\frac{f(x) - g(x)}{f(x) + g(x)} = \frac{4x}{2x^2 + 2} = \frac{2x}{x^2 + 1}$ (4)

Let $f(x) = x^2 + 2x + 1$ and $g(x) = x^2 - 2x + 1$

$f(x) + g(x) = 2x^2 + 2$

$\frac{f(x) + g(x)}{f(x) - g(x)} = \frac{2x^2 + 2}{4x} = \frac{x^2 + 1}{2x}$

Sample Calculation based on This Research

Assume the following parameters are known or can be suitably approximated:

Ship

- L = Length between perpendiculars, ft.
- B = Waterline beam, ft.
- H = Normal draft, ft.
- Δ = Normal displacement, lb.
- i_B = Angle from base line to stem, radians
- β = Angle of normal to bow plating with respect to centerline plane, radians
- v_1 = Velocity of ship immediately prior to initial contact, ft/sec.
- α = Waterplane coefficient, dimensionless
- LCF = Distance from amidships to center of flotation (+ if forward, - if aft), ft.
- LCG = Distance from amidships to center of gravity (+ if forward, - if aft), ft.
- KG = Height of center of gravity above base line, ft.
- d = Height of propeller hub above base line, ft.
- T_{BOL} = Bollard thrust at same rpm as that needed to maintain v_1 at approach, lb.
- GM_L = Longitudinal Metacentric Height, ft.

Diagram illustrating the forces on a projectile

Define the following quantities and show or state their units:

Symbol

Unit

1 = Length between two points, l

2 = Velocity, v

3 = Acceleration, a

4 = Time interval, t

5 = Angle of launch with respect to horizontal

6 = Angle of arrival with respect to horizontal

Plane, region

7 = Velocity of this projectile when it reaches the ground

8 = Resistance coefficient, dimensionless

9 = Distance from launch to center of gravity (+ if forward)

$- l \sin \theta$, l

10 = Distance from landing to center of gravity (+ if forward)

$- l \cos \theta$, l

11 = Height of center of gravity above base line, h

12 = Height of projectile above base line, H

13 = Roll angle at any time t needed to maintain v

at approach, α

14 = Longitudinal resistance height, h_r

Ice

f_k = Coefficient of kinetic friction between ice and ship,
dimensionless.

f_s = Coefficient of static friction between ice and ship,
dimensionless.

σ = Compressive failure stress of ice (during the local
crushing) lb/ft²

Values will be selected (or assumed) based on the C.G.C. Westwind.

$$L = 250.0 \text{ ft.}$$

$$B = 62.0 \text{ ft.}$$

$$H = 25.75 \text{ ft.}$$

$$\Delta = (2240)(5300) = 11.88 \times 10^6 \text{ lb.}$$

$$i_B = 30^\circ = \frac{30}{57.3} = 0.523 \text{ radians}$$

$$\beta = 50.8^\circ = \frac{50.8}{57.3} = 0.886 \text{ radians}$$

$$v_1 = 6.0 \text{ ft/sec}$$

$$\alpha = 0.724$$

$$LCF = -4.3 \text{ ft (assumed)}$$

$$LCG = -3.3 \text{ ft (assumed)}$$

$$KG = 22.75 \text{ ft (assumed)}$$

$$d = 6.75 \text{ ft (assumed)}$$

The first part of the problem is to find the
 value of $\sin^{-1}(\frac{1}{2})$. We know that $\sin(\frac{\pi}{6}) = \frac{1}{2}$
 and $\sin(\frac{5\pi}{6}) = \frac{1}{2}$. Since the range of the
 sine function is $[-\frac{\pi}{2}, \frac{\pi}{2}]$, the only angle
 in this range for which $\sin(x) = \frac{1}{2}$ is $x = \frac{\pi}{6}$.
 Therefore, $\sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$.

The second part of the problem is to find the
 value of $\cos^{-1}(\frac{1}{2})$. We know that $\cos(\frac{\pi}{3}) = \frac{1}{2}$
 and $\cos(\frac{2\pi}{3}) = -\frac{1}{2}$. Since the range of the
 cosine function is $[0, \pi]$, the only angle
 in this range for which $\cos(x) = \frac{1}{2}$ is $x = \frac{\pi}{3}$.
 Therefore, $\cos^{-1}(\frac{1}{2}) = \frac{\pi}{3}$.

$$\sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$$

$$\cos^{-1}(\frac{1}{2}) = \frac{\pi}{3}$$

$$\sin^{-1}(\frac{\sqrt{3}}{2}) = \frac{\pi}{3}$$

$$\cos^{-1}(\frac{\sqrt{3}}{2}) = \frac{\pi}{6}$$

$$\sin^{-1}(\frac{\sqrt{3}}{2}) = \frac{\pi}{3}$$

$$\cos^{-1}(\frac{\sqrt{3}}{2}) = \frac{\pi}{6}$$

- (a) $\sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}$
- (b) $\cos^{-1}(\frac{1}{2}) = \frac{\pi}{3}$
- (c) $\sin^{-1}(\frac{\sqrt{3}}{2}) = \frac{\pi}{3}$
- (d) $\cos^{-1}(\frac{\sqrt{3}}{2}) = \frac{\pi}{6}$

$$T_{\text{BOL}} = 50.0 \times 10^3 \text{ lbs} \quad (38)$$

(Note: 49.6 rpm at 0 towrope pull gives 6 knots 10 ft/sec

49.6 rpm at 0 ft/sec gives T_{BOL})

$$GM_L = 240 \text{ ft}$$

$$f_k = 0.20 \text{ (assumed)}$$

$$f_s = 0.80$$

$$\sigma = 347 \text{ psi} = 50,000 \text{ lb/ft}^2$$

See (21)

$$2000 - 1000 = 1000 = 10^3$$

1000 is the number of times the number 1000 is repeated in the number 2000

$$1000 = 10^3$$

$$1000 = 10^3$$

$$(1000)^2 = 10^6 = 10^3 \times 10^3$$

$$1000 = 10^3$$

$$1000 \times 1000 = 10^3 \times 10^3 = 10^6$$

$$(10^3)^2$$

BP = 250.000 B = 62.000 H = 25.750 DIS = 11880000.000
 BA = .523 SA = .886 VI = 6.000 AL = .724
 CF = -4.300 CG = -3.300 GK = 22.750 D = 6.750
 TB = 50000.000 CM = 240.000 FK = .200 FS = .800
 SIG = 50000.000

XM = .3992E 06	ZM = .6867E 06	(B66)(B67)
RG = .5500E 02	THM = .1797E 10	(B71)(B68)
DP = .8265E 09	DH = .3975E 06	(B69)(B70)
TF = .7205E 06	PL = .2109E 05	(B72)(B33)
P2 = .2167E 05		(B39)
A1 = .1797E 10	E1 = .8265E 09	(B56)(B56)
C1 = .2851E 10	P3 = .2844E 07	(B56)(B51)
DL = .1024E 09		(B56)
AL1 = .2300E 00	BE1 = .1239E 01	(B57)(B57)
AA1 = .3922E-01	AA2 = .2409E -01	(B57)(B57)
A2 = .6867E 06	B2 = .3975E 06	(B40)(B40)
CL = .7205E 06	DL = .7802E 06	(B40)(B40)
AL2 = .2894E 00	BE1 = -.1405E 01	(B61)(B61)
BEL = .1405E 01	BB2 = .1630E 01	(B61)(B61)

T = -0.			
TH = -.00000	THD = -.00000	THDD = .00000	(B58)(B55)(B59)
Z = .00000	ZD = .00000	ZDD = -.00000	(B62)(B63)(B64)
X = 0.	XD = 6.00000	XDD = -0.	(B41)(B40)(B39)
FXC = .00000E 00	FZC = .00000E 00		(B47)(B48)
TAGA = -.00000	GAX = 128.30000	GAZ = 3.00000	(B65)(B20)
	DIF = .57655		

T = .05000			
TH = .00000	THD = .00000	THDD = .00014	
Z = -.00000	ZD = -.00005	ZDD = -.00281	
X = .30000	XD = 5.99992	XDD = -.00475	
FXC = .18978E 04	FZC = .19504E 04		
TAGA = .00006	GAX = 128.29999	GAZ = 3.00000	DIF = .57649

.10000		
.00000	.00002	.00056
-.00001	-.00037	-.01114
.59997	5.99937	-.01902
.75906E 04	.78010E 04	

* Numbers in parentheses indicate equation numbers of Appendix B.

.00046	128.29988	2.99999	.57609
.15000			
.00000	.00006	.00125	
-.00005	-.00125	-.02479	
.89991	5.99786	-.04278	
.17077E 05	.17550E 05		
.00155	128.29938	2.99995	.57500
.20000			
.00001	.00015	.00220	
-.00015	-.00294	-.04359	
1.19979	5.99493	-.07604	
.30354E 05	.31196E 05		
.00366	128.29806	2.99985	.57290
.25000			
.00002	.00029	.00340	
-.00036	-.00569	-.06734	
1.49959	5.99009	-.11880	
.47419E 05	.48734E 05		
.00710	128.29530	2.99964	.56948
.30000			
.00004	.00049	.00484	
-.00074	-.00975	-.09584	
1.79929	5.98288	-.17102	
.68267E 05	.70159E 05		
.01218	128.29032	2.99926	.56442
.35000			
.00007	.00078	.00651	
-.00136	-.01535	-.12887	
2.09887	5.97280	-.23272	
.92893E 05	.95467E 05		
.01922	128.28217	2.99864	.55742
.40000			
.00012	.00115	.00840	
-.00230	-.02271	-.16623	
2.39831	5.95938	-.30386	
.12129E 06	.12465E 06		
.02852	128.26979	2.99770	.54819

DATE	DESCRIPTION	AMOUNT	BALANCE
1900
1901
1902
1903
1904
1905
1906
1907
1908
1909
1910
1911
1912
1913
1914
1915
1916
1917
1918
1919
1920

.45000			
.00018	.00162	.01050	
-.00366	-.03204	-.20768	
2.69759	5.94211	-.38442	
.15345E 06	.15770E 06		
.04037	128.25193	2.99634	.53643
.50000			
.00028	.00220	.01279	
-.00554	-.04354	-.25302	
2.99670	5.92049	-.47440	
.18936E 06	.19461E 06		
.05507	128.22725	2.99446	.52186
.55000			
.00041	.00290	.01525	
-.00806	-.05740	-.30201	
3.29561	5.89402	-.57376	
.22903E 06	.23537E 06		
.07293	128.19424	2.99194	.50416
.60000			
.00057	.00373	.01789	
-.01133	-.07380	-.35442	
3.59429	5.86214	-.68247	
.27242E 06	.27997E 06		
.09428	128.15132	2.98867	.48304
.65000			
.00078	.00469	.02067	
-.01548	-.09290	-.41001	
3.89275	5.82431	-.80052	
.31954E 06	.32839E 06		
.11943	128.09677	2.98452	.45817
.70000			
.00104	.00580	.02359	
-.02066	-.11485	-.46856	
4.190904	5.77993	-.92786	
.37037E 06	.38063E 06		
.14874	128.02878	2.97934	.42920

2700	2700 2700 2700	2700 2700 2700	2700 2700 2700
2800	2800 2800 2800	2800 2800 2800	2800 2800 2800
2900	2900 2900 2900	2900 2900 2900	2900 2900 2900
3000	3000 3000 3000	3000 3000 3000	3000 3000 3000
3100	3100 3100 3100	3100 3100 3100	3100 3100 3100
3200	3200 3200 3200	3200 3200 3200	3200 3200 3200
3300	3300 3300 3300	3300 3300 3300	3300 3300 3300
3400	3400 3400 3400	3400 3400 3400	3400 3400 3400
3500	3500 3500 3500	3500 3500 3500	3500 3500 3500
3600	3600 3600 3600	3600 3600 3600	3600 3600 3600
3700	3700 3700 3700	3700 3700 3700	3700 3700 3700
3800	3800 3800 3800	3800 3800 3800	3800 3800 3800
3900	3900 3900 3900	3900 3900 3900	3900 3900 3900
4000	4000 4000 4000	4000 4000 4000	4000 4000 4000

.75000			
.00136	.00705	.02664	
-.02702	-.13980	-.52983	
4.48886	5.72840	-1.06446	
.42490E 06	.43667E 06		
.18261	127.94544	2.97298	.39576
.80000			
.00175	.00846	.02979	
-.03470	-.16788	-.59357	
4.78648	5.66904	-1.21029	
.48311E 06	.49650E 06		
.22146	127.84478	2.96530	.35743
.85000			
.00221	.01003	.03304	
-.04386	-.19919	-.65956	
5.08378	5.60114	-1.36531	
.54499E 06	.56009E 06		
.26578	127.72475	2.95614	.31373
.90000			
.00276	.01177	.03636	
-.05467	-.23387	-.72756	
5.38074	5.52391	-1.52947	
.61052E 06	.62744E 06		
.31613	127.58323	2.94533	.26410
.95000			
.00339	.01367	.03975	
-.06730	-.27198	-.79734	
5.67735	5.43648	-1.79274	
.67968E 06	.69852E 06		
.37319	127.41804	2.93270	.20789
1.00000			
.00413	.01574	.04318	
-.08193	-.31363	-.86867	
5.97359	5.33788	-1.88507	
.75246E 06	.77331E 06		
.43777	127.22699	2.91807	.14429

1.05000			
.00497	.01799	.04665	
-.09873	-.35887	-.94131	
6.26942	5.22699	-2.07641	
.82884E 06	.85180E 06		
.51088	127.00782	2.90127	.07231

T= 1.10000			
TH= .00593	THD= .02041	THDD= .05014	
Z= -.11788	ZD= -.40777	ZDD= -1.01505	
X= 6.56484	XD= 5.10254	XDD= -2.27670	
FXC= .90879E 06	FXC= .93397E 06		
TAGA= .59378	GAX= 126.75827	GAZ= 2.88212	DIF= -.00930

T2= 1.09430			
TH2= .00582	THD2= .02013	THDDD2= .04974	
Z2= -.11569	ZD2= -.40220	ZDD2= -1.00665	
X2= 6.53119	XD2= 5.11671	XDD2= -2.25389	
FXC2= .89968E 06	FZC2= .92461E 06		
TAGA2= .58433	GAX2= 126.78669	GAZ2= 2.88431	DIF2= .00000

AS= .489195E 00	BS= .448067E 00	(B80)(B81)
P4= .133318E 03	P5= .219200E 01	(B87)(B99)

All= -.399168E 06	B11= -.838691E 04	}	(B95)
C11= .000000E 00	A12= .758452E 06		
B12= .439022E 06	C12= .795773E 06		
A13= .000000E 00	B13= .000000E 00		
C13= .280060E 06	D1= -.533230E 05		

A21= .000000E 00	B21= -.127029E 06	}	(B100)
C21= -.238980E 06	A22= -.892475E 08		
A23= -.179685E 10	B23= -.826493E 09		
C23= -.292629E 10	D2 = -.223974E 07		

A31= .000000E 00	B31= .000000E 00	}	(B103)
C31= .100000E 01	A32= .000000E 00		
B32= .000000E 00	C32= .171112E 01		
A33= .000000E 00	B33= .000000E 00		
C33= -.214063E 03	D3 = .508733E 01		

DL1= - .2453E 07
 DL2= - .2695E 07
 DL3= - .5332E 05
 D21= .5504E 05
 D22= - .1327E 06
 D23= - .2240E 07
 D33= .5087E 01

(E104)

DD4= - .1022E 17
 DD3= - .6601E 16
 DD2= - .1421E 17
 DD1= - .2184E 16
 DDO= - .2343E 16

(E109)

U13= - .6672E 17
 U12= - .9522E 17
 U11= - .1204E 18
 U10= - .7252E 17
 U09= - .1344E 17

(E108)

U23= .1182E 16
 U22= .4958E 16
 U21= .4045E 16
 U20= .7220E 16
 U19= .1591E 15

(E152)

U33= - .5946E 14
 U32= - .2483E 15
 U31= - .1924E 15
 U30= - .2292E 15
 U29= - .5826E 13

(E166)

O = 1.0000E -03

TOT= -.4964E-01 (E123)

C= .2000E-02

TOT= -.4869E-01 (E123)

C= .3000E-02

TOT= -.4775E-01 (E123)

C= .4000E-02

TOT= -.4681E-01 (E123)

.5000E-02

-.4588E-01

.6000E-02

-.4496E-01

.7000E-02

-.4404E-01

(1000)	10	1000	1000
(1000)	11	1000	1000
(1000)	12	1000	1000
(1000)	13	1000	1000
(1000)	14	1000	1000
(1000)	15	1000	1000
(1000)	16	1000	1000
(1000)	17	1000	1000
(1000)	18	1000	1000
(1000)	19	1000	1000
(1000)	20	1000	1000
(1000)	21	1000	1000
(1000)	22	1000	1000
(1000)	23	1000	1000
(1000)	24	1000	1000
(1000)	25	1000	1000
(1000)	26	1000	1000
(1000)	27	1000	1000
(1000)	28	1000	1000
(1000)	29	1000	1000
(1000)	30	1000	1000
(1000)	31	1000	1000
(1000)	32	1000	1000
(1000)	33	1000	1000
(1000)	34	1000	1000
(1000)	35	1000	1000
(1000)	36	1000	1000
(1000)	37	1000	1000
(1000)	38	1000	1000
(1000)	39	1000	1000
(1000)	40	1000	1000
(1000)	41	1000	1000
(1000)	42	1000	1000
(1000)	43	1000	1000
(1000)	44	1000	1000
(1000)	45	1000	1000
(1000)	46	1000	1000
(1000)	47	1000	1000
(1000)	48	1000	1000
(1000)	49	1000	1000
(1000)	50	1000	1000
(1000)	51	1000	1000
(1000)	52	1000	1000
(1000)	53	1000	1000
(1000)	54	1000	1000
(1000)	55	1000	1000
(1000)	56	1000	1000
(1000)	57	1000	1000
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(1000)	59	1000	1000
(1000)	60	1000	1000
(1000)	61	1000	1000
(1000)	62	1000	1000
(1000)	63	1000	1000
(1000)	64	1000	1000
(1000)	65	1000	1000
(1000)	66	1000	1000
(1000)	67	1000	1000
(1000)	68	1000	1000
(1000)	69	1000	1000
(1000)	70	1000	1000
(1000)	71	1000	1000
(1000)	72	1000	1000
(1000)	73	1000	1000
(1000)	74	1000	1000
(1000)	75	1000	1000
(1000)	76	1000	1000
(1000)	77	1000	1000
(1000)	78	1000	1000
(1000)	79	1000	1000
(1000)	80	1000	1000
(1000)	81	1000	1000
(1000)	82	1000	1000
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(1000)	89	1000	1000
(1000)	90	1000	1000
(1000)	91	1000	1000
(1000)	92	1000	1000
(1000)	93	1000	1000
(1000)	94	1000	1000
(1000)	95	1000	1000
(1000)	96	1000	1000
(1000)	97	1000	1000
(1000)	98	1000	1000
(1000)	99	1000	1000
(1000)	100	1000	1000

.6300E-01	
-.3312E-02	
.6400E-01	
-.2750E-02	
.6500E-01	
-.2192E-02	
C= .6600E-01	
TOT= -.1639E-02	(R123)
C= .6700E-01	
TOT= -.1092E-02	(R123)
C= .6800E-01	
TOT= -.5491E-03	(R123)
C= .6900E-01	
TOT= -.1134E-04	(R123)
C= .7000E-01	
TOT= .5215E-03	(R123)
C= .6902E-01	
AL3= .4460E-01	(R124)
BE3= .4464E 00	(R125)
AL4= .2785E 00	(R126)
BE4= .1031E 01	(R127)
G3= .2013E 00	(R134)
G4= .1140E 01	(R134)
A4X= .6531E 01	
A3X= .9321E 01	(R111)
A2X= .1178E 02	
ALX= .7098E 01	(R11)
AOX= .1315E 01	
CLX= .5736E 01	(R136)
DLX= .7955E 00	(R137)
D2X= .5615E 01	(R138)
D3X= .3808E 01	(R139)
D4X= .5872E 01	(R140)
C6X= .3877E-02	(R141)
C5X= .5152E 01	(R142)
C4X= -.3342E-01	(R143)
C3X= .8289E 00	(R144)
P23X= .3700E 00	
P13X= .5115E 01	
P24X= -.3444E-01	
P14X= .1318E-01	(R148)

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	1	

A4Z=	- .1157E 00	
A3Z=	- .4853E 00	
A2Z=	- .3959E 00	
A1Z=	- .7068E 00	
A0Z=	- .1557E-01	(154)
C1Z=	- .6788E-01	(R156)
D1Z=	- .4781E-01	
D2Z=	- .4414E 00	
D3Z=	- .3-15E 00	
D4Z=	- .6922E 00	(R157)
C6Z=	.1777E 00	(R158)
C5Z=	- .6388E 00	(R159)
C4Z=	- .9902E-01	(R160)
C3Z=	.5121E-01	(R161)
P23Z=	.2286E-01	
P13Z=	-.6411E 00	(R162)
P24Z=	- .1020E 00	
P14Z=	.2053E 00	
A4T=	.5820E-02	
A3T=	.2431E-01	
A2T=	.1884E-01	(R168)
A1T=	.2243E-01	
A0T=	.5703E-03	
C1T=	.2486E-02	(R170)
D1T=	.3334E-02	
D2T=	.2270E-01	
D3T=	.1538E-01	(R171)
D4T=	.2190E-01	
C6T=	.1439E-02	(R172)
C5T=	.1896E-01	(R173)
C4T=	- .9476E-03	(R174)
C3T=	.4281E-02	(R175)
P23T=	.1911E-02	
P13T=	.1877E-01	
P24T=	-.9766E-03	(R176)
P14T=	.1703E-02	
Total Time=	1.09430	T= -0.

X=	6.53119	XD=	5.10089	XDD=	-.59423	(B149)	(B150)	(B151)
Z=	-.11569	ZD=	-.41053	ZDD=	.03023	(B163)	(B164)	(B165)
TH=	.00582	THD=	.02055	THDD=	-.00253	(B177)	(B178)	(B179)
FBZS=	.23022E 06	WRAT=	.003230	VAX=	5.04163	(B92)	(B214)	(B183)

Test 1= - .3564E-01 (B95)
Test 2= - .3187E-01 (B100)
Test 3= - .2980E-06 (B103)

These values are of the order of 1/100 of 1 o/o error in the solution of the simultaneous differential equations for sliding.

Total Time=	1.19430	T=	.10000		
X=	7.03814	XD=	5.03647	XDD=	-.69383
Z=	-.15662	ZD=	-.40814	ZDD=	.01790
TH=	.00786	THD=	.02027	THDD=	-.00310
FBZS=	.26219E 06	WRAT=	.003678	VAX=	4.97885

-.6396E-01
-.3438E 01
-.3576E-06

Total Time=	1.29430	T=	.20000		
X=	7.53816	XD=	4.96220	XDD=	-.79118
Z=	-.19736	ZD=	-.40690	ZDD=	.00713
TH=	.00987	THD=	.01993	THDD=	-.00364
FBZS=	.29387E 06	WRAT=	.004123	VAX=	4.90635

-.8838E-01
-.3687E 01
-.3576E-06

Total Time=	1.39430	T=	.30000		
X=	8.03026	XD=	4.87832	XDD=	-.88605
Z=	-.23803	ZD=	-.40665	ZDD=	-.00193
TH=	.01185	THD=	.01954	THDD=	-.00415
FBZS=	.32525E 06	WRAT=	.004563	VAX=	4.82435

-.1299E 00
-.2937E 01
-.4172E-06

Total Time=	1.49430	T=	.40000		
X=	8.51351	XD=	4.78508	XDD=	-.97824
Z=	-.27872	ZD=	-.40722	ZDD=	-.00917
TH=	.01378	THD=	.01910	THDD=	-.00464
FBZS=	.35626E 06	WRAT=	.004998	VAX=	4.73311

(100)	(100)	(100)	(100)	(100)	(100)	(100)	(100)	(100)
(100)	(100)	(100)	(100)	(100)	(100)	(100)	(100)	(100)
(100)	(100)	(100)	(100)	(100)	(100)	(100)	(100)	(100)
(100)	(100)	(100)	(100)	(100)	(100)	(100)	(100)	(100)

It is noted that the above figures include the amount of the 100% interest in the company.

(100)
(100)
(100)

(100)	(100)	(100)	(100)
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(100)	(100)	(100)	(100)
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(100)	(100)	(100)	(100)
(100)	(100)	(100)	(100)

-.1597E 00
-.4688E 01
-.5960E- 06

Total Time =	1.59430	T =	.50000		
X=	8.98698	XD=	4.68276	XDD=	-1.06756
X=	-.31950	ZD=	-.40842	ZDD=	-.01453
TH=	.01566	THD=	.01861	THDD=	-.00510
FBZS=	.38686E 06	WRAT=	.005427	VAX=	4.63288

-.2139E 00
-.3187E 01
-.4768E- 06

Total Time=	1.69430	T=	.60000		
X=	9.44977	XD=	4.57167	XDD=	-1.15384
Z=	-.36042	ZD=	-.41007	ZDD=	-.01795
TH=	.01750	THDD=	.01808	THDD=	-.00553
FBZS=	.41700E 06	WRAT=	.005850	VAX=	4.52395

-.2529E 00
-.2687E 01
-.5960E- 06

Total Time=	1.79430	T=	.70000		
X=	9.90103	XD=	4.45210	XDD=	-1.23688
Z=	-.40152	ZD=	-.41195	ZDD=	-.01943
TH=	.01928	THD=	.01751	THDD=	-.00593
FBZS=	.44663E 06	WRAT=	.006266	VAX=	4.40662

-.2979E 00
-.4688E 01
-.6557E- 06

Total Time=	1.89430	T=	.80000		
X=	10.33992	XD=	4.32441	XDD=	-1.31654
Z=	-.44281	ZD=	-.41389	ZDD=	-.01895
TH=	.02100	THD=	.01689	THDD=	-.00630
FBZS=	.47567E 06	WRAT=	.006673	VAX=	4.28121

-.3604E 00
-.5187E 01
-.5960E- 06

Total Time=	1.99430	T=	.90000		
X=	10.76565	XD=	4.18892	XDD=	-1.39266
Z=	-.48429	ZD=	-.41568	ZDD=	-.01657
TH=	.02266	THD=	.01625	THDD=	-.00664
FBZS=	.50407E 06	WRAT=	.007072	VAX=	4.14805

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-.4160E 00
-.3687E 01
-.5960E- 06

Total Time=	2.09430	T=	1.00000	
X=	11.17745	XD=	4.04600	XDD=-1.46510
Z=	-.52593	ZD=	-.41714	ZDD=-.01232
TH=	.02425	THD=	.01557	THDD=-.00694
FBZS=	.53176E 06	WRAT=	.007460	VAX= 4.00748

-.4692E 00
-.5187E 01
-.4172E- 06

Total Time=	2.19430	T=	1.10000	
X=	11.57461	XD=	3.89602	XDD=-1.53374
Z=	-.56770	ZD=	-.41808	ZDD=-.00630
TH=	.02577	THD=	.01486	THDD=-.00722
FBZS=	.55867E 06	WRAT=	.007838	VAX= 3.85988

-.5410E 00
-.6187E 01
-.7153E- 06

Total Time=	2.29430	T=	1.20000	
X=	11.95644	XD=	3.73938	XDD=-1.59846
Z=	-.60953	ZD=	-.41834	ZDD=.00141
TH=	.02722	THD=	.01412	THDD=-.00746
FBZS=	.58473E 06	WRAT=	.008203	VAX= 3.70562

-.6050E 00
-.6187E 01
-.5960E- 06

Total Time=	2.39430	T=	1.30000	
X=	12.32228	XD=	3.57646	XDD=-1.65916
Z=	-.65134	ZD=	-.41775	ZDD=.01068
TH=	.02859	THD=	.01337	THDD=-.00767
FBZS=	.60986E 06	WRAT=	.008556	VAX= 3.54507

-.6641E 00
-.4187E 01
-.5960E- 06

Total Time=	2.49430	T=	1.40000	
X=	12.67153	XD=	3.40768	XDD=-1.71576
Z=	-.69304	ZD=	-.41616	ZDD=.02138
TH=	.02989	THD=	.01259	THDD=-.00784
FBZS=	.63400E 06	WRAT=	.008894	VAX= 3.37863

-.7231E 00
-.4187E 01
-.5960E- 06
Total Time= 2.59430 T= 1.50000
X= 13.00363 XD= 3.23345 XDD= -1.76816
Z= -.73453 ZD= -.41343 ZDD= .03338
TH= .03111 THD= .01180 THDD= -.00799
FBZS= .65707E 06 WRAT= .009218 VAX= 3.20672

-.7988E 00
-.4187E 01
-.3576E- 06
Total Time= 2.69430 T= 1.60000
X= 13.31806 XD= 3.05419 XDD= -1.81631
Z= -.77569 ZD= -.40944 ZDD= .04651
TH= .03225 THD= .01099 THDD= -.00811
FBZS= .67900E 06 WRAT= .009526 VAX= 3.02974

-.8623E 00
-.5187E 01
-.5384E- 06
Total Time= 2.79430 T= 1.70000
X= 13.61432 XD= 2.87034 XDD= -1.86015
Z= -.81638 ZD= -.40410 ZDD= .06061
TH= .03331 THD= .01018 THDD= -.00821
FBZS= .69972E 06 WRAT= .009816 VAX= 2.84811

-.9185E 00
-.4187E 01
-.4768E- 06
Total Time= 2.89430 T= 1.80000
X= 13.89198 XD= 2.68231 XDD= -1.89963
Z= -.85646 ZD= -.39730 ZDD= .07550
TH= .03429 THD= .00935 THDD= -.00827
FBZS= .71916E 06 WRAT= .010089 VAX= 2.66226

-.9795E 00
-.4187E 01
-.3576E- 06
Total Time= 2.99430 T= 1.90000
X= 14.15066 XD= 2.49056 XDD= -1.93473
Z= -.89578 ZD= -.38897 ZDD= .09102
TH= .03518 THD= .00853 THDD= -.00831
FBZS= .73727E 06 WRAT= .010343 VAX= 2.47262

Year	1950	1951	1952	1953
1950	1000	1000	1000	1000
1951	1000	1000	1000	1000
1952	1000	1000	1000	1000
1953	1000	1000	1000	1000
1954	1000	1000	1000	1000
1955	1000	1000	1000	1000
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2007	1000	1000	1000	1000
2008	1000	1000	1000	1000
2009	1000	1000	1000	1000
2010	1000	1000	1000	1000
2011	1000	1000	1000	1000
2012	1000	1000	1000	1000
2013	1000	1000	1000	1000
2014	1000	1000	1000	1000
2015	1000	1000	1000	1000
2016	1000	1000	1000	1000
2017	1000	1000	1000	1000
2018	1000	1000	1000	1000
2019	1000	1000	1000	1000
2020	1000	1000	1000	1000

-.1033E 01
-.4187E 01
-.3576E- 06
Total Time= 3.09430 T= 2.00000
X= 14.38999 XD= 2.29551 XDD= -1.96543
Z= -.93420 ZD= -.37908 ZDD= .10699
TH= .03599 THD= .00769 THDD= -.00833
FBZS= .75397E 06 WRAT= .010578 VAX= 2.27962

-.1090E 01
-.4187E 01
-.2980E- 06
Total Time= 3.19430 T= 2.10000
X= 14.60966 XD= 2.09762 XDD= -1.99171
Z= -.97155 ZD= -.36757 ZDD= .12322
TH= .03672 THD= .00686 THDD= -.00832
FBZS= .76921E 06 WRAT= .010791 VAX= 2.08370

-.1140E 01
-.6187E 01
-.3576E- 06
Total Time= 3.29430 T= 2.20000
X= 14.80943 XD= 1.89732 XDD= -2.01357
Z= -1.00766 ZD= -.35443 ZDD= .13955
TH= .03736 THD= .00603 THDD= -.00829
FBZS= .78295E 06 WRAT= .010984 VAX= 1.88530

-.1187E 01
-.4187E 01
-.1788E- 06
Total Time= 3.39430 T= 2.30000
X= 14.98906 XD= 1.69505 XDD= -2.03103
Z= -1.04238 ZD= -.33966 ZDD= .15579
TH= .03792 THD= .00520 THDD= -.00824
FBZS= .79512E 06 WRAT= .011155 VAX= 1.68486

-.1246E 01
-.4187E 01
.1192E- 06
Total Time= 3.49430 T= 2.40000
X= 15.14839 XD= 1.49126 XDD= -2.04411
Z= -1.07554 ZD= -.32328 ZDD= .17179
TH= .03840 THD= .00438 THDD= -.00818
FBZS= .80568E 06 WRAT= .011303 VAX= 1.48282

-.1275E 01
-.5187E 01
-.1192E- 06
Total Time= 3.59430 T= 2.50000
X= 15.28728 XD= 1.28637 XDD= -2.05284
Z= -1.10698 ZD= -.30532 ZDD= .18737
TH= .03880 THD= .00357 THDD= -.00809
FBZS= .81461E 06 WRAT= .011428 VAX= 1.27962

-.1324E 01
-.5187E 01
.2384E- 06
Total Time= 3.69430 T= 2.60000
X= 15.40564 XD= 1.08083 XDD= -2.05725
Z= -1.13655 ZD= -.28582 ZDD= .20240
TH= .03912 THD= .00276 THDD= -.00799
FBZS= .82186E 06 WRAT= .011530 VAX= 1.07568

-.1367E 01
-.4187E 01
.4768E- 06
Total Time= 3.79430 T= 2.70000
X= 15.50344 XD= .87506 XDD= -2.05740
Z= 01.16410 ZD= -.26486 ZDD= .21673
TH= .03935 THD= .00197 THDD= -.00788
FBZS= .82740E 06 WRAT= .011608 VAX= .87144

-.1377E 01
-.6187E 01
.5960E- 06
Total Time= 3.189430 T= 2.80000
X= 15.58066 XD= .66949 XDD= -2.05334
Z= -1.18948 ZD= -.24250 ZDD= .23022
TH= .03951 THD= .00119 THDD= -.00775
FBZS= .83123E 06 WRAT= .011661 VAX= .66734

-.1403E 01
-.2188E 01
.8345E- 06
Total Time= 3.99430 T= 2.90000
X= 15.63735 XD= .46453 XDD= -2.04514
Z= -1.21255 ZD= -.21885 ZDD= .24276
TH= .03959 THD= .00042 THDD= -.00761
FBZS= .83332E 06 WRAT= .011691 VAX= .46378

-.1417E 01
-.5187E 01
.8345E- 06
Total Time=
X= 15.67360
Z= -1.23320
TH= .03960
FBZS= .83366E 06

4.09430 T= 3.00000
XD= .26060 XDD= -2.03286
ZD= -.19399 ZDD= .25425
THD= -.00033 THDD= -.00746
WRAT= .011696 VAX= .26119

-.1430E 01
-.6187E 01
.1073E- 05
Total Time=
X= 15.68952
Z= -1.25131
TH= .03953
FBZS= .83226E 06

4.19430 T= 3.10000
XD= .05809 XDD= -2.01659
ZD= -.16804 ZDD= .26458
THD= -.00107 THDD= -.00731
WRAT= .011676 VAX= .05997

-.1462E 01
-.4187E 01
.1431E- 05
Total Time=
X= 15.68528
Z= -1.26678
TH= .03938
FBZS= .82911E 06

4.29430 T= 3.20000
XD= -.14259 XDD= -1.99641
ZD= -.14111 ZDD= .27367
THD= -.00179 THDD= -.00714
WRAT= .011632 VAX= -.13948

-.1454E 01
-.2188E 01
.1550E- 05
Total Time=
X= 15.68597
Z= -1.26607
TH= .03939
FBZS= .82931E 06

4.28930 T= 3.19500
XD= -.13260 XDD= -1.99751
ZD= -.14248 ZDD= .27325
THD= -.00176 THDD= -.00715
WRAT= .011635 VAX= -.12955

-.1442E 01
-.4187E 01
.1550E- 05
Total Time=
X= 15.68661
Z= -1.26535
TH= .03940
FBZS= .82950E 06

4.28430 T= 3.19000
XD= -.12261 XDD= -1.99860
ZD= -.14384 ZDD= .27282
THD= -.00172 THDD= -.00716
WRAT= .011637 VAX= -.11963

-.1451E 01
-.4187E 01
.1550E- 05
Total Time= 4.27930 T= 3.18500
X= 15.68719 XD= -.11262 XDD= -1.99968
Z= -1.26463 ZD= -.14521 ZDD= .27239
TH= .03941 THD= -.00169 THDD= -.00716
FBZS= .82969E 06 WRAT= .011640 VAX= -.10969

-.1446E 01
-.5187E 01
.1550E- 05
Total Time= 4.27430 T= 3.18000
X= 15.68773 XD= -.10262 XDD= -2.00075
Z= -1.26390 ZD= -.14657 ZDD= .27196
TH= .03942 THD= -.00165 THDD= -.00717
FBZS= .82988E 06 WRAT= .011643 VAX= -.09975

-.1453E 01
-.4187E 01
.1431E- 05
Total Time= 4.26930 T= 3.17500
X= 15.68822 XD= -.09261 XDD= -2.00181
Z= -1.26317 ZD= -.14793 ZDD= .27152
TH= .03943 THD= -.00162 THDD= -.00718
FBZS= .83006E 06 WRAT= .011645 VAX= -.08980

-.1432E 01
-.5187E 01
.1669E- 05
Total Time= 4.26430 T= 3.17000
X= 15.68866 XD= -.08260 XDD= -2.00287
Z= -1.26242 ZD= -.14928 ZDD= .27108
TH= .03943 THD= -.00158 THDD= -.00719
FBZS= .83024E 06 WRAT= .011648 VAX= -.07985

-.1445E 01
-.4187E 01
.1550E- 05
Total Time= 4.25930 T= 3.16500
X= 15.68905 XD= -.07258 XDD= -2.00391
Z= -1.26167 ZD= -.15064 ZDD= .27063
TH= .03944 THD= -.00154 THDD= -.00720
FBZS= .83041E 06 WRAT= .011650 VAX= -.06990

-.1447E 01
-.3187E 01
.1669E-05
Total Time= 4.25430 T= 3.16000
X= 15.68938 XD= -.06256 XDD= -2.00494
Z= -1.26092 ZD= -.15199 ZDD= .27019
TH= .03945 THD= -.00151 THDD= -.00721
FBZS= .83058E 06 WRAT= .011652 VAX= -.05994

-.1452E 01
-.4187E 01
.1431E-05
Total Time= 4.24930 T= 3.15500
X= 15.68967 XD= -.05253 XDD= -2.00597
Z= -1.26015 ZD= -.15334 ZDD= .26974
TH= .03946 THD= -.00147 THDD= -.00721
FBZS= .83074E 06 WRAT= .011655 VAX= -.04997

-.1452E 01
-.4187E 01
.1431E-05
Total Time= 4.24430 T= 3.15000
X= 15.68991 XD= -.04250 XDD= -2.00698
Z= -1.25938 ZD= -.15469 ZDD= .26928
TH= .03946 THD= -.00144 THDD= -.00722
FBZS= .83090E 06 WRAT= .011657 VAX= -.04000

-.1425E 01
-.4187E 01
.1431E-05
Total Time= 4.23930 T= 3.14500
X= 15.69010 XD= -.03246 XDD= -2.00799
Z= -1.25861 ZD= -.15603 ZDD= .26883
TH= .03947 THD= -.00140 THDD= -.00723
FBZS= .83106E 06 WRAT= .011659 VAX= -.03003

-.1472E 01
-.2188E 01
.1311E-05
Total Time= 4.23430 T= 3.14000
X= 15.69023 XD= -.02242 XDD= -2.00898
Z= -1.25782 ZD= -.15738 ZDD= .26837
TH= .03948 THD= -.00136 THDD= -.00724
FBZS= .83121E 06 WRAT= .011661 VAX= -.02005

Yield	100%	100%	100%	100%
Loss	0%	0%	0%	0%
Cost	100%	100%	100%	100%
Profit	0%	0%	0%	0%

Yield	100%	100%	100%	100%
Loss	0%	0%	0%	0%
Cost	100%	100%	100%	100%
Profit	0%	0%	0%	0%

Yield	100%	100%	100%	100%
Loss	0%	0%	0%	0%
Cost	100%	100%	100%	100%
Profit	0%	0%	0%	0%

Yield	100%	100%	100%	100%
Loss	0%	0%	0%	0%
Cost	100%	100%	100%	100%
Profit	0%	0%	0%	0%

Yield	100%	100%	100%	100%
Loss	0%	0%	0%	0%
Cost	100%	100%	100%	100%
Profit	0%	0%	0%	0%

-.1461E 01
 -.1187E 01
 .1431E- 05
 Total Time= 4.22930 T= 3.13500
 X= 15.69032 XD= -.01237 XDD= -2.00997
 Z= -1.25703 ZD= -.15872 ZDD= .26790
 TH= .03948 THD= -.00133 THDD= -.00725
 FBZS= .83135E 06 WRAT= .011663 VAX= -.01006

Test 1 = -.1437E 01
 Test 2 = -.6187E 01
 Test 3 = .1311E- 05

State 3 values

Total Time= 4.22930 T3= 3.13500
 X3= 15.69032 XD3= -.01237 XDD3= -2.00997
 Z3= -1.25703 ZD3= -.15872 ZDD3= .26790
 TH3= .03948 THD3= -.00133 THDD3= -.00725
 FBZ3= .83135E 06 WRAT3= .011663 VAX3= -.01006

GAX3= .1176E 03 (B87)
 GAZ3= .11743E 01 (B86)
 Q1= .1186E 03 (B192)
 A4= -.2788E- 05
 B4= .1629E 03 (B194)
 C4= -.1418E 09

State 4 values

X4= 15.69032 Z4= -1.26647 TH4= .03940 (X3)(B186)(B193)
 Vertical Force at Bow = .88404E 06 (B195)
 White Ratio = .012402 (B214)

Extracting Thrust = .31676E 06 (B202)
 Ratio of Extracting Thrust to Bollard Thrust IS 6.335



thesW5553

Dynamically developed force at the bow o



3 2768 001 95073 6

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