Performance analysis of a LINK-16/JTIDS compatible waveform with noncoherent detection, diversity and side information

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PERFORMANCE ANALYSIS OF A LINK-16/JTIDS COMPATIBLE WAVEFORM WITH NONCOHERENT DETECTION, DIVERSITY AND SIDE INFORMATION

by

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September 2009

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The Joint Tactical Information Distribution System (JTIDS) is a hybrid frequency-hopped, direct sequence spread spectrum system. A (31, 15) Reed-Solomon (RS) code is used for forward error correction coding, and cyclical code-shift keying (CCSK) $M$-ary modulation is used, where each encoded symbol represents five bits. In this thesis, a compatible waveform consistent with the existing JTIDS channel waveform is analyzed. The system considered uses the same (31, 15) RS encoding as JTIDS. The coded symbols are then transmitted on the in-phase (I) and quadrature (Q) components of the carrier using 32-ary orthogonal signaling with 32-chip baseband waveforms and detected noncoherently. For noncoherent detection, only one five bit symbol is transmitted on both the I and Q components of the carrier, so the data throughput for noncoherent detection is $\frac{1}{2}$ that of coherent detection. This alternative waveform is consistent with the direct sequence waveform generated by JTIDS. A sequential diversity of two, consistent with the double-pulse structure of JTIDS, is also assumed, and performance is examined both for the case of linear, soft diversity combining and also for soft diversity combining with side information. The type of side information considered is perfect side information (PSI).

Based on the analysis, we conclude that the double-pulse structure outperforms the single-pulse structure for the alternative JTIDS waveform under examination. Moreover, pulsed-noise interference (PNI) degrades the performance of the receiver, but the use of PSI reduces the effect of PNI.
PERFORMANCE ANALYSIS OF A LINK-16/JTIDS COMPATIBLE
WAVEFORM WITH NONCOHERENT DETECTION, DIVERSITY AND SIDE
INFORMATION

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ABSTRACT

The Joint Tactical Information Distribution System (JTIDS) is a hybrid frequency-hopped, direct sequence spread spectrum system. A (31, 15) Reed-Solomon (RS) code is used for forward error correction coding, and cyclical code-shift keying (CCSK) M-ary modulation is used, where each encoded symbol represents five bits. In this thesis, a compatible waveform consistent with the existing JTIDS channel waveform is analyzed. The system considered uses the same (31, 15) RS encoding as JTIDS. The coded symbols are then transmitted on the in-phase (I) and quadrature (Q) components of the carrier using 32-ary orthogonal signaling with 32-chip baseband waveforms and detected noncoherently. For noncoherent detection, only one five bit symbol is transmitted on both the I and Q components of the carrier, so the data throughput for noncoherent detection is ½ that of coherent detection. This alternative waveform is consistent with the direct sequence waveform generated by JTIDS. A sequential diversity of two, consistent with the double-pulse structure of JTIDS, is also assumed, and performance is examined both for the case of linear, soft diversity combining and also for soft diversity combining with side information. The type of side information considered is perfect side information (PSI).

Based on the analysis, we conclude that the double-pulse structure outperforms the single-pulse structure for the alternative JTIDS waveform under examination. Moreover, pulsed-noise interference (PNI) degrades the performance of the receiver, but the use of PSI reduces the effect of PNI.
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EXECUTIVE SUMMARY

Network-centric warfare (NCW) is an emerging theory of war applied in modern military operations in order to improve their effectiveness. To achieve this, NCW applies information age concepts to speed communications and increase situational awareness through networking. As a result, NCW transforms the efficient flow of information into combat power by linking allied forces across the battlefield, thus enabling them to employ more effective decision making during military operations.

One of the most demanding requirements for the developers of NCW is to achieve communications interoperability within a multinational coalition. Interoperability is achieved with digital datalinks, which are the platforms used by modern military forces in order to exchange information with each other. If interoperability is effectively applied, digital datalinks represent a significant tool in implementing NCW theory.

Link-16 is a widely used digital datalink system which uses the Joint Tactical Information Distribution System (JTIDS) as its communications component. JTIDS employs several techniques in order to improve performance. The error detection and correction (EDAC) mechanism used is (31, 15) Reed-Solomon (RS) encoding. The (31, 15) notation implies that 16 redundant symbols are added for every 15 symbols of actual data, thus providing a block of 31 symbols. The signal is modulated at baseband using an $M$-ary modulation technique called cyclic code-shift keying (CCSK) that is compatible with direct sequence spread spectrum. CCSK modulates 5-bit interleaved symbols previously produced by the RS encoding into 32-chip pseudo-random sequences. Finally, these sequences are transmitted using minimum-shift keying (MSK) modulation.

This thesis presents the analysis of a JTIDS/Link-16 compatible waveform, 32-ary orthogonal signaling with (31, 15) RS coding, detected noncoherently. Sequential diversity of two, consistent with the double-pulse structure of JTIDS, is considered. The type of side information considered is perfect side information (PSI), which is not practical but gives an idea of the best performance that can be obtained when pulse-noise interference (PNI) is present.
Initially, this thesis investigates the performance of the JTIDS compatible waveform with a diversity of two detected noncoherently in an additive white Gaussian noise (AWGN) environment. A comparison conducted between the double-pulse structure waveform and the single-pulse structure waveform shows that the double-pulse structure outperforms the single-pulse structure, thus, verifying that diversity is an effective technique employed in noisy environments.

The effects of AWGN and PNI on the performance of the JTIDS compatible waveform for noncoherent demodulation are also examined. The numerical results show that for large values of $\rho \ (0.5 \leq \rho \leq 1)$, where $\rho$ is the fraction of time the PNI is on, the required signal-to-pulsed-noise interference ratio ($E_b / N_t$) increases as $\rho$ decreases for a fixed $P_b$. Moreover, it was observed that the absolute performance of the receiver for various values of $\rho$ degrades in an analogous manner as values of signal-to-noise-ratio ($E_b / N_o$) decreases.

Finally, this thesis examined the potential gains from PSI in an AWGN and PNI environment. It was shown that the existence of PSI is beneficial to tactical datalink users since PSI reduces the effect of PNI. In all cases investigated, when PSI was available there was an improvement in the performance of the receiver compared to when PSI was not available.
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I would also like to thank the Hellenic Navy for giving me the opportunity to have an enlightening educational experience at the Naval Postgraduate School.

I dedicate this work to my father, Panagiotis Kagioglidis. His concrete moral values and ideals, along with his scientific background, have always been an inspiration to me.
I. INTRODUCTION

A. OVERVIEW

Network-centric warfare (NCW) is an emerging theory of war applied in modern military operations in order to improve their effectiveness. To achieve this, NCW applies information age concepts to speed communications and increase situational awareness through networking [1]. As a result, NCW transforms the efficient flow of information into combat power by linking allied forces across the battlefield, thus enabling them to employ more effective decision making during military operations [2].

One of the most demanding requirements for the developers of NCW is to achieve interoperability among the communications systems of allied militaries. According to the NATO Glossary of Terms and definitions, interoperability can be defined as the ability of systems to provide services to and accept services from other systems and to use the services so exchanged to enable them to operate effectively together [3]. Digital datalinks are the platforms used by modern military forces in order to exchange information with each other. If interoperability is effectively applied, digital datalinks represent a significant tool in implementing NCW theory.

For many years several NATO forces have used tactical data links, such as Link-11 and Link-4A. Link-16 was introduced later in order to improve certain technical and operational capabilities of the already existing tactical data links [4]. Link-16 is a Communications, Navigation and Identification (CNI) system intended to exchange surveillance and Command and Control (C2) information among various C2 platforms and weapons platforms in order to enhance the varied missions of the multiple services. It provides multiple access, high capacity, jam resistant, digital data and secure voice CNI information to a variety of platforms [5].

The Joint Tactical Information Distribution System (JTIDS) is the communications component of JTIDS/Link-16. From the definition used in [5] the “JTIDS is a joint-service system which provides a reliable, secure, jam resistant, high-capacity, CNI capability through the use of direct-sequence, spread-spectrum, frequency-
hopping, and error detection and correction techniques.” The error detection and correction (EDAC) mechanism employs (31, 15) Reed-Solomon (RS) encoding. The (31, 15) notation means that 16 error detection and correction symbols are added for every 15 symbols of actual data, thus providing a message of 31 symbols. The signal is modulated at baseband by using an $M$-ary modulation technique called cyclic code shift keying (CCSK) that is compatible with direct sequence spread spectrum. CCSK modulates the 5-bit interleaved symbols previously produced by the RS encoding technique into 32-chip pseudo-random sequences. Finally, these sequences are transmitted using minimum-shift keying (MSK) modulation [4].

B. LITERATURE REVIEW

Since 1991, military specialists have striven to speed up the transmission and the reception of digital information. This has translated into an increase in demand for throughput, which is the transmission capacity for any given channel on a network [1]. A limited throughput results in an overall low data rate. The JTIDS/Link-16 operates in the L-band (from 969 MHz to 1206 MHz). In comparison with earlier tactical datalinks, such as Link-11 and Link-4A, JTIDS/Link-16 has a higher throughput [4], but limited data throughput in general is its most significant drawback. This stems from the fact that a frequency-hopped spread spectrum signal, which is used to increase jam resistance, occupies only a small fraction of the available bandwidth at any time. As a result, there is a significant reduction in throughput relative to the overall system bandwidth. Moreover, the transmitted messages also include, in addition to the data payload, an additional number of overhead bits. Thus, the amount of actual data transmitted is even more reduced [6].

There are several programs in progress that intend to introduce some enhancements in order to overcome the weakness of JTIDS/Link-16, as mentioned above. Among them is a program called Enhanced Throughput, which takes advantage of modern channel encoding techniques in order to increase JTIDS’s throughput. Specialists speculate that the improved throughput will be a maximum of 1.1 Mbps, which is almost ten times higher than the currently available throughput [7]. However, it is doubtful that
this maximum rate is actually going to be achieved in practice. This is because the maximum speeds for wireless communications are usually achieved when signals are transmitted between immobile stations that are close to each other and in an RF environment clear from interference. Since JTIDS is usually employed by moving platforms, often operating at a long distance from each other, and in an environment with possible interference, the actual data rates are expected to be lower than the estimated maximum rate [7].

It should be noted that very-high-speed commercial wireless technologies are continuously introduced in the market. Several NCW experts envision the adaptation of these new technologies by military communication systems. A prime example is the effort to incorporate IP-based waveforms into tactical datalinks. These new waveforms present wideband networking features, thus providing a seamless capacity of data transmission [8]. An IP-based Wideband Networking Waveform (WNW) has already been developed within the Joint Tactical Radio System (JTRS). However, evaluation tests conducted by the United States Air Force (USAF) and the United States Navy aviation community have shown that the WNW is incompatible with fighter combat [8].

There continues to be an active search for an IP-based waveform able to meet military communication needs. All efforts, however, are still in an experimental level or in a demonstration mode. For example, the Tactical Targeting Networking Technology (TTNT) developed under the auspices of the U.S. Defense Advanced Research Projects Agency has introduced a system that incorporates an IP-based waveform able to provide a 2Mbit/s throughput. The key advantage of this system is that data can be transmitted with much lower latency periods compared to the WNW. The major drawbacks of TTNT, however, are its high cost and the absence of interoperability with systems used by coalition partners (Link-16 is still used by the majority of NATO allies) [8]. Another example is a system called Flexible Access Secure Transfer (FAST). FAST makes use of sophisticated software able to transform the Link-16 waveform into an IP-based signal which is wideband but still compatible with the existing terminals used by all Link-16 users. The software used by FAST eliminates the pre-arranged time slots used previously
in Link-16 and introduces instead a variable and adaptive data rate which enhances the overall throughput, thus allowing a transmission rate of 1.1 Mbit/s. The FAST program is still in a test and demonstration mode [8].

Since the incorporation of new wireless technologies in military applications is still a vision that remains to become reality sometime in the future, present efforts to improve JTIDS/Link-16 are timely and cost effective. Accordingly, there are several studies which strive to somehow modify either the existing JTIDS/Link-16 waveform or the EDAC mechanism in order to improve the overall throughput and reliability. For example, [9] presents a comparison of the performance of a CCSK waveform with an orthogonal waveform, and [10] provides an analysis of different forward error correction (FEC) techniques for high-rate direct sequence spread spectrum. An analytical approximation for the probability of symbol error of CCSK is derived in [11], but according to [12] the performance obtained is optimistic by about 2 dB.

C. THESIS OBJECTIVE

This thesis examines an alternative waveform, which utilizes (31, 15) RS encoding as in the current JTIDS. The coded symbols are then transmitted on the in-phase (I) and quadrature (Q) component of the carrier using 32-ary orthogonal signaling with 32 chip baseband waveforms such as Walsh functions and detected noncoherently. For noncoherent detection, only one five bit symbol is transmitted on both the I and Q component of the carrier, so the data throughput for noncoherent detection is ½ that of coherent detection. A sequential diversity of two, consistent with the double-pulse structure of JTIDS, is also assumed, and performance will be examined both for the case of linear, soft diversity combining but also for soft diversity combining with side information. The type of side information to be considered is perfect side information. To the best of the author’s knowledge, the analysis of a compatible waveform obtained by replacing CCSK with 32-ary orthogonal signal detected noncoherently and taking into account the effect of pulsed-noise interference (PNI) on this waveform and a sequential diversity of two has not been previously investigated.
D. THESIS OUTLINE

This thesis is organized as follows. The first chapter is the introduction and explains why the topic of this thesis is important and provides a review of the relevant literature. Chapter II provides a series of basic concepts that the reader needs to have in mind in order to be able to understand the analysis of the compatible JTIDS waveform which follows in the next chapters. Chapter III presents the results of the performance analysis of the compatible JTIDS waveform with noncoherent detection. The analysis takes into account a purely additive white Gaussian noise (AWGN) environment. In Chapter IV, the analysis is performed for an AWGN environment with the addition of PNI. Chapter V examines the performance of the compatible JTIDS waveform in an AWGN and PNI environment with perfect side information (PSI). Chapter VI contains a comparison of the performance of 32-ary orthogonal signaling with (31, 15) RS encoding noncoherently detected with the original JTIDS waveform both for AWGN only, as well as both AWGN and PNI. Finally, Chapter VII presents the conclusions based on the results derived from the analysis in the previous chapters.
This chapter provides a series of basic concepts that the reader needs to have in mind in order to be able to understand the analysis of the compatible JTIDS waveform, which follows in the next chapters.

A. **M-ARY ORTHOGONAL SIGNALS**

When M-ary signaling is used, the processor of a digital communication system accepts \( k \) coded information bits at a time. In the process, the system’s modulator produces one of \( M = 2^k \) distinct waveforms \( s_m(t), m = 1,2,...,M \) in order to represent symbol \( m \). For \( k > 1 \), M-ary signaling can be considered a waveform coding procedure [13]. By definition, orthogonal signals are a set of signals \( s_m(t), m = 1,2,...,M \) with equal energy such that [14]

\[
\langle s_m(t), s_n(t) \rangle = \int_{-\infty}^{\infty} s_m(t)s_n^*(t)dt = \begin{cases} E, & m=n \\ 0, & m \neq n \end{cases},
\]

where \( \langle \cdot, \cdot \rangle \) denotes the inner product of the signals \( s_m(t) \) and \( s_n(t) \), and \( E \) is the signal energy when \( m = n \). For orthogonal signaling, error performance is improved (the required signal-to-noise ratio \( (E_s/N_0) \) is reduced for a fixed \( P_s \)) as \( k \) increases at the expense of bandwidth [13]. An M-ary orthogonal signal can be received either coherently or noncoherently. In this thesis, only noncoherent detection is considered.

In an AWGN environment, the waveform of an M-ary orthogonal signal can be represented by

\[
s_f(t) = \sqrt{2}A_c c_m(t)\cos(2\pi f_c t + \theta) + n(t), \quad 0 \leq t \leq T_s
\]

where \( n(t) \) is AWGN noise with power spectral density (PSD) \( N_0/2 \), the phase reference \( \theta \) is unknown for noncoherent detection and \( c_m(t), m = 1,2,...,M \), is an orthogonal, baseband waveform that represents symbol \( m \). By design, the \( M \) baseband waveforms are orthogonal over periods of \( T_s \) [15]
\[ T \int_{0}^{T} c_m(t)c_n(t)dt = \begin{cases} T_s & \text{for, } m=n \\ 0 & \text{for, } m\neq n \end{cases} . \]  

(2.3)

A block diagram of a noncoherent \( M \)-ary orthogonal baseband waveform demodulator is shown in Figure 1 [15].

In the presence of AWGN, the integrator outputs \( x_m(iT_s) \) for each branch of the receiver can be represented as independent Gaussian random variables \( X_m, X_{m'}, m = 1,2,\ldots,M \). The expected value of the in-phase integrator outputs are given by [15]

\[
\bar{X}_m = \frac{2 \sqrt{2} A}{T_s} \int_{0}^{T} c_m(t)c_n(t) \cos(2\pi f_s t + \theta) \cos(2\pi f_s t) dt \\
= \begin{cases} \sqrt{2} A \cos \theta & \text{for, } n=m \\ 0 & \text{for, } n\neq m \end{cases} .
\]  

(2.4)

and the expected value of the quadrature integrator outputs by

\[
\bar{X}_{m'} = \frac{2 \sqrt{2} A}{T_s} \int_{0}^{T} c_m(t)c_{n'}(t) \cos(2\pi f_s t + \theta) \cos(2\pi f_s t) dt \\
= \begin{cases} -\sqrt{2} A \sin \theta & \text{for, } n=m \\ 0 & \text{for, } n\neq m \end{cases} .
\]  

(2.5)

The noise power at the integrator outputs are given by the variances of \( X_m \) and \( X_{m'} \), \( m = 1,2,\ldots,M \), and the variances are expressed as [15]

\[
\sigma_{X_1}^2 = \sigma_{X_2}^2 = \ldots = \sigma_{X_M}^2 = \sigma^2 = N_0 / T_s .
\]  

(2.6)

The conditional probability density functions (pdfs) for the random variables \( V_m \) that represent the output of the \( m^{th} \) branch when the signal corresponding to symbol \( m \) is transmitted is given by the noncentral chi-squared probability density function with two degrees of freedom [15]:
\[ f_{v_m}(v_m | m) = \frac{1}{2\sigma^2} \exp\left[ -\left(\frac{v_m + 2A_c^2}{2\sigma^2}\right) \right] I_o\left( \frac{A_c \sqrt{2v_m}}{\sigma^2} \right) u(v_m) \] (2.7)

where \( I_o(\bullet) \) is the modified Bessel function of the first kind and order zero and \( u(\nu) \) is the unit step function.

The conditional pdfs for the random variables \( V_n \) that represent the outputs of each branch when the signal frequency \( f_m \) is transmitted \( (m \neq n) \) are given by the central chi-squared pdf with two degrees of freedom [15]:

\[ f_{v_n}(v_n | m) = \frac{1}{2\sigma^2} \exp\left[ -\frac{v_n}{2\sigma^2} \right] u(v_n) \] (2.8)

since \( I_o(0) = 1 \).

---

**Figure 1.** Block diagram of a noncoherent \( M \)-ary orthogonal baseband waveform demodulator [From: [15]].
B. PERFORMANCE OF M-ARY ORTHOGONAL SIGNALING IN AWGN

In the presence of AWGN with power spectral density $N_0/2$, the probability of channel symbol error for noncoherent $M$-ary orthogonal signaling is [15]

$$p_s = \sum_{n=1}^{M-1} \frac{(-1)^{n+1}}{n+1} \binom{M-1}{n} \exp \left[ -\frac{nE_s}{(n+1)N_0} \right]$$  \hspace{1cm} (2.9)

where $E_s$ is the average energy per channel symbol, $E_s = A_v^2 T_s$, where $A_v^2$ is the average received signal power, and $T_s$ is the symbol duration.

Alternatively, the probability of channel symbol error can be evaluated by performing the union bound approach [13]:

$$p_s \leq \frac{M-1}{2} \exp \left( -\frac{E_s}{2N_0} \right)$$  \hspace{1cm} (2.10)

C. PERFORMANCE IN BOTH AWGN AND PULSED-NOISE INTERFERENCE

In recent years, there has been increasing activity in the area of pulsed intentional electromagnetic interference (IEMI) also known as “EM Terrorism.” By definition, IEMI is the “Intentional malicious generation of electromagnetic energy introducing noise or signals into electric and electronic systems, thus disrupting, confusing or damaging these systems for terrorist or criminal purposes [16].” IEMI is most effective upon data communications systems since they are usually designed upon expectations of a wideband Gaussian noise environment, rather than an intense pulsed interfering signal [17].

It is known that an intentional interferer (jammer) that targets a frequency-hopping system will achieve maximum interference (highest average probability of error) by focusing its power so as to jam a fraction of the hops unless the jammer is so powerful that it can spread its power across the whole bandwidth, thus causing errors for all hops [18]. A pulse-noise jammer transmits pulses of bandlimited Gaussian noise having a high peak power [13]. The general consideration of this thesis is the degree of impact that a
pulse-noise jammer has on a communication system. More specifically, we will examine the performance of a compatible JTIDS/Link-16 waveform in a AWGN and pulsed-noise interference (PNI) environment.

In an AWGN environment, the noise signal that reaches the receiver is assumed to be uniformly spread across the spectrum and time-independent. These parameters are not always valid in the case of PNI. In this thesis, the AWGN and PNI are assumed to be statistically independent, and the PNI is modeled as Gaussian noise \[14\]. In the presence of both AWGN and PNI, the total noise power at the receiver integrator outputs is given by \[19\]

\[
\sigma_X^2 = \sigma_{WG}^2 + \sigma_I^2
\]  

(2.11)

where \(\sigma_{WG}^2 = N_o / T_s\), \(\sigma_I^2 = N_I / \rho T_s\), \(N_I\) is the PSD of barrage noise interference and \(\rho\) is a fraction of time that an interferer is switched on. In the case of \(\rho = 1\), the jammer is continuously on and is referred to as barrage noise interference.

When PNI is present, the average probability of symbol error is given by the expression

\[
P_s = \Pr(\text{Interferer is ON}) p_s(\text{AWGN+PNI}) + \Pr(\text{Interferer is OFF}) p_s(\text{AWGN})
\]  

(2.12)

where we assume that a symbol is either completely free of PNI or the entire symbol is affected by PNI \[19\]. Since \(\Pr(\text{Interferer is ON})=\rho\),

\[
P_s = \rho p_s(\text{AWGN + PNI}) + (1-\rho) p_s(\text{AWGN})
\]  

(2.13)

where \(p_s(x)\) represent the probability of symbol error for condition as it is defined by \(x\).

D. PERFORMANCE WITH DIVERSITY

Designers of anti-jam communication systems aim at forcing a jammer to expend its resources over a wide-frequency band for a maximum time and from a diversity of sites. The most commonly used design options are diversity techniques \[13\]. Moreover, diversity techniques are based on the assumption that receivers make errors when the channel attenuation is large, i.e., when the channel is fading. Accordingly, if we send to
the receiver several replicas of the same information signal transmitted over fading channels independent of each other, the probability that all the replica signals will fade at the same time is significantly reduced [14].

One of the prevalent methods used to produce these replicas employs frequency diversity by the use of frequency-hopping spread-spectrum techniques [13]. This means that the same information signal is transmitted on \( L \) carriers, where the separation between successive carriers equals or exceeds the coherence bandwidth \( (\Delta f)_c \) of the channel. The separation of frequency by \( (\Delta f)_c \) is a form of block-interleaving the bits in a repetition code in order to make these bits statistically independent and, thus, obtain independent errors [14].

The Link-16 message data can be transmitted with either a single-pulse structure (without diversity) or a double-pulse structure. The latter increases the anti-jam capability of the link since it provides a diversity of \( L = 2 \). When the double-pulse structure is used, JTIDS is a hybrid direct sequence/fast-frequency hopping (DS/FFH) spread spectrum system with sequential diversity \( L = 2 \) since each symbol is transmitted twice on two different carrier frequencies [20].

For a system with diversity of order \( L \), where each diversity signal is received independently, the probability that \( i \) of \( L \) receptions are affected by PNI can be represented as [19]

\[
Pr(i \ of \ L \ pulses \ jammed) = \binom{L}{i} \rho^i (1 - \rho)^{L-i} \quad (2.14)
\]

where there are \( \binom{L}{i} \) different ways that \( i \) of \( L \) receptions can be jammed and \( \rho \) is the fraction of time when the channel is affected by PNI. The probability of symbol error for a system with diversity of \( L \) is

\[
P_s = \sum_{i=0}^{L} [Pr(i \ of \ L \ signals \ jammed) \cdot p_s(i)] \quad (2.15)
\]

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where \( p_s(i) \) is the conditional probability of channel symbol error in the event that \( i \) of \( L \) diversity receptions are affected by PNI. Substituting (2.14) into (2.15), we get

\[
P_s = \sum_{i=0}^{L} \binom{L}{i} \rho^i (1-\rho)^{L-i} p_s(i) \quad (2.16)
\]

**E. FORWARD ERROR CORRECTION CODING**

Designers of digital communications and storage systems take under serious consideration the integrity of the received data. When EDAC strategies are required, designers often use high reliability techniques such as forward error correction (FEC) coding. The most significant advantage of FEC coding is that the digital system that employs it can detect and reconstruct an erroneous transmitted message at the receiver without requiring retransmissions [21].

Modern coding theory has resulted in several block codes for FEC, including the Reed-Solomon (RS) codes. RS codes have the ability to detect and correct both random errors (typically caused by additive noise), and burst errors (errors that occur sequentially in time and as groups and typically caused by media defects in digital storage systems). Moreover, RS codes are easily and economically applied to both high and low data rate systems, which make them popular with designers [21].

The FEC used by JTIDS/Link-16 is (31, 15) RS coding, a linear, nonbinary code. In order to be consistent with the original JTIDS/Link-16 waveform, the compatible JTIDS/Link-16 waveform, examined by this thesis, also employs (31, 15) RS coding for error detection and correction. For nonbinary codes, instead of bits symbols are generated. Each of these symbols represents \( m \) bits and the number of different symbols required are \( M = 2^m \). An \((n, k)\) RS encoder, takes \( k \) information symbols (\( mk \) information bits) and generates \( n \) coded symbols (\( mn \) coded bits) [22].

The probability of decoder, or block, error for a \( t \)-symbol error correcting, nonbinary block code with maximum likelihood decoding is upper bounded by [22]
\[ P_E \leq \sum_{i=t+1}^n \binom{n}{i} p_s^i (1 - p_s)^{n-i} \]  
(2.17)

or

\[ P_E \leq 1 - \sum_{i=0}^{t} \binom{n}{i} p_s^i (1 - p_s)^{n-i} \]  
(2.18)

where equality holds for either a perfect code or a bounded distance decoder, and \( p_s \) is the probability of coded, or channel, symbol error.

For \( M \)-ary orthogonal modulation with \( M = 2^m \) and RS codes with hard decision decoding employed, we obtain the probability of information bit error as [22]

\[ P_b \approx \frac{n+1}{2n^2} \sum_{i=t+1}^n \binom{n}{i} p_s^i (1 - p_s)^{n-i}. \]  
(2.19)

F. PERFECT-SIDE INFORMATION

When a communications system is under jamming attack, it is very useful for the receiver to have some type of side information. One type of side information is when the receiver knows the jammer state, i.e., when the jammer is on or off. This knowledge can be acquired from channel measurements of noise power levels in adjacent frequency bands [14]. Moreover, perfect side information (PSI) is the certain knowledge of which specific hops are jammed and which are not [20].

For a system with a diversity of \( L \), where the diversity receptions are received independently, PSI can be considered as a means to reduce the effect of PNI. For a diversity of two, when both received symbols in the repetitive pulses are unaffected by PNI, they are combined and demodulated as usual. If either of the diversity receptions is affected by PNI, the receiver discards the PNI-affected symbol and makes a decision based on the remaining diversity reception affected only by AWGN. When both diversity receptions are affected by PNI, the receiver combines the two receptions and makes a decision. PSI requires at least a diversity of two and can improve system performance in a PNI environment where \( \rho < 1 \) [23].
G.  CHAPTER SUMMARY

The first part of this chapter offered an introduction to $M$-ary orthogonal signals. It then addressed the necessary concepts required to examine the performance of a compatible JTIDS/Link-16 waveform which consists of 32-ary orthogonal signaling with (31, 15) RS coding, where the detection will be performed noncoherently. Other considerations, such as the impact of jamming in the form of pulsed-noise interference, along with an anti-jamming technique, namely diversity, were addressed as well. In the later part of this chapter, the concepts of forward error correction coding and perfect side information were also introduced. In the next chapter, the performance of a compatible JTIDS waveform that utilizes (31, 15) RS coding with $M$-ary orthogonal modulation transmitted over a channel with only AWGN is examined.
III. PERFORMANCE ANALYSIS OF NONCOHERENT 32-ARY ORTHOGONAL SIGNALING WITH (31, 15) RS ENCODING IN AN AWGN ENVIRONMENT

A. PERFORMANCE IN AWGN

In this chapter, we examine the performance of a JTIDS compatible 32-ary orthogonal signaling with (31, 15) RS encoding in an AWGN environment by analyzing the bit error versus the signal-to-noise ratio. As mentioned in the previous chapter, the Link-16 message data can be transmitted with either a single-pulse structure or a double-pulse structure. The waveform under examination belongs to the second category since it employs a diversity of two. We are going to perform a comparison between the probability of bit error of the single-pulse structure Link-16 waveform (no diversity) and the double-pulse structure waveform in order to determine if there are any improvements.

The probability of channel symbol error for noncoherent \( M \)-ary orthogonal signaling in an AWGN environment is provided by (2.9). In terms of average bit energy, it can be written as

\[
P_s = \sum_{n=1}^{M-1} \frac{(-1)^{n+1}}{n+1} \binom{M-1}{n} \exp \left[ \frac{-n m E_b}{(n+1) N_o} \right]
\]  

(3.1)

since \( E_s = mE_b \), where \( m \) is the number of bits per symbol.

When FEC coding is used, the probability of channel symbol error with no diversity is given by [12]

\[
P_s = \sum_{n=1}^{M-1} \frac{(-1)^{n+1}}{n+1} \binom{M-1}{n} \exp \left[ \frac{-n m E_b}{(n+1) N_o} \right]
\]  

(3.2)

or

\[
P_s = \sum_{n=1}^{M-1} \frac{(-1)^{n+1}}{n+1} \binom{M-1}{n} \exp \left[ \frac{-n m r E_b}{(n+1) N_o} \right]
\]  

(3.3)
where $E_b = rE_b$ is the average energy per coded bit, and $r = k / n$ is the code rate, indicating there are $k$ information symbols per $n$ coded symbols.

When the compatible noncoherent $M$-ary orthogonal signaling with a diversity of two is employed, we need to define the new channel symbol error. Since the double structure is used, JTIDS functions as a hybrid DS/FFH spread spectrum system. The performance of FFH $M$-ary orthogonal signaling with $L > 1$ and soft decision demodulation is the same as that for a noncoherent $M$-ary orthogonal signaling system with $L$-fold time diversity and soft decision demodulation. A block diagram of a noncoherent FFH $M$-ary frequency-shift keyed (MFSK) detector with soft decision demodulation is shown in Figure 2. Mathematically, the performance of this receiver is identical to that of an $M$-ary orthogonal baseband receiver with linear, soft decision combining.

![Figure 2. Noncoherent FFH/MFSK receiver with linear, soft decision combining.](image)
The analog outputs, \( V_{1_k}, V_{2_k}, \ldots, V_{M_k} \), of each square-law detector for each diversity reception are added and they give the decision statistics \( V_1, V_2, \ldots, V_M \). The latter are compared with each other in order to decide whether the signal corresponding to a particular branch is present or not. For \( L \) diversity receptions the conditional pdfs for the random variables \( V_1 \) and \( V_2 \) are given by (symbol 1 is assumed to be transmitted) [24]

\[
 f_{V_1}(v_1 | 1) = \frac{v_1^{(L-1)/2}}{2\sigma^2 (2LA_c^2)^{(L-1)/2}} \exp \left[ -\frac{(v_1 + 2LA_c^2)}{2\sigma^2} \right] I_{L-1} \left( \frac{A_c \sqrt{2L}v_1}{\sigma^2} \right) u(v_1) \quad (3.4)
\]

where \( I_{L-1}(\bullet) \) is the modified Bessel function of the first kind and order \( L-1 \),

and

\[
 f_{V_2}(v_2 | 1) = \frac{v_2^{L-1}}{2\sigma^2 \cdot (L-1)!} \exp \left[ -\frac{v_2}{2\sigma^2} \right] u(v_2) \quad . \quad (3.5)
\]

We use an indirect approach where we obtain the probability of not making a symbol error and subtract that result from one [15] to get

\[
 p_s = 1 - p_e = 1 - \int_0^\infty f_{V_1}(v_1 | 1) \left[ \int_0^1 f_{V_2}(v_2 | 1) dv_2 \right]^{M-1} dv_1 \quad (3.6)
\]

Substituting (3.4) and (3.5) into (3.6), we obtain an expression for the probability of channel symbol error with a diversity of two as

\[
 p_s = 1 - \sum_{p=0}^{M-1} (-1)^p \binom{M-1}{p} \binom{p}{q} q! \exp \left( -\frac{p}{p+1} \frac{rmL_E c}{N_o} \right) \sum_{w=0}^{q} \frac{1}{w!} \binom{q+1}{q-w} \frac{1}{(p+1)^{w+q+2}} \left( \frac{rmL_E c}{N_o} \right)^w \quad (3.7)
\]

or, alternatively,

\[
 p_s = 1 - \sum_{p=0}^{M-1} (-1)^p \binom{M-1}{p} \binom{p}{q} q! \exp \left( -\frac{p}{p+1} \frac{rmE_b}{N_o} \right) \sum_{w=0}^{q} \frac{1}{w!} \binom{q+1}{q-w} \frac{1}{(p+1)^{w+q+2}} \left( \frac{rmE_b}{N_o} \right)^w \quad . \quad (3.8)
\]
since $E_b = LE_c$, where $E_b$ is the overall average energy per information bit and is related to the average signal energy per diversity reception $E_c$ by a factor of $L$.

For $M = 2$, $m = 5$, $L = 1$, and $r = k / n = 15 / 31$ the expressions (3.3) and (3.8) are simplified to, for no diversity,

$$
p_s = \sum_{n=1}^{31} \frac{(-1)^{n+1}}{n+1} \binom{31}{n} \exp \left[ \frac{-2.419E_b}{n+1N_o} \right] \tag{3.9}
$$

and, for a diversity of two $(L = 2)$,

$$
p_s = 1 - \sum_{p=0}^{M-1} (-1)^p \binom{31}{p} \sum_{q=0}^{p} q! \exp \left( \frac{-p}{p+1} \frac{2.419E_b}{N_o} \right) \sum_{w=q}^{\infty} \frac{1}{w!} \left( \frac{q+1}{w+q+1} \right) \frac{1}{(p+1)w^2} \left( \frac{2.419E_b}{N_o} \right)^w. \tag{3.10}
$$

As mentioned earlier, for RS codes and $M$-ary orthogonal modulation with $M = 2^m$ and hard decision decoding, we obtain the probability of information bit error using (2.19), repeated here for convenience:

$$
P_b \approx \frac{n+1}{2n^2} \sum_{i=n+1}^{n} \binom{n}{i} p_s^i (1-p_s)^{n-i}. \tag{3.11}
$$

**B. NUMERICAL RESULTS**

The performance of noncoherent 32-ary orthogonal signaling with (31, 15) RS coding for both the original single structure waveform with no diversity and the double-pulse structure waveform, equivalent to a diversity of two, is shown in Figure 3.
Figure 3. Performance of noncoherent 32-ary orthogonal signaling with (31, 15) RS encoding for both the single-pulse and the double-pulse structure in AWGN.

At $P_b = 10^{-5}$, the required $E_b / N_o$ for the double-pulse structure is about 3.6 dB, while the $E_b / N_o$ required for the single-pulse structure is about 5.9 dB. Thus, the compatible waveform which employs a diversity of two outperforms the single-pulse waveform by 2.3 dB at $P_b = 10^{-5}$ in AWGN.
C. CHAPTER SUMMARY

In this chapter, the performance of the compatible JTIDS waveform with a diversity of two detected noncoherently in an AWGN environment was investigated. In order to examine the potential improvements of the new waveform, a comparison was conducted between the double-pulse structure waveform and the single-pulse structure waveform. The key in this analysis was to derive an expression for the probability of channel symbol error of the compatible waveform. The comparison, as expected, showed that the double-pulse structure outperforms the single-pulse structure, thus, verifying that diversity is an effective technique employed in noisy environments. In the next chapter, the performance of the compatible JTIDS waveform for noncoherent demodulation in both AWGN and PNI are examined.
IV. PERFORMANCE ANALYSIS OF NONCOHERENT 32-ARY ORTHOGONAL SIGNALING WITH (31, 15) RS ENCODING IN AN AWGN AND PULSED-NOISE INTERFERENCE ENVIRONMENT AND A DIVERSITY OF TWO

A. PERFORMANCE IN AWGN AND PNI

In this chapter, we examine the performance of the receiver in the presence of PNI and AWGN. For practical reasons, we model PNI as Gaussian noise that is turned on and off periodically. If $\rho$ represents the fraction of time that the PNI is turned on, then $(1 - \rho)$ represents the fraction of time that the PNI is turned off where $0 < \rho \leq 1$. In this kind of noisy environment, received symbols are affected by two different levels of noise power since some of the symbols are affected only by AWGN and the rest by both AWGN and PNI. If the one-sided PSD of the AWGN is $N_o$ and the one-sided PSD of barrage noise interference is $N_i$, then $N_i / \rho$ is the PSD of the PNI since we assume that average interference power is independent of $\rho$.

For a channel with a diversity of $L$ in an environment of both AWGN and PNI, the probability of channel error is given by (2.16). Accordingly, for a diversity of two, we have

$$P_s = \sum_{i=0}^{2} \left[ \binom{2}{i} \rho^i (1 - \rho)^{2-i} p_s(i) \right] = (1 - \rho)^2 p_s(0) + 2\rho (1 - \rho) p_s(1) + \rho^2 p_s(2)$$

(4.1)

where $p_s(0)$ is the conditional probability of channel symbol error when PNI is not present in either diversity reception and is expressed as

$$p_s(0) = 1 - \sum_{p=0}^{M-1} \left[ \sum_{q=0}^{p} \binom{p}{q} q! \exp \left( -p \frac{r m E_0}{N_o} \right) \sum_{w=0}^{q} \frac{1}{w!} \left( q+w \right) \frac{1}{\left( p+1 \right)^{q+w+2}} \left( \frac{m E_0}{N_o} \right)^w \right].$$

(4.2)
The conditional probability of channel symbol error \( p_s(2) \) when both diversity receptions suffer PNI is obtained as follows. The random variables \( V_1 \) and \( V_2 \) are obtained from (3.4) and (3.5), respectively, by replacing \( \sigma^2 \) with \( \sigma_T^2 \) to get
\[
f_{V_1}(v_1 | 1) = \frac{v_1^{(L-1)/2}}{2\sigma_T^2 (2LA_T^2)^{(L-1)/2}} \exp \left[ -\left( \frac{v_1 + 2LA_T^2}{2\sigma_T^2} \right) \right] I_{L-1}\left( \frac{A_T \sqrt{2Lv_1}}{\sigma_T^2} \right) u(v_1)
\] (4.3)
and
\[
f_{V_2}(v_2 | 1) = \frac{v_2^{L-1}}{2\sigma_T^2 (L-1)!} \exp \left[ -\frac{v_2}{2\sigma_T^2} \right] u(v_2)
\] (4.4)
where
\[
\sigma_T^2 = \sigma_s^2 + \sigma_o^2 = \frac{N_o}{T_s} + \frac{N_f}{\rho T_s}
\] (4.5)
and \( N_o \) and \( N_f \) are the one-sided PSDs of AWGN and PNI, respectively.

We use the indirect method, as in the previous chapter, in order to derive the conditional probability \( p_s(2) \), and for \( L = 2 \) we get:
\[
p_s(2) = 1 - \sum_{p=0}^{M-1} (-1)^p \binom{M-1}{p} \sum_{q=0}^{p} \binom{p}{q} q! \exp \left[ -\frac{p}{p+1} \left( \frac{E_h}{N_o} \right)^{-1} + \frac{1}{\rho} \left( \frac{E_f}{N_f} \right)^{-1} \right] \relphantimes \sum_{w=0}^{q} \frac{1}{w!} \left( \frac{q+1}{p+1} \right)^{q+w} 2^{q+w} \left( \frac{1}{\left( \frac{mE_h}{N_o} \right)^{-1} + \frac{1}{\rho} \left( \frac{mE_f}{N_f} \right)^{-1}} \right).
\] (4.6)

The conditional probability of channel symbol error \( p_s(1) \) when only one of the diversity receptions suffers PNI is obtained as follows. The probability density function
of the decision statistic $V_i$ for $i$ hops experiencing pulsed noise interference (symbol 1 is assumed to be transmitted) is given by [25]

$$f_{V_i}(v_i | 1) = \frac{1}{2\sigma_r^2} \exp \left[ -\frac{1}{2\sigma_r^2} (v_i + A_t^2) \right] I_o \left( \frac{A \sqrt{v_i}}{\sigma_r^2} \right) \bigotimes \frac{1}{2\sigma_o^2} \exp \left[ -\frac{1}{2\sigma_o^2} (v_i + A_t^2) \right] I_o \left( \frac{A \sqrt{v_i}}{\sigma_o^2} \right). \quad (4.7)$$

For $i = 1$ and $L = 2$, we get:

$$f_{V_1}(v_1 | 1) = \frac{1}{2\sigma_r^2} \exp \left[ -\frac{1}{2\sigma_r^2} (v_1 + A_t^2) \right] I_o \left( \frac{A \sqrt{v_1}}{\sigma_r^2} \right) \bigotimes \frac{1}{2\sigma_o^2} \exp \left[ -\frac{1}{2\sigma_o^2} (v_1 + A_t^2) \right] I_o \left( \frac{A \sqrt{v_1}}{\sigma_o^2} \right). \quad (4.8)$$

Let us define the components of this convolution as:

$$f_{V_{1a}}(v_1 | 1) = \frac{1}{2\sigma_r^2} \exp \left[ -\frac{1}{2\sigma_r^2} (v_1 + A_t^2) \right] I_o \left( \frac{A \sqrt{v_1}}{\sigma_r^2} \right) \quad (4.9)$$

and

$$f_{V_{1b}}(v_1 | 1) = \frac{1}{2\sigma_o^2} \exp \left[ -\frac{1}{2\sigma_o^2} (v_1 + A_t^2) \right] I_o \left( \frac{A \sqrt{v_1}}{\sigma_o^2} \right). \quad (4.10)$$

Evaluating (4.8) in the decision statistic domain requires the use of the definition of convolution, that is

$$f_{V_i}(v_i | 1) = \int_0^{V_i} f_{V_{1a}}(v_1 | 1) \times f_{V_{1b}}(v_1 - t | 1) dt$$

$$= \int_0^{V_i} \left\{ \frac{1}{2\sigma_r^2} \exp \left[ -\frac{1}{2\sigma_r^2} (v_1 + A_t^2) \right] I_o \left( \frac{A \sqrt{v_1}}{\sigma_r^2} \right) \right\} \times \left\{ \frac{1}{2\sigma_o^2} \exp \left[ -\frac{1}{2\sigma_o^2} (v_1 - t + A_t^2) \right] I_o \left( \frac{A \sqrt{v_1 - t}}{\sigma_o^2} \right) \right\} dt \quad (4.11)$$

where $t$ is a dummy variable. The analytic evaluation of this integral is not possible to the best of the authors’ knowledge.

Transforming this probability density function to the Laplace domain yields the Laplace transform of the probability density function for decision statistic $V_1$:
\[
F_{V_2}(s \mid 1) = \left[ \frac{1}{1 + 2\sigma_r^2 s} \right] \exp \left( -\frac{A^2 s}{1 + 2\sigma_o^2 s} \right) \left[ \frac{1}{1 + 2\sigma_o^2 s} \right] \exp \left( -\frac{A^2 s}{1 + 2\sigma_o^2 s} \right), \quad (4.12)
\]

for which no known closed form solution exists [25].

In the same manner as in the case of \( V_1 \), the probability density function of decision statistic \( V_2 \) for one hop experiencing pulsed noise interference is given by

\[
f_{V_2}(v_2 \mid 1) = \left\{ \frac{1}{2\sigma_r^2} \exp \left( -\frac{v_2^2}{2\sigma_r^2} \right) \right\} \otimes \left\{ \frac{1}{2\sigma_o^2} \exp \left( -\frac{v_2^2}{2\sigma_o^2} \right) \right\}. \quad (4.13)
\]

Let us define the components of this convolution as:

\[
f_{V_2\text{a}}(v_2 \mid m) = \frac{1}{2\sigma_r^2} \exp \left( -\frac{v_2^2}{2\sigma_r^2} \right), \quad (4.14)
\]

and

\[
f_{V_2\text{b}}(v_2 \mid 1) = \frac{1}{2\sigma_o^2} \exp \left( -\frac{v_2^2}{2\sigma_o^2} \right) \quad (4.15)
\]

Transforming these probability density functions to the Laplace domain, we get the Laplace transform of the probability density function for decision statistic \( V_2 \):

\[
F_{V_2}(s \mid 1) = \left[ \frac{1}{2\sigma_r^2 s + 1} \right] \left[ \frac{1}{2\sigma_o^2 s + 1} \right]. \quad (4.16)
\]

The inverse Laplace transform of (4.16) is:

\[
f_{V_2}(v_2 \mid 1) = \frac{1}{2(\sigma_o^2 - \sigma_r^2)} \left[ \exp \left( -\frac{v_2^2}{2\sigma_o^2} \right) - \exp \left( -\frac{v_2^2}{2\sigma_r^2} \right) \right] \quad (4.17)
\]

which is the result we would obtain by using the definition of convolution for the evaluation of (4.13). Thus,
\[ f_{v_2}(v_2 \mid 1) = \int_0^{V_2} f_{v_2}(v_2 \mid 1) f_{v_2}(v_2 - u \mid 1) du \]

\[ = \int_0^{V_2} \left\{ \frac{1}{2\sigma_T^2} \exp\left( -\frac{v_2^2}{2\sigma_T^2} \right) \frac{1}{2\sigma_o^2} \exp\left( -\frac{(v_2 - u)^2}{2\sigma_o^2} \right) \right\} du \]

\[ = \frac{1}{2(\sigma_o^2 - \sigma_T^2)} \left[ \exp\left( -\frac{v_2^2}{2\sigma_o^2} \right) - \exp\left( -\frac{v_2^2}{2\sigma_T^2} \right) \right] \]  

(4.18)

where \( u \) is a dummy variable, eliminated by the inverse Laplace transform evaluation (an important simplification in the process of our analysis).

In order to derive an expression for the probability of symbol error, once again we use the indirect approach, repeated here for convenience:

\[ p_s = 1 - p_c = 1 - \int_0^\infty f_{v_1}(v_1 \mid 1) \left[ \int_0^\infty f_{v_2}(v_2 \mid 1) dv_2 \right]^{M-1} dv_1 . \]  

(4.19)

Substituting (4.11) and (4.17) into (4.19), it yields

\[ p_s(1) = 1 - \int_0^\infty \int_0^{V_1} \left\{ \frac{1}{2\sigma_T^2} \exp\left( -\frac{1}{2\sigma_T^2}(v_1 + \xi) \right) I_{v_1} (\frac{A\sqrt{v_1}}{\sigma_T}) - \frac{1}{2\sigma_o^2} \exp\left( -\frac{1}{2\sigma_o^2}(v_1 - \xi) \right) I_{v_1} (\frac{A\sqrt{v_1}}{\sigma_o}) \right\} dt \]

\[ \times \int_0^{V_1} \left\{ \frac{1}{2(\sigma_o^2 - \sigma_T^2)} \left[ \exp\left( -\frac{v_2}{2\sigma_o} \right) - \exp\left( -\frac{v_2}{2\sigma_T} \right) \right] \right\}^{M-1} dv_2 dv_1 \]  

(4.20)

The integral over \( v_2 \) can be evaluated, and with a combination of substitutions of variables we eliminate the dummy variable \( t \) and reach an expression suitable for evaluating the probability of symbol error (we make use of the expressions \( \gamma_f = E_b / N_f \) and \( \gamma_b = E_b / N_o \) for the sake of space):
\[ p_i(l) = 1 - \int_0^\infty \int_0^y \frac{1}{1 + \rho \frac{\gamma_b}{\gamma_I}} \exp \left\{ -\omega - \frac{1}{4} \rho \gamma_b + \left[ -\frac{1}{4} \rho \gamma_b - y + \omega \right] \left( 1 + \rho \frac{\gamma_b}{\gamma_I} \right) \right\} I_0 \left( \frac{2 \omega \gamma_b}{1 + \rho \gamma_b} \right) \]

\[ \times I_0 \left( \frac{1}{1 + \rho \frac{\gamma_b}{\gamma_I}} \sqrt{2(y - \omega) \rho \gamma_b} \right) d\omega \left\{ 1 + \frac{1}{1 + \rho \frac{\gamma_b}{\gamma_I}} \right\} \left( -y - \frac{1}{1 + \rho \frac{\gamma_b}{\gamma_I}} \right) dy. \quad (4.21) \]

**B. NUMERICAL RESULTS**

Substituting the conditional probabilities of channel symbol errors for number of jammed diversity receptions \( i = 0, 1, 2 \) given by (4.2), (4.6) and (4.21) into (4.1), we derive the expression for the probability of information bit error. The performance for different values of \( \rho \) with \( E_b / N_o = 11.3 \) dB is shown in Figure 4.
We observe that the value of $E_b/N_l$ increases as $\rho$ decreases, at least for comparatively large values of $\rho$. For example, $\rho=1$ gives $E_b/N_l=8.4$ dB, compared to $\rho=0.7$, which gives $E_b/N_l=10.1$ dB. It can also be seen that at $P_b=10^{-5}$, varying $\rho$ from 1 to 0.5 does not degrade the receiver performance significantly as compared to barrage jamming ($\rho=1$). The degradation is 1.7 dB and 1.5 dB for $\rho=0.7$ and $\rho=0.5$, respectively. For smaller values of $\rho$, such as $\rho=0.3$, however, the performance of the receiver is improved compared to larger values of $\rho$. At $P_b=10^{-5}$, the performance for
$\rho = 0.3$ is equal to the performance of barrage jamming. However, compared to $\rho = 0.5$ there is an improvement in performance of about 1.5 dB.

The performance of the receiver with $E_b / N_o = 10$ dB is shown in Figure 5.

![Figure 5](image_url)

Figure 5. Performance of 32-ary orthogonal signaling with (31, 15) RS encoding for different values of $\rho$ in both AWGN and PNI with $E_b / N_o = 10$ dB.

We observe that the degradation in performance due to PNI as compared to barrage jamming is 2.6 dB at $P_b = 10^{-5}$ for both $\rho = 0.7$ and $\rho = 0.5$. This means an increase of about 1.1 dB compared to the previous case. For $\rho = 0.3$, there is also degradation in performance compared to $\rho = 1$ of about 1.2 dB, which is also worse.
compared to the performance with $E_b / N_o = 11.3$ dB. Moreover, the absolute performance for various values of $\rho$ is worse in the case with $E_b / N_o = 10$ dB versus the one with $E_b / N_o = 11.3$ dB. In numerical terms, this is a degradation of 0.9 dB for $\rho = 1$ to about 2.2 dB for $\rho = 0.3$ at $P_b = 10^{-5}$.

In Figure 6, the performance of the receiver with $E_b / N_o = 9$ dB is shown.

![Figure 6. Performance of 32-ary orthogonal signaling with (31, 15) RS encoding for different values of $\rho$ in both AWGN and PNI with $E_b / N_o = 9$ dB.](image.png)
In this case, the degradation in performance due to PNI as compared to barrage jamming is 6.3 dB and 9.4 dB at $P_b = 10^{-5}$ for $\rho = 0.7$ and $\rho = 0.5$, respectively. Not only is there a greater degradation compared to the previous cases ($E_b / N_o = 11.3$ dB and $E_b / N_o = 10$ dB) but a rapid degradation in the performance between $\rho = 0.7$ and $\rho = 0.5$, a phenomenon not observed before, can be seen. Moreover, the performance of the receiver for $\rho = 0.3$ is worse compared to barrage jamming by about 0.9 dB, as opposed to our previously examined cases. Here again, the absolute performance for various values of $\rho$ is worse compared to those obtained for greater values of $E_b / N_o$. Hence, we can say that with $E_b / N_o = 9$ dB the receiver reaches a critical point at which its performance is more vulnerable to PNI, a fact which is confirmed when smaller values of $E_b / N_o$ are examined.

The numerical results for Figures 4, 5 and 6 are summarized in Table 1.

Table 1. Performance of 32-ary orthogonal signaling with (31, 15) RS encoding for different values of $\rho$ in both AWGN and PNI when $P_b = 10^{-5}$.

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$E_b / N_o = 11.3$ dB</th>
<th>$E_b / N_o = 10$ dB</th>
<th>$E_b / N_o = 9$ dB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_b / N_I$ dB</td>
<td>$E_b / N_I$ dB</td>
<td>$E_b / N_I$ dB</td>
</tr>
<tr>
<td>1</td>
<td>8.4</td>
<td>9.3</td>
<td>10.4</td>
</tr>
<tr>
<td>0.7</td>
<td>10.1</td>
<td>12</td>
<td>16.9</td>
</tr>
<tr>
<td>0.5</td>
<td>9.9</td>
<td>12</td>
<td>19.9</td>
</tr>
<tr>
<td>0.3</td>
<td>8.4</td>
<td>10.6</td>
<td>14.9</td>
</tr>
</tbody>
</table>
C. CHAPTER SUMMARY

In this chapter, the effects of AWGN and PNI on the performance of the compatible JTIDS waveform for noncoherent demodulation were examined. This required the evaluation of the conditional probabilities of channel symbol error for $i = 0, 1$ and 2 hops affected by pulsed-noise interference. The analysis was focused on deriving a formula for $p_s(1)$ (conditional probability of channel symbol error for $i = 1$ hop experiencing PNI). Computational difficulties in obtaining the probability density function of decision statistic $V_1$, in both the decision statistic and Laplace domains, mandated that we use algebraic substitutions in order to transform the given integrals into simpler ones. Even so, it is not clear that all computational problems have been solved, since the results obtained for $\rho \leq 0.3$ are not as expected. This will remain an area for future research.

The numerical observations made are summarized as follows. For comparatively large values of $\rho$ ($0.5 \leq \rho \leq 1$), it was shown that the required $E_b/N_t$ increases as $\rho$ decreases for a fixed $P_b$. A significant degradation was observed in the performance of the receiver when $E_b/N_o = 9$ dB for all values of $\rho < 1$ (even for $\rho = 0.3$). This was a hint that for smaller values than $E_b/N_o = 9$ dB the degradation would increase rapidly, a fact which was confirmed when such cases were examined. Finally, the absolute performance of the receiver for various values of $\rho$ degrades in an analogous manner as values of $E_b/N_o$ decrease. In the next chapter, the performance of the compatible JTIDS waveform for noncoherent demodulation is examined when perfect-side information is available in an AWGN and PNI environment.
In this chapter, we examine the performance of the receiver in the presence of PNI and AWGN with perfect side information (PSI). As mentioned previously, PSI is the certain knowledge of which specific hops are jammed and which are not and is a means to reduce the effect of PNI. For practical reasons, we model PNI as Gaussian noise that is turned on and off periodically.

A. PERFORMANCE IN AWGN AND PNI WITH PSI

For a channel with a diversity of two in an environment of both AWGN and PNI with PSI, the probability of channel symbol error is given by (2.16). Accordingly, for a diversity of two we have

\[ P_s = (1 - \rho)^2 p_s(0) + 2\rho (1 - \rho) p_s(1) + \rho^2 p_s(2) \]  

(5.1)

where \( p_s(0) \), \( p_s(1) \), and \( p_s(2) \) are the conditional probabilities of channel symbol error for \( i = 0, 1, 2 \) hops corrupted by PNI, respectively. These are given by

\[
p_s(i) = 1 - \sum_{p=0}^{M-1} \left[ \left( \sum_{q=0}^{P-1} \frac{(-p \cdot m \cdot E_c)}{(p+1) \cdot N_o} \right)^{q+1} \frac{1}{(q-w)!} \left( \frac{m - \frac{LE_c}{N_o + (iN_f/L_p)}}{m} \right)^w \right],
\]

or

\[
p_s(i) = 1 - \sum_{p=0}^{M-1} \left[ \left( \sum_{q=0}^{P-1} \frac{(-p \cdot m \cdot E_c)}{(p+1) \cdot N_o} \right)^{q+1} \frac{1}{(q-w)!} \left( \frac{m - \frac{E_b}{N_o + (iN_f/L_p)}}{m} \right)^w \right],
\]

(5.2)

since \( E_b = LE_c \).
Analytically, $p_s(0)$ is the conditional probability of channel symbol error when PNI is not present in either diversity reception and is given by (5.3) for $i = 0$ and $L = 2$ as follows

$$p_s(0)=1-\sum_{p=0}^{M-1}(-1)^p\left(\sum_{q=0}^{p}\binom{p}{q}\right) q! \exp\left(-\frac{p}{p+1} \frac{mE_b}{N_0}\right) \sum_{w=0}^{q+1}(q+1)!\frac{1}{(p+1)^{w+2}} \left(\frac{mE_b}{N_0}\right)^w$$  (5.4)

where the RS code $r = 15 / 31$, $m = 5$ and $M = 32$.

For the conditional probability of channel symbol error $p_s(1)$ when only one of the diversity receptions is affected by PNI, the corrupted reception is discarded as a result of the advantageous knowledge of PSI. The diversity reception that we take into account is affected by AWGN only. In this case, the conditional error probability of channel symbol error is given by (5.2) for $L = 1$ (only one pulse received, since the other is discarded) and $i = 0$ (no pulse affected by PNI) to get

$$p_s(1)=1-\sum_{p=0}^{M-1}(-1)^p\left(\sum_{q=0}^{p}\binom{p}{q}\right) q! \exp\left(-\frac{p}{p+1} \frac{mE_b}{2N_0}\right) \sum_{w=0}^{q+1}(q+1)!\frac{1}{(p+1)^{w+2}} \left(\frac{mE_b}{2N_0}\right)^w$$  (5.5)

or

$$p_s(1)=1-\sum_{p=0}^{M-1}(-1)^p\left(\sum_{q=0}^{p}\binom{p}{q}\right) q! \exp\left(-\frac{p}{p+1} \frac{mE_b}{N_0+(N_f/\rho)}\right) \sum_{w=0}^{q+1}(q+1)!\frac{1}{(p+1)^{w+2}} \left(\frac{mE_b}{N_0+(N_f/\rho)}\right)^w$$  (5.6)

in terms of average bit energy.

Finally, $p_s(2)$ is the conditional probability of channel symbol error when both diversity receptions are corrupted by PNI and is given by (5.3) for $i = 2$ and $L = 2$ as follows:

$$p_s(2)=1-\sum_{p=0}^{M-1}(-1)^p\left(\sum_{q=0}^{p}\binom{p}{q}\right) q! \exp\left(-\frac{p}{p+1} \frac{mE_b}{N_0+(N_f/\rho)}\right) \sum_{w=0}^{q+1}(q+1)!\frac{1}{(p+1)^{w+2}} \left(\frac{mE_b}{N_0+(N_f/\rho)}\right)^w$$  (5.7)
or

\[ p_s(2) = 1 - \sum_{p=0}^{M-1} (-1)^p \left( \begin{array}{c} M-1 \\ p \end{array} \right) \sum_{q=0}^{p} \binom{p}{q} q! \exp \left( \frac{-p}{p+1} \left( \frac{1}{r m E_b N_o} \right)^{-1} + \frac{1}{\rho} \left( \frac{r m E_b N_f}{N_f} \right)^{-1} \right) \]

\times \sum_{w=0}^{q} \frac{1}{w!} \left( \frac{q+1}{q-w} \right) \frac{1}{(p+1)^{w+q+2}} \left( \frac{1}{(r m E_b N_o)^{-1}} + \frac{1}{\rho} \left( \frac{r m E_b N_f}{N_f} \right)^{-1} \right)^w. \quad (5.8) \]

Substituting (5.4), (5.6) and (5.8) into (5.1), we derive the probability of channel symbol error.

**B. NUMERICAL RESULTS**

1. **Performance with PSI when** \( E_b / N_o = 15 \) dB

   The performance of 32-ary orthogonal signaling in an AWGN and PNI environment with PSI for \( E_b / N_o = 15 \) dB is shown in Figure 7.
Figure 7. Performance of 32-ary orthogonal signaling with (31, 15) RS encoding with PSI for different values of $\rho$ in both AWGN and PNI with $E_b/N_0 = 15$ dB.

It can be seen $E_b/N_0$ increases as $\rho$ decreases for $0.5 \leq \rho \leq 1$. For example, $\rho = 1$ gives $E_b/N_0 = 7.2$ dB, compared to $\rho = 0.7$ which gives $E_b/N_0 = 7.9$ dB. It can also be observed that at $P_b = 10^{-5}$, varying $\rho$ from 1 to 0.5 does not degrade the receiver performance significantly as compared to barrage jamming ($\rho = 1$). The degradation is 0.5 dB and 0.7 dB for $\rho = 0.7$ and $\rho = 0.5$, respectively. For smaller values of $\rho$, such as $\rho = 0.3$, however, the performance of the receiver is improved. At $P_b = 10^{-5}$, compared to barrage jamming there is an improvement of about 1.1 dB.
2. **Performance with and without PSI when $E_b/N_o = 10$ dB**

In Figure 8, the performance of the receiver with and without PSI for $E_b/N_o = 10$ dB is presented.

![Figure 8](image_url)

Figure 8. Performance of 32-ary orthogonal signaling with (31, 15) RS encoding with and without PSI for different values of $\rho$ in both AWGN and PNI with $E_b/N_o = 10$ dB.

The numerical results for Figure 8 are shown in Table 2.
Table 2. Performance of 32-ary orthogonal signaling with (31, 15) RS encoding with and without PSI for different values of $\rho$ in both AWGN and PNI, with $E_b / N_o = 10$ dB at $P_b = 10^{-5}$.

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$E_b / N_o = 10$ dB at $P_b = 10^{-5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$E_b / N_i$ dB</td>
</tr>
<tr>
<td></td>
<td>PSI</td>
</tr>
<tr>
<td>1</td>
<td>9.3</td>
</tr>
<tr>
<td>0.7</td>
<td>10.3</td>
</tr>
<tr>
<td>0.5</td>
<td>10.4</td>
</tr>
<tr>
<td>0.3</td>
<td>9</td>
</tr>
</tbody>
</table>

*a. Comparison of Performances with $E_b / N_o = 15$ dB and $E_b / N_o = 10$ dB*

We observe that when $E_b / N_o = 10$ dB the degradation in performance due to PNI with PSI as compared to barrage jamming is about 1 dB at $P_b = 10^{-5}$ for both $\rho = 0.7$ and $\rho = 0.5$. This means an increase of about 0.4 dB compared to the performance with $E_b / N_o = 15$ dB. The performance for $\rho = 0.3$ is better compared to the performance of barrage jamming (9 dB for $\rho = 0.3$ versus 9.3 dB for $\rho = 1$). However, this slight improvement is by far lesser compared to the performance with $E_b / N_o = 15$ dB. Moreover, the absolute performance for various values of $\rho$ is worse in the case with $E_b / N_o = 10$ dB versus the one with $E_b / N_o = 15$ dB. In numerical terms, this is a degradation of 2 dB for $\rho = 1$ to about 4 dB for $\rho = 0.7$ and $\rho = 0.5$ at $P_b = 10^{-5}$.
b. **Comparison of Performances between the Receptions with PSI and without PSI when** $E_b / N_o = 10$ dB

When $\rho = 1$ (barrage jamming), there is no difference in performance whether PSI is used or not. This is because the probability of channel symbol error is solely dependent on the conditional probability $p_s(2)$, which is the case where both diversity receptions are corrupted by PNI and is the same both with and without PSI. For $\rho = 0.7$ and $\rho = 0.5$, there is an improvement in performance with PSI compared to the one without PSI of about 1.7 dB at $P_b = 10^{-5}$. This is an expected outcome since PSI is a technique used to reduce the effect of PNI. It can also be observed that in both cases (with and without PSI) the plots for $\rho = 0.7$ and $\rho = 0.5$ converge at $P_b = 10^{-5}$. For $\rho = 0.3$, we have $E_b / N_i = 9$ dB and 10.6 dB for PSI and no PSI, respectively. As expected, there is a gain in the case with PSI, which is about 1.6 dB. Thus, for all values of $\rho < 1$ under examination, there is an improvement in performance when PSI is available.

3. **Performance with and without PSI for** $E_b / N_o = 9$ dB

The performance of 32-ary signaling in an AWGN and PNI environment with and without PSI for $E_b / N_o = 9$ dB is shown in Figure 9.

a. **Comparison of Performances with**

$E_b / N_o = 15$ dB,

$E_b / N_o = 10$ dB and $E_b / N_o = 9$ dB

When $E_b / N_o = 9$ dB, the degradation in performance due to PNI as compared to barrage jamming is 3.3 dB and 6.2 dB at $P_b = 10^{-5}$ for $\rho = 0.7$ and $\rho = 0.5$, respectively. Not only is there a greater degradation compared to the previous cases ($E_b / N_o = 15$ dB and $E_b / N_o = 10$ dB), but it can also be seen that there is a rapid degradation in the performance between $\rho = 0.7$ and $\rho = 0.5$, a phenomenon, which was also observed when the presence of PNI without PSI was examined in Chapter IV.
Moreover, the performance of the receiver for $\rho = 0.3$ is worse compared to barrage jamming by about 2.6 dB at $P_b = 10^{-5}$. A comparison with $E_b / N_o = 10$ dB for $\rho = 0.3$ shows an expected degradation, which is about 4.2 dB. Here again, the absolute performance for various values of $\rho$ is worse compared to those obtained for greater values of $E_b / N_o$. In a manner similar to the case without PSI, we can say that
with $E_b / N_o = 9$ dB the receiver reaches a critical point at which performance is vulnerable to PNI. It is also observed that for values of $E_b / N_f$ greater than 20 dB, the performances with and without PSI start to converge.

**b. Comparison of Performances between the Receptions with PSI and without PSI when $E_b / N_o = 9$ dB**

Here again, when $\rho = 1$ (barrage jamming) there is no difference in performance whether PSI is used or not. For $\rho = 0.7$ and $\rho = 0.5$, there is an improvement in performance with PSI compared to the one without PSI of about 2.9 dB and 3.9 dB, respectively, at $P_b = 10^{-5}$. Thus, it is shown again that the use of PSI reduces the effect of PNI. For $\rho = 0.3$, we have $E_b / N_f = 13.2$ dB and 14.8 dB for PSI and no PSI, respectively, at $P_b = 10^{-5}$. As expected, there is a gain in the case with PSI, which is about 1.6 dB. This means that for all values of $\rho < 1$ under examination there is an improvement in performance when PSI is available.

The numerical results of Figure 9 are presented in Table 3.

Table 3. Performance of 32-ary orthogonal signaling with (31, 15) RS encoding with and without PSI for different values of $\rho$ in both AWGN and PNI, with $E_b / N_o = 9$ dB at $P_b = 10^{-5}$.

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$E_b / N_f$ dB</th>
<th>$E_b / N_f$ dB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PSI</td>
<td>No PSI</td>
</tr>
<tr>
<td>1</td>
<td>10.6</td>
<td>10.6</td>
</tr>
<tr>
<td>0.7</td>
<td>13.9</td>
<td>16.8</td>
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<td>0.5</td>
<td>16</td>
<td>19.9</td>
</tr>
<tr>
<td>0.3</td>
<td>13.2</td>
<td>14.8</td>
</tr>
</tbody>
</table>
C. CHAPTER SUMMARY

In this chapter, the effects of AWGN and PNI with PSI on the performance of the compatible JTIDS waveform for noncoherent demodulation were examined. This required the evaluation of the conditional probabilities of channel symbol error for $i = 0, 1$ and 2 hops affected by pulsed-noise interference. The key point of this analysis was that the knowledge of PSI simplified the evaluation of the conditional probability of channel symbol error for $i = 1$ hop experiencing PNI, since the specific hop was discarded.

The numerical observations made are summarized as follows. For comparatively large values of $\rho$ ($0.5 \leq \rho \leq 1$), it was shown that the required $E_b/N_t$ for $P_b = 10^{-5}$ increases as $\rho$ decreases. We recall that the same observation was made when the presence of PNI without PSI was examined in the previous chapter. Moreover, the absolute performance of the receiver for various values of $\rho$ degrades in an analogous manner as values of $E_b/N_o$ decreases. In all cases investigated for $\rho < 1$, when PSI was available there was an improvement in the performance of the receiver compared to when PSI was nonexistent. This confirmed the fact that having PSI is valuable in order to reduce the effect of PNI. The comparison between $E_b/N_o = 10$ dB and $E_b/N_o = 9$ dB for all values of $\rho$ illuminated an increased degradation in the performance of the receiver for $E_b/N_o = 9$ dB. This implied that for smaller values than $E_b/N_o = 9$ dB the degradation would increase rapidly, a fact which was confirmed when such cases were examined.

In the next chapter, this thesis will offer a summary of the conclusions made in Chapters III, IV and V, as well as recommendations for future work.
VI. CONCLUSIONS AND FUTURE WORK

A. CONCLUSIONS

This thesis presented the analysis of a JTIDS/Link-16 compatible waveform, utilizing 32-ary orthogonal signaling with (31, 15) RS coding and noncoherent detection. A sequential diversity of two, consistent with the double-pulse structure of JTIDS, was also considered. The type of side information considered was perfect side information.

Initially, the performance of the compatible JTIDS waveform with a diversity of two and detected noncoherently in an AWGN environment was investigated. A comparison was conducted between the double-pulse structure waveform and the single-pulse structure waveform. As expected, it was shown that the double-pulse structure outperforms the single-pulse structure, thus, verifying that diversity is an effective technique employed in noisy environments.

Consequently, the effects of both AWGN and PNI on the performance of the JTIDS compatible waveform for noncoherent demodulation were examined. This required the evaluation of the conditional probabilities of channel symbol error for \(i = 0, 1\) and 2 hops affected by pulsed-noise interference. The numerical results showed that for large values of \(\rho (0.5 \leq \rho \leq 1)\) the required \(E_b/N_0\) increases as \(\rho\) decreases for a fixed \(P_b\). The analysis also showed that with \(E_b/N_0 = 9\) dB the degradation increases rapidly for values of \(\rho\) smaller than 1. Finally, it was observed that the absolute performance of the receiver for various values of \(\rho\) degrades in an analogous manner as \(E_b/N_0\) decreases.

It was demonstrated that the existence of PSI is beneficial to tactical datalink users, since PSI reduces the effect of PNI. In order to evaluate the gains as a result of PSI, we derived an expression for the performance of the JTIDS compatible waveform in an AWGN plus PNI environment. This required the evaluation of the conditional probabilities of channel symbol error for \(i = 0, 1\) and 2 hops affected by pulsed-noise interference. The knowledge of PSI simplified the evaluation of the conditional
probability of channel symbol error for $i=1$ hop experiencing PNI. The numerical analysis showed that for comparatively large values of $\rho$, $E_b/N_t$ increases as $\rho$ decreases. Moreover, the absolute performance of the receiver for various values of $\rho$ degrades in an analogous manner as values of $E_b/N_o$ decreases. In all cases investigated with $\rho < 1$, when PSI was available there was an improvement in the performance of the receiver compared to when PSI was nonexistent.

Summarizing, the general fact that the double-pulse structure outperforms the single-pulse structure was demonstrated for the JTIDS compatible waveform under examination. PNI degrades the performance of the receiver, but the existence of PSI is beneficial for tactical datalink users, since it reduces the effect of PNI. It is known that the JTIDS/Link-16 waveform is currently received noncoherently. Given the fact that the compatible JTIDS waveform examined in this thesis is noncoherently detected and employs a double-pulsed structure that outperforms the single-pulse structure waveform (also noncoherently detected), the analysis offered in this thesis could potentially be beneficial for JTIDS/Link-16 designers.

B. FUTURE RESEARCH AREAS

We examined a compatible JTIDS/Link-16 waveform that consists of 32-ary orthogonal signaling with $(31, 15)$ RS coding detected noncoherently. This waveform employs a double-pulse structure that provides an improvement in performance over the single-pulse structure JTIDS/Link-16 waveform.

We observed that the examination of the effects of AWGN and PNI on the performance of the JTIDS compatible waveform for noncoherent demodulation presented some computational difficulties. The numerical results showed that the techniques used to overcome these computational difficulties provided reasonable results for a specific range of $\rho$, that is $1 \leq \rho \leq 0.3$. However, the results obtained for $\rho < 0.3$ were not as expected. Accordingly, this remains an area for future research.

Possible future work could also be to examine changing the code rate while keeping the block length the same in order to see if the throughput can be increased.
without reducing performance. Moreover, a concatenated code in place of the (31, 15) RS block code could be used, perhaps using a (31, 15) RS code as the inner code, to see if performance can be improved without decreasing throughput significantly.
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