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Passive and active sonar prosecution of diesel submarines by nuclear submarines

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PASSIVE AND ACTIVE SONAR PROSECUTION OF DIESEL SUBMARINES BY NUCLEAR SUBMARINES

by

Erik J. Nelson

March 2008

Thesis Advisor: Arnold Buss
Second Reader: Thomas Lucas

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This study analyses the trend for initial detection times using both passive and active sonar during submarine–on-submarine operations. Specifically, it simulates a nuclear powered submarine (SSN) searching for a diesel submarine in an environment where the SSN has a speed advantage and active sonar detection ranges exceed passive sonar detection ranges. The simulation uses a mover-sensor discrete event application of SIMKIT, developed by Professor Arnold Buss.

The simulation results show that initial detection times of a search follow an exponential trend as a function of SSN speed, diesel submarine speed, detection ranges, ping interval and detection probability. As a result, as detection ranges continue to decrease due to increases in sound quieting technology, initial detection times during a submarine search will increase exponentially. This can render a passive sonar prosecution ineffective when combating a modern diesel submarine. Should an SSN use active sonar, initial detection times can be significantly reduced, especially if combined with an effective search path. The threat to the SSN of using active sonar can be mitigated by judicious consideration of ping interval and search speed with detection probability and active detection ranges. All values used to arrive at the conclusions stated are notional, and no classified information sources were consulted as part of this work.
PASSIVE AND ACTIVE SONAR PROSECUTION OF DIESEL SUBMARINES
BY NUCLEAR SUBMARINES

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ABSTRACT

This study analyses the trend for initial detection times using both passive and active sonar during submarine-on-submarine operations. Specifically, it simulates a nuclear powered submarine (SSN) searching for a diesel submarine in an environment where the SSN has a speed advantage and active sonar detection ranges exceed passive sonar detection ranges. The simulation uses a mover-sensor discrete event application of SIMKIT, developed by Professor Arnold Buss.

The simulation results show that initial detection times of a search follow an exponential trend as a function of SSN speed, diesel submarine speed, detection ranges, ping interval and detection probability. As a result, as detection ranges continue to decrease due to increases in sound quieting technology, initial detection times during a submarine search will increase exponentially. This can render a passive sonar prosecution ineffective when combating a modern diesel submarine. Should an SSN use active sonar, initial detection times can be significantly reduced, especially if combined with an effective search path. The threat to the SSN of using active sonar can be mitigated by judicious consideration of ping interval and search speed with detection probability and active detection ranges. All values used to arrive at the conclusions stated are notional, and no classified information sources were consulted as part of this work.
DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the planner.
# TABLE OF CONTENTS

I. INTRODUCTION ........................................................................................................1  
A. PROBLEM STATEMENT .............................................................................1  
B. RESEARCH FOCUS .......................................................................................2  
C. OUTLINE .........................................................................................................3  
D. EXPECTED OUTCOME OF RESEARCH ..................................................4  

II. RELEVANT SUBMARINE DESCRIPTIONS .........................................................5  
A. SUBMARINES OF THE UNITED STATES ................................................5  
   1. Los Angeles Class Submarine ................................................................5  
   2. Seawolf Class Submarine ....................................................................5  
   3. Virginia Class Submarine ...................................................................6  
B. DIESEL SUBMARINES .................................................................................6  
   1. Kilo Submarine ....................................................................................6  
   2. Song Submarine ...................................................................................6  

III. SIMULATION MODEL AND SCENARIO .............................................................7  
A. SIMULATION MODEL .................................................................................7  
   1. SIMKIT .................................................................................................7  
   2. Submarine Representation ....................................................................7  
   3. Sonar Detection Representation .........................................................8  
   4. Parameter Input ...................................................................................10  
   3. Model Intent .......................................................................................12  
B. SCENARIO DESCRIPTION ........................................................................12  

IV. SIMULATION ANALYSIS ......................................................................................15  
A. RANDOM PASSIVE SEARCH ...............................................................15  
B. RANDOM ACTIVE SEARCH ....................................................................21  
C. SEARCH PATH EFFECTS ON TIME TO INITIAL DETECTION ......25  
D. PING INTERVAL ANALYSIS ....................................................................29  
   1. Ping Interval vs. SSN Speed ..............................................................30  
   2. Ping Interval vs. Detection Probability ............................................30  
   3. Ping Interval vs. Active Range ..........................................................32  

V. CONCLUSIONS ........................................................................................................33  

APPENDIX .............................................................................................................35  

BIBLIOGRAPHY .........................................................................................................47  

INITIAL DISTRIBUTION LIST .........................................................................................49
LIST OF FIGURES

Figure 1. Visual Representation of Initial Model Generation ..............................................11
Figure 2. Visual Representation of Initial Model Detection .................................................12
Figure 3. Passive Random Search Detection Time ..............................................................17
Figure 4. Full Regression of Passive Random Search .........................................................19
Figure 5. Regression of Passive Random Search Without Kilo Speed ..................................20
Figure 6. Active Random Search Detection Times .............................................................22
Figure 7. Full Regression of Active Random Search .........................................................24
Figure 8. Active vs. Passive Detection Times .....................................................................28
Figure 9. Detection Time vs. Ping Interval .........................................................................29
Figure 10. Ping Interval and SSN Speed vs. Detection Time ...............................................30
Figure 11. Ping Interval and Detect Prob. Vs. Detection Time .............................................31
Figure 12. Ping Interval and Active Range vs. Detection Time .........................................32
Figure 13. Linear Passive Sonar Regression .....................................................................40
Figure 14. Log-Linear Passive Sonar Regression ..............................................................41
Figure 15. Multivariate Analysis in Log-Linear Passive Model .........................................42
Figure 16. Linear Active Sonar Regression ........................................................................43
Figure 17. Log-Linear Active Sonar Regression ...............................................................44
Figure 18. Multivariate Analysis in Log-Linear Active Model ...........................................45
LIST OF TABLES

Table 1. Random Active Search Results.................................................................16
Table 2. Random Active Search Results.................................................................22
Table 3. Comparison of Search Paths .................................................................26
Table 4. Comparison of Random and Spiral-Out Search Paths.............................27
Table 5. Ping Interval and Speed vs. Detection Time..............................................46
Table 6. Ping Interval and Active Detection Range vs. Detection Time.................46
EXECUTIVE SUMMARY

There has been an increase in production worldwide of modern diesel submarines. Nations have found that these ships can provide a relatively inexpensive ability to both defend local coastlines and provide a credible anti-surface warfare threat. Traditionally, the nuclear submarine fleet (SSNs) has been trained to passively detect and track opposing submarine forces. As these passive detection ranges continue to decrease with modern sound-silencing techniques, the ability to detect these diesel submarines diminishes, and it must be determined how to properly utilize SSNs to search for these threats.

This thesis uses discrete event simulation to observe trends in both passive and active sonar detection. The simulation recreates a submarine–on-submarine operation, with an SSN searching for a diesel submarine until initial detection occurs. The simulation is performed using SIMKIT, a software simulation tool developed by Professor Arnold Buss of the Naval Postgraduate School.

The initial analysis focuses on performing a random search of a diesel submarine using first passive sonar detection, then active sonar detection. This analysis shows that initial detection time follows an exponential relationship between detection ranges and submarines’ operating speed. Additionally, ping interval and probability of a ping resulting in detection contribute to the exponential effect when using active sonar. Due to this relationship, it can be concluded that as passive detection ranges continue to decrease, the time needed to locate a target submarine will increase exponentially. It also shows that as the detection range of active sonar exceeds that of passive sonar by a factor determined by the conditions in which active sonar is implemented, it rapidly reduces the time to initial detection.

The second part of the analysis explores how the search path affects the time to initial detection when using active sonar. It was found that a "lawnmower" search path—that is, boundary to boundary sweeps as the submarine travels down an area—did not improve performance above that of a random search. Though diesel submarine speed is
relatively slow when submerged, it has sufficient speed in some instances to cross the perimeter of an area previously searched by the searching submarine, especially with long ping intervals and lower detection probabilities. However, a "spiral-out" search path was found to significantly reduce initial detection time by making it less likely for a diesel submarine to enter previously searched areas. This is due to the perimeter of the searched area being initially small and increasing linearly as the search progresses.

The final analysis involves how ping interval is affected by other ships’ parameters during a search. If the Commanding Officer deems it necessary to minimize the ping interval to be able to minimize the risk to his own ship through the use of active sonar, other factors will change the effects of the exponential contribution of ping interval to detection time. With a long detection range, a reduction in the SSN's search speed as well as a moderate to high detection probability will allow for a lengthening of the time between pings with a relatively small increase in detection time.

The conclusions of the thesis are:

- The initial time to detect a target is an exponential function based on detection range, speed of the participants, and in the case of active sonar, ping interval and detection probability. As a result, as the passive detection range of diesel submarines continues to decrease, the average detection time will increase exponentially. A point exists where a passive prosecution is no longer feasible, and either other assets need to be employed or an active search could be performed to maintain a reasonable detection time.

- The speed of a submerged diesel submarine is limited; however, it is sufficient to make many search patterns no more effective than a random search if the diesel can enter previously searched areas. A "spiral-out" path can provide an additional decrease in detection time, as it provides an initially small perimeter that the diesel can cross in order to enter this previously searched area. The simulations suggest that an active search
can yield detection times at a fraction of passive searches, but real world exercises are needed to determine the breakpoint at which this occurs.

- If the Commanding Officer deems it necessary to reduce ping interval to minimize the threat of being counter-detected, the following factors (listed in order of priority) will mitigate the effect of the exponential increase in detection time:

  1. A long active detection range
  2. Traveling at slower search speeds
  3. A moderate to high detection probability
ACKNOWLEDGMENTS

This research and thesis could not have been completed without the encouragement and effort provided by my thesis advisor, Associate Professor Arnold Buss. I am extremely grateful for his extended patience in the development and analysis of the model on which this thesis is based.

I would also like to thank Senior Lecturer Jeff Kline for guiding my thesis subject, and for his assistance locating interested agencies.

Additionally, Professor Samuel Buttrey's assistance in the statistical analysis of the simulation's output was invaluable. Without his assistance, the analysis of the simulation would not have been nearly as successful.
I. INTRODUCTION

This chapter summarizes the thesis. It includes the problem statement, the focus of the research performed, and the expected outcome of the analysis.

A. PROBLEM STATEMENT

When a submarine is conducting search operations in an attempt to detect another submarine, it is generally conducted using passive sonar. This method detects the target submarine once its radiated noise level exceeds that of the ambient noise of the environment by way of a receiver mounted on the searching vessel. Another method that is available is the use of active sonar, which emits a signal into the environment that reflects off a target submarine and is received by the searching submarine. This places the searching submarine at risk, as this signal is also generally heard by the target, who may then evade or act against the searcher. With modern diesel submarines, however, the radiated noise it emits is generally very small, making passive detection difficult. During events in which detection of these submarines is vital or the threat of the diesel submarine is relatively low, active sonar is a tactical option that must be considered during the prosecution of diesel submarines.

Currently, many nations are developing or purchasing modern diesel-electric submarines that are extremely quiet when submerged and operating on the battery. Recent technological advances allow them to remain submerged for extended periods of time at low speeds. Their low radiated noise has reduced the passive detection ranges to the point that the effectiveness of passive prosecution is substantially reduced, especially in areas of high ambient noise.

The current fleet of U.S. nuclear powered attack submarines (SSN) is equipped with active sonar. As stated previously, active sonar effectively broadcasts the location of the SSN, placing it at risk if the diesel submarine is able to obtain the bearing of the SSN based on the radiated noise. The slow speed of the diesel submarine makes evasion difficult, but, if equipped with a capable weapons system, can allow the diesel to fire
upon the SSN. Given this risk, active search is still a viable option for specific high-threat situations involving a diesel submarine or against those diesels that are less capable of successfully engaging a modern nuclear-powered submarine.

B. RESEARCH FOCUS

This thesis uses discrete event simulation to analyze the use of passive and active sonar to search for and detect diesel submarines by SSNs. The proliferation of diesel submarines throughout the world poses significant challenges to the U.S. dominance of the sea. Though unable to travel and operate in vast expanses of the ocean as effectively as nuclear powered submarines, diesel submarines are adept at patrolling local littoral waters and provide a credible Anti-Submarine Warfare (ASW) and Anti-Surface Warfare (ASUW) threat. Though incurring risk, an active sonar prosecution is viable in several scenarios to reduce the time to locate diesel submarines. Examples of these include but are not limited to:

- The deployment of significant numbers of diesel submarines by a nation in which, though a state of war does not exist, tensions exist. This assumes the opposing nation has not authorized the diesel submarines to engage any tracking submarines.

- Diesel submarines are equipped with weapons that cannot effectively engage SSNs. For example, their torpedoes are not effective against SSN countermeasures or are designed/designated for a high priority surface target.

- A nation sponsoring terrorism has supplied a diesel submarine to terrorists which intend to conduct operations in the immediate future.

The lack of use of active sonar in recent years has likely led to atrophy in Commanding Officers’ experience in effectively using active sonar, specifically in the knowledge of the benefit (reduction in detection times) that can be gained, and the proper employment of active sonar.
An analysis of the trends of detection times using both passive and active sonar provides a guide to the Commanding Officer such that future options have a greater chance of success in detecting diesel submarines. The analysis also provides information on how to reduce the risk of counter-detection by minimizing the frequency at which a ping needs to be emitted using an active sonar search.

C. OUTLINE

This thesis is organized into five chapters. Chapter I is divided into four sections. The first section describes the problem statement. The second section explains the focal point of the research. The third section gives a brief overview of each chapter in this thesis. The fourth section explains the expected research outcomes.

Chapter II explores the different participants that are considered in this study. A description of the current U.S. SSN fleet is given. Selected diesel submarines common in the world and their capabilities are also discussed.

Chapter III defines the discrete event simulation and scenario. In the first section, the simulation program is explained, along with the parameters and the assumptions designated for the simulation. The second section discusses the scenario used as the basis of the specific simulation.

Chapter IV covers the analysis of the simulation results, and contains four sections. The first section describes the passive sonar search of a diesel submarine. The result of the first section is used as the baseline for further evaluation of following searches. The second section analyzes the active sonar search and compares it to those of the passive sonar search. The third section analyzes a simulation of an active sonar search using different search paths. The fourth section examines ping interval, and how altering ping interval and associated search parameters affects initial detection.

Chapter V summarizes the results obtained from simulations. The conclusions provide a guideline for future sonar searches, and areas of further research are identified.

The Appendix includes results from the simulation runs and the regressions performed on the data.
D. EXPECTED OUTCOME OF RESEARCH

The thesis explains the trend of initial detection rates as a function of the parameters of passive and active sonar searches. The results can be used as a guide in future sonar searches by Commanding Officers to provide a better understanding of how search type, parameters and actions will affect the time to detect a target submarine. It can also be used by senior naval personnel to determine which assets can be effectively utilized to conduct a search for diesel submarines.
II. RELEVANT SUBMARINE DESCRIPTIONS

In order to establish context, this chapter provides a brief introduction of the submarines considered in the analysis. This chapter is not intended to thoroughly examine each submarine and its capabilities, but rather to explain relative performances and provide a background for the model.

A. SUBMARINES OF THE UNITED STATES

1. Los Angeles Class Submarine

The Los Angeles Class submarine is the mainstay of the U.S. Navy's attack submarine fleet, with 46 currently commissioned.\textsuperscript{1,2} This will remain in effect in the foreseeable future, as its replacement, the Virginia Class submarine, is currently at a production rate of two per year.\textsuperscript{3}

2. Seawolf Class Submarine

There are three Seawolf Class submarines in the U.S. Navy, with no further construction planned. This submarine class was designed to conduct Cold-War era submarine warfare, and is faster, better armed, and has an improved sonar system compared to the Los Angeles Class submarine.\textsuperscript{4}


\textsuperscript{4} Saunders, 883.
3. **Virginia Class Submarine**

Virginia Class submarines will eventually replace Los Angeles Class submarines as the primary attack submarine in the U.S. fleet. Though not as fast and well-armed as the Seawolf Class, its modular design allows it to act in multiple roles in most ocean environments.5

**B. DIESEL SUBMARINES**

1. **Kilo Submarine**

Countries that maintain a Kilo class submarine in service include China and Iran. This dual-hulled submarine has an at-sea endurance of 45 days, and can perform multiple missions, from mine-laying to anti-naval operations. It is one of the significant assets sold by Russia to foreign nations.6

2. **Song Submarine**

In October 2006, a Song class submarine surfaced within five miles of the aircraft carrier Kitty Hawk (CV-63). The Song Class is the first modern-era Chinese-built submarine, and is armed with wake homing torpedoes and anti-ship cruise missiles. The incident emphasized the threat that quiet diesel submarines pose to carrier strike groups.7

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III. SIMULATION MODEL AND SCENARIO

This chapter explains the model that is used in the thesis. The program used in the simulation, the construction of each entity, how parameters are implemented in the model, and the scenario are discussed. Assumptions that are made in the model are explicitly stated.

A. SIMULATION MODEL

1. SIMKIT

The simulations performed in this thesis were developed using the Java-based program SIMKIT, developed by Professor Arnold Buss of the Naval Postgraduate School. The discrete event simulations execute an event graph, in which subsequent actions are appended onto an event list. These actions are continuously ordered and performed chronologically, and time periods without events do not consume computational overhead. The benefit of discrete event simulation is the ability to perform multiple simulations in a fraction of the time of equivalent time-step simulations, since the model calculations are performed after each event rather than after each time increment. In this thesis, over 36,000 distinct simulations were performed, representing millions of total simulated hours.

2. Submarine Representation

The submarines are represented as uniform linear movers. A more complicated representation is not necessary in this model for two reasons: First, the time scale of this model (based on the size of the simulation area and speed of the vessels involved) do not require a detailed analysis of acceleration, deceleration, advance, and transfer. Second, the ability to change depth is not analyzed in the model because any change in depth can be represented as a new two-dimensional problem, as depth changes should only have an effect on the sonar detection range and the probability of receiving a signal.
A uniform linear mover starts its move at position $x$ at time $t_o$ and starts to move with velocity $v$. As a result, its location at time $t$ is $x + (t - t_o)v$. As a result, the position of the mover does not have to be explicitly stored at all times in the model, but rather can be calculated as needed.

A separate mover manager maintains the scheduling of all movements in the simulation associated with the specific mover. As such, this manager is able to store a set of waypoints that the mover is to follow during the conduct of the simulation.

Though submerged diesel submarines can operate above 10 knots, this places a significant drain on the battery and cannot be sustained for long periods of time. Therefore, the simulation assumes the diesel submarine travels between two and five knots. The SSN is assumed to have a speed advantage over the diesel submarine, and will travel between five and fifteen knots (fifteen knots giving a 3:1 speed advantage over the diesel's highest speed).

For the random passive and active sonar searches, each submarine is randomly generated in the simulation area. The Kilo performs random maneuvers during all simulations. This occurs even if the Kilo is pinged by active sonar. Though the Kilo may determine that it is being pinged, it is assumed that it is unable to resolve the bearing from which the ping originates. The SSN also maneuvers randomly during the initial active searches to provide a baseline for further analysis, in which definitive search paths during active sonar prosecution are performed.

3. Sonar Detection Representation

Both active and passive sonar systems rely on the principle of signal excess. In all underwater detection, detection range relies on multiple factors depending on the target position, searcher position, and ocean characteristics. Bottom depths, ocean bottom contour, composition of the ocean floor, target aspect, and several other factors all have an effect on detection range.

In passive systems, a receiver is mounted to the hull of the searching submarine. As radiated noise leaves the target submarine, its intensity decreases as a result of
spreading and attenuation losses. A receiver mounted on the searching submarine receives the signal and is processed to be evaluated by a sonar operator. If the radiated noise level is sufficiently greater than the ambient noise level, detection will occur. This detection relies on the operator being able to isolate and recognize the radiated noise as that of the target submarine. In passive sonar detection, continuous monitoring of the passive sonar system leads to very quick detection once the source (diesel) travels within the range that the signal-to-noise ratio is above that required for detection. As a result, passive sonar can be reasonably represented as a cookie-cutter sensor. The detection ranges vary widely between the types of submarines and operating environments, and the simulations assume a notional detection range between 0.5 and 3 nautical miles (nm).

With active sonar, the SSN will emit a ping (radiated noise) that will travel, reflect off a target, and return to the SSN. If the return signal is sufficiently strong, it will be read as a detection event. Pings also suffer from spreading and attenuation losses. Though this limits the range that active sonar can be effectively used, the source signal generated is such that its effective range will be greater than passive range.

Clutter such as sea mounts and biologics can distort, reduce, or confuse the signal such that detection is not certain even if the target is within the range required for detection. Increased training can assist in the ability to discriminate a return signal as the target, but cannot eliminate all errors. As a result, when a ping occurs and the target is within the detection range of the SSN, there is an associated probability that the ping both returns to the SSN and is then recognized as the target. It is assumed that the Commanding Officer would recognize and not operate active sonar in a poor acoustic environment. It is also assumed that active sonar detection ranges will be greater than passive detection ranges. The simulation uses an active detection range between 5nm and 15nm, a ping interval between 0.01 and 1 hour, and a detection probability of 0.3 to 1.0. Baffle zones are not considered in the simulation, in order to maintain this thesis’s unclassified status. This should not affect results, as the speed advantage of the SSN makes detection of the diesel submarine very unlikely to occur in such an area.

In this simulation, the sensor is a separate object that is attached to the mover. The sensor maintains a list of contacts that have been detected, as well as maintaining the
parameters needed to calculate if a detection event occurs. A separate referee is implemented that will determine whether a detection occurs as a mover enters the range of the sensor. Similarly to a mover manager, the referee calculates and stores the times at which a mover will enter detection range, be detected, remain undetected, and exit detection range. As the movers change course, these times are recalculated and updated on the event list.

4. Parameter Input

As submarine speeds, detection ranges, ping intervals, and detection probabilities all act as a continuous range, there are virtually unlimited combinations of these parameters. In order to efficiently ensure that the possible range of these combinations is considered, a Nearly Orthogonal Latin Hypercube (NOLH) was used to input the parameters of the simulation. This provides uniform samples for the marginal distribution of each single input, with the main effects being nearly orthogonal to each other (allowing for greater distinction of these effects). Each data point in the NOLH design was replicated for 200 trials (simulations).

A general representation of the model is shown in Figures 1 and 2. These were obtained directly from the passive detection model to aid in the visual representation of the model. The blue square represents the SSN, and the red square represents the Kilo. The yellow circle surrounding the SSN represents the sensor's range--in this case either passive or active sonar. Each submarine will continue to maneuver until the Kilo penetrates the yellow detection range as shown in Figure 2. In the case of passive sonar, detection will then occur. If the sensor represents active sonar, detection will only occur when a ping is scheduled and a drawn random number is then compared to detection probability. An example of the coding used for this simulation is included in the Appendix.

---

Figure 1. Visual Representation of Initial Model Generation
3. Model Intent

The discrete event simulation construction is optimistic when compared to real world scenarios. Any best possible outcome for a flexible model that can cover multiple current and future scenarios requires having an active detection range that is significantly high. As a result, the area that the simulation covers is a 125nm x 125nm area. It should also be noted that the initial detection times are not predictive, but rather show trends and relationships between factors and tactics.

B. SCENARIO DESCRIPTION

A U.S. Los Angeles class submarine has been tasked with locating and tracking a Kilo submarine prior to the commencement of a carrier strike group exercise. The Kilo has recently been deployed and is believed to be located in an area in which a carrier task
group will be conducting exercises. The exercise will be conducted in a 125nm x 125nm box, and will not include ASW operations. There has been an increase in tension with the nation that has deployed the Kilo, and due to the sensitive nature of the exercise it is desired that any external presence be identified prior to the decision to initiate the exercise. The Kilo has orders to remain in the area unless detected due to the high priority its government places on the mission. The Kilo is unable to resolve the bearing of any active sonar it detects, and as such will move randomly in the area to avoid developing any pattern that can be exploited by searchers.
IV. SIMULATION ANALYSIS

This chapter contains the results and analysis of the various simulations performed in the thesis. The initial simulations include baseline passive and active sonar searches that provide a baseline for further analysis. Later simulations analyze the performance of search patterns in relation to detection times. The final portion of this chapter examines how to effectively utilize active sonar when a minimum ping interval is desired.

A. RANDOM PASSIVE SEARCH

The simulation involves both the SSN and the diesel submarine moving randomly in the designated area. Though a random search path is not optimal, it has historically been shown to be a good representation of reality.\(^9\) This is due to the fact that target speed, though small, is sufficient to provide some evasion capability. This causes most exhaustive search paths to perform no better than a random search. Both submarines are generated at a random position in the simulation area. The SSN travels at a constant speed between 5 and 15 knots, the Kilo travels at a constant speed between 2 and 5 knots, and passive detection range varies from 0.5 to 3 nm. The Kilo does not respond if pinged, which may be interpreted as an inability to localize the direction of the ping received. Each data point was replicated 200 times, and the results of the simulation are shown in Table 1.

---

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<td>808.765</td>
<td>770.54226</td>
</tr>
<tr>
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<td></td>
<td></td>
<td><strong>643.015</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Overall Std Dev</strong></td>
<td></td>
<td></td>
<td><strong>596.585</strong></td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Random Active Search Results

The mean value of detection times over the range of submarine speeds and detection ranges is 643 hours. The standard deviations of the results are not constant, suggesting heteroscedasticity. Additionally, the values of the standard deviations are almost equal to those of the mean detection times. This is a property of the exponential distribution. Figure 3, which charts the detection times as a function of passive detection range, also suggests that detection time looks similar to general exponential decay.
Since the detection time is not a linear function, a linear regression is not appropriate for this data. As a result, an intrinsically linear regression is performed by taking the natural log of initial detection times. The results of the regression are shown in Figure 4. The initial model that includes all factors shows the p-value for the Kilo speed is 0.148. Though common sense would dictate that the Kilo’s speed will affect the time it will be detected, the model suggests that the other factors have a much greater effect, to the point of making the Kilo's speed negligible. A subsequent regression, shown in Figure 5, was performed without the Kilo’s speed. Removing this factor did not change the amount of variation that is explained by the regression (R squared remains at .2324), suggesting that the Kilo’s speed did not explain any more of the variation in detection times. The F statistic p value is essentially zero, implying a statistically significant model. Both passive detection range and SSN speed have individual t statistic p-values of essentially zero, which strongly suggests that these factors are important in predicting initial detection times, with passive detection range being the most important factor. The R squared states that the model only explains 23% of the variability in detection times, but this was expected due to the random submarine location generation and random courses of the submarines involved over such a large area. The formula resulting from the regression is:
Initial Detection Time (hrs) =

\[ 2916.1 e^{7.978 - 0.1 \times (SSN \text{ Speed, knts}) - 0.567 \times (\text{Passive Detection Range, nm})} \]

The conclusion drawn from this equation is that as detection range decreases, the time to initial detection will increase exponentially. This can be partially offset by increasing SSN search speed, but it cannot be overcome. As a result, as the detection range of diesel submarines continue to decrease, it is likely that a passive search using an SSN will be ineffective and will not find the diesel submarine in any reasonable period of time.
Figure 4. Full Regression of Passive Random Search
Figure 5. Regression of Passive Random Search Without Kilo Speed
B. RANDOM ACTIVE SEARCH

This SSN will now use active sonar to search for the Kilo in the area. The Kilo and SSN will move randomly in the area, with the SSN ping interval and detection probability being set at a constant value. The simulation stops upon initial detection of the Kilo. The detection range of active sonar is 5 to 15 nm as compared to the 0.5 to 3 nm of passive sonar. The lower bound was selected because active detection range will always be greater than passive detection range. This will take into account the Commanding Officer’s use of active sonar only if its range is significantly greater than passive detection range. The higher bound was selected to give a 5:1 advantage of maximum passive detection range and maintain the area of simulation at a reasonable size. SSN and Kilo speed and limitations will remain the same as in the passive search.

The results, shown in Table 2, again suggest an exponential trend of average initial detection time. The mean time to initial detection is 110.74 hours, as opposed to 643 hours for the passive random search. Figure 6 also suggests that an exponential trend is reasonable.
Table 2. Random Active Search Results

<table>
<thead>
<tr>
<th>SSN Speed (knots)</th>
<th>Kilo Speed (knots)</th>
<th>Active Range (nm)</th>
<th>Detection Probability</th>
<th>Ping Interval (hours)</th>
<th>Average Initial Detect Time (hours)</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>4</td>
<td>8.8</td>
<td>0.48</td>
<td>0.94</td>
<td>200.81185</td>
<td>195.5241582</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>10.6</td>
<td>0.3</td>
<td>0.32</td>
<td>71.03755</td>
<td>71.5955215</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>7.5</td>
<td>0.74</td>
<td>0.81</td>
<td>133.14265</td>
<td>138.0111488</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
<td>15</td>
<td>0.69</td>
<td>0.13</td>
<td>35.69445</td>
<td>38.76185187</td>
</tr>
<tr>
<td>14</td>
<td>3</td>
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</tr>
<tr>
<td>8</td>
<td>3</td>
<td>13.1</td>
<td>0.34</td>
<td>0.75</td>
<td>78.49365</td>
<td>75.59156854</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>8.1</td>
<td>0.91</td>
<td>0.44</td>
<td>95.82585</td>
<td>97.72846919</td>
</tr>
<tr>
<td>14</td>
<td>4</td>
<td>14.4</td>
<td>0.87</td>
<td>0.63</td>
<td>28.6538</td>
<td>27.6565906</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
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<td>0.65</td>
<td>0.51</td>
<td>72.67396985</td>
<td>70.00585123</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
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<td>0.83</td>
<td>0.07</td>
<td>87.25525</td>
<td>98.2287408</td>
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<td>0.69</td>
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<td>52.07474125</td>
</tr>
<tr>
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<td>5</td>
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<td>0.56</td>
<td>0.2</td>
<td>36.0078</td>
<td>39.35277176</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>5</td>
<td>0.61</td>
<td>0.88</td>
<td>501.518</td>
<td>452.1313694</td>
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<td>6</td>
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<td>13.8</td>
<td>0.78</td>
<td>1</td>
<td>63.06215</td>
<td>62.74986837</td>
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<tr>
<td>12</td>
<td>4</td>
<td>6.9</td>
<td>0.96</td>
<td>0.26</td>
<td>75.79385</td>
<td>81.09363257</td>
</tr>
<tr>
<td>13</td>
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<td>11.9</td>
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<td>0.57</td>
<td>65.37125</td>
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<tr>
<td>6</td>
<td>3</td>
<td>5.6</td>
<td>0.43</td>
<td>0.38</td>
<td>219.14805</td>
<td>211.6105996</td>
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<tr>
<td><strong>Overall Mean</strong></td>
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<td></td>
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<td>110.743</td>
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</tr>
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<td></td>
<td></td>
<td>108.107</td>
<td></td>
</tr>
</tbody>
</table>

Figure 6. Active Random Search Detection Times

A full regression performed using the natural log of initial detection time, as shown in Figure 7, shows an F statistic p value of essentially zero, and all factors have a t
statistic p-value less than 0.05. The R squared states the model explains only 20% of the variability in the simulation. Though yielding 3% less than the passive search, more factors are involved, and there is some probability the Kilo will not be detected even if it is pinged inside of the active detection range. The most important factor is active detection range, followed by ping interval, detection probability, SSN speed, and Kilo speed, respectively. The regression results in the following equation to predict initial detection time:

\[
\text{Initial Detection Time (hrs)} = 437 \times e^{0.173 \times (\text{Active Range, nm}) + 1.02 \times (\text{Ping Interval, hrs}) - 0.86 \times (\text{Detect Prob}) - 0.055 \times (\text{SSN Spd, knts}) + 0.047 \times (\text{Kilo Spd, knts})}
\]
Figure 7. Full Regression of Active Random Search

- $R^2$
- F-significance p value
- Individual p-values
This regression suggests that the exponential effect of ping interval, though important to the model, can be delayed (up to a point) without a significant increase in initial detection time. This is due to the diminishing returns inherent in the exponential distribution. As ping interval is shortened, it will yield sequentially less reduction in initial detection time. This is further explored in Section D.

C. SEARCH PATH EFFECTS ON TIME TO INITIAL DETECTION

The previous model was performed with the SSN conducting a random search. This is not an optimal search configuration, and any Commanding Officer performing a search would undoubtedly choose a specific patrol path to conduct it. There are multiple methods in which a sonar search can be conducted. This thesis considers two such search paths. The paths were selected based on capabilities of the SSN and ease of use.

The lawnmower path of the SSN begins the search in the southwest corner of the area and conducts a west–to-east sweep, traveling some distance north, and then conducting an east to west sweep. This pattern is continued until the entire area is searched.

The next path approximates a "spiral-out" search. The SSN will start at the center of the area, and then travel outward in areas of expanding squares until the entire area is searched.

A "spiral-in" search path is also possible but was not included in the simulation because the SSN's speed advantage was not sufficient to be able to trap the Kilo within the area of diminishing squares.¹⁰

Table 3 shows the average results at each trial point involving the different search paths used, including the random search. With the same input parameters and limitations as the random active sonar search, the lawnmower path performance was on par with the random search path. This suggests that the Kilo's speed is sufficient to reduce the performance of the lawnmower search to be no better than a random search. The spiral-

¹⁰ Washburn, Search and Detection, 1-11.
out search path, however, performed better than a random search at every data point. Table 4 shows an overall reduction in detection time of 20%.

Table 3. Comparison of Search Paths

<table>
<thead>
<tr>
<th>SSN Speed (knots)</th>
<th>Kilo Speed (knots)</th>
<th>Act Rng (nm)</th>
<th>Det Prob</th>
<th>Ping Interval (hours)</th>
<th>Avg Time to Init. Detect Using Random Search</th>
<th>Avg Time To Init. Detect Using Lawnmower Search</th>
<th>Avg Time to Init. Detect Using Spiral Out</th>
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</thead>
<tbody>
<tr>
<td>15</td>
<td>4</td>
<td>8.8</td>
<td>0.48</td>
<td>0.94</td>
<td>200.81185</td>
<td>207.5831</td>
<td>141.19825</td>
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<tr>
<td>8</td>
<td>5</td>
<td>10.6</td>
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<td>0.32</td>
<td>71.03755</td>
<td>77.57815</td>
<td>64.4391</td>
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<tr>
<td>9</td>
<td>2</td>
<td>7.5</td>
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<td>0.81</td>
<td>133.14265</td>
<td>131.60635</td>
<td>117.37425</td>
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<tr>
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<td>3</td>
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<td>5</td>
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<td>40.9402</td>
<td>29.1944</td>
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<td>150.4501</td>
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<td>Overall Mean 110.74396</td>
<td>122.3584588</td>
<td>87.57010294</td>
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26
Table 4. Comparison of Random and Spiral-Out Search Paths

<table>
<thead>
<tr>
<th>Data Point</th>
<th>Avg Time to Init. Detect Using Random Search</th>
<th>Avg Time to Init. Detect Using Spiral Out</th>
<th>Percent Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200.81185</td>
<td>141.19825</td>
<td>29.69%</td>
</tr>
<tr>
<td>2</td>
<td>71.03755</td>
<td>64.4391</td>
<td>9.29%</td>
</tr>
<tr>
<td>3</td>
<td>133.14265</td>
<td>117.37425</td>
<td>11.84%</td>
</tr>
<tr>
<td>4</td>
<td>35.69445</td>
<td>33.32905</td>
<td>6.63%</td>
</tr>
<tr>
<td>5</td>
<td>67.2388</td>
<td>48.61055</td>
<td>27.70%</td>
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<tr>
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<td>19.12%</td>
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<tr>
<td>7</td>
<td>95.82585</td>
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</tr>
<tr>
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<td>20.83%</td>
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<td>87.25525</td>
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<td>16.75%</td>
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<tr>
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<td>50.9184</td>
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<td>6.04%</td>
</tr>
<tr>
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<td>36.0078</td>
<td>29.1944</td>
<td>18.92%</td>
</tr>
<tr>
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<td>501.518</td>
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<td>16.83%</td>
</tr>
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<td>57.1936</td>
<td>9.31%</td>
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<td>32.12%</td>
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</tr>
<tr>
<td>Overall</td>
<td>110.74396</td>
<td>87.57010294</td>
<td>19.79%</td>
</tr>
</tbody>
</table>

It was previously noted that the Kilo's speed is the factor that causes an exhaustive search to decline into a random search's performance. This is due to the Kilo's ability to enter previously searched areas while the SSN continues conducting its assigned search pattern. In a lawnmower search path, the perimeter that the Kilo can cross to enter the previously searched area is the length of the operational area (125nm). In the spiral-out search path, this perimeter begins at zero and then increases linearly as the search is conducted. As a result, it is difficult in the first part of the simulation for the Kilo to enter the previously searched area. This yields a more efficient search as well as a reduction of initial detection time.

In order to compare an active sonar search using a spiral-out technique to the passive sonar search, simulation runs were performed to chart the time to initial detection using active sonar as a percentage of time to a passive sonar search. The simulations
used the same submarine speeds and a ratio of active detection range to passive detection range. Figure 8 shows the result of the simulation runs, with the following input parameters:

SSN Speed=15 knots
Kilo Speed=3 knots
Passive Detection Range=2nm
Active detection probability=0.75
Ping Interval=0.25 hours
Active Range is varied from 2*(passive det. Range) to 7*(passive det. range)

![Active vs Passive Detection Time](chart.png)

Figure 8. Active vs. Passive Detection Times

Initially, when active sonar range is twice that of passive sonar range, the passive search performs better. This is because active sonar pings occasionally miss the Kilo unless the SSN is constantly pinging or active sonar has a detection probability of 1. As active sonar range increases, there is a significant reduction of search times that follow an exponential trend. When active sonar range is five times that of passive sonar range, the active search detects the Kilo in 25% of the time that a passive search would take. Though this is not predictive of the real world situation, the results imply that an active
search can detect a submarine in a fraction of the time than a passive search takes. Real-world exercises and data could determine the breakpoint at which the active sonar search will outperform the passive sonar search.

D PING INTERVAL ANALYSIS

It has been shown that initial detection time has an exponential relationship with ping interval. This means that as the time between pings increases, there is very little initial loss in detection time. This effect builds, however, such that eventually initial detection time increases substantially. This is shown in Figure 9. SSN speed was set to 10 knots, to the Kilo’s 3 knots, active detection range is 10nm, and the probability that a ping inside the detection range leads to detection is 0.75. 200 simulations were then performed on a series of ping intervals that is increased by 0.25 hours.

![Detection Time vs. Ping Interval](chart.png)

**Figure 9. Detection Time vs. Ping Interval**

Since initial detection time is not only a function of ping interval, the other factors may be adjusted to mitigate the exponential increase in detection time versus ping interval. This will deteriorate the best case scenario of initial detection time, but due to uncertainties in a real life situation it may be appropriate in order to gain the reduction of risk associated with less frequent pinging.
1. Ping Interval vs. SSN Speed

The first, and easiest, parameter for the SSN to change during search is its speed. Additional simulations to those performed in section D were conducted—once with SSN speed being reduced to 5 knots, and another with search speed increasing to 10 knots. All other parameters are held constant. Figure 10 shows the results of the simulations.

![Ping Interval and SSN Speed vs. Detection Time](image)

Figure 10. Ping Interval and SSN Speed vs. Detection Time

With a very short ping interval, a speed of 15 knots provides the best initial detection time at 29.46 hours. Over a ping interval range of 1 to 1.75, the initial detection time increases from approximately 71 hours to over 520 hours. Alternately, at 10 knots the initial detection time does not increase above 200 hours until a 2 hour ping interval is reached, and at 5 knots it is not reached until the ping interval is 2.75 hours. If a Commanding Officer feels that a threat exists and is willing to sacrifice the best initial detection time for a reduction in ping interval, he should reduce SSN search speed.

2. Ping Interval vs. Detection Probability

One factor for a Commanding Officer to consider in his decision to use active sonar is the probability of detection if the Kilo is within active sonar range. For
example, if the SSN operates in a high clutter environment and the detection probability is lowered, it should be understood how much detection time will change if ping interval is reduced.

Figure 11 shows the results of simulation runs similar to those performed in Section 1. However, instead of varying SSN speed, detection probability is varied from 0.4 to 0.9. Varying the detection probability does not have the same impact on detection time as SSN speed had. As long as sufficient looks are performed on the Kilo as it is within detection range, there will be a moderately high probability of detecting the Kilo. For example, assume the SSN is able to ping the Kilo twice as it is in detection range. The probability the SSN can detect the Kilo in these two looks is:

\[
\text{Prob(Detected in two looks)} = \text{Prob(Detected in the First Look)} + \text{Prob(Missed in First Look)} \times \text{Prob(Detected in Second Look)}
\]

As a result, at a detection probability of 0.4, the overall chance of detecting the Kilo in two looks is 64%. At a detection probability of 0.7 and 0.9, this chance increases to 91% and 99% respectively. The ping interval in these simulations are generally short enough such that multiple looks at the Kilo in detection range is obtained, and detection probability does not have as large an affect on detection time as ping interval is varied.

Figure 11. Ping Interval and Detect Prob. Vs. Detection Time
3. Ping Interval vs. Active Range

The last factor known to the Commanding Officer is the estimated active detection range. Comparably to detection probability, if the active range is considered low and the Commanding Officer feels it tactically necessary to reduce ping interval, there needs to be an understanding of how this will affect active detection time. Figure 12 shows the effect of changing ping interval as active detection range changes from 5nm to 15nm.

![Ping Interval and Range vs. Detection Time](image)

Figure 12. Ping Interval and Active Range vs. Detection Time

Active range is extremely significant in exacerbating the effects of the exponential distribution on detection time. If active range is significant, it takes a drastic delay in ping interval to see the exponential effect. Conversely, a relatively short detection range will result in a very rapid increase in detection time if ping interval is delayed. Therefore, active range is the primary factor the Commanding Officer should consider if he decides to employ active sonar.

Statistically, the model derived in Sections A and B are not a good fit, as they only explain approximately 20% of the variability in the model. Further research, including more scenarios and possibly additional factors would be needed to attempt to explain more of variability in the scenario. The graphs do strongly suggest and exponential relationship of the factors in regards to initial detection time, and this relationship still should be included in any decision to use active sonar when detecting diesel submarines.
V. CONCLUSIONS

The following synopsis provides the outcome of the simulations performed in this thesis:

- The performance graphs of the simulations do strongly suggest an exponential pattern, implying that the time to initially detect a target is an exponential function based on detection range, speed of the participants, and in the case of active sonar ping, interval and detection probability. As a result, as the passive detection range of diesel submarines continues to decrease, the average detection time will increase exponentially. There is a point where a passive prosecution is no longer feasible, and either other assets need to be employed, or an active search could be performed to maintain a reasonable detection time.

- The speed of a submerged diesel submarine is limited; however, it is sufficient to make many search patterns no more effective than a random search if the diesel can enter previously searched areas. A "spiral-out" path can provide an additional decrease in detection time, as it provides an initially small perimeter that the diesel submarine can cross in order to enter this previously searched area. The simulations suggest that an active search can yield detection times at a fraction of passive searches, but real world exercises are needed to determine the breakpoint at which this occurs.

- If the Commanding Officer deems it necessary to reduce ping interval to minimize the threat of being counter-detected, the following factors (listed in order of priority) will mitigate the effect of the exponential increase in detection time:
  1. A long active detection range
  2. Traveling at slower search speeds
  3. A moderate to high detection probability
To advance the study of active sonar employment, further research is needed to identify additional tactics to optimize its performance. Such studies include identifying a set of possible actions once detection occurs. For example, if engagement with the diesel submarine is desired, there must be a determination whether to rapidly engage the diesel or to continue tracking and obtain better tactical positioning. A proper combination of active and passive sonar tracking can also be explored.

The regression model performed in this thesis only explains approximately 20% of the variability that appears in the simulations. As a result, further research is needed to attempt to better explain this variability. These may include additional scenarios and additional factors. Future work in this field should include an attempt to build a better statistical model that better explains the variability shown. Additional scenarios could include factors that were not considered in this thesis, and may also contain situations that would not occur in a real world encounter. An incremental increase in the number of replications could identify whether the randomness of the model or the situation being modeled overcomes the factors due to its stochastic nature.

The modern diesel submarine provides significant challenges to the U.S. naval fleet. Not only does it pose a significant threat to high value units such as aircraft carriers, but the relatively cheap cost of each diesel submarine is such that they may be deployed in significant numbers to overwhelm small numbers of high quality units such as nuclear submarines. The ability to detect diesel submarines early provides the best ability to successfully defend against any threat that may be posed.
public class ActiveSearchRandom {

    /**
     * The class random number generator will produce a different random number
     * at the start of each run. The parameters are the number of replications,
     * SSN speed, Diesel Speed, Active detection range, probability of
     * detection, and ping interval which can be overridden using the arguments.
     * *
     * @param args The values of the number of replications run, SSN speed, Diesel Speed, Active
     * Detection Range, Detection Probability, Ping Interval,
     * */

    public static void main(String args[]) {

        double simTime = 0.0;
        double totalSimTime = 0.0;
        int numberReplications = 200;
        double ssnSpeed = 0.0;
        double dieselSpeed = 0.0;
        double activeRange = 0.0;
        double probDetect = .0;
        double pingInterval = .0;
        if (args.length == 6) {
            numberReplications = Integer.parseInt(args[0]);
            ssnSpeed = Double.parseDouble(args[1]);
            dieselSpeed = Double.parseDouble(args[2]);
            activeRange = Double.parseDouble(args[3]);
            probDetect = Double.parseDouble(args[4]);
            pingInterval = Double.parseDouble(args[5]);
        }

        SensorTargetMediatorFactory.addMediator(ActiveSonarSensor2.class, UniformLinearMover.class, ActiveSonarMediator2.class);

        RandomVariate randPosit = RandomVariateFactory.getInstance("Uniform", new Object[] { new Double(0.0),
        new Double(125.0) });
        Point2D.Double ssnStart = new Point2D.Double(randPosit.generate(), randPosit.generate());
        Point2D.Double dieselStart = new Point2D.Double(randPosit.generate(), randPosit.generate());

        // Create the diesel and the SSN as uniform linear movers.
        Mover badGuy = new UniformLinearMover("Diesel", dieselStart, dieselSpeed);
        Mover goodGuy = new UniformLinearMover("SSN", ssnStart, ssnSpeed);

        // Attach the passive sonar to the SSN
        Sensor ssnActive = new ActiveSonarSensor2(goodGuy, activeRange,
// Register the movers to the Referee
SensorTargetReferee activeRef = new SensorTargetReferee();
activeRef.register(badGuy);
activeRef.register(ssnActive);

goodGuy.moveTo(new Point2D.Double(randPosit.generate(), randPosit.generate()));
badGuy.moveTo(new Point2D.Double(randPosit.generate(), randPosit.generate()));

// Have the Diesel and SSN to move randomly
RandomVariate[] rv = new RandomVariate[] {
    RandomVariateFactory.getInstance("Uniform", new Object[] {0.0, 125.0}),
    RandomVariateFactory.getInstance("Uniform", new Object[] {0.0, 125.0})
};
RandomLocationMoverManager goodGuyMM = new RandomLocationMoverManager(goodGuy, rv);
RandomLocationMoverManager badGuyMM = new RandomLocationMoverManager(badGuy, rv);
goodGuyMM.setStartOnReset(true);
badGuyMM.setStartOnReset(true);
SandboxFrame frame = new SandboxFrame("Passive Sensor Platform");
frame.setSize(1000, 1000);
((PingThread) frame.getControlPanel().getController()).setMillisPerSimtime(50);
frame.getSandbox().setOrigin(new Point2D.Double(50, 150));
frame.addMover(badGuy, Color.red);
frame.addMover(goodGuy, Color.blue);
frame.addSensor(ssnActive, Color.orange);
PropertyChangeFrame pcf = new PropertyChangeFrame();
ssnActive.addPropertyChangeListener("detection", pcf);
ssnActive.addPropertyChangeListener("undetection", pcf);
frame.setLocation(1, 1);
frame.setVisible(true);
pcf.setLocation(frame.getLocationOnScreen().x + frame.getWidth(), frame.getLocationOnScreen().y);
pcf.setVisible(true);
for (int i=0;i<numberReplications;i++){
    Schedule.reset();
    Schedule.setVerbose(false);
    Schedule.stopOnEvent(1, "Detection", Contact.class);

    Schedule.startSimulation();

    simTime = Schedule.getSimTime();
    DecimalFormat form = new DecimalFormat("##.##");
    System.out.println(form.format(simTime));
}
public class PassiveSearchRandom {

/**
 * The class random number generator will produce a different random number
 * at the start of each run. The parameters are the number of replications,
 * SSN speed, Diesel Speed, Passive detection range, and probability of
 * detection, and can be overridden using the arguments.
 *
 * @param args  The values of the number of replications, SSN
 * Speed, Diesel Speed, and Passive sonar detection range.
 */

public static void main(String args[]) {

    double simTime = 0.0;
    int numberReplications = 200;
    double ssnSpeed = 15.0;
    double dieselSpeed = 3.0;
    double passiveRange = 2.0;

    if (args.length == 4) {
        numberReplications = Integer.parseInt(args[0]);
        ssnSpeed = Double.parseDouble(args[1]);
        dieselSpeed = Double.parseDouble(args[2]);
        passiveRange = Double.parseDouble(args[3]);
    }

    SensorTargetMediatorFactory.addMediator(CookieCutterSensor.class,
        UniformLinearMover.class, CookieCutterMediator.class);

    RandomVariate randPosit = RandomVariateFactory.getInstance("Uniform",
        new Object[] { new Double(0.0),
        new Double(125.0) });

    Point2D.Double ssnStart = new Point2D.Double(0.0, 0.0);
    Point2D.Double dieselStart = new Point2D.Double(randPosit.generate(),
        randPosit.generate());

    Mover badGuy = new UniformLinearMover("Diesel", dieselStart,
        dieselSpeed);
    Mover goodGuy = new UniformLinearMover("SSN", ssnStart, ssnSpeed);

    // Attach the passive sonar to the SSN
    Sensor ssnPassive = new CookieCutterSensor(passiveRange, goodGuy);

    // Register the movers to the Referee
    SensorTargetReferee passiveRef = new SensorTargetReferee();
    passiveRef.register(badGuy);
    passiveRef.register(ssnPassive);

    goodGuy.moveTo(new Point2D.Double(0.0, 0.0));
    badGuy.moveTo(new Point2D.Double(randPosit.generate(),
        randPosit.generate()));

    // Have the Diesel and SSN to move randomly

}
RandomVariate[] rv = new RandomVariate[] {
    RandomVariateFactory.getInstance("Uniform", new Object[] {
        0.0, 125.0
    }),
    RandomVariateFactory.getInstance("Uniform", new Object[] {
        0.0, 125.0
    })
};

// RandomLocationMoverManager goodGuyMM=new RandomLocationMoverManager(goodGuy, rv);
PatrolMoverManager goodGuyMM = new PatrolMoverManager(goodGuy, pathlawn);
RandomLocationMoverManager badGuyMM = new RandomLocationMoverManager(badGuy, rv);

goodGuyMM.setStartOnReset(true);
badGuyMM.setStartOnReset(true);

SandboxFrame frame = new SandboxFrame("Passive Sensor Platform");
frame.setSize(150, 150);
((PingThread) frame.getControlPanel().getController()).setMillisPerSimtime(50);

frame.getSandbox().setOrigin(new Point2D.Double(0, 160));
frame.addMover(badGuy, Color.red);
frame.addMover(goodGuy, Color.blue);
frame.addSensor(ssnPassive, Color.orange);
PropertyChangeFrame pcf = new PropertyChangeFrame();

ssnPassive.addPropertyChangeListener("detection", pcf);
ssnPassive.addPropertyChangeListener("undetection", pcf);

frame.setLocation(1, 1);
frame.setVisible(true);
pcf.setLocation(frame.getLocationOnScreen().x + frame.getWidth(), frame.getLocationOnScreen().y);
pcf.setVisible(true);

for (int i=0; i<numberReplications; i++) {
    Schedule.reset();
    Schedule.setVerbose(false);
    Schedule.stopOnEvent(1, "Detection", Contact.class);

    Schedule.startSimulation();
    simTime = Schedule.getSimTime();
    DecimalFormat form = new DecimalFormat("##.##");
    System.out.println(form.format(simTime));
}
Figure 13. Linear Passive Sonar Regression
Figure 14. Log-Linear Passive Sonar Regression
### Multivariate Analysis in Log-Linear Passive Model

#### Correlations

<table>
<thead>
<tr>
<th></th>
<th>LN (Avg Time Init Det)</th>
<th>SSN Spd (hours)</th>
<th>Kilo Spd (hours)</th>
<th>Pass Rng (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LN (Avg Time Init Det)</td>
<td>1.0000</td>
<td>-0.2762</td>
<td>-0.1193</td>
<td>-0.3935</td>
</tr>
<tr>
<td>SSN Spd (hours)</td>
<td>-0.2762</td>
<td>1.0000</td>
<td>0.0688</td>
<td>0.0012</td>
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<tr>
<td>Kilo Spd (hours)</td>
<td>-0.1193</td>
<td>0.0688</td>
<td>1.0000</td>
<td>0.2000</td>
</tr>
<tr>
<td>Pass Rng (nm)</td>
<td>-0.3935</td>
<td>0.0012</td>
<td>0.2000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

#### Scatterplot Matrix

![Scatterplot Matrix](image)

Figure 15. Multivariate Analysis in Log-Linear Passive Model
Figure 16. Linear Active Sonar Regression
Figure 17. Log-Linear Active Sonar Regression
Figure 18. Multivariate Analysis in Log-Linear Active Model

<table>
<thead>
<tr>
<th>SSN Speed (knots)</th>
<th>Kil Speed (knots)</th>
<th>Active Range (nm)</th>
<th>Det Prob</th>
<th>Ping Intv</th>
<th>LN (Initial Detection Time)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0000</td>
<td>-0.1492</td>
<td>0.0233</td>
<td>0.0270</td>
<td>0.0230</td>
<td>-0.1241</td>
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<tr>
<td>-0.1492</td>
<td>1.0000</td>
<td>0.0470</td>
<td>-0.0267</td>
<td>-0.0648</td>
<td>0.0149</td>
</tr>
<tr>
<td>0.0233</td>
<td>0.0470</td>
<td>1.0000</td>
<td>-0.0028</td>
<td>-0.0004</td>
<td>-0.3505</td>
</tr>
<tr>
<td>0.0270</td>
<td>-0.0267</td>
<td>-0.0028</td>
<td>1.0000</td>
<td>0.0001</td>
<td>-0.1274</td>
</tr>
<tr>
<td>0.0230</td>
<td>-0.0648</td>
<td>-0.0004</td>
<td>0.0001</td>
<td>1.0000</td>
<td>0.2022</td>
</tr>
<tr>
<td>-0.1241</td>
<td>0.0149</td>
<td>-0.3595</td>
<td>-0.1274</td>
<td>0.2022</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

**Correlations**

**Scatterplot Matrix**
<table>
<thead>
<tr>
<th>Ping Interval (hrs)</th>
<th>Td (hrs), Speed=5 kts</th>
<th>Td (hrs), Speed=10 kts</th>
<th>Td (hrs), Speed=15 kts</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>76.1287</td>
<td>40.79985</td>
<td>29.4612</td>
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<tr>
<td>0.25</td>
<td>77.1503</td>
<td>43.8965</td>
<td>33.52365</td>
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<tr>
<td>0.5</td>
<td>77.29365</td>
<td>54.5613</td>
<td>39.8998</td>
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<tr>
<td>0.75</td>
<td>81.87565</td>
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<td>51.9809</td>
</tr>
<tr>
<td>1</td>
<td>92.8883</td>
<td>66.065</td>
<td>70.9209</td>
</tr>
<tr>
<td>1.25</td>
<td>109.0508</td>
<td>73.5598</td>
<td>121.8946</td>
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<tr>
<td>1.5</td>
<td>110.41015</td>
<td>95.6014</td>
<td>269.48165</td>
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<tr>
<td>1.75</td>
<td>136.82545</td>
<td>157.3704</td>
<td>520.1694</td>
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<tr>
<td>2</td>
<td>153.43715</td>
<td>233.63715</td>
<td></td>
</tr>
<tr>
<td>2.25</td>
<td>160.4353</td>
<td>320.5972</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>281.05375</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5. Ping Interval and Speed vs. Detection Time

<table>
<thead>
<tr>
<th>Ping Interval (hrs)</th>
<th>Td (hrs), Prob=.4</th>
<th>Td (hrs), Prob=.7</th>
<th>Td (hrs), Prob=.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>44.50285</td>
<td>40.79985</td>
<td>39.62365</td>
</tr>
<tr>
<td>0.25</td>
<td>50.16305</td>
<td>43.8965</td>
<td>43.2795</td>
</tr>
<tr>
<td>0.5</td>
<td>70.1109</td>
<td>54.5613</td>
<td>49.00745</td>
</tr>
<tr>
<td>0.75</td>
<td>83.89485</td>
<td>57.44255</td>
<td>52.1533</td>
</tr>
<tr>
<td>1</td>
<td>102.1648</td>
<td>66.065</td>
<td>54.3191</td>
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<tr>
<td>1.25</td>
<td>144.7518</td>
<td>73.5598</td>
<td>62.2855</td>
</tr>
<tr>
<td>1.5</td>
<td>181.58175</td>
<td>95.6014</td>
<td>83.13195</td>
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<td>321.32825</td>
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<td>2.25</td>
<td>320.5972</td>
<td>271.86135</td>
<td>271.86135</td>
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<tr>
<td>2.5</td>
<td>455.89275</td>
<td>359.43825</td>
<td>395.43825</td>
</tr>
</tbody>
</table>

Table 6. Ping Interval and Active Detection Range vs. Detection Time
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