Applying Bayesian belief networks in Sun Tzu's Art of war

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APPLYING BAYESIAN BELIEF NETWORKS IN SUN TZU’S *ART OF WAR*

by

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December 2004

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Applying Bayesian Belief Networks in Sun Tzu’s Art of War

The principles of Sun Tzu’s Art of War have been widely used by business executives and military officers with much success in the realm of competition and conflict. However, when conflict situations arise in a highly stressful environment coupled with the pressure of time, decision makers may not be able to consider all the key concepts when forming their decisions or strategies. Therefore, a structured reasoning approach may be used to apply Sun Tzu’s principles correctly and fully. Sun Tzu’s principles are believed to be able to be modeled mathematically; hence, a Bayesian Network model (a form of mathematical tool using probability theory) is used to capture Sun Tzu’s principles and provide the structured reasoning approach. Scholars have identified incompleteness in Sun Tzu’s appreciation of information in war and his application of secret agents. This incompleteness resulted in circular reasoning when both sides of the conflict apply his principles. This circular reasoning can be resolved through the use of advanced probability theory. A Bayesian Network Model however, not only provides a structured reasoning approach, but more importantly, it can also resolve the circular reasoning problem that has been identified.
APPLYING BAYESIAN BELIEF NETWORKS IN SUN TZU’S ART OF WAR

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ABSTRACT

The principles of Sun Tzu’s Art of War have been widely used by business executives and military officers with much success in the realm of competition and conflict. However, when conflict situations arise in a highly stressful environment coupled with the pressure of time, decision makers may not be able to consider all the key concepts when forming their decisions or strategies. Therefore, a structured reasoning approach may be used to apply Sun Tzu’s principles correctly and fully. Sun Tzu’s principles are believed to be able to be modeled mathematically; hence, a Bayesian Network model (a form of mathematical tool using probability theory) is used to capture Sun Tzu’s principles and provide the structured reasoning approach. Scholars have identified incompleteness in Sun Tzu’s appreciation of information in war and his application of secret agents. This incompleteness resulted in circular reasoning when both sides of the conflict apply his principles. This circular reasoning can be resolved through the use of advanced probability theory. A Bayesian Network Model not only provides a structured reasoning approach, but more importantly, it can also resolves the circular reasoning problem identified.
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I. INTRODUCTION

A. INTRODUCTION

Sun Tzu's *Art of War* was written around 500 B.C, and it is one of the pre-eminent works on military strategy. The *Art of War* underlies the science of combat that covers the craft of deception, interpreting terrain, the movement of materiel and men and the discipline and motivation of troops (Griffith, 1971). In the period of the Warring States, where hundreds of wars were fought between the separate states of China, the desire of each individual state was either to dominate or to survive (Wing, 1988). Battles were no longer primitive. They evolved and became directed efforts. Armies had professional troops and no longer advanced unsupported. Detailed plans and signals were formulated to coordinate battle maneuver. It was at this period where the art of military tactics was born (Griffith, 1971).

Sun Tzu realized the importance of war to the state and hence devoted himself to the formulation of the *Art of War*, which contains thirteen chapters of rational writings for the planning and conduct of military operations. The *Art of War* not only deals with the stratagems and transitory techniques of war, but is a systematic written work to guide rulers and generals in an intelligent thought process to conduct a successful war. In the *Art of War*, thorough appreciation of factors affecting war are observed: the influence of supply on the conduct of operations; the relationship of the sovereign to his generals; the moral, emotional, and the intellectual qualities of a good general; and in battle organization, maneuver, control, terrain, and weather factors (Griffith, 1971).

Sun Tzu is well aware of the economic implications of war on the state and wisely noted that no state has ever benefited from a protracted war; hence, he downplays the conduct of war through sheer military might, and instead, places his emphasis on skillful strategizing to win war without bloodshed, limiting the use of armed force only when such approaches have failed. His perception of winning a war is as such: “To win one hundred victories in one hundred battles is not the acme of skill. To subdue the enemy without fighting is the supreme excellence” (Griffith, 1971).
The importance of intelligence and deception are greatly emphasized in the Art of War for the formulation of strategies, where careful planning is based on sound information about the enemy that would lead to effective military decisions, and deception through simulation and dissimulation should be utilized by generals to confuse and delude the enemy of their dispositions and intent (Griffith, 1971).

Sun Tzu’s writing may have been initially written for rulers and generals engaging in interstate conflicts that manifested into wars. In our present time, conflict (also in the form of competition) no longer exists in the realm of the battlefield alone, it is in politics, businesses, sports, and games, where principles of his Art of War is being applied.

Sun Tzu’s Art of War has been widely used by both business executives and military officers with much success (McNeilly, 1996). However, successful application of Sun Tzu’s principles comes through constant practice and time, and through lessons learned from historical case studies. Each of the thirteen chapters in the Art of War contains lists of important principles that need to be identified and applied. To successfully apply the philosophies in the Art of War, one needs to be cognizant of all these principles. More often than not the conflict situation is a highly stressful environment, which when coupled with the pressure of time, creates an environment where the decision maker faces great difficulty in applying the Art of War. This may result in his inability to consider all the key principles where decision or strategy is formed, only regarding those principles that he deems relevant. The strategies formulated would then have a degree of bias toward certain principles (which the decision maker is more comfortable with), contributed from personal preference, stress, and time pressures. There is however, no easy structural approach in the formulation of decisions or strategies using Sun Tzu’s Art of War to allow all principles to be considered. In actual decision making, one can be certain that not all principles in the Art of War may be applicable or required; only those that are relevant in the context of the situation will be considered. The problem identified is how a decision maker could be able to quickly differentiate and choose the relevant principles when he is in a stressful environment with limited time.
The principles in *Art of War* being widely used in businesses and militaries are now common knowledge; hence the possibility that in a conflict between two parties, both may be using Sun Tzu’s teachings against each other. Niou and Ordeshook (1994), in their attempt to examine how Sun Tzu anticipated the implication of contemporary game theory, postulates that such a situation can result in the possibility of occurrence of circular reasoning, where both parties may reason in a “he thinks that I think” cycle. This circular reasoning is seen in the *Art of War* in the chapter regarding the employment of secret agents. Sun Tzu said that the secret agent of the enemy should be employed as double agent (Griffith, 1971), but one will not be able to determine if this double agent will become a triple agent. For if both sides of the conflict are using Sun Tzu’s principles in employing newly recruited double agents, circular reasoning could result as both parties start doubting the loyalty of this double agent. As there is nothing in Sun Tzu’s work that addresses the possibility of circular reasoning in military conflict, Niou and Ordeshook (1994) concluded that Sun Tzu’s treatment of information in conflict is incomplete, and that resolving such circular reasoning requires the application of modern advanced probability theory and mathematics.

**B. HYPOTHESIS**

Many aspects of Sun Tzu’s principles of *Art of War* can be modeled with the use of mathematical tools. The thirteen chapters of his work that assisted military and business decision makers in affairs of conflict consist of many elements, and in time of confrontation may present themselves as uncertainty variables. A Bayesian Network is a mathematical concept that falls within the field of Decision Theory, and is used for decision making in uncertainty. I hypothesize that this concept presented by Decision Theory will allow me to model Sun Tzu’s *Art of War*. The result of this model will provide a structured approach that will allow military decision makers to better utilize the principles in the *Art of War* and to resolve the possibility of circular reasoning that arises with imperfect information.
C. THESIS GOAL

The intent of this thesis is to model the principles of the *Art of War* mathematically. A Bayesian Network is the mathematical tool that is selected for the creation of the models representing the principles in the *Art of War*. A Bayesian Network approach allows a user to represent causal relationship variables that are contained in a philosophical argument or hypothesis, where such causal relationship can be easily represented graphically. The graphical representation has the form of a cognitive/causal map where nodes and arrows are used to capture the causal relationship (Bielza & Shenoy, 1999). Causality is an important underlying assumption in the creation of a causal map; hence, its applicability in the modeling of the *Art of War* will be covered in chapter three.

A Bayesian Network is a directed acyclic graph having nodes representing variables and directional arcs representing the dependence relations among variables. A Bayesian Network uses the principle of probability to associate the relationship of all the variables. The probabilistic relationship among the nodes allows for numerical calculations to determine the outcomes based on instantiations of variables in a decision problem. The usage and concept of Bayesian Networks will be further discussed in chapter three. In this thesis, the software *Netica* developed by Norsys Software Corporation will be used to construct the Bayesian Network necessary for the modeling.

The model developed will be used to assist decision makers in better utilizing Sun Tzu’s *Art of War* through a systematic and structured manner, and will also help to resolve the problem of circular reasoning that arises from the usage of information from the *Art of War*. The application of Bayesian Networks in the model will have the concepts in the principles of the *Art of War* embedded, which in turns allows one to take into consideration all known and uncertain variables, and through a probabilistic process that encompasses both qualitative and quantitative analysis to reach the best desired outcome or objectives. Chapter three of this thesis will cover the methods of how Bayesian Networks are used to create the models, chapter four on the actual modeling, and chapter five on resolving circular reasoning.
The end result of this thesis hopes to give military personnel deeper insights on Sun Tzu’s *Art of War*, and to be able to further appreciate the dynamics of warfare.
II. BACKGROUND

A. A SUMMARY OF SUN TZU’S ART OF WAR

Sun Tzu is believed to have lived in China around 500 B.C. in the state of Chi during the period of Warring States, which was a period of great instability in China. Neighboring states fought against one another for the purpose of survival or dominance. It was also a time where iron technology flourished, resulting in the expansion of weapon arsenals that led to greater lethality. Wars were no longer fought in a primitive fashion. Professional armies of soldiers were formed and the complexity of battles fought increased—where armies’ maneuvers were coordinated via detailed planning conducted through the use of signals and flags—and logistic trains were developed to support and sustain the armies throughout a battle. War was costly and had a huge impact on the economy and survivability of states, therefore kings engaged professional strategists to lead them to victory. In this era of constant struggles between the states of an emerging China, Sun Tzu realized the tremendous importance of war to the state and hence devoted himself to the formulation of the Art of War, which contains thirteen chapters of rational writings for the planning and conduct of military operations. This chapter of the thesis will contain a summary of the thirteen chapters of the Art of War translated by Griffith (1971) to give the reader a better appreciation of the model that is developed.

1. Chapter One - “Estimates”

The first chapter of the Art of War, Sun Tzu begins by emphasizing the importance of estimating, as the process of estimation allows the commander to weigh the chances of winning a war before committing himself and his troops. As war is always costly for the state, careful calculations on the advantages of both waging or avoiding war have to be made. No attempt should be made to engage in war if the chance of winning is not favorable. Therefore, in order to conduct estimation prior to war, a commander has to consider five fundamental factors: Moral influence, which reflects the harmony between the people of the state and their leaders; weather, which reflects the interaction of natural forces and the conduct of military operations; terrain, which reflects the ease or difficulty of the ground conditions in military operations; command, which reflects the attributes of a commander in terms of wisdom, sincerity, humanity, courage, and
strictness; and doctrine which reflects the delegation of authority to subordinates within the military organization in terms of command and control of forces and logistical support.

2. Chapter Two - “Waging War”

In the second chapter, Sun Tzu states that victory is the objective of war and one should strive to achieve it in the shortest time. There would be negative economic and moral impact on the state if war is prolonged, as Sun Tzu states that no country has ever benefited from a protracted war. Therefore, to prevent war from being prolonged, a commander must understand the danger of waging war, motivate his troops and make them realize the rewards of winning and the consequences of a prolonged or losing battle. He also emphasizes that treating any captives of war well is a means of strengthening the state.

3. Chapter Three - “Offensive Strategy”

Sun Tzu’s principle of war is to defeat the enemy by capturing them intact (which will reserve the strength of one’s own forces). Destroying them is not preferred. “For one to win one hundred battles is not the acme of skill. To subdue the enemy without fighting is the acme of skill”. The way to achieve this is to attack the enemy’s strategy; first through the use of plots. If this fails, only then should one engage the enemy. A commander must craft his plan of attack by assessing the enemy’s strength in terms of materiel and men.

4. Chapter Four - “Disposition”

Sun Tzu highlights the two aspects of warfare: defense and attack. An invincible army lies in defense and a victorious one, in attack. When an army should use defensive or attacking strategies depends on its strength; defend when inadequate and attack when abundant. Victory is not gained by fighting the enemy and hoping to win, but rather, it should be gained before even fighting the enemy. Therefore, positive winning policies should be formulated and practiced. A commander must be aware of these two aspects of war and the capability of his forces so as to create a situation of winning before he engages his enemy.
5. **Chapter Five - “Energy”**

The ability to gain victory over the enemy in battle is to be able to use different types of forces effectively. Sun Tzu mentions the use of normal and extraordinary forces, where normal forces are used to engage the enemy while extraordinary forces are used to win. There are countless ways by which these two forces can be combined to gain a winning strategy; therefore, a commander should never demand victory from his men; instead, he should seek victory from the situation he creates.

6. **Chapter Six - “Weaknesses and Strengths”**

In this chapter, Sun Tzu illustrates the usage of strength and weakness in terms of attack and defense. To weaken the enemy in defense is to prevent the enemy from knowing your intent and place of attack. When the enemy is forced to defend himself from attack everywhere he is weak everywhere. The commander then has to focus his strength and attack the enemy’s weaker few. When in defense, be unpredictable so that the enemy does not know where to attack. The best strategy to engage the enemy is to know the enemy’s plan, all the while being unpredictable to circumvent the enemy from laying plans against you. Therefore, to achieve victory, a commander has to be unpredictable and modify his tactics in battle in accordance with the enemy’s situation.

7. **Chapter Seven - “Maneuver”**

In the conduct of war, Sun Tzu regards maneuvering as the most difficult aspect of war. The commander has to understand the forces that he is commanding (in terms of capabilities), communications (as the means to achieve battle coordination), and logistical support; because military maneuver is conducted with these factors in mind and the challenge is to balance them. When a commander pursues the enemy without considering these factors, he runs the risk of losing his supplies as well as the division of his forces (as vigorous troops will arrive first and the feeble straggle behind). Careful considerations of these factors will enable the commander to defeat his enemy without unnecessary sacrificing of his troops.

8. **Chapter Eight - “The Nine Variables”**

The nine variables of tactics as determined by Sun Tzu are as follows:

You should not encamp in low lying ground.

In communicating ground, unite with your allies.
You should not linger in desolate ground.

In enclosed ground, resourcefulness is required.

In death ground, fight.

There are some roads not to follow,
some troops not to strike,
some cities not to assault,

and some ground not to be contested.

Therefore, a commander who is unable to understand the tactics suitable to these nine variable situations will be unable to use his troops effectively. With this in mind, the commander has to take into account both favorable and unfavorable factors; where he uses favorable ones to make his plan feasible and unfavorable ones to resolve difficulties.

9. Chapter Nine - “Marches”

In this chapter, Sun Tzu highlights a list of ‘where’ and ‘when’ factors for engaging the enemy. For example he states that it is disadvantageous to fight the enemy uphill, and not to engage the enemy when they show signs of desperation (as they will fight decisively till death). Based on the nine variables, the commander must assess how to use his forces to fight his enemy and when to fight them.

10. Chapter Ten - “Terrain”

Terrain is an important factor in battle; therefore careful appreciation of the ground has to be conducted. Sun Tzu classifies grounds according to their nature: accessible, entrapping, indecisive, constricted, precipitous, and distant. Assessable ground is terrain on which both sides can traverse, thus the side that occupies it first has the advantage of convenient supplies for his battle. Entrapping ground is easy to get out from but difficult to return to. Do not engage a prepared enemy from such grounds. A ground that is disadvantageous for both sides is indecisive, do not engage the enemy. Constricted ground is advantageous to the side that occupies it first as the enemy can be trapped by sealing the passes in such grounds, but do not pursue an enemy that occupies such ground. On precipitous ground, fight the enemy from sunny heights and avoid the enemy that occupies such grounds. In the situation of distant ground, when at a distance
from an enemy of equal strength, avoid engaging him on his own ground as it will be
disadvantageous to you. These are the principles to the six different grounds; the
commander has to study them carefully as they can greatly affect the outcome of battles.


In this chapter, Sun Tzu adds on to the discussion in the previous chapter of the
varieties of ground that he has observed. Apart from the nature of the ground, Sun Tzu
further classifies them as: dispersive, frontier, key, communicating, focal, serious,
difficult, encircled, and death. The definitions of these classifications is as follows:
Dispersive is when a commander fights in his own territory; frontier is when shallow
penetration into enemy territory has been made; key is advantageous to both sides;
communicating is ground that is assessable to both sides; focal is when both sides are
enclosed by other states; serious is when the army has penetrated deep into hostile
territory; difficult is when marches is hindered by terrain; encircled is when the enemy
can use his situation to constrict us; and death is when survival can only be achieved
through courage of desperation. Knowing these definitions allows the commander to
both avoid the potential pitfalls that are presented by the terrain by which they might
conduct their battle and to use them to his advantage.

12. Chapter Twelve - “Attack by Fire”

Sun Tzu provides five methods of attack through the use of fire, and they are: to
burn personnel, to burn store, to burn equipment, to burn arsenals, and to use fire as
incendiary missiles. When fire is used, the commander must prepare to take advantage of
the panic that it will cause the enemy and attack should be timed and coordinated
accordingly. Fire is an asset to war and the commander who knows how to utilize it is
intelligent and strong.

13. Chapter Thirteen - “Employment of Secret Agents”

A commander who possesses foreknowledge defeats his enemies. Foreknowledge
is the knowing of the situation of the enemy. The men that have knowledge of the enemy
are employed as secret agents, and their role is to collect information on the enemy.
There are five types of secret agents: native, the locals of the enemy; inside, administrative people within the enemy’s government; double, enemy’s spies that have
been recruited; expandable, agents that are fed with false information to serve the
commander’s purpose; and living, agents that have survived and brought back information on the enemy. Employment and treatment of these agents are crucial as the information drawn from them affect every move made by an army.

The thirteen chapters in Sun Tzu’s *Art of War* as a whole can be regarded as the science for both management and conduct of military operations. One of the key principles that was not summarized as part of the thirteen chapters (but was constantly mentioned) is Sun Tzu’s emphasis on the use of deception in the conduct of military operations, which is applied to most of his chapters involving the usage of information. He states that information should be manipulated by the commander according to situation to create advantages for himself against the enemy, and is done through simulation and dissimulation. To conclude, the goal of Sun Tzu’s *Art of War* is to allow commanders to formulate strategies to defeat their enemies without physical engagement.

B. A DISCUSSION ON NIOU & ORDESHOOK’S A GAME THEORETIC INTERPRETATION OF SUN TZU’S THE ART OF WAR

The authors of this paper, Niou and Ordeshook, presume that Sun Tzu’s *Art of War* is a codification of the insights of an era skilled at strategy and tactics; an understanding of Sun Tzu’s work is believed to provide a better understanding of the essentials of conflict. They concur that Sun Tzu offers insight to the realm of conflict, but they believe that modern understanding of conflict has progressed beyond Sun Tzu; and that modern advances of analysis such as decision theory and its underlying mathematical theories have aided in refining our thinking with regards to strategic analysis. Scholars interested in understanding Sun Tzu’s work therefore have to analyze his writings in the context of such advances to maximize his writings’ contemporary relevance; and this is the key issue that their paper addresses.

Niou and Ordeshook then introduce game theory as their main theory of strategic behavior as part of the modern advances which they mention. They view game theory as a mathematical tool that can be used to isolate abstract principles of decision making when choice is dependent on the decision of others in a situation where everyone is aware of their mutual interdependence. Game theory like the *Art of War* involve the analysis of pure conflict, therefore, like the *Art of War* (as mentioned in chapter one) it is not
restricted only to the realm of strategic military planning. Both game theory and *Art of War* address our understanding of strategy; but Niou and Ordeshook believe that game theory offers generality and mathematical precision that allows the ascertainment of logical coherence of ideas about strategic interaction; while “Sun Tzu provides a specific application of general principles, and demonstrates the art of rendering logical and abstract reasoning practical”. Hence, Niou and Ordeshook’s intent is to review the essential components of game theory to explore the consistency of Sun Tzu’s writing with the theory. Niou and Ordeshook are skeptical about Sun Tzu’s writing and feel that he might not have anticipated all the nuances of strategic interaction that formal mathematical reasoning reveals in today’s modern world; and they intend to search for the aspects where Sun Tzu fails to account fully for what is known in today’s understanding on strategic choice.

Sun Tzu’s emphasis on secret agents provides an enormous strategic advantage over the enemy as knowledge on the enemy’s strategy is known beforehand. The role of a secret agent is to allow a decision maker to condition his actions based on information of the enemy, so as to render sequential moves rather than simultaneous (in the field of game theory). A sequential game is played by choosing one’s strategy using foreknowledge, but the advantages of foreknowledge are threatened when both sides use agents to feed each other false information. Sun Tzu states that the enemy’s agents should be bribed and treated well, but when both sides are adopting the same principle it raises the question that the double could be a triple agent. This results in the dilemma of circular reasoning that game theory tries to resolve. Here, the players are concerned with what their opponents believe about them as a function of their choices; hence, the choices selected not only manipulate outcomes directly, but also indirectly through the manipulation of opponents’ beliefs. Niou and Ordeshook reckon that resolving circular reasoning in this situation requires the use of advanced principles of probability theory and mathematics; therefore, this is where they regard Sun Tzu’s treatment of information in the realm of conflict incomplete.

Based on this incompleteness that Niou and Ordeshook highlight, the thesis attempts to use Bayesian Network to resolve circular reasoning in Sun Tzu’s usage of information and to overcome the manipulation of beliefs.
III. MODELING METHODOLOGY

A. CAUSALITY

Before I proceed with the methodology of modeling used in this thesis, it is important that I discuss the notion of causality and its underlying assumption in the model that is to be developed.

In scientific practices, references to the word causality (the idea of cause and effect) are made very frequently, but yet causation is one of the most controversial subjects in the philosophy of science (Druzdel & Simon, 1993). One of the earlier and most important contributions to the concept of causality was by Hume (1739), who claims that causes do not logically entail their effects and argues that casual relation cannot be observed; it is but an idea that exists only in the mind. It is a mental conjunction of empirical entities where the mental thought moves from a cause to its effects; it is the subjective feeling within the mind that formalizes a connection for causality which he regards as a criterion for causation, and that causes are invariably followed by their effects (Marini & Singer, 1988). This is the Regularity Theory of causation. Looking from a modern perspective of causation by Pearl, “causation is a language with which one can talk efficiently about certain structures or relevance relationships, with the objective of separating the relevant from the superfluous” (1988, p.18).

In the various branches of social science, the seeking of genuine cause is a priority as it is deemed as the basis for understanding social phenomena and the formulation of an explanatory science (Marini & Singer, 1988). The strength and applicability of causality as defined by Marini & Singer (1988) is as follows:

With causal knowledge it is often possible to predict events in the future or new observations and to exercise some measure of control over events. It is knowledge of causes that makes intervention for the production of desired effects possible (p.347-348).

With this definition, an understanding of causality could give rise to a diversified and flexible research approach where the subject matter considerations dictate the kind of evidence/cause that should be sought to establish a foundation for causal inference; and
regardless of the research approach, the degree of belief in a causal hypothesis relies on the strength of the evidence/cause available to support it (Marini & Singer, 1988). Given the usefulness of causation in research, when applied it has to be conformed to a set of boundaries and limitations: First, causal relationships are always identified against the background of some causal context, and specification of the context is crucial to the interpretation of an observed relationship; second, causes are often disjunctions of conjunctions, failure to consider the conjunctive properties of relationships within the context of interest may lead to failure to detect a causal relationship; “third, there are different types of causes and each requires different approaches to empirical analysis; fourth, human reasoning is purposive, hence, the temporal ordering of thoughts may not be a cue of causal direction” (Marini & Singer, 1988).

In scientific research work, a single cause rarely produces an effect (if it does happen, it is referred to as an inus condition); usually a number of causes is required to produce an effect. Therefore, it is not common that a causal relation is observed to be as solely factor A causing event B, but it is usually a combination of multiple factors such as A, C and D causing B, which is known as conjunctive plurality of causes (Marini & Singer, 1988). The disjunctive plurality of causes that may produce an effect is therefore called a causal structure. In order to form a causal structure, the connection between the cause and its effect has to be established, and this is done through analyzing the causal relations in terms of a causal process which explains the cause and effect relationship, where a causal process is the means by which structure and order are propagated from one state in a certain time to another state and time (Marini & Singer, 1988). In a complex causal structure, causal relation may be mediated. For example, factor A causes B and B causes C, in this illustration, we say that A causes C but is mediated by B. The analysis of the causal process not only reveals the connection of the causal relationship but also the direction as seen from this illustration. Causal relation is therefore asymmetric and transitive, where in the above illustration, when it is true that A causes C through the mediation of B, it is false to say C causes A.
B. PICTORIAL REPRESENTATION OF CAUSALITY

Causality used to formulate hypotheses, theories, and laws has been also perceived as a composition of independent and dependent variables. The following are a list of definitions by Van Evera (1997) that we need to determine before proceeding.

- **Variable** - A Concept that can have various values.
- **Independent variable** - A variable framing the causal phenomenon of a causal theory or hypothesis—which is the *cause* as discussed previously.
- **Dependent variable** - A variable framing the caused phenomenon of a causal theory or hypothesis—which is the *effect* as discussed previously.
- **Intervening variable** - A variable framing the intervening phenomenon included in a causal theory’s explanation. It is caused by the independent variable and cause the dependent variable—which is the *mediating factor* as discussed previously.
- **Condition variable** - A variable framing an antecedent condition, which helps to create the conjunctive plurality of causes.
- **Law** - An observed regular relationship between two phenomena. Law can be in the form of deterministic framing invariant relationship or probabilistic framing probabilistic relationship.
- **Hypothesis** - A conjectured relationship between two phenomena. It can be of two types, causal or non-causal.
- **Theory** - A causal law or causal hypothesis, together with an explanation of the causal law or hypothesis that explicates how a factor causes an event.
- **Explanation** - The causal laws or hypothesis that connects the cause to the phenomenon being caused, showing how causation occurs.

With this set of definitions, Van Evera provides the necessary tools to represent causality in pictorial forms. Van Evera (1997) claims that a theory is nothing more than a set of connected causal laws or hypothesis, and can be depicted in an arrowed diagram. In
Figure 1, A is the theory’s independent variable, B is the dependent variable. Letters q and r represent intervening variables, and hence Figure 1 depicts the theory’s explanation.

\[ A \rightarrow q \rightarrow r \rightarrow B \]

Figure 1. Theory’s explanation (Van Evera, 1997).

Condition variables can be added to the explanation when required, and is achieved in a pictorial form by using a symbol “x”. To give an explanation a higher resolution more intervening variable and condition variables can be added to form a chain theory.

\[ Y \rightarrow Z \rightarrow A \rightarrow q \rightarrow r \rightarrow s \rightarrow t \rightarrow B \]

\[ X \rightarrow C \]
\[ D \]

Figure 2. Chain Theory (Van Evera, 1997).

The pictorial approach adopted by Van Evera (1997) provides scientists in the research field with a valuable visualization technique in their search for causation. The picture clearly illustrates a crucial element in causality that is directionality; where Van Evera (1997) claims that a theory that cannot be represented by an arrowed diagram is not a theory and needs further reframing to become a theory. The pictorial approach has also illustrated the asymmetric and transitivity characteristics of causality as discussed above.

C. CAUSAL MAP

“Causal maps are cognitive maps that represent the causal knowledge of subjects in a specific domain” (Nadkarni & Shenoy, 2001), they are in many ways similar to pictorial representation of causality as they are directed acyclic graphs that represent cause-effect relations. With a clear understanding of Van Evera’s pictorial representation of causality, one will be able to better appreciate and understand the usage and construction of causal maps. Causal maps are useful decision tools for decision makers,
they “have been used extensively in areas of policy analysis and management sciences to represent salient points, knowledge, and conditions that influence decision making” (Nadkarni & Shenoy, 2001). The usefulness of a causal map is that it allows decision makers to make qualitative interpretation of a decision problem from analyzing the causal structure in the map, where the decision problem is captured by the concepts (represented by variables) in the map (Nadkarni & Shenoy, 2001); or as Pearl (1988) puts it, “causal maps are attractive mainly because they provide effective data structures for representing empirical knowledge”, and they play an essential role in the decomposition of complex problem to provide a vivid representation of sets of variables that are relevant to each other (Pearl, 2000).

A causal map is made up of three crucial components: First, the nodes in a map represent the causal concept; second, links (represented by unidirectional arrows) represent the causal connection among causal concepts, and the links can have either a positive or negative influence on the effect concept; third is the strength of a concept representing the causal value of a causal connection (Nadkarni & Shenoy, 2001). Figure 3 is a causal map that consists of three causal concepts, Knowing the Enemy, Knowing Yourself, and Winning the Battle. These causal concepts are linked through causal connections. In the map, the arrow is pointing towards the concept that represents effect and the tail end of the arrow is touching the concept that represents cause, capturing the antecedent consequence relation between the two concept; and in this example both causes have a positive influence on the effect. In fact, Figure 3 is representing one of Sun Tzu’s famous teachings:

Know the enemy know yourself; in a hundred battles, you will never be defeated. When you are ignorant of the enemy but know yourself, your chances of winning or losing are equal. If ignorant both of your enemy and yourself, you are sure to be defeated in every battle (Griffith, 1971).
Figure 3. Causal map of Sun Tzu’s teaching.

Figure 3 and Van Evera’s pictorial representation of causality have illustrated the primary emphasis in causal maps, that is, to identify the dependence between variables in terms of explanation and theoretical relationships (Nadkarni & Shenoy, 2001).

Huff (1990) identified that causal maps make three key assumptions on cognition (in the context of decision making): Firstly, the decision problem can be described and understood through causal associations; secondly, causality is the primary form of post-hoc explanation of decision outcome; and thirdly, choice among alternative decision actions involves causal relations (Nadkarni & Shenoy, 2001). There are different techniques that have been employed in the construction of causal maps, in this thesis, the open method based on personal construct method and textual analysis approach will be used (Nadkarni & Shenoy, 2001). These two are chosen as Sun Tzu’s *Art of War* can be viewed as a set of written principles that are based on cause-effect reasoning.

**D. PROBABILISTIC CAUSATION**

In Regularity Theory, causes are invariably followed by their effects, however, not all causation follows this theory. There are instances where imperfect regularities exist where causes do not have deterministic effects but probabilistic ones. Marini & Singer (1988) describes such imperfect regularities as follows:
…the traditional notion of necessity and sufficiency in causation pertain to complex scenarios, often involving a disjunction of conjunctions that we rarely, if ever, know fully, our elliptical understanding of these scenarios results in the observation of probabilistic regularities between identifiable “causes” and their effects…because complex regularities are seldom, if ever fully known, we are usually in a position to formulate only incomplete propositions reflecting them, from which inferences can be made with probability about the relation between a cause and its effect (p. 356-357).

If we were to look at the case of smoking causing lung cancer, it has been recognized that not all smokers developed lung cancer. Rather, smokers develop lung cancer at a much higher rate than non smokers. Therefore, in this case smoking does not necessarily cause lung cancer but it can be inferred that smoking raises the possibility of contracting this particular disease. This is the central idea of probabilistic causation, where causes raise the probability of their effects; and it is important to note that an effect may still occur in the absence of a cause or even fail to occur in its presence (Hitchcock, 2002).

The idea of probabilistic causation is expressed using the calculus of probability as it can be used to handle context dependent information and covers closely many aspect of plausible human reasoning (Pearl, 1988). Pearl (1988) states that:

Causation is listed as one of the four basic primitives of the language of probability because it is an indispensible tool for structuring and specifying probabilistic knowledge and because the semantics of causal relationships are preserved by the syntax of probability manipulation; no auxiliary devices are needed to force conclusions to conform with people’s conception of causation.

Through the use of conditional probability, where for example the probability of A given the presence of B is represented by P(A|B), for one to understand the concept of probabilistic causation that A raises the probability of B, in Probability Theory it would be represented in the form of P(B|A) is greater than P(B| not-A) (Hitchcock, 2002).

E. BAYESIAN BELIEF NETWORKS

Given the usefulness of causal maps that has been discussed, however, they exhibit two distinct shortcomings when they get complex: Firstly, identifying the level of
uncertainty in variables is important in making inference, but causal maps do not model the uncertainty that is associated with the decision variables within their causal structure as all the variables have the same level of certainty; secondly, causal maps only provide a static representation of decision variables. However, they do not reflect the level of beliefs decision makers have on the decision variables, neither are they able to capture the dynamics of information flow when decision makers gained new information on certain variables represented in the maps (Nadkarni & Shenoy, 2001).

These shortcomings however could be overcome through the use of Bayesian Networks that are based on Probability Theory. Bayesian Networks, like causal maps, are directed acyclic graphs; the main differences between them are: each variable in a Bayesian Network represents a random variable or uncertainty quantity that takes on two or more values (these values are called state space that consists of mutually exclusive and exhaustive values of the variable); the directed arrows or arcs represent direct causal influences between variables, but unlike causal maps, the strengths of these influences are quantified through the use of conditional probabilities (as discussed above) (Pearl, 1988 p. 50-51). Therefore, “Bayesian Networks allows us to use causal maps to make inferences for decision making” (Nadkarni & Shenoy, 2001).

From the perspective of probability, a Bayesian Network is a detailed description of a joint probability distribution of two or more variables in terms of conditional distributions for each variable (as illustrated in Figure 4). As such, Bayesian Networks in general do not necessarily possess causal relations (Nadkarni & Shenoy, 2001). Causal relations are present when Bayesian Networks are used to model causal maps, where the dependence relations in the Bayesian Networks are causal; and such networks are usually known as Bayesian Belief Networks or Causal Belief Networks. A Bayesian Network in itself “emphasizes three aspects: the subjective nature of the input information, the reliance on Baye’s conditioning as the basis for updating information, and the distinction between causal and evidential modes of reasoning” (Pearl, 2000, p.14). Bayesian Networks, when used as Bayesian Belief Networks, capture the uncertainty associated with the variables in the map, and thus allows sensitivity analysis to be carried out on variables of interest (Nadkarni & Shenoy, 2001).
F. CAUSALITY AND BAYESIAN BELIEF NETWORKS IN SUN TZU’S <i>ART OF WAR</i>

Controversy over probabilistic causation and its applicability to represent the notion of causality in science has yet to be resolved. We can identify that such controversy has its impact only in scientific research work, where scientists try to use causality as the basis for their hypothesis or theory. In this thesis, causality is not used to prove any hypothesis, theory, or explanation; it is used to construct Bayesian Belief Networks through a knowledge based approach that uses causal knowledge of domain experts—where the domain expert is Sun Tzu and his <i>Art of War</i>. In order to achieve the goal of this thesis which is to model Sun Tzu’s <i>Art or War</i>, I will decompose his principles in the <i>Art of War</i> into concepts, and their relevancy using the causal mapping approach; and lastly, Bayesian Belief Networks will be used to model the causal relationship of his principles—which are the expert knowledge.

G. BAYESIAN NETWORK

“A Bayesian Network is a graphical model that encodes relationships among variables of interest”; it is used to model causal relationships to gain understanding about a problem domain. As it possesses both causal and probabilistic semantics, it is an ideal representation for combining prior knowledge and data (Nadkarni & Shenoy, 2004). Bayesian Networks capture both the qualitative and quantitative aspects of a decision maker. The qualitative aspect (as discussed above) represents the causal relationship of a
decision problem through a directed acyclic graph in which nodes represent variables and
directed arcs/arrows describe the conditional independence relations embedded in the
network; while the quantitative aspect represents the degree of belief of a decision maker,
where the dependence relations are expressed in terms of conditional probability
distribution for each variable in the network (Nadkarni & Shenoy, 2004). As mentioned
earlier, each variable has a possible set of values called state space which consists of
mutually exclusive and exhaustive values of the variable. In a Bayesian Network, which
is used to represent a causal map, the cause is referred to as the parent and the effect as
the child. Therefore, for each variable in a Bayesian Network there is a table of
conditional probability distributions, one for each configuration of states of its parent
(Nadkarni & Shenoy, 2004). An illustration of the network’s probability distribution is in
Figure 5.

Figure 5. Bayesian Network with conditional probabilities distributions.

A Bayesian Network uses probability calculus, therefore, when the conditionals of
each variable are multiplied, the joint probability distribution for all the variables in the
network is observed (Nadkarni & Shenoy, 2004). In the following example, say
“sunshine (S) and rain (R) allow photosynthesis (P) to occur and therefore, the plant
grows (G)”, therefore,

\[
P(S,R,P,G) = P(S) \times P(R) \times P(P|S,R) \times P(G|P)
\]

for the same joint probability distribution, the axiom of total probability shows that

\[
P(S,R,P,G) = P(S) \times P(R|S) \times P(P|S,R) \times P(G|S,R,P).
\]
By comparing the two joint probability equations, we can conclude that R is independent of S as \( P(R|S) = P(R) \), and G is conditional independent of S and R given P as \( P(G|S,R,P)=P(G|P) \). The conditional independent relationships between the four variables as shown have been demonstrated by the use of probability equations, but such relationships could also be easily drawn from a Bayesian Network as shown below.

![Bayesian Network Diagram](image)

Figure 6. Illustrating conditional independence using Bayesian Network.

The acyclic nature of the Bayesian Network here shows the presence of a sequence as S R P C. Using this sequence that contains all the variables, it can be said that each variable in the sequence is conditionally independent of its predecessors given its parents (Nadkarni & Shenoy, 2004). The absence of an arrow joining S and R tells that S is independent of R; and that the absence of arrows from S to G and R to G tells that G is independent of S and R given P.

Bayesian Networks allow probabilistic inference that is based on evidence propagation, where evidence propagation refers to the computation of marginal probabilities of variables of interest (conditioned on the arbitrary configurations of other variables) which constitute the observed evidence (Nadkarni & Shenoy, 2004). With this feature, a Bayesian Network can be used to make inferences about the variables in the model. The conditionals in a Bayesian Network represent the prior joint distribution of the variables. If the states of some of the variables are observed, then such observations can have a probability scale where 1 is assigned to the observed states and 0 to the
unobserved ones; the product of all the marginal probabilities in the model will result in the posterior joint distribution of the variables. Hence, the joint distribution of variables changes every time new evidence about the variables is learned, allowing conclusions to be drawn based on this new evidence (Nadkarni & Shenoy, 2004).

H. MODELING METHODOLOGY

The modeling approach for this thesis has been adopted from the knowledge based approach, developed by Nadkarni and Shenoy (2004), where the causal knowledge of a domain expert is used for constructing Bayesian Belief Networks. They laid out a set of systematic procedures for modeling causal relations and their subsequent transformation into Bayesian Belief Networks. The procedure for the modeling consists of four main steps: Data elicitation, derivation of causal maps, modification of causal maps for constructing of Bayesian Belief Networks, and derivation of the parameters of Bayesian Belief Networks. Briefly, data elicitation is first drawn from a domain expert, and is followed by a detailed analysis of the context through a systematic content analyzing technique to map the causal relations within the expert’s domain, leading to the creation of the causal map. With the causal map created, further modifications are made to transform it to a Bayesian Belief Network. Lastly, variables of the Bayesian Belief Network are determined using probabilistic encoding techniques (Nadkarni & Shenoy, 2004). In the following sections, a detailed discussion of the four main steps of the modeling will be covered.

1. Data Elicitation

In the first step, the domain information is elicited from the expert. There are two different types of data elicitation techniques that can be used to capture the domain information, they are: structured and unstructured (Nadkarni & Shenoy, 2004). In this thesis the expert domain that I am trying to model is Sun Tzu’s *Art of War*, Sun Tzu has clearly stated all his principles and hence there is no need for an unstructured approach to elicit data, which is meant for unknown and ill structured domains. In the structured technique, experts are provided with a list of predefined concepts and are asked to specify the direction and signs between the concepts; this technique is thus more suitable for conforming and validating expert knowledge (Nadkarni & Shenoy, 2004). In this thesis,
the concepts and signs between them are drawn by closely examining the written principles in the *Art of War*, and the method of extracting them will be discussed in the following paragraphs.

2. **Derivation of Causal Maps**

Before creating a causal map, it is necessary to identify the causal relation in Sun Tzu’s written text. Causal statements (as discussed above) are statements in the written text that contain the cause-effect relationship, and they link the different concepts through causal connectors (Nadkarni & Shenoy, 2004). Literature on logic suggests that man’s perception of cause-effect relation are based on two types of reasoning: deductive and abductive; where a deductive reasoning is reasoning done from the cause to its effect, in the direction of causation, and a abductive reasoning is reasoning done from the effect to its cause, in the opposite direction of causation (Nadkarni & Shenoy, 2004). Determining the reasoning of a causal statement allows us to determine the direction of the connectors in the causal map. However, Nadkarni and Shenoy (2004) note that the common perceptions by most individuals of causal relationships between concepts are based on language which may lead to an abductive reasoning; where such causal statements will be misrepresented in a causal map, in which the direction of the arrow is pointing from the effect to its cause. Therefore, Nadkarni and Shenoy (2004) emphasize that in deriving causal maps, the focus should be placed on the reasoning underlying the causal statements, rather than the language used. This would then be the basis of analysis in Sun Tzu’s *Art of War*.

With the above argument, to identify the causal statements in a narrative, the causal connectors have to be first identified, and the identification of these connectors, suggested by Nadkarni and Shenoy (2004) can be done by developing a comprehensive dictionary list of words that can be considered as causal connectors such as “if-then”, “because”, “therefore” and so on. Upon identifying the causal connectors, the causal phrases would be determined. The subsequent phase of this step is to break down the causal statements into their respective components: causal phrases, connectors, and effect phrases (Nadkarni & Shenoy, 2004). This approach could also be reinforced by Van Evera’s (1997) way of viewing causality, by determining first the variables in the causal statements, then further distinguishing these variables into independent and dependent
variables. Upon distinguishing the independent and dependent variables, we could then determine the direction of the arrow and hence derive the causal map.

The last phase in this step is to refine the causal phrases, connectors, and effect phrases into coded causal maps. In the previous phase, the components of the causal statements allow us to formulate a raw causal map that captures the causal relations. However, such raw maps hinder analysis as they are too complex to be represented in graphical forms. Nadkarni and Shenoy (2004) use a process called aggregation, where the process determines the part of a causal text to be coded and the words to use in the coding scheme. The advantages of aggregation is that it allows the generalized concepts in the raw causal maps to be used “to move the coded text beyond explicitly articulated ideas to implied or tactic ideas”, and “it can also be used to avoid misclassification of concepts due to peculiar wording” (Nadkarni & Shenoy, 2004) in the modeling of Sun Tzu’s *Art of War*. In the process of coding, decisions have to be made on which words in the raw causal phrases to retain and delete, or which part of the phrases need to be reworded; and this process requires the modeler of the *Art of War* to conduct his own interpretations. The final coded concept may be of a single word, a composite of words, or a complex phrase (Nadkarni & Shenoy, 2004). Once the aggregation process is completed, the modeler has to review the coded concepts to ensure that they capture the meanings of the causal phrases to prevent a misrepresentation of the raw causal maps and the coded causal maps. Therefore, the final product of a coded causal map represents a network of concepts formed from causal statements depicting the directionality of causation and the positive or negative relations between the concepts (Nadkarni & Shenoy, 2004). Figure 7 depicts the derivation process of causal map.
3. Constructing Bayesian Belief Networks

Upon completing the coded causal maps, modification of these maps is required to construct Bayesian Belief Networks. To achieve this, Nadkarni and Shenoy (2004) suggest that the modification needs to address four major issues: “conditional independencies, reasoning underlying the link between concepts, distinction between direct and indirect relations, and eliminating circulation relations”.

Of the four issues, reasoning underlying concepts has been discussed earlier; I will now proceed with the issue on direct and indirect relations. In the process of deriving causal maps, distinctions between direct and indirect relations of the concepts are not covered. Nadkarni and Shenoy (2004) state that “a direct link between two
concepts in the causal map does not guarantee a direct relationship between two concepts”, and “it just implies a relation between the two concepts that can be either direct or indirect”. The distinction between direct and indirect relationships between causal concepts is important as it: helps the modeler to understand the nature of relations between variables, removes redundant arrows in the causal map that increase complexity of the representation, and allows the identification of conditional independencies in causal maps. In the previous example used to illustrate a complex casual structure, where A causes B and B causes C; using this illustration to illustrate direct and indirect relationships, it is said that A is directly related to B and A is indirectly related to C. Therefore in a causal map, A will have arrows pointing towards B and C, but in a Bayesian Belief Network, A will only be pointing towards B (as in Figure 8.)

![Causal Map and Bayesian Belief Map](image)

Figure 8. Distinguishing between direct and indirect relations (After Nadkarni & Shenoy, 2004)

As stated earlier, causal maps are directed graphs and have an acyclic structure. Therefore circular relations or causal loops in a causal map destroy the acyclic structure of the causal map, and violate the acyclic graphical structure required by a Bayesian Network (Nadkarni & Shenoy, 2004). In the modeling process it is thus crucial to remove any circular reasoning in the causal maps when transforming them to Bayesian Belief Networks. Causal loops exist because of coding mistakes or they may represent dynamic relations between variables across different timeframes. Coding mistakes could be resolved through clarifying the causal linkages between variables in the form of reasoning (deductive versus abductive reasoning) and relations (direct versus indirect).
As for circular relations resulting from different timeframes, disaggregating the variables into their different timeframes would solve the problem (Nadkarni & Shenoy, 2004).

Structured method such as adjacency matrices can be used to address these four issues. An example of an adjacency matrix is shown in Table 1. The rows in the table represent causes and the columns represent effect. The modeler will have to enter the relationship between two concepts being compared into the matrix by using these symbols “0”, “+”, and “-”. The symbol “0” represents that there is no relation between the two concepts compared; “+” represents that there is a positive relation between the two concepts compared; and “-” represents that there is a negative relation between the two concepts compared. This causal statement used in Figure 7 will be used to illustrate the usage of adjacency matrices in Table 1.

### Table 1. Adjacency Matrix (After Nadkarni & Shenoy, 2004).

<table>
<thead>
<tr>
<th>Causes</th>
<th>Sunshine</th>
<th>Rain</th>
<th>Photosynthesis</th>
<th>Caterpillars</th>
<th>Plant Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunshine</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Rain</td>
<td>0</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Photosynthesis</td>
<td>+</td>
<td>+</td>
<td></td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>Caterpillars</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Plant Growth</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

4. **Derivation of the Parameters of Bayesian Belief Networks**

The final step is to derive the parameters of the Bayesian Belief Networks. Up to this point, the concepts that are captured do not reflect the uncertainty that they are associated with, all the concepts are still assumed to have the same level of uncertainty; hence, inference of concepts in the network for decision analysis is not possible. “A Bayesian Belief Network allows decision maker to make inferences on the different [concepts] in the network based on the information about other variables in the network”; therefore, to allow for inferences to take place, there is a need to assess the uncertainty that is associated with each concept and their interactive effects with multiple causal concepts on the effect concepts (Nadkarni & Shenoy, 2004). In the usage of Bayesian
Networks, both terms, concepts and variables, refer to the same parameter in a Bayesian Network. For the sake of consistency, from this point onwards I will use the term ‘variable’ in place of ‘concept’. The parameters of the Bayesian Belief Network are determined by identifying the state space of each variable in the network and the conditional probabilities associated with the variables in the network. State space in the context of Bayesian Belief Networks is referring to the state of each variable that is observed, and state space is identified from the definition of each concept. It is important to note that the meanings that are associated with the variables of the Bayesian Belief Network are not universal as they depend solely on the perception of the expert that is modeled (in this thesis the meaning associated to the variables would be based on interpretation of Sun Tzu’s *Art of War*); and precise definition of the variables will specify the scope (or state space) of each variable which will allow the making of inferences in a Bayesian Belief Network (Nadkarni & Shenoy, 2004). The example used in Figure 7 will be used to illustrate the concept of state space. For example Rain perceived by a decision maker could have state space of “High”, “Low”, or “None”, and each of these states has a different level of impact on the effect variable which enables inferences to be made.

The uncertainty of the variables in the Bayesian Belief Network is determined through measuring the decision maker’s degree of belief for the states of each variable or variables that conditioned on the states of their parent variables, and the process of measuring degree of belief is referred to as probability assessment (Nadkarni & Shenoy, 2004). Probability assessment is an important part of decision analysis, and is performed in the context of a specific decision framework; it also improves a decision maker’s awareness of his state of information and provides a clear means for inference about uncertainty (Spetzler & Von Holstein, 1975). Spetzler and Von Holstein (1975) suggest that decision analysis usually involves three phases: deterministic, probabilistic, and informational. In the deterministic phase, a decision problem is structured by defining the relevant variables and assigning values to possible states in the variables. When in the “probabilistic phase, uncertainty is explicitly incorporated into the analysis by assigning probability distributions to the important variables”; and these distributions are obtained by assessing the judgment of the decision maker’s knowledge about the
problem, where the judgments are transformed into probability distributions that express the uncertainty of the various states in the variables (Spetzler & Von Holstein, 1975). The informational phase concerns the economic values of information, determined through calculating the worth of reducing uncertainty about each of the important variables in the problem, which is not within the scope of this thesis and will not be further discussed. The focus in this step is therefore limited to the deterministic and probabilistic phases. The representation of uncertainty using probability assessment to represent the decision maker’s degree of belief is thus a subjective interpretation; therefore probabilities in Bayesian networks are interpreted subjectively as degrees of belief to produce Bayesian Belief Networks (Clemen & Reilly, 2001).

There are different methods of assessing the uncertainties for the variables, and the method used depends solely on the decision maker’s preference. In this thesis the equivalent lottery method from Clemen and Reilly (2001) will be used. In the equivalent lottery method, the decision maker is asked to weigh the importance of each state that is contained in a variable through the use of utility values, with varying values from 0 to 100. Once the decision maker has completed weighing the different states, he is then asked to compare two lotteries like games, each of which result in the utility value that he had decided, and he has to respond by specifying the points on the probability scale (zero to one) while the utility values of each state remain fixed. The probability assessment of the state is determined once he feels that the two lotteries are indifferent and that the total of probabilities of all the states must sum to one. The degree of belief for each of the state in the variable assessed would be the probability values that are determined in lottery 2. Illustration of the equivalent lottery method is shown in Figure 9.
Figure 9. Probability Assessment.

With this modeling methodology laid out, modeling of Sun Tzu’s *Art of War* can be accomplished; and in chapter four, I will be demonstrating the process of converting Sun Tzu’s principles into causal maps and from causal maps to Bayesian Belief Networks.
IV. IMPLEMENTATION OF MODEL

A. MODELING SUN TZU’S “ESTIMATES”

The intent of this chapter is to demonstrate the modeling methodology that is discussed in chapter three, to then apply this methodology in the domain of Sun Tzu’s *Art of War*, and through this demonstration, prove the hypothesis which was discussed in chapter one—where Sun Tzu’s principles in his *Art of War* can be captured using a mathematical model. The rest of this chapter will illustrate the conversion of Sun Tzu’s *Art of War* into a causal map and from a causal map to a Bayesian Belief Network. The creation of the Bayesian Belief Network is done with a commercial software called *Netica*. It is not the purpose of this thesis to capture the entire aspect of the *Art of War*; therefore, for ease and clarity of proving the hypothesis, only Chapter One of Sun Tzu’s work “Estimates” will be modeled. Lastly, with the created Bayesian Belief Network model, a sensitivity analysis of the model will be conducted to determine the critical concept (or variable) of the model that will have the greatest influence on the outcome, and finally, the chapter ends with a simple illustration to show the overall usefulness of Bayesian Belief Networks.

1. Data Elicitation

In this modeling process, Sun Tzu is considered as our expert and for the purpose of this thesis we deemed his writings sufficient to possess the knowledge for developing the Bayesian Belief Network model; therefore, in this data elicitation process, data is elicited directly from his writings that are translated by Griffith (1971). The following extract from Sun Tzu’s “Estimates” is used as the data for the model that will be developed.

War is a matter of vital importance to the State; the province of life or death; the road to survival or ruin. It is mandatory that it be thoroughly studied.

Therefore, appraise it in terms of the five fundamental factors and make comparisons of the seven elements later named. So you may assess its essentials.

The first of these factors is moral influence; the second, weather; the third, terrain; the fourth, command; and the fifth, doctrine.
By moral influence I mean that which causes the people to be in harmony with their leaders, so that they will accompany them in life and unto death without fear of mortal peril.

By weather I mean the interaction of natural forces; the effects of winter’s cold and summer’s heat and the conduct of military operations in accordance with the seasons.

By terrain I mean distances, whether the ground is traversed with ease of difficulty, whether it is open or constricted, and the chances of life or death.

By command I mean the general’s qualities of wisdom, sincerity, humanity, courage and strictness.

By doctrine I mean organization, control, assignment of appropriate ranks to officers, regulation of supply routes, and the provision of principal items used by the army.

There is no general who has not heard of these five matters. Those who master them win; those who do not are defeated.

Therefore in laying plans compare the following elements, appraising them with the utmost care.

If you say which ruler possesses moral influence, which commander is the more able, which army obtains the advantages of nature and the terrain, in which regulations and instructions are better carried out, which troops are the stronger;

Which has the better trained officers and men;

And which administers rewards and punishments in a more enlightened manner;

I will be able to forecast which side will be victorious and which defeated.

If a general who heeds my strategy is employed he is certain to win. Retain him! When one who refuses to listen to my strategy is employed, he is certain to be defeated. Dismiss him!

Having paid heed to the advantages of my plans, the general must create situations which will contribute to their accomplishment. By ‘situations’ I mean that he should act expeditiously in accordance with what is advantageous and so control the balance.
2. Derivation of Causal Map

In this step, the data that is elicited from the previous step is processed through a four phase procedure (which has been illustrated in chapter 3) to derive a causal map that represents the underlying principles in Sun Tzu’s “Estimates”. The detailed process of deriving this causal map is illustrated in the following paragraphs.

a. Identification of Causal Statement in the Text

The first phase of this step is to identify the causal statements that are contained in the expert’s domain; therefore, the causal statements drawn are based subjectively on the appreciation of Sun Tzu’s Art of War by the decision maker (or modeler) in this thesis. The following is the list of causal statements that are drawn from the “Estimates”.

Table 2. Identified Causal Statements

<table>
<thead>
<tr>
<th>No.</th>
<th>Causal Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Appraise war in terms of is moral influence, weather, terrain, command, and doctrine.</td>
</tr>
<tr>
<td>2</td>
<td>Moral influence causes the people to be in harmony with its leaders and affects the outcome of a war.</td>
</tr>
<tr>
<td>3</td>
<td>Command is affected by the general’s qualities of wisdom, sincerity, humanity, courage and strictness.</td>
</tr>
<tr>
<td>4</td>
<td>Doctrine is affected by organization, control, assignment of appropriate ranks to officers, regulation of supply routes, and the provision of principal items used by the army.</td>
</tr>
</tbody>
</table>

b. Causal Components

Having identified the causal statements, this next phase of this process is to break up the causal statements into their respective components as listed in the following table.

Table 3. Causal Components (After Nadkarni & Shenoy, 2004).

<table>
<thead>
<tr>
<th>No.</th>
<th>Causal Phrase</th>
<th>Causal Connector</th>
<th>Effect Phrase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Moral influence, weather, terrain, command, and doctrine</td>
<td>Appraise</td>
<td>War</td>
</tr>
<tr>
<td>2</td>
<td>Moral influence</td>
<td>Causes</td>
<td>People to be in harmony with its leaders</td>
</tr>
<tr>
<td>3</td>
<td>People in harmony with its leaders</td>
<td>Affects</td>
<td>The outcome of war</td>
</tr>
<tr>
<td>4</td>
<td>General’s qualities of wisdom, sincerity, humanity, courage and strictness</td>
<td>Affects</td>
<td>Command ability</td>
</tr>
<tr>
<td>5</td>
<td>Organization, control, assignment of appropriate ranks to officers, regulation of supply routes, and the provision of principal items used by the army</td>
<td>Affects</td>
<td>Doctrine of war</td>
</tr>
</tbody>
</table>
c. Coding Scheme

In this phase, the raw concept phrases that are identified in the above table are simplified through further refinement into coded concepts listed in the following table.

Table 4. Coding Scheme (After Nadkarni & Shenoy, 2004).

<table>
<thead>
<tr>
<th>No.</th>
<th>Raw Phrase</th>
<th>Coded Concept</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>War</td>
<td>War</td>
</tr>
<tr>
<td>2</td>
<td>Moral Influence</td>
<td>Moral Influence</td>
</tr>
<tr>
<td>3</td>
<td>Weather conditions</td>
<td>Weather</td>
</tr>
<tr>
<td>4</td>
<td>Terrain conditions</td>
<td>Terrain</td>
</tr>
<tr>
<td>5</td>
<td>Command ability</td>
<td>Command</td>
</tr>
<tr>
<td>6</td>
<td>Doctrine of war</td>
<td>Doctrine</td>
</tr>
<tr>
<td>7</td>
<td>People to be in harmony with its leaders</td>
<td>Harmony</td>
</tr>
<tr>
<td>8</td>
<td>The outcome of war</td>
<td>War</td>
</tr>
<tr>
<td>9</td>
<td>Wisdom in handling challenges of war</td>
<td>Wisdom</td>
</tr>
<tr>
<td>10</td>
<td>Sincerity towards the people</td>
<td>Sincerity</td>
</tr>
<tr>
<td>11</td>
<td>Humanity towards the people</td>
<td>Humanity</td>
</tr>
<tr>
<td>12</td>
<td>Courage to face the challenges of war</td>
<td>Courage</td>
</tr>
<tr>
<td>13</td>
<td>Strictness towards the troops</td>
<td>Strictness</td>
</tr>
<tr>
<td>14</td>
<td>Organization, control, assignment of appropriate ranks to officers.</td>
<td>Command and Control (C2)</td>
</tr>
<tr>
<td>15</td>
<td>Regulation of supply routes, and the provision of principal items used by the army</td>
<td>Resource allocation</td>
</tr>
</tbody>
</table>

d. The Final Coded Causal Map

In the final phase of this step, the causal map representing the principles of Sun Tzu’s “Estimates” is created using the coded concepts and the causal relationships between the concepts that are derived in Tables 3 and 4. The following figure is a causal map representation of “Estimates”.

Figure 10. Coded Causal Map of Sun Tzu’s “Estimates”.

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3. Construction of Bayesian Belief Networks

In this step of the modeling process, further refinement of the causal map is conducted by distinguishing the difference between direct and indirect relationships among the variables. This distinction process is done using the adjacency matrix method (as discussed in chapter three). With the completion of this matrix, it ensures that the created Bayesian Network is acyclic.

Table 5. Adjacency Matrix for Sun Tzu’s “Estimates” (After Nadkarni & Shenoy, 2004).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Wisdom</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Sincerity</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Humanity</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Courage</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Strictness</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Resource Allocation</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Moral Influence</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Weather</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Terrain</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Command</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Doctrine</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Harmony</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>War</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

The transformation of the causal map to a skeleton structure of the Bayesian Network is now completed and is shown in Figure 11. When observed, only a slight change of the causal map has taken place, the arrow indicating an indirect relationship is removed from between Moral Influence and War.
4. Derivation of Parameters

In this step of the transformation process, the state spaces and their utility values, which are associated with each variable of the model, will be determined by the decision maker (a practitioner of the Art of War) using the method described in chapter three; and the states determined are based solely on the decision maker’s perception of the meaning of the variable. The following table shows the states determined for each variable of the Bayesian Belief Network model.

Table 6. Determination of the states of each variable (After Nadkarni & Shenoy, 2004).

<table>
<thead>
<tr>
<th>No</th>
<th>Concept</th>
<th>Description</th>
<th>States</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>War</td>
<td>The potential outcome when a state engages in war</td>
<td>Win, Lose</td>
</tr>
<tr>
<td>2</td>
<td>Moral</td>
<td>Moral influence of a leader affects the peoples’ commitment to war</td>
<td>High, Low</td>
</tr>
<tr>
<td>3</td>
<td>Weather</td>
<td>Weather affects the conduct of war</td>
<td>For us, Against us</td>
</tr>
<tr>
<td>4</td>
<td>Terrain</td>
<td>Terrain affects the conduct of war</td>
<td>For us, Against us</td>
</tr>
<tr>
<td>5</td>
<td>Command</td>
<td>Commander’s ability to fight the war</td>
<td>High, Moderate, Low</td>
</tr>
<tr>
<td>6</td>
<td>Doctrine</td>
<td>Rules that form the basis for the conduct of war</td>
<td>Effective, Ineffective</td>
</tr>
<tr>
<td>7</td>
<td>Harmony</td>
<td>The relationship between the people and the leadership</td>
<td>In harmony, not in harmony</td>
</tr>
<tr>
<td>8</td>
<td>Wisdom</td>
<td>Level of intelligence, coupled with the knowledge and experience needed to make good decision</td>
<td>High, Low</td>
</tr>
<tr>
<td>9</td>
<td>Sincerity</td>
<td>The genuineness of commander’s feeling toward his troops</td>
<td>Sincere, Not sincere</td>
</tr>
<tr>
<td>10</td>
<td>Humanity</td>
<td>Commander’s level of kindness and compassion towards his troops</td>
<td>High, Low</td>
</tr>
<tr>
<td>11</td>
<td>Courage</td>
<td>Commander’s ability to overcome danger, difficulty, fear, and uncertainty.</td>
<td>High, Low</td>
</tr>
<tr>
<td>12</td>
<td>Strictness</td>
<td>Commander’s attitude and ability to maintain</td>
<td>Strict, Slack</td>
</tr>
</tbody>
</table>
The next phase of this step is to determine the utility values of each state by the decision maker. After the values are determined the decision maker will have to determine the probability associated with each of the states to establish the probability distribution tables. This is done using the lottery method described in chapter three. The assessment of all the values for all states and the probability of all the variables will not be shown, as it is not necessary for the purpose of this thesis; instead, only the following illustration on the assessment of the variable, “Wisdom”, will be done. In this illustration, the decision maker has assessed the values for states “High” and “Low” to be 100 and 50 respectively. When asked to assess the probability associated to each state, he has assigned probabilities of 0.8 and 0.2 to the respective two states, and with these probability scales assigned, his perception of the two lotteries is indifferent.

![Assessing the probabilities for the states in variable “Wisdom”](image)

Upon completing the assessment of all the probability association with all the states in the model, the decision maker will have generated the conditional distribution tables for all the variables, thus completing the creation of the Bayesian Belief Network. The following is the complete decision maker’s Bayesian Belief Network of Sun Tzu’s “Estimates” created by *Netica*.

<table>
<thead>
<tr>
<th></th>
<th>troop discipline</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>Command and Control (C2)</td>
</tr>
<tr>
<td>14</td>
<td>Resource allocation.</td>
</tr>
</tbody>
</table>
Figure 13. Bayesian Belief Network for Sun Tzu’s “Estimates” created using Netica.

With this Bayesian Belief Network, the decision maker captures Sun Tzu’s principles into the model. When using the principles of “Estimates”, the decision maker (in making the decision to go to war) will have to consider all the critical concepts in the model to reach his decision. With the model in hand and upon completing his assessments, he will be able to see the probabilistic results of his expected outcome. In the above figure, the model shows the decision maker that his probability of winning the war is 73.5%, indicating a high chance of winning. Instead of using his gut feeling in applying Sun Tzu’s principle, the model quantifies his assessed outcome to allow for better decision making. When placed in a stressful environment, coupled with the pressure of time, the decision maker is guided by the model to derive his decision in a structured manner without failing to consider all the crucial variables.

B. SENSITIVITY ANALYSIS

The Bayesian Belief Network model not only quantifies a decision maker’s assessed outcome, but also allows the decision maker to determine which variables in his assessment have the greatest impact on his assessed outcome. Using Netica to create the model allows the decision maker to determine which variables have the greatest influence on this assessed outcome. With this information, the decision maker could refocus his resources in further evaluating these variables to recalculate a more refined assessed outcome. In the Bayesian Belief Network model that is created in Figure 13, the variable
“War” is of highest importance to the decision maker, therefore, he would want to know which variables in the model would affect it most. The analyzed result (using the sensitivity analysis tool in Netica) for this model is shown in the following table.

<table>
<thead>
<tr>
<th>Node</th>
<th>Mutual Info</th>
<th>Quadratic Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>War</td>
<td>0.86263</td>
<td>0.2039227</td>
</tr>
<tr>
<td>Weather</td>
<td>0.03354</td>
<td>0.0096</td>
</tr>
<tr>
<td>Terrain</td>
<td>0.02861</td>
<td>0.0084</td>
</tr>
<tr>
<td>Doctrine</td>
<td>0.02861</td>
<td>0.0084</td>
</tr>
<tr>
<td>Harmony</td>
<td>0.02861</td>
<td>0.0084</td>
</tr>
<tr>
<td>Moral</td>
<td>0.01627</td>
<td>0.0048</td>
</tr>
<tr>
<td>C2</td>
<td>0.01041</td>
<td>0.003024</td>
</tr>
<tr>
<td>Command</td>
<td>0.00887</td>
<td>0.0026426</td>
</tr>
<tr>
<td>Resource Allocation</td>
<td>0.00466</td>
<td>0.001344</td>
</tr>
<tr>
<td>Humanity</td>
<td>0.00076</td>
<td>0.0002186</td>
</tr>
<tr>
<td>Courage</td>
<td>0.00036</td>
<td>0.0001017</td>
</tr>
<tr>
<td>Wisdom</td>
<td>0.00027</td>
<td>0.0000777</td>
</tr>
<tr>
<td>Sincerity</td>
<td>0.00025</td>
<td>0.0000705</td>
</tr>
<tr>
<td>Strictness</td>
<td>0.00019</td>
<td>0.0000536</td>
</tr>
</tbody>
</table>

From Table 7, the analyzed result shows that variable “Weather” –with the highest quadratic score after the variable of interest—has the most influence on “War”. Therefore, this sensitivity analysis results prompts the decision maker to reassess the “Weather”, or to allocate his resources to further evaluate this variable to improve his decision outcome. (The mathematics behind the generation of this result will not be discussed as that is not the focus of the thesis.)

C. INERENCE FOR DECISION MAKING

With the results from the sensitivity analysis, the decision maker may want to further determine his assessed variables to improve his decision. In the above model, the decision maker first assessed the “Weather” variable with 60% probability that is “for him” and 40% that is “against him”. If for example, his resources were to come back with a detailed assessment on the “Weather”, the probability distribution for this variable will be different (or updated). If say, the assessment came back with a probability of
90% and 10% for the respective states, the model will be updated and the assessed outcome for variable “War” will be affected (as shown in the following figure). With this new update, the inferred outcome on “War” is therefore different with a higher winning probability of 77.5%; and with this new inferred outcome, the decision maker can change his decision accordingly. If again, the decision maker’s resource were to be able to confirm with certainty, for example the “Terrain” and is sure that it is “for us”, the probability assigned to this state will be ‘1’ and ‘0’ will be assigned for other states associated with this variable. The confirmation of a state on a variable will again update the model to provide the decision maker with most updated probable outcome (as shown in Figure 11).

With this model, the decision maker—prior to the execution of his decision—can continuously run the cycle of sensitivity analysis and update the model as and when new or additional information is available, so as to give him the most up to date assessment of the outcome.

Bayesian Belief Network model with a reassessed variable.
Bayesian Belief Network model with a confirmed variable state.

Figure 14. Inference using the Bayesian Belief Network.
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V. RESOLVING CIRCULAR REASONING

A. INFORMATION AND SECRET AGENTS IN THE ART OF WAR

In Sun Tzu’s Art of War, he constantly emphasizes the importance of information in the conduct of war. Information in all key aspects, such as knowing the battle terrain to knowing the enemy’s intent, plays a crucial role in the commander’s implementation of his courses of action. With this information at hand, the commander could maneuver his army to an advantageous position and defeat his enemy. Sun Tzu regards such information in war as foreknowledge. With foreknowledge, a commander will be able to know when, where, and how to attack his enemy using the right amount of forces necessary to defeat his enemy. Sun Tzu’s says foreknowledge cannot be gained through superstitious means or calculation, rather, it has to be gained through the use of men who know the enemy situation; and such men are employed as secret agents. Sun Tzu continues by stating that employment and treatment of these men are crucial as information drawn from them affects every move the army makes. There are five types of secret agents that are identified by Sun Tzu: native, inside, double, expendable, and living. The employment of doubled agents could be viewed as the most difficult and useful—as they are enemy agents recruited to spy against the enemy. In terms of usefulness, these agents could provide the commander with invaluable information on the enemy and inject false information to the enemy. However, difficulty arises when both sides of the conflict employ doubled agents. In such a situation, the commander faces the dilemma of trust regarding the information that he received from his doubled agents.

Niou and Ordeshook (1994) recognize the usefulness of Sun Tzu’s usage of foreknowledge in terms of a game theoretical approach. Having foreknowledge in analyzing conflict with game theory is to have the liberty of a sequential move option. The commander knows the moves of the enemy and his courses of action are conditioned on the enemy’s first move. With such sequential move, the commander will always be in an advantageous position. Sun Tzu’s foreknowledge breaks down when this sequential move option is disrupted, i.e. when both sides of the conflict employ double agents against each other. Therefore, the commanders of both sides question themselves regarding the true nature of the double agents, and query if they have become triple
agents. Triple agent in this context refers to a failed recruitment on the opponent’s agent, by which the double agent has deceived his new master when actually he is still working for his previous master. If this reasoning cycle continues, it will result in circular reasoning as Niou and Ordeshook (1994) have highlighted.

B. RESOLVING CIRCULAR REASONING USING BAYESIAN BELIEF NETWORK

A commander when faced with the dilemma of circular reasoning is placed into a state of uncertainty with the information that he has received. This dilemma has been encountered by earlier practitioners of Sun Tzu’s Art of War as they realized the implication of using doubled agents. In Griffith (1971) he quotes Mei Yao-ch’en stating, “Take precautions against the spy having been turned around” and also, Tu Mu cautioning the use of secret agents.

Tu Mu further states that a commander “must be deep and subtle[,] then [he] can assess the truth or falsity of the spy’s statements and discriminate between what is substantial and what is not”. In Tu Mu’s approach, he would want the commander to evaluate the spy and make a stand to believe or not to believe the information he brings. If this approach is adopted, it will break the circular reasoning that is encountered by the commander. The downside of this approach is however, detrimental to the state. If the commander believes and conditions his courses of action based on the spy’s input, and if the spy turns out to be a triple agent, the commander would likely be defeated; but if the spy is loyal, and the commander chooses not to believe him, his army could also be defeated or be trapped in an unwanted prolonged war. The commander is caught in a losing situation for either decision.

Instead of being trapped in a circular reasoning loop or a losing decision, the commander can assess the degree of belief that he has in the spy to formulate his courses of action and this degree of belief can be incorporated into a Bayesian Belief Network model. To illustrate how a commander can utilize the degree of belief he has in the spy to break the circular reasoning, the following Sun Tzu principle will be used. Sun Tzu says,
Know the enemy know yourself; in a hundred battles, you will never be defeated. When you are ignorant of the enemy but know yourself, your chances of winning or losing are equal. If ignorant both of your enemy and yourself, you are sure to be defeated in every battle (Griffith, 1971).

From this principle, knowing the enemy is important to win the battle. If we were to raise the question of how a commander knows his enemy, the answer would be through his agents. Therefore, the agents provide the commander with information—the foreknowledge—of the enemy to allow him to defeat them; and we can assume that among the agents used, a portion could be doubled agents. With this argument, the causal relationship between agents and knowledge on enemy is established. The following is a set of simple Bayesian Belief Network models which will be used to illustrate Tu Mu’s approach and how a commander uses his degree of belief in his doubled agents to finalize his courses of action.

The above illustration indicates the probabilistic outcomes based on Tu Mu’s way of assessing the double agents. If the agent is providing false information and the commander bases his courses of action on the agent’s information, he would engage his enemy, and is bound to be defeated; or misses the chance of defeating the enemy if otherwise.
In this second model, the commander is made to assess his degree of belief in the
doubled agents that he employed, and through this model, the commander can make his
decision of going to battle based on the assessed outcome. From his assessment, the
model shows that his probability of winning is 73%, thus influencing the commander’s
decision to engage his enemy. However, the probabilistic outcome of the model still does
not tell the commander whether he should use or discard the information from his double
agents to formulate his courses of action. The way to interpret this model would be that
the commander formalizes his courses of action based on his degree of belief. The
courses of action should be divided according to his degree of belief, in which 70% of the
engagement plan should be based on the doubled agents information and 30% not. With
such division of the plan of actions, the 30% of the plan could be seen as the contingency
of the execution. If the doubled agents turned out to be providing false information, the
commander could potentially avoid a total defeat, or he may surprise his enemy with the
contingency plan and still win the battle.

Niou and Ordeshook (1994) correctly identify the incompleteness in Sun Tzu’s
treatment of information, and state that this could potentially be overcome through the
use of probability theory. The above illustration made using Bayesian Belief Network
model (using probability calculus) has demonstrated that circular reasoning resulting
from usage of doubled agents can be resolved.
VI. CONCLUSION

The purpose of this thesis is two-fold: firstly, to prove my hypothesis that Sun Tzu’s principles in his *Art of War* can be modeled mathematically and secondly, to use this created model to provide a structured approach for applying his principles. In this thesis, modeling of Sun Tzu’s principles has been demonstrated through the use of Bayesian Networks where the resulting model is known as a Bayesian Belief Network. Prior to the process of modeling Sun Tzu’s *Art of War*, the notion of causality was introduced as the underlying basis for the modeling approach. Even though there are many controversies over the philosophical aspect of causality, they do not affect the modeling approach, because the perception of Sun Tzu’s principles is revealed through the understanding of cause-effect relationship. Sun Tzu’s principles are however regarded as the expert knowledge which a decision maker uses in formulating his decisions or strategies.

Using causality and the modeling methodology that was described, Sun Tzu’s principles are first represented in a graphical form of a causal map. The causal map that is created only shows the static causal relationships of the concepts that are embedded in Sun Tzu’s principle. It does not capture the uncertainties and beliefs that a decision maker has on the concepts that are to be considered, neither does it allow the decision maker to make any inference in aid of his decision making. The result of such shortcomings is resolved through further transformation, from a causal map to a Bayesian Belief Network. In this thesis, a Bayesian Belief Network model has been created to capture the content of chapter one in Sun Tzu’s *Art of War* “Estimates” for illustration purposes. The model demonstrated the ability for the decision maker to analyze all the concepts (in the form of variables) in the principles and provides him with a probabilistic outcome on the concept of his concern to make his decision. The model also allows him to conduct sensitivity analysis on the concepts that affect the probabilistic outcome of his concern which in turn allows him to refine his decision. With the knowledge of knowing the concept that has the greatest influence on his decision, the decision maker could use his resources to further reevaluate or confirm the state of that concept to improve his decision outcome.
The creation of this Bayesian Belief Network model has provided a structured approach for the decision maker to consider all the concepts embedded in Sun Tzu’s principles. In formulating one’s own varied decisions or strategies, we can overcome the issue of failing to consider crucial concepts when in a stressful environment or under the pressures of time constraint.

The next issue that this thesis addresses is the incompleteness of Sun Tzu’s usage of information and secret agents. This incompleteness results in circular reasoning when both parties of a conflict use Sun Tzu’s principles against each other—highlighted by Niou and Ordeshook (1994). This thesis demonstrated how a Bayesian Belief Network can be applied to resolve the challenge of circular reasoning by letting the decision maker assess the degree of belief that he has in the information that is provided by the double agent. Through the degree of belief the decision maker breaks the chain of circular reasoning and formulates his decision accordingly to derive his best possible outcome based on the uncertainty of information provided by the double agent.

The concept of modeling through knowledge based approach using Bayesian Belief Network in this thesis captures only a specific domain of a decision frame. This decision frame is a Bayesian Belief Network model that captures the concepts required only for a specific situation; and at this stage the Bayesian Belief Network used is thus created manually. The challenge to further improve the use of Bayesian Belief Network model of this thesis would be to aid the decision maker in mixed or cross domains decision situation, where he requires different principles to be combined, and the decisions made through the used of one model may be used as inputs to a subsequent model. The combination of different principles requires the use of different Bayesian Belief Network models, therefore, when faced with mixed domains decision situation, combinations of the different models would have to be carried out. The challenge then would be to automate the selection of the different models, combine them, and reconstruct them into a new Bayesian Belief Network model for the decision maker.
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