Damping modeling strategy for naval ship system

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DAMPING MODELING STRATEGY FOR NAVAL SHIP SYSTEM

by

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The damping modeling strategy for naval ship system is presented for ship shock transient time-domain analysis. The Complex Exponential Method is used for extraction of modal parameters in time-domain. Inverse Fourier Transform of Mobility form for general viscous damping model is used to verify the calculated modal parameters. Rayleigh damping parameters are calculated using modal frequency and modal damping ratios. The statistical characteristics of Rayleigh damping parameters are quantified and evaluated in each categorized area: keel, bulkhead and deck. Then the Rayleigh damping parameters are recommended for ship shock response prediction. The damping studies were conducted using 2000 ms data based on DDG 53 Ship Shock Trial.
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ABSTRACT

The damping modeling strategy for naval ship system is presented for ship shock transient time-domain analysis. The Complex Exponential Method is used for extraction of modal parameters in time-domain. Inverse Fourier Transform of Mobility form for general viscous damping model is used to verify the calculated modal parameters. Rayleigh damping parameters are calculated using modal frequency and modal damping ratios. The statistical characteristics of Rayleigh damping parameters are quantified and evaluated in each categorized area: keel, bulkhead and deck. Then the Rayleigh damping parameters are recommended for ship shock response prediction. The damping studies were conducted using 2000 ms data based on DDG 53 Ship Shock Trial.
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I. INTRODUCTION

Mechanical energy transforms into heat and dissipates in all vibrating systems. There are many energy dissipation mechanisms that contribute to the damping in the structure system. Some of these mechanisms are: fluid resistance and coupling, internal friction (material damping), and friction at a joint. All of these dissipation mechanisms have been shown to be a function of many variables, including a structure’s shape or geometry, its material properties, temperature, frequency, boundary conditions, and different excitation energy levels. Usually over 90 percent of the inherent damping associated with fabricated build-up structures originates in the mechanical joints (Beards and Woodwat, 1985). These mechanical joints are friction joints, which dissipate energy during the vibration of a structure. Reducing the contact force in bolted structural connections can reduce system vibration amplitudes by enhancing joint damping capacity (Shin et al., 1991).

Naval ship structure systems have mostly welded joints and all stiffeners are also welded to hull plates, decks and bulkheads. The ship system also has many energy dissipation sources such as long cable trays, hangers, snubbers, the surrounding fluid coupled with ship hull, etc. The ship system damping is measurable, but difficult to quantify (Rutgerson, 2002). In conjunction with ship-shock simulation based transient analysis, time-domain representation of system damping is desirable using the frequency-domain characteristics of damping. The damping studies used for analysis were conducted using 2 sec data from the DDG 53 Ship Shock Trials.

The goal of this study is to present the damping model in Rayleigh damping form of a naval ship system for ship shock transient time domain analysis. In this study, the Complex Exponential Method is used for extraction of modal parameters in the time domain. The Inverse Fourier Transform of Mobility form of the general viscous damping model verifies the calculated modal parameters. Two factors in the Rayleigh damping
model are calculated using modal frequency and modal damping ratios. The statistical characteristics of two Rayleigh factors are quantified in each categorized area. Then the spatially dependent Rayleigh damping model is investigated and a model to be used in shock transient analysis is recommended.
II. THEORY OF GENERAL VISCOUSLY DAMPED SYSTEM

The general equation of motion for a MDOF (Multi Degree Of Freedom) system with viscous damping and harmonic excitation is:

\[ [M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{f\} \quad (1) \]

In the above equation, \([M]\) is the system mass matrix, \([K]\) is the system stiffness matrix, \([C]\) is the system damping matrix, \({x}\) is the system response vector and \({f}\) is the forcing vector.

We consider first the case where there is zero excitation in order to determine the natural modes of the system and to this end, we assume a solution to the equations of motion which has the form

\[ \{x\} = \{x\} e^{st} \quad (2) \]

Substituting this into the appropriate equation of motion gives:

\[ \left( s^2 [M] + s [C] + [K] \right) \{x\} = \{0\} \quad (3) \]

the solution of which constitutes a complex eigenvalue problem. In this case, there are 2N eigen values, \(s\), in complex conjugate pairs. (This is an inevitable result of the fact that all the coefficients in the matrices are real and thus any characteristic values, or roots, must either be real or occur in complex conjugate pairs.) There is an
eigenvector corresponding to each of these eigenvalues, but these also occur as complex conjugates. Hence we can describe the eigensolution as:

$$s_r, s_r^* \text{ and } \{s\}_r, \{s^*\}_r \quad r = 1, N$$

(4)

It is customary to express each eigenvalues $s_r$ in the form

$$s_r = \omega_r (-\zeta_r + i\sqrt{1-\zeta_r^2})$$

(5)

where $\omega_r$ is the ‘natural frequency’ and $\zeta_r$ is the critical damping ratio for that mode. Sometimes, the quantity $\omega_r$ is referred to as the ‘undamped natural frequency’ but this is not strictly correct, except in the case of proportional damping (or, of course, of a single degree of freedom system).

The eigen solution possesses orthogonality. In order to examine these we must first note that any eigenvalue/eigenvector pair satisfies the equation

$$\left(s_r^2 [M] + s_r [C] + [K]\right) \{\psi\}_r = \{0\}$$

(6)

and then we pre-multiply this equation by $\{\psi\}_q^T$ so that we have:

$$\{\psi\}_q^T \left(s_r^2 [M] + s_r [C] + [K]\right) \{\psi\}_r = \{0\}$$

(7)
A similar expression to (6) can be produced by using $\lambda_q$ and $\{\psi\}_q$:

$$\left(s_q^2 [M] + s_q [C] + [K]\right) \{\psi\}_q = \{0\}$$  \hspace{1cm} (8)

which can be transposed, taking account of the symmetry of the system matrices, to give:

$$\{\psi\}_q^T \left(s_q^2 [M] + s_q [C] + [K]\right) = \{0\}^T$$  \hspace{1cm} (9)

If we now post-multiply this expression by $\{\psi\}_r$ and subtract the result from that in Equation (7), we obtain:

$$\left(s_r^2 - s_q^2\right) \{\psi\}_q^T [M] \{\psi\}_r + \left(s_r - s_q\right) \{\psi\}_q^T [C] \{\psi\}_r = 0$$  \hspace{1cm} (10)

and provided $s_r$ and $s_q$ are different, this leads to the first of a pair of orthogonality equations:

$$\left(s_r + s_q\right) \{\psi\}_q^T [M] \{\psi\}_r + \{\psi\}_q^T [C] \{\psi\}_r = 0$$  \hspace{1cm} (11)

A second equation can be derived from the above expressions as follows:

Multiply (7) by $s_q$ and (9) by $s_r$ and subtract one from the other to obtain:
These two equations - (11) and (12) – constitute the orthogonality conditions of the system and it is immediately clear that they are far less simple. However, it is interesting to examine the form they take when the modes $r$ and $q$ are found as a complex conjugate pair. In this case, we have that

\[ s_q = \omega_r (\zeta_r - i\sqrt{1 - \zeta_r^2}) \]  

and also that,

\[ \{\psi\}_q = \{\psi^*\}_r \]  

Inserting these into Equation (11) gives

\[ -2\omega_r \zeta_r \{\psi^*\}_r^T [M] \{\psi\}_r + \{\psi^*\}_r^T [C] \{\psi\}_r = 0 \]  

from which we obtain:

\[ 2\omega_r \zeta_r \frac{\{\psi^*\}_r^T [C] \{\psi\}_r}{\{\psi^*\}_r^T [M] \{\psi\}_r} = \frac{c_r}{m_r} \]  

Similarly, inserting (13) and (14) into (12) gives
\[ \omega_r^2 \{\psi^*_r\}^T_r [M] \{\psi\}_r - \{\psi^*_r\}^T_r [K] \{\psi\}_r = 0 \]  

(17)

from which

\[
\omega_r^2 = \frac{\{\psi^*_r\}^T_r [K] \{\psi\}_r}{\{\psi^*_r\}^T_r [M] \{\psi\}_r} = \frac{k_r}{m_r} \tag{18}
\]

In these expressions, \( m_r, k_r, \) and \( c_r \) may be described as modal mass, stiffness and damping parameters respectively although the meaning is slightly different to that used in the other systems.

A. FORCED RESPONSE ANALYSIS

Returning to Equation (1), and assuming a harmonic response:

\[ \{x(t)\} = \{x\} e^{i\omega t} \tag{19} \]

we can write the forced response solution directly as

\[ \{x\} = [\{K\} - \omega^2 [M] + i\omega [C]]^{-1} \{f\} \tag{20} \]
but this expression is not particularly convenient for numerical application. Define a new coordinate vector \( \{y\} \), which is of order 2N, and which contains both the displacements \( \{x\} \) and the velocities \( \{\dot{x}\} \):

\[
\{y\} = \begin{pmatrix} x \\ \dot{x} \end{pmatrix}_{(2N\times1)}
\]  

Equation (1) can then be written as:

\[
[C : M]_{N\times2N} \{\dot{y}\}_{2N\times1} + [K : 0] \{y\} = \{0\}_{N\times1}
\]  

However, in this form we have N equations and 2N unknowns and so we add an identity equation of the type:

\[
[M : 0] \{\ddot{y}\} + [0 : -M] \{y\} = \{0\}
\]  

which can be combined to form a set of 2N equations

\[
\begin{pmatrix} C & M \\ M & 0 \end{pmatrix} \{\ddot{y}\} + \begin{pmatrix} K & 0 \\ 0 & -M \end{pmatrix} \{y\} = \{0\}
\]  

which can be simplified to:
These equations are now in a standard eigenvalue form and by assuming a trial solution of the form \( \{y\} = \{y\}e^{\lambda t} \), we can obtain the 2N eigenvalues and eigenvectors of the system, \( \lambda_r \) and \( \{\theta\}_r \), which together satisfy the general equation:

\[
(\lambda_r [A] + [B])\{\theta\}_r = \{0\}; \quad r = 1, 2N
\]  

These eigen properties will, in general, be complex although for the same reasons as previously they will always occur in conjugate pairs. They possess orthogonality properties, which are simply stated as

\[
\{\theta\}^T [A] \{\theta\} = \begin{bmatrix} \cdots a_r, \cdots \end{bmatrix}
\]

\[
\{\theta\}^T [B] \{\theta\} = \begin{bmatrix} \cdots b_r, \cdots \end{bmatrix}
\]

and which have the usual characteristic that

\[
\lambda_r = -\frac{b_r}{a_r} \quad r = 1, 2N
\]

Now we may express the forcing vector in terms of the new coordinate system as:
\[\{P\}_{2N\times1} = \begin{bmatrix} f \\ 0 \end{bmatrix}\]  \hspace{1cm} (29)

and assuming a similarly harmonic response and making use of the previous development of a series form expression of the response. We may write:

\[
\begin{bmatrix} x \\ i\omega x \end{bmatrix} = \sum_{r=1}^{2N} \frac{\{\theta\}_r^T \{p\} \{\theta\}_r}{a_r(i\omega - s_r)}
\]  \hspace{1cm} (30)

However, because the eigenvalues and eigenvectors occur in complex conjugate pairs, this last equation may be written as:

\[
\begin{bmatrix} x \\ i\omega x \end{bmatrix} = \sum_{r=1}^{N} \left( \frac{\{\theta\}_r^T \{p\} \{\theta\}_r}{a_r(i\omega - s_r)} + \frac{\{\theta\}_r^*^T \{p\} \{\theta\}_r^*}{a_r^*(i\omega - s_r^*)} \right)
\]  \hspace{1cm} (31)

At this stage, it is convenient to extract a single response parameter, say \(x_j\), resulting from a single force such as \(f_k\) - the receptance frequency response function, \(\alpha_j^k\), and in this case Equation (31) leads to:

\[
\frac{x_j}{f_k} = \alpha_j^k(\omega) = \sum_{r=1}^{N} \left( \frac{\theta_j r \theta_k}{a_r(i\omega - s_r)} + \frac{\theta_j^* r \theta_k^*}{a_r^*(i\omega - s_r^*)} \right)
\]  \hspace{1cm} (32)

or,
\[
\alpha_j^k(\omega) = \sum_{r=1}^{N} \left( \frac{r A_{jk}}{(i\omega - s_r)} + \frac{r A_{jk}^*}{(i\omega - s_r^*)} \right)
\]  

(34)

where, \( r A_{jk} = \frac{r \theta_j r \theta_k}{a_r} \), \( r A_{jk}^* = \frac{r \theta_j^* r \theta_k^*}{a_r^*} \)

Equation (34) describes the displacement response at ‘j’ degree of freedom under excitation at ‘k’ degree of freedom of general viscously damped system in frequency domain. If Fourier Transform is made onto Equation (34), we can get the Impulse Response Function \( h_{jk}(t) \) of this system.

\[
h_{jk}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \sum_{r=1}^{N} \left( \frac{r A_{jk}}{(i\omega - s_r)} + \frac{r A_{jk}^*}{(i\omega - s_r^*)} \right) e^{i\omega t} d\omega
\]

(35)

If we let \((i\omega - s_r)\) as \(iz\), then, by Cauchy’s Integral Formula, it becomes:

\[
\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{r A_{jk}}{(i\omega - s_r)} e^{i\omega t} d\omega = \frac{r A_{jk} e^{s_i t}}{2\pi i} \int_{-\infty}^{\infty} \frac{e^{izt}}{z} dz = r A_{jk} e^{s_i t}
\]

(36)

and similarly,

\[
\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{r A_{jk}^*}{(i\omega - s_r^*)} e^{i\omega t} d\omega = \frac{r A_{jk}^* e^{s_i^* t}}{2\pi i} \int_{-\infty}^{\infty} \frac{e^{izt}}{z} dz = r A_{jk}^* e^{s_i^* t}
\]

(37)
From Equations (36) and (37), the Impulse Response Function of a general viscous damping system can be written as:

\[
h_{jk}(t) = \sum_{r=1}^{N} \left( r A_{jk} e^{\theta_{r,t}} + r A_{jk}^* e^{\bar{\theta}_{r,t}} \right) \tag{38}
\]

The forced response can then be calculated as Equation (39) in time domain.

\[
x_j = \int_{0}^{t} f_k(\tau) h_{jk}(t - \tau) d\tau \tag{39}
\]
III. PROCEDURE OF DAMPING CALCULATION FROM MEASURED DATA

A. MODAL PARAMETER EXTRACTION

In Equation (34), $r_j$, $A_{jk}$, $A_{jk}^*$, $s_r$, $s_r^*$ are called modal parameters, $r_j$, $A_{jk}$ are called modal constants, and eigenvalues $s_r$, $s_r^*$ contain information of modal properties such as modal frequency $\omega_r$ and modal damping ratio $\zeta_r$. Calculation of damping ratio from the measured shock trial data needs utilizing modal parameter extraction methods, the simple 3-dB (half power) bandwidth measurement or logarithmic decay rate calculation can be incorrect in real cases, because, in usual cases, real measurement data not only contain noise components but also have many closely coupled frequency components.

Basically there are two groups of techniques in the field of experimental modal analysis. One is related to frequency domain analysis methods that use Frequency Response Function of measured input and output data. This group of methods is widely used, from single degree of freedom circle fitting to complex multi-degree of freedom fitting methods. And the other is related to time domain analysis techniques that use Impulse Response Function as analysis data.

In this study, the time domain method is used because the measured useful shock trial data sets are too short in time to obtain sufficient frequency resolution in the frequency domain, and there is only one event, shot data, thus the averaging process cannot be done. Complex Exponential Method (CEM), one of the effective modal parameter extraction methods in the time domain, is used to extract modal parameters.

The 2 second long measured data has been used in this study and the effective frequency span is limited from 3 Hz to 250 Hz. The original measured data has been band-pass filtered from 2 Hz to 250 Hz to avoid long-term trends in low frequencies and to remove unwanted high frequency noise components.
1. Complex Exponential Method (CEM)

In the field of experimental modal analysis, a term Receptance is widely used to describe the ratio of displacement response to excitation force, and the term Mobility is used in describing the ratio of velocity response to excitation force. The Receptance (displacement/force) $\alpha(\omega)$ of a general viscously damped system, Equation (34), can be rewritten as Equation (40), complex eigenvalue $s_r$ is as in Equation (40.a),

$$\alpha(\omega) = \sum_{r=1}^{N} \frac{A_r}{j\omega - s_r} + \frac{A_r^*}{j\omega - s_r^*} ; \quad s_r = -\omega_r\zeta_r + j\omega_r\sqrt{1-\zeta_r^2} \quad (40.a)$$

or,

$$\alpha(\omega) = \sum_{r=1}^{2N} \frac{A_r}{j\omega - s_r} ; \quad s_r \Rightarrow s_r^* , \quad A_r \Rightarrow A_r^* , \text{ for } r > N \quad (40.b)$$

we can get velocity $v = Ve^{j\omega t}$ by time differentiating displacement $x = Xe^{j\omega t}$, that is,

$$v = V(\omega)e^{j\omega t} = j\omega Xe^{j\omega t} \quad ( )$$

and Mobility(velocity/force) $Y(\omega)$ can be related to $\alpha(\omega)$(displacement/force),
The corresponding Impulse Response Function (IRF) can be obtained by taking Inverse Fourier Transform (IFT) of the Receptance $\alpha(\omega)$ as Equations (35) through (38),

$$Y(\omega) = j\omega\alpha(\omega) \quad (41)$$

By time differentiating, the velocity form of IRF can be expressed as, 

$$h(t) = \sum_{r=1}^{2N} A_r e^{s_r t} \quad (42)$$

Hereafter, the exponential term will be simplified using the following notation.

$$e^{s_r \Delta t} \rightarrow V_r \quad (45)$$

Thus for the j-th sample data, the Equation (43) becomes,
\[ \dot{h}_j = \sum_{r=1}^{2N} A_r s_r V_r^j \]  

(46)

which, when extended to the full data set of \( q \) samples \((j=1,2,\ldots,q)\), gives:

\[
\begin{align*}
\dot{h}_0 &= s_1 A_1 + s_2 A_2 + \cdots + s_{2N} A_{2N} \\
\dot{h}_1 &= V_1 s_1 A_1 + V_2 s_2 A_2 + \cdots + V_{2N} s_{2N} A_{2N} \\
\dot{h}_2 &= V_1^2 s_1 A_1 + V_2^2 s_2 A_2 + \cdots + V_{2N}^2 s_{2N} A_{2N} \\
&\vdots \\
\dot{h}_q &= V_1^q s_1 A_1 + V_2^q s_2 A_2 + \cdots + V_{2N}^q s_{2N} A_{2N}
\end{align*}
\]

(47)

Provided that the number of sample points \( q \) exceeds \( 4N \), this equation can be used to set up an eigenvalue problem, the solution yields the complex natural frequencies contained in the parameters \( V_1, V_2, \ldots \).

Multiply each equation in (47) by a coefficient, \( \beta_j \) to form the following set of equations:

\[
\begin{align*}
\beta_0 \dot{h}_0 &= \beta_0 A_1 + \beta_0 A_2 + \cdots + \beta_0 A_{2N} \\
\beta_1 \dot{h}_1 &= \beta_1 V_1 A_1 + \beta_1 V_2 A_2 + \cdots + \beta_1 V_{2N} A_{2N} \\
\beta_2 \dot{h}_2 &= \beta_2 V_1^2 A_1 + \beta_2 V_2^2 A_2 + \cdots + \beta_2 V_{2N}^2 A_{2N} \\
&\vdots \\
\beta_q \dot{h}_q &= \beta_q V_1^q A_1 + \beta_q V_2^q A_2 + \cdots + \beta_q V_{2N}^q A_{2N}
\end{align*}
\]

(48)

Adding all equations in (48) vertically results in,
\[ \sum_{i=0}^{q} \beta_i \dot{h}_i = \sum_{j=1}^{2N} (A_j \sum_{i=0}^{q} \beta_i V_{j,i}) \]  
\[ (49) \]

The coefficients \( \beta_i \) s are taken to be the coefficients in the polynomial equation,

\[ \beta_0 + \beta_1 V + \beta_2 V^2 + \beta_3 V^3 + \cdots + \beta_q V^q = 0 \]  
\[ (50) \]

The roots are \( V_1, V_2, \cdots, V_q \).

Next, the values of the \( \beta \) coefficients are to be sought in order to determine the roots of Equation (50) - values of \( V_r \) - and hence the system natural frequencies. Now, recalling that \( q \) is the number of degrees of freedom of the system model. It is now convenient to set these two parameters to the same value, i.e. let \( q = 2N \).

If we find \( \beta \) coefficients that make Equation (50) fulfilled, then Equation (50) can be expressed as,

\[ \sum_{j=0}^{2N} \beta_j V_r^j = 0 \quad ; \quad r = 1, 2N \]  
\[ (51) \]

And thus every term on the right-hand side of Equation (49) is zero.
Rearranging Equation (52), by moving the last term of left-hand side to right-hand side,

\[ \sum_{i=0}^{2N} \beta_i \dot{h}_i = 0 \] (52)

\[ \sum_{i=0}^{2N-1} \beta_i \dot{h}_i = -\dot{h}_{2N} \quad \text{by setting } \beta_{2N} = 1 \] (53)

Repeat the process from (44) to (53) using different set of IRF data points and further choose the new data set that overlaps considerably with the first set – In fact, for all but one item.

Successive applications of this procedure lead to a full set of 2N equations:

\[
\begin{bmatrix}
\dot{h}_0 & \dot{h}_1 & \dot{h}_2 & \ldots & \dot{h}_{2N-1} \\
\dot{h}_1 & \dot{h}_2 & \dot{h}_3 & \ldots & \dot{h}_{2N} \\
\vdots & \vdots & \vdots & \ldots & \vdots \\
\dot{h}_{2N-1} & \dot{h}_{2N} & \dot{h}_{2N+1} & \ldots & \dot{h}_{4N-2}
\end{bmatrix}
\begin{bmatrix}
\beta_0 \\
\beta_1 \\
\vdots \\
\beta_{2N-1}
\end{bmatrix}
= -
\begin{bmatrix}
\dot{h}_{2N} \\
\dot{h}_{2N+1} \\
\vdots \\
\dot{h}_{4N-1}
\end{bmatrix}
\] (54.a)

or,

\[ \begin{bmatrix} \dot{h} \end{bmatrix}_{2N \times 2N} \{\beta\}_{2N \times 1} = -\{\dot{h}\}_{2N \times 1} \] (54.b)
The unknown coefficients \( \{\beta\} \) can be found from Equation (54). Now the values of \( V_1, V_2, \ldots, V_{2N} \) can be determined using Equation (50) and subsequently the system natural frequencies can be found using the following relationship.

\[
V_r = e^{s_r \Delta t}
\]  

(55)

Using Equation (47), corresponding modal constants \( A_1, A_2, \ldots, A_{2N} \) can be calculated, this may be written as,

\[
\begin{bmatrix}
1 & 1 & 1 & \cdots & 1 \\
V_1 & V_2 & V_3 & \cdots & V_{2N} \\
V_1^2 & V_2^2 & V_3^2 & \cdots & V_{2N}^2 \\
\vdots & \vdots & \vdots & \cdots & \vdots \\
V_1^{2N-1} & V_2^{2N-1} & V_3^{2N-1} & \cdots & V_{3}^{2N-1}
\end{bmatrix}
\begin{bmatrix}
A_1s_1 \\
A_2s_2 \\
A_3s_3 \\
\vdots \\
A_{2N}s_{2N}
\end{bmatrix}
= 
\begin{bmatrix}
\dot{h}_0 \\
\dot{h}_1 \\
\dot{h}_2 \\
\vdots \\
\dot{h}_{2N-1}
\end{bmatrix}
\]  

(56)

or

\[
[V] \{A\} = \{\dot{h}\}
\]  

(57)
2. Verification of Extracted Modal Parameters

The modal parameters calculated according to the mentioned procedure can be verified by comparing synthesized time histories to the originally measured time histories. Mobilities can be determined by frequency differentiation, multiplied by \( j\omega \), from Equations (40) and (41),

\[
\hat{Y}(\omega) = \sum_{r=1}^{N} \frac{j\omega A_r}{j\omega - s_r} + \frac{j\omega A_r^*}{j\omega - s_r^*} ; \quad s_r = -\omega_r \zeta_r + j\omega_r \sqrt{1 - \zeta_r^2} \quad (58)
\]

or

\[
\hat{Y}(\omega) = \sum_{r=1}^{2N} \frac{j\omega A_r}{j\omega - s_r} ; \quad s_r \Rightarrow s_r^* , \quad A_r \Rightarrow A_r^* \quad \text{for} \quad r > N \quad (59)
\]

In Equations (58) and (59), extracted modal parameters \( A_r , A_r^* , s_r , s_r^* \) are used to calculate the synthesized Frequency Response Function in frequency domain. And by inverse Fast Fourier Transform (IFFT), taking the real part of the results, synthesized IRF can be calculated and compared to original time histories. Frequency bandwidth \( \Delta f \) should be multiplied during calculation procedure to generate band level.

\[
\{\hat{h}\} = \text{Real Part of Inverse FFT}(\Delta f \hat{Y}) \quad (60)
\]
3. Calculation of Rayleigh Damping

Rayleigh damping is a kind of general proportional damping model. It assumes that damping matrix \([C]\) in Equation (1) can be represented as linear combination of the mass matrix and stiffness matrix. Then the damping matrix can be easily decoupled to the modal damping matrix. Using the Rayleigh damping representation, the damping matrix can be represented as,

\[
[C] = \alpha[M] + \beta[K]
\]  \hspace{1cm} (61)

or by using mass normalized modal matrix \([\varphi]\),

\[
[\varphi]^T [C] [\varphi] = [2\omega_r \zeta_r]_{diag} = \alpha I + \beta \omega_r^2
\]  \hspace{1cm} (62)

By using Equation (62) for all \(2N\) modes, following \(2N\) equations can be set.

\[
\begin{align*}
\alpha + \beta \omega_1^2 &= 2\omega_1 \zeta_1 \\
\alpha + \beta \omega_2^2 &= 2\omega_2 \zeta_2 \\
\vdots \\
\alpha + \beta \omega_{2N}^2 &= 2\omega_{2N} \zeta_{2N}
\end{align*}
\]  \hspace{1cm} (63)

or

\[
[W]_{2N \times 2} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = [Z]_{2N \times 1}
\]  \hspace{1cm} (64)
If 2N is larger than 2, then Equation (64) becomes over-determined, with 2 unknowns and 2N equations. By post-multiplying the transpose matrix of [W] to both sides of Equation (64), we can get Equation (65).

\[
[W]_{{2\times 2N}}^T [W]_{{2N\times 2}} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = [W]_{{2\times 2N}}^T \{Z\}_{{2N\times 1}} \tag{65}
\]

Then two Rayleigh parameters \( \alpha \) and \( \beta \) are calculated as,

\[
\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = ([W]_{{2\times 2N}}^T [W]_{{2N\times 2}})^{-1} [W]_{{2\times 2N}}^T \{Z\}_{{2N\times 1}} \tag{66}
\]

As a final step, modal damping ratios for each vibration modes are calculated in each categorized area of ship.

\[
\zeta = \alpha \frac{1}{2\omega} + \beta \frac{\omega}{2} \tag{67}
\]
IV. RESULTS OF MODAL PARAMETER EXTRACTION

A. VERIFICATION RESULTS

Modal parameters for a total of 773 sets of measured data are calculated for each measuring position and direction.

The following figures show some results of the modal parameter extraction method according to the aforementioned procedure. Figure 1, Figure 3, and Figure 5 show the original measured data sets, while Figure 2, Figure 4, and Figure 6 show synthesized curves with the calculated modal parameters $\omega_r$, $\zeta_r$, and $A_r$. The black line shows parts of the original signal between the 125 msec point and the 1200 msec point. The red line stands for the synthesized curve of those parts. The 125 msec parts of each of the measured data sets are not included in analysis to avoid being mixed with the effect of excitation signal. Also the latter parts that contain the secondary excitation are not included in the analysis. This secondary excitation can be seen on Figure 1, Figure 3, and Figure 5 around the 1250 msec point.

Figure 1. Measured Data at A2001AI
Figure 2. Synthesized Results at A2001A (between 125 msec and 1200 msec)

Figure 3. Measured Data at A2004A

Figure 4. Synthesized Results at A2004A (between 125 msec to 1200msec)
B. RESULTS OF RAYLEIGH DAMPING CALCULATION

A total of 773 data sets were categorized into 67 area groups, based on the location of the measuring sensor installation, as well as by the direction of measurement, which were the athwartship and vertical directions of the ship.

Figure 7 through Figure 10 show some typical results of the Rayleigh Damping coefficients calculated according to the aforementioned procedure, using Equations (61) through (67). Each black square point is a mode. The red lines represent the regenerated modal damping ratio using $\alpha$ and $\beta$ calculated in least-square sense.
Each figure tagged as ‘Original’ in caption is from the initial step results, whereas the others are of the final results, which have been modified by eliminating the unrealistic and noisy components. From the ‘Original’ figures, we can identify that some of the results are scattered and contain damping ratios, which cannot be regarded as reasonable against the physical sense. Some modifications to the above results have been made. The unreasonable damping data points have been removed from curve fitting $\alpha$ and $\beta$. The modes of which modal constants $A_r$ are seriously less than one thousandth of the maximum value in each measuring position have been removed. The modes that contain damping ratios, which are greater than 0.5 have been removed. Likewise, the modes with damping ratios that contain a great deal of scattered from the initial curve-fitted have been also removed. The final results are shown in both of linear scales and logarithmic scales.

The figures presented in Appendix A illustrate the damping calculation results for the remaining areas that were studied.

Figure 7. Modal Damping Ratio at Area 6, Athwartship Direction (Original)
Figure 8. Modal Damping Ratio at Area 6, Athwartship Direction (Modified)

Figure 9. Modal Damping Ratio at Area 6, Vertical Direction (Original)

Figure 10. Modal Damping Ratio at Area 6, Vertical Direction (Modified)
C. CURVE-FITTED RAYLEIGH DAMPING $\alpha$ AND $\beta$ FOR EACH AREA

The Curve-fitted Rayleigh damping coefficients, $\alpha$ and $\beta$, for each categorized area are presented in Table 1 and Table 2, for the athwartship and vertical directions, respectively. Figure 2 is a profile view of the DDG 51 Arleigh Burke Class Destroyer. This drawing shows the major transverse frame positions of the ship.

![Profile view of DDG 51 Arleigh Burke Class Destroyer](image)

Figure 11. Transverse Frame Locations of the DDG 51 Class Destroyer

| Area No. | Deck | Frame | Athwartship Position | $\alpha$ | $\beta$ | Number of modes used in curvefitting $\alpha$ and $\beta$
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>56</td>
<td>4’p-2’s</td>
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<td>1.22E-06</td>
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<td>CL-5’s</td>
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<td>4.69E-06</td>
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</tr>
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<td>37</td>
</tr>
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Table 1. Rayleigh Damping Results for the Athwartship Direction by Area Group
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<tr>
<th>Area No.</th>
<th>Deck</th>
<th>Frame</th>
<th>Athwartship Position</th>
<th>Alpha</th>
<th>Beta</th>
<th>Number of modes used in curvefitting α and β</th>
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Table 2. Rayleigh Damping Results for the Vertical Direction by Area Group
<table>
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<th>Frame</th>
<th>Athwartship Position</th>
<th>Alpha</th>
<th>Beta</th>
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<td>Alpha</td>
<td>Beta</td>
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<td>4°s-5's</td>
<td>2.02E+01</td>
<td>2.84E-05</td>
<td>4</td>
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</table>

Table 3 shows the weighted mean value of α for each of the directions of motion. The weighting factor, shown in the rightmost column of Table 1, is defined as the number of modes used in the curve-fitting α and β. Thus, it can be concluded that weighted mean values of α are similar in both directions.

Table 4 shows the weighted mean value of β for each direction of motion, the weighting factor is the rightmost column of Table 2, the number of modes used in curve-fitting α and β. It can be concluded that weighted mean values in athwartship direction are slightly larger than those of the vertical direction. Equation 68 is used to calculate the mean α coefficient,

\[
\alpha_{\text{mean}} = \frac{\sum_{i=1}^{M} \alpha_i N_i}{\sum_{i=1}^{M} N_i}
\]  

(68)
where the variable M stand for the of areas to be considered.

Table 3. Weighted Mean of $\alpha$

<table>
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<tr>
<th>Athwartship Direction</th>
<th>Vertical Direction</th>
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<tbody>
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</table>

Table 4. Weighted Mean of $\beta$

<table>
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<th>Vertical Direction</th>
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</thead>
<tbody>
<tr>
<td>2.82E-06</td>
<td>2.09E-06</td>
</tr>
</tbody>
</table>

Figure 12. Rayleigh Damping Coefficient $\alpha$ for Athwartship Direction on Deck 1
Figure 13. Rayleigh Damping Coefficient $\alpha$ for Vertical Direction on Deck 1
V. EFFECTS OF DAMPING TO SHIP SHOCK RESPONSES

In the 1994, the USS JOHN PAUL JONES (DDG 53) was chosen as the representative ship of the DDG 51 Arleigh Burke Class Destroyer and subsequently subjected to a series of shock trials. Some seven years later in the summer of 2001, similar ship shock trials were conducted on the USS WINSTON S. CHURCHILL (DDG 81). This latter series of live fire tests was performed on the 31st ship in the same class due to the significant design changes incorporated into the Flight IIA version of this ship. Some of the significant changes that were found in DDG 81 included an extension in the ship’s overall length and the additional of two helicopter hangers.

Starting with a highly complex finite element model of the ship and the surrounding fluid mesh, shown in Figure 14, the shock response velocity was calculated for various locations throughout the ship using the modeling and simulation process outlined in Figure 15 [4].

Figure 14. DDG 81 Coupled Fluid-Structure Model
The results of this process were compared with the actual ship shock trial sensor data obtained during the 2001 Live Fire Testing and Evaluation.

Additionally the shock velocity response plots obtained from the aforementioned process were compared against the earlier conducted simulations that used the Rayleigh Damping Coefficients previously used in the DDG 53 modeling and simulation effort, which occurred during the mid 1990’s at the Naval Postgraduate School. The values for these coefficients are listed in Table 5.
Table 5. Rayleigh Damping Coefficient for 4% & 8% Proportional Damping

<table>
<thead>
<tr>
<th>Damping Value</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4%</td>
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<td>4.99E-5</td>
</tr>
<tr>
<td>8%</td>
<td>4.93</td>
<td>9.89E-5</td>
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</tbody>
</table>

In this case the Rayleigh parameters ($\alpha$, $\beta$) were based on evaluation of the damping values at two given points, 5 Hz and 250 Hz, which cover the range of data which was required for subsequent comparisons.

Figure 16. Rayleigh Damping Values (in Linear Scale)
The following series of velocity response plots compares the Rayleigh damping coefficients, $\alpha$ and $\beta$, presented in Tables 3 and 4 with coefficients that were used in previous studies conducted on the DDG 53 and DDG 81, which appear in Table 5.

Russell’s Error Factor [5-7] was chosen as a means of comparing the velocity response data against the actual ship shock trial data. For the purpose of this study, an established set of acceptance criteria was taken from the work accomplished in 2003 on the DDG 81 Ship Shock Simulation [4]. These values are presented in Table 6.

<table>
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<tr>
<th>RC</th>
<th>Acceptance Criteria</th>
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<td>$\leq 0.15$</td>
<td>Excellent</td>
</tr>
<tr>
<td>$0.15 &lt; RC &lt; 0.28$</td>
<td>Good</td>
</tr>
<tr>
<td>$\geq 0.28$</td>
<td>Poor</td>
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</table>
Figure 18. Sample Vertical Velocity Response: Deck Sensor

Figure 19. Sample Vertical Velocity Response: Deck Sensor
Figure 20. Sample Vertical Velocity Response: Keel Sensor

Figure 21. Sample Vertical Velocity Response: Bulkhead Sensor
As the velocity response plot comparisons in Figure 18 through Figure 21 show, there is a much better correlation between the NPS damping values and the ship shock trial data, than with the fixed 4% damping. For the sensors examined of which the approximate location of each is indicated on the time history plots by a red dot, a Russell’s Comprehensive (RC) error correlation factor was computed. The mean RC for the 4% Damping cases was 0.25 while in comparison when the new NPS damping values from Table 3 and Table 4 were used, the mean RC value was only 0.18. Recalling that by Russell’s correlation criteria, a value below 0.15 is considered an excellent correlation, the simulations using the new NPS damping values consistently show better overall correlation and an average reduction of approximately 25% in deviation from the recorded ship shock trial data versus those using the fixed 4% damping. Table 7 illustrates a sampling of the supporting data.

Table 7. Comparison of Russell’s Error Factor for DDG 81Shot 2 (vertical direction)

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Node</th>
<th>Mounting Type</th>
<th>Location (in)*</th>
<th>Shock Trial Data vs. 4% Damping (LS-DYNA/USA DATA (&lt;250HZ))</th>
<th>Shock Trial Data vs. NPS Damping (LS-DYNA/USA DATA (&lt;250HZ))</th>
</tr>
</thead>
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<td>V2002V</td>
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<td>Deck</td>
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<td>0.0679 0.2175 0.2019</td>
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<td>0.1260 0.1932 0.2016</td>
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<td>Keel</td>
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<td>0.0009 0.1692 0.15</td>
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<tr>
<td>V2125V</td>
<td>222436</td>
<td>Bulkhead</td>
<td>3504 375 390</td>
<td>0.1651 0.1936 0.2255</td>
<td>0.0214 0.1914 0.1707</td>
</tr>
</tbody>
</table>

Russell Error Correlation
- >0.25 Poor
- <0.15 Excellent

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Node</th>
<th>Mounting Type</th>
<th>Location (in)*</th>
<th>Shock Trial Data vs. 4% Damping (LS-DYNA/USA DATA (&lt;250HZ))</th>
<th>Shock Trial Data vs. NPS Damping (LS-DYNA/USA DATA (&lt;250HZ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>V2002V</td>
<td>142481</td>
<td>Deck</td>
<td>4656 24 85</td>
<td>0.1974 0.2175 0.2975</td>
<td>0.0679 0.2175 0.2019</td>
</tr>
<tr>
<td>V2008V</td>
<td>210894</td>
<td>Deck</td>
<td>4004 176 171</td>
<td>0.1207 0.2068 0.256</td>
<td>0.1260 0.1932 0.2016</td>
</tr>
<tr>
<td>V2035V</td>
<td>330769</td>
<td>Keel</td>
<td>1152 135 193</td>
<td>0.1643 0.1849 0.2192</td>
<td>0.0009 0.1692 0.15</td>
</tr>
<tr>
<td>V2125V</td>
<td>222436</td>
<td>Bulkhead</td>
<td>3504 375 390</td>
<td>0.1651 0.1936 0.2255</td>
<td>0.0214 0.1914 0.1707</td>
</tr>
</tbody>
</table>

* Referenced to the G&C NASTRAN Model coordinate origin located at the stern. In the Y-direction, port is positive from the centerline.

Figure 22 is a graphical representation of the data presented in Table 7. Notice that the Russell’s Comprehensive correlation factor for the simulations using the NPS damping values are all in the excellent or highly acceptable range, while the results from the simulations performed using the fixed 4% damping values are only marginally acceptable or fall outside of the acceptable range all together. Note as well that there is considerable improvement in the accuracy of the magnitude component of the Russell’s correlation in the simulations using the NPS damping values, as demonstrated by the grouping of points nearer the ordinate.
Figure 22. Russell’s Error Factor for Selected Sensors of DDG 81 Shot 2

In this small but representative sampling of data points from various locations throughout the ship, comparison of the Russell’s error correlation shows that using the NPS damping values tends to improve the accuracy of the simulation from 20% to 30%. These results are presented in Table 8.

Table 8. Relative % Change in RC for NPS Damping versus 4% Damping Case

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Node</th>
<th>Ship Compartment Location</th>
<th>Percent Relative Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>V2008VI</td>
<td>210894</td>
<td>4th Deck</td>
<td>32.13%</td>
</tr>
<tr>
<td>V2002V</td>
<td>142489</td>
<td>4th Deck</td>
<td>21.86%</td>
</tr>
<tr>
<td>V2035V</td>
<td>330769</td>
<td>3rd Deck</td>
<td>31.57%</td>
</tr>
<tr>
<td>V2125V</td>
<td>222436</td>
<td>1st Deck</td>
<td>24.30%</td>
</tr>
<tr>
<td>Average Improvement in Correlation</td>
<td></td>
<td></td>
<td>27.47%</td>
</tr>
</tbody>
</table>
VI. CONCLUSIONS

Rayleigh damping representation for modeling ship system damping has been investigated based on the ship shock trial data. Based on the results of studies, a set of Rayleigh damping parameters ($\alpha$ and $\beta$) are recommended for ship shock response predictions. The results of investigation also indicate that (i) the system damping is largely affected by mass driven ($\alpha$[M]), and (ii) the damping decreases as frequency increases as we commonly understood.
LIST OF REFERENCES

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(in Linear Scale)  (in Logarithmic Scale)

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(in Linear Scale)   (in Logarithmic Scale)

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(in Linear Scale)

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APPENDIX B. MODAL PARAMETER EXTRACTION PROGRAM LIST IN TIME DOMAIN

This program uses IMSL libraries that are included in Microsoft Power Station.

use msimslmd
implicit double precision (a-h, o-z)
dimension tt(4000), vv(4000), hh(4000,200), hh2n(4000), betaq(201), coeff(201), &
  coefre(0:201), coefim(0:201), rootre(200), rootim(200)
dimension hht(200,4000), hhtth(200,200), hht2n(200)
double complex root(200), freg(6000), vvregm(6000)
dimension xr(200), wr(200), vvreg(4000), vvregi(4000)
dimension arn(200), nonq(1000)
double complex aai(4000,200), aaat(200,4000), ah(4000), aaatah(200), &
  aaataaa(200,200), cmodal(201), ar(201), cw
character*25 ifiles(1000), pfiles(1000), gfiles(1000), sensor(1000), sensorid
LOGICAL COMPL

pi=4.0d0*datan(1.0d0)
tfinal=2000.0d0
nf=773 ! no. of data sets 774
! nq=170 ! total mode no. to be considered
open(unit=1, file='modifieddat.lst') ! input time data file list
do 1881 ifl=1,nf
  read(1,*) sensor(ifl), nonq(ifl)
  ifiles(ifl)=trim(sensor(ifl))//'_2ms.dat'
pfiles(ifl)=trim(sensor(ifl))//'_modal.dat'
gfiles(ifl)=trim(sensor(ifl))//'_regen.dat'
1881 continue
do 8888 ifl=467,nf
  mm=1500 ! data points to be used for calculation
  ng=nonq(ifl)
  write(*,*), ifl, ' --> ', ifiles(ifl)
  open(unit=3, file=ifiles(ifl), err=8888)
  open(unit=4, file=pfiles(ifl))
  open(unit=6, file=gfiles(ifl))
!
! Velocity Data input, time is in miliseconds.
!
do j=1,mm
  read(3,*) tt(j), vv(j)
  if(tt(j).ge.tfinal) goto 1788
enddo
1788  mm=j-1
!
  dt=(tt(mm)-tt(1))/dfloat(mm-1)/1000.0d0 ! time interval in seconds
  write(*,*), dt
!
! Calculate [H]{beta}={h}
!
m2=mm-ng-1
do i=1,m2
  ii=i-1
  hh2n(i)=-vv(ii+ng+1)
do j=1,ng
  hh(i,j)=vv(ii+j)
enddo
enddo
\begin{verbatim}
  hht(1:nq,1:m2)=transpose(hh(1:m2,1:nq))
  hhthh(1:nq,1:nq)=matmul(hht(1:nq,1:m2),hh(1:m2,1:nq))
  hht2n(1:nq)=matmul(hht(1:nq,1:m2),hh2n(1:m2))
  CALL DLSARG (nq, hhthh, 200, hht2n, 1, betaq)

  betaq(nq+1)=1.0d0
  compl=.FALSE.
  coefre(0:nq)=betaq(1:nq+1)
  CALL BAUPOL(coefre,coefim,nq,COMPL,ROOTRE,ROOTIM,NUMIT)
  root(1:nq)=dcmplx(rootre(1:nq), rootim(1:nq))

  do i=1,nq
    sr(i)=cdlog(root(i))/dt
    wr(i)=cdabs(sr(i))
    xr(i) = -dble(sr(i))/wr(i)
  enddo

  cmodal=dcmplx(0.0d0,0.0d0)
  m2=mm-1
  do i=1,m2
    ah(i)=vv(i)
    do j=1,nq
      aaa(i,j)=root(j)**dfloat(i-1)
    enddo
  enddo
  aaat(1:nq,1:m2)=transpose(aaa(1:m2,1:nq))
  aaataaa(1:nq,1:nq)=matmul(aaat(1:nq,1:m2),aaa(1:m2,1:nq))
  aaatah(1:nq)=matmul(aaat(1:nq,1:m2),ah(1:m2))
  CALL DLSACG (nq, aaataaa, 200, aaatah, 1, cmodal)

  sensorid=sensor(ifl)
  do i=1,nq
    cmodal(i)=cmodal(i)/sr(i)
  enddo

  armax=0.0d0
  arn=0.0d0
  do i=1,nq
    if(cdabs(cmodal(i)).gt.armax) armax=cdabs(cmodal(i))
  enddo
  arn(1:nq)=cdabs(cmodal(1:nq))/armax

  sensorid=sensor(ifl)
  do i=1,nq
    cmodal(i)=cmodal(i)/sr(i)
  enddo

  fspan=1.0d0/dt/2.0d0
  wspan=2.0d0*pi*fspan
  dw=wspan/(dfloat(mm/2)-1.0d0)
  freg=dcmplx(0.0d0,0.0d0)
  do i=2,mm/2+1
    w=dfloat(i-1)*dw
    cw=dcmplx(0.0d0,w)
\end{verbatim}
do j=1,nq
    freg(i)=freg(i)+cmodal(j)*cw/(cw-sr(j))
enddo

do i=2, mm/2
    ii=mm-i+2
    freg(ii)=dconjg(freg(i))
enddo

freg=freg*dw/(2.0d0*pi)
!
! Calculate synthesized time history by inverse FFT
!
CALL DFPTCB (mm, freg, vvregm)
vvregr=0.0
do i=2,mm
    ii=mm-i+1
    vvregr(i)=dble(vvregm(i))
    vvregi(i)=dimag(vvregm(i))
enddo
!
! Output synthesized time histories
!
do i=2,mm/2
    write(6,200) tt(i), vvregr(i),vvregi(i), vv(i)
200        format(5(1x,e15.7))
enddo
!
! Sort and Output all modal parameters
!
call sortmodal(nq, wr, xr, cmodal, arn)
do i=1,nq
    write(4,100) i, wr(i)/(2.0d0*pi), xr(i), cmodal(i), arn(i)
100        format(1i5, 5(1x, e14.6))
enddo
close(unit=3)
close(unit=4)
close(unit=6)
8888 continue
877  format(5(1x,e15.7))
400  format(5(1x,e15.7))
close(unit=1)
close(unit=2)
close(unit=5)
stop
end
!
!----------------------------------------------------------------------------
!  Subroutine to calculate damping curve and standard deviation
!----------------------------------------------------------------------------
!
Subroutine meansd nto, totalfreq, totaldamp, rayleigh, raywgt, &
    ffreq, fdamp1, fdamp2, sdray, sdwgt)
implicit double precision (a-h,o-z)
dimension totalfreq(2000), totaldamp(2000), ffreq(250), fdamp1(250), &
    fdamp2(250)
dimension rayleigh(2), raywgt(2)
pi=4.0d0*datan(1.0d0)
sdray=0.0d0
sdwgt=0.0d0
do i=1,nto
  w=2.0d0*pi*totalfreq(i)
  x=rayleigh(1)/2.0d0/w+rayleigh(2)/2.0d0*w
  sdray=sdray+(totaldamp(i)-x)**2.0d0
  x=raywgt(1)/2.0d0/w+raywgt(2)/2.0d0*w
  sdwgt=sdwgt+(totaldamp(i)-x)**2.0d0
enddo
sdray=sdray/nto
sdwgt=sdwgt/nto
do i=1,250
  f=dfloat(i)
  ffreq(i)=f
  w=2.0d0*pi*f
  fdamp1(i)=rayleigh(1)/2.0d0/w+rayleigh(2)/2.0d0*w
  fdamp2(i)=raywgt(1)/2.0d0/w+raywgt(2)/2.0d0*w
enddo
return
end
!
!----------------------------------------------------------------------------
! Sort total
!----------------------------------------------------------------------------
!
subroutine sorttotal(itt, ffreq, ddamp, ar)
!
implicit double precision (a-h,o-z)
ntt=itt
do i=1,ntt-1
  do j=i+1, ntt
    if(ffreq(i).gt.ffreq(j)) then
      ddum=ffreq(i)
      ffreq(i)=ffreq(j)
      ffreq(j)=ddum
      ddum=ddamp(i)
      ddamp(i)=ddamp(j)
      ddamp(j)=ddum
      ddum=ar(i)
      ar(i)=ar(j)
      ar(j)=ddum
    endif
  enddo
enddo
return
end
!
!----------------------------------------------------------------------------
subroutine sortmodal(nq, wr, xr, cmodal, arn)
!
implicit double precision (a-h,o-z)
double complex cmodal(2000), cdum
do i=1,nq-1
  do j=i+1, nq
    if(wr(i).gt.wr(j)) then
      cdum=wr(i)
      wr(i)=wr(j)
      wr(j)=cdum
      cdum=xr(i)
      xr(i)=xr(j)
      xr(j)=cdum
      cdum=arn(i)
      arn(i)=arn(j)
      arn(j)=cdum
    endif
  enddo
enddo
return
end
ddum=wr(i)
    wr(i)=wr(j)
    wr(j)=ddum
    ddum=xr(i)
    xr(i)=xr(j)
    xr(j)=ddum
    ddum=arn(i)
    arn(i)=arn(j)
    arn(j)=ddum
    cdum=cmodal(i)
    cmodal(i)=cmodal(j)
    cmodal(j)=cdum
  endif
  enddo
enddo
return
end
!
!----------------------------------------------------------------------
subroutine FindAr(nq,mm,dt,vv,sr,cmodal)
!----------------------------------------------------------------------
! to find modal constants with frequency domain least square method
implicit double precision (a-h,o-z)
dimension vv(2000)
double complex sr(201),cmodal(201),t(2000),ss(2000,200), &
   st(200,2000),stf(200),sts(200,200),ff(2000),wi
pi=4.0d0*datan(1.0d0)
do i=1,mm
  t(i)=dcmplx(vv(i),0.0d0)
enddo
CALL DFFTCF (mm, t, ff)
m=mm/2
df=1.0d0/(dt*dfloat(mm-1))
dw=2.0d0*pi*df
ff(1:mm)=ff(1:mm)/dfloat(mm)
do i=1,m+1
  w=dfloat(i-1)*dw
  wi=dcmplx(0.0d0,w)
  do j=1,nq
    ss(i,j)=wi/(wi-sr(j))*df
  enddo
endo
do i=mm,m+2,-1
  w= dfloat(mm-i+1)*dw
  wi=dcmplx(0.0d0,-w)
  do j=1,nq
    ss(i,j)=wi/(wi-sr(j))*df
  enddo
endo
st(1:nq,1:mm)=transpose(ss(1:mm,1:nq))
stf(1:nq)=matmul(st(1:nq,1:mm),ff(1:mm))
sts(1:nq,1:nq)=matmul(st(1:nq,1:mm),ss(1:mm,1:nq))
CALL DLSACG (nq, sts, 100, stf, 1, cmodal)
return
end
!
!----------------------------------------------------------------------
SUBROUTINE BAUPOL(COEFFRE, COEFIM, N, COMPL, ROOTRE, ROOTIM, NUMIT)

!*****************************************************************
! Without knowing any approximations of the roots, this
! SUBROUTINE finds N approximate values Z(I), I=1, ..., N for
! the N zeros of a polynomial PN of degree N with real or
! complex coefficients.
! The polynomial is described as follows:
! PN(Z)=COEF(0)+COEF(1)*Z+COEF(2)*Z**2+...+COEF(N)*Z**N,
! with COEF(I) = (COEFRE(I),COEFIM(I)) for I=0, ..., N (complex
! coefficients).

! INPUT PARAMETERS:

! COEFS : (N+1)-vector COEFS(0:N) containing the real
! part of each coefficient of the polynomial PN in
! DOUBLE PRECISION.
! COEFIM : (N+1)-vector COEFIM(0:N) containing the imaginary
! part of each coefficient of the polynomial PN in
! DOUBLE PRECISION.
! N : degree of the polynomial PN.
! COMPL : boolean variable :
! COMPL=.TRUE. , if the coefficients are COMPLEX. 
! COMPL=.FALSE. , if the coefficients are REAL.

! OUTPUT PARAMETERS:

! ROOTRE : N-vector ROOTRE(1:N) containing the approximate
! real parts of the computed zeros of PN in
! DOUBLE PRECISION.
! ROOTIM : N-vector ROOTIM(1:N) containing the approximate
! imaginary parts of the computed zeros of PN in
! DOUBLE PRECISION.
! NUMIT : maximum number of iteration steps.

! Initializing the iteration step counter NUMIT and the error
! bound GAMMA.
NUMIT=0
GAMMA=5.0D-18
IF(N .EQ. 1) THEN

CALL CDIV(-COEFFRE(0), -COEFIM(0), COEFFRE(1), COEFIM(1))
RETURN
ELSE
   N1=N+1
!
! Scaling via SCALFC.
!
   DO 10 I=1,N1
      E(I)=ABSCOM(COEFRE(N1-I),COEFIM(N1-I))
   10 CONTINUE
   CALL MCONST(FMACHP,INFINY,SMALNO,BASE)
   BND=SCALFC(N1,E,FMACHP,INFINY,SMALNO,BASE)
   IF(BND .EQ. 1.0D0) THEN
!
! Normalizing, in case scaling by SCALFC did not normalize the coefficients.
!
      MAX=0.0D0
      MIN=1.0D+300
      DO 20 I=N,0,-1
         X=ABSCOM(COEFRE(I),COEFIM(I))
         IF(X .GT. MAX) MAX=X
         IF(X .LT. MIN .AND. X .NE. 0.0D0) MIN=X
      20 CONTINUE
      BND=DSQRT(MAX*MIN)
      BND=1.0D0/BND
   END IF
   DO 30 K=1,N1
      B(2*K-1)=COEFRE(N1-K)*BND
      B(2*K)=0.0D0
      IF(COMPL) B(2*K)=COEFIM(N1-K)*BND
   30 CONTINUE
!
! Calculating the I-th zero of PN.
!
   X0=0.0D0
   Y0=0.0D0
   DO 40 I=1,N
      L=2*(N+2-I)
      DO 50 K=1,L
         A(K)=B(K)
      50 CONTINUE
!
   CALL BAUZRO(X0,Y0,N+1-I,GAMMA,XNEW,YNEW,NUMIT,A,B,C)
   ROOTRE(I)=XNEW
   ROOTIM(I)=YNEW
   X0=XNEW
   Y0=-YNEW
   40 CONTINUE
END IF
RETURN
END
!
SUBROUTINE BAUZRO(X0,Y0,N,GAMMA,XNEW,YNEW,NUMIT,A,B,C)
!
!****************************************************************
!                                                               *
! This SUBROUTINE calculates a zero of a polynomial PN with      *
! complex coefficients.                                         *
! We solve the equation PN(Z)/PN'(Z)=0                           *
!
via Newton's method with spiralization and extrapolation to improve convergence. The initial approximation \((X_0 + iY_0)\) can be chosen arbitrarily.

**INPUT PARAMETERS:**

- \(X_0\): real part of the initial approximation.
- \(Y_0\): imaginary part of the initial approximation.
- \(N\): degree of the polynomial \(P_N\).
- \(\Gamma\): error bound.

**OUTPUT PARAMETERS:**

- \(X_{NEW}\): real part of the computed zero of \(P_N\).
- \(Y_{NEW}\): imaginary part of the zero of \(P_N\).
- \(NUM\): maximum number of iteration steps.

**INPUT PARAMETERS:**

```
IMPLICIT DOUBLE PRECISION (a-h,o-z)
DIMENSION A(500),B(500),C(500)
LOGICAL ENDIT
```

```fortran
IF(N .EQ. 2) THEN

! Solving the remaining 2nd degree polynomial exactly.

CALL CDIV(A(3),A(4),A(1),A(2),P1RE,P1IM)
CALL CDIV(A(5),A(6),A(1),A(2),Q1RE,Q1IM)
P12RE=-P1RE/2.0D0
P12IM=-P1IM/2.0D0
RA1RE=P12RE*P12RE-P12IM*P12IM
RA1IM=2.0D0*P12RE*P12IM
RARE=RA1RE-Q1RE
RAIN=RA1IM-Q1IM
IF(RAIN .EQ. 0.0D0) THEN
  IF(RARE .LT. 0.0D0) THEN
    ! Purely imaginary root.
    RTIM=DSQRT(-RARE)
    RTRE=0.0D0
    XNEW=P12RE
    YNEW=P12IM+RTIM
    RETURN
  ELSE
    ! Real root.
    RTRE=DSQRT(RARE)
    RTIM=0.0D0
    XNEW=P12RE+RTRE
    YNEW=P12IM
    RETURN
  END IF
ELSE
  !
END IF
```

ELSE
! Complex root.
!
RABE=ABSCOM(RARE,RAIM)
IF(RARE .GT. 0.0D0) THEN
RTRE=DSQRT(0.5D0*(RABE+RARE))
IF(RAIM .LT. 0.0D0) RTRE=-RTRE
RTIM=0.5D0*RAIM/RTRE
ELSE
RTIM=DSQRT(0.5D0*(RABE-RARE))
RTRE=0.5D0*RAIM/RTIM
END IF
XNEW=P12RE+RTRE
YNEW=P12IM+RTIM
RETURN
END IF
ELSE IF(N .EQ. 1) THEN
!
! Polynomial of 1st degree.
!
XNEW=P12RE-RTRE
YNEW=P12IM-RTIM
RETURN
ELSE
I=0
ENDIT=.FALSE.
RHO=DSQRT(GAMMA)
BETA=10.0D0*GAMMA
QR=0.1D0
QI=0.9D0
XNEW=X0
YNEW=Y0
CALL COMHOR(XNEW,YNEW,N,GAMMA,UNEW,VNEW,UDNEW,VDNEW, &
UDDNEW,VDDNEW,BD,BDD,A,B,C)
NUMIT=NUMIT+1
PN=ABSCom(UNEW,VNEW)
IF(PN .LE. BD) THEN
RETURN
ELSE
PBOLD=2.0D0*PN
DZMIN=BETA*(RHO+ABSCom(XNEW,YNEW))
10 PSB=ABSCom(UDNEW,VDNEW)
!
! Spiralization.
!
IF(PN .LT. PBOLD) THEN
DZMAX=1.0D0+ABSCom(XNEW,YNEW)
NUMIT=NUMIT+1
H1=UDNEW*UDDNEW+VDNEW*VDDNEW
H2=2.0D0*UDNEW*VDNEW-UNEW*UDDNEW+VNEW*VDDNEW
IF(PSB .GT. 10.0D0*BD .AND. &
ABSCom(H1,H2) .GT. 100.0D0*BD*BD) THEN
!
! Applying Newton's method.
!
I=I+1
IF(I .GT. 2) I=2
U=UNEW*UDNEW-VNEW*VDNEW
V = UNEW*VNEW+VNEW*UNEW
CALL CDIV(-U,-V,H1,H2,DX,DY)
IF (ABSCOM(DX,DY) .GT. DZMAX) THEN
  H = DZMAX/ABSCOM(DX,DY)
  DX = DX*H
  DY = DY*H
  I = 0
END IF
IF (I .EQ. 2 .AND. ABSCOM(DX,DY) .LT. DZMIN/RHO .AND. & ABSCOM(DX,DY) .GT. 0.0D0) THEN
  ! Extrapolation.
  I = 0
  CALL CDIV(XNEW-XOLD,YNEW-YOLD,DX,DY,H3,H4)
  H3 = 1.0D0+H3
  H1 = H3*H3-H4*H4
  H2 = 2.0D0*H3*H4
  CALL CDIV(DX,DY,H1,H2,H3,H4)
  IF (ABSCOM(H3,H4) .LT. 50.0D0*DZMIN) THEN
    DX = DX+H3
    DY = DY+H4
  END IF
END IF
XOLD = XNEW
YOLD = YNEW
PBOLD = PBNEW
ELSE
  ! In a neighborhood of a saddle point.
  I = 0
  H = DZMAX/PBNEW
  DX = H*UNEW
  DY = H*VNEW
  XOLD = XNEW
  YOLD = YNEW
  PBOLD = PBNEW
  CALL COMHOR(XNEW+DX,YNEW+DY,N,GAMMA,U,V,H,H1, & H2,H3,H4,H5,A,B,C)
  IF (DABS(ABSCOM(U,V)/PBNEW-1.0D0) .LE. RHO) THEN
    DX = 2.0D0*DX
    DY = 2.0D0*DY
    GOTO 20
  END IF
END IF
ELSE
  I = 0
  NUMIT = NUMIT+1
  H = QR*DX-QI*DY
  DY = QR*DY+QI*DX
  DX = H
END IF
IF (ENDIT) THEN
  IF (ABSCOM(DX,DY) .LT. 0.1D0*BDZE) THEN
    XNEW = XNEW+DX
    YNEW = YNEW+DY
  END IF
CALL COMHOR(XNEW, YNEW, N, GAMMA, UNEW, VNEW, UDDNEW, VDDNEW, BD, BDD, A, B, C)
RETURN
ELSE
XNEW=XOLD+DX
YNEW=YOLD+DY
DZMIN=BETA*(RHO+ABS(XNEW, YNEW))
CALL COMHOR(XNEW, YNEW, N, GAMMA, UNEW, VNEW, UDDNEW, VDDNEW, BD, BDD, A, B, C)
PBNEW=ABS(UNEW, VNEW)
IF(PBNNEW .EQ. 0.0D0) THEN
RETURN
ELSE IF(ABS(DX, DY) .GT. DZMIN .AND. &
PBNNEW .GT. BD) THEN
GOTO 10
ELSE
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ENDIF
ENDER SUBROUTINE COMHOR(X, Y, N, GAMMA, U, V, UD, UD, UDD, VDD, BDP, BDPD, A, B, C)

IMPLICIT DOUBLE PRECISION (a-h,o-z)
DIMENSION A(500),B(500),C(500)
C(1)=A(1)
B(1)=A(1)
C(2)=A(2)
B(2)=A(2)
BDPD=ABSCOM(A(1),A(2))
BDP=BDPD
MS=N-1
M=N
J=N
NM2P1=N*2+1
DO 10 K=3,NM2P1,2
   J=J-1
   H1=X*B(K-2)-Y*B(K-1)
   H2=Y*B(K-2)+X*B(K-1)
   B(K)=A(K)+H1
   B(K+1)=A(K+1)+H2
   H3=ABSCOM(A(K),A(K+1))
   H4=ABSCOM(H1,H2)
   H=H3
   IF(H3 .LT. H4) H=H4
   IF(H .GT. BDP) THEN
      BDP=H
      M=J
   END IF
END IF
IF(K .EQ. NM2P1) THEN
   GOTO 20
ELSE
   H1=X*C(K-2)-Y*C(K-1)
   H2=Y*C(K-2)+X*C(K-1)
   C(K)=B(K)+H1
   C(K+1)=B(K+1)+H2
   H3=ABSCOM(B(K),B(K+1))
   H4=ABSCOM(H1,H2)
   H=H3
   IF(H3 .LT. H4) H=H4
   IF(BDPD .LT. H) THEN
      BDPD=H
      MS=J-1
   END IF
END IF
10 CONTINUE
20 CONTINUE
U=B(2*N+1)
V=B(2*N+2)
UD=C(2*N-1)
VD=C(2*N)
H=ABSCOM(X,Y)
IF(H .NE. 0.0D0) THEN
   BDP=BDP*FLOAT(M+1)*H**M
   BDPD=BDPD*FLOAT(MS+1)*H**MS
ELSE
   BDP=ABSCOM(U,V)
   BDPD=ABSCOM(UD,VD)
ENDIF
BDP=BDP*GAMMA
BDPD=BDPD*GAMMA
IF(N .GT. 1) THEN
   H1=C(1)
   H2=C(2)
   NM2M3=N*2-3
   DO 30 K=3,NM2M3,2
      H=C(K)+X*H1-Y*H2
      H2=C(K+1)+Y*H1+X*H2
      H1=H
   30 CONTINUE
   UDD=2.0D0*H1
   VDD=2.0D0*H2
   RETURN
ELSE
   UDD=0.0D0
   VDD=0.0D0
   RETURN
ENDIF
END

DOUBLE PRECISION FUNCTION ABSCOM(X,Y)
!
!****************************************************************
! This FUNCTION-subroutine calculates the absolute value of a   *
! complex number (X+I*Y).                                     *
!                                                               *
! INPUT PARAMETERS:                                             *
! =================                                             *
! X        : real part of the complex number.                   *
! Y        : imaginary part of the complex number.              *
!                                                               *
! OUTPUT PARAMETER:                                             *
! =================                                             *
! ABSCOM   : absolute value of the complex number.              *
!
!
IMPLICIT DOUBLE PRECISION (a-h,o-z)

IF(X .NE. 0.0D0 .OR. Y .NE. 0.0D0) THEN
   IF(DABS(X) .GE. DABS(Y)) THEN
      ABSCOM=DABS(X)*DSQRT(Y/X/Y/X+1.0D0)
      RETURN
   ELSE
      ABSCOM=DABS(Y)*DSQRT(X/Y/Y+1.0D0)
      RETURN
   END IF
ELSE
   ABSCOM=0.0D0
   RETURN
END IF
END

SUBROUTINE MCONST(FMACHP, INFINY, SMALNO, BASE)

!****************************************************************
!*                                                               *
!* This subroutine sets up some constants that are machine        *
!* dependent.                                                    *
!*                                                               *
!* OUTPUT PARAMETERS:                                            *
!* ==================                                            *
!* FMACHP   : machine constant for DOUBLE PRECISION.             *
!* INFINY   : largest representable floating-point number.       *
!* SMALNO   : smallest representable floating-point number.      *
!* BASE     : base of the floating-point number system used to    *
!*            represent machine numbers.                         *
!*                                                               *
!* Description of the auxiliary variables:                       *
!* ===========                                                   *
!* I        : number of digits of the floating-point mantissa     *
!* of DOUBLE PRECISION numbers.                                 *
!* M        : largest allowed exponent.                          *
!* N        : smallest allowed exponent.                         *
!*                                                               *
!* IMPLICIT DOUBLE PRECISION (a-h,o-z)
!* DOUBLE PRECISION INFINY
!* BASE=8.0D0
!* I=29
!* M=322
!* N=293
!* FMACHP=1.0D0
!* 10 FMACHP=0.5D0*FMACHP
!* IF(1.0D0 .LT. 1.0D0+FMACHP) GOTO 10
!* FMACHP=2.0D0*FMACHP
!* INFINY=BASE*(1.0D0-BASE**(-I))*BASE**(M-1)
!* SMALNO=(BASE**(N+3))/BASE**3
!* RETURN
!* END
DOUBLE PRECISION FUNCTION SCALFC(NN, PT, FMACHP, INFINY, SMALNO, BASE)

This FUNCTION-subroutine calculates a scaling factor which is used to scale the polynomial coefficients.

INPUT PARAMETERS:

- NN : 1 + the degree of the polynomial.
- PT : nn-vector PT(1:NN) containing the absolute values of the polynomial's coefficients.
- FMACHP : machine constant for DOUBLE PRECISION.
- INFINY : largest representable floating-point number.
- SMALNO : smallest representable floating-point number.
- BASE : base for the floating-point number system used by the machine.

OUTPUT PARAMETER:

- SCALFC : scaling factor.

IMPLICIT DOUBLE PRECISION (a-h,o-z)
DIMENSION PT(NN)
DOUBLE PRECISION MAX, MIN , INFINY
HI=DSQRT(INFINY)
LO=SMALNO/FMACHP
MAX=0.0D0
MIN=INFINY
DO 10 I=1,NN
   X=PT(I)
   IF(X .GT. MAX) MAX=X
   IF(X .NE. 0.0D0 .AND. X .LT. MIN) MIN=X
10 CONTINUE
SCALFC=1.0D0
IF(MIN .GE. LO .AND. MAX .LE. HI) THEN
   RETURN
ELSE
   X=LO/MIN
   IF(X .GT. 1.0D0) THEN
      SC=X
      IF(INFINY/SC .GT. MAX) SC=1.0D0
   ELSE
      SC=1.0D0/(DSQRT(MAX) * DSQRT(MIN))
   END IF
   L=DLOG(SC)/DLOG(BASE)+0.5D0
   SCALFC=BASE**L
   END IF
RETURN
END
SUBROUTINE CDIV(A,B,C,D,X,Y)

!*****************************************************************
! This SUBROUTINE performs a complex division
!(X+I*Y) := (A+I*B)/(C+I*D).
!*****************************************************************

INPUT PARAMETERS:
A : real part of the numerator.
B : imaginary part of the numerator.
C : real part of the denominator.
D : imaginary part of the denominator.

OUTPUT PARAMETERS:
X : real part of the quotient.
Y : imaginary part of the quotient.

NOTE: If the denominator's real and imaginary parts are both equal to zero, the program is aborted with a detailed error message.

*****************************************************************

IMPLICIT DOUBLE PRECISION (a-h,o-z)
IF(C .NE. 0.0D0 .OR. D .NE. 0.0D0) THEN
  IF(A .NE. 0.0D0 .OR. B .NE. 0.0D0) THEN
    IF(DABS(A) .GT. DABS(B)) THEN
      U=A
      AM=1.0D0
      AN=B/A
    ELSE
      U=B
      AM=A/B
      AN=1.0D0
    END IF
    IF(DABS(C) .GT. DABS(D)) THEN
      V=C
      P=1.0D0
      Q=D/C
    ELSE
      V=D
      P=C/D
      Q=1.0D0
    END IF
    F=U/V
    V=P*P+Q*Q
    U=(AM*P+AN*Q)/V
    X=U*F
    U=(-AM*Q+AN*P)/V
    Y=U*F
  ELSE
    X=(A/C+B/D)*AM
    Y=(B/C-A/D)*AN
  END IF
ELSE
  X=A/C
  Y=B/C
END IF
RETURN
ELSE
   X=0.0D0
   Y=0.0D0
   RETURN
END IF
ELSE
   WRITE(*,*) 'DIVISION BY ZERO IN SUBROUTINE CDIV'
   STOP
END IF
END
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