We consider Research Problem 6.94 in Graham, Knuth, and Patashnik’s Concrete Mathematics, and solve it by using bivariate exponential generating functions. The family of recurrence relations considered contains many cases of combinatorial interest. We give a complete classification of the partial differential equations satisfied by the exponential generating functions, and solve them in all cases. We also show that the recurrence relations defining the combinatorial numbers in this problem display an interesting degeneracy that we study in detail.