

Linear conductance oscillations in quantum wires and stripes with Rashba interaction

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

2010 J. Phys.: Conf. Ser. 248 012016

(<http://iopscience.iop.org/1742-6596/248/1/012016>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 161.111.180.103

The article was downloaded on 26/01/2012 at 16:19

Please note that [terms and conditions apply](#).

Linear conductance oscillations in quantum wires and stripes with Rashba interaction

M. Magdalena Gelabert¹ and Llorenç Serra^{1,2}

¹ Departament de Física, Universitat de les Illes Balears, E-07122 Palma de Mallorca, Spain

² Institut de Física Interdisciplinària i Sistemes Complexos IFISC (CSIC-UIB), Universitat de les Illes Balears, E-07122 Palma de Mallorca, Spain

E-mail: magdalena.gelabert@uib.es, llorens.serra@uib.es

Abstract. An analysis of the linear conductance of 2D quantum wires and stripes with Rashba spin-orbit interaction and attached to spin polarized leads is presented. Differences and similarities between both systems are highlighted. We discuss the dependence of the conductance with both energy and Rashba intensity.

1. Introduction

Almost twenty years after the theoretical proposal of a spin transistor by Datta and Das [1] the underlying physical principle of conductance oscillation induced by electrical tuning of the spin-orbit coupling has been proved in experiment [2]. This spin-orbit coupling is due to the asymmetry of the vertical (z) confinement felt by a 2D electron gas lying in the xy plane. It is known as Rashba interaction [3] and its tunability with electric gates has been demonstrated in quantum wells [4] and quantum wires [5]. Although the subject of conductance oscillations induced by Rashba coupling has been a central topic in spintronics for some time (see Ref. [6] for a review) the experiment by Koo *et al* spurred on a renewed interest [7, 8, 9, 10].

The physics of quantum wires with spin-orbit coupling differs depending on whether the coupling is uniform or localized in space. With an extended and uniform coupling the system is characterized by a modified subband structure, as compared to the case without Rashba coupling. A characteristic of this modified subband structure is the existence of anticrossing points in the presence of in-plane magnetic fields [11, 12]. An interesting consequence of the anticrossings is the existence of anomalous conductance plateaus due to the subband maxima and minima. Indeed, whenever the Fermi energy crosses a local extremum the number of propagating modes increases or decreases by one unit, with a corresponding increase or decrease in conductance. Quite remarkably, these anomalous conductance variations have been recently measured in a GaAs quantum wire with transport carried by holes [13].

A localized Rashba interaction, restricted to a small region of a quantum wire, is predicted to generate peculiar conductance dips due to the existence of quasibound states. This physical behavior was dubbed the Fano-Rashba effect in Ref. [14] due to the similarity with the well-known Fano resonances in electron scattering by atoms [15]. Since in real devices the spin-orbit channel is usually attached to ferromagnets an inhomogeneous Rashba coupling is a more realistic description than an extended one for small-sized devices.

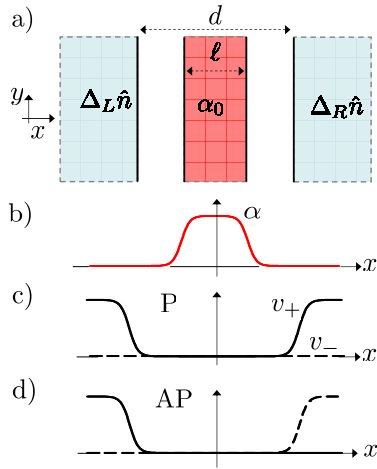


Figure 1. (Color online) Sketch of the physical system (a) and of the spatial variation of Rashba intensity $\alpha(x)$ (b) and of the spin-dependent potentials $v_{\pm}(x)$ (c,d).

In this work we discuss differences and similarities in the conductance oscillations of quantum wires and stripes with localized Rashba interaction, two distinct cases that we treated separately in Refs. [8, 9]. Other related works dealing with spin-orbit wires and stripes are Refs. [16, 17, 18, 19, 20, 21, 22, 23]. While a quantum wire is characterized by a transverse confinement, which we assume to be parabolic, a stripe has a vanishing confinement in the transverse direction. Transport in a quantum wire is carried by a discrete set of transverse modes that are coupled by the Rashba interaction. On the contrary, a stripe has a continuum of transverse momenta q , uncoupled with each other since the Rashba interaction preserves this quantum number. We shall discuss the physical implications of these two behaviors on the linear conductance and the polarization of the transmitted current.

2. Model

Our model considers a 2D electron gas in the xy plane with transport occurring along x and with the possibility of a transverse confining potential $v_t(y)$. We call *stripe* the system described by a vanishing $v_t(y)$, while $v_t(y) = m\omega_0^2 y^2/2$ corresponds to a parabolic *wire* characterized by a confinement energy $\hbar\omega_0$. The reader is addressed to Refs. [8, 9] for full details of the model while here it is just sketched for the sake of completeness. The system Hamiltonian reads

$$\mathcal{H} = -\frac{\hbar^2}{2m_0} \left(\frac{d^2}{dx^2} + \frac{d^2}{dy^2} \right) + v_t(y) + \Delta(x) \hat{n} \cdot \vec{\sigma} + |\Delta(x)| + \mathcal{H}_R, \quad (1)$$

where \mathcal{H}_R is the Rashba Hamiltonian,

$$\mathcal{H}_R = \frac{1}{\hbar} \left(\alpha(x) p_y \sigma_x - \frac{1}{2} \{ \alpha(x), p_x \} \sigma_y \right). \quad (2)$$

Polarized leads in the direction of \hat{n} are described by means of a Zeeman field $\Delta(x)$ that couples to the spin vector $\vec{\sigma}$. The functions determining the Hamiltonian are $\Delta(x)$ and $\alpha(x)$. These quantities take a constant value in the three parts of our system: left contact (L), central region and right contact (R), and they vary smoothly, described by a Fermi-type function, at the interfaces. We denote by m_0 the conduction-band effective mass of the semiconductor and by α_0 the Rashba intensity of the central region. The Zeeman field in contact c , where $c = L, R$, is denoted by Δ_c , respectively. The case of parallel polarized contacts (P) corresponds to $\Delta_L = \Delta_R \equiv \Delta_0$, while the case of antiparallel polarizations (AP) corresponds to $\Delta_L = -\Delta_R \equiv \Delta_0$, where the total Zeeman splitting is $2\Delta_0$. For simplicity, Δ_0 is assumed

equal in both contacts. We use the notation $\hat{n}P$ and $\hat{n}AP$ to indicate parallel and antiparallel configurations along a certain direction \hat{n} . Figure 1 shows a sketch of the model with the variation of the Rashba intensity $\alpha(x)$. It also shows the potentials v_s , for $s = \pm$ spins, defined as

$$v_s(x) = s\Delta(x) + |\Delta(x)|, \quad (3)$$

where $|\Delta(x)|$ is a scalar gate potential aligning the band bottom of the different regions. Notice that in the P configuration the $s = -$ spin sees no potential at all while $s = +$ is confined by a potential well of width d . On the contrary, in the AP configuration both spins feel a potential step, but in opposite contacts. As we will discuss below, these differences in potential landscape for $+$ and $-$ spins greatly influence the transport properties of the stripe with polarized contacts.

3. Linear conductance and polarization

We calculate the linear conductance G from left to right contacts when an infinitesimal potential bias is applied along x . In the Landauer formalism of scattering G is given by the quantum transmission T at the Fermi energy [24]. In practice, however, we have to differentiate the quantum wire and the stripe systems due to the discrete and continuous character of the transverse modes, respectively. In terms of the conductance quantum $G_0 = e^2/h$, Landauer formula for the quantum wire reads

$$\text{(wire)} \quad G = G_0 \sum_{ns, n's'} T_{n's', ns}, \quad (4)$$

where ns and $n's'$ are labeling transverse mode and spin in L and R contacts, respectively. On the other hand, for the stripe the corresponding formula for the conductance per unit of transverse length L_y reads

$$\text{(stripe)} \quad \frac{G}{L_y} = \frac{G_0}{4\pi} \sum_{ss'} \left\{ \kappa_{Ls} \int_{-\pi/2}^{\pi/2} d\theta |\cos \theta| T_{s's}(\kappa_{Ls}, \theta) + \kappa_{Rs} \int_{\pi/2}^{3\pi/2} d\theta |\cos \theta| T'_{s's}(\kappa_{Rs}, \theta) \right\}, \quad (5)$$

where κ_{cs} is the Fermi wavevector in contact $c = L, R$ and spin s , while θ gives the polar angle of the vectorial momentum for the incident electron. We shall also calculate the relative polarization p of the transmitted current, defined as $p = G_p/G$, where the *polarized conductance* G_p takes into account the spin of the current in the R terminal. Its expression for the quantum wire and the stripe are straightforward extensions of Eqs. (4) and (5).

4. Results

This section presents our results for the linear conductance of wires and stripes, Eqs. (4) and (5), using the method of Refs. [8, 9]. We shall discuss the dependence with energy and Rashba intensity for various orientation of the Zeeman fields in the contact regions.

4.1. Energy dependence

Figures 2 and 3 display a comparison of $G(E)$ for a stripe (S) and a wire (W). Black symbols correspond to the results without Rashba coupling while gray (red in color) symbols are for an intensity of $\alpha_0 = 0.3E_U L_U$, where E_U and L_U denote energy and length unit, respectively. As in Refs. [8] and [9] we choose a units system in which $\hbar = m_0 = 1$. Additionally, for the stripe we choose the energy unit $E_U = \Delta_0$ while for the wire we take $E_U = \hbar\omega_0$. In terms of these units it is $L_U = \sqrt{\hbar^2/m_0 E_U}$.

In the quantum stripe (left panels in Figs. 2 and 3) the energy $2\Delta_0$ signals the threshold from only one propagating spin ($E < 2\Delta_0$) to both spins propagating in the contacts ($E > 2\Delta_0$).

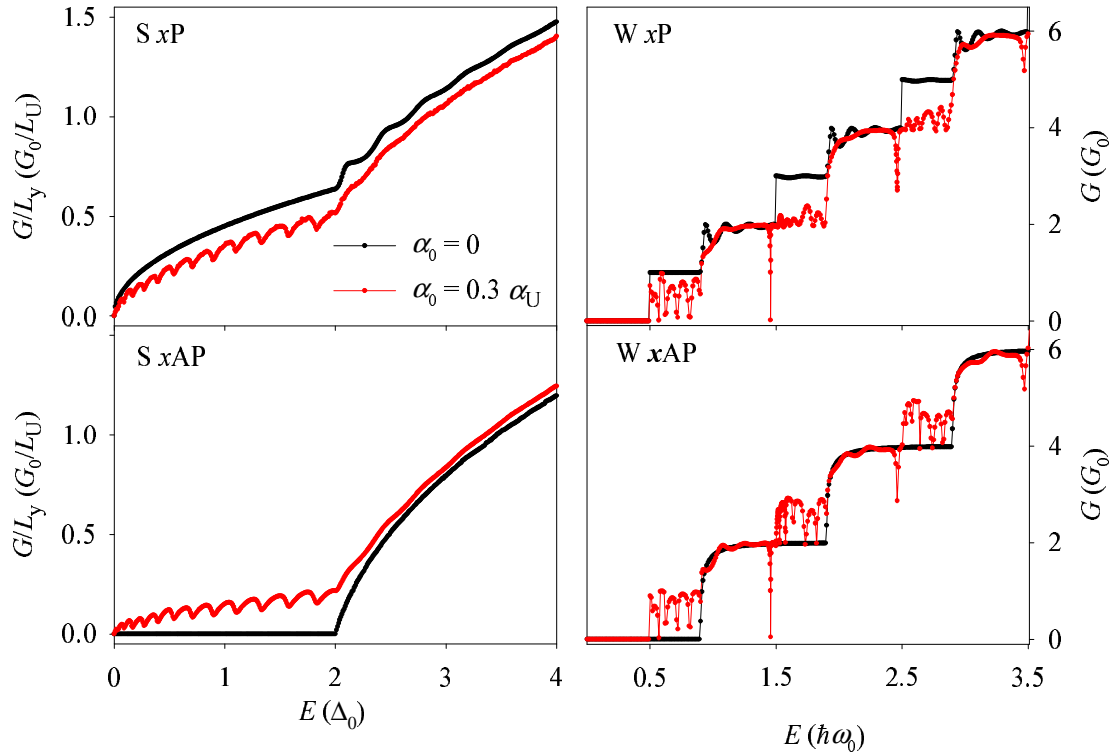


Figure 2. (Color online) Conductance of the stripe (S, left) and wire (W, right) systems for polarization of the contacts along x . Upper and lower rows correspond to parallel (P) and antiparallel (AP) orientations, respectively. We take $\ell = 8L_U$ (length of the Rashba region), and $\Delta_0 = 0.2\hbar\omega_0$ in the wire case. Black symbols are for vanishing Rashba coupling while gray (red) ones correspond to $\alpha_0 = 0.3\alpha_U$. The unit of Rashba coupling is given by $\alpha_U = \sqrt{\hbar^2\Delta_0/m_0}$ and $\sqrt{\hbar^3\omega_0/m_0}$ for the quantum stripe and wire, respectively.

This transition reflects in an abrupt enhancement of conductance when the energy exceeds the threshold. In addition, there are conductance oscillations of two types: for x -oriented contacts the system displays Fano oscillations below the threshold $2\Delta_0$ due to the Rashba-induced coupling between the propagating spin and the quasibound states of the opposite evanescent spin. Above threshold there are Ramsauer oscillations due to the potential steps in the polarized leads. Notice, however, that in AP configurations the Ramsauer oscillations are greatly reduced and, also, that in y -polarization the Fano oscillations are absent. Quite remarkably, while the conductance of P configurations is enhanced by the Rashba coupling with respect to the spin-orbit-free case, i.e., red-gray symbols are higher than black ones, this situation is reversed in AP configurations. This behavior can be interpreted as a destruction of the spin-valve behavior due to the mixing induced by the Rashba coupling.

Let us now focus on the wire conductances shown in the right panels of Figs. 2 to 3. In the absence of spin-orbit coupling the conductance is characterized by a staircase appearance; each step corresponding to the activation of additional transverse modes. In P configurations steps for up and down spins are shifted by an energy $2\Delta_0$ and, therefore, the conductance jumps by G_0 from one step to the next. In the AP case the corresponding increments are doubled, $2G_0$. With the addition of the Rashba coupling (red-gray symbols) we see that in general the

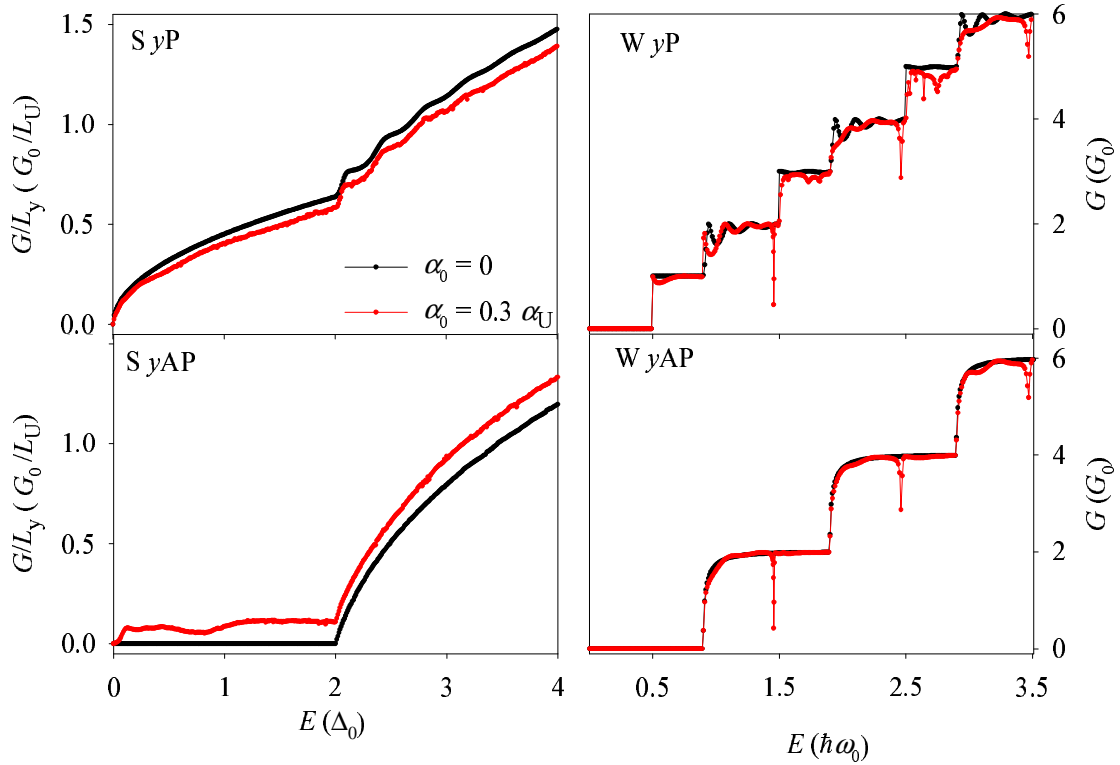


Figure 3. (Color online) Same as Fig. 2 for y configuration of the contacts.

conductance displays more oscillation. As a reminiscence of the perfectly clean wires we refer to the energy interval $[n - 1/2, n + 1/2]\hbar\omega_0$ as the n -th *conductance plateau*. Comparing with the stripe case (left panels) we notice that the wire conductance for each conductance plateau resembles the result of the stripe. Looking, for instance, the result in Fig. 2 for the second plateau $[1.5, 2.5]\hbar\omega_0$ we see at the beginning Fano oscillations followed by a sudden increase in conductance for $E \approx 1.9\hbar\omega_0$, quite similar to the stripe behavior. Interestingly, a qualitative difference with the stripe can be seen at the end of the plateau as a pronounced conductance dip. This is again a Fano resonance but is qualitatively different from those seen at the beginning of the plateau. It originates in a quasibound state shifted by a negative energy from the next plateau by the Rashba coupling. Notice that this is different with respect to the Fano resonances at the beginning of the plateau which stem from quasibound states induced by the polarized contacts. These wire conductance structures are repeated almost regularly in each plateau.

The results for other configurations of the polarized contacts show analogous similarities between wire and stripe conductances for each conductance plateau. Notice, in particular that in y orientation the Fano oscillations at the beginning of the plateaus are absent while those at the end are still present. It is also worth mentioning that the yAP configuration displays the less-structured conductance of all, the Rashba interaction only inducing dips at energies $E \approx (n + 1/2)\hbar\omega_0$ in this case.

Figure 4 displays the energy dependence of the polarization p for selected cases to illustrate the polarization mechanism of the localized Rashba coupling. The stripe is characterized by its full polarization $|p| = 1$ for energies below the Zeeman gap $2\Delta_0$. When exceeding this threshold, the polarization decreases in absolute value and tends to zero for high enough energies. Ramsauer oscillations can be clearly seen for energies slightly above threshold. Comparing black and gray

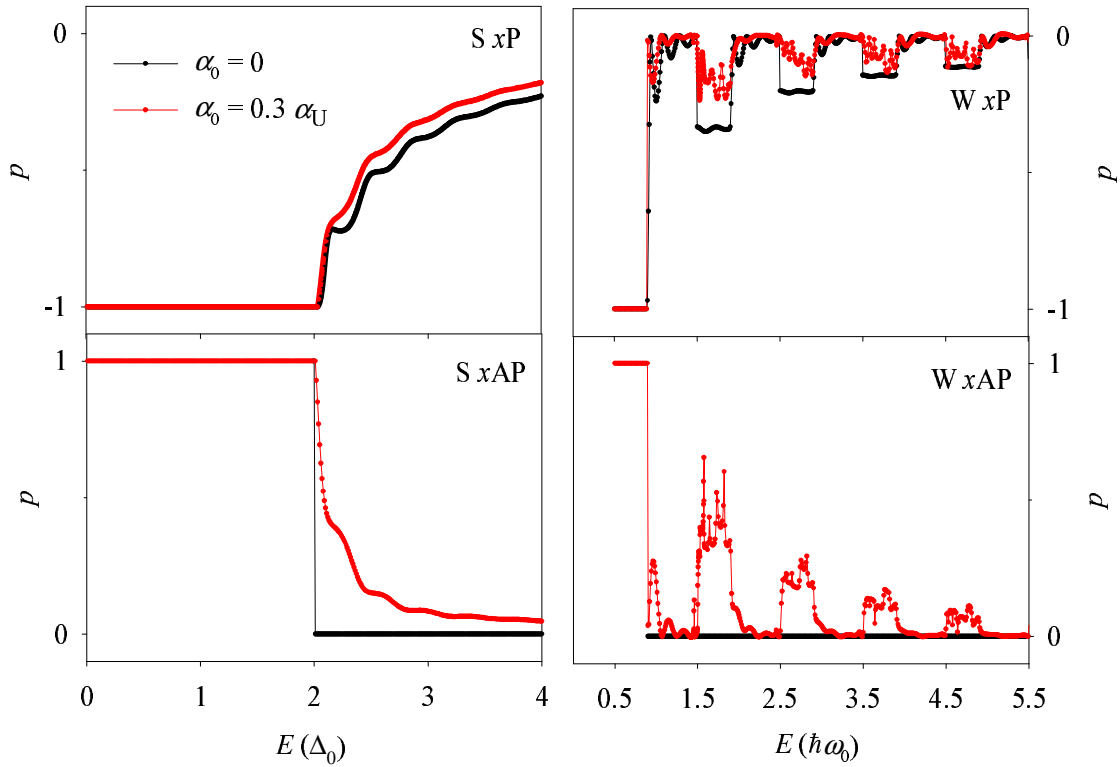


Figure 4. (Color online) Polarization of the transmitted current, defined as $p = G_p/G$, where $G_p = \sum_{s'} s' G_{s's}$ is the polarized conductance. As in Fig. 2, black and gray (red) symbols correspond to the absence and presence of Rashba interaction, respectively. The configuration of the contacts for each panel is also labeled as in Fig. 2.

(red) symbols, we notice that the main effect of the Rashba interaction (red-gray symbols) in the stripe geometry is to smooth the transition from full to vanishing polarization in AP configuration.

Focusing now on the wire polarization of the transmitted current, displayed in the right panels of Fig. 4, we notice that each conductance-plateau-region, $[n - 1/2, n + 1/2]\hbar\omega_0$, is again a qualitative repetition of the stripe behavior. The P configuration is characterized by a transition from high absolute polarization at the beginning of the plateau to low polarization towards the plateau end. There is an overall tendency to decrease the polarization when the energy increases and conspicuous oscillations are superimposed on the general trend. In agreement with our preceding analysis of the conductance, we can associate the oscillations in the high- $|p|$ part of each plateau with Fano resonances due to quasibound states and those on the vanishing tail with Ramsauer potential oscillations. The Fano resonances are quite narrow and, although not seen in the discrete set of plotted symbols of Fig. 4, polarization can be actually reversed with respect to the main trend when the energy is very close to some resonances. An approximate expression for the polarization, not taking into account conductance oscillations, is obtained from the number of up (n_+) and down (n_-) propagating channels as $|p| \approx (n_+ - n_-)/(n_+ + n_-)$. This simplified formula is in qualitative agreement with the behavior shown in the right panels of Fig. 4 when counting the number of propagating modes for a given energy.

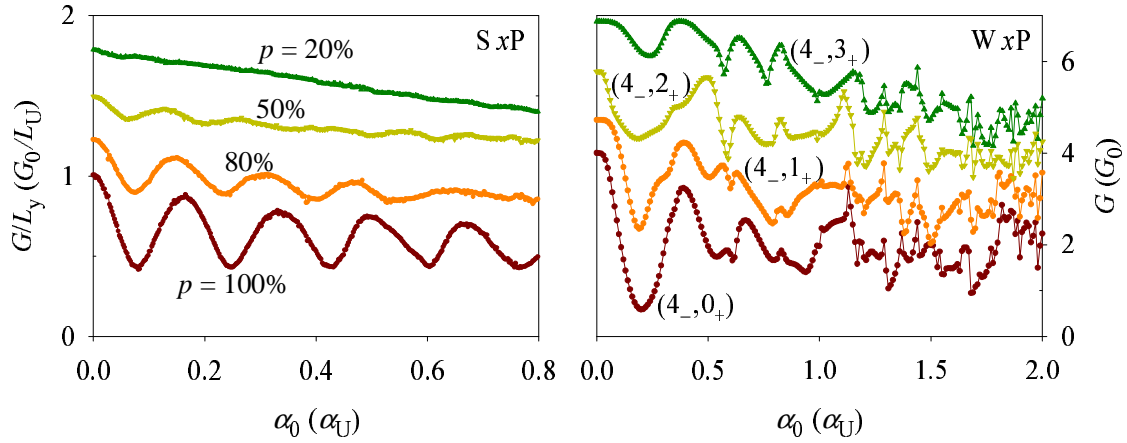


Figure 5. (Color online) Conductance as a function of α_0 in stripe (left) and wire (right) geometries. The polarization of the contacts is indicated for the stripe, while for the wire (n_-, n_+) gives the number of propagating down and up spin modes.

4.2. Dependence on Rashba coupling intensity

The Datta-Das spin transistor relies on the oscillatory character of the conductance as a function of Rashba intensity. The present subsection analyzes this dependence of the conductance, at a fixed energy, in quantum wires and stripes. In particular, we explore the robustness of the oscillations when $E > 2\Delta_0$ and both spins can propagate in the contacts, i.e., the regime of partially polarized contacts. Figure 5 displays the conductance in both systems. In the stripe the incident-current polarization can be varied in a continuous way simply by increasing the energy above the Zeeman barriers of the contacts. For the wire, however, we always have an integer number of spin-down and spin-up propagating modes (n_-, n_+) and, as mentioned above, approximate polarizations $|p| \approx (n_+ - n_-)/(n_+ + n_-)$. Of course, partial reflections and transmissions can lead to deviations from this simplified expression.

For the stripe, shown in the left panel of Fig. 5, at full polarization ($p = -1$) the conductance displays damped oscillations with α_0 . Remarkably, when the polarization of the leads is reduced the oscillating behavior is greatly quenched and it is fully washed out for polarizations below 20%. In the wire geometry (right panel) the behavior is less regular. At full polarization there is a clear initial oscillation, but as α_0 increases the conductance exhibits an irregular or *disordered* behavior. When the polarization is reduced the initial oscillation is heavily distorted, but is more robust than in the stripe case. At large α_0 's the region of irregular conductance is not qualitatively modified when the polarization is reduced.

5. Conclusions

In this work we have compared the linear conductance of a parabolic wire and a stripe with a localized Rashba interaction and polarized contacts. The spin splitting of the polarized contacts is modeled by means of Zeeman fields. For energies below the Zeeman energy gap the spin-selective barriers of the contacts induce the formation of quasibound states. These quasibound states couple with propagating states via the Rashba coupling and manifest as Fano resonances of the conductance for both stripe and wire systems. In the case of the wire these Fano resonances appear at the beginning of each conductance plateau. At the end of each plateau we also find a second class of Fano resonances, conductance dips, due to quasibound states which are not induced by the polarized leads, but they originate in the Rashba interaction alone. For energies above the Zeeman gap the conductance shows Ramsauer oscillations due to the underlying

potential. We have also discussed the variations for x and y orientation of the polarized contacts in both parallel and antiparallel configurations.

As a function of α_0 the stripe conductance shows a damped oscillating behavior which is not robust for partial polarizations; i.e., it becomes monotonous for polarizations below $\approx 20\%$. The wire conductance is qualitatively similar but with a more robust initial oscillation. At high values of α_0 the wire conductance is characterized by an irregular behavior that we attribute to the existence of many resonances.

Acknowledgments

We acknowledge R. López and D. Sánchez for valuable discussions. This work was supported by the Grant No. FIS2008-00781.

References

- [1] Datta S and Das B 1990 *Appl. Phys. Lett.* **56** 665
- [2] Koo H C, Kwon J H, Eom J, Chang J, Han S H and Johnson M 2009 *Science* **325** 1515
- [3] Rashba E I 1960 *Fiz. Tverd. Tela* (Leningrad) **2**, 1224 [1960 *Sov. Phys. Solid State* **2**, 1109]
- [4] Nitta J, Akazaki T, Takayanagi H and Enoki T 1997 *Phys. Rev. Lett.* **78**, 1335
- [5] Engels G, Lange J, Schäpers Th and Lüth H 1997 *Phys. Rev. B* **55**, R1958
- [6] Zutic I, Fabian J and Das Sarma S 2004 *Rev. Mod. Phys.* **76** 323
- [7] Agnihotri P and Bandyopadhyay S 2010 *Physica E* **42**, 1736
- [8] Gelabert M M, Serra Ll, Sánchez D and López R 2010 *Phys. Rev. B* **81**, 165317
- [9] Gelabert M M and Serra Ll 2010 *ArXiv:1005.2480* (unpublished)
- [10] Zainuddin A N M, Hong S, Siddiqui L and Datta S 2010 *Arxiv:1001.1523* (unpublished)
- [11] Nesteroff J A, Pershin Yu V and Privman V 2004 *Phys. Rev. B* **69**, 121306(R)
- [12] Serra Ll, Sánchez D and López R 2005 *Phys. Rev. B* **72**, 235309
- [13] Quay C H L, Hughes T L, Sulpizio J A, Pfeiffer L N, Baldwin K W, West K W, Goldhaber-Gordon D and de Picciotto R 2010 *Nature Phys.* **6**, 336
- [14] Sánchez D and Serra Ll 2006 *Phys. Rev. B* **74**, 153313
- [15] Fano U 1961 *Phys. Rev.* **124**, 1866
- [16] Matsuyama T, Hu C M, Grundler D, Meier G and Merkt U 2002 *Phys. Rev. B* **65**, 155322
- [17] Pala M G, Governale M, König J and Zülicke U 2004 *Europhys. Lett.* **65**, 850
- [18] Shelykh I A and Galkin N G 2004 *Phys. Rev. B* **70**, 205328
- [19] Nikolić B and Souma S 2005 *Phys. Rev. B* **71**, 195328
- [20] Zhang L, Brusheim P and Xu H Q 2005 *Phys. Rev. B* **72**, 045347
- [21] Jeong J S and Lee H W 2006 *Phys. Rev. B* **74**, 195311
- [22] Perroni C A, Bercioux D, Marigliano Ramaglia V and Cataudella V 2007 *J. Phys.: Condens. Matter* **19**, 186227
- [23] Wan J, Cahay M and Bandyopadhyay S 2007 *J. App. Phys* **102**, 034301 [erratum 2007 *J. App. Phys* **102**, 099902]
- [24] Datta S 2002 *Electronic transport in mesoscopic systems* (Cambridge UP)