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PRE-PROCESSING GEODETIC DATA OF THE VOLCANIC AREA OF TEIDE TO MONITORING DEFORMATIONS

por

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SUMMARY

A statistical analysis of data observations in geodetic networks, made before adjustment, provides a means for error control and allows us to ensure a previous accuracy of data. After computing the corrections for systematics effects, distance measurements and horizontal and vertical angles observations are statisticaly analized. Four field campaigns data are analized and a comparison of the results is made in order to detect potential movements in the Caldera of Teide area.

1. INTRODUCTION

The precise geodetic techniques are a decisive tool in the quantitative determination and mathematical formulation of possible crustal deformations in geologically active zones with small dimensions.

The Institute of Astronomy and Geodesy (UCM-CSIC) is developing a project of observation and study of deformations in a precision network in the caldera of Teide, formed by 17 survey points (Sevilla and Martín, 1986). The comparison of results obtained at the different campaigns made, enables us to detect possible deformations, whenever they are satisfactorily controled in precision and reliability.

At present, four observation campaigns in summers of 1984, 1985, 1986 and 1987 have been carried out. The measurements of distances, vertical and horizontal angles between the 17 survey points were done using second geodetic order EDM instrument and theodolite. Therefore, we can only expect to detect variations if they are greater than the instrumental precision limits. We set them in 7 mm for distances, 2" for horizontal angles, and 20 mm for height differences calculated from zenital angles and distances.

In the observation of a geodetic network a great amount of data are obtained. After computing the corrections for systematic effects, distance measurements and horizontal and vertical angles observations are statisticaly analized using hypothesis testing.

2. DISTANCE MEASUREMENTS

The electro-magnetic distance measurement between two survey points gives i series of n_i observations. Since all observations should be homogeneuous, they must be corrected for atmospheric and geometrical conditions using appropriate expressions according to the precision of instrument used. From all series of measurements made we must obtain the value of the distance measured and some estimate of the accuracy of this determination.

2.1. Correction for meteorological conditions

Meteorological corrections arise from two causes (Burnside, 1982).

a) Correction due to atmospheric effects on the velocity of propagation of the electromagnetic wave

The correction, $\Delta_1 d$, to the distance provided by the instrument, d_0, can be obtained by

$$\Delta_1 d = d - d_0 = - \frac{k\,(1 - k)\,d_0^3}{12\,R^2} , \qquad (2.1)$$

k being the coefficient of refraction and R the radius of curvature of the ray path.

The accurary of a distance measurement is directly related to the accuracy of the refraction coefficient determination. Empirical values are required for a precise determination; formulae to calculate the refraction coefficient from the refractive index for a given set of meteorological conditions provides less accurate values.

b) Correction for path curvature

The correction for path curvature to get the straight distance, d', from the arc distance, d, is given by

$$\Delta_2 d = d' - d = - k \frac{d^3}{24\,R^2} \qquad (2.2)$$

k being the coefficient of refraction, and R the radius of curvature of the ray path. In practice, for short distances, R may be taken as the earth mean radius.

2.2. Geometric correction for height difference between distancemeter and prism

The corrected distance d^c for path inclination with distancemeter at height H_D and prism at height H_P is given by

$$d = (d' - \Delta H \sin z) - \Delta H \cos z , \tag{2.3}$$

z being the zenital angle, $\Delta H = H_D - H_P$, the height difference between distancemeter and prism, and d' the corrected distance for atmospheric effects.

$$d' = d_0 + \Delta_1 d + \Delta_2 d \tag{2.4}$$

with $\Delta_1 d$ and $\Delta_2 d$ given by (2.1) and (2.2) respectively.

2.3. Statistical analysis of distance measurements before adjustment

After correcting the data for systematic effects, we make use of hypothesis testing to analyze the distance measurements, this enables us to check the internal precision of instrument and the precision of determination.

From the n_i lectures, d_{ij}, $j = i, \ldots, n_i$, of each series i, all of then supossed with the same weight, we calculate for the i series the mean value, $\bar{d}_i$, and the mean square error, s_i, by

$$\bar{d}_i = \frac{\sum_{j=1}^{n_i} d_{ij}}{n_i} , \tag{2.5}$$

and

$$s_i = \sqrt{\frac{\sum_{j=1}^{n_i} (\bar{d}_i - d_{ij})^2}{n_i - 1}} \tag{2.6}$$

This is obtained for each series $i = i, \ldots, k$, with k number of series.

The short time employed in the observation of one series, enables us to suppose that corrections are constant for each series of distances, so corections given thereinbefore are applied to the mean values obtained in (2.5), thus we get d^c_i.

From the corrected values, d_i^c, we obtain the mean value of all series

$$\bar{d} = \frac{\sum_{i=1}^{k} d_i^c}{k}$$

the mean square error of all series

$$s = \sqrt{\frac{\sum_{i=1}^{k} (\bar{d} - d_i^c)^2}{k - 1}}, \qquad (2.8)$$

and the mean square error of $\bar{d}$

$$s_{\bar{d}} = \frac{s}{\sqrt{k}} .$$

Now, we make use of hypothesis testing in order to ensure the previous accuracy of data. (Bjerhammar, 1973)

The Variance test

a) Individually applied to each series, test the internal precision of the instrument, that we set to $\sigma_0 = 1$ mm. We compute.

$$y = \frac{(n_i - 1)\, s_i^2}{\sigma_0^2},$$

where s_i^2 given by (2.6) is the estimated variance of the i-series with n_i elements.

The primary (null) hypothesis is

$H_0: \sigma^2 \leq \sigma_0^2$.

The alternative hypothesis is

$H_1: \sigma^2 > \sigma_0^2$.

Then, the primary hypothesis is rejected if

$y > \chi_{n_i - 1, \alpha}$,

$\chi_{n_i - 1, \alpha}$ being the χ^2-distribution on $n_i - 1$ degrees of freedom at the significance level α, usually 5%.

b) Applied to the set of all series, test the precision in the determination , that we set to σ = 7 mm. In this case, we use in a similar way the value

$$y = \frac{(k - 1)\, s^2}{\sigma_0^2},$$

with s given by (2.8), and rejecting the null hypotesis if

$$y > \chi^2_{k-1,\alpha} \,.$$

The Tau test

If the variance test is rejected, further investigations have to be made to detect gross errors. This can be done with the Tau-Test (Pope, 1975). If

$$\left| \frac{d_{ij} - \bar{d}_i}{\sqrt{\frac{n_i - 1}{n_i}}\; s_i} \right| \geq \tau_{n_i - 1, \alpha/2} \,,$$

the corresponding observation, d_{ij}, is rejected, $\tau_{n_i -1,\alpha/2}$ being the τ-distribution at the significance level $\alpha/2$.

The Bartlett-test

Applied to the set of all series test the equalitity of the variances. The primary hypothesis is

$$H_0:\ \sigma_1^2 = \dots = \sigma_k^2 .$$

We define

$$y = \frac{n_0 \log s_0^2 - \sum_{i=1}^{k} (n_i - 1) \log s_i^2}{1 + \dfrac{\sum_{i=1}^{k} \dfrac{1}{n_i} - \dfrac{1}{n}}{3(k - 1)}},$$

where

$$n_0 = \sum_{i=1}^{k} (n_i - 1) ,$$

$$n = \sum_{i=1}^{k} n_i ,$$

$$s_0^2 = \frac{\sum_{i=1}^{k} (n_i - 1)\, s_i^2}{n_0} ,$$

with s_i^2 given by (2.6).

Then, if

$$y > \chi^2_{k-1,\alpha} ,$$

H_0 is rejected.

Supposing that the distance series obtained in different epochs are independent we can use the classical F-Test in several repeated tests checking two series in the following way.

Primary hypothesis $\sigma_1^2 = \sigma_2^2$,
Alternative hypothesis $\sigma_1^2 \neq \sigma_2^2$.

Taking $s_1 > s_2$, we compute

$$y = \frac{s_1^2}{s_2^2} ,$$

then, H_0 is rejected when y exceeds the theoretical value for a Fisher distribution at a significance level of $\alpha/2$, with n_1-1 and n_2-1 degrees of freedom.

The Student-test

To analyze the compatibility of the different series of observations in the same determination, we compute the quantity

$$y = \frac{(\bar{d}_1 - \bar{d}_2)}{\sigma} \sqrt{\frac{n_1\, n_2}{n_1 + n_2}} ,$$

with $\bar{d}_1$ y $\bar{d}_2$ given by (2.5) for two different series, and assuming that the two series have the same variance σ = 7 mm tested before.

The null hypothesis $H_0 : \bar{d}_1 = \bar{d}_2$ is rejected, if

$$|y| > t_{n_1+n_2-2,\alpha/2} ,$$

where t is the value of the Student distribution with n_1+n_2-2 degrees of freedom.

3.- ZENITHAL ANGLES OBSERVATION

For each line sight between two survey points we have different pairs (RC, LC) of readings, every one must be corrected for systematic effects. The main disturbing cause is the refraction, that we can correct using the simple formula of geodetic refraction.

$$\Delta z = z' - z^0 = k \sin z^0, \qquad (3.1)$$

k being the coefficient of refraction. Appart from this, we must consider the geometric correction, β , for hight difference, ΔH, between theodolite and target,

$$z = z' - \beta , \qquad (3.2)$$

with β given by

$$\beta = \sin^{-1} \left(\frac{\Delta H}{d'} \sin z' \right).$$

After correcting the zenithal angles using (3.1) and (3.2), we obtain the mean value of the n_c consecutive readings (RC, LC)

$$z_i^R = \frac{\sum_{m=1}^{n_c} z_m^R}{n_c} ,$$

$$z_i^L = \frac{\sum_{m=1}^{n_c} z_m^L}{n_c} ,$$

to remove the eclimetric error we calculate

$$z_i = z_i^L + \frac{360^0 - (z_i^R + z_i^L)}{2} ,$$

Finally, from these results, the mean value of the n_R determinations made is obtained

$$z = \frac{\sum_{i=1}^{n_R} z_i}{n_R} , \qquad (3.3)$$

with its mean square error

$$e = \sqrt{\frac{\sum_{i=1}^{n_R} (z_i - z)^2}{n_R - 1}} \qquad (3.4)$$

3.1. Statistical analysis of zenithal angles

To test the precision in the zenithal angle determinations, that according to the precision of the theodolite used we set to $\sigma_0 = 2''$, a variance test can be used in thefollowing way, we compute

$$y = \frac{(n_R - 1)\, e^2}{\sigma_0^2} ,$$

where e^2, given by (3.4), is the estimated variance of the n_R determinations.

The primary hypothesis

$$H_0 : \sigma^2 = \sigma_0^2$$

is rejected if

$$y > \chi^2_{n_R - 1, \alpha} ,$$

where $\chi^2_{n_R - 1, \alpha}$ is the value of the χ^2-distribution at a significance level of α with $n_R - 1$ degrees of freedom.

3.2. Height differences

From observation data of distances, d, and zenithal angles, z, we can determine height differences between survey points by

$$\Delta h = d \cos(z - \theta/2) \sec \theta/2, \qquad (3.5)$$

with

$$\theta = L/R ,$$

L being the reduced distance, calculated by (Heiskanen, Moritz, 1967)

$$L = \sqrt{(d^2 - \Delta h^2) / (1 + h_1/R)(1 + h_2/R)} \quad ,$$

where h_1 and h_2 are the heights of the conected points and Δh their difference. This calcul is made by iterations.

The propagation law of the variances gives the precision in height difference determination as

$$\sigma^2_{\Delta h} = \left[\cos(z- \theta/2)\sec \theta/2\right]^2 \sigma^2_d + \left[d \sin(z-\theta/2) \sec \theta/2\right]^2 \sigma^2_z ,$$

σ^2_d and σ^2_z being the variances of the distance and the zenital, respectively.

4.- AZIMUTHAL ANGLES

In every station-survey point, from the n_c consecutive readings to a survey point i, obtained in both positions of instrument (RC and LC) we compute the mean values

$$l^R = \frac{\sum_{m=1}^{n_c} l^R_m}{n_c} ,$$

$$l^L = \frac{\sum_{m=1}^{n_c} l^L_m}{n_c} ,$$

then, the reading for direction i is

$$l_i = \frac{l^R + (l^L - 200^\circ)}{2} ,$$

and the angle formed by two directions i, j is obtained by

$$\bar{\alpha}_{ij} = l_j - l_i .$$

Finally, from the n_R determinations made we obtain the mean value

$$\alpha_{ij}^{0} = \frac{\sum_{m=1}^{n_R} \bar{\alpha}_{ij}^{m}}{n_R}, \tag{4.1}$$

and the mean square error

$$e = \sqrt{\frac{\sum_{m=1}^{n_R} (\bar{\alpha}_{ij}^{m} - \alpha_{ij}^{0})^2}{n_R - 1}}, \tag{4.2}$$

As for every station point observations corresponds to more angles than those theoretically required to determine the orientation of the directions of the horizont round, we establish a station adjustment for every survey point which permits us to obtain unique adjusted values of the angles of the horizon round.

The adjustment has been carried out by means of the least-squares model of observations taking as unknown parameters the corrections δl_i to a set of provisional directions. The corresponding mathematical model can be written as

$$A \underline{x} - \underline{t} = \underline{v} \tag{4.3}$$

$\underline{x} = (\delta l_1, \ldots \delta l_n)^T$ being the vector of unknown parameters,

$\underline{t} = (t_1, \ldots t_m)^T$ the vector of observation constants,

$\underline{v} = (v_1, \ldots v_m)^T$ the vector of residual errors, and

$A = (a_{ks})$ the design matrix, with

$$a_{ks} = 0 \text{ for } s = 1, \ldots n, \; s \neq i, \; s \neq j,$$

$$a_{ki} = -1$$

$$a_{kj} = 1.$$

The desing matrix A has a deficiency of rank equal to 1, coming from the indetermination in the orientation. To get the best linnear mininum bias estimation, we take the mininum norm least-squares solution using inner constrains, easily applicable to our problem. Therefore, we add to our system the relation (Blaha, 1971).

$$D\,\underline{x} = 0, \tag{4.4}$$

verifying

$$A\,D^T = 0,$$

(the supercrit T means transposition).

In this case relation (4.4) reduces to

$$(\overbrace{1, \ldots, 1}^{n})\,\underline{x} = 0 \tag{4.5}$$

To solve both set of equations (4.3) and (4.5) we use the elimination constrains procedure, the vector of unkown parameters is decomposed in two blocks (Perelmuter, 1979)

$$\underline{x} = \begin{vmatrix} x_1 \\ \underline{x}_2 \end{vmatrix},$$

$\underline{x}_2$, with n-1 independent unknowns and x_1 with the other one. Consequently the other vectors and matrix are also decomposed, and the solution is given by

$$\begin{aligned} \hat{\underline{x}}_2 &= (\bar{A}^T P \bar{A})^{-1}\,\bar{A}^T P \bar{\underline{t}}\,, \\ \hat{x}_1 &= -D_1^{-1} D_2\,\hat{\underline{x}}_2\,, \end{aligned} \tag{4.6}$$

where

$$\bar{A} = A_2 - A_1 D_2,$$

$$\bar{\underline{t}} = \underline{t} \qquad ,$$

with P being the weight matrix of observations.

The residuals are obtained as

$$\hat{\underline{v}} = (A_2 + A_2\,D_2^T\,D_2)\,\hat{\underline{x}}_2 - \underline{t}\,.$$

The variance of unit weight is

$$\hat{\sigma}_0^2 = \frac{\hat{\underline{v}}^T P\,\hat{\underline{v}}}{m-n+1}\,,$$

and the covariance matrix being

$$\Sigma_{\hat{x}_1\hat{x}_1} = \hat{\sigma}_0^2\, Q_{\hat{x}_1\hat{x}_1} = D_1^{-1} D_2 (\bar{A}^T\ P\ \bar{A})^{-1} D_2^T (D_1^{-1})^T,$$

$$\Sigma_{\hat{x}_1\hat{\underline{x}}_2} = \hat{\sigma}_0^2\, Q_{\hat{x}_1\hat{\underline{x}}_2} = -\ D_1^{-1} D_2 (\bar{A}^T\ P\ \bar{A})^{-1}\ ,$$

$$\Sigma_{\hat{\underline{x}}_2\hat{x}_1} = \hat{\sigma}_0^2\, Q_{\hat{\underline{x}}_2\hat{x}_1} = -(\bar{A}^T\ P\ \bar{A})^{-1} D_2^T (D_1^{-1})^T,$$

$$\Sigma_{\hat{\underline{x}}_2\hat{\underline{x}}_2} = \hat{\sigma}_0^2\, Q_{\hat{\underline{x}}_2\hat{\underline{x}}_2} = (\bar{A}^T\ \ P\ \bar{A}\ )^{-1}\ ,$$

$$\Sigma_{\hat{\underline{v}}\hat{\underline{v}}} = \hat{\sigma}_0^2\, Q_{\hat{\underline{v}}\hat{\underline{v}}} = P^{-1} -\ \ \bar{A}\ (\bar{A}\ ^T P\ \bar{A})^{-1} \bar{A}^T.$$

4.1. Statistical analysis of azimutal angles

After that the station adjustment is accomplished, the following tests are applied in order to assesing the quality of data. First, the validity of the variance of unit weight is tested with the F-distribution. If

$$y = \max\left(\frac{\hat{\sigma}_0^2}{\sigma_0^2}\ ,\ \frac{\sigma_0^2}{\hat{\sigma}_0^2}\right) > F_{m-n+1,\infty\ ,\alpha}\quad ,$$

then the hypothesis H_0: $\hat{\sigma}_0^2 = \sigma_0^2$ is rejected. If H_0 is not accepted further investigations are made to locate possible outliers. Each observation is tested with the Pope-Test. We define (Pope, 1976)

$$y = \frac{\hat{v}_i}{\sigma_{\hat{v}_i}}\quad ,$$

$\sigma_{\hat{v}_i}$ being the variance of $\hat{v}_i$ taken off from $\Sigma_{\hat{v}\hat{v}}$. If

$$|y| \geq \tau_{m-n+1,\ \alpha/2}\ ,$$

then, with τ being the Tau-distribution, the i-observation is rejected and have to be investigated for possible errors. Due to correlation a single observation error mostly cause more than one rejection, the largest y points to the most suspected observation.

Two FORTRAN programs have been developed to perform the correction for systematic effects and the statistical analysis of data. AEDIS for the treatment of distance measurements and zenithal angles observations, and AEANG for horizontal angles including the station adjustment.

Data available after pre-processing are shown in the table 1.

TABLE 1

DATA / CAMPAIGN	DISTANCES AND ZENITHALS	AZIMUTHAL ANGLES	TRIANGLES
1984	70	69	21
1985	82	83	30
1986	82	202	33
1987	81	283	33

Data after pre-processing for the different campaigns

5. RESULTS OF THE COMPARISON BETWEEN CAMPAIGNS

In order to determine possible temporal variations we have made use of two criteria of comparison : a deterministic method and a statistical method. First, we apply a difference operator in all combinations, then, and independly, assuming that the four campaigns data are normally distributed, a statistical analysis using hypothesis testing is carried out.

5.1. Numerical results

A direct comparison of different observations campaigns by means of two FORTRAN V programs have been performed. ADORDIS for the treatment of distances and zenithal angles and ADORANG for the horizontal angles. Numerical differences between the four campaigns are obtained applying the operator D, defined by the matrix.

$$D = \begin{vmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & 0 & 1 \end{vmatrix} ,$$

to the vector, $\underline{l}$ of the different campaigns results

$$\underline{l}^t = [l_1, l_2, l_3, l_4] ,$$

so, the observed differences, $\underline{t}$, are given by

$$\underline{t} = D\,\underline{l} , \qquad (5.1)$$

with the covariance matrix

$$C_{tt} = D\, C_{ll}\, D^T ,$$

C_{ll}, being the covariance matrix of the observations vector $\underline{l}$.

For each two station points numerical results are obtained as shows Table 2

TABLE 2

VISUAL: 6- 8

(d)	(D)	(m_d)	(d')	(D)	($m_{d'}$)	(d-d')	(d_m)	(D)	(m_{dm})
947041.2	14.8	4.7	947052.3	-4.1	1.1	-11.1	947046.7	5.4	4.8
947056.0	-7.0	.7	947048.2	-10.7	.4	7.8	947052.1	-8.9	.8
947049.0	.7	.6	947037.5	13.7	1.5	11.5	947043.3	7.2	1.6
947049.7	7.8	.3	947051.2	-14.8	.6	-1.5	947050.5	-3.5	.7
	-6.3			3.0				-1.7	
	8.5			-1.1				3.7	
947118.7	9.2	4.7	947129.5	-7.2	1.1	-10.8	947124.1	1.0	4.8
947127.9	-3.6	.7	947122.3	-5.3	.4	5.6	947125.1	-4.5	.8
947124.3	-1.9	.6	947117.0	11.7	1.5	7.3	947120.6	4.9	1.6
947122.4	5.6	.3	947128.7	-12.5	.6	-6.3	947125.5	-3.5	.7
	-5.5			6.4				.5	
	3.7			-.8				1.5	

947202.9	9.2	4.7	947213.7	-7.2	1.1	-10.8	947208.3	1.0	4.8
947212.1	-6.0	.7	947206.5	-7.7	.4	5.6	947209.3	-6.9	.8
947206.1	-1.8	.6	947198.8	11.8	1.5	7.3	947202.5	5.0	1.6
947204.3	3.2	.3	947210.6	-14.9	.6	-6.3	947207.5	-5.9	.7
	-7.8			4.1				-1.9	

(z)	(D)	(m_h)	(z')	(D)	($m_{z'}$)	(c)	(θ)	(c-θ)
89 10 1.7	1.3	.9	90 50 28.7	5.2	1.3	30.4	30.7	-.3
89 10 3.0	-5.9	1.1	90 50 33.9	-6.6	.5	36.9	30.7	6.3
89 9 57.1	-1.8	1.0	90 50 27.3	.8	.5	24.5	30.7	-6.2
89 9 55.3	-4.6	.7	90 50 28.1	-1.3	.6	23.5	30.7	-7.2
	-7.7			-5.8				
	-6.4			-.5				

(h)	(D)	(m_h)	(h')	(D)	($m_{h'}$)	(h+h')	(h_m)	(D)	(m_{hm})
13838.6	-6.0	4.1	-13837.5	-23.9	6.0	1.1	13838.0	8.9	7.3
13832.6	27.0	5.1	-13861.3	30.2	2.3	-28.7	13847.0	-1.6	5.5
13859.6	8.2	4.6	-13831.1	-3.8	2.3	28.5	13845.4	6.0	5.1
13867.9	21.0	3.2	-13834.9	6.3	2.8	32.9	13851.4	7.4	4.2
	35.3			26.4				4.4	
	29.3			2.5				13.4	

NUMERICAL RESULTS OF COMPARISON BETWEEN CAMPAIGNS

Block I, distances measured. Block II, distances corrected for meteorological conditions. Block III, distances corrected with the distancemeter constants. Block IV, zenithal angles. Block V, height differences.

Explanation of symbols: D = differences between campaigns, d = distance, m_d = m.s.e. of the distance, d'= reciprocal distance, d_m = mean distance, m_{dm} = m.s.e. of the mean distance, Z = zenithal distance, m_z = m.s.e. of the series of zenithals for the line sight, Z'= reciprocal zenithal distance, $m_{z'}$= m.s.e. of, the reciprocal zenital distance, C = misclosure for zenithals, θ = sloping for verticals, h = height difference, m_h = m.s.e. of height difference, h' = reciprocal height difference, $m_{h'}$ = m.s.e of the reciprocal height difference, h_m = mean value for the height difference, m_{hm} = m.s.e. of the mean height difference.

From the analysis of distances comparisons made we get some interesting conclusions. First, results of blocks I and II show the atmospheric effects. Variations for meteorological conditions between campaigns remain globally constants (observations have been made in the same year epoch), but diurnal variations are remarkable being atmospheric corrections essential and pointing out that empirical determinations of the refraction coefficient will be necessary to obtain a local atmospheric model if more accurate instruments would be used.

Greatest differences of distances between campaings (Block III) are about 9 mm, being in mean 5 mm, no systematic effects appear which enables us to detect some deformation. Obviosly, a first comparison of results with appropiate criteria can be useful as an external control of observations theirself.

Block IV of table 2 corresponds to zenithal angles. As direct and reciprocal zenithal angles, corrected but no simultaneous, are individually compared, differences of block IV can not be concludings, although they are acceptables. Results are more important as an external control to detect possible blunders.

The height differences calculated from distances and zenithal angles are in block V. Differences are about 25 mm and have been analyzed to detect some broad-scale vertical deformation in a survey point, but it must be say that vertical deformations in the Caldera of Teide are studied with the high precision levelling data of an independent network.

Results for azimuthal angles are shown in Table 3

TABLE 3

(A)	(D)	(m_A)	(B)	(D)	(m_B)	(C)
ANGLE : 6- 2- 8						
45 36 46.2	6.6	1.5	45 36 48.7	.1	4.2	-9.7
45 36 52.7	-3.0	2.8	45 36 48.7	.0	4.0	8.0
45 36 49.8	-1.8	1.4	45 36 48.7	-.8	1.2	2.2
45 36 48.0	3.6	1.4	45 36 48.0	.1	.2	1.0
	-4.7			-.8		
	1.8			-.7		
ANGLE : 3- 2- 6						
48 23 50.4	1.6	1.5	48 23 52.9	-2.0	4.3	-9.7
48 23 51.9	-2.8	1.4	48 23 50.9	-.1	2.7	8.0
48 23 49.1	2.5	1.4	48 23 50.8	1.1	1.3	2.2
48 23 51.6	-1.2	1.4	48 23 51.9	-2.1	.2	1.0
	-.3			.9		
	1.3			-1.0		
. . .	. . .	. . .	. . .	. . .	. . .	. . .

Explanation of symbols

A : horizontal angle observed

m_A : m.s.e.of the horizontal angle

D : differences between campaigns

B : horizontal angle after the station adjustment

m_B : m.s.e. for the station adjustment

C : misclosure for horizon round of the survey point

After station adjustment horizontal angles differences are on average about 1.5". Higher values have been detected as originated in bad conditions, and therefore no deformations have been found.

For the analysis of horizontal angles, misclosure of network triangles have been calculated. As angles of a triangle have been observed in an independent horizont round we have another important external control of observations.

5.2. Statistical analysis

In order to determine with realibility criteria significant variations in the geodetic network for detecting possible crustal deformations we have made use of hypothesis testing for the comparison of different observation campaigns

As results of measurements made in different campaigns we obtain the following values of a same observable l an of their variance s^2

Campaign	$i, \quad i = 1, 2, \ldots, k,$
Observable value	$l_i = \sum_{j=1}^{n_i} l_{ij}/n_i \ ,$
Variance	$s_i^2 = \sum_{j=1}^{n_i} (l_{ij} - l_i)^2 \ / \ (n_i - 1) \ ,$
Sample size	$n_i .$

Observables can be any of the geodetic elements considered before (distances, angles, height differences, etc).

We assume that these values came from a normal population with expected value μ and standard deviation σ, $N(\mu, \sigma^2)$

Tests for the equality of variances

We must investigate if results of observations made in different epochs are samples from Normal distributions with the same variance and therefore they can be compared with reliability criteria.

Let us consider $(l_i, \ s_i^2)$, $i = 1, \ldots k$, samples from normal distributions, $N(\mu_i, \ \sigma_i^2)$, supposed independents.

To test the equality of all variances we applied the Bartlett test. The null hypothesis H_0 is that

$$H_0 : \sigma_1^2 = \sigma_2^2 = \ldots\ldots = \sigma_k^2 \ ,$$

we define

$$y = \frac{n_0 \log s_0^2 - \sum_{i=1}^{k} (n_i - 1) \log s_i^2}{1 + \frac{\sum_{i=1}^{k} \frac{1}{n_i} - \frac{1}{n}}{3(k-1)}},$$

where

$$n_0 = \sum_{i=1}^{k} (n_i - 1),$$

$$n = \sum_{i=1}^{k} n_i ,$$

$$s_0^2 = \frac{\sum_{i=1}^{k} (n_i - 1)\, s_i^2}{n_0},$$

then, at a significance level of α, H_0 is rejected if

$$y > \chi^2_{k-1,\alpha},$$

where, as usual, $\chi^2_{k-1,\alpha}$ refers to the chi-squared distribution on k-1 degrees of freedom.

If H_0 is rejected, then every two variances are checked in several repeated test. Results of two campaigns $(1_i,\ s_i^2)$ and $(1_j, s_j^2)$ are supposed samples from normal distributions $N(\mu_i,\ \sigma_i^2)$ and $N(\mu_j,\ \sigma_j^2)$.

Quantities

$$\frac{(n_i - 1)\, s_i^2}{\sigma} \quad \text{and} \quad \frac{(n_j - 1)\, s_j^2}{\sigma},$$

are statistically independents and distributed as chi-squared on n_i-1 and n_j-1 degrees of freedom, then

$$y = \max (s_i^2,\ s_j^2) \ /\ (\ \min (s_i^2\ ,\ s_j^2)\ ,$$

has the F-distribution on n_i-1 and n_j-1 degrees of freedom.

The assumption

$$H_0 : \sigma_i^2 = \sigma_j^2 = \sigma\ ,$$

is rejected if

$$y > F_{n_i-1,n_j-1,\alpha/2} \ .$$

Bartlett test can be accepted even the test on the two variances comparison was not due to some systematic cause.

Test for the equality of several means

To detect significant differences between the observations made at different epochs, the following test on the equality of means have been used.

Let (l_i, s_i^2) and (l_j, s_j^2) be samples from Normal distributions, $N(\mu_i, \sigma_i^2)$ and $N(\mu_j, \sigma_j^2)$. If variances are assumed to be equal, $\sigma_i^2 = \sigma_j^2$, as they have been tested before with the Bartlett Test, the quantity.

$$y = \frac{(l_i - l_j) - (\mu_i - \mu_j)}{\sqrt{(n_i-1)\, s_i^2 + (n_j-1)\, s_j^2}} \sqrt{\frac{n_i n_j (n_i + n_j - 2)}{n_i + n_j}} \ ,$$

has a Student t-distribution on n_i+n_j-2 degrees of freedom.

Then, the null hypothesis

$$H_0: \ \mu_i = \mu_j \ ,$$

is rejected, if

$$y = \frac{l_i - l_j}{\sqrt{(n_i - 1)\, s_i^2 + (n_j - 1)\, s_j^2}} \sqrt{\frac{n_i n_j (n_i + n_j - 2)}{n_i + n_j}} \ ,$$

verify

$$|y| > t_{n_i+n_j-2,\alpha/2} \ .$$

If H_0 is rejected we can conclude that the geodetic network is deformed.

If all campaigns are supposed to have the same precision, $s_i = s_j = s$, statistic y reduces to

$$y = \frac{l_i - l_j}{s} \sqrt{\frac{n_i n_j}{n_i + n_j}} .$$

If variances are not equal $\sigma_i^2 = \sigma_j^2$, we would use the Fisher-Behrans statistic (Lloyd, 1984)

$$y = \frac{l_j - l_j - (\mu_i - \mu_j)}{\sqrt{\frac{s_i^2}{n_i} + \frac{s_j^2}{n_j}}} ,$$

which are asymptotically distribuyted as a Student t on

$$\left(\frac{s_i}{n_i} + \frac{s_j}{n_j} \right) / \left(\frac{s_i^4}{n_i(n_i + 1)} + \frac{s_j^4}{n_j(n_j+1)} \right) - 2 ,$$

degrees of freedom.

Two FORTRAN programs, AEORDIS and AEORANG, have been developed to perform the statistical analysis of the different observation campaigns. Results are obtained as show Table 4.

TABLE 4

VISUAL 6 - 8

DISTANCES		M.S.E.	(e)	
947208.3	(1-2) ACEPTED	4.8	9.0	(1-2) ACEPTED
947209.3	(2-3) ACEPTED	.8	7.1	(2-3) ACEPTED
947202.5	(3-4) ACEPTED	1.6	7.4	(3-4) ACEPTED
947207.5	(1-3) ACEPTED	.7	7.0	(1-3) ACEPTED
	(2-4) ACEPTED			(2-4) ACEPTED
	(1-4) ACEPTED			(1-4) ACEPTED

BARTLETT TEST ACEPTED

ZENITHALS		M.S.E.	(e)	
89 10 1.7	(1-2) ACEPTED	.9	2.5	(1-2) ACEPTED
89 10 3.0	(2-3) REJECTED	1.1	3.0	(2-3) ACEPTED
89 9 57.1	(3-4) ACEPTED	1.0	2.8	(3-4) ACEPTED
89 9 55.3	(1-3) ACEPTED	.7	2.0	(1-3) ACEPTED
	(2-4) REJECTED			(2-4) ACEPTED
	(1-4) REJECTED			(1-4) ACEPTED

BARTLETT TEST ACEPTED

HEIGHT DIFFERENCES		M.S.E. (e)		
13838.0	(1-2) ACEPTED	7.3	23.	(1-2) ACEPTED
13847.0	(2-3) ACEPTED	5.5	19.	(2-3) ACEPTED
13845.4	(3-4) ACEPTED	5.1	18.	(3-4) ACEPTED
13851.4	(1-3) ACEPTED	4.2	16.	(1-3) ACEPTED
	(2-4) ACEPTED			(2-4) ACEPTED
	(1-4) ACEPTED			(1-4) ACEPTED

BARTLLET TEST ACEPTED

STATISTICAL ANALYSIS OF DIFFERENT OBSERVATIONS CAMPAIGNS

(e) m.s.e. modulated to the theoretical accuracy range

6. CONCLUSIONS

The direct comparison of distance measurements and angles observations in a geodetic network repeatedly observed is an useful method for detecting crustal deformations as well an external control of observations. By other hand, the statistical analysis enables us to determine with realibility criteria variations in the geodetic network.

Differences obtained in the comparison between the four observation campaigns in the Caldera of Teide are of the same order than instrumental precision limits. Therefore it can not be concluded that significant variations have originated.

Results discussed here correspond only to the analysis made before adjustment. Network adjustment and linear hypothesis testing of the results of adjustment and the mathematical model used are treated in other paper.

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