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An evaluation of the uncertainty associated with the measurement of the geomagnetic field with a D/I fluxgate theodolite

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Abstract
An analysis is presented of the sources of uncertainty in measuring the angular elements of the geomagnetic field with the D/I fluxgate theodolite on the basis of the Guide to the Expression of Uncertainty in Measurement published by the International Organization for Standardization (ISO). Along with the uncertainty associated with random effects, the habitual measurement procedure evidences the existence of systematic effects that are often ignored in the daily observatory practice. Special emphasis has been put in the development of a plausible theoretical scheme to explain the origin of such effects, and a series of procedures are proposed to find out their actual sources, as well as several recommendations with the final aim to improve the accuracy of the observations. Other effects, which seem to not strictly suit the traditional classification into either systematic or random, are also analysed. Some of the results obtained have been applied to the absolute instruments in use at the Livingston Island Geomagnetic Observatory and at certain European observatories. Systematic contributions on uncertainty are difficult to outline in a general case, since they depend on each particular instrument. On the other hand, an accurate estimation of the uncertainty associated to random effects has been obtained, concluding that their magnitude does not generally exceed 0.1 arc-minutes for an experienced observer, for either Declination or Inclination.

Keywords: uncertainty, absolute measurement, fluxgate magnetometer

1. Introduction

Geomagnetic observatories are installations where the geomagnetic field vector has been observed during an extended period of time, providing the scientific community and users with accurate data which will be used for geophysical studies and for producing geomagnetic field models (e.g. Macmillan and Maus 2005).

The absolute measurements of the angular geomagnetic field elements (Declination, D, and Inclination, I), to which any variation data is referred, are at present universally being made with the D/I fluxgate theodolite (from now on DI-flux, figure 1). There are several methods used to measure the D and I angles with the DI-flux (Jankowski and Sucksdorff 1996, or Kerridge 1988). Although a similar analysis could be applied to these methods, the one employed in this study is the normal one, based on determining the plane perpendicular to the magnetic field vector through the search for zeros on the sensor electronics display unit (see, for example, Jankowski and Sucksdorff 1996, pp 88, 89 and 95). For both Declination and Inclination four null readings are made: $\varphi_{E\uparrow}$, $\varphi_{W\uparrow}$, $\varphi_{E\downarrow}$, $\varphi_{W\downarrow}$, $\theta_{N\downarrow}$, $\theta_{S\uparrow}$, $\theta_{N\uparrow}$, $\theta_{S\downarrow}$, i.e., combinations of fluxgate sensor above (↑) or below (↓) the telescope, and pointing towards

East or West (for D), North or South (for I). The angle φ is related to horizontal readings, while θ to vertical ones.

But the absolute condition of the DI-flux does not exempt its measurements from being affected by limited accuracy. The quantities D and I do not depend on environmental circumstances other than those strictly related to the geomagnetic field; in consequence, the measurement procedure has to correct for those possible external circumstances that potentially affect the measurements. These circumstances are referred in the Guide to the Expression of Uncertainty in Measurement (ISO 1993) as to *influence quantities.* The analysis of the sources of uncertainty presented following is made on the basis of this guide, which is commonly known as GUM.

From the usual measuring procedure (exposed below) an estimation of those influence quantities, traditionally termed as *errors* (namely collimation errors and zero-field offset), is obtained, being most of them typified in the literature and well-known (Trigg 1970, Bitterly *et al* 1984, Kring Lauridsen 1985, Kerridge 1984, 1988). But the analysis of long series of measurements with the DI-flux reveals the existence of further influence quantities, whose origin has been suggested by some authors (Kring Lauridsen 1985, or Rasson 2005), although a supporting theory remains to be established. In this sense, one of the aims of the present work is to extend the existing theoretical scheme, proposing a series of observable quantities and procedures to scrutinize their actual sources in order to either correct them, eliminate them, or at least estimate their associated effect for a complete evaluation of uncertainty in measurement of D and I. Following the behaviour of these quantities provides valuable information on possible instrument defects, incorrect procedures, or validity of certain individual observations. The efficiency of the proposal will be illustrated with results obtained from the Livingston Island Geomagnetic Observatory (LIV, 62º 40' S, 60º 24' W), which is operated by the authors, but also from Ebre (where the authors are based on) and some other European observatories.

On the other hand, an analysis of the random variables affecting the mentioned measurement procedure and an estimation of its uncertainty has been made following the precepts of the GUM.

2. Systematic effects on the measurements with the DI-flux

Although a separation into systematic and random contributions to the uncertainty is not recommended in the GUM, it has been considered opportune to make such distinction because of the difficulty of estimating the systematic contribution, as it will be argued below.

If measurements with the DI-flux were exempted from errors of any type, any single horizontal reading would suffice to determine D, while any vertical one to determine I. The other readings would then be obtained in an obvious manner, adding or subtracting either 180º or 360º; but in a real case these values are obtained approximately. This could be due to the existence of random effects influencing the measurements, but when a "long" series of data is analysed, systematic deviations from the ideal situation are observed. Traditionally, these deviations have been attributed to lack of collimation between the optical axis and the fluxgate sensor and to the zero-field offset (Stuart 1972, or Forbes 1987). However, it is noteworthy that other causes could produce the same effect on data, although the stated ones are, in fact, the most likely to happen in a general case. In fact, the measurement procedure assumes the existence of additional quantities influencing the measurements other than those strictly related to the magnetic field.

The amount of available information in the measuring process depends on the number of readings carried out, each one being associated to one equation. In the case dealt with here, two equations are referred to the reference mark readings: $\varphi_{M\uparrow}$, $\varphi_{M\downarrow}$, with two associated unknowns: H_M (azimuth of the mark in the theodolite reference frame) and δ_{Ret} (the azimuth "collimation error" of the cross-hairs marked on the telescope reticle). The abovementioned eight readings $\varphi_{E\uparrow}$, $\varphi_{W\uparrow}$, $\varphi_{E\downarrow}$, $\varphi_{W\downarrow}$, $\theta_{N\downarrow}$, $\theta_{S\uparrow}$, $\theta_{N\uparrow}$ and $\theta_{S\downarrow}$ are referred to the magnetic field, being five the number of unknowns: D and I are the required angles, while δ_{Fl}, ε_{Fl} (the so-called azimuth and dip collimation errors of the fluxgate sensor) and T_o (the zero-field offset) account for the abovementioned influence quantities assumed in the measuring process. This gives a total of ten equations in seven unknowns; the first two are commonly solved independently, being the solution unique. On the other hand, an over-determined set of eight magnetic-field-related equations ((3) to (10)) in five unknowns (D, I, δ_{Fl}, ε_{Fl} and T_o) is obtained (the footnotes apply here and in the rest of the text):

$$\varphi_{M\uparrow} = H_M - \frac{\delta_{Ret}}{\sin\theta_{M\uparrow}} \quad ^{1} \tag{1}$$

$$\varphi_{M\downarrow} = H_M + \frac{\delta_{Ret}}{\sin\theta_{M\uparrow}} \mp 180^{o} \quad ^{2} \tag{2}$$

$$\varphi_{E\uparrow} = 90^{o} + D - \left(AM_{GN} - H_M\right) - \delta_{Fl} + \varepsilon_{Fl}\tan I - \frac{180^{o}}{\pi}\frac{T_o}{H} + n360^{o} \quad ^{3} \tag{3}$$

$$\varphi_{W\uparrow} = 90^{o} + D - \left(AM_{GN} - H_M\right) - \delta_{Fl} - \varepsilon_{Fl}\tan I + \frac{180^{o}}{\pi}\frac{T_o}{H} \pm 180^{o} + n360^{o} \quad ^{3,4} \tag{4}$$

$$\varphi_{E\downarrow} = 90^{o} + D - \left(AM_{GN} - H_M\right) + \delta_{Fl} - \varepsilon_{Fl}\tan I - \frac{180^{o}}{\pi}\frac{T_o}{H} \pm 180^{o} + n360^{o} \quad ^{3,4} \tag{5}$$

$$\varphi_{W\downarrow} = 90^{o} + D - \left(AM_{GN} - H_M\right) + \delta_{Fl} + \varepsilon_{Fl}\tan I + \frac{180^{o}}{\pi}\frac{T_o}{H} + n360^{o} \quad ^{3} \tag{6}$$

$$\theta_{N\downarrow} = 270^{o} - I - \varepsilon_{Fl} + \frac{180^{o}}{\pi}\frac{T_o}{F} \pm 90^{o} \quad ^{3,5} \tag{7}$$

$$\theta_{S\uparrow} = 90^{o} - I - \varepsilon_{Fl} - \frac{180^{o}}{\pi}\frac{T_o}{F} \pm 90^{o} \quad ^{3,5} \tag{8}$$

$$\theta_{N\uparrow} = 90^{o} + I - \varepsilon_{Fl} - \frac{180^{o}}{\pi}\frac{T_o}{F} \mp 90^{o} \quad ^{3,5} \tag{9}$$

$$\theta_{S\downarrow} = 270^{o} + I - \varepsilon_{Fl} + \frac{180^{o}}{\pi}\frac{T_o}{F} \mp 90^{o} \quad ^{3,5}, \tag{10}$$

where $\theta_{M\uparrow}$ stands for the "sensor up" vertical reading of the fixed mark and AM_{GN} for its azimuth (from the Geographic North). The integer n situates D in the desired interval, i.e., (-180º, 180º] or [0º, 360º), while F and H are approximate values of the total and horizontal components of the geomagnetic field. As well, I as appearing in equations (3) to (6) is an approximate value of the magnetic Inclination.

There therefore exist multiple solutions for a given unknown with different appropriate combinations of five equations; but if the reported unknowns (excepting D and I) were the

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[1] All the angles in the formulas are expressed in degrees.
[2] The upper sign is valid when $\varphi_{M\uparrow} > \varphi_{M\downarrow}$; the lower otherwise.
[3] Change T_o sign if the sensor gives a positive reading when the telescope is directed towards the north.
[4] The upper sign is valid when $\varphi_{E\uparrow}$ belongs to the interval (0º, 180º); the lower otherwise.
[5] The upper sign is valid for the Northern hemisphere, while the lower for the Southern.

only influence quantities involved with the measurements, it can be concluded that such combinations should coincide in their results. Obviously, this may not be the case due to the existence of random effects and their inherent associated uncertainty. If this hypothesis is made, an optimum estimate of the *measurands* (see GUM) D and I would be obtained from the application of the well-known expressions (the superscripts in the following equations remit to the footnotes corresponding to the first time the same superscript appears):

$$D = \frac{\varphi_{E\uparrow} + \varphi_{W\uparrow} + \varphi_{E\downarrow} + \varphi_{W\downarrow}}{4} + \left(AM_{GN} - H_M\right) - 90^\circ \mp 90^\circ - n360^\circ \;\; ^{4} \tag{11}$$

$$I = \frac{-\theta_{N\downarrow} - \theta_{S\uparrow} + \theta_{N\uparrow} + \theta_{S\downarrow}}{4} \pm 90^\circ \;\; ^{5} \tag{12}$$

Following the precepts of the GUM, the left hand side of the above equations are *output quantities*, whereas the right hand side expresses its functional relationship f with the *input quantities*: $\varphi_{E\uparrow}$, $\varphi_{W\uparrow}$, $\varphi_{E\downarrow}$, $\varphi_{W\downarrow}$, AM_{GN}, H_M (for D), $\theta_{N\downarrow}$, $\theta_{S\uparrow}$, $\theta_{N\uparrow}$, $\theta_{S\downarrow}$ (for I). The contribution of the input quantities to the uncertainty of the measurement is analysed in section 3. In reality, a complete definition of the measurands should take into account their time-dependence, and therefore D and I in the above equations should contain information about the point in time the measurements were actually taken, i.e., D as appearing in equation (3) should be replaced by $D(t_{E\uparrow})$, etc. In consequence, D and I as appearing in equations (11) and (12) should also be replaced by the arithmetic means $\overline{D}$ and $\overline{I}$ of $D(t_i)$ and $I(t_i)$, respectively. Equations (11) and (12) are valid approximations only if magnetic field variations are insignificant during the time the whole process lasts, being their left hand side the values assigned to the measurands at the time the absolute measurement was made (the time assigned to D is then typically the mean of the four t_i corresponding to Declination, and similarly for Inclination). Nevertheless, in general it is worth defining the quantities:

$$\begin{aligned}\overline{D} - \overline{D}_{\mathrm{var}} &= \frac{\varphi_{E\uparrow} + \varphi_{W\uparrow} + \varphi_{E\downarrow} + \varphi_{W\downarrow}}{4} - \overline{D}_{\mathrm{var}} + \left(AM_{GN} - H_M\right) - 90^\circ \mp 90^\circ - n360^\circ = \\ &= \frac{\varphi'_{E\uparrow} + \varphi'_{W\uparrow} + \varphi'_{E\downarrow} + \varphi'_{W\downarrow}}{4} + \left(AM_{GN} - H_M\right) - 90^\circ \mp 90^\circ - n360^\circ \;\; ^{4}\end{aligned} \tag{13}$$

$$\overline{I} - \overline{I}_{\mathrm{var}} = \frac{-\theta_{N\downarrow} - \theta_{S\uparrow} + \theta_{N\uparrow} + \theta_{S\downarrow}}{4} - \overline{I}_{\mathrm{var}} \pm 90^\circ = \frac{-\theta'_{N\downarrow} - \theta'_{S\uparrow} + \theta'_{N\uparrow} + \theta'_{S\downarrow}}{4} \pm 90^\circ \;\; ^{5}, \tag{14}$$

where $\varphi'_{E\uparrow} \equiv \varphi_{E\uparrow} - D_{var}(t_{E\uparrow})$, $\varphi'_{W\uparrow} \equiv \varphi_{W\uparrow} - D_{var}(t_{W\uparrow})$, $\varphi'_{E\downarrow} \equiv \varphi_{E\downarrow} - D_{var}(t_{E\downarrow})$, $\varphi'_{W\downarrow} \equiv \varphi_{W\downarrow} - D_{var}(t_{W\downarrow})$, $\theta'_{N\downarrow} \equiv \theta_{N\downarrow} + I_{var}(t_{N\downarrow})$, $\theta'_{S\uparrow} \equiv \theta_{S\uparrow} + I_{var}(t_{S\uparrow})$, $\theta'_{N\uparrow} \equiv \theta_{N\uparrow} - I_{var}(t_{N\uparrow})$ and $\theta'_{S\downarrow} \equiv \theta_{S\downarrow} - I_{var}(t_{S\downarrow})$, being $D_{var}(t_{E\uparrow})$, $D_{var}(t_{W\uparrow})$, $D_{var}(t_{E\downarrow})$ and $D_{var}(t_{W\downarrow})$ ($I_{var}(t_{N\downarrow})$, $I_{var}(t_{S\uparrow})$, $I_{var}(t_{N\uparrow})$ and $I_{var}(t_{S\downarrow})$) the Declination (Inclination) values obtained from a variometer at the instants the E↑, W↑, E↓ and W↓ (N↓, S↑, N↑ and S↓) readings are produced, respectively, and $\overline{D}_{\mathrm{var}}$ ($\overline{I}_{\mathrm{var}}$) their mean value. The introduction of the variometric terms to both sides of equations (11) and (12) permits defining the two new quantities in the left hand side of equations (13) and (14), corresponding to the observed baseline values for Declination and Inclination, with a greater *reproducibility* (see GUM), instead of the time-dependent quantities D and I.

Following with the hypothesis that the discrepancies observed among different linear combinations of equations (3) to (10) are attributable to random effects, the observed baseline values would be randomly distributed around the "true" baselines. But when the time evolution of the five unknowns obtained from these linear combinations is plotted, a systematic deviation between them is still often observed (figure 2), revealing the existence of further systematic effects arising from influence quantities that have not been taken into account in the measurement procedure.

2.1 Analysis of additional influence quantities of systematic effect

Kring Lauridsen (1985) suggested the existence of magnetic field gradients causing a systematic effect observed as a discrepancy between the mentioned linear combinations, which would be rendered by the intrinsic limitation of the DI-flux to situate the fluxgate sensor in the same physical position before and after the telescope is reversed. If a measurable gradient exists, we can assert that its source is close to the sensor location. Hence, let us now analyse the different possibilities related to a magnetized body (MB) in the sensor surrounding, causing on this element an extra field superimposed to the natural one, which will be generically denoted as $\mathbf{B_{MB}}$:

a) If this source has a natural origin, e.g., strong magnetization of the surface rocks, there is no reason to consider that the obtained values for D and I are incorrect, since in fact they correspond (assuming linearity of $\mathbf{B_{MB}}$) to the natural magnetic field at the axes cross (figure 1). In order to avoid such disturbing gradients, an alternative location for the DI-flux should be found.
b) If the gradient source lies in the pier on which the instrument is mounted or, in general, in any object fixed with respect to the absolute building, the (constant) vector $\mathbf{B_{MB}}$ causes artificial values being measured. We will call $\mathbf{B_{MS}}$ (the subscript stands for magnetization of the surrounding) the vector $\mathbf{B_{MB}}$ in this particular case.
c) If the origin is in the DI-flux itself, the disturbing effect depends on the location and nature of the magnetization source. Let us separate the theodolite in three parts: the mounting, the alidade and the telescope (figure 1).

If the gradient is due to a light theodolite mounting magnetization, the results are similar to those of case b), with the difference that the effect is now subject to a rotation of the mounting (i.e., $\mathbf{B_{MB}}$ rotates accordingly).

If the gradient is due to the alidade, the effect of $\mathbf{B_{MB}}$ depends on the type of magnetization. The magnetic field produced by both paramagnetic and diamagnetic materials is assumed to be insignificant at the position of the sensor. On the other hand, solving the general problem related to the effects of a ferromagnetic material, like a small piece of iron, is complex in a general case and would imply an excessive number of variables for our purposes; instead, let us evaluate the consequences on the DI-flux readings of a small, permanently magnetized ferromagnetic body (i.e., with permanent magnetization):

Let us call $\mathbf{B_{MA}}(\mathbf{r})$ (a new particular case of $\mathbf{B_{MB}}$) the disturbance field produced by a small magnetization (as compared with the natural magnetic field) of the alidade evaluated at the point $\mathbf{r}$ in a geographical reference frame centred at the axes cross, and estimate its effects at the different positions of the sensor, $\mathbf{r}_{E\uparrow}$, $\mathbf{r}_{W\uparrow}$, … Let us define $\mathbf{B_{MA}}(\mathbf{r}_{E\uparrow}) \equiv \mathbf{B_{MA}} + \boldsymbol{\delta}\mathbf{B_{MA}}$ and $\mathbf{B_{MA}}(\mathbf{r}_{W\downarrow}) \equiv \mathbf{B_{MA}} - \boldsymbol{\delta}\mathbf{B_{MA}}$ as the disturbing magnetic field at the sensor positions $E_\uparrow$ and $W_\downarrow$, respectively. It is easy to see from figure 3 that when turning the alidade about face in order to carry out the $W_\uparrow$ and $E_\downarrow$ readings, the vectors $\mathbf{B_{MA}} + \boldsymbol{\delta}\mathbf{B_{MA}}$ and $\mathbf{B_{MA}} - \boldsymbol{\delta}\mathbf{B_{MA}}$ (fixed in the alidade reference frame) are rotated 180º around the vertical axis, so that their horizontal projections are inverted. Accordingly, let us define $D_{MA} \pm \delta D_{MA}$ as the additional terms produced by the magnetization of the alidade in the $E_\uparrow$ (upper sign) and $W_\downarrow$ (lower sign) positions respectively, where D_{MA} is the magnetic field at some point (close to the axes cross) between the sensor positions up and down. With the above reasoning, it is easy to show that the additional Declination in the sensor positions $W_\uparrow$ and $E_\downarrow$ are $-(D_{MA} \pm \delta D_{MA})$ respectively, since only horizontal projections must be considered.

A similar argument can be applied to the change concerning I. Now, the disturbing field components along the direction of increasing I must be considered: we will call them $I_{MA}^{a} \pm \delta I_{MA}^{a}$ when the $N_\downarrow$ (lower sign) and $S_\uparrow$ (upper sign) readings are performed. Later on, the $N_\uparrow$ and $S_\downarrow$ readings are made, being the initial vectors again rotated 180º around the vertical axis. With the help of a diagram similar to that of figure 3 (from a "side view" and considering vertical projections of the concerned vectors) it can be easily seen that new components of these vectors along the direction of increasing I must be defined; let us call them $I_{MA}^{b} \pm \delta I_{MA}^{b}$, respectively.

If the gradient source lies inside the telescope, the effect of $\mathbf{B_{MB}}$ (which will now be referred as to $\mathbf{B_{MT}}$) depends again on the type of magnetization, being considerably complex in the general case of ferromagnetism. For simplicity, we will only consider the consequences of a permanently magnetized body. As the telescope is fixed with respect to the sensor, we can imagine that the magnetic field produced by the magnetization of the telescope, $\mathbf{B_{MT}}$ (see table 1 for a summary of terms), is frozen in this element, and its projections must be considered. It is easy to show that if D_{MT} is the additional Declination when the $E_\uparrow$ and $E_\downarrow$ readings are produced, $-D_{MT}$ is that corresponding to both $W_\uparrow$ and $W_\downarrow$. Regarding Inclination, if $-I_{MT}^{a}$ is the additional term due to the magnetized body when the $N_\downarrow$ reading is made, I_{MT}^{a} is that affecting $S_\uparrow$, whereas a new parameter has to be introduced for both $N_\uparrow$ (I_{MT}^{b}) and $S_\downarrow$ ($-I_{MT}^{b}$) readings.

2.2 An extended scheme proposal

With this parameterization, a straightforward scheme can be developed which would explain the discrepancies among the previous linear combinations (with the disadvantage of having introduced more unknowns than equations).

$$H_M = \frac{\varphi_{M\uparrow} + \varphi_{M\downarrow}}{2} \pm 90^{\circ} \quad (15)$$ [2]

$$\frac{\delta_{Ret}}{\sin\theta_{M\uparrow}} = -\left(\frac{\varphi_{M\uparrow} - \varphi_{M\downarrow}}{2} \mp 90^{\circ}\right) \equiv Az\,\mathrm{Re}t \quad (16)$$ [2]

$$\bar{D} + D_{MS} - \bar{D}_{\mathrm{var}} = \frac{\varphi'_{E\uparrow} + \varphi'_{W\uparrow} + \varphi'_{E\downarrow} + \varphi'_{W\downarrow}}{4} + \left(AM_{GN} - H_M\right) - 90^{\circ} \mp 90^{\circ} - n360^{\circ} \equiv$$ [4]
$$\equiv Abs - Var\_D \quad (17)$$

$$\delta_{Fl} - \delta D_{MS} = \frac{-\varphi'_{E\uparrow} - \varphi'_{W\uparrow} + \varphi'_{E\downarrow} + \varphi'_{W\downarrow}}{4} \equiv Az\_MS \quad (18)$$

$$\varepsilon_{Fl} + \frac{D_{MA}}{\tan I} = \frac{\varphi'_{E\uparrow} - \varphi'_{W\uparrow} - \varphi'_{E\downarrow} + \varphi'_{W\downarrow} \pm 360}{4\tan I} \equiv Dip\_MA\_D \quad (19)$$ [4]

$$\frac{180^{\circ}}{\pi}\left(\frac{T_o}{F}\right) - \left(\delta D_{MA} + D_{MT}\right)\cos I = \frac{-\varphi'_{E\uparrow} + \varphi'_{W\uparrow} - \varphi'_{E\downarrow} + \varphi'_{W\downarrow}}{4}\cos I \equiv Off\_MI\_D \quad (20)$$ [3]

$$\bar{I} + I_{MS} + \frac{I^{a}_{MA} + I^{b}_{MA}}{2} - \bar{I}_{\mathrm{var}} = \frac{-\theta'_{N\downarrow} - \theta'_{S\uparrow} + \theta'_{N\uparrow} + \theta'_{S\downarrow}}{4} \pm 90^{\circ} \equiv$$ [5]
$$\equiv Abs - Var\_I \quad (21)$$

$$\delta I_{MS} + \frac{\left(\delta I^{a}_{MA} + I^{a}_{MT}\right) + \left(\delta I^{b}_{MA} + I^{b}_{MT}\right)}{2} = \frac{\theta'_{N\downarrow} - \theta'_{S\uparrow} + \theta'_{N\uparrow} - \theta'_{S\downarrow}}{4} \equiv MS\_MI \quad (22)$$

$$\varepsilon_{Fl} + \frac{I^{a}_{MA} - I^{b}_{MA}}{2} = 180^{\circ} - \frac{\theta'_{N\downarrow} + \theta'_{S\uparrow} + \theta'_{N\uparrow} + \theta'_{S\downarrow}}{4} \equiv Dip\_MA\_I \quad (23)$$

$$\frac{180^{\circ}}{\pi}\left(\frac{T_o}{F}\right) \mp \left(\frac{\left(\delta I^{a}_{MA} + I^{a}_{MT}\right) - \left(\delta I^{b}_{MA} + I^{b}_{MT}\right)}{2}\right) = \mp\left(\frac{\theta'_{N\downarrow} - \theta'_{S\uparrow} - \theta'_{N\uparrow} + \theta'_{S\downarrow}}{4} - 90^{\circ}\right) \equiv$$ [3,5]
$$\equiv Off\_MI\_I\,, \quad (24)$$

where D_{MS} and I_{MS} are the contributions to D and I due to the effect of a magnetization in the instrument surrounding, either of natural or artificial origin, and evaluated at the point where the theodolite axes cross, while δD_{MS} and δI_{MS} are the corresponding semi-differences evaluated at the sensor positions "up" and "down".

This represents a scheme that extends previously reported formulas, e.g., those of Jankowski and Sucksdorff (1996). Additionally, as indicated by Matzka and Hansen (2006), one must be aware of the sign mistakes in equations (5.7) and (5.8) of Jankowski and Sucksdorff, which are the simplified versions of our equations (19) and (20), respectively. From the above equations, three different sets can actually be distinguished: equations (15) and (16) refer to the reference mark and can be considered belonging to an independent procedure; equations (17) to (20) refer to Declination, while equations (21) to (24) refer to Inclination measurements. As well, a set of additional observable quantities are derived:

AzRet is defined as the value of eq. (16) and it depends on the azimuth "collimation error" of the cross-hairs marked on the telescope reticle. A sudden variation of this quantity can occur when the collimation is manually corrected or the instrument changed, as each particular instrument has its own particular collimation. Any other variation is suspect of reflecting a further anomaly.

Abs-Var_D, eq (17), essentially corresponds to the difference between the absolute and variometric value of Declination. The interpolation of its experimental values gives rise to the Declination baseline, which is assumed to not vary significantly in a period of several days or weeks; hence, a non-random, short-term variation of this quantity would suggest an anomaly (e.g., any change in a potentially magnetized surrounding, implicit in the D_{MS} term) if measurements are always taken at the same point, since it is ideally transparent to a change of instrument and observer, but not to a change of DI-flux (or variometer) location.

The same applies to *Abs-Var_I*, defined as equation (21). The only difference comes from the undesirable term appearing in this equation relative to the magnetization of the alidade, which would make it theoretically sensible to a change of instrument.

Az_MS is the value of equation (18). This quantity can vary when the sensor - optical axis collimation is corrected, when the instrument is changed, or when the DI-flux is translated to a place with different external gradients (term with the MS subscript).

MS_MI is the ideally null value of equation (22), denoting the existence of general gradients, either from the surrounding or the instrument itself (alidade and/or telescope); in the first case, this quantity will be subject to a change of location, while in the second to an instrument change.

Dip_MA_D and *Dip_MA_I*, defined as equations (19) and (23) respectively, depend on the "collimation error" and on two different magnetization, ideally null functions of the alidade, which theoretically make them distinguishable. These quantities are sensible to a correction of the collimation and to an instrument change (since the term MA is particular of each instrument), but not to either a change of observer or location.

Similarly with *Off_MI_D* and *Off_MI_I*, defined as equations (20) and (24), respectively. In these cases, a variation is expected when the zero-field offset is manually corrected or when the temperature change is significant.

Finally, *MA* corresponds to the difference between the right hand side of equations (19) and (23), and *MI* to that between equations (20) and (24). They indicate hypothetical permanent magnetizations of the DI-flux, being zero their ideal value.

Of course, the unknown terms with subscripts MS and MA in equations (17) and (21) are ideally zero; if this is so, these equations reduce to the expressions (13) and (14), which permit to establish the baseline for the D and I magnetic elements.

In the general scheme described in this section we have not considered all types of magnetization and body sizes within the DI-flux potentially causing erroneous values of D and I. In this sense, it does not intend to be a comprehensive model and, in fact, different causes can be found to lead to a same effect (i.e., a same value for a given quantity). Instead, it offers a theoretical basis and a systematic procedure to alert of some critical aspects regarding the DI-flux behaviour, as small deficiencies of a particular instrument, of its operation, suitability of its location, etc., for further checking in case of systematic deviations from the indicated ideal values. It is important to remark that such information is directly obtained from the habitual measurement procedure.

In view of the above, it is not possible to estimate a correction for residual (i.e., other than those assumed in the measurement procedure itself) systematic effects in a general case, nor even concerning a particular instrument if further tests are not carried out. Nevertheless, as stated the expected value of the quantities *MA*, *MI* and *MS_MI* is zero in an ideal case; for this reason, these quantities will be referred as to *control* quantities or control observables. Since the input quantities of the control observables are the same as those of the baseline quantities (*Abs-Var_D* and *Abs-Var_I*), with a series of hypothesis it is possible to show that their values can themselves provide a coarse idea of the magnitude of the absolute value of the systematic error committed. Nevertheless, it is remarkable that the uncertainty associated to such estimations may be as great as their own value.

Table 2 presents the value of the control observables obtained from several reference European observatories for a given period of time, along with those at LIV.

3. Random effects

Other errors exist that unavoidably concern the individual measurements, but being processed within a long (ideally infinite) series of measurements are cancelled, as their expected value is zero. They correspond to the random errors, and their effects on measurements are referred as to random effects. Their recognized sources and associated uncertainties are discussed in the following paragraphs.

When the measurand is not measured directly, but instead it is determined from other quantities, it is viewed as an output quantity depending on different input quantities through a functional relationship *f*. Our output quantities are established as being the left hand side of equations (13) and (14), i.e., $\overline{D}-\overline{D}_{\mathrm{var}}$ and $\overline{I}-\overline{I}_{\mathrm{var}}$, while their right hand side expresses its functional dependence with the uncorrelated input quantities: φ_i, θ_i, $D_{var}(t_i)$, $I_{var}(t_i)$, AM_{GN}, H_M. The expressions of the *combined variance* (see GUM) $u_c^2(\overline{D}-\overline{D}_{\mathrm{var}})$ and $u_c^2(\overline{I}-\overline{I}_{\mathrm{var}})$ follow from equations (13) and (14):

$$u_c^2\left(\overline{D}-\overline{D}_{\mathrm{var}}\right)=\sum_{i=1}^{4}\left(\frac{\partial(\overline{D}-\overline{D}_{\mathrm{var}})}{\partial\varphi_i}u(\varphi_i)\right)^2+\sum_{i=1}^{4}\left(\frac{\partial(\overline{D}-\overline{D}_{\mathrm{var}})}{\partial D_{\mathrm{var}}(t_i)}u\left(D_{\mathrm{var}}(t_i)\right)\right)^2+ +\left(\frac{\partial(\overline{D}-\overline{D}_{\mathrm{var}})}{\partial H_M}u(H_M)\right)^2+\left(\frac{\partial(\overline{D}-\overline{D}_{\mathrm{var}})}{\partial AM_{GN}}u(AM_{GN})\right)^2= =\sum_{i=1}^{4}\left(\frac{u(\varphi_i)}{4}\right)^2+\sum_{i=1}^{4}\left(\frac{u\left(D_{\mathrm{var}}(t_i)\right)}{4}\right)^2+u^2(H_M)+u^2(AM_{GN}), \tag{25}$$

$$u_c^2\left(\overline{I}-\overline{I}_{\mathrm{var}}\right)=\sum_{i=1}^{4}\left(\frac{\partial\left(\overline{I}-\overline{I}_{\mathrm{var}}\right)}{\partial\theta_i}u(\theta_i)\right)^2+\sum_{i=1}^{4}\left(\frac{\partial\left(\overline{I}-\overline{I}_{\mathrm{var}}\right)}{\partial I_{\mathrm{var}}(t_i)}u\left(I_{\mathrm{var}}(t_i)\right)\right)^2= =\sum_{i=1}^{4}\left(\frac{u(\theta_i)}{4}\right)^2+\sum_{i=1}^{4}\left(\frac{u\left(I_{\mathrm{var}}(t_i)\right)}{4}\right)^2 \tag{26}$$

An evaluation of the standard uncertainty of the input quantities is required, which in turn may themselves be viewed as measurands or output quantities depending on other input quantities.

3.1 Instrument limited resolution

The way the angles are read with the DI-flux, or the resolution of the display unit are limitations imposed by the instrument itself and, in general, depend on the DI-flux model in use. An evaluation of the uncertainty $u(\varphi_i)$ of φ_i demands an analysis of its measurement process. Let us illustrate how to find $u(\varphi_{E\uparrow})$. Equation (3) was obtained assuming both perfect horizontality of the telescope and exact zero output from the fluxgate sensor. If this hypothesis is not completely fulfilled, and instead θ - 90º and T are the small values of the telescope angle departing from 90º and the fluxgate output signal, respectively, it can be shown that equation (3) is modified in the following way:

$$\varphi_{E\uparrow} = 90^{\circ} + D(t_{E\uparrow}) - (AM_{GN} - H_M) - \delta_{Fl} + (\varepsilon_{Fl} + \theta - 90^{\circ})\tan I + \\ + \frac{180^{\circ}}{\pi}\frac{T - T_o}{H} + n360^{\circ} \tag{27}$$

Before going on, it is worth distinguishing between a) the mere reading φ of the horizontal circle corresponding to a previously given direction of the telescope, and b) the horizontal reading, e.g., $\varphi_{E\uparrow}$, as a result of a measurement process implying the orientation of the telescope to a specified direction (E↑) and afterwards reading the horizontal circle. In the first case, only the uncertainty associated to the horizontal circle reading, $u(\varphi)$, must be taken into account, whilst in the second case, the uncertainties associated to the different individual processes leading to the telescope orientation also contribute to the final uncertainty, $u(\varphi_{E\uparrow})$. In fact, when the $E_\uparrow$ reading is measured with the Zeiss THEO 015B, several operations are actually made:

- Firstly, the telescope must be aligned horizontally, which implies centring the vertical circle reading (90º) between graduation marks. A type A (see GUM) evaluation has revealed an experimental standard deviation $s(\theta) \approx 2''$ (= $u(\theta)$) for this operation due to the theodolite's limited resolution.
- Next, a zero in the electronic console must be obtained by rotating the theodolite horizontally. The resolution of the unit in use at LIV (an ELSEC 810A) is 1 nT, for which a rectangular distribution has been assumed, implying a type B standard deviation $u(T) = (1\text{ nT})/\sqrt{12} \approx 0.3$ nT (see GUM) around the expected zero.
- Finally, the mere reading of the horizontal circle must be made, which also generates an uncertainty $u(\varphi) = s(\varphi) \approx 2''$.

In view of the above process, the following dependences arise:

$$\varphi_{E\uparrow} = \varphi^0_{E\uparrow} + (\theta_m - \delta\theta - 90^{\circ})\tan I + \frac{180^{\circ}}{\pi}\frac{T_m - \delta T}{H} + \delta\varphi , \tag{28}$$

where $\varphi_{E\uparrow}$ is the input quantity of equation (13) or, equivalently, the result of a measurement considering it as the output quantity of equation (28), depending on the input quantities $\delta\theta$, δT and $\delta\varphi$, described below. $\varphi^0_{E\uparrow}$ is the (unknown) "true" value of the horizontal circle assuming perfect horizontality of the telescope and (exactly) zero output from the fluxgate, and $\delta\varphi$ is the (unknown) random error associated to the horizontal circle reading, which alternatively can be defined as the result of the measurement in ideal conditions (i.e., perfect horizontality of the telescope and zero output) minus its "true" value, $\varphi^0_{E\uparrow}$. T_m is the *measured* fluxgate output, i.e., the zero value appearing in the console, while θ_m is the vertical angle reading, i.e., the 90º 00' 00" value sought. Finally, $\delta T = T_m - T$ and $\delta\theta = \theta_m - \theta$ stand for the (unknown) errors associated to the previous quantities. To propagate the uncertainties of the input quantities, they can all be assumed to be independent, so that they are added in quadrature:

$$u_c^2(\varphi_{E\uparrow}) = \left(\frac{\partial\varphi_{E\uparrow}}{\partial\delta\theta}u(\delta\theta)\right)^2 + \left(\frac{\partial\varphi_{E\uparrow}}{\partial\delta T}u(\delta T)\right)^2 + \left(\frac{\partial\varphi_{E\uparrow}}{\partial\delta\varphi}u(\delta\varphi)\right)^2 = \\ = (u(\delta\theta)\tan I)^2 + \left(\frac{180^{\circ}}{\pi}\frac{u(\delta T)}{H}\right)^2 + u^2(\delta\varphi) \quad (29)$$ [6]

At LIV, where I is -55º 43' and H is 20200 nT, the combined standard uncertainty associated to $\varphi_{E\uparrow}$, $u_c(\varphi_{E\uparrow})$, is 5", while it is between 4" and 7" for most of the mid-latitude observatories, increasing as we approach the magnetic poles.

Similar arguments can be applied to the remaining horizontal readings, $\varphi_{W\uparrow}$, $\varphi_{E\downarrow}$ and $\varphi_{W\downarrow}$, since the process is actually repeated four times. The same uncertainty is therefore obtained.

As for the uncertainty associated to the vertical readings, equations equivalent to (27), (28) and (29) for $\theta_{N\downarrow}$ are, respectively:

$$\theta_{N\downarrow} = 270^{\circ} - I - \varepsilon_{FI} - \frac{180^{\circ}}{\pi}\frac{T - T_o}{F} \pm 90^{\circ} \quad (30)$$

$$\theta_{N\downarrow} = \theta_{N\downarrow}^0 - \frac{180^{\circ}}{\pi}\frac{T - \delta T}{F} + \delta\theta\,, \quad (31)$$

$$u_c^2(\theta_{N\downarrow}) = \left(\frac{\partial\theta_{N\downarrow}}{\partial\delta T}u(\delta T)\right)^2 + \left(\frac{\partial\theta_{N\downarrow}}{\partial\delta\theta}u(\delta\theta)\right)^2 = \\ \left(\frac{180^{\circ}}{\pi}\frac{u(\delta T)}{F}\right)^2 + u^2(\delta\theta), \quad (32)$$

leading to a combined standard uncertainty $u_c(\theta_{N\downarrow})$ = 2.6" at LIV, where F is 35600 nT (being between 2" and 3" for most of the mid-latitude observatories). As well, the same standard uncertainty is obtained for the remaining vertical readings, $\theta_{S\uparrow}$, $\theta_{N\uparrow}$ and $\theta_{S\downarrow}$.

The uncertainty associated to the measurement of the azimuth of the mark from the theodolite reference frame, $u(H_M)$, is directly given as 2.5" by the manufacturer's instructions manual (see Carl Zeiss 1987). Nevertheless, a type A evaluation gives a similar result.

The azimuth of the mark in a geographic reference frame, AM_{GN}, is usually determined by means of independent measurements and the information to evaluate its uncertainty depends on each particular case. As this quantity is measured once and its value taken as a constant, attempts should be made to reduce its uncertainty, since its (unknowable) residual error will play the role of a systematic error in D measurements. Thus for instance, if both the DI-flux location and the reference mark, separated by a certain distance, are determined from GPS measurements, it is necessary to carry out a sufficiently large number of measurements in each point so as to reduce the type A uncertainty associated to the mean. In general, it will be assumed that $u(AM_{GN})$ is small as compared with the other uncertainties in equation (25).

3.2 Variometer inaccuracy

Finally, estimation of the uncertainties related to the measurement of D and I with a variometer, $u(D_{var}(t_i))$ and $u(I_{var}(t_i))$, is required. As before, many types of magnetometers may act as variometers, each one possessing its own accuracy. In many observatories, tri-axial fluxgate sensors working together with proton magnetometers are progressively being imposed for this purpose. Manufacturer's "total accuracy" given for these devices is typically 0.1 nT, for which a rectangular distribution will be considered, conferring a standard

[6] We will make no distinction in use of $u(x)$ and $u(\delta x)$, since in fact the uncertainty associated to a measurand is the uncertainty associated to its error.

uncertainty $u(X_{var}) = u(Y_{var}) = u(Z_{var}) = 0.06$ nT. It is immediate to see that the uncertainty associated to Declination measurements is $u(D_{var}) = 180º/\pi \cdot u(Y_{var})/H$, while that for Inclination is $u(I_{var}) = 180º/\pi \cdot u(Y_{var})/F$. At LIV, $u(D_{var}) = 0.6"$ and $u(I_{var}) = 0.35"$, while in most of the mid-latitude observatories these quantities do not surpass 1" and 0.5", respectively. In principle, all the $u(D_{var}(t_i))$ are equivalent, and they will be generically referred as to $u(D_{var})$.

3.3 Compendium

Once in possession of the uncertainties of the input values of equations (25) and (26), the combined standard uncertainty of the output quantities immediately follows:

$$u_c\left(\overline{D}-\overline{D}_{\text{var}}\right)=\sqrt{4\left(\frac{u(\varphi_{E\uparrow})}{4}\right)^2+4\left(\frac{u(D_{\text{var}})}{4}\right)^2+u^2(H_M)+u^2(AM_{GN})}= \tag{33}$$

$$=\sqrt{\frac{1}{4}\left(\left(u(\delta\theta)\tan I\right)^2+\left(\frac{180^{\circ}}{\pi}\frac{u(\delta T)}{H}\right)^2+u^2(\delta\varphi)+u^2(D_{\text{var}})\right)+u^2(H_M)}\,,$$

$$u_c\left(\overline{I}-\overline{I}_{\text{var}}\right)=\sqrt{4\left(\frac{u(\theta_{N\downarrow})}{4}\right)^2+4\left(\frac{u(I_{\text{var}})}{4}\right)^2}= \tag{34}$$

$$=\frac{1}{2}\sqrt{\left(\frac{180^{\circ}}{\pi}\frac{u(\delta T)}{F}\right)^2+u^2(\delta\theta)+u^2(I_{\text{var}})}$$

With the assumptions made here, it is worth remarking that only the terms related to $u(\delta\varphi)$, $u(\delta\theta)$, $u(\delta T)$ and specially $u(H_M)$ play an important role. The values obtained for $u_c(\overline{D}-\overline{D}_{\text{var}})$ and $u_c(\overline{I}-\overline{I}_{\text{var}})$ at LIV are 3.1" and 1.3", respectively, whereas these uncertainties would not surpass 4" and 2" for most of the mid-latitude observatories (with the same assumptions). Equations (33) and (34) have been written explicitly so that each observatory can introduce its own estimations, which will depend on the type of DI-flux and variometer in use, geomagnetic latitude and other particular circumstances.

Once the baselines have been established, Declination and Inclination are found by adding the respective variometric values. Absolute observations use to be made at least once in a week, depending on the observatory baseline stability; this procedure implies an implicit assumption: typical periods of baseline variation are usually no shorter than two weeks (in fact, the double of the sampling period). In consequence, all the absolute observations made in a time interval (typically a fraction of the period T of variation) in which the baselines are supposed to not significantly vary can provide independent information on the uncertainties $u_c(\overline{D}-\overline{D}_{\text{var}})$ and $u_c(\overline{I}-\overline{I}_{\text{var}})$, particularly by computing the standard deviation of the experimental values around the baselines. Moreover, if n measurements were made in such interval, a type A evaluation would conclude that the uncertainty associated to the final observatory *values* of D and I is typically $\sigma/n^{1/2} \approx u_c/n^{1/2}$ (if the random effects exposed so far were the only ones influencing the measurements).

4. Other effects

Certain effects derived from the incorrect operation of the DI-flux, or due to an instrument malfunction, cannot be strictly considered to have either a systematic or a random nature. Their magnitudes are practically impossible to define since they can depend on the fault associated with the particular instrument or the care taken by the person who handles it. Some of these effects are discussed below.

4.1 Consequences of the imperfect theodolite levelling

The vertical circle in Zeiss 010, 015 and 020 theodolites most commonly used in magnetic observatories work using a pendulum arrangement which permits automatic correction of vertical readings from imperfect theodolite levelling, taking the gravity direction as a reference. The maximum range this pendulum can compensate for a Zeiss THEO 015B is 4'; if the degree of mis-levelling is greater than this the results will be in error.

Also, a small error in the horizontal reading of the Declination reference mark is systematically produced when the theodolite is not perfectly levelled. This error also depends on the height above or below the theodolite the fixed mark is observed, being completely removed if both lie on the same horizontal plane.

In practice, the theodolite is seldom levelled to the accuracy required for *D* and *I* measurements; for this reason, the observer should check the telescope horizontality (i.e., the vertical reading equal to either 90º or 270º) before each one of the four *D* readings. Suppose the observatory sequence for *D* is $E_\uparrow$, $W_\uparrow$, $E_\downarrow$ and $W_\downarrow$. If the imperfect levelling in a real case is not assumed, one could think that when performing the $W_\uparrow$ reading after the $E_\uparrow$ reading, the telescope is maintained in a plane really horizontal, since the telescope has not been explicitly rotated around the horizontal axis. In reality, the telescope would be separated by a certain angle δv from this plane (so that the vertical reading would be 90º + δv), and the null field would be found at an angle $\varphi_{W\uparrow} + \delta v \cdot \tan I$. Similarly for what concerns the $W_\downarrow$ reading after performing the $E_\downarrow$ reading, where the null would be found at $\varphi_{W\downarrow} + \delta v \cdot \tan I$. If we, thus, assume that the horizontal position is necessarily verified before making the E (↑ and ↓) readings, it is immediate to check that the additional term $\delta v \cdot \tan I /2$ coming from the W readings appears in the right hand side of equation (17) for *Abs-Var_D*, while the term $\delta v \cdot \sin I /2$ appears in the right hand side of equation (20) for *Off_MI_D* (for more details, see Curto and Sanclement, 2001). As the quantity *Abs-Var_D* constitutes the observed baseline values for *D*, this fault would affect systematically (assuming a constant angle δv) the final value of this magnetic element. Moreover, as mentioned below in section 5, the effect described in this paragraph is also observed as an anomalous value of the quantity *MI*.

Finally, it is worth remarking that if a theodolite type other than those using a pendulum arrangement is used, further terms arising from mis-levelling must be considered in section 3.

4.2 Sliding or dragging of the theodolite horizontal plate

If during the course of an observation the theodolite is violently handled, sliding or dragging of its horizontal circle can occur, resulting in an invalid *D* measurement. This problem can be detected if the four *D* readings are in between (in the time sequence) the two readings of the fixed mark, $\varphi_{M\uparrow}$ and $\varphi_{M\downarrow}$, because an anomalous value of *AzRet* would then be produced. For this reason it is best to rotate the theodolite smoothly.

4.3 Fluxgate sensor manipulation in the course of a measurement

For vertical rotations care should be taken to hold the telescope during the time it is unclamped, avoiding in this way the telescope-sensor assembly to fall down due to the sensor weight. Moreover, handling the sensor in the course of a measurement could modify the fluxgate-telescope collimation, resulting in a bad observation.

4.4 Timing errors

If GPS time is not available in either the absolute building or the device(s) governing the automatic acquisition of variometer(s), timing errors should be considered. This effect greatly depends on the state of natural disturbance of the field, which in general may depend on the observatory location, hour of observation, position in the solar cycle, etc. Each individual observatory has to evaluate this effect; otherwise, the uncertainties associated to the measurements can surpass those given at the end of section 3, especially when the natural field is disturbed. In this last case of magnetic disturbance, it is critical to write down the exact time (including seconds) in which the null fluxgate output has been obtained for each reading.

4.5 The human factor

The human factor is an extra effect to be taken into account since, although research is under way to provide a fully automatic DI-flux (Van Loo and Rasson 2006, Auster *et al* 2006), the measuring procedure has not been automated up to now. Although some of the effects exposed above in this section can actually be regarded as "human" effects (subsection 4.1 and 4.3), our intention is to give an account of the reduction observed in the standard deviation of the measurements carried out by a given observer as it acquires experience, and they are probably due to a greater incidence of the cited effects when the observing habits have not yet been attained. But this effect does not only concern inexperienced observers; in fact, a mistake as a result of a lack of concentration can also be their source, and they unavoidably appear immerse in the data. They could be treated as random because they can influence in both a positive or negative sense, but they cannot in general be included in the uncertainty evaluation, since they are expected to be somewhat sporadic. In this case, efforts should be made to identify and reject the corresponding data. As shown in the following section, the observable quantities defined in section 2 can be of help for this task. It is difficult to give an idea of their magnitude beforehand, because they depend on the experience, care and personal abilities of each observer.

4.6 Uncorrected readings

Correction for natural field variations to each one of the eight readings corresponding to the magnetic field by means of variometer data has been assumed, as proposed in the paragraph following equations (11) and (12) in section 2. But if these two equations are used instead of (13) and (14), a random effect appears due to the field variation during the time the whole process of measurement lasts. It is difficult to assign an uncertainty to this effect, since it depends on the state of disturbance of the field, augmenting as a rule with the observatory geomagnetic latitude. On the contrary, observer's experience reduces the uncertainty as the measurement process takes less time.

4.7 Compendium

At the end of section 3, it was said that an idea of the magnitude of the uncertainties $u_c(\overline{D}-\overline{D}_{\mathrm{var}})$ and $u_c(\overline{I}-\overline{I}_{\mathrm{var}})$ can be obtained by computing the standard deviation of the observed baseline values around the adopted baselines. In reality, some of the effects cited in this section unavoidably contribute to the standard deviation, in such a way that such dispersion provides us with an idea of the magnitude of their superimposition. At LIV, a type A evaluation carried out with a long series of measurements from a skilled observer establishes this dispersion as 5" for *D* (figure 4), and about 3" for *I*. These results are in good agreement with the general lines established in this text, and it can be concluded that the superimposition of the unevaluated effects exposed in section 4 (human effects, timing errors, etc.) are of the order of 3-4" at this particular station.

5. Examples using experimental data

While some of the effects dealt with in section 4 are difficult to quantify, systematic effects go unnoticed if the measurements are not contrasted with those from any other source, as those from another observer or a different DI-flux with identical external conditions. Nevertheless, the observable quantities exposed in section 2 can help in the task of detecting systematic errors. In this section a series of actual results are given, obtained by applying these quantities to the instruments used in several observatories. In particular, the following examples are aimed at showing what information can be deduced from them.

Figure 5 shows the progress of the quantity *Abs-Var_D* at LIV, including a change of observer performing the measurements. Precisely when this change was produced (around measurement 24) a sudden jump appears, indicating that the anomaly associated (see section 2.2) can only be due to a systematic procedure fault by one of the observers (the remaining external conditions were identical). A scatter reduction beyond this point can also be seen because of a greater experience of observer "b" (i.e., a lower "human effect").

Both phenomena also appear in the *MI* quantity at the same epoch (figure 6), where some features departing from the ideal situation can be observed:

Firstly, the dispersion of the data around a supposed "average" is due to the random effects discussed in section 3; but let us pay our attention to the two distant points (measurement numbers 8 and 15). Considering the amount of available data, their probability of occurrence (regardless of the distribution governing the experimental data) is actually low, and they seem likely to be due to some other additional fault; consequently, we must consider them as two candidates to be rejected during the baseline production (see figures 6, 7 and section 4.5).

Secondly, a discontinuity appears in this quantity, from a value of around 0.5' to a value close to the ideal zero, along with a decrease of the dispersion. These changes seem to reinforce that observer "a" was making some kind of systematic fault in his procedure, since it is unlikely that the instrument magnetization (*MI*) could have suddenly changed.

In order to identify the reasons for the jump, figure 7 shows a plot of the quantities *Off_MI_D* and *Off_MI_I* for the same time interval. As stated, ideally $MI = 0$, and hence *Off_MI_D* = *Off_MI_I*, which is actually detected for observer "b" (within the uncertainty), but not for "a". On the other hand, no discontinuity is seen in the transition of the quantity *Off_MI_I* between observers "a" and "b". All these facts suggest that the undesired systematic effect is produced because observer "a" did not check the telescope horizontal level when making the four Declination readings (see Sec. 4.1). The fact that different causes can lead to a same value for a given observable quantity (e.g., $MI \neq 0$ can indicate either an instrument magnetization or an erroneous systematic observing procedure) is an example of what has been stated at the end of section 2, i.e., the proposed scheme is not complete. In this case, the comparison with a second observable quantity and the additional information that a change of observer was produced has been fundamental to determine the actual source.

Other tests compare the quantities obtained from two different instruments. In Figs. 8 and 9 *MS_MI* and *MA* for both the DI-flux normally used at LIV (DI 1) and a spare one (DI 2) are plotted. DI 2 appears to be closer to the ideal value zero in both cases, indicating its better behaviour, while DI 1 probably presents a slight magnetic pollution.

The same observable quantities described in section 2.2 have been plotted for the DI-fluxgates in use at some other observatories, showing that for the most relevant quantities (see Table 2) their values are close to the ideal zero within several arc-seconds. Let us analyse with more detail the absolute instrument deployed at EBR. This observatory shows almost ideal values for the *MS_MI* quantity, indicating the suitability of the habitual DI-flux location as for gradients arising from a magnetization in the surroundings. In figure 10, *MA* is plotted, where a systematic offset of 0.2 arc-minutes from the ideal zero is independently found at EBR by two different observers, pointing towards a small anomaly in the instrument itself. Further tests (see section 6) showed that the vertical pendulum is not working correctly, and hence the measurements rely on the theodolite levelling, based only on the bubble levels. It can be shown that the pendulum malfunction can cause the mentioned anomaly in *MA*.

Another remarkable phenomenon is the annual cycle appearing in some observatories when plotting both *Dip_MA_D* and *Dip_MA_I* observable quantities (figure 11 for the case of DOU), probably originating from movement of the sensing element in response to temperature changes. Such long-term variations do not have any repercussion on data.

It is worth mentioning the near-ideal values of the control quantities shown by the Zeiss DI-flux in use at DOU Observatory (Table 2). In figure 12 a histogram shows that the frequency of the *MI* values around its mean at this observatory approximately fits a Gaussian distribution.

In spite of the sources proposed in section 2 causing the systematic effects, the authors wish to note the impossibility of knowing *a priori* the actual origin of the slight anomalies causing the non-ideal values of some instruments appearing in Table 2; instead, certain values of these quantities (i.e., when differing a few tenths of an arc-minute from their ideal value) should act as an alert to start a series of further checks, as recommended in section 6, in order to establish such origin.

6. Recommendations for the reduction of errors

A series of proposals could be suggested in an attempt to improve the accuracy of the measurements with the DI-flux, minimising some of the negative effects shown in the preceding sections. Most are well-known and will not be mentioned here (see Newitt *et al* 1996); others are related to the sources described or come out from the contributions derived from this work:

(1) The correct pendulum work (see section 4.1) can be checked by slightly unlevelling the DI-flux and verifying that the vertical reading of a given reference mark has not changed.

(2) It is preferable to not carry out absolute measurements when the magnetic field is disturbed (section 4.4).

(3) Correction of the readings for the natural field variation by means of variometer data will help to reduce random errors (section 4.6). This procedure was applied at LIV, showing a reduction of the global uncertainty of around 30%.

(4) Regarding the scheme proposed in section 2, we recommend to evaluate the DI-flux behaviour by plotting the control quantities *MS_MI*, *MA* and *MI* for a series of measurements. If their value is not randomly distributed around zero, the next step will depend on each particular case.

i) A non-zero value of *MS_MI* can indicate the presence of gradients.
   (a) Natural gradients are evidenced by bringing a sample of the surface rocks near the sensor.
   (b) The presence of artificial sources fixed with respect to the normal DI-flux location can be checked by monitoring the *MS_MI* quantity where one is sure that no magnetic artificial sources exist. If a zero is now found, proceed to check the absolute building (pier, static objects…); if the same non-ideal value is instead found, probably the source lies in the DI-flux itself. The possibility of a magnetized mounting is checked by removing this part and moving it relative to the sensor; otherwise, probably *MI* will show non-ideal values, too.

ii) If a non-zero systematic value of *MA* and/or *MI* is found, it is advisable to ask a skilled observer to perform a series of measurements in order to compare and detect procedure faults. A second test consists of substituting the examined DI-flux by another (magnetically clean if possible) in the same pier; if the second DI-flux presents the ideal value, probably the first one has some kind of magnetization. In this case, a major revision of the instrument should be carried out in the lab.

(5) In the observatory routine it is advisable to plot the *Abs-Var_D* and *Abs-Var_I* quantities along with *MS_MI*, *MA* and *MI*, watching over possible discontinuities and spikes. If this is the case, the problem can affect either the DI-flux or the variometer. The rest of quantities defined in section 2.2 can provide valuable information just in case an anomaly is found in these five observables.

7. Conclusions

The DI-flux is an excellent instrument to reliably undertake the absolute measurements of Declination and Inclination, and for this reason it has become the international standard. Nevertheless, a series of effects limit its accuracy.

Regarding systematic effects, it is in general affected by "collimation errors" and zero-field offset, which are compensated in the normal procedure. Undesired systematic effects can be detected with a series of control measures. Their quantification depends on many factors:

instrument, observer, location…, but if a magnitude is to be given, it can be obtained from these control observables.

Random effects are minimized by a long series of measurements. Their magnitude is larger for *D* than for *I*, but the associated uncertainty usually does not exceed 0.1' for an experienced observer.

Other effects that potentially affect the DI-flux such as a bad deployment, horizontal plate sliding or timing errors are of more difficult quantification.

The International Association of Geomagnetism and Aeronomy (IAGA) organizes biannual Workshops on *Geomagnetic Observatory Instruments, Data Acquisition and Processing*. Their value in helping to identify problems with instruments by having direct intercomparisons between instruments brought to a single location is remarkable. The results of the most recent one, held at Belsk Observatory, show that 70% of the *D* measurements obtained from the participating observatories are within a range 28 arc-seconds wide, while that for *I* is 10 arc-seconds wide. This gives an idea of the discrepancy between instruments and of the accuracy attained with the DI-fluxgate theodolites. However, it must be taken into account that experimental errors (and especially those derived of the "human" effects) are usually overweighed during these measurement sessions. An idea of the magnitude of all types of contribution to random effects (i.e., those discussed in section 3 and some of section 4) can be obtained through the mean of the standard deviations corresponding to each series of measurements, being 14 arc-seconds for *D* and 6 for *I*, although the short series of measurements from just three-day sessions impedes a robust analysis.

In this work the monitoring of a series of observable quantities has been suggested. These can provide pointers to assist in the assessment of errors and of their possible origin, and how some effects may be corrected. Also, some useful procedures have been suggested which will keep uncertainty to a minimum.

The main function of the DIflux is to provide absolute values of *D* and *I* which are used to generate baselines for the variometer data. Even though a great deal of global magnetic field information is currently obtained from satellites, long term precise modelling of the main field still relies on the worldwide network of magnetic observatories to provide continuous accurate vector data. The long term accuracy of this work therefore depends on accurate absolute observations with observers regularly checking and comparing their instruments and results, for which the procedures recommended here will help ensure success.

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Figure captions

Figure 1. DI-flux model THEO 015B Zeiss Jéna in use at LIV, with its most important parts.

Figure 2. Systematic difference between two independent *I* values at LIV, obtained through two different combinations (either using vertical readings with the sensor above, $I_{\uparrow}$, or below the telescope, $I_{\downarrow}$) of the fundamental equations.

Figure 3. Diagram showing the horizontal component of the different magnetic field vectors concerning a measurement of Declination for the four positions of the sensor in case of a magnetization in the alidade. $\mathbf{B_{nat}}$ stands for the undisturbed or natural magnetic field; $\pm(\mathbf{B_{MA}} + \boldsymbol{\delta}\mathbf{B_{MA}})_H$ and $\mp(\mathbf{B_{MA}} - \boldsymbol{\delta}\mathbf{B_{MA}})_H$ stand for the horizontal projections of the alidade disturbance field in the sensor positions E↑, W↑, E↓, W↓ respectively, which disturb *D* by the amount $\pm(D_{MA} + \delta D_{MA})$ and $\mp(D_{MA} - \delta D_{MA})$. The search for zeros actually implies aligning the sensor in the directions $\varphi_{E\uparrow}(\mathbf{B_{tot}})$, $\varphi_{W\uparrow}(\mathbf{B_{tot}})$,… (instead of the ideal $\varphi_{E\uparrow}(\mathbf{B_{nat}})$, $\varphi_{W\uparrow}(\mathbf{B_{nat}})$,…) perpendicular to the total magnetic field, $\mathbf{B_{tot}}$, which is the sum of both contributions.

Figure 4. Dispersion of the observed baseline values around the adopted baseline for the *D* element at LIV during the 2004-2005 summer survey.

Figure 5. *Abs-Var_D* for two different observers at LIV during the Antarctic survey 2004-2005. A jump between the measurements performed by observer "a" (diamonds) and "b" (squares) can be seen. The values for the first one are clearly higher and they show more scatter than the second ones. The greater dispersion can be attributed to a lack of experience, while the different mean values are due to some systematic fault in the procedure by one of the two observers.

Figure 6. Behaviour of the quantity *MI* at LIV during the Antarctic survey 2004-2005. The two outlying points are candidates to be rejected.

Figure 7. Same as figures 5 and 6 for both *Off_MI_D* (diamonds for observer "a", triangles for observer "b") and *Off_MI_I* (squares for observer "a", crosses for observer "b") quantities. The same outlying points of figure 6 are also observed here.

Figure 8. *MS_MI* for two different DI-flux (diamonds and squares) instruments deployed alternately on the LIV fundamental pillar.

Figure 9. Same as Figure 8 for the quantity *MA*.

Figure 10. *MA* control observable for one-year data at EBR. The grey band represents the uncertainty associated to its mean. Determinations for both Observer "c" (diamonds) and "d" (crosses) roughly coincide.

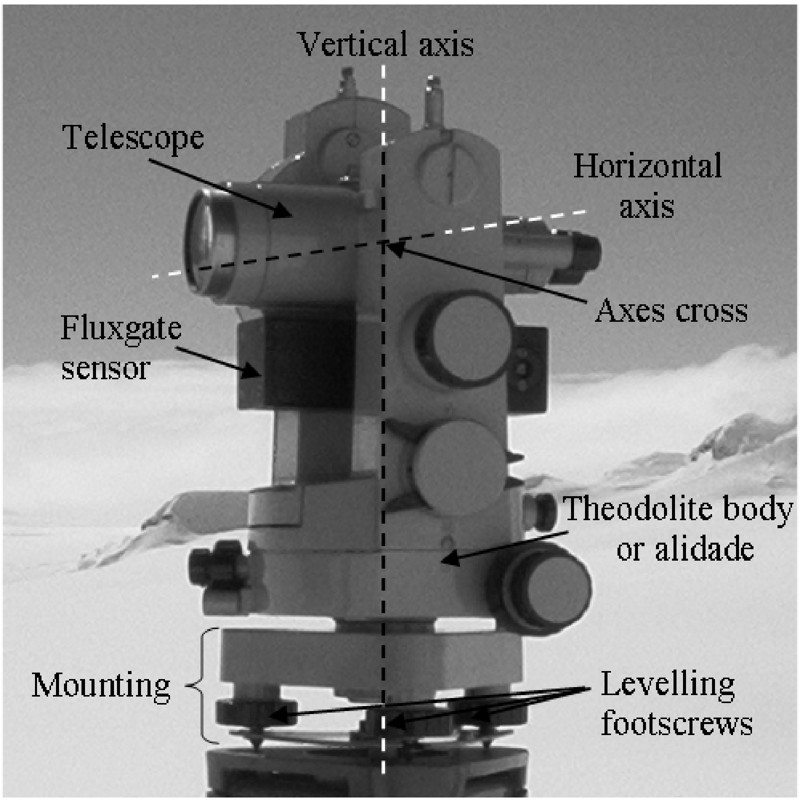
Figure 11. *Dip_MA_D* (squares) and *Dip_MA_I* (crosses) quantities at DOU, showing a periodicity of circa 1 year and more than 20" amplitude.

Figure 12. Histogram showing the relative proportion of measurements falling into categories 1" wide for a given observable quantity (*MI* in this case). The grey curve corresponds to a Gaussian distribution.

Table captions

Table 1. Summary of the different terms proposed in the scheme of section 2, along with a short description.

Table 2. Control observables have been calculated for several observatories during different periods of time: LIV for the 2005-06 survey, EBR for the year 2005, DOU for the period Nov 2003 – June 2006 and both SPT and NGK for January – May 2006. Their mean value is presented along with the uncertainty associated to that mean, obtained through a type A evaluation. Note that the ideal mean value is zero.


Vertical axis
Telescope
Horizontal axis
Axes cross
Fluxgate sensor
Theodolite body or alidade
Mounting
Levelling footscrews
Figure 1

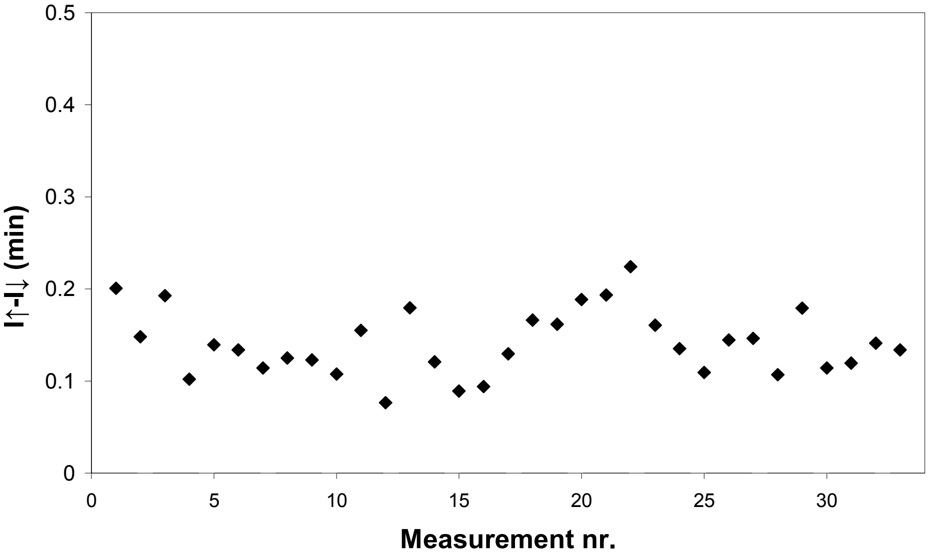


Figure 2

Top view:

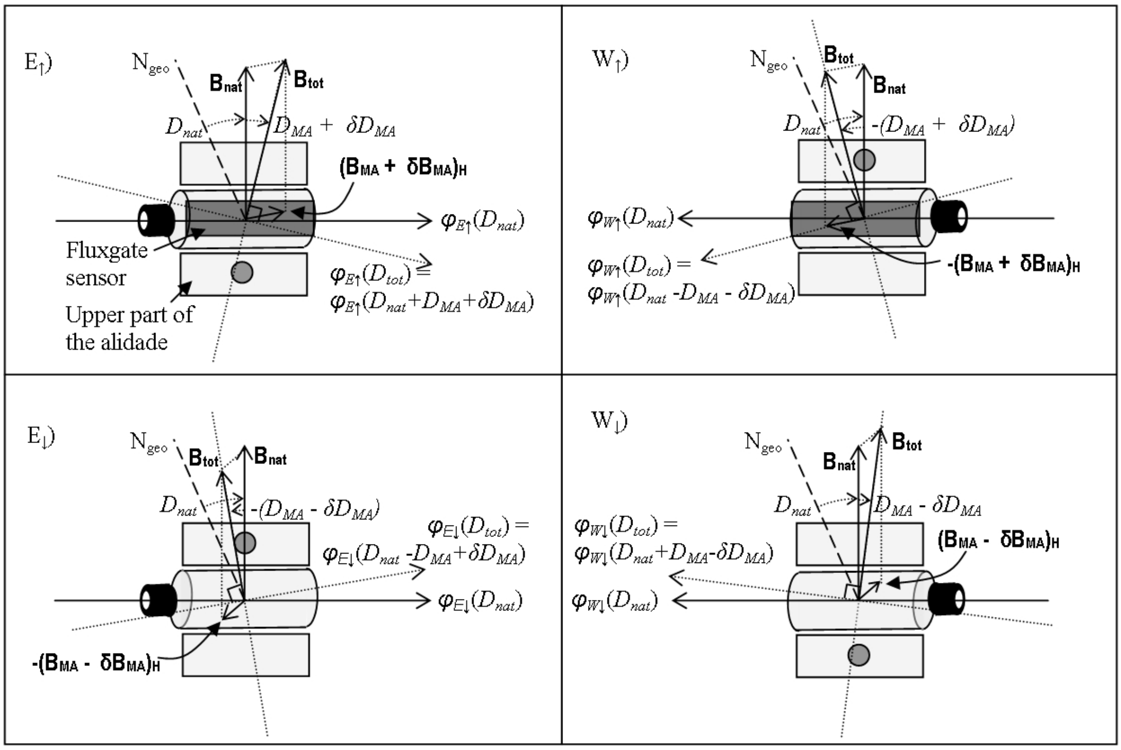
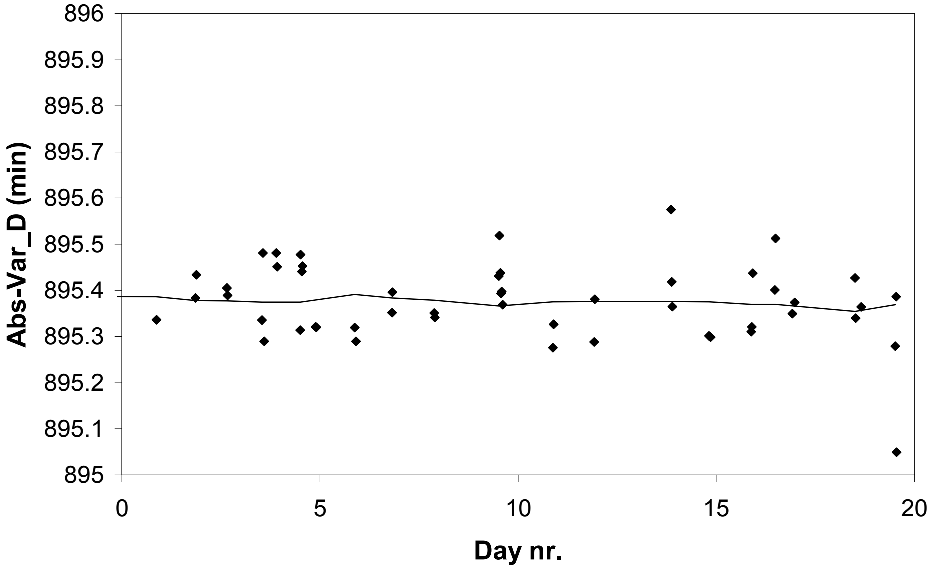
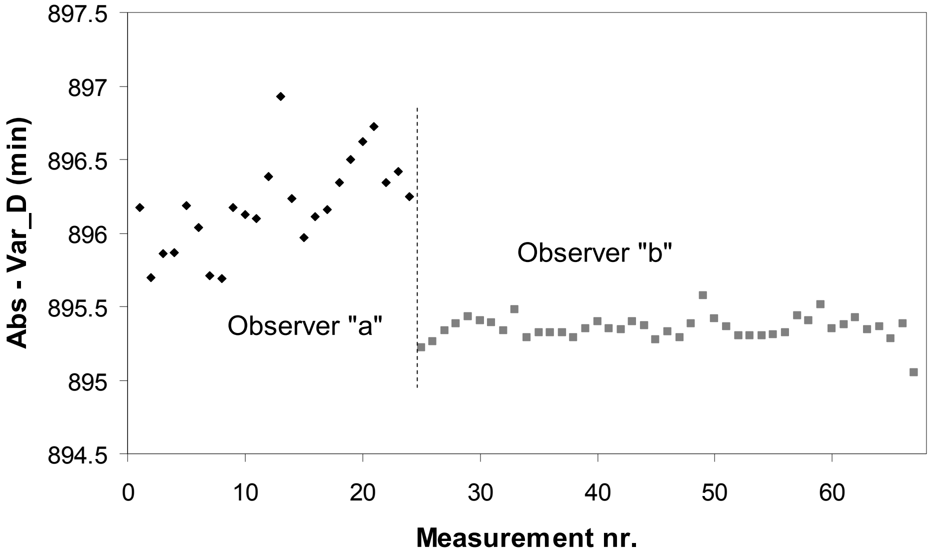


Figure 3



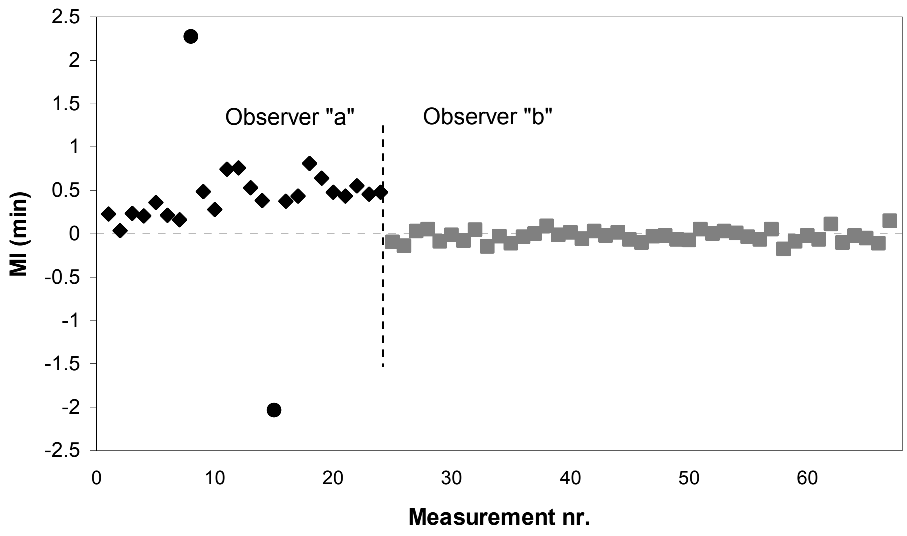
896
895.9
895.8
895.7
895.6
895.5
895.4
895.3
895.2
895.1
895
Abs-Var_D (min)
0
5
10
15
20
Day nr.

Figure 4



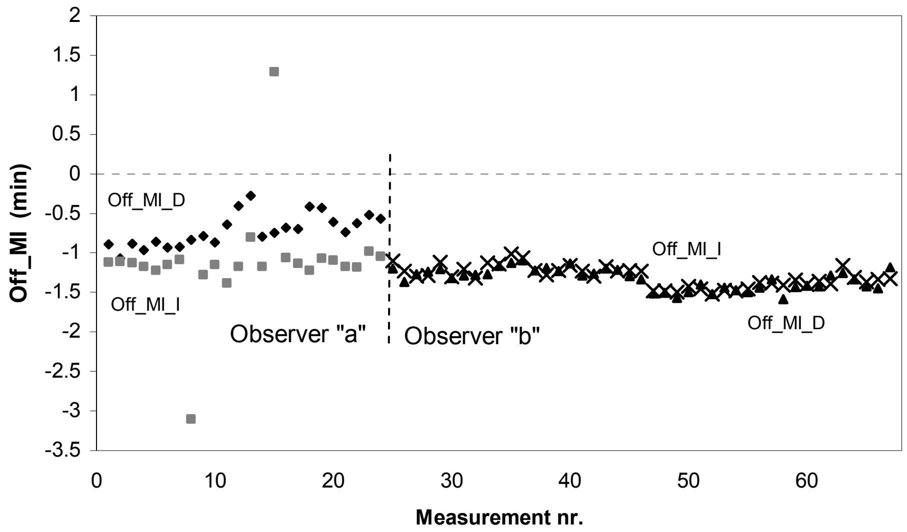
897.5
897
896.5
896
895.5
895
894.5
0
10
20
30
40
50
60
Abs - Var_D (min)
Measurement nr.
Observer "a"
Observer "b"

Figure 5



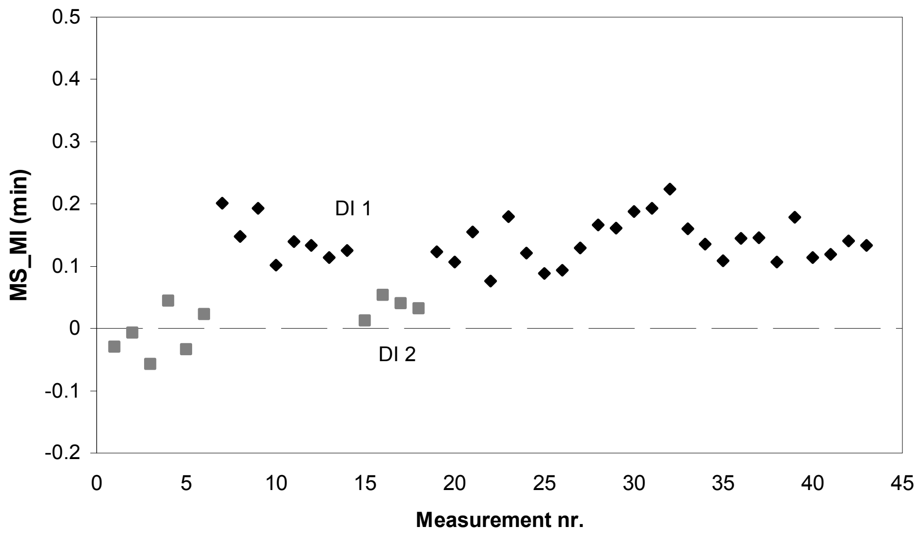
Observer "a"
Observer "b"
MI (min)
Measurement nr.
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
10
20
30
40
50
60

Figure 6



2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
Off_MI (min)
Off_MI_D
Off_MI_I
Observer "a"
Observer "b"
Off_MI_I
Off_MI_D
0
10
20
30
40
50
60
Measurement nr.

Figure 7



MS_MI (min)
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
DI 1
DI 2
0
5
10
15
20
25
30
35
40
45
Measurement nr.

Figure 8

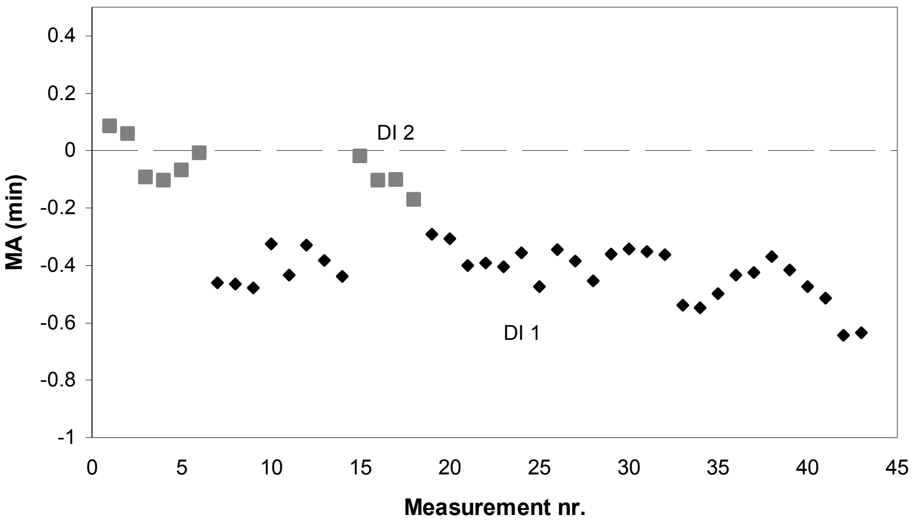
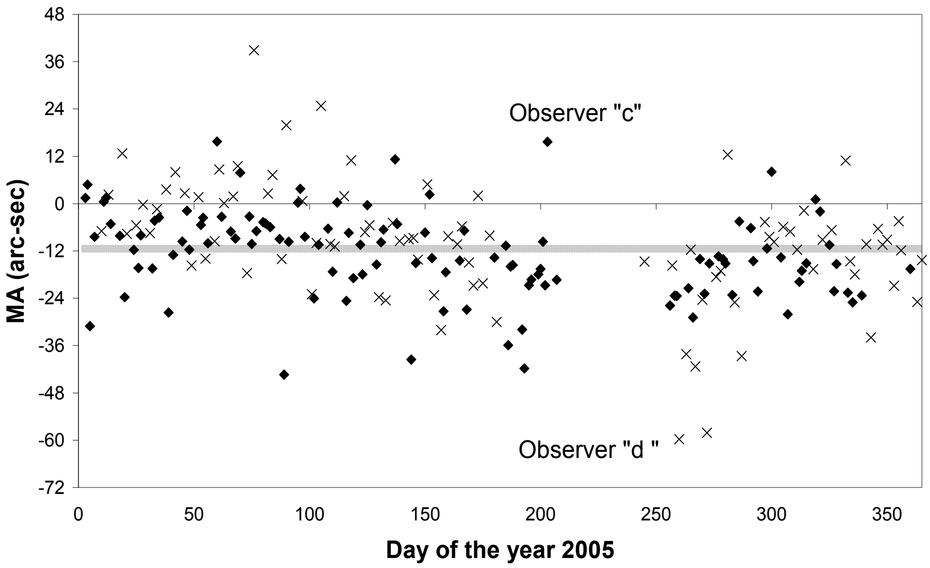
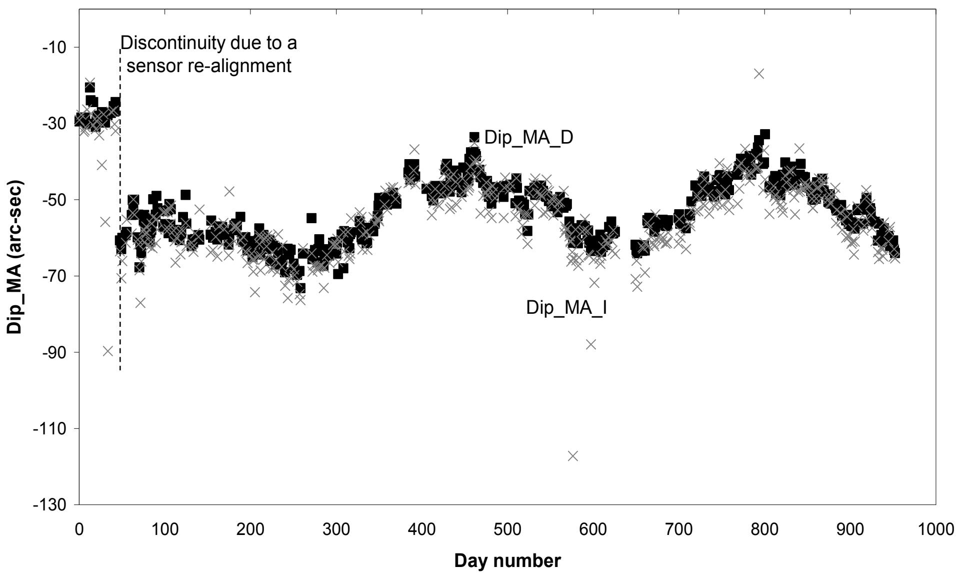

0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
MA (min)
DI 2
DI 1
0
5
10
15
20
25
30
35
40
45
Measurement nr.

Figure 9



48
36
24
12
0
-12
-24
-36
-48
-60
-72
MA (arc-sec)
Observer "c"
Observer "d "
0
50
100
150
200
250
300
350
Day of the year 2005

Figure 10



Discontinuity due to a
sensor re-alignment
Dip_MA_D
Dip_MA_I
Dip_MA (arc-sec)
-10
-30
-50
-70
-90
-110
-130
0
100
200
300
400
500
600
700
800
900
1000
Day number

Figure 11

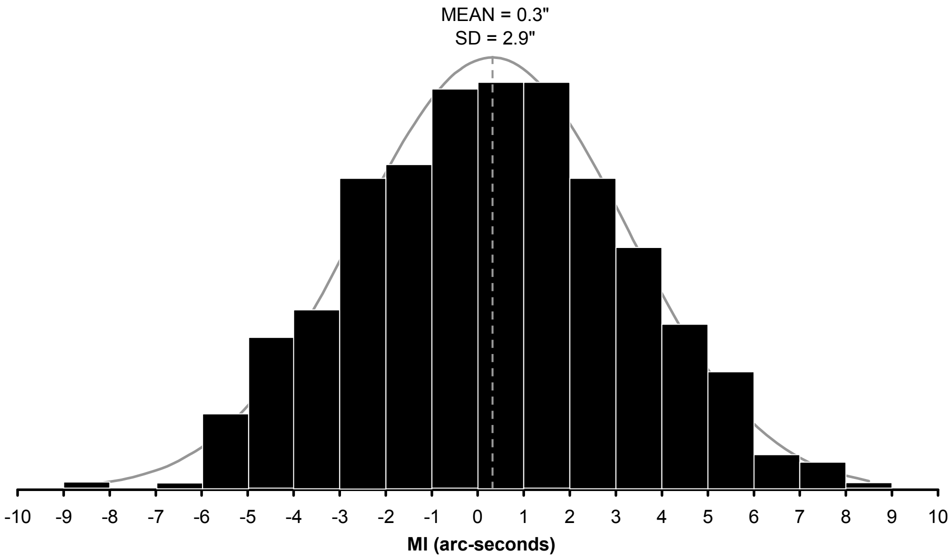


Figure 12

Term	Description	Derived terms
$\mathbf{B}_{MB}$	Disturbing magnetic field produced by a magnetized body in general. Several particular cases can be distinguished depending on its location:	$\mathbf{B}_{MS}$, $\mathbf{B}_{MA}$, $\mathbf{B}_{MT}$
$\mathbf{B}_{MS}$	The magnetized body (MB) lies in the DI-flux surrounding	D_{MS}, δD_{MS}, I_{MS}, δI_{MS}
$\mathbf{B}_{MA}$	The magnetized body (MB) lies in the DI-flux alidade	D_{MA}, δD_{MA}, $I_{MA}{}^{a}$, $\delta I_{MA}{}^{a}$, $I_{MA}{}^{b}$, $\delta I_{MA}{}^{b}$
$\mathbf{B}_{MT}$	The magnetized body (MB) lies in the DI-flux telescope	D_{MT}, $I_{MT}{}^{a}$, $I_{MT}{}^{b}$
D_{MS}, I_{MS}	Disturbing contribution to Dec. and Inc. due to a magnetized body in the surroundings; evaluated near the axes cross.	
δD_{MS}, δI_{MS}	Difference of the surrounding contributions between sensor positions "up" and "down".	
D_{MA}	Disturbing contribution to Dec. due to a magnetized alidade; evaluated near the axes cross.	
$I_{MA}{}^{a}$, $I_{MA}{}^{b}$	Contribution to Inc. due to a magnetized alidade for the two possible orientations (*a* and *b*) of this part; evaluated near the axes cross.	
δD_{MA}, $\delta I_{MA}{}^{a}$, $\delta I_{MA}{}^{b}$	Difference of the alidade contributions between sensor positions "up" and "down".	
D_{MT}	Disturbing contribution to Dec. due to a magnetized telescope.	
$I_{MT}{}^{a}$, $I_{MT}{}^{b}$	Contribution to Inc. due to a magnetized telescope for the two possible orientations of the alidade.	

Table 1

Observatory	Control observable		
	MS_MI (arc-sec)	*MA* (arc-sec)	*MI* (arc-sec)
LIV	2.8 ± 0.2	1.5 ± 0.7	5.3 ± 0.5
EBR	-1.0 ± 0.3	-11.4 ± 0.9	3.9 ± 0.6
DOU	1.6 ± 0.3	2.1 ± 0.2	0.3 ± 0.1
SPT	-0.1 ± 0.3	-1.3 ± 0.5	8.1 ± 0.6
NGK	11.2 ± 0.2	-0.2 ± 0.2	-10.3 ± 0.2

Table 2