Mixing structures in the Mediterranean Sea from Finite-Size Lyapunov Exponents

Francesco d'Ovidio, Vicente Fernández, Emilio Hernández-García, and Cristóbal López[1]

[1] *Instituto Mediterráneo de Estudios Avanzados IMEDEA (CSIC-UIB), Campus Universitat de les Illes Balears, E-07122 Palma de Mallorca, Spain.*

We characterize horizontal mixing and transport structures in the surface circulation of the Mediterranean Sea, as obtained from a primitive equation circulation model. We calculate the Finite Size Lyapunov Exponents (FSLEs) of the velocity data set, which gives a direct measure of the local stirring. By proper election of the FSLE parameters, we focus on the mesoscale structures, locating a number of vortices embedded in an intricate network of high-stretching lines. These lines control transport in the system. At the edge of the vortices, a dense tangle of line intersections appears, identifying strong mixing. The spatial distribution of FSLEs, averaged over one year, allows to classify areas in the Mediterranean basin according by their mixing activity. The space average of FSLEs on selected geographical regions gives a measure for quantifying and comparing the mixing seasonal variability.

Introduction

Horizontal transport and mixing processes are central to the study of the physical, chemical, and biological dynamics of the ocean. Correct understanding and precise modelling of them are relevant from a theoretical viewpoint and crucial for a range of practical issues, such as plankton dynamics or the fate of pollutant spills. In this regard, the last few years have seen the appearance of interesting new developments [*Mariano et al.,* 2002] on the Lagrangian description of transport and mixing phenomena, many of them coming from the area of nonlinear dynamics. Such approaches do not aim at predicting individual tracer trajectories, but at locating spatial structures that are known from dynamical systems theory to act as templates for the whole flow [*Ottino,* 1989, *Wiggins,* 1992]. Such structures (attractors, saddles, manifolds, ...) have been used since many years ago for classifying the evolution of trajectories in abstract dynamical systems. However, when put in a fluid dynamics context, they gain a new and direct physical meaning, corresponding for instance to avenues and barriers to transport, vortex boundaries, or lines of strong stretching. In particular, chaotic motions such as the ones occurring in the turbulent ocean are characterized by complex intersection of stretching and contracting manifolds at the so called *hyperbolic points*: regions of fluid initially compact in the proximity of these points become elongated along the stretching directions and then folded, leading to the typical filamental and convoluted structures that are common in satellite pictures of water temperature or chlorophyll, as well as in laboratory experiments. From the point of view of transport, such regions are characterized by a strong mixing: trajectories of initially close particles are quickly separated along the stretching directions, and fluid of different origins is inserted in between.

Until recently, the power of these novel Lagrangian approaches has been mainly relegated to mathematical systems or simplified workbench models, since the required detailed knowledge of the velocity field was not readily available in real geophysical situations. However, in the last decades the situation has dramatically changed, with a rapidly increasing amount of data available from Lagrangian drifters [*Mariano et al.,* 2002], satellite measurements [*Halpern,* 2000], and especially from detailed computer models [*Haidvogel and Beckmann,* 1998, *Dietrich,* 1997]. This has paved the way to a growing number of geophysical applications, among which we mention studies of the Loop Current in the gulf of Mexico [*Kuznetsov et al.,* 2002], or the characterization of dispersion properties in the Adriatic Sea from drifter experiments [*Lacorata et al.,* 2001]. The methodology is not restricted to ocean circulation, but it is also being developed in atmospheric dynamics [*Joseph and Legras,* 2002], and mantle convection [*Farnetani and Samuel,* 2003]. Indeed, new Lagrangian approaches are very appealing for geophysical applications, since they can be used as a tool for automatically extracting transport structures underlying raw Eulerian velocity data.

In this Letter we characterize mixing strength at the mesoscale in different areas of the Mediterranean Sea by means of a Lagrangian technique, the Finite Size Lyapunov Exponents method. The technique also identifies dynamical objects that organize the transport, and relevant coherent structures.

Numerical Data

We produce velocity data from the DieCAST ocean model (*Dietrich for Center Air Sea Technology*), adapted to the Mediterranean basin with a horizontal resolution of 1/8 degrees (approximately 10 km) and 30 vertical levels (see [*Fernández et al.,* 2004] for details). The DieCAST model is a z-level primitive equation model based on the hydrostatic, incompressible, and rigid lid approximations. The model has been integrated for 20 years being forced by yearly repeating monthly climatological atmospheric forcing. Using such climatological forcing the model reproduces well the general surface circulation and many of the important features of the observed annual cycle of the Mediterranean Sea [*Fernández et al.,* 2004]. Due to the adequate horizontal resolution and the numerical characteristics of the model, basically high order numerics and low numerical and physical dissipation [*Dietrich,* 1997], the numerical simulations reproduce a great number of mesoscale structures, that are required for the present study.

In this work we analyze the daily model output of velocity field obtained from the last three years of simulation and corresponding to the second vertical horizontal layer (in this way we avoid the strong dependence on wind stress of the first model layer). This layer has a vertical extent of 11.60 m, centered at a depth of 16 m. The two-dimensional horizontal velocity field on this layer is not exactly incompressible, but it is very close to this situation since typical vertical velocities in the ocean are four orders of magnitude smaller than horizontal ones (10^{-5} vs 10^{-1} m/s). Thus, points at which fluid particles diverge horizontally along particular directions receive fluid along other directions, so that stretching is linked to mixing. On the other hand, within the Finite-Size Lyapunov exponent (FSLE) method used in this Paper and presented in the next sections to estimate transport at the mesoscale, fluid particle trajectories need to be integrated only while they undergo horizontal displacements of the order of 100 km, i.e. during 1-10 days (see Section of Results). Estimating an effective or average vertical velocity for this time as the spatial average of the vertical velocity field in horizontal regions of that size, one obtains an effective vertical velocity of $0.1 - 0.7$ m/day. Thus, during the time of integration, most particles do not leave the horizontal layer considered. In consequence, restricting the study to horizontal motion on a single model layer is a good description of the full transport processes for the space and time scales relevant to mesoscale processes within the FSLE approach.

Finite Size Lyapunov Exponents (FSLEs)

A common way to quantify the stretching by advection is by means of the standard Lyapunov exponents. They are defined as the exponential rate of separation, averaged over infinite time, of fluid parcels initially separated infinitesimally. In realistic situations (such as the case of the Mediterranean Sea where boundaries at finite distance strongly influence the circulation) the infinite-time limit in the definition makes the Lyapunov exponent a quantity of limited practical use. Recently, the Finite Size Lyapunov Exponent (FSLE) has been introduced [*Aurell et al,* 1997, *Artale et al.,* 1997] in order to study non-asymptotic dispersion processes, which is particularly appropriate to analyze transport in closed areas. FSLEs have been used for two complementary goals: for characterizing dispersion processes [*Lacorata et al.,* 2001], and for detecting and visualizing Lagrangian structures (e.g., transport barriers or vortex boundaries) [*Koh and Legras,* 2002]. Here we will focus mainly in the second use, but we will also introduce measures of dispersion and mixing based on the Lagrangian structures detected.

The FSLE technique appears to be ideally suited for oceanographic applications, being the mathematical analogous of a floater experiment: a set of tracers, with some initial mutual distances, are followed in time as they are transported by integrating the velocity field (we use a bilinear interpolation to assign velocities to points that are not model grid points; thus the velocity field is effectively smooth at scales below 1/8 of degree). The FSLE is inversely proportional to the time at which two tracers reach a prescribed separation. More precisely, $\lambda(\mathbf{x}, t, \delta_0, \delta_f)$, the FSLE at position $\mathbf{x}$ and time t, is computed from the time τ it takes for a trajectory starting at time t at a distance δ_0 from $\mathbf{x}$ to reach a separation δ_f from the reference trajectory that started at $\mathbf{x}$:

$$\lambda(\mathbf{x}, t, \delta_0, \delta_f) \equiv \frac{1}{\tau} \log \frac{\delta_f}{\delta_0}. \qquad (1)$$

In order to characterize the strongest separation (and the fastest convergence along the complementary direction), λ is selected as the maximum among the four values obtained when the initial separation δ_0 is chosen along four orthogonal directions.

The FSLE depends on the choice of two length scales: the initial separation δ_0 and the final one δ_f. Here, we are interested in the spatial distribution of FSLEs, and thus we calculate them at points $\mathbf{x}$ located on a grid of spacing Δx. In this case, a simple argument shows the convenience of using a value of δ_0 close to the intergrid spacing Δx: If one chooses δ_0 much smaller than Δx, all the points of a stretching manifold laying further than δ_0 from any grid point are not tested, and thus the method gives only a rather discontinuous sampling of the structure. On the other hand, if δ_0 is much larger than Δx, the same stretching manifold is detected ("smeared") on several grid points, with a loss in spatial resolution. Since we are interested in mesoscale structures, the other length, δ_f, will be chosen as $\delta_f = 1$ degree, i.e., separations of about 110 Km. In this way the FSLE represents the inverse time scale for mixing up fluid parcels between length scales δ_0 and δ_f.

Results

The spatial distribution of FSLEs for a particular day of the simulation (10th of June of the first year of the data set) can be seen in Fig. 1. Typical values are in the order of $0.1 - 0.6$ $days^{-1}$, corresponding to mixing times for mesoscale distances of $1.7 - 10$ $days$. As observed in previous works, maximum values of the distribution organize in lines [*Joseph and Legras,* 2002, *Koh and Legras,* 2002] that provide good approximations to repelling material lines (which are in turn stable manifolds of hyperbolic moving points) [*Joseph and Legras,* 2002]. These lines organize the transport processes in the basin. Spatial structures ranging from the small scales to the ones typical of mesoscale vortices are clearly identified. Computing such picture for every day of the year and taking then the time average, one can obtain a map of regions in the Mediterranean with different mixing activity (fig. 2). As expected, the Southern part of the basin appears more active, especially close to the North African coast. In fig. 3 we show more in detail the FSLEs of the area in the small box of fig. 1. In the core of the eddies one has low values of the FSLEs (i.e., low dispersion rates); on the contrary, the largest values of the FSLEs can be found in the outer part of the eddies, where the stretching of the fluid parcels is particularly important.

Note that in some regions of the vortex cores, chaotic tangles are still observed as local maxima of the FSLE distribution. These maxima are, however, not strong. In fact, even if the stretching is locally very high, the requirement for two points to diverge for more than $\delta_f = 110km$ gives a low value of λ to such finer structures, since such distance is bigger than the size of the vortex that acts as a containing barrier most of the time. This is an example of a useful property of the FSLE technique: it allows to restrict the analysis to the structures relevant for transport among selected lengthscales only.

Additional sets of coherent structures and organizing lines can be obtained by computing the FSLEs from trajectory integration *backwards in time*. Maxima in the new distribution identify lines of maximum compression, approximating attracting material lines or unstable manifolds of hyperbolic moving points [*Joseph and Legras,* 2002]. Since stable and unstable manifolds cannot be crossed by particle trajectories, such lines strongly constrain and determine fluid motion.

Calculating in this way the FSLEs in a region of strong mixing, we unveil the tangle of stretching and compressing lines in which vortices are embedded (fig. 4). These lines also define the directions of transport. Lobes arising from intersections of stretching and compressing lines at a vortex edge indicate where transport in and from the vortex takes place, whereas tangencies among them provide barriers to transport. In Fig. 4 some intersections of stretching and compressing lines are indicated as black dots. These identify Lagrangian hyperbolic points (and their motion define hyperbolic trajectories). Such points correspond to areas with strong mixing activity: fluid is advected here along a compression line and then dispersed away along the stretching line.

This dynamical picture suggests a quantitative measure of mixing in a prescribed area A: one can define de quantity $M_{\pm}(t) \equiv \left\langle \sqrt{\lambda_+ \lambda_-} \right\rangle_A$ where λ_+ and λ_- are the FSLEs in the forward and in the backwards time direction, and the average is the spatial average over the area A. This quantity is large only where hyperbolic points are present. The time dependence of this quantity when the area A is the whole Mediterranean is shown in fig. 5, characterizing the seasonal variations of mixing. Maximum values, of the order of 0.13 $days^{-1}$, are attained in winter. Because of the approximate incompressible character of the horizontal flow, the temporal variations of forward and backward FSLEs are strongly correlated, and one expects that the same information can be obtained from just one of the FSLEs. Thus one can define a simpler measure of mixing in an area as $M_+(t) = \langle \lambda_+ \rangle_A$. We show in fig. 5 that, as expected, it contains essentially the same information as $M_{\pm}$, but at variance with it, it could in principle be measured from floater experiments. It is thus a more convenient characterization of mixing strength. As a further example, we compare (fig. 6) the temporal behavior of $M_+(t)$ in two regions (the areas North and South of the Balearic islands, corresponding to the boxes in fig. 2) where we expect (from fig. 2) to see a very different mixing activity. The higher activity in the Southern part, where the Algerian current is present, is confirmed. In addition, seasonal fluctuations are smaller in the Northern part.

Conclusions

The FSLEs provide a direct method for computing simultaneously the mixing activity and the coherent structures that control transport at a given scale. Interestingly, the method is based on the evolution of the relative separation between two passive tracers, and thus numerical results directly suggest floater-based experiments for verification. In this work we have analyzed with this method horizontal velocity data from a Mediterranean computer simulation. Different mixing behavior between geographical regions, and at different seasons, is readily characterized. Finally, we point out that the strong horizontal stirring associated with hyperbolic points should also have important biological consequences. In the direction of recent works [*Martin,* 2003], it would be interesting to compare the Lagrangian structures presented here with productivity, patchiness, or other measures obtained from biological distributions.

Acknowledgements

We acknowledge financial support from MCyT of Spain and FEDER under projects REN2001-0802-C02-01/MAR (IMAGEN) and BFM2000-1108 (CONOCE). C.L. is a *Ramón y Cajal* research fellow (MCyT of Spain).

[*Artale et al.,* 1997] Artale, V., G. Boffetta, A. Celani, M. Cencini, and A. Vulpiani, Dispersion of passive tracers in closed basins: Beyond the diffusion coefficient *Phys. Fluids* 9: 3162–3171, 1997.

[*Aurell et al,* 1997] Aurell, E., G. Boffetta, A. Crisanti, G. Paladin, and A. Vulpiani. Predictability in the large: an extension of the concept of Lyapunov exponent. *J. Phys. A*, 30:1–26, 1997.

[*Dietrich,* 1997] Dietrich, D. E., Application of a modified "a" grid ocean model having reduced numerical dispersion to the gulf of mexico circulation. *Dyn. of Atm. and Oceans*, 27:201–217, 1997.

[*Farnetani and Samuel,* 2003] Farnetani, C.G., and H. Samuel. Lagrangian structures and stirring in the Earth's mantle. *Earth Planet. Sci. Lett.*, 206:335–348, 2003.

[*Fernández et al.,* 2004] Fernández, V., D.E. Dietrich, R.L. Haney and J. Tintoré. Mesoscale, seasonal and interannual variability in the Mediterranean Sea using a numerical ocean model. *Progress in Oceanography* (In Press), 2004.

[*Haidvogel and Beckmann,* 1998] Haidvogel, D.B., A. Beckmann, *Numerical Ocean Circulation Modeling*, Imperial College Press, London, 1999.

[*Halpern,* 2000] Halpern, D., (editor). *Satellites, Oceanography and Society.* Elsevier, Amsterdam, 2000.

[*Joseph and Legras,* 2002] Joseph, B., and B. Legras. Relation between kinematic boundaries, stirring, and barriers for the Antartic Polar vortex. *J. Atm. Sci.*, 59:1198–1212, 2002.

[*Koh and Legras,* 2002] Koh, T., and B. Legras. Hyperbolic lines and the stratospheric polar vortex. *Chaos*, 12:382–394, 2002.

[*Kuznetsov et al.,* 2002] Kuznetsov, L., M. Toner, A.D. Kirwan, Jr., C.K.R.T. Jones, L.H. Kantha and J. Choi. Interactions of the Loop Current and adjacent rings in the Gulf of Mexico. *J. Mar. Res.* 60:405–429 (2002).

[*Lacorata et al.,* 2001] Lacorata, G., E. Aurell, and A. Vulpiani. Drifter dispersion in the adriatic sea: Lagrangian data and chaotic model. *Annales Geophysicae*, 19:121–129, 2001.

[*Mariano et al.,* 2002] Mariano, A. J., A. Griffa, T.M. Özgökmen, E. Zambianchi. Lagrangian analysis and predictability of coastal and ocean dynamics 2000. *J. Atm. Ocean. Tech. 19*, 1114–1125, 2002.

[*Martin,* 2003] Martin, A.P., Phytoplankton patchiness: the role of lateral stirring and mixing. *Progress in Oceanography*, 57:125–174, 2003.

[*Ottino,* 1989] Ottino, J.M., *The kinematics of mixing: stretching, chaos, and transport*, Cambridge Univ. Press, Cambridge (UK), 1989.

[*Wiggins,* 1992] Wiggins, S., *Chaotic transport in dynamical systems*, Springer Verlag, New York, 1992.

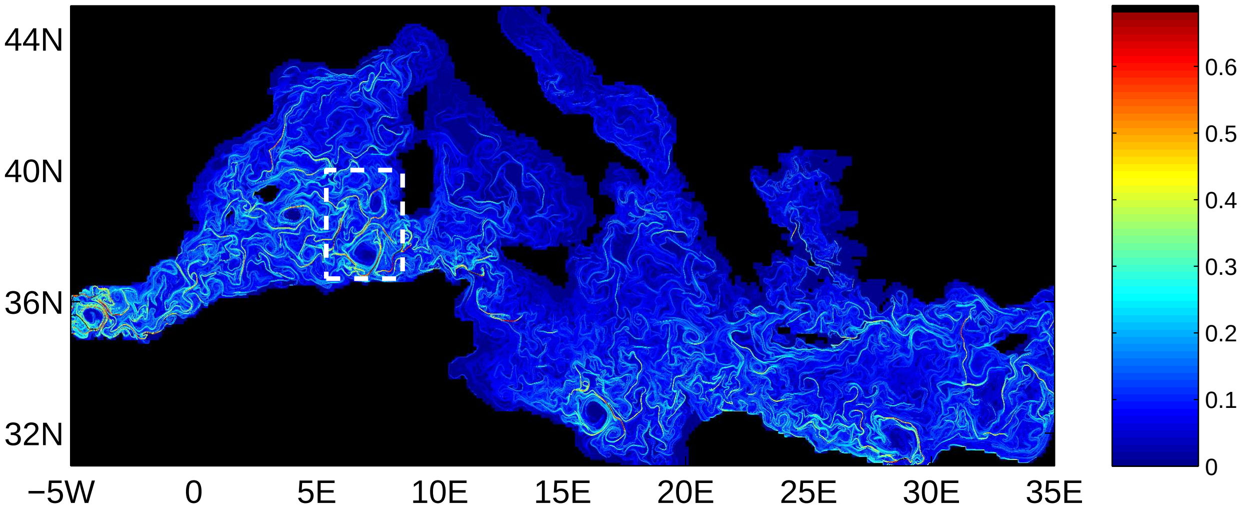

FIG. 1: FSLE spatial distribution for the whole Mediterranean on a specific day (10th of June of the first simulation year). $\Delta x = \delta_0 = 0.02$ degrees. Mesoscale structures and vortices can be clearly seen. Units for FSLEs are day^{-1}. A zoom of the indicated box is presented in fig. 3.

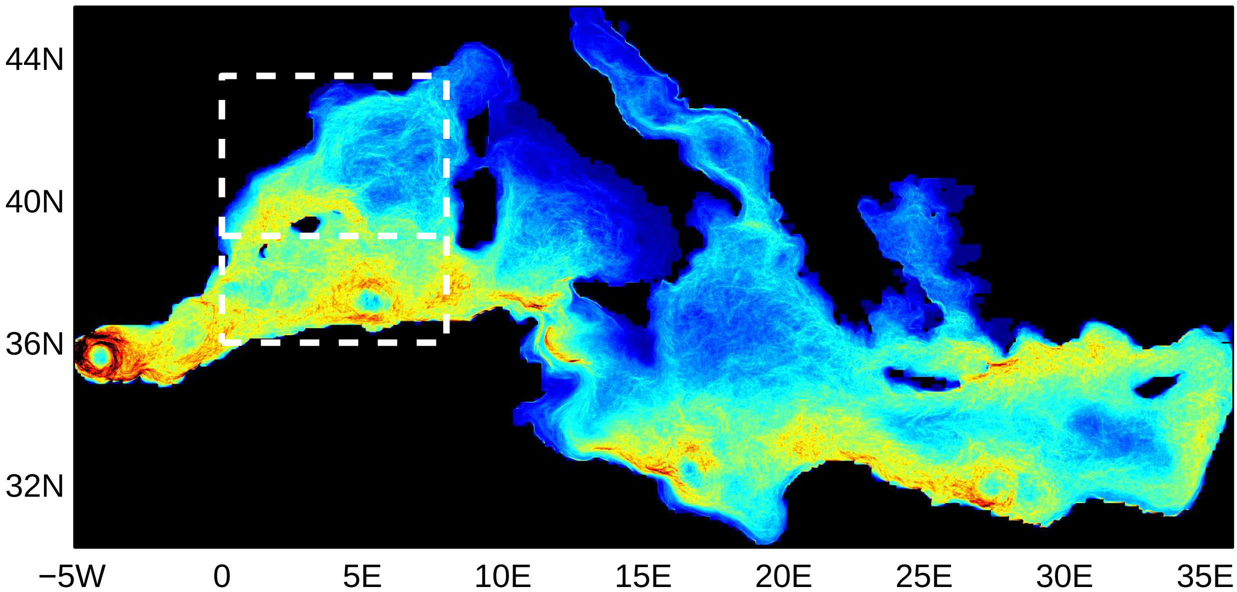

FIG. 2: Time average (for the first simulation year) of the FSLEs in the whole Mediterranean basin. Geographical regions of different mixing activity appear. Colors as in fig. 1.

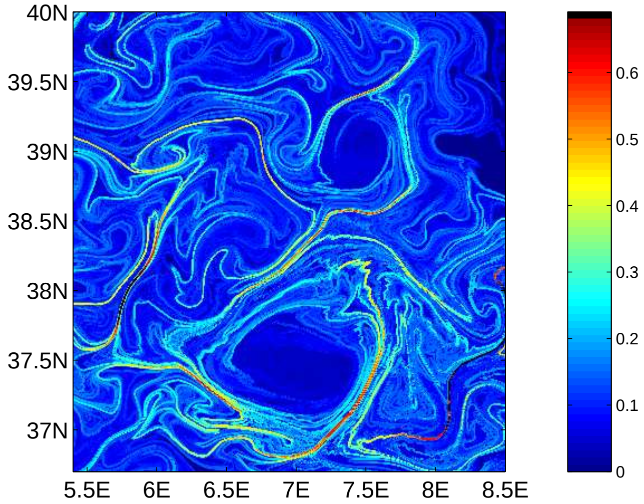

FIG. 3: Enlarged plot of the small box shown in fig. 1.

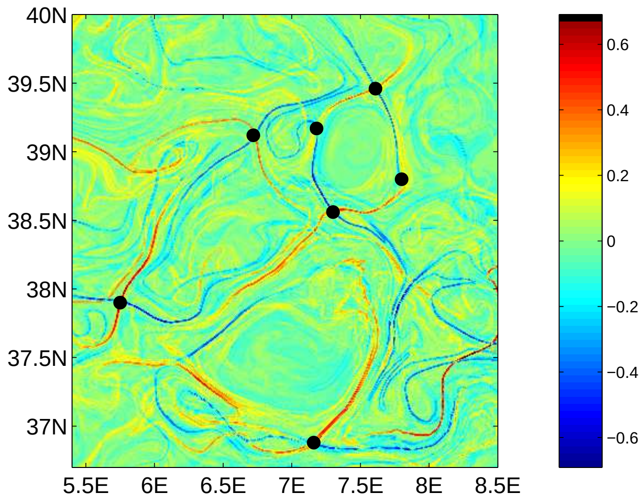

FIG. 4: FSLEs calculated from forward (displayed as positive values) and backwards (displayed as negative values) integrations in time. A region with strong mixing appears organized by a tangle of stretching and compressing manifolds. Such lines organize the flow. The black dots indicate some of the hyperbolic points that are located at the intersections of the lines.

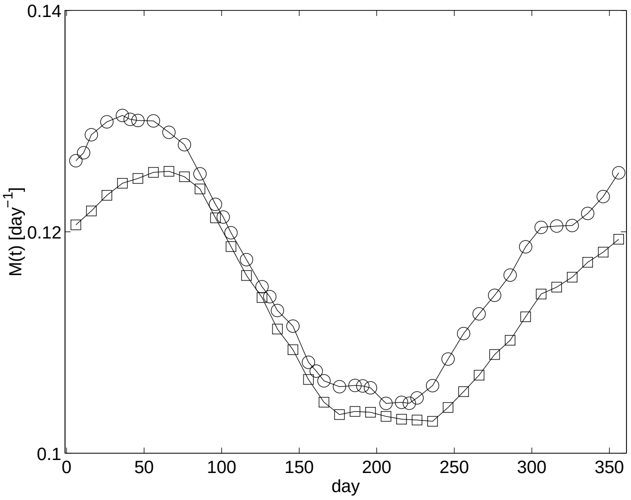

FIG. 5: Temporal evolution of the mixing measures $M_+(t)$ (circles), and of $M_\pm(t)$ (squares) for the whole Mediterranean Sea during one simulation year. They display a very similar behavior, with maximum values in winter.

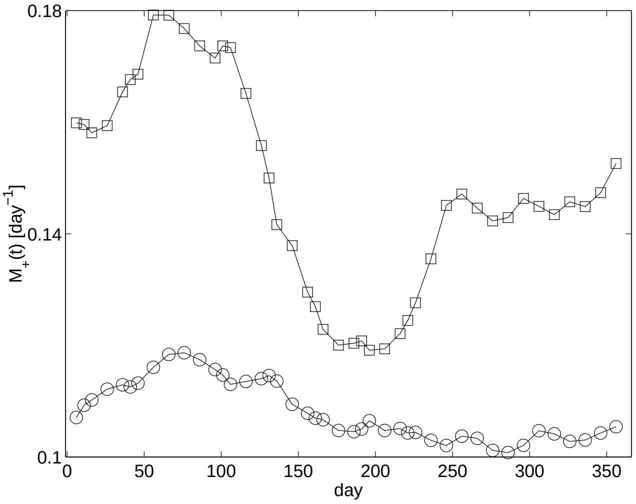

FIG. 6: $M_+(t)$ during one year for the Algerian current (squares), which corresponds to the area delimited by the southern box in fig. 2, and the north of the Balearic islands (circles), which is the northern box in fig. 2.