# net in Spain

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## 1. INTRODUCTION

The Spanish first order gravity net REDGRAES was established in 1973 with the measurements made by combined groups from the IGN (Instituto Geográfico Nacional, Spain) and the DMATC (Defense Mapping Agency Topographic Center, USA).

The gravity data were processed by DMATC and results were reported to ING, (1). Since 1974 to 1977, a scientific cooperation was established between the IGN and the present Instituto de Astronomía y Geodesia (IAG) which proceed to the analysis of the REDGRAES, and the study of the calibration line and the regional densification trips to get an automatic procedure for computing the gravity anomalies in Spain and the global study of the Spanish gravimetry, (20).

Recently these results have been revised to obtain a data bank of gravity anomalies to be used, in cooperation with the «Instituto di Topografia, Fotogrammetria e Geofisica» (Milano), in the gravimetric geoid determination. As results of this analysis possible errors in the first computation and the lack of a statistical analysis have been detected. So, a new adjustment of the Spanish gravity net has been planned using the IGSN 71 stations sited in Spain.

## 2. OBSERVATION DATA

Following the projected gravity survey plans, observations of REDGRAES were made in ladder sequence ABCDDCBA, and in some cases with additional measurements in rest stations.

Four LaCoste&Romberg gravimeters model G (n. 41, 69, 115, 301) were used making consecutive measurements with all of them in every station. For

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each gravimeter two visual readings were noted with the time, but without data of atmospheric pressure, air temperature and orientation of the gravimeter. Transport of gravimeters was made by car in the Peninsula, and by aircraft in Baleares Islands connections.

Station coordinates were taken off from the available cartography, heights are of different classes (indicated with a code) caming from precision levelling to map interpolation.

Figure 2.1 shows the Spanish gravimetric net, stations are marked with squares, every itinerary with a circle and IGSN 71 stations with hexagons. REDGRAES consists of 72 stations distributed in 32 trips. Number of observations is 1091, with 729 repeated stations. The m.s.e. of the mean drift is $\mp$ 0.01 mGal, and the m.s.e. of observation is $\mp$ 0.02 mGal.

We have made a pre-processing data to detect possible errors in the coordinates, in the heights of stations, in times or in gravimetric reading. As control for this analysis we make use of the drift and its mean square error, a chronological control of observations, the regression between free air anomalies and heights, centrality and dispersion measures of gravity values and anomalies, and repeated observations.

As results of this pre-processing we get information about the number of stations involved, number of trips in the network, number of gravvimeters participating, number of measurements done, number of revisiting stations and redundancies and a check of stations are all connected.

## 2.1. Transformation of dial readings to milligals. Calibration factor

Let $m$ be a dial reading and $l$ the corresponding milligal value. The calibration function $F$ gives,

$$l = F(m)$$

The problem is to model this function determining its parameters by the calibration of the instrument.

Several models for the function $F$ have been established. For LaCoste-&Romberg gravimeters manufacturer gives the calibration table for each gravimeter. By interpolation in this table we can determine from $m$ the corresponding relative gravity value $z$. Then, we must transform the $z$ value to a real scale in milligals using the scale factor

$$m \xrightarrow{\text{table}} z \xrightarrow{\text{scale}} l^r$$

Also, the scale factor must be modelated. Usually, (7), (13), (26), this can be done in the form

$$l^r = \sum_{i=1}^{n} c_i z^i + \sum_{j=1}^{m} a_j \cos (v_j z + \varphi_j) \qquad [1]$$

with polynomial and periodic parts.

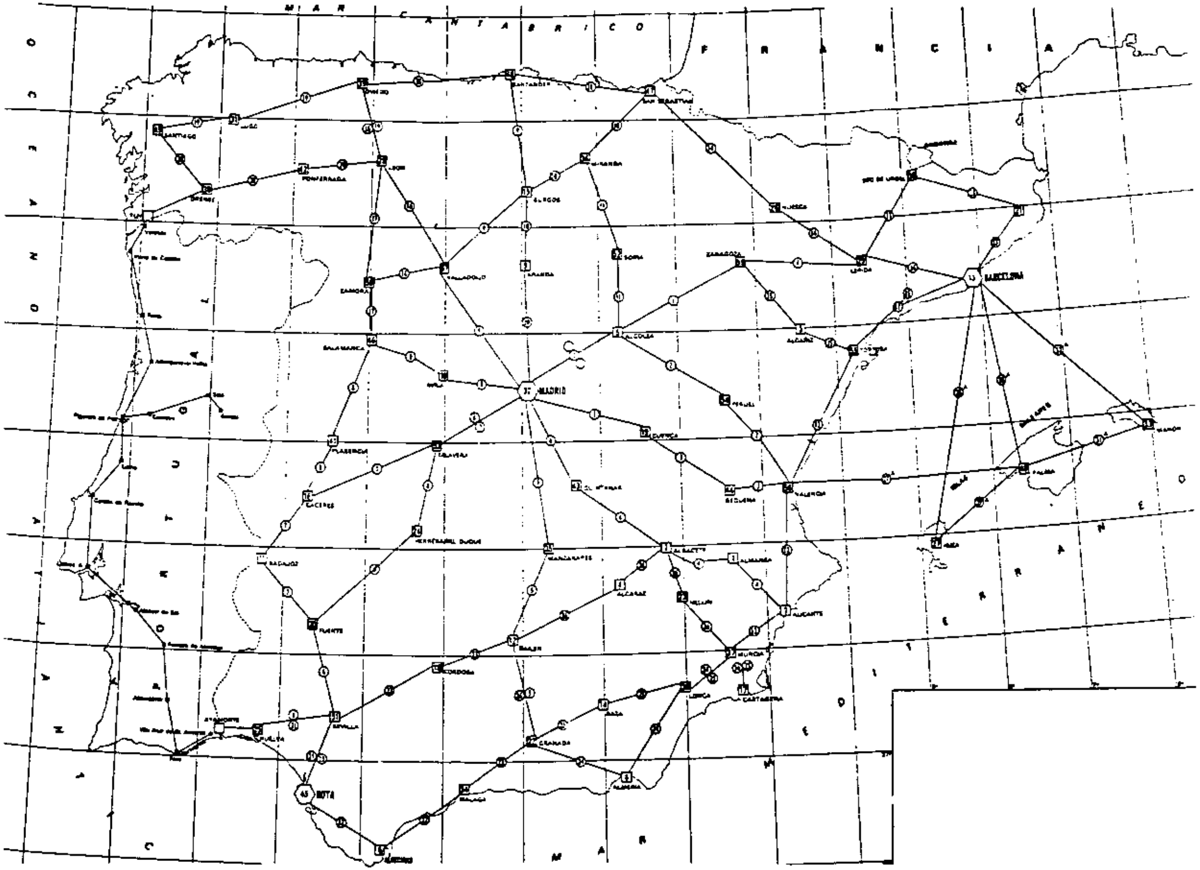


Figure 2.1.—Spanish first order gravity net.

In practice, for short rank of measurements, this model can be reduced to one with a lineal term and two or three periodical terms,

$$l^m = f_0 + f(1 + \Delta f) z + \sum_{j=1}^{2} a_j \cos (v_j z + \varphi_j) \qquad [2]$$

Knowing the factors in this formula we can get the milligals value. Really, we only know the f value (manufacturer gives $f=1$ and $\Delta f=0$) which can have been modified by calibration in a calibration line with absolute gravity measurements. This procedure enables us to determine some periodic coefficients too.

After all, we have three values for the observable $l$

$l^r$ real value of observation in milligals, (unknown)

$l^m$ theoretical value of observation according to the scale model,

$l^c$ calculated value with a provisional scale factor,

$$l^c = fz \qquad [3]$$

being related by,

$$\begin{aligned} l^r \simeq l^m &= f_0 \;\; f(1 + \Delta f) z + \sum_{j=1}^{2} a_j \cos (v_j z + \varphi_j) \\ &= f + l^c + \Delta f \, l^c + \sum_{j=1}^{2} a_j \cos (v_j z + \varphi_j) \end{aligned}$$

and without periodic part

$$l^e = f_o + l^c + \Delta f \, l^c \qquad [4]$$

## 3. SYSTEMATIC EFFECTS. CORRECTIONS

Precise gravity observations must be corrected of all known systematic effects of different kind: Earth tides, polar motion, vertical gradient of the gravity (height of gravimeter above the bench mark), temperature variations, air pressure, influence of the atmospheric pressure on the gravimeter, variations of the groundwater level, changes in the magnetic azimuth, variations in the battery voltage, microsismic effects, long period movements of the crust, mechanic vibrations, etc.

The modelation and evaluation of some of these effects are complicated, and the local effects are difficult to study. Nevertheless, for absolute

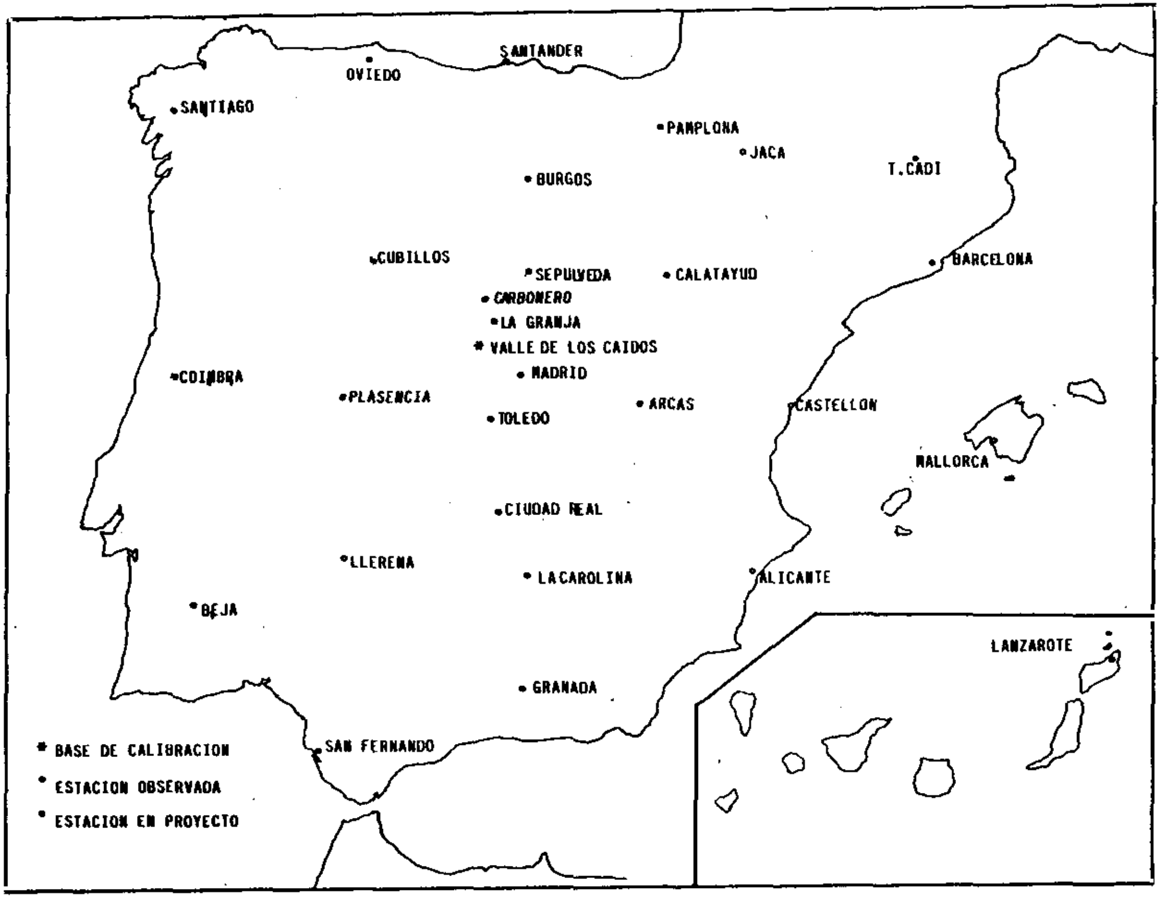


Figure 3.1.—Spanish earth tides network.

gravimetry some of them have been investigated (6), (7), (27). Corrections for absolute measurements which can be applied to relative measurements too, are reviewed in (4). The more important effects that can be corrected are:

### 3.1. Earth tides

At present several precise methods for computing the vertical component of earth tides are available (8), (9), (12), (24), (25), (34), (35). In our work we have used the Cartwright-Tayler-Eden development supplemented by the International Center of Earth Tides with 505 tidal constituents, which provides a precision of 0.1 $\mu Gal$.

As in Spain we have the data of an Earth tides net with 19 stations well distributed (30), see figure 3.1, the $\delta$ and $\kappa$ parameters of the wave groups 01, P1, K1, N2, M2, S2 and M3 have been interpolated from the results obtained in the harmonic analysis of tide observations; the other waves have been modified by the factors calculated from the Molodensky I model (31).

The direct constant part of the tidal gravity effect has been considered following the resolutions of the International Association of Geodesy, (4).

$$\delta_g = -4.83 + 15.73 \sin^2\psi - 1.59 \sin^4\psi \qquad \mu Gal,$$

$\psi$ being the geocentric latitud.

### 3.2. Polar Motion

Another temporal variation in the observed gravity is due to the variation of the Earth rotation axis. To correct this effect we use the Wahr formula (32),

$$\delta_g = 1.164\ 10^8\, \omega^2\, a\ 2\ sin\,\varphi \cos\varphi\ (x \cos\lambda - y \sin\lambda) \qquad \mu Gal,$$

where $x$ and $y$ are the pole coordinates in the IERS system, $\omega$ is the angular velocity of rotation of the Earth, $\omega = 7292115\ 10^{-11}$ rad $s^{-1}$, $a$ is the semimayor axis, $a = 6378136\ m$ and $\varphi$, $\lambda$ are the coordinates of the station. (As for observations of 1973 concerned we have taken the CIO System).

### 3.3. Vertical Gradient

This correction is to refer the gravity measurement to the bench mark of the station. We use the formula for the vertical gradient of the normal gravity,

$$\delta_g = 0.3086\ h\ \mu\text{Gal},$$

$h$ being the height of the instrument in mm, which must be known with a precision of 3 mm. Using the normal gradient instead of the real gradient, the error is about 20 %.

### 3.4. Air pressure

Effects of air pressure variations in gravity measurements are of two kinds: atraction of air masses in the point station and crustal deformations due to these masses. Following (10) and (28), the correction is,

$$\delta_g = 0.30\ 10^{-10}\ \delta p\ \mathrm{m\,s^{-2}}\ ,$$

where,

$$\delta p = P_a - P_n\ ,$$

$P_a$ being the observed pressure and $P_n$ the normal pressure calculated as,

$$P_n = 1.01325\ 10^5\ (1 - 0.0065\ H/288.15)^{5.2559}\ ,$$

$H$ being the station height in meters. This correction has not been applied for the 1973 data.

Other systematic effects are analized in (7).

## 4. OBSERVATIONAL DATA VALUES AND APPROXIMATE GRAVITY VALUES FOR STATIONS

Approximate gravity values for the stations of the net, which will be used in the adjustment, are calculated from dial readings.

Corrected and adjusted in mean results are classified with the following procedure.

First, dial readings $m$ are converted to his value in milligal $l^c$ by interpolating in the calibration tables supplied by manufacturer and the available value of the scale factor. Then, values $l^c$ [3] are the initial values of observations. By other hand, observation times, usually given in date, hour and minute are transformated in fraction of year, to get a continuos series of Universal Time values.

With these times and other data, corrections given in section 3 are calculated and applied to the $(l^c)$ values.

All these corrections are represented by $c_s$.

Wiht these times and other data, corrections given in section 3 are calculated and applied to the ($l^c$) values.

Thus, we have two kinds of observations, the calculated ones (numerical values), from [3],

$$l = l^c + c_s \,, \qquad [5]$$

and the modelates ones, from [2]

$$l^s = l^m + c_s \,, \qquad [6]$$

that in the restricted model [4] are given by,

$$l^s = f_0 + l^c + \Delta f \, l^c + c_s \,. \qquad [7]$$

## 4.1. Modelling of gravimeter drift

Instrumental drift must be taken into account when observations made by the same gravimeter are accumulated. For short ties, drift of LaCoste & Romberg gravimeters can be considered as a lineal function of time. More complicated models used to obtain the estimated values do not show significative differences with the lineal model (17). Then, we write,

$$D(t) = D_0 + d \; t \,, \qquad [8]$$

where $D_0$ is the accumulative drift up to $t = 0$, (the same for a series of measurements), and $d$ the rate of the drift, taken the same for each gravimeter and each trip, $t$ is the time elapsed from the initial time.

Then, a gravimetric observation at time $t$ is modelled by adding to expression [7] the model [8],

$$l^d = f_0 + l^c + \Delta f l^c + c_s + D_0 + dt \qquad [9]$$

This expression will be taken into account for the design of the network adjustment model.

Nevertheless, in order to calculate well approximate gravity values in the observation stations, we need to know a numerical value of observation itself, which means that we need a numerical value of parameters $D_0$ and $d$ if we want to consider the drift. In this case iteration procedure in the adjustment can be avoided.

Under the hipothesis that gravimeter drift is the time variation of corrected measurements made in the same station, we can determine $d$.

Let $m_i$, $m_i'$ be two measurements made with the same gravimeter in the same station and belonging to the same trip in $t$ and $t'$ times respectively, so from [5] and [9] we have the values,

$$l_i^d = f_0 + l_i + \Delta f\, l_i^c + D_0 + dt_i \,,$$

$$l'^d_i = f_0 + l_i' + \Delta f\, l'^c_i + D_0 + dt_i' \,,$$

as $\Delta f$ is almost zero, and $l_i^c$ very close to $l'^c_i$ and futhermore $l_i^d = l'^d_i$ we can write,

$$d = (l_i - l_i') \,/\, (t_i' - t_i) \,.$$

Let us consider $m$ revisiting stations ($l_i$, $l_i'$ observation in times $t_i$, $t_i'$, $i = 1, 2, \ldots m$), then we can get $m$ calculated values of $d$. In these conditions our problem is to find the optimum value of $d$ and its corresponding precision. We establish the following least squares model of observations equations.

$$(t_i' - t_i)\, d - (l_i - l_i') = v_i \,.$$

The normal equation for the single unknown $d$ is (21),

$$\sum_{i=1}^{m} (t_i' - t_i)^2 \hat{d} - \sum_{i=1}^{m} (l_i - l_i')(t_i' - t_i) = 0$$

from this equation we get the best lineal unbiased estimation of the drift $\hat{d}$.

The estimated residual errors are,

$$\hat{v}_i = (t_i' - t_i)\, \hat{d} - (l_i - l_i') \,,$$

and the aposteriori variance of unit weight is,

$$\hat{\sigma}_0^2 = \frac{1}{m-1} \sum_{i=1}^{m} \left[ (t_i' - t_i)\, \hat{d} - (l_i - l_i') \right]^2 .$$

Now, the cofactor of $\hat{d}$ is directly obtained from the normal equation as,

$$q_{\hat{d}} = 1 \;/\; \sum_{i=1}^{m} (t_i' - t_i)^2 \,,$$

with this, the variance of $\hat{d}$ is given by,

$$\hat{\sigma}_{\hat{d}}^2 = \hat{\sigma}_0^2 \;/\; \sum_{i=1}^{m} (t_i' - t_i)^2 \,,$$

The ajusted observations are now,

$$\hat{l}_i - \hat{l}_i' = (t_i' - t_i)\, \hat{d} \,,$$

and theirs aposteriori variances,

$$\hat{\sigma}^2_{\hat{l}_i - \hat{l}_i'} = \hat{\sigma}_{\hat{d}}^2 \,(t_i' - t_i)^2 \,,$$

### 4.2. Removing the static drift

It has been proved that gravimeters work in different way in rest of during transport, (11), (23). In the first case, for instance during the night in a trip, we always have a measurement before rest (end of transport) and other measurement before the next transport (end of rest). In the second case the measurements are made between stations and the behavior of gravimeter must be controled.

To obtain the drift we consider the variations when the gravimeter is operating or in transport only. Therefore, we remove of the measurements the static drift, calculated with the data of the begining and the end of rest, subtracting their effect from the next to the final observed values, the same thing is done with the corresponding time tag.

### 4.3. Calculating the gravity

Let $G_I$ be the reference gravity of the initial station of the trip, $l_I$ the observation in this station $P_I$ in time $t_I$, $l_j$ the observation in station $P_j$ in time

$t_j$, and $\hat{d}$ the rate of the drift in the trip, then the approximated gravity in station $P_j$, $j = 2, ..., n$ is given by,

$$g_j^0 = G_1 + l_j - l_1 - (t_j - t_1)\,\hat{d}\,, \qquad [10]$$

and its corresponding mean square error by,

$$m_j = \sigma_{\hat{d}}\,(t_j - t_1)\,.$$

As references values in the initial stations of the trips we take the IGSN 71 gravity values; when the initial station do not belong to IGSN 71 we take the result of a previous calcul through another trip linked to the IGSN 71; this connection is always possible.

If we get several values of gravity in a station due to redundancy of observations, to get a sigle value we take the arithmetic mean, and the same is done if we have measurements of different gravimeters. The resulting values of this procedure will be taken as approximate values of gravity in the network adjustment.

## 5. ADJUSTMENT MODELS

With the establishment of the International Gravity Standardization Network IGSN 71 and the moderns National Base Networks, several mathematical models for adjustment of gravimetric networks have been developed, (2), (6), (11), (13), (14), (15), (17), (18), (23), (29), (33), (36).

Two fundamental models with different options to study the influence of the parameters will be considered: the model of observation equations and the mixed model. In the first the observables are differences of observed gravity between two consecutive stations in the same trip for every used gravimeter, and in the second they are observed gravities in each station. Here we present the first model.

### 5.1. **Model of observation equations**

In general, the observed gravity at a station $P$, taken into account [1], [6], and [8] can be written as,

$$l_p^d = \sum_{i=0}^{n} c_i z^i + \sum_{j=1}^{m} a_j \cos(\nu_j z + \varphi_j) + D_c + d\,t + c_s\,,$$

The particular conditions of the observations of REDGRAES lead us to consider the restricted model [9] with [5],

$$l_p^d = f_0 + l_i + D_0 + dt_i + \Delta f\, l_i^c + s_p , \qquad [11]$$

where $s_p$ is a parameter to include unknown possible additional systematic effects.

In the same way, for another station $Q$ we have,

$$l_q^d = f_0 + l_j + D_0 + dt_j + \Delta f\, l_j^c + s_q , \qquad [11]$$

So, the difference of observed gravity is,

$$l_p^d - l_q^d = l_i - l_j + d(t_i - t_j) + \Delta f\,(l_i^c - l_j^c) + (s_p - s_q) . \qquad [12]$$

Let $g_p$, $g_q$ be the gravity values of stations $P$, $Q$, then the observation equation is written as,

$$(g_p - g_q) - (l_p^d - l_q^d) = v . \qquad [13]$$

If we take approximate values of gravity $g_p^0$, $g_q^0$, using [10] we can write,

$$\begin{aligned} g_p &= g_p^0 + \delta g_p , \\ g_q &= g_q^0 + \delta g_q , \end{aligned} \qquad [14]$$

Therefore, with [12], [13] and [14] we can establish the *model of observation equation* in the following way,

$$\begin{aligned} & s_q - s_p + \delta g_p - \delta g_q + d_k (t_j - t_i) + \Delta f_l \,(l_j^c - l_i^c) - \\ & - [(l_i - l_j) - (g_p^0 - g_q^0)] = v_r , \end{aligned} \qquad [15]$$

with,

$s_p$, $s_q$ systematic parameters, $p, q = 1, ..., n_e$,

$\delta g_p$, $\delta g_q$, correcting parameters to the approximate values of gravity in stations $p, q = 1, ..., n_e$,

$d_k$ parameters of drift, one parameter for each gravimeter and each trip, $k=1, ..., n_{gi}$,
$t_j - t_i$ time interval of measurement,
$\Delta f_l$ parameter for correction calibration factor, $l=1, ..., n_g$,
$l_i - l_j$ diferences of raw observations,
$l_j^c - l_i^c$ diferences of observations corrected for known systematic effects,
$g_p^0 - g_q^0$ difference of approximate gravity values,
$v_r$ residuals, $r=1, ..., m$,
$n_e$ number of stations,
$n_g$ number of gravimeters,
$n_i$ number of trips,
$n_{gi}$ $=n_g n_i$,
$m$ number of observed differences of gravity.

## 5.2. Control equations

The control of a gravimetric net is achieved if absolute sites are avalaible. These stations, which give the gravimetric reference to the network, will be considered as fix stations or weight reference stations (at present, absolute measuurements made in the same station with different instruments give significative discrepances). Absolute base points are of great importance because the quality of a gravity network depends on the accuracy and distribution of absolute base points (5).

For the other reference stations (IGSN 71), according to the suggested procedure (16) for readjusting existing sub-nets, we introduce the gravity as weighted observations in the observation equations. Thus, for each fixed station we applied the condition $\delta g_f = 0$, and for each weighted station we add the equation,

$$\delta g_R = v_R, \qquad R = 1, 2, ..., m_R, \qquad [16]$$

which is more or less weighted depending how close the network will be fitted to these $m_R$ points. For large weight the network is fit to these points, and for small weight the network is less deformed and some corrections to reference gravity values come out.

In any case, excep in the free network adjustment, a reference station (fixed or weighted) will be necessary to avoid the rank deficiency in origin in the design matrix.

In the final results of the adjustment of the Spanish first order gravity net we have taken the following IGSN 71 weighted stations

| | | |
|---|---|---|
| MADRID C | N.1032 | $g_M^0 = 979955.61$, |
| BARCELONA J | N.1013 | $g_B^0 = 980306.23$. |

### 5.3. Reference Gravimeter

The approximate values of correction calibration factor $\Delta f$ of all gravimeters are taken equal to zero. If we want to fix the scale of the network by a particular gravimeter we must use the condition $\Delta f=0$ for this gravimeter. It is not necessary the reference gravimeter has been used in all stations of the network. Weighted equations for all scales can be also added in the form,

$$\Delta f_G = v_G, \qquad G=1, ..., m_G, \tag{17}$$

with a weight more or less large according to the closeness to each $m_G$ gravimeter scale we want the scale of the network remains.

If we have only considered a fix or reference station, except in the free network adjustment, a reference scale factor (fixed or weighted) will be necessary to avoid the rank deficiency in scale in the design matrix.

If we take at least two reference stations, the scale of the network is given by the gravity of these stations, and the scale factors of all gravimeters can be determined in the adjustment. Particulary, if these two stations are absolute sites we treat with a real calibration of instrument.

The use of differences of gravity as observations presents the following advantages: the parameter $D_0$ of drift is missing, we have removed as many parameters as trip by gravimeter number, also the factor $f_0$ disappears, in equal number as gravimeters; and static drifts are removed.

In the final results of the adjustment of the REDGRAES we have taken as reference scale factor the gravimeter number 115. (See Section 5.5).

### 5.4. Matricial Formulation

The adjustment model of observation equations according to [15], [16] and [17] can be written in the form,

$$A\underline{x} - \underline{t} = \underline{v}, \tag{18}$$

where $A$ is the design matrix given by,

$$\begin{bmatrix} A_S & A_E & A_D & A_{\Delta f} \\ 0 & A_R & 0 & 0 \\ 0 & 0 & 0 & A_G \end{bmatrix} \tag{19}$$

the different block being,

$$A_S = \begin{bmatrix} -1 & 1 & 0 & \cdots\cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots\; -1 & 1 \end{bmatrix}, \quad A_E = \begin{bmatrix} 1 & -1 & 0 & \cdots\cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots\cdots & 1 & -1 \end{bmatrix},$$

$$A_D = \begin{bmatrix} t_2 & -t_1 & 0 & \cdots\cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots\cdots & t_n & -t_{n\;1} \end{bmatrix}, \quad A_{\Delta f} = \begin{bmatrix} l_2 & -l_1 & 0 & \cdots\cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots\cdots & l_n & -l_{n\;1} \end{bmatrix}$$

$$A_R = \begin{bmatrix} \cdots & 1 & \cdots\cdots\cdots & & \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots\cdots\cdots & & & 1 & \cdots \end{bmatrix}, \quad A_G = \begin{bmatrix} \cdots\cdots\cdots & & & 1 & \cdots \\ \cdots & 1 & \cdots\cdots\cdots & & \\ \cdots\cdots\cdots & & & 1 & \cdots \end{bmatrix},$$

with dimensions,

$$\dim A_s = (m,\ n_{gi}), \qquad \dim A_E = (m,\ n_g),$$
$$\dim A_D = (m,\ n_{gi}), \qquad \dim A_{\Delta f} = (m,\ n_g),$$
$$\dim A_R = (m_R,\ n_{gi}), \qquad \dim A_G = (m_G,\ n_g),$$

The parameter vector $\underline{x}$ is,

$$\underline{x} = (s_1,\ \ldots\ s_{n_e},\ \delta g_1,\ \ldots\ \delta g_{n_e},\ d_1,\ \ldots\ d_{n_{gi}},\ \Delta f_1,\ \ldots\ \Delta f_{n_g})^T,$$

and the constants of observations vector $\underline{t}$ is,

$$\underline{t} = \left[[l_i - l_j - g_p^0 + g_q^0]_1,\ \ldots,\ [l_i - l_j - g_p^0 + g_q^0]_m,\ \ldots\ 0\ \ldots\ 0\right]^T,$$

with dimensions,

$$\dim \underline{x} = (2n_e + n_{gi} + n_g,\ 1),$$

$$\dim \underline{t} = (m + m_R + m_g,\ 1),$$

*Regular network*

In this case matrix $A$ is a full rank regular matrix and the number of degrees of freedom is equal to the number of equations minus the number of unknowns. It is necesary at least to have fixed one station and one gravimeter or to add one equation of type [16] and another of type [17].

*Free network*

In this case we can have a deficiency of rank equal to 1 or 2. If we have fixed the scale with a gravimeter to solve the mathematical model we add the inner constrain,

$$\sum_{k=1}^{n_e} \delta g_k = 0,$$

in the second case, if the scale has not been fixed, we also add the equation,

$$\sum_{k=1}^{n_e} \delta g_c \, g_i^0 = 0.$$

The free adjustment has some advantages, as we do not take any privileged station (or gravimeter) all gravity data are corrected. This kind of adjustment will be used to study the precision of different observation groups, and to detect outliers.

## 5.5. Stochastic model

Taking as observables gravity differences the apriori covariance matrix for one gravimeter is (6),

$$\Sigma_{l_i - l_j} = \sigma_0^2 \begin{bmatrix} 2 & -1 & \cdots & & 0 \\ -1 & 2 & -1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & & -1 & 2 \end{bmatrix},$$

$\sigma_0^2$ being the apriori variance. This correlation has influence in the estimated variances for the adjusted gravity values. However, due to it is a small value in practice it is not considered (6), (13), (26), (29). So, we take as weighting matrix the diagonal matrix $P = \sigma_0^2 \, \Sigma^{-1} = Q^{-1}$, being $Q$ the cofactor matrix.

By individual adjustments of measurements of each gravimeter and several surveying areas also, we have got the results given in Table I.

TABLE I

| I | II | III | IV | V | VI | VII | VIII | IX | X | XI |
|---|---|---|---|---|---|---|---|---|---|---|
| 301 | 200 | 3 | 1 | 20 | 26.4 | 46.0214 | 22 | 53 | 23 | 26 |
| 115 | 208 | 0 | 0 | 20 | 15.2 | 47.3163 | 12 | 24 | 14 | 14 |
| 69 | 202 | 1 | 1 | 20 | 18.7 | 43.9071 | 15 | 38 | 17 | 18 |
| 41 | 204 | 0 | 0 | 20 | 16.8 | 45.6084 | 14 | 35 | 15 | 16 |

I NUMBER OF GRAVIMETER, II NUMBER OF OBSERVATION, III NUMBER OF REJECTED OBSERVATIONS, IV ITERATION NUMBER, V APRIORI STANDARD DEVIATION, VI APOSTERIORI STANDARD DEVIATION, VII TRACE OF THE CO-FACTOR MATRIX OF PARAMETERS, VIII MAXIMUM STANDARD DEVIATION OF PARAMETERS, IX MAXIMUM RESIDUAL, X MAXIMUM STANDARD DEVIATION OF RESIDUALS, XI MAXIMUM STANDARD DEVIATION OF OBSERVATIONS. (UNITS ARE $\mu$GAL.).

Therefore, to homogenize the network we have adopted the following weight factors

$$
\begin{aligned}
F_B\ 301 &= 0.57 \Rightarrow 0.329,\\
F_B\ 115 &= 1.73 \Rightarrow 1.0\ \ ,\\
F_B\ \ 69 &= 1.14 \Rightarrow 0.659,\\
F_B\ \ 41 &= 1.42 \Rightarrow 0.821,
\end{aligned}
$$

and a priori standard deviation of unit weight of $\sigma_0 = 20$ *μgal.*

For reference stations, weight is about 1 and for scale factors about 25.000.

In the statistical analysis after adjustment the following assumptions must be tested: 1) Residuals are normally distributed, 2) The mathematical expectation of $\underline{v}$ is equal to zero, 3) the apriori variance of unit weight must be consistent with the results of adjustment.

## 5.6. **Estimated values**

The least squares solution of the mathematical model [18] is given by, (21)

$$\underline{\hat{x}} = (A'PA)^{-1}\, A'P\underline{t}, \qquad [20]$$

the residuals are obtained as,

$$\underline{\hat{v}} = A\underline{\hat{x}} - \underline{t} = [A'(A'PA)^{-1}\, A'P - I]\ \underline{t}\,. \qquad [21]$$

The variance of unit weight is,

$$\hat{\sigma}_0^2 = \frac{(\hat{\underline{v}}' P \hat{\underline{v}})}{f}, \qquad [22]$$

where $f = m$-rank$(A)$ is the number of degrees of freedom of adjustment.

And de covariance matrix are,

$$\Sigma_{\underline{\hat{x}}\underline{\hat{x}}} = \hat{\sigma}_0^2 \, Q_{\underline{\hat{x}}\underline{\hat{x}}} = \hat{\sigma}_0^2 \, (A'PA)^{-1}, \qquad [23]$$

for parameters,

$$\Sigma_{\underline{\hat{v}}\underline{\hat{v}}} = \hat{\sigma}_0^2 \, Q_{\underline{\hat{v}}\underline{\hat{v}}} = \hat{\sigma}_0^2 \, [\, Q - A\,(A'PA)^{-1} A' \,], \qquad [24]$$

for residuals, and,

$$\Sigma_{\hat{l}_i - \hat{l}_j} = \hat{\sigma}_0^2 \, Q_{\Delta\hat{l}\Delta\hat{l}} = \hat{\sigma}_0^2 \, [A\,(A'PA)^{-1} A'\,], \qquad [25]$$

for observations. $Q_{\underline{\hat{x}}\underline{\hat{x}}}$, $Q_{\underline{\hat{v}}\underline{\hat{v}}}$, $Q_{\Delta\hat{l}\Delta\hat{l}}$ are the corresponding cofactor matrix.
The estimated values of the final adjustment are shown in Section 8.

## 6. **Statistical Analysis**

The statistical analysis of adjustment results is made in order to,

— check the hypothesis of the Gauss-Markov model adopted,
— detect outliers,
— study the reliability of the network,
— find the optimum model.

So, after that the adjustment is accomplished, the following test are applied (22).

### 6.1 $X^2$ — *test of normality of residuals*

We applied this test to the vector $\hat{\underline{v}}$ with a classification of 20 intervals at a significance level of $\alpha = 0.05$.
For the final results of REDGRAES we have

COMPUTED VALUE = 23.876
CHI VALUE = 27.593

### 6.2 F-*test of the variance of unit weight*

We define

$$y = \max\left[\frac{\hat{\sigma}_0^2}{\overline{\sigma}_0^2},\ \frac{\sigma_0^2}{\overline{\sigma}_0^2}\right],$$

the null hypotesis

$$H_0 : \hat{\sigma}_0^2 = \sigma_0^2 ,$$

is rejected if

$$y > F_{f,\infty,\alpha}$$

where $F$ is the value of the $F$-distribution with $f$ and $\infty$ degreess of freedom at a significance level $\alpha$.

For the final results of REDGRAES we have

| | |
|---|---|
| APRIORI VARIANCE | = 0.00040 |
| APOSTERIORI VARIANCE | = 0.00036 |
| $Y$-STATISTIC VALUE | = 0.905 |
| $F_{613,\infty,0.05}$ | = 1.099 |

### 6.3. *t-test for systematic errors*

We define the statistic,

$$y = \frac{\hat{W}_m}{\sigma_{\hat{W}_m}},$$

where $\hat{W}_m$ is the mean value of the standardized residuals and $\sigma_{\hat{W}_m}$ their mean square error, then the hypothesis,

$H_0$: there are not systematic errors $(E[v]=0)$

is rejected if,

$$y > t_{m-1,\alpha}$$

with $t$ being the Student-distribution with $m-1$ degrees of freedom.

For the final results of REDGRAES we have

Y-STATISTIC VALUE = 0.088
$t_{812,0.05}$ = 1.963

### 6.4 Pope test to detect outliers

We applied this test to the standardized residuals,

$$\hat{v}_i = \hat{v}_i \;/\; \hat{\sigma}_{\hat{v}_i}, \qquad [26]$$

where $\hat{v}_i$ are the residuals given by [21] and $\hat{\sigma}_{\hat{v}_i}$ the variance of $\hat{v}_i$ taken off from $\Sigma_{\hat{v}\hat{v}}$ [24]. If

$$|\hat{v}_i| \geq \tau_{m-n+1,\,\alpha/2}$$

$\tau$ being the Tau-distribution (19), the corresponding observation (two consecutive gravimeter readings) is rejected, and must be investigated for possible errors. If it is not locate the error cause, alternatively each gravimeter reading is eliminated, if $H_0$ is in both cases rejected, the two measurements are eliminated.

In the REDGRAES adjustment 2 observations have been rejected.

### 6.5. Reliability parameters

Following theory of Baarda (3), we compute the reliability parameters, Baarda coefficient,

$$q_B = q_{\hat{i}} \;/\; q_l \qquad [27]$$

External local reliability for parameters,

$$r_x = \sqrt{\frac{q_B}{1 - q_B}} \quad 3.44, \qquad [28]$$

Internal local reliability for observations,

$$r_\tau = \sqrt{\frac{1}{1 - q_B}} \quad 3.44, \qquad [29]$$

Minimum error detectable,

$$\Delta_m = r_\tau \, \sigma_0 \, q_l^{1/2}, \qquad [30]$$

Effect of the minimum error detectable on a residual,

$$\Delta_v = (1 - q_B)\,\Delta_m\,, \qquad [31]$$

where $q_l$ is the diagonal element of the apriori cofactor matrix of observations $Q$ and $q_{\hat{l}}$ the aposteriori corresponding element of $Q_{\Delta\hat{l}}$.

The reliability parameters give us information about the possibility to detect outliers. Following Baarda tables, in ordinary cases ($A0 = 0.001$, $B0 = 20$, $\lambda_0^{1/2} = 3.44$) for good controled observations, $0.6 < q_B < 0.9$, $5 < r_x < 12$, $6.5 < r_t < 13$. For lower values, observations (or parameters) are very well controled. This can be used not only for control but to get a optinum design criteria of the network.

The trip 132 Gra. 301 (3 observations) has been rejected not by the Tau-Test, but by its large reliability parameters (11.457, 10.928).

This happened only in this case due to a false connection between stations, having taken the same measured value as initial and final values (static drift include) in contraposition with the connections made with the others gravimeters in the same trip. Thus, it is shown the utility of the reliability parameters.

### 6.6 **Extreme values**

For the control of the different estimate vectors in the adjustment process the following values are computed: sume of elements, mean value, standard deviation, maximun value and its position, minimum value and its position, number of zeros (See Table IV).

For the final results of REDGRAES adjustment see Section 8.

## 7. DIFFERENT MODELS COMPARISON

Several mathematical models of adjustment have been considered to study the influence of different parameters.

Using the same set of observations (acepted in the statistical analysis of section 6), the same stations, the same datum and the same unit of weight, sixteen different models with different number of unknowns have been adjusted (In Table II, asteriscs correspond to considered parameters).

The parameters $G$ (one for each gravimeter) have been added to the models to take into account some systematic effects of the gravimeters and to have a procedure to standardize the measurementes made with any gravimeter to a reference gravimeter o set of gravimeters.

All these models are regular, having taken as weighted station MADRID $C$ and as weighted scale factor the gravimeter 115 according to [16] and [17] respectively.

TABLE II

Adjustments.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $s_p$ | | * | | * | | * | | * | | * | | * | | * | | * |
| $s_q$ | | * | | * | | * | | * | | * | | * | | * | | * |
| $g_p$ | * | * | * | * | * | * | * | * | * | * | * | * | * | * | * | * |
| $g_q$ | * | * | * | * | * | * | * | * | * | * | * | * | * | * | * | * |
| $d$ | | | * | * | | | | | * | * | * | * | | | * | * |
| $\Delta f$ | | | | | * | * | | | * | * | | | * | * | * | * |
| $G$ | | | | | | | * | * | | | * | * | * | * | * | * |

To compare the differents models, we establish the following statistical testing in terms of the linear general hypothesis. Let $A$ and $B$ be two of these models, the model $B$ with the new parameters, which influence we want to check and that would be equal to zero if they have not any important significance. We take the model $B$ (with all parameters)

$$A_B \underline{x} - \underline{t}_B = \underline{v}_B,$$

and the linear general hypothesis,

$$\underline{H} \underline{x}_c - \underline{h} = \underline{O},$$

where $\underline{x}_c$ are the added parameters that if they are zero will be $H = I$, and $\underline{h} = \underline{O}$.

Solving the model $B$ (with all the parameters) and the model $A$ (without the added parameters) we can compute the quadratic forms $(\hat{v}' P \hat{v})_B$, $(\hat{v}' P \hat{v})_A$. If we define the estimate,

$$y = \frac{(\hat{v}' P \hat{v})_A - (\hat{v}' P \hat{v})_B}{(\hat{v}' P \hat{v})_B} \frac{f}{s}, \qquad [32]$$

where $f$ is the number of degrees of freedom of the $B$ adjustment, and $s$ the number or added parameters; the null hypothesis,

$H_O$ : The added parameters are no significatives, can be tested in the following way, if

$$y > F_{s,f,\alpha}$$

where $F_{s, f, \alpha}$ is the value of the *F*-distribution on *s* and *f* degrees of freedom, the null hypothesis is rejected and can be concluded that the model *A* can be improved. If the null hypothesis is accepted, the model *A* is corrected and at the significance level $\alpha$, nothing succeed by adding new parameters.

Table III and Table IV show the concerning results of the some of the different adjustments numbered in Table II.

TABLE III

| *Adjustments number* | *1* | *3* | *5* | *9* | *13* | *15* |
|---|---|---|---|---|---|---|
| Number of station ........................ | 72 | 72 | 72 | 72 | 72 | 72 |
| Number of fixed stations .................. | 0 | 0 | 0 | 0 | 0 | 0 |
| Number of fixed scales .................... | 0 | 0 | 0 | 0 | 0 | 0 |
| Number of weighted reference stations ........ | 1 | 1 | 1 | 1 | 1 | 1 |
| Number of weighted reference scales ......... | 1 | 1 | 1 | 1 | 1 | 1 |
| Number of gravimeters .................... | 4 | 4 | 4 | 4 | 4 | 4 |
| Number of trips .......................... | 125 | 125 | 125 | 125 | 125 | 125 |
| Number of the reference gravimeter .......... | 0 | 0 | 0 | 0 | 0 | 0 |
| Number of systematic unkonwns ............ | 0 | 0 | 0 | 0 | 0 | 0 |
| Number of station unknowns ............... | 72 | 72 | 72 | 72 | 72 | 72 |
| Number of drift unkknowns ............... | 0 | 125 | 0 | 125 | 0 | 125 |
| Number of scale unknowns ................. | 0 | 0 | 4 | 4 | 4 | 4 |
| Number of normalitation unknowns ......... | 0 | 0 | 0 | 0 | 4 | 4 |
| Total number of unknowns ................ | 72 | 197 | 76 | 201 | 80 | 205 |
| Total number of observations .............. | 815 | 815 | 815 | 815 | 815 | 815 |

| *Adj. Num.* | *1* | *3* | *5* | *9* | *13* | *15* |
|---|---|---|---|---|---|---|
| I | 1.0452 | 1.0839 | 0.9314 | 0.9524 | 0.9327 | 0.9540 |
| II | −21.5974 | 1.6972 | −21.6461 | 1.7696 | −20.9453 | 1.4150 |
| III | −0.4351 | 0.0992 | −0.4363 | 0.1044 | −0.4186 | 0.0955 |
| IV | 811.7459 | 726.0754 | 641.0784 | 556.9259 | 639.3358 | 555.1670 |
| V | 0.5177 | 0.4582 | 0.4376 | 0.3790 | 0.4364 | 0.3777 |
| VI | 0.0209 | 0.0217 | 0.0186 | 0.0190 | 0.0187 | 0.0191 |
| VII | 90.5581 | 90.6946 | 104.7022 | 104.8407 | 108.1236 | 108.3636 |
| VIII | 0.0234 | 0.0243 | 0.0225 | 0.0230 | 0.0229 | 0.0234 |
| IX | 743 | 618 | 739 | 614 | 735 | 610 |
| X | 3.9782 | 4.0772 | 3.9780 | 4.0830 | 3.9779 | 4.0740 |

I = Variance factor.
II = Sum of weighted residuals.
III = Sum of residuals.
IV = Sum of squares of weighted residuals.

V = Sum of squares of residuals.
VI = Aposteriori standard deviation.
VII = Trace of the parameters cofactor matrix.
VIII = Mean standard deviation of parameters.
IX = Number of degrees of freedom.
X = Tau-Value.

TABLA IV

| *1* | *3* | *5* | *9* | *13* | *15* |
|---|---|---|---|---|---|
| 5.875 | 10.477 | 5.378 | 9.284 | 5.565 | 9.410 |
| 0.007 | 0.013 | 0.007 | 0.011 | 0.007 | 0.012 |
| 0.001 | 0.004 | 0.001 | 0.003 | 0.001 | 0.003 |
| 0.015 ( 3) | 0.036 (814) | 0.013 ( 1) | 0.032 (814) | 0.013 ( 1) | 0.032 (814) |
| 0.005 (178) | 0.006 (456) | 0.004 (294) | 0.005 (456) | 0.004 (294) | 0.006 (473) |

STATISTICAL ANALYSIS OF THE M.S.E. OF OBSERVATIONS

| | | | | | |
|---|---|---|---|---|---|
| -21.60 | 1.697 | -21.646 | 1.770 | -20.945 | 1.415 |
| 0.027 | 0.002 | -0.027 | 0.002 | -0.026 | 0.002 |
| 0.998 | 0.944 | 0.887 | 0.827 | 0.886 | 0.826 |
| 3.293 (774) | 3.521 (774) | 2.969 (774) | 3.202 (774) | 2.907 (774) | 3.141 (774) |
| -3.610 (216) | -3.259 (793) | -3.185 (669) | -3.101 (184) | -3.034 (669) | -3.064 (184) |

STATISTICAL ANALYSIS OF WEIGHTED RESIDUALS

| | | | | | |
|---|---|---|---|---|---|
| -0.435 | 0.099 | -0.436 | 0.104 | -0.419 | 0.096 |
| 0.001 | 0.000 | 0.001 | 0.000 | 0.001 | 0.000 |
| 0.025 | 0.024 | 0.023 | 0.022 | 0.023 | 0.022 |
| 0.115 (774) | 0.123 (774) | 0.104 (774) | 0.112 (774) | 0.101 (774) | 0.110 (774) |
| -0.089 ( 25) | -0.077 (184) | -0.087 ( 25) | -0.076 (184) | -0.087 ( 25) | -0.076 (184) |

STATISTICAL ANALYSIS OF RESIDUALS

| | | | | | |
|---|---|---|---|---|---|
| 20.690 | 19.513 | 18.398 | 17.102 | 18.374 | 17.075 |
| 0.025 | 0.024 | 0.023 | 0.021 | 0.023 | 0.021 |
| 0.006 | 0.006 | 0.006 | 0.006 | 0.006 | 0.006 |
| 0.036 (196) | 0.037 (480) | 0.032 (196) | 0.033 (480) | 0.032 (196) | 0.033 (480) |
| 0.015 ( 3) | 0.010 (812) | 0.013 ( 81) | 0.009 (812) | 0.013 ( 1) | 0.009 (812) |

STATISTICAL ANALYSIS OF THE M.S.E. OF THE RESIDUALS

| | | | | | |
|---|---|---|---|---|---|
| -23.09 | 2.446 | -26.058 | 2.860 | -25.294 | 2.383 |
| -0.028 | 0.003 | -0.032 | 0.004 | 0.031 | 0.003 |
| 0.100 | 1.014 | 0.997 | 1.004 | 0.998 | 1.004 |
| 3.188 (774) | 3.526 (774) | 3.258 (774) | 3.690 (774) | 3.190 (774) | 3.620 (774) |
| -3.653 (216) | -3.620 (216) | -3.675 (669) | -3.476 (184) | -3.536 (669) | -3.439 (184) |

STATISTICAL ANALYSIS OF THE STANDARDIZED RESIDUALS

| | | | | | |
|---|---|---|---|---|---|
| 2939. | 3265. | 2945. | 3273. | 2953. | 3284. |
| 3.607 | 4.006 | 3.614 | 4.017 | 3.624 | 4.030 |
| 0.137 | 0.509 | 0.136 | 0.513 | 0.136 | 0.516 |
| 4.865 ( 3) | 11.390 (814) | 4.865 ( 81) | 11.458 (814) | 4.685 ( 1) | 11.488 (814) |
| 3.468 (196) | 3.492 (480) | 3.468 (196) | 3.493 (480) | 3.476 (196) | 3.510 (480) |

INTERNAL LOCAL RELIABILITY FOR PARAMETERS

| | | | | | |
|---|---|---|---|---|---|
| 844. | 1595. | 866. | 1612. | 894. | 1635. |
| 1.036 | 1.958 | 1.063 | 1.980 | 1.098 | 2.007 |
| 0.345 | 0.802 | 0.338 | 0.807 | 0.231 | 0.805 |
| 3.440 ( 3) | 10.858 (814) | 3.440 ( 81) | 10.929 (814) | 3.440 ( 1) | 10.961 (814) |
| 0.436 (196) | 0.610 (480) | 0.436 (196) | 0.603 (480) | 0.502 (196) | 0.695 (480) |

EXTERNAL LOCAL RELIABILITY FOR OBSERVATIONS

*The different rows are: the sum, the mean value, the standard deviation, the maximum value and the minimum value, respectively.*

Several tests have been applied to check the different models of adjustment. First, consecutives odd and even models are compared for studing if systematic parameters for stations are present.

| *TEST* | *CHECKED MODELS* | *AIMS* |
|---|---|---|
| n. 1 | 1 and 2 | studing if systematic parameters for stations are present. General study without specifying systematic parameters for stations. |
| n. 2 | 3 and 4 | idem excepting the drift. |
| n. 3 | 5 and 6 | idem excepting the scale factor. |
| *n. 4* | *7 and 8* | *idem excepting systematism of gravimeter.* |
| n. 5 | 9 and 10 | idem excepting drift and scale factor. |
| n. 6 | 11 and 12 | idem excepting drift and systematism of gravimeter. |
| n. 7 | 13 and 14 | idem excepting scale and systematism of gravimeter. |
| n. 8 | 15 and 16 | idem excepting drift, scale and systematism of gravimeter. |

Other comparisons made between different models of adjustment are

| *TEST* | *CHECKED MODELS* | *AIMS* |
|---|---|---|
| n. 9 | 1 and 3 | to analysis the drift with respect to a model without scale parameter. |
| n.10 | 5 and 9 | to analysis the drift with respect to a model with scale parameter. |
| n.11 | 1 and 5 | to analysis the scale factors with respect to a model without drift parameter. |

| TEST | CHECKED MODELS | AIMS |
|---|---|---|
| n.12 | 3 and 9 | to analysis the scale factors with respect to a model with drift parameter. |
| n.13 | 9 and 15 | to analysis the gravimeter systematisms with respect to a model with scale and drift parameters. |

Table V shows the aims and the results of the analysis of different adjustments.

TABLE V

| TEST | NULL HYPOTHESIS | S | f | STATISTIC | F-VALUE | RESULT |
|---|---|---|---|---|---|---|
| 1-8 | Station systematic parameters are not significatives. | 72 | 538 | 0.512 | 1.316 | ACCEPTED |
| 9 | Driff parameters are not significatives | 125 | 618 | 0.588 | 1.245 | ACCEPTED |
| 10 | Driff parameters are not significatives | 125 | 614 | 0.742 | 1.245 | ACCEPTED |
| 11 | Scale parameters are not significatives | 4 | 739 | 49.184 | 2.384 | REJECTED |
| 12 | Scale parameters are not significatives | 4 | 614 | 46.621 | 2.386 | REJECTEC |
| 13 | Gravimeter parameters are not significatives | 4 | 610 | 0.483 | 2.387 | ACCEPTED |

From Tables III, IV and V we get the following conclusions

a) The models with systematic parameters of station (even models) improve the stochastic behaviour of the adjustment ( $\hat{v}^T P\ \hat{v}$, $\hat{\sigma}_0^2$ ,etc.) but the precision of the results are degraded ($TR\ Q_{xx'}$, $\sigma_{\hat{x}m}$). It is not necesary to take them into account; if observations were sufficient precise will be better to take systematic parameters related to time-station but this can be a problem with the solving system.

b) The scale factor parameters are very significatives in the adjustment model and must be always considered.

c) The drift parameters (one parameter for each gravimeter and trip) are not significatives. Moreover, in the models with drift parameters the reliability of the observations with large time interval decrease, so the large time interval links in a gravity net must be avoided. Nevertheless, in the models without drift parameters the sum of weighted residuals increase considerably, so we take them into account.

d) The models with systematic parameters of gravimeter produce the same effect as case b) but more attenuate.

For all these reasons in our final adjustment we use the model number 9 with gravity, scale and drift parameters.

*Datum comparison*

In our gravimetric network we have four IGSN 71 stations, the origin can be defined fixing one station. We have data from 4 gravimeters and the scale can be defined fixing the scale factor for one gravimeter. If we fix two stations the origin and the scale are defined. So, there are many possibilities for defining the datum.

IGSN 71 stations have not enough precision to define the scale, however, according to the analysis of the different gravimeters made, the gravimeter number 115 can be used to define the scale.

When results from absolute gravimeters have been warrantied, they could be used to define the scale.

After that the scale has been determined we can investigate the consistency of the IGSN 71 reference stations. For this, we use as well the linear general hypothesis. Let us consider the adjustment model 9 with one (no more) reference stations fixed, and the hypothesis

$$x_R = 0,$$

where $x_R$ are the station parameters (corrections to the approximate gravity values) corresponding to the remaining IGSN 71 stations.

The estimate is defined as in [32], taken as model A the model with all stations fixed, and as model B without the stations to check. Now, $f$ is the number of degrees of freedom of model $A$ and $s$ the number of stations to controle. Some results are shown in Table VI.

TABLE VI

| *MODEL* | *FIXED STATIONS* | $\Delta M$ | $\Delta B$ | $\Delta R$ | $\Delta P$ | $\hat{v}^T P \hat{v}$ | $f$ |
|---|---|---|---|---|---|---|---|
| A | MADRID C<br>BARCELONA J | 0 | 0 | -181 | 57 | 561.9976 | 616 |
| B | MADRID C | 0 | -63 | -171 | 24 | 556.3229 | 617 |
| C | BARCELONA J | 63 | 0 | -108 | 87 | 556.9259 | 617 |

$\Delta M$ = Correction to IGSN 71 gravity value of MADRID C.
$\Delta B$ = Correction to IGSN 71 gravity value of BARCELONA J.
$\Delta R$ = Correction to IGSN 71 gravity value of ROTA K.
$\Delta P$ = Correction to IGSN 71 gravity value of MALLORCA J.

Testing the models $A$ and $B$ (or $A$ and $C$),

$$\text{STATISTIC VALUE} = 5.610$$
$$F_{1,116,0.05\ \text{VALUE}} = 3.854$$

the null hipothesis is rejected: The IGSN 71 reference stations are *not consistent* with the REDGRAES observations. This can be check by comparison of the corrections $\Delta M$, $\Delta B$, $\Delta R$, $\Delta P$ with the standard deviation of the final adjustment of the net.

From all comparisons made, we conclude that the best model of adjustment of the first order gravity net in Spain is the model 9 (with gravity, scale and drift parameters), with gravimeter 115 as a reference gravimiter (weighted condition) and with the stations Madrid C and Barcelona J as weighted reference stations. This is the model we use which results are presented in the following section 8.

## 8. RESULTS

### I. General information about the adjustment

| | |
|---|---|
| Number of stations .......................... | 72 |
| Number of fixed stations ..................... | 0 |
| Number of fixed scales ....................... | 0 |
| Number of weighted reference stations ......... | 2 |
| Number of weighted reference scales ........... | 1 |
| Number of gravimeters ....................... | 4 |
| Number of trips ............................ | 124 |
| Number of the reference gravimeter ............ | 0 |
| Number of systematic unknowns .............. | 0 |
| Number of station unknowns ................. | 72 |
| Number of drift unknowns ................... | 124 |
| Number of scale unknowns ................... | 4 |
| Number of normalitation unknowns ........... | 0 |
| Total number of unknowns ................... | 200 |
| Total number of observations ................ | 813 |

### II. Table of observations (Only the two first trips)

| Station Number -- Station Name | Date | Time | Row Data | Earth Tide | Pole | Fine Data |
|---|---|---|---|---|---|---|
| 41 102 | | | | | | |
| 1032 Madrid C | 3 573 | 8.175 | 3592.84551 | −0.01250 | −0.00180 | 3592.83122 |
| 1035 Manzanares B | 3 573 | 12.658 | 3540.64496 | 0.17940 | −0.00180 | 3540.82255 |
| 1012 Bailén B | 3 573 | 16.033 | 3523.91675 | 0.03850 | −0.00177 | 3523.95349 |

| *Station Number* | *Station Name* | *Date* | *Time* | *Row Data* | *Earth Tide* | *Pole* | *Fine Data* |
|---|---|---|---|---|---|---|---|
| 1022 | Granada B | 3 573 | 20.167 | 3290.43430 | -0.09610 | —0.00177 | 3290.33644 |
| 1022 | Granada B | 4 573 | 7.850 | 3290.42387 | -0.06180 | —0.00177 | 3290.36030 |
| 1012 | Bailén B | 4 573 | 11.383 | 3523.84839 | 0.13560 | —0.00178 | 3523.98221 |
| 1035 | Manzanares B | 4 573 | 14.733 | 3540.72114 | 0.14940 | —0.00182 | 3540.86872 |
| 1032 | Madrid C | 4 573 | 18.575 | 3592.91962 | -0.04600 | —0.00182 | 3592.87180 |
| 69 | 102 | | | | | | |
| 1032 | Madrid C | 3 573 | 8.183 | 3545.75653 | -0.01160 | —0.00180 | 3545.74313 |
| 1035 | Manzanares B | 3 573 | 12.400 | 3493.49864 | 0.17980 | —0.00180 | 3493.67664 |
| 1012 | Bailén B | 3 573 | 16.100 | 3476.82045 | 0.03440 | —0.00177 | 3476.85309 |
| 1022 | Granada B | 3 573 | 20.150 | 3243.25623 | -0.09620 | —0.00177 | 3243.15826 |
| 1022 | Granada B | 4 573 | 7.808 | 3243.23146 | -0.06400 | —0.00177 | 3243.16569 |
| 1012 | Bailén B | 4 573 | 11.317 | 3476.71932 | 0.13240 | —0.00178 | 3476.84994 |
| 1035 | Manzanares B | 4 573 | 14.633 | 3493.55901 | 0.15300 | —0.00182 | 3493.71019 |
| 1032 | Madrid C | 4 573 | 18.550 | 3545.77665 | -0.04460 | —0.00182 | 3545.73023 |

## III. **Solutions**

Compensación de la red gravimétrica española de primer orden.

| *Núm.* | *Estación* | *Gravedad inicial* | *Corrección* | *Grav. Compensada* | *E.M.C.* |
|---|---|---|---|---|---|
| 1 | 1001 Almansa B | 979880.570 | 0.059 | 979880.629 | 0.018407 |
| 2 | 1002 Albacete B | 979885.940 | 0.038 | 979885.978 | 0.017833 |
| 3 | 1003 Alcañiz | 980145.110 | 0.035 | 980145.145 | 0.015952 |
| 4 | 1004 Alcaraz B | 979809.510 | 0.043 | 979809.553 | 0.019647 |
| 5 | 1005 Alcolea B | 979952.850 | 0.040 | 979952.890 | 0.015770 |
| 6 | 1006 Algeciras B | 979750.990 | 0.071 | 979751.061 | 0.021732 |
| 7 | 1007 Alicante B | 980026.500 | 0.036 | 980026.536 | 0.016149 |
| 8 | 1008 Almería B | 979904.300 | 0.061 | 979904.361 | 0.018541 |
| 9 | 1009 Aranda B | 980070.730 | 0.032 | 980070.762 | 0.015833 |
| 10 | 1010 Avila B | 979923.480 | 0.039 | 979923.519 | 0.017183 |
| 11 | 1011 Badajoz B | 980037.670 | 0.025 | 980037.695 | 0.017006 |
| 12 | 1012 Bailén B | 979886.680 | 0.060 | 979886.740 | 0.017821 |
| 13 | 1013 Barcelona J | 980306.230 | -0.022 | 980306.208 | 0.015409 |
| 14 | 1113 Barcelona AP | 980305.740 | 0.007 | 980305.747 | 0.015917 |
| 15 | 1213 Barcelona UNIV | 980297.780 | 0.011 | 980297.791 | 0.016141 |
| 16 | 1313 Barcelona NHO | 980313.690 | 0.018 | 980313.708 | 0.018232 |
| 17 | 1014 Baza B | 979655.960 | 0.120 | 979656.080 | 0.023428 |
| 18 | 1015 Burgos B | 980140.390 | 0.008 | 980140.398 | 0.015670 |
| 19 | 1016 Cáceres B | 979999.780 | 0.021 | 979999.801 | 0.016598 |
| 20 | 1017 Cartagena B | 980018.160 | 0.040 | 980018.200 | 0.017583 |
| 21 | 1018 Córdoba B | 979935.160 | 0.052 | 979935.212 | 0.017966 |
| 22 | 1019 Cuenca B | 979881.460 | 0.113 | 979881.573 | 0.017687 |
| 23 | 1020 Fuente B | 979912.980 | 0.037 | 979913.017 | 0.017873 |
| 24 | 1021 Gerona B | 980330.860 | -0.002 | 980330.858 | 0.017813 |

Compensación de la red gravimétrica española de primer orden.

| *Núm.* | *Estación* | *Gravedad inicial* | *Corrección* | *Grav. Compensada* | *E.M.C.* |
|---|---|---|---|---|---|
| 25 | 1022 Granada B | 979653.050 | 0.087 | 979653.137 | 0.022951 |
| 26 | 1023 Hellín B | 979877.200 | 0.054 | 979877.254 | 0.018642 |
| 27 | 1024 Herrera B | 979986.350 | 0.022 | 979986.372 | 0.016984 |
| 28 | 1025 Huelva B | 979970.390 | 0.054 | 979970.444 | 0.018372 |
| 29 | 1026 Huesca B | 980214.490 | 0.009 | 980214.499 | 0.016578 |
| 30 | 1027 Ibiza B | 980127.210 | 0.026 | 980127.236 | 0.015766 |
| 31 | 1028 León B | 980159.180 | 0.022 | 980159.202 | 0.016220 |
| 32 | 1029 Lérida B | 980250.630 | 0.007 | 980250.637 | 0.016081 |
| 33 | 1030 Lorca B | 979878.710 | 0.049 | 979878.759 | 0.018487 |
| 34 | 1031 Lugo B | 980346.490 | 0.019 | 980346.509 | 0.019719 |
| 35 | 1032 Madrid C | 979955.610 | 0.022 | 979955.632 | 0.015409 |
| 36 | 1132 Madrid A NP 26 | 979966.560 | 0.003 | 979966.563 | 0.020489 |
| 37 | 1232 Madrid N | 979981.340 | −0.022 | 979981.318 | 0.024606 |
| 38 | 1332 Madrid M | 979992.480 | −0.018 | 979992.462 | 0.027839 |
| 39 | 1033 Mahón B | 980229.260 | 0.023 | 980229.283 | 0.016148 |
| 40 | 1034 Málaga B | 979900.170 | 0.046 | 979900.216 | 0.018883 |
| 41 | 1035 Manzanares B | 979903.560 | 0.053 | 979903.613 | 0.017416 |
| 42 | 1036 Miranda B | 980251.600 | 0.002 | 980251.602 | 0.016698 |
| 43 | 1037 Murcia B | 979993.850 | 0.038 | 979993.888 | 0.016828 |
| 44 | 1038 Orense B | 980313.000 | 0.004 | 980313.004 | 0.019695 |
| 45 | 1039 Oviedo B | 980415.740 | −0.008 | 980415.732 | 0.019736 |
| 46 | 1040 Mallorca J | 980163.100 | −0.114 | 980162.986 | 0.015243 |
| 47 | 1140 Mallorca K | 980161.580 | 0.057 | 980161.637 | 0.017228 |
| 48 | 1041 Plasencia B | 980055.740 | 0.018 | 980055.758 | 0.016703 |
| 49 | 1042 Ponferrada B | 980218.690 | 0.030 | 980218.720 | 0.018020 |
| 50 | 1043 Quintanar B | 979928.330 | 0.059 | 979928.389 | 0.017055 |
| 51 | 1044 Requena B | 979934.340 | 0.082 | 979934.422 | 0.017115 |
| 52 | 1045 Rota B | 979849.630 | 0.068 | 979849.698 | 0.019532 |
| 53 | 1145 Rota K | 979851.310 | −0.152 | 979851.158 | 0.021098 |
| 54 | 1046 Salamanca B | 980046.600 | 0.026 | 980046.626 | 0.016214 |
| 55 | 1047 S. Sebastián B | 980439.190 | −0.014 | 980439.176 | 0.019855 |
| 56 | 1048 Santander B | 980497.330 | −0.028 | 980497.302 | 0.021263 |
| 57 | 1049 Santiago B | 980401.090 | −0.009 | 980401.081 | 0.021087 |
| 58 | 1050 Seo de Urgel B | 980135.690 | 0.014 | 980135.704 | 0.016356 |
| 59 | 1051 Sevilla B | 979937.230 | 0.050 | 979937.280 | 0.017827 |
| 60 | 1052 Soria B | 980028.560 | 0.011 | 980028.571 | 0.016218 |
| 61 | 1053 Talavera B | 980029.950 | 0.044 | 980029.994 | 0.015580 |
| 62 | 1054 Teruel B | 979905.800 | 0.061 | 979905.861 | 0.017230 |
| 63 | 1055 Tortosa B | 980217.030 | 0.029 | 980217.059 | 0.015556 |
| 64 | 1056 Valencia B | 980113.410 | 0.049 | 980113.459 | 0.014970 |
| 65 | 1057 Valladolid B | 980097.300 | 0.018 | 980097.318 | 0.015506 |
| 66 | 1058 Zamora B | 980139.550 | 0.023 | 980139.573 | 0.016079 |
| 67 | 1059 Zaragoza B | 980223.200 | 0.021 | 980223.221 | 0.015876 |
| 68 | 1060 Aduana NAP | 980037.110 | 0.017 | 980037.127 | 0.019379 |
| 69 | 1160 Portugal NP 47 | 980037.220 | 0.023 | 980037.243 | 0.019371 |
| 70 | 1061 Deriva 1 | 980428.700 | −0.010 | 980428.690 | 0.020977 |
| 71 | 1062 Ayamonte B | 979979.540 | 0.045 | 979979.585 | 0.019973 |
| 72 | 1063 Vila Real NP | 979977.420 | 0.048 | 979977.468 | 0.023605 |

Compensación de la red gravimétrica española de primer orden.

| *Itinerario* | *Gravímetro* | *Deriva* | *E.M.C.* | *Itinerario* | *Gravímetro* | *Deriva* | *E.M.C.* |
|---|---|---|---|---|---|---|---|
| 101 | L&R 115 | 0.009797 | 0.008918 | 113 | L&R 301 | −0.002126 | 0.008448 |
| 102 | L&R 41 | −0.000951 | 0.002252 | 114 | L&R 41 | −0.002686 | 0.002189 |
| 102 | L&R 69 | 0.001359 | 0.002520 | 114 | L&R 69 | −0.002833 | 0.002441 |
| 102 | L&R 115 | −0.000202 | 0.002040 | 114 | L&R 115 | 0.000967 | 0.001982 |
| 102 | L&R 301 | 0.009911 | 0.003671 | 114 | L&R 301 | −0.000703 | 0.003477 |
| 103 | L&R 41 | −0.000368 | 0.002061 | 115 | L&R 41 | −0.003604 | 0.002070 |
| 103 | L&R 69 | −0.001831 | 0.002294 | 115 | L&R 69 | 0.001964 | 0.002304 |
| 103 | L&R 115 | 0.002661 | 0.001862 | 115 | L&R 115 | 0.000716 | 0.001863 |
| 103 | L&R 301 | −0.001632 | 0.001952 | 115 | L&R 301 | 0.000415 | 0.003326 |
| 104 | L&R 41 | −0.000317 | 0.002258 | 116 | L&R 41 | −0.001818 | 0.001982 |
| 104 | L&R 69 | −0.001955 | 0.002452 | 116 | L&R 69 | −0.000461 | 0.002198 |
| 104 | L&R 115 | 0.004078 | 0.002001 | 116 | L&R 115 | 0.000496 | 0.001791 |
| 104 | L&R 301 | −0.003063 | 0.003459 | 116 | L&R 301 | −0.000524 | 0.003251 |
| 105 | L&R 41 | 0.000292 | 0.002044 | 117 | L&R 41 | 0.014457 | 0.007065 |
| 105 | L&R 69 | 0.001209 | 0.002268 | 117 | L&R 69 | 0.005534 | 0.008259 |
| 105 | L&R 115 | 0.001457 | 0.001843 | 117 | L&R 115 | 0.009960 | 0.006483 |
| 105 | L&R 301 | −0.002870 | 0.002986 | 117 | L&R 301 | 0.009170 | 0.011096 |
| 106 | L&R 41 | −0.000550 | 0.002867 | 118 | L&R 41 | −0.001914 | 0.002175 |
| 106 | L&R 69 | −0.000857 | 0.003200 | 118 | L&R 69 | −0.002896 | 0.002426 |
| 106 | L&R 115 | 0.001475 | 0.002601 | 118 | L&R 115 | 0.002329 | 0.001956 |
| 106 | L&R 301 | 0.001362 | 0.004542 | 118 | L&R 301 | −0.001067 | 0.003422 |
| 107 | L&R 41 | −0.002673 | 0.002300 | 119 | L&R 41 | −0.000214 | 0.001702 |
| 107 | L&R 69 | −0.003444 | 0.002582 | 119 | L&R 69 | −0.000727 | 0.001900 |
| 107 | L&R 115 | 0.001925 | 0.002101 | 119 | L&R 115 | 0.001634 | 0.001549 |
| 107 | L&R 301 | 0.002974 | 0.003650 | 119 | L&R 301 | 0.001298 | 0.002320 |
| 106 | L&R 41 | 0.000201 | 0.002093 | 120 | L&R 41 | −0.000737 | 0.002041 |
| 108 | L&R 69 | −0.000264 | 0.002307 | 120 | L&R 69 | −0.001194 | 0.002269 |
| 108 | L&R 115 | 0.001913 | 0.001874 | 120 | L&R 115 | 0.002331 | 0.001819 |
| 108 | L&R 301 | −0.000003 | 0.003285 | 120 | L&R 301 | −0.000187 | 0.002788 |
| 109 | L&R 41 | −0.002778 | 0.002173 | 121 | L&R 41 | −0.002125 | 0.002417 |
| 109 | L&R 69 | −0.000315 | 0.002391 | 121 | L&R 69 | 0.000453 | 0.002698 |
| 109 | L&R 115 | 0.001339 | 0.001950 | 121 | L&R 115 | 0.000251 | 0.002188 |
| 109 | L&R 301 | −0.002050 | 0.003408 | 121 | L&R 301 | −0.001438 | 0.003829 |
| 110 | L&R 41 | −0.000343 | 0.002574 | 122 | L&R 41 | −0.005015 | 0.003832 |
| 110 | L&R 69 | −0.000446 | 0.002776 | 122 | L&R 69 | −0.000584 | 0.004266 |
| 110 | L&R 115 | 0.003277 | 0.002267 | 122 | L&R 115 | −0.001613 | 0.003471 |
| 110 | L&R 301 | 0.000210 | 0.002041 | 122 | L&R 301 | 0.001254 | 0.006019 |
| 111 | L&R 41 | 0.001220 | 0.002259 | 123 | L&R 41 | −0.001929 | 0.004310 |
| 111 | L&R 69 | −0.000067 | 0.002518 | 123 | L&R 69 | −0.000584 | 0.004808 |
| 111 | L&R 115 | 0.003909 | 0.002051 | 123 | L&R 115 | −0.000788 | 0.003925 |
| 111 | L&R 301 | −0.000995 | 0.003575 | 123 | L&R 301 | 0.000406 | 0.006888 |
| 112 | L&R 41 | −0.000273 | 0.002557 | 124 | L&R 41 | −0.002639 | 0.003374 |
| 112 | L&R 69 | −0.000840 | 0.002824 | 124 | L&R 301 | 0.003605 | 0.005354 |
| 112 | L&R 115 | 0.001245 | 0.002305 | 124 | L&R 69 | 0.000718 | 0.003903 |
| 112 | L&R 301 | −0.001301 | 0.004013 | 124 | L&R 115 | 0.004140 | 0.003167 |
| 113 | L&R 41 | −0.000322 | 0.005324 | 125 | L&R 41 | −0.002854 | 0.002216 |
| 113 | L&R 69 | −0.000101 | 0.005958 | 125 | L&R 69 | −0.000660 | 0.002471 |
| 113 | L&R 115 | 0.001933 | 0.004837 | 125 | L&R 115 | 0.000359 | 0.002020 |

Compensación de la red gravimétrica española de primer orden.

| *Itinerario* | *Gravímetro* | *Deriva* | *E.M.C.* | *Itinerario* | *Gravímetro* | *Deriva* | *E.M.C.* |
|---|---|---|---|---|---|---|---|
| 125 | L&R 301 | 0.004363 | 0.003523 | 129 | L&R 69 | 0.000004 | 0.002596 |
| 126 | L&R 41 | 0.000020 | 0.002625 | 129 | L&R 115 | 0.000823 | 0.002104 |
| 126 | L&R 69 | 0.000227 | 0.002918 | 129 | L&R 301 | 0.002430 | 0.003651 |
| 126 | L&R 115 | 0.004636 | 0.002375 | 130 | L&R 41 | −0.007860 | 0.004201 |
| 126 | L&R 301 | 0.004964 | 0.004099 | 130 | L&R 69 | 0.004222 | 0.004683 |
| 127 | L&R 41 | 0.000692 | 0.002683 | 130 | L&R 115 | −0.001514 | 0.003782 |
| 127 | L&R 69 | −0.000674 | 0.003004 | 130 | L&R 301 | −0.011591 | 0.006575 |
| 127 | L&R 115 | −0.000838 | 0.002434 | 131 | L&R 41 | −0.006854 | 0.007765 |
| 127 | L&R 301 | 0.007334 | 0.004243 | 131 | L&R 69 | −0.002562 | 0.008859 |
| 128 | L&R 41 | −0.000813 | 0.002305 | 131 | L&R 115 | 0.000805 | 0.006872 |
| 128 | L&R 69 | 0.001974 | 0.002571 | 131 | L&R 301 | −0.010711 | 0.011929 |
| 128 | L&R 115 | 0.000825 | 0.002078 | 132 | L&R 41 | 0.005017 | 0.008644 |
| 129 | L&R 301 | 0.006926 | 0.003643 | 132 | L&R 69 | −0.003344 | 0.009503 |
| 129 | L&R 41 | −0.000697 | 0.002311 | 132 | L&R 115 | 0.000398 | 0.007993 |

Compensación de la red gravimétrica española de primer orden.

| *Gavímetro* | *Coef. lineal factor escala* | *E.M.C.* | *Cte. normalización* | *E.M.C.* |
|---|---|---|---|---|
| L&R 41 | −0.00017609 | 0.000038 | S/C | — |
| L&R 69 | −0.00016806 | 0.000039 | S/C | — |
| L&R 115 | 0.00002994 | 0.000035 | S/C | — |
| L&R 301 | −0.00013938 | 0.000042 | S/C | — |

## IV. Statistic of observations

Compensación de la red gravimétrica española de primer orden.

Errores medios cuadráticos de las observaciones Hoja 1

| *N. Obs.* | *NPE* | *NPV* | *Diferencia Gravedad Observada* | *Diferencia Gravedad Compensada* | *Diferencia* | *Peso de la Observación* | *E.M.C. de la Observación* |
|---|---|---|---|---|---|---|---|
| 1 | 1032 | 1132 | 10.9210 | 10.9230 | −0.0020 | 50.0000 | 0.0157 |
| 2 | 1132 | 1232 | 14.7430 | 14.7416 | 0.0014 | 50.0000 | 0.0157 |
| 3 | 1232 | 1332 | 11.1366 | 11.1360 | 0.0006 | 50.0000 | 0.0152 |
| 4 | 1332 | 1232 | −11.1534 | −11.1540 | 0.0006 | 50.0000 | 0.0152 |
| 5 | 1232 | 1132 | −14.7637 | −14.7651 | 0.0014 | 50.0000 | 0.0157 |
| 6 | 1132 | 1032 | −10.9347 | −10.9328 | −0.0020 | 50.0000 | 0.0157 |
| 7 | 1032 | 1035 | −52.0087 | −51.9933 | −0.0153 | 45.3045 | 0.0120 |
| 8 | 1035 | 1012 | −16.8691 | −16.8652 | −0.0038 | 45.3045 | 0.0102 |
| 9 | 1012 | 1022 | −233.6170 | −233.5944 | −0.0226 | 45.3045 | 0.0108 |

| *N. Obs.* | *NPE* | *NPV* | *Diferencia Gravedad Observada* | *Diferencia Gravedad Compensada* | *Diferencia* | *Peso de la Observación* | *E.M.C. de la Observación* |
|---|---|---|---|---|---|---|---|
| 10 | 1022 | 1012 | 233.6219 | 233.5968 | 0.0251 | 45.3045 | 0.0099 |
| 11 | 1012 | 1035 | 16.8865 | 16.8937 | −0.0072 | 45.3045 | 0.0101 |
| 12 | 1035 | 1032 | 52.0031 | 51.9742 | 0.0289 | 45.3045 | 0.0111 |
| 13 | 1032 | 1035 | −52.0665 | −52.0994 | 0.0329 | 40.5894 | 0.0126 |
| 14 | 1035 | 1012 | −16.8236 | −16.7661 | −0.0574 | 40.5894 | 0.0115 |
| 15 | 1012 | 1022 | −233.6948 | −233.7424 | 0.0476 | 40.5894 | 0.0117 |
| 16 | 1022 | 1012 | 233.6842 | 233.7315 | −0.0473 | 40.5894 | 0.0107 |
| 17 | 1012 | 1035 | 16.8603 | 16.8490 | 0.0112 | 40.5894 | 0.0108 |
| 18 | 1035 | 1032 | 52.0200 | 52.0175 | 0.0025 | 40.5894 | 0.0120 |
| 19 | 1032 | 1035 | −52.0220 | −52.0273 | −0.0053 | 50.0000 | 0.0109 |
| 20 | 1035 | 1012 | −16.8943 | −16.9168 | 0.0225 | 50.0000 | 0.0103 |
| 21 | 1012 | 1022 | −233.5892 | −233.5836 | −0.0056 | 50.0000 | 0.0097 |
| 22 | 1022 | 1012 | 233.6056 | 233.6149 | −0.0093 | 50.0000 | 0.0095 |
| 23 | 1012 | 1035 | 16.8712 | 16.8691 | 0.0021 | 50.0000 | 0.0096 |
| 24 | 1035 | 1032 | 52.0336 | 52.0489 | −0.0153 | 50.0000 | 0.0103 |
| 25 | 1032 | 1035 | −52.1133 | −52.1609 | −0.0476 | 28.6793 | 0.0161 |
| 26 | 1035 | 1012 | −16.9612 | −17.0067 | 0.0454 | 28.6793 | 0.0163 |
| 27 | 1012 | 1022 | −233.6441 | −233.6188 | −0.0253 | 28.6793 | 0.0145 |
| 28 | 1022 | 1012 | 233.5827 | 233.5647 | 0.0180 | 28.6793 | 0.0145 |
| 29 | 1012 | 1035 | 16.9321 | 17.0232 | −0.0911 | 28.6793 | 0.0145 |
| 30 | 1035 | 1032 | 51.9988 | 52.0072 | −0.0085 | 28.6793 | 0.0150 |
| 31 | 1032 | 1019 | −74.0745 | −74.0786 | 0.0041 | 45.3045 | 0.0109 |
| 32 | 1019 | 1044 | 52.8684 | 52.8769 | −0.0085 | 45.3045 | 0.0103 |
| 33 | 1044 | 1056 | 179.0414 | 179.0130 | 0.0284 | 45.3045 | 0.0098 |
| 34 | 1056 | 1040 | 49.5536 | 49.5707 | −0.0171 | 45.3045 | 0.0089 |
| 35 | 1040 | 1140 | −1.3562 | −1.3635 | 0.0073 | 45.3045 | 0.0084 |
| 36 | 1140 | 1040 | 1.3296 | 1.3095 | 0.0200 | 45.3045 | 0.0082 |
| 37 | 1040 | 1056 | −49.5505 | −49.5666 | 0.0161 | 45.3045 | 0.0086 |
| 38 | 1056 | 1044 | −179.0496 | −179.0315 | −0.0181 | 45.3045 | 0.0091 |
| 39 | 1044 | 1019 | −52.8501 | −52.8431 | −0.0070 | 45.3045 | 0.0104 |
| 40 | 1019 | 1032 | 74.0780 | 74.0823 | −0.0043 | 45.3045 | 0.0123 |

## IV. Statistic of residuals

Compensación de la red gravimétrica española de primer orden.

Errores medios cuadráticos de los residuales Hoja 1

| *NO* | *NPE* | *NPV* | *Residuo sin Pon* | *E.M.C. del residuo* | *Residuo Tipificado* | *EMD* | *E.S.R.* | *QB* | *RX* | *RT* | *FLE* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1032 | 1132 | 0.0020 | 0.0108 | 0.1817 | 0.12 | 0.04 | 0.68 | 4.99 | 6.06 | 0.68 |
| 2 | 1132 | 1232 | −0.0014 | 0.0107 | −0.1312 | 0.12 | 0.04 | 0.68 | 5.05 | 6.11 | 0.68 |
| 3 | 1232 | 1332 | −0.0006 | 0.0114 | −0.0531 | 0.11 | 0.04 | 0.64 | 4.58 | 5.73 | 0.64 |
| 4 | 1332 | 1232 | −0.0006 | 0.0114 | −0.0531 | 0.11 | 0.04 | 0.64 | 4.58 | 5.73 | 0.64 |
| 5 | 1232 | 1132 | −0.0014 | 0.0107 | −0.1312 | 0.12 | 0.04 | 0.68 | 5.05 | 6.11 | 0.68 |
| 6 | 1132 | 1032 | 0.0020 | 0.0108 | 0.1817 | 0.12 | 0.04 | 0.68 | 4.99 | 6.06 | 0.68 |

| *NO* | *NPE* | *NPV* | *Residuo sin Pon* | *E.M.C. del residuo* | *Residuo Tipificado* | *EMD* | *E.S.R.* | *QB* | *RX* | *RT* | *FLE* |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | 1032 | 1035 | 0.0153 | 0.0172 | 0.8921 | 0.09 | 0.06 | 0.33 | 2.41 | 4.20 | 0.40 |
| 8 | 1035 | 1012 | 0.0038 | 0.0183 | 0.2085 | 0.09 | 0.07 | 0.24 | 1.92 | 3.94 | 0.29 |
| 9 | 1012 | 1022 | 0.0226 | 0.0180 | 1.2586 | 0.09 | 0.07 | 0.27 | 2.07 | 4.02 | 0.32 |
| 10 | 1022 | 1012 | −0.0251 | 0.0185 | −1.3520 | 0.09 | 0.07 | 0.22 | 1.83 | 3.90 | 0.27 |
| 11 | 1012 | 1035 | 0.0072 | 0.0184 | 0.3908 | 0.09 | 0.07 | 0.23 | 1.88 | 3.92 | 0.28 |
| 12 | 1035 | 1032 | −0.0289 | 0.0178 | −1.6189 | 0.09 | 0.06 | 0.28 | 2.14 | 4.05 | 0.34 |
| 13 | 1032 | 1035 | −0.0329 | 0.0198 | −1.6630 | 0.10 | 0.07 | 0.29 | 2.19 | 4.08 | 0.44 |
| 14 | 1035 | 1012 | 0.0574 | 0.0204 | 2.8099 | 0.10 | 0.07 | 0.24 | 1.93 | 3.93 | 0.36 |
| 15 | 1012 | 1022 | −0.0476 | 0.0203 | −2.3437 | 0.10 | 0.07 | 0.25 | 1.99 | 3.97 | 0.38 |
| 16 | 1022 | 1012 | 0.0473 | 0.0209 | 2.2667 | 0.10 | 0.08 | 0.21 | 1.77 | 3.87 | 0.32 |
| 17 | 1012 | 1035 | −0.0112 | 0.0208 | −0.5394 | 0.10 | 0.08 | 0.21 | 1.78 | 3.87 | 0.32 |
| 18 | 1035 | 1032 | −0.0025 | 0.0201 | −0.1245 | 0.10 | 0.07 | 0.26 | 2.05 | 4.00 | 0.40 |
| 19 | 1032 | 1035 | −0.0053 | 0.0156 | −0.3409 | 0.08 | 0.06 | 0.33 | 2.39 | 4.19 | 0.33 |
| 20 | 1035 | 1012 | −0.0225 | 0.0160 | −1.4008 | 0.08 | 0.06 | 0.29 | 2.20 | 4.08 | 0.29 |
| 21 | 1012 | 1022 | 0.0056 | 0.0164 | 0.3395 | 0.08 | 0.06 | 0.26 | 2.04 | 4.00 | 0.26 |
| 22 | 1022 | 1012 | 0.0093 | 0.0165 | 0.5654 | 0.08 | 0.06 | 0.25 | 1.99 | 3.97 | 0.25 |
| 23 | 1012 | 1035 | −0.0021 | 0.0164 | −0.1272 | 0.08 | 0.06 | 0.26 | 2.02 | 3.99 | 0.26 |
| 24 | 1035 | 1032 | 0.0153 | 0.0160 | 0.9586 | 0.08 | 0.06 | 0.29 | 2.22 | 4.10 | 0.29 |
| 25 | 1032 | 1035 | −0.0476 | 0.0290 | −1.6398 | 0.14 | 0.10 | 0.23 | 1.90 | 3.93 | 0.71 |
| 26 | 1035 | 1012 | −0.0454 | 0.0289 | −1.5729 | 0.14 | 0.10 | 0.24 | 1.94 | 3.95 | 0.74 |
| 27 | 1012 | 1022 | 0.0253 | 0.0299 | 0.8472 | 0.13 | 0.11 | 0.19 | 1.67 | 3.82 | 0.58 |
| 28 | 1022 | 1012 | −0.0180 | 0.0299 | −0.6030 | 0.13 | 0.11 | 0.19 | 1.66 | 3.82 | 0.58 |
| 29 | 1012 | 1035 | 0.0911 | 0.0298 | 3.0549 | 0.13 | 0.11 | 0.19 | 1.67 | 3.83 | 0.58 |
| 30 | 1035 | 1032 | 0.0085 | 0.0296 | 0.2866 | 0.13 | 0.11 | 0.21 | 1.75 | 3.86 | 0.62 |
| 31 | 1032 | 1019 | −0.0041 | 0.0179 | −0.2296 | 0.09 | 0.06 | 0.27 | 2.10 | 4.03 | 0.33 |
| 32 | 1019 | 1044 | 0.0085 | 0.0183 | 0.4632 | 0.09 | 0.07 | 0.24 | 1.94 | 3.95 | 0.29 |
| 33 | 1044 | 1056 | −0.0284 | 0.0186 | −1.5285 | 0.09 | 0.07 | 0.22 | 1.81 | 3.89 | 0.26 |
| 34 | 1056 | 1040 | 0.0171 | 0.0190 | 0.8998 | 0.08 | 0.07 | 0.18 | 1.61 | 3.80 | 0.22 |
| 35 | 1040 | 1140 | −0.0073 | 0.0192 | −0.3802 | 2.08 | 0.07 | 0.16 | 1.50 | 3.75 | 0.20 |
| 36 | 1140 | 1040 | −0.0200 | 0.0193 | −1.0361 | 0.08 | 0.07 | 0.15 | 1.46 | 3.74 | 0.19 |
| 37 | 1040 | 1056 | −0.0161 | 0.0192 | −0.8337 | 0.08 | 0.07 | 0.17 | 1.53 | 3.77 | 0.20 |
| 38 | 1056 | 1044 | 0.0181 | 0.0189 | 0.9552 | 0.08 | 0.07 | 0.19 | 1.66 | 3.82 | 0.23 |
| 39 | 1044 | 1019 | 0.0070 | 0.0173 | 0.3856 | 0.09 | 0.07 | 024 | 1.95 | 3.95 | 0.30 |
| 40 | 1019 | 1032 | 0.0043 | 0.0171 | 0.2507 | 0.09 | 0.06 | 0.34 | 2.47 | 4.24 | 0.41 |

*NO* number of order, *NPE* number of initial station, *NPV* number of final station, *RESIDUO SIN PON* given by (21), *EMC DEL RESIDUO* given by (24), *RESIDUO TIPIFICADO* given by (26), *EMD* given (30), *ESR* given by (31), *QB* given by (27), *RX* given by (28), *RT* given by (29).

## VI. Stochastic characteristics of the adjustment

| | |
|---|---|
| VARIANCE FACTOR .................................. | 0.9515 |
| SUM OF WEIGHTED RESIDUALS ...................... | 2.3957 |
| SUM OF RESIDUALS .................................. | 0.1289 |
| SUM OF SQUARES OF WEIGHTED RESIDUALS ...... | 554.9338 |
| SUM OF SQUARES OF RESIDUALS .................. | 0.3768 |
| APOSTERIORI STANDARD DEVIATION ............. | 0.0190 |

TRACE OF THE PARAMETERS COFACTOR MATRIX . 66.0951
MEAN STANDARD DEVIATION OF PARAMETERS ... 0.0182
NUMBER OF DEGREES OF FREEDOM ............... 613
TAU-VALUE ........................................ 3.9725

VII. **Extreme values**

| | *Sun* | *Mean Value* | *Standard Deviation* | *Maximun (Num)* | *Minimum (Num)* |
|---|---|---|---|---|---|
| Corrections to observations | −0.129 | 0.000 | 0.022 | 0.077 (184) | −0.113 (774) |
| M.S.E. of observations | 9.228 | 0.011 | 0.003 | 0.029 (277) | 0.005 (456) |
| *Residuals* | *0.129* | *0.000* | *0.022* | *0.133 (774)* | *−0.077 (184)* |
| M.S.E. of residuals | 17.045 | 0.021 | 0.006 | 0.033 (480) | 0.009 (812) |
| Standardized Residuals | 2.504 | 0.003 | 1.003 | 3.733 (774) | −3.508 (184) |
| Internal Local Reliability | 3258.5 | 4.008 | 0.443 | 7.300 (277) | 3.492 (480) |
| External Local Reliability | 1600.0 | 1.968 | 0.745 | 6.439 (277) | 0.600 (480) |

INTERNAL MEAN RELIABILITY OF THE NET 0.24
EXTERNAL MEAN RELIABILITY OF THE NET 0.38

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