The fate of bilingualism in a model of language competition

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In the general context of dynamics of social consensus, we study an agent based model for the competition between two socially equivalent languages, addressing the role of bilingualism and social structure. In a regular network, we study the formation of linguistic domains and their interaction across the boundaries. We analyse also a small world social structure, in order to capture the effect of long range social interactions. In both cases, a final scenario of dominance of one language and extinction of the other is obtained, but with smaller times for extinction in the latter case. In addition, we compare our results to our previous work on the agent based version of Abrams-Strogatz model.

1 Introduction

Language competition occurs today worldwide. Different languages coexist within many societies and the fate of a high number of them in the future is worrying: most of the 6000 languages spoken today are in danger, with around 50% of them facing extinction in the current century. Even more striking is the distribution of speakers, since 4% of the languages are spoken by 96% of the world population, while 25% have fewer than 1000 speakers. New pidgins and creoles are also emerging, but their number is relatively small compared with the language loss rate [1]. In this scenario, and beyond Weinreich 's *Languages in Contact* [2], numerous sociolinguistic studies have been published in order to: (1) reveal the level of endangerment of specific languages [3]; (2) find a common pattern that might relate language choice to ethnicity, community identity or the like [4]; and (3) claim the role played by social networks in the dynamics of language competition, which has given rise to the monographic issue [5].

In the recent years, language competition, which studies the dynamics of language use and learning due to social interactions, has also been tackled with from a different approach. Abrams and Strogatz model for the dynamics of endangered languages [6] has triggered a coherent effort to understand the mechanisms of language dynamics outside the traditional linguistic research. Their study considers a two-state society, that is, one in which there are speakers of either a language A or a language B. This seminal work, as well as others along the same line [7, 8], belongs to the general class of studies of population dynamics based on nonlinear ordinary differential equations for the populations of speakers of different languages. In addition, other studies implement discrete agent based models with speakers of many [9, 10] or few languages [11, 12], as reviewed in [9, 13]. These studies consider social structures modelled by complex networks in which agents are connected with one another.

Language competition, then, belongs to the general class of processes that can be modelled by the interaction of heterogeneous agents as an example of collective phenomena in problems of social consensus [14]. The question in a consensus problem is to establish when the dynamics of a set of interacting agents that can choose among several options leads to a consensus in one of these options, or when a state with several coexisting social options prevails. In this respect, a specific feature of language dynamics is that agents can share two of the social options that are chosen by the agents in the consensus dynamics. In the present work, these are the bilingual agents, that is, agents that use both language A and B, who have been claimed to play a relevant role in the evolution of multilingual societies [8, 15, 16].

Following Milroy [17], we expect that social structure might also be an important factor in language competition, and, therefore, we first study the dynamics in a regular lattice and, secondly, we consider a small world network. This way we will provide a quantitative analysis that is wanting in the field of sociolinguistics, as noted by de Bot and Stoessel [18]. A regular lattice structure captures a society where interactions are based on geographical proximity. However, it has been shown that social networks are far from being regular, and they are not totally random either [19]. Quite on the contrary, what has been found as a main character in most of the real social networks analysed (e.g., mobile phone calls [20]) is the small world phenomenon, which describes the effect of long range social interactions among the agents. Therefore, an analysis of language competition in a small world network is crucial to understand the role of these long range interactions in this process.

In this work we are interested in the emergent phenomena appearing as a result of a self-organized dynamics in the case of two equally prestigious competing languages. With the aim of elucidating possible mechanisms that could stabilize the coexistence of these languages, we wish to discuss the role of bilingual individuals and social structure in the process of language competition. In this way, this paper complements our previous analysis of the agent-based version of Abrams-Strogatz two-state model [12].

This paper is, then, structured as follows. In Section 2, and along the lines of the original proposal by Minett and Wang [16], we study an agent based model that incorporates bilingual agents. Section 3 shows the effects of local interactions in a two dimension regular network, while in Section 4 we analyse the dynamics of language competition in a small world social structure. Finally, Section 5 offers the main conclusions.

2 The Bilinguals Model

We consider a model of two socially equivalent (i.e equally prestigious) competing languages in which an agent i sits in a node within a network of N individuals and has k_i neighbours. It can be in three possible states: A, agent using [3] language A; B, agent using language B; and AB, bilingual agent using both languages, A and B.

The state of an agent evolves according to the following rules: at each iteration we first choose one agent i at random, and, then, we compute the local densities of language users of each linguistic community in the neighbourhood of agent i: σ_i^l (l=A, B, AB; i=$1, N$; $\sigma_i^A + \sigma_i^B + \sigma_i^{AB} = 1$). The agent i changes its state of language use according to the following transition probabilities:

$$p_{i,A\to AB} = \frac{1}{2}\sigma_i^B \ , \qquad p_{i,B\to AB} = \frac{1}{2}\sigma_i^A \tag{1}$$

$$p_{i,AB\to B} = \frac{1}{2}(1 - \sigma_i^A) \ , \qquad p_{i,AB\to A} = \frac{1}{2}(1 - \sigma_i^B). \tag{2}$$

Equation (1) gives the probabilities for an agent to move away from a monolingual community to the bilingual community AB. They are proportional to the density of monolingual speakers of the other language in its neighbourhood. On the other hand, equation (2) gives the probabilities for an agent to move from the bilingual community towards one of the monolingual communities. Such probabilities are proportional to the density of speakers of the adopting language including bilinguals ($1 - \sigma_i^l = \sigma_i^j + \sigma_i^{AB}$, l, j=A, B; $l \neq j$). It is important to note that a change from being monolingual A to monolingual B or vice versa always implies an intermediate step through the bilingual community. The transition probabilities (1) and (2) are fully symmetric under the exchange of A and B, which is consistent with the fact that both languages are socially equivalent in terms of prestige.

In a fully connected society, and in the limit of infinite population (in the mean field approximation), this model can be described by coupled differential equations for the total population densities ($\Sigma_A, \Sigma_A, \Sigma_{AB}$):

$$\frac{\mathrm{d}\Sigma_A}{\mathrm{d}t} = \frac{1}{2}[(1 - \Sigma_A + (\Sigma_B)^2 - 2\Sigma_B), \tag{3}$$

$$\frac{\mathrm{d}\Sigma_B}{\mathrm{d}t} = \frac{1}{2}[(1 - \Sigma_B + (\Sigma_A)^2 - 2\Sigma_A) \tag{4}$$

[3] Note that we always refer to language use rather than competence.

The analysis of these mean field equations, (3) and (4), shows the existence of three fixed points: two are stable and equivalent, and correspond to states of monolingual dominance of one of the languages and extinction of the other communities: $(\Sigma_A, \Sigma_B, \Sigma_{AB})^*_1 = (1, 0, 0)$; $(\Sigma_A, \Sigma_B, \Sigma_{AB})^*_2 = (0, 1, 0)$; and the other one is unstable, with all three communities being present, corresponding to unstable coexistence, $(\Sigma^*_A, \Sigma^*_B, \Sigma^*_{AB})$, with $\Sigma^*_l \neq 0$ $(l = A, B, AB)$.

On the other hand, the Abrams-Strogatz model for the particular case of socially equivalent languages, also considered in the mean field approximantion, leads to a simple differential equation:

$$\frac{\mathrm{d}\Sigma_A}{\mathrm{d}t} = 0, \tag{5}$$

predicting that any initial given density of speakers of one language would persist forever. However, numerical results for the agent based model for the Abrams-Strogatz dynamics [12] indicate a very different behaviour due to finite size fluctuation effects, with one of the two languages eventually becoming dominant, as we discuss in the following sections.

Generally speaking, mean field equations do not give an appropriate description of finite size communities, or societies which are not fully connected. Therefore, we consider here how the above simple mean field description is modified or complemented by finite size, local effects, social structure and by the presence of bilingual agents.

In our simulations we use random asynchronous node update: at each time step a single node is randomly chosen and updated according to the transition probabilities (1) and (2). We normalize time so that in every unit of time each node has been updated on average once, i.e., a unit of time includes N time steps. If not otherwise specified, we start from random initial conditions: random distribution of 1/3 of the population being monolingual A, 1/3 monolingual B and 1/3 bilingual AB.

For a quantitative description of the emergence and dynamics of linguistic spatial domains we use the ensemble average interface density $\langle\rho\rangle$ as an order parameter. This is defined as the density of links joining nodes in the network which are in different states [21, 14]. The ensemble average, indicated as $\langle\cdot\rangle$, denotes average over realizations of the stochastic dynamics starting from different random distributions of initial conditions. For the random initial conditions: $\langle\rho(t = 0)\rangle = 2/3$; a given node has probability 2/3 of being connected to a node in a different state. This is valid for any network among whose nodes there are no correlations in the random initial distribution of states.

During the time evolution, the decrease of ρ from its initial value describes the ordering dynamics, where linguistic spatial domains, in which agents are in the same state, grow in time. The minimum value $\rho = 0$ corresponds to a stationary configuration in which all the agents belong to the same linguistic community.

3 Local Effects in a Regular Network

To study the effects of local interactions in a partially connected society we first consider a situation in which the agents are located in the sites of a two-dimensional regular network, with periodic boundary conditions, interacting with their four nearest neighbours.

In Figure 1-*left* we show the time evolution for a typical realization: language A takes over the system, while language B faces extinction. In the long run, as languages are socially equivalent, both A and B face extinction with probability 1/2. We observe an early very fast decay of the interface density and of the total density of bilingual speakers, followed by a stage of fluctuations of these quantities around a small value until one of the languages starts to dominate, and the bilinguals disappear together with the language that faces extinction.

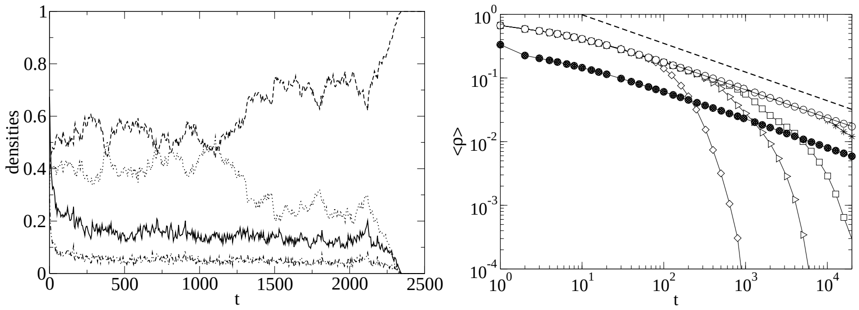

Fig. 1. *Left*: Time evolution of the total densities of speakers belonging to each linguistic community, Σ_l (l= A, B, AB), and the interface density, ρ, for the bilinguals model in a two-dimensional regular lattice. One realization in a population of $N = 400$ agents is shown. From top to bottom: Σ_A (dashed line), Σ_B (dotted line), ρ (solid line), Σ_{AB} (dot-dashed line) *Right*: Time evolution of the averaged interface density $\langle\rho\rangle$ for the bilinguals model in a two-dimensional regular lattice for different system sizes. Empty symbols: from left to right: $N = 10^2$ ($\diamond$), 20^2 ($\triangleright$), 30^2 ($\square$), 100^2 ($*$), 300^2 ($\circ$). The averaged global density of bilingual agents, $\langle\Sigma_{AB}\rangle$, for $N = 300^2$ agents is also shown ($\bullet$). Averages are calculated over 100-1000 realizations depending on the system size. Dashed line for reference: $\langle\rho\rangle \sim t^{-0.45}$

In Figure 1-*right* we show the time evolution of the average interface density and of the total density of bilinguals, averaged over different realizations. For the relaxation towards one of the absorbing states (dominance of one language) both the average interface density and the average density of bilinguals decay following a power law with the same exponent, $\langle\rho\rangle \sim \langle\Sigma_{AB}\rangle \sim t^{-0.45}$ [22]. This indicates that the evolution of the average density of bilinguals is correlated with the interface dynamics. During this stage, spatial domains of each monolingual community are formed and grow in size. This is known in the physics literature as *coarsening*. Eventually a finite size fluctuation occurs (as the one shown in Figure 1-*left*) so that the whole system is driven to an absorbing state in which one linguistic community prevails.

In Figure 2-*top*, for a typical realization, we display snapshots of the temporal evolution of the spatial distributions of agents, where the formation and growth of the monolingual communities can be observed. However bilingual spatial domains are never formed. During an early fast stage of the dynamics, bilingual agents place themselves in the boundaries between the two monolingual communities. This explains the finding that the bilingual density follows the same power law as the average density of interfaces. For the growth law of the typical length of a monolingual spatial domain we find an exponent near 0.5, which corresponds to domain growth dominated by interface curvature. Curvature driven growth is a mechanism by which boundaries tend to stretch, leading small domains to collapse, swallowed by the surrounding larger ones: agents feel the social pressure of the majority, and change their language use according to it.

For a typical realization, the average time to reach an absorbing monolingual state, τ, can be estimated to scale as $\tau \sim N$ (N being the total number of agents) since we find $\langle \rho \rangle \sim t^{-\gamma}$, with $\gamma \simeq 0.5$, and when the last speaker disappears $\langle \rho \rangle \sim 1/N^{1/d}$ (d is the dimensionality of the lattice). However, there exist simulations which get trapped in stripe-like metastable states. When taking into account these realizations, the characterisitic time τ scales as $\tau \sim N^{1.8}$ [22].

In Figure 2-*bottom* we show evidence of the intrinsic instability of bilingual communities: an initial bilingual domain disintegrates into smaller monolingual spatial domains very fast, and bilinguals are just placed at the interfaces. Therefore, it follows from the present model that even when initial bilingual domains exist, these are not sustained: societies with knowledge of two languages tend to end up using just one of them, even if they are ideally socially equivalent. The role of bilinguals is to link different monolingual domains, communicating with both of them, but they eventually disappear together with the monolingual community that gets extinct.

To summarize the effect of the bilingual agents, we must first recall the results obtained in the analysis of the agent based model for the Abrams-Strogatz dynamics for equivalent languages, where bilinguals are not taken into account [12]. For a two-dimensional regular lattice, this study shows that the average interface density decays following a power law $\langle \rho \rangle \sim t^{-0.11}$ (consistent with a logarithmic decay), the dependence of the lifetime with the system size scales as $\tau \sim N \ln(N)$, and the domain boundaries follow noisy interface dynamics, without well defined boundaries of the monolingual domains. All these results are compatible with voter model dynamics [23, 14], an extensively studied imitation model where agents copy the state of a randomly chosen neighbour. Comparing the results obtained in Section 3 for the bilinguals model with the results explained above for the agent based model for the Abrams-Strogatz dynamics, it follows that the presence of bilingual agents yields a much faster process for the growth of monolingual spatial domains

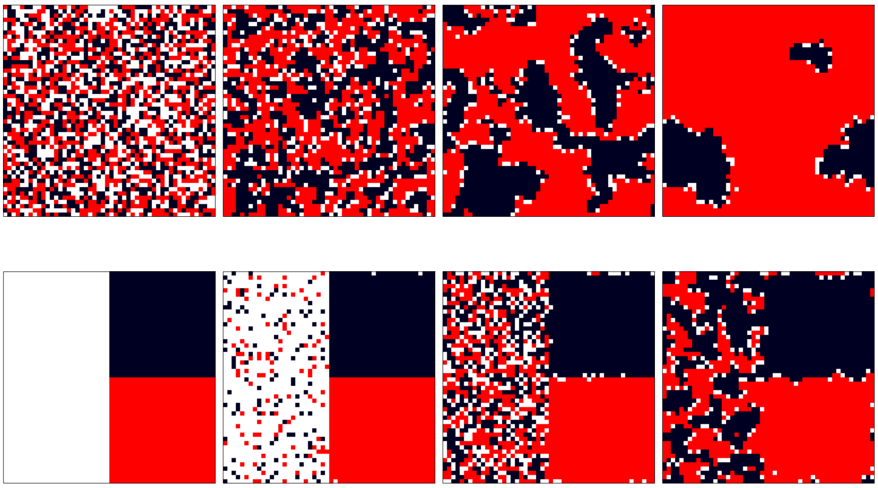

Fig. 2. *Top*: Random initial conditions: snapshots of a typical simulation of the dynamics in a regular lattice of 2500 individuals. t=0, 2, 20, 200 from left to right. *Bottom*: Disintegration of an initial bilingual community in a regular network: 2500 individuals. t=0, 5, 50, 500 from left to right. Grey: monolinguals A, black: monolinguals B, white: bilinguals

(faster coarsening) but they also favour a longer persistence of the two languages before one faces extinction (larger average time for large fixed N). These two effects follow from the role played by the thin film of bilingual agents placed at the boundaries of the monolingual domains. The boundaries become well defined and they evolve driven by curvature analogously to what happens in majority driven models like the Ising model with Glauber Dynamics at T=0 [24]. A two-state model based on memory effects [25] finds similar interface dynamics to the one shown in the present paper.

However, the dynamics continues to have intrinsic stochastic ingredients (as in the voter model) so that the system does not get trapped in frozen configurations in which coexistence of the two linguistic communities would be maintained forever. This kind of configurations have been obtained in a different model for political opinion dynamics, which deals also with three possible states for the agents [26].

4 Social Structure Effects: Small World Network

To study the effect of long range social interactions in the network, we next consider the dynamics of language competition on a small world network constructed following the algorithm by Watts & Strogatz [19]: starting from a two-dimensional regular lattice, we rewire each of the links at random with probability p, obtaining, in this way, long range interactions throughout the network.

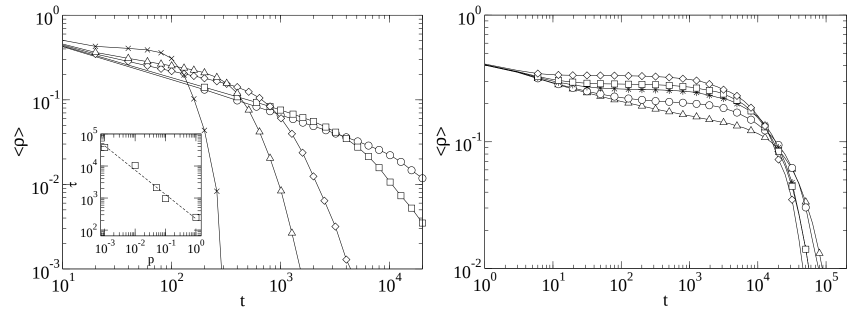

Fig. 3. Time evolution of the average interface density $\langle\rho\rangle$ in small world networks with different values of the rewiring parameter p. For the sake of comparison, we show the case $p = 0$, which corresponds to a regular network, and the case $p = 1$, corresponding to a random network. *Left*: bilinguals model: from left to right: p=1.0 ($\times$), 0.1 ($\triangle$), 0.05 ($\diamond$), 0.01 ($\square$), 0.0 ($\circ$). The inset shows the dependence of the average time to reach an absorbing monolingual state τ with the rewiring parameter p. The dashed line corresponds to the power law fit $\tau \sim p^{-0.76}$. Population of $N = 100^2$ agents. Averages taken over 500 realizations. *Right*: agent based model for the Abrams-Strogatz dynamics: from up to bottom, p=1.0 ($\diamond$), 0.1 ($\square$), 0.05 ($*$), 0.01 ($\circ$), 0.0 ($\triangle$). Population of $N = 100^2$ agents. Averages taken over 900 realizations.

In Figure 3-$left$ we show the evolution of the average interface density for different values of p. We observe here a dynamical stage of coarsening with a power law decrease of $\langle\rho\rangle$ followed by a fast decay to a state of dominance of one of the languages caused by a finite size fluctuation. In the range of intermediate values of p properly corresponding to a small world network, increasing the rewiring parameter p has two main effects: i) the coarsening process is notably slower, so that monolingual spatial domains grow slower, and ii) the average time to reach an absorbing monolingual state τ drops following a power law: $\tau \sim p^{-0.76}$ (inset of Figure 3-$left$), so that the monolingual absorbing state is reached much earlier as the network becomes disordered with many long range connections among agents. We would also like to point out that, during the dynamical stage of coarsening, the monolingual communities have similar size, while the total density of bilinguals is much smaller.

To understand the role of bilingual agents, the above results should be compared with the ones in Figure 3-$right$ for the agent based model for Abrams-Strogatz dynamics in which there are no bilingual agents. In contrast to the model with bilinguals, intermediate values of p stop the coarsening process leading to dynamical metastable states characterized by a plateau regime for the average interface density. However the lifetime of these states is not very sensitive to the value of p, with the average time τ being just slightly smaller than the one obtained in a regular lattice (in which $p = 0$). This is a different effect from the strong dependence on p found for these average times when bilingual agents are at work. Comparing now results of the simulations with bilingual agents and without them for the same value of p, we observe that bilingual agents produce a faster coarsening, as it happened in a regular network. However, the average time to reach the absorbing monolingual state becomes much smaller, which represents the opposite effect to the one caused by bilingual agents in a large regular network.

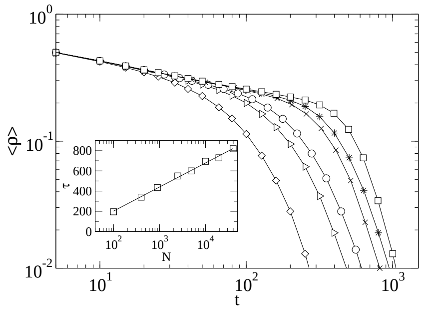

Fig. 4. Time evolution of the averaged interface density, $\langle\rho\rangle$, for different values of the population size, N, in a small world network with $p = 0.1$. $N = 10^2$ ($\diamond$), 20^2 ($\triangleright$), 30^2 ($\circ$), 70^2 ($\times$), 100^2 ($*$), 200^2 ($\Box$); from left to right. Averaged over 1000 realizations in 10 different networks. Inset: dependence of the characterisitc time to reach an absorbing monolingual state τ with the system size: $\tau \sim \ln N$.

For a fixed value of the rewiring parameter p, the effect of different system sizes is analyzed in Figure 4. There we observe that the initial stage of coarsening process is grossly independent of system size, but the average time τ scales with the system size N as $\tau \sim \ln(N)$. When bilingual agents are not taken into account one obtains $\tau \sim N$ [12]. Again, this indicates that, contrary to the results obtained in the regular lattice, the presence of bilinguals leads to a faster extinction of one of the languages in a society with many individuals. This is due to the large effect of the small world structure when bilinguals are present: the monolingual domains evolving by mean curvature that were formed in the regular lattice grow to a much lesser extent due to the existence of shortcuts or long range links in the network that avoid the formation of big monolingual communities. However, dynamics through noisy interfaces, as the one that exists when there are no bilingual agents, does not feel, to the same extent, the presence of these shortcuts.

In summary, the social structure described by a small world network becomes more relevant when bilingual agents are considered and they cause a faster decay to a state in which one linguistic community disappears.

5 Summary and Conclusions

We have studied an agent based model for the competition between two languages, introducing bilingual individuals in the dynamics. In this respect, this work comes to fill the gap left by, on the one hand, earlier studies about language competition using population dynamics [6, 8], and, on the other hand, agent-based studies that disregard bilinguals [12]. This approximation to the social reality of language competition complements other studies found in traditional sociolinguistic literature, e.g. [27].

We find that, within the assumptions discussed here, with linear transition probabilities of language shift (proportional to densities of speakers), bilingualism and social structure are found not to be efficient mechanisms to stabilize language coexistence or maintenance in a finite population. Quite on the contrary, we find that, after a stage of formation and growth of monolingual spatial domains, the asymptotic state is one in which one of the linguistic communities gets extinct together with the bilingual agents.

As for the role of bilinguals in the model analysed, we have found that bilingual agents are not able to form stable communities. Instead, they place themselves at the boundaries between the monolingual spatial domains, favouring communication between them. In addition, we would like to emphasize the fact that even if there are initial bilingual communities, they disintegrate partially into smaller monolingual domains. Meanwhile, the remaining bilinguals play, once again, the role of links at the boundaries between monolingual communities. Therefore, the present model leads to the conclusion that societies with knowledge of two languages tend to end up splitting into communities where just one language is used, at the same time that one of the languages faces extinction in the long run.

This work has also contributed to the study of language competition as far as social structure is concerned. Thus, we have shown that the effect of bilingual agents in the dynamics of language competition is not the same in a regular social network or in a partially disordered network with long range connections. In a regular network, their presence accelerates the rate of growth of monolingual spatial domains, but slows down the process of final extinction of one of the languages. On the contrary, on a small world network, bilingual agents destroy a metastable state of dynamical coexistence or language maintenance causing a slow growth of monolingual spatial domains. At the same time, they lead to a much faster rate of extinction of one of the linguistic communities. Therefore, the small world effect, which is characteristic of the current interconnected societies, might be an ingredient which accelerates language extinction. This effect might be related to an overall globalization process in which not only languages, but also whole cultures tend to homogeneity rather than diversity. Globalization has indeed been pointed out as the trigger of the emergence of a linguistic variety that Crystal calls *World Standard Spoken English* [28].

To sum up, according to the model analysed, in which two languages are socially equivalent in terms of prestige, the presence of bilinguals in a small world network accelerates language death. This could lead to the conclusion that active measures in favour of bilingualism might not result in the conservation of linguistic diversity. However, the model analysed lacks important mechanisms acting in the dynamics of language competition. One of them involves the emergence of new linguistic varieties resulting from code-switching [29], or from further evolutions of language contact which givie rise to pidgins or creoles. Another mechanism worth of interest in language competition is bounded rational choice. This considers that agents take into account cost and benefit when changing language use, in which the unequal prestige of languages or the environment in which they are used can play a relevant role.

Therefore, we may conclude that, within its limited framework, our study contributes to paving the way towards a more comprehensive characterization of language dynamics, helping to clarify the crucial role of bilinguals, and the different effects of social structure in the complex world of language competition.

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