Superflow in Solid Helium

Abhishek Roy

December 16, 2004

Abstract

The small mass and significant zero point motion of solid ^{4}He make it very different from its conventional counterparts. In the seventies it was suggested that at very low temperatures solid helium might flow like a superfluid while remaining a crystal. This hidden "supersolid" phase was subsequently almost ruled out both by experiments and theoretical calculations. However, in 2004 Kim and Chan reported that when rotated at temperatures below 230mK a fraction of solid ^{4}He does indeed refuse to be dragged around by its walls, a famous effect hitherto only seen in its superfluid counterpart. In this paper, I will describe the experiment, its consequences and possible theoretical explanation.

Kim and Chan's Experiment

I will start directly with the experiment. The authors used a torsional oscillator with an annular channel which was filled with high-purity ^{4}He and solidified by increasing the pressure. A driving motor and lock-in amplifier were used to make the rod oscillate and measure its resonant frequency. As the temperature was lowered below 230mK it was found that the resonance period decreased steadily before saturating at about 40mK. Since the period is given by $2\pi\sqrt{\frac{I}{G}}$ this implies a decrease in the moment of inertia I. The authors concluded that part of the solid helium was decoupling from the oscillations of the cavity. In other words, in the rest frame of the rod a small fraction of helium was was flowing without friction.

The superfluid fraction $f_s = \rho_s/\rho$ was found by taking the measured moment of inertia and using the relation,

$$I(T) = I_{classical}(1 - f_s(t))$$

It depends both on the temperature and the maximum velocity of the oscillator. For all pressures from 26 bars (close to the melting point) to 65 bars f_s increased as the temperature is decreased reaching a maximum that ranges from .75% to 1.7% depending on the pressure. The authors speculate that the variation in f_s was caused by crystalline imperfections and grain boundaries in the sample. The dependence of f_s on the maximum velocity of the walls is shown in the attached figure. It suggests that there is a (pressure dependent) critical velocity ω_c and the supersolid can only decouple during the part of the cycle where the velocity is less than ω_c.

As a control, a similar setup was run with a Magnesium barrier in the annulus. If superflow was indeed occurring around the axis of rotation previously, the barrier would now prevent it, and there would be no decrease in the resonance period. Actually a small decrease (upto 2% of that in the unblocked channel) in the resonance period was observed as the temperature was lowered. The authors attribute this to the decoupling caused by *irrotational* flow of solid ^{4}He. When liquid ^{4}He was used in the blocked channel, the fractional decoupling observed was comparable to the solid case, supporting this explanation.

Technicalities

Previously the same researchers had reported superflow in solid helium confined in porous Vycor glass, but there were concerns that the effect there could be purely due to the "liquid" behaviour of Helium adsorbed on the surface of the pores. The present paper is the first observation on of superflow in *bulk* solid helium. A similar experiment by Bishop et. al. [9] found no such effect. It should be noted that the purity of the ^{4}He is very important since only 441 ppm of ^{3}He was enough to quench the supersolid phase in the Vycor experiment.
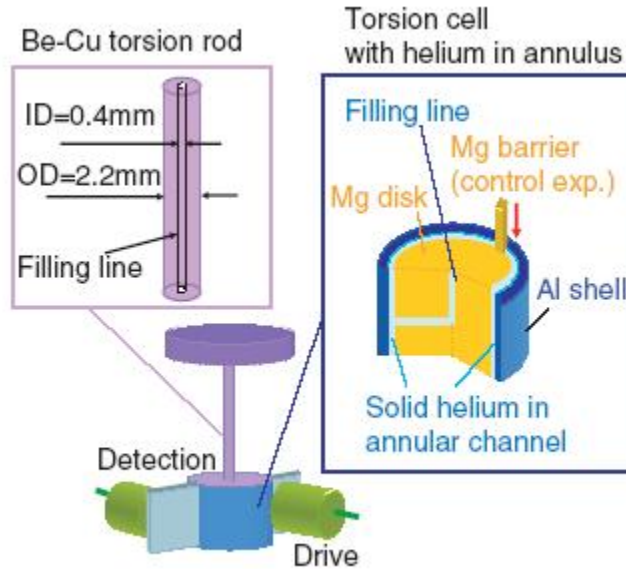
Figure 1: Experimental Setup - from [1]

This could be one reason for the failure of previous efforts (a comprehensive list is given by Meisel [9]) to detect a supersolid phase.

Theory

This section rests on much shakier ground since by the pigeonhole principle, one or more of the authors mentioned below must be wrong. I have tried to summarise as much of each approach that I could understand. The last two papers cited are unpublished and I am certainly not qualified to review them. Caveat emptor.

Leggett's Argument

In 1970 Leggett [4] suggested that a solid could have the property of *non-classical rotational inertia* just like superfluid helium. He proposed an experiment very similar to the one described above and gave an argument leading to an upper bound on the supersolid fraction ρ_s/ρ. I will briefly sketch his argument here, due to its historical importance.

Consider the n-particle ground state wavefunction of a crystalline bosonic solid confined
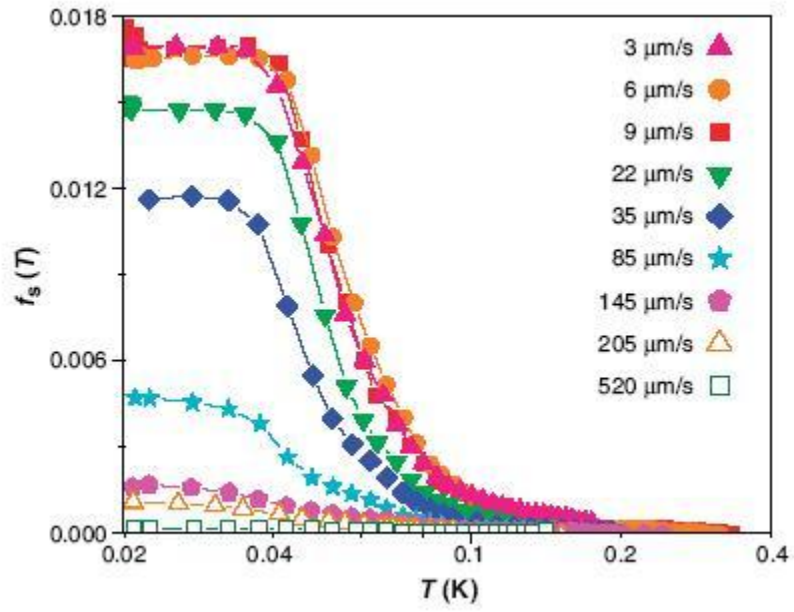
Figure 2: The superfluid fraction at 41 bars - from [3]. The lines are labelled by the maximum velocity of the rotating chamber

to a thin annulus of radius R in a cylinder at $T = 0$. Our Hamiltonian is the standard,

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_{j=1}^{n}\nabla_j^2 + \frac{1}{2}\sum_{j,k=1}^{n}U(\vec{r_j} - \vec{r_k}) + \sum_{j=1}^{N}V(\vec{r_j})$$

In cylindrical coordinates Ψ is periodic, satisfying the condition,

$$\Psi(r_1, z_1, \theta_1; \cdots r_j, z_j, \theta_j + 2\pi; \cdots r_n, z_n, \theta_n) = \Psi(r_1, z_1, \theta_1; \cdots r_j, z_j, \theta_j; \cdots r_n, z_n, \theta_n)$$

for each i. Now if we rotate the enclosing surface by an angular velocity ω the Hamiltonian becomes time dependent but we can simplify things by putting,

$$\theta_j = \theta_j' + \omega t \tag{1}$$
$$\Psi(\theta_j, t) = \Psi'(\theta_j', t)e^{im\omega R^2 \sum_i \theta'(t)/\hbar} \tag{2}$$

with a new boundary condition,

$$\Psi(r_1, z_1, \theta_1; \cdots r_j, z_j, \theta_j + 2\pi; \cdots r_n, z_n, \theta_n) = \tag{3}$$
$$e^{-2\pi i m R^2 \omega/\hbar}\Psi(r_1, z_1, \theta_1; \cdots r_j, z_j, \theta_j; \cdots r_n, z_n, \theta_n) \tag{4}$$

This unitary transformation works if we ignore the width d of the annulus compared to its radius. The Hamiltonian is time independent in the new coordinates and we could variationally find the minimum expectation value $E(\omega)$. If we assume that as in a superfluid,

$$\Delta E = E_o - E(\omega) = \frac{1}{2}(\rho_s/\rho)I_o\omega^2$$

where I_o is the classical moment of inertial, then we could calculate ρ_s/ρ. If the system is to avoid superfluidity $E(\omega)$ should be independent of ω; in other words we should be able to vary the ground state wave-function to satisfy obey the boundary condition (3) without changing the expection value of the energy [1]. One way to achieve this to modify the phase of Ψ in regions where it almost vanishes and leaving it untouched elsewhere. This will work if the wavefunction is sufficiently "disconnnected", i. e. any path from θ_i to $\theta_i + 2\pi$ goes through a region where Ψ essentially vanishes. This "localisation" of Ψ prevents superfluidity in a conventional solid.

Leggett then suggests a variational scheme of the form,

$$\Psi = \exp\left\{\sum_{i=1}^{n}\phi(\vec{r_i}; \omega)\right\}\Psi_o(\vec{r_1} \cdots \vec{r_n}) \tag{5}$$

---

[1] or at least have $|E(\omega) - E_o| < \hbar^2/mR^2$ as expected from ordinary superfluidity

where Ψ_o and ϕ are real and ϕ satisfies the required boundary condition

$$\phi(\theta + 2\pi) = \phi(\theta) - 2\pi m R^2 \omega/\hbar$$

We get

$$\Delta E = \hbar^2/2m \int (\nabla\phi)^2 \rho(\vec{r})d\vec{r}$$

where $\rho(\vec{r})$ is the single particle probability density. Further computation depends on the exact form of ρ.

Estimates and Calculations   A numerical calculation was performed by Saslow, [6] who took a crystal lattice with $\rho(\vec{r})$ the sum of gaussians centered at each lattice point,

$$\rho(\vec{r}) = \sum_{sites} \rho_{site}, \quad \rho_{site}(r) = (\pi b)^{3/2} \exp(-r^2/b^2)$$

If a is the distance between the lattice sites then intuitively ρ_s/ρ should increase with b/a i. e. more overlap between neighbouring sites. That is indeed what was obtained (graph attached).

Guyer [5] interpreted Leggett's mechanism to be the cooperative tunneling of pairs of particles resulting in motion of the single-particle density function. From NMR and X-ray scattering data, he estimated that $\rho_s/\rho \approx 10^{-6}$ and a minimum temperature below 0.1mK putting it almost outside the reach of experiment. This analysis was generally regarded as definitive before Kim and Chan's results.

Chester and others

Chester suggested that a system of strongly interacting bosons could exhibit Bose-Einstein condensation and crystalline ordering at the same time. He based his conjecture on a theorem of Onsager and Penrose which says that provided the multiparticle wavefunction is a Jastrow state, BEC results in the thermodynamic limit. A Jastrow state is given by,

$$\Psi = \exp(-\frac{1}{2}\sum_{i \neq j} u(r_i j))$$

where the conditions on $u(r)$ state roughly speaking that it has a hard core, is bounded below, and falls off sufficiently quickly. The theorem holds even after we add to u a term for the zero-point energy of long-wavelength density waves,

$$\chi(r) = \frac{1}{N}\sum_{0 < k < k_c} e^{i\vec{k}\cdot\vec{r}}\frac{2mc}{\hbar k}$$
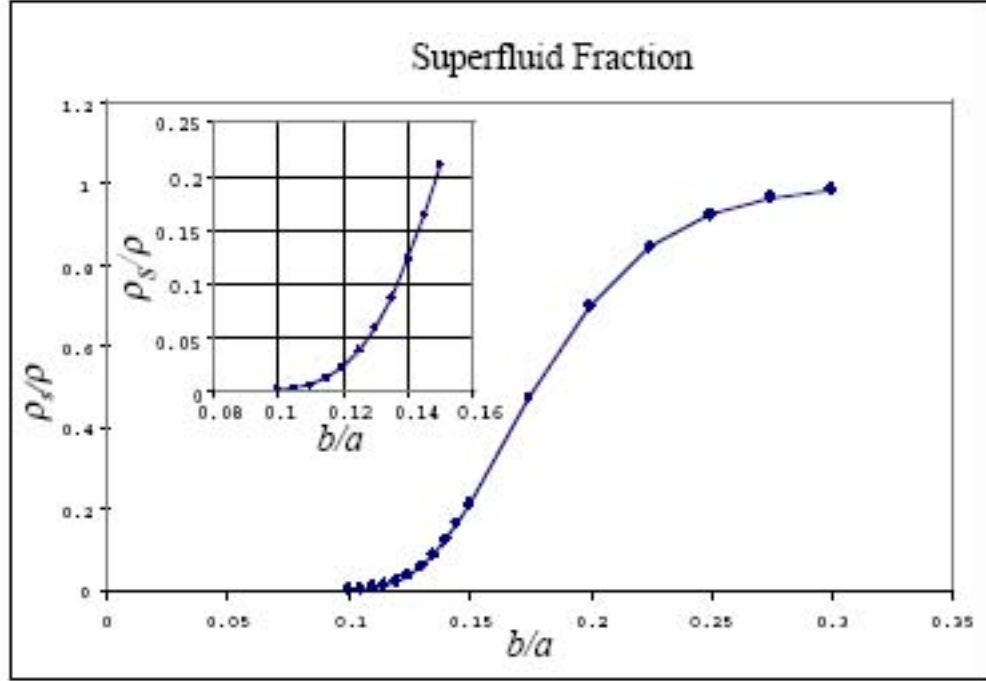
6

Figure 3: Calculated superfluid fraction by Leggett's method - from [7]

Next an effective temperature is introduced by,

$$u(r) + \chi(r) \rightarrow \frac{u(r) + \chi(r)}{T_{eff}}$$

The resulting probability distribution is identical to the Gibbs distribution for N particles interacting via a two-body potential. Chester now reasons that the Gibbs distribution is applicable to a very wide range of systems over all phases. So it is possible that a sufficiently low temperature the appropriate u for ^{4}He will give rise to crystalline ordering. This is ruled by another theorem of Onsager and Penrose but the result doesn't hold if there is macroscopic concentration of vacancies in the lattice sites at $T = 0$. Then BEC and crystalline ordering *could* occur simultaneously. This is similar to the speculation of Andreev and Lifshitz[15] that frozen vacancies and defects at zero temperature could condense and lead to a supersolid phase.

Estimates   From observations of vacancy-waves in ^{4}He, Guyer [5] deduces that that number of vacancies at $T = 0$ for N particles is less than $10^{-14}N$. Meisel's review [9] states that all

experimental evidence argues for only thermally *activated* vacancies whose concentration in negligible below 0. 2K. Finally Ceperley [11] notes that that according to simulations Jastro wavefunctions don't crystallize easily and their transition density is far removed from the experimental value, so it is unwise to base predictions on them

Naysayers

In 1987, Pollock and Ceperley [13] had worked out a detailed model for calculating the superfluid densities and momentum distribution using path integrals. When applied to ^{4}He, their model gives agrees well with experiment and has the advantage of not requiring a trial wave function or ad hoc approximations. After Kim and Chan's results, Ceperly [11] revisited the problem and concluded numerically that exchange processes in solid helium are very localised and cannot possibly lead to superflow.

Prokof'ev and Svistunov [10] also used the above approach to reach a similar conclusion. They speculate that the Kim and Chan's results arise from a heterogenous sample that has solid microcrystallites with superfluid interfaces. According to them, this would explain the broad distribution of transition temperatures in the data and also the vanishing of of ρ_s at T_c with zero derivative intead of the $(\frac{T_c-T}{T_c})^{0.671}$ expected from bulk superfluidity.

Conclusion

It seems that the question of the supersolid phase will only be settled one way or the other experimentally. Future experiments will probably improve on the homogeneity of the sample and verify whether the phenomenon is actually an equilibrium effect (in the present case the the chamber was oscillating as the temperature was lowered). Observation of the quantisation of circulation, if possible, could be decisive. Perhaps a future ESM student will report on the answer.

References

[1] E. Kim and M. H. W. Chan, *Science* 305 1941, 2004

[2] E. Kim and M. H. W. Chan, *Nature* 427 225, 2004

[3] A. J. Leggett, *Science* 305 1921

[4] A. J. Leggett, *Phys. Rev. Lett.* 25 1543, 1970

[5] R. A. Guyer, *Phys. Rev. Lett.* 26 174, 1971

[6] W. M. Saslow, *Phys. Rev. Lett.* 36 1151, 1975

[7] W. M. Saslow, cond-mat/0407166

[8] G. V. Chester, *Phys. Rev. A* 2 256, 1969

[9] M. W. Meisel, *Physica B* 178, 121, 1992

[10] N. Prokof'ev and B. Svistunov, cond-mat/0409472

[11] D. M. Ceperley and B. Bernu, cond-mat/0409336

[12] W. Kohn and D. Sherrington, *Rev. Mod. Phys.* 42 1

[13] E. L. Pollock and D. M. Ceperley, *Phys. Rev. B* 36 8343, 1987

[14] M. Tiwari and A. Datta, cond-mat/0406124

[15] A. F. Andreev and I. M. Lifshitz, *JETP* 29 1107, 1969