

Frequency Adaptive Repetitive Control of Grid-connected Inverters

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Abstract—Grid-connected inverters (GCI) are widely used to feed power from renewable energy distributed generators into smarter grids. Repetitive control (RC) enables such inverters to inject high quality fundamental-frequency sinusoidal currents into the grid. However, digital RC which can get approximately zero tracking error of any periodic signal with known integer period in steady-state, cannot exactly track or reject periodic signal of frequency variations. Thus digital RC would lead to a significant power quality degradation of GCIs when grid frequency varies and causes periodic signal with non-integer periods. In this research paper a frequency adaptive repetitive control scheme (FARC) at a predefined sampling rate is proposed to deal with all types of periodic signal of variable frequency. A fractional delay filter which is based on Lagrange interpolation is used to estimate the fractional period terms in RC. This proposed FARC controller offers the fast, during process modification of fractional delay and fast revise of filter parameters, and then provides GCIs with a simple but very accurate real-time frequency adaptive control solution to the injection of high quality sinusoidal current under grid frequency variations. A case study a three-phase GCI is conducted to testify the validity of the proposed strategy.

Keywords—Repetitive control; Grid connected inverter; Frequency variation;

I. INTRODUCTION

Grid-connected inverters (GCIs) are widely used to feed power from renewable energy distributed generators into smarter grids. Repetitive control (RC) which is based on the internal model principle, can get approximately zero error tracking of all periodic signals with known frequency in steady-state [1-5]. Repetitive control (RC) enables such inverters to inject high quality fundamental frequency sinusoidal currents into the grid [6-10]. The most advanced controllers are often realized in digital or discrete form. The conventional repetitive controller in digital form is given as $z^{-N}/(1-z^{-N})$. It is capable of tracking all periodic signals with integer periods of $N = T/T_s$ where T being the reference signal time period, T_s corresponds to the sampling time and N being the order of the repetitive controller [2,6]. The grid frequency varies with the random generation-load imbalance, intermittent renewable distributed generators DGs would increase grid frequency fluctuations, e.g. in E.ON grid codes for DGs, the extreme grid frequency variation range is up to (46.5, 53.5) Hz [2]. Therefore grid voltages and currents are

time-varying and fractional period. Conventional RC are not capable of exactly tracking or rejecting non-integer period reference signals/disturbances, and lead to intolerable performance degradation of GCIs when grid frequency varies and causes periodic signals with fractional periods. GCIs are quite sensitive to the grid frequency variation.

This paper reports some preliminary results based on a frequency adaptive repetitive control (FARC) scheme at some predefined sampling rate to track/reject a variable frequency periodic. FARC is based on the fractional delay (FD) filter design theory. FARC divides the order of RC in two parts; integer part and fractional part where the fractional part is realized by FD filters.

The remaining paper is organized as follow; Section II explicitly describes the design and analysis of a FARC controller. In section III FARC control of a three-phase grid connected inverter is implemented. Simulation results are given in section IV. Finally, conclusion is presented in section V.

II. FREQUENCY ADAPTIVE REPETITIVE CONTROL

Fig. 1 represents the typical control system in closed-loop with a plug-in digital conventional repetitive controller (CRC), where $R(z)$ is the reference input signal, $Y(z)$ is the output signal, $E(z) = R(z) - Y(z)$ is the actuating signal, $D(z)$ is the unknown and undesired disturbance, $G_p(z)$ is the plant, $G_c(z)$ is the conventional feedback controller, $G_r(z)$ is a feed-forward plug-in CRC, k_r is the proportional and integral gain, $U_r(z)$ is the output of the CRC controller, $G_f(z)$ is a phase lead filter to overcome the practical delays of the closed-loop system and $Q(z) = a_1z + a_0 + a_1z^{-1}$ with $2a_1 + a_0 = 1$ is a low pass filter to enhance robustness of the system.

The plug-in CRC in Fig. 1 can be expressed as [1-3,7,8]:

$$G_{rc}(z) = k_r \frac{z^{-N} Q(z)}{1 - Q(z) z^{-N}} G_f(z) \quad (1)$$

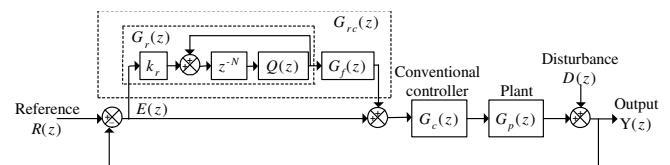


Fig. 1: Plug-in repetitive control system

Where $N = \lceil f_s / f \rceil \in N$ where, f is the fundamental frequency of the reference signal $R(z)$. It can also be the frequency of unknown disturbance $D(z)$ and f_s is the sampling frequency, N is the order of the repetitive control system. The poles of the repetitive control system $G_r(z)$ are located around harmonic frequencies $2\pi mf$ with $m = 0, 1, 2, \dots, M$ ($M = N/2$ for even N and $M = (N-1)/2$ for odd N) [7-9]. It is very noticeable that amplitude of $G_r(z)$ at all harmonic frequencies $2\pi mf$ below the Nyquist frequency reaches ∞ if the low pass filter has unity magnitude response *i.e.* $|Q(z)| = 1$. Thus CRC achieves minimum (≈ 0) error tracking during steady-state operation at all frequencies (fundamental and harmonics) below Nyquist frequency if the low pass filter has unity magnitude and its order N is integer.

However, in variable-frequency cases the controller order N is usually non-integer when the sampling frequency f_s is fixed, *i.e.* the fundamental frequency of the $R(z)$ and/or $D(z)$ is non-integer [2]. The CRC controllers with order truncated to the nearest integer value of N is unable to perfectly follow or reject non-integer period signals or unwanted and unknown disturbances; because the gains at the harmonic frequencies shifts away from actual harmonic frequencies, and thus lead to significant loss of performance [11-13].

For example, in case of a CRC for periodic signals of fundamental frequency $f = 50$ Hz where integer order $N = 100$, the sampling frequency is 5 kHz, $k_r = 0.2$, $Q(z) = 0.1z^1 + 0.8z^0 + 0.1z^{-1}$ and the magnitude of the CRC at the 5th order harmonic frequency (250 Hz) is 20.232; if the fundamental frequency of periodic signals changes from 50 Hz to 50 ± 0.2 Hz, the corresponding magnitude at 5th order harmonic frequency will drop 1.580 at 250 ± 1 Hz. This example clearly shows that integer order CRC is not capable of tracking or eliminating fractional order periodic signals.

In practical implementations, it is impossible to implement a delay z^{-N} with non-integer values of N . According to the fractional delay filters (FD) design theory [2], the best way to achieve fractional delays is by the use of integer order FD filters. Supposing $z^{-N} = z^{-N_i + F}$ where $N_i = \text{int}[N]$ *i.e.* the integer part of the controller order N and $F = N - N_i$ and ($0 \leq F < 1$) is the non-integer part of the controller order N , the fractional delay z^{-F} can be closely approximated by a finite impulse response Lagrange interpolator as follows [2]:

$$z^{-F} \approx \sum_{k=0}^n A_k z^{-k} \quad (2)$$

Where $k = 0 : 1 : n$ and the Lagrange interpolator coefficients represented by A_k are calculated as follow:

$$A_k \approx \prod_{\substack{i=0 \\ i \neq k}}^n \frac{F-i}{k-i} \quad k = 0, 1, 2, \dots, n \quad (3)$$

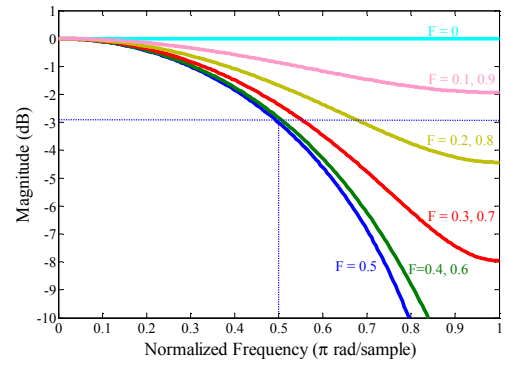


Fig.2: The magnitude response of Lagrange interpolation based filter with $n=1$

In case of $n = 1$ in Eq. (2) a linear interpolation $z^{-F} = A_0 + A_1 z^{-1}$ where $A_0 = 1-F$ and $A_1 = F$ is obtained. Lagrange interpolation is one of the simplest method to design a fractional delay filter that can approximate any given fractional delay. Computational complexity of the Lagrange interpolation is also reasonably low.

Quick online calculation of the coefficients of fractional delay and their quick apprise is of supreme significance for real-time controllers used in power converters. The Lagrange interpolation needs only a few multiplications and additions for updating coefficients.

Fig. 2 represents the magnitude response of the first order FD filter for a fractional delay range $F = 0, 0.1, 0.2, \dots, 0.9$. This FD filter is based on Lagrange interpolation. It has been observed that the FD filters with $n=1$ provide a fairly accurate approximation of the fractional delay z^{-F} at lower frequencies. Thus first order FD filters provide a bandwidth of 50% Nyquist frequency.

Generally, infinite impulse response filters (IIR) exhibit the similar characteristics in frequency domain with a fewer number of mathematical operations than an FIR filter. Unluckily, the IIR filters design and frequency response is way more complex than the analogous FIR filters. It may also lead to additional finite word length problem due to round off errors, cycles limit and likely uncertainty in the quantization of coefficients (predominantly if the coefficients were updated on-line). Moreover, transient problems may occur in actual-time implementations. Thus updating the filter coefficient becomes really difficult. Consequently the IIR FD filters are usually not suggested for power converters applications.

Substituting Eq. (2) and (3) into Eq. (1), a FARC is achieved:

$$G_{fr}(z) = k_r \frac{z^{-N_i} \left(\sum_{k=0}^n A_k z^{-k} Q(z) \right)}{1 - z^{-N_i} \left(\sum_{k=0}^n A_k z^{-k} Q(z) \right)} G_f(z) \quad (4)$$

The FARC of Eq. (4) is equivalent to the CRC of Eq. (1) when the fractional part F is zero. Thus, the FARC of Eq. (4) is capable of tracking or elimination of all types of periodic signal having random frequency (integer or non-integer). Replacing the $G_r(z)$ of Fig. 1 with the proposed FARC

$G_p(z)$, the transfer function of the actuating or error signal $Y(z)/R(z)$ can be derived as follow:

$$\frac{Y(z)}{R(z)} = \frac{1 - \left(z^{-N_i} \sum_{k=0}^n A_k z^{-k} \right)}{1 + G_p(z)G_c(z)} \times \frac{1}{1 - z^{-N_i} \left[Q(z) \left(\sum_{k=0}^n A_k z^{-k} \right) (1 - k_r G_f(z)H(z)) \right]} \quad (5)$$

where

$$H(z) = \frac{G_p(z)G_c(z)}{1 + G_p(z)G_c(z)}$$

From Eq. (5) it is clear that the FARC control system is asymptotically stable during closed-loop operation if and only if these conditions are fulfilled:

- The roots of $1 + G_c(z)G_p(z) = 0$ lie inside unity circle;
- The roots of $1 - z^{-N_i} \sum_{k=0}^n A_k z^{-k} Q(z) (1 - k_r G_f(z)H(z)) = 0$ are inside the unit circle, then

$$|Q(z)| \left| \sum_{k=0}^n A_k z^{-k} \right| |1 - k_r G_f(z)H(z)| < 1$$

$$\forall z = e^{j\omega}, 0 \leq \omega \leq \pi/T$$

Clearly, the stability criteria given above for the FARC system is compatible with the CRC control system. The design of k_r , $Q(z)$ and phase lead compensator $G_f(z)$ for a FARC system are not different from their design for CRC. The FARC control scheme provides a competent high performance control solutions to deal with periodic signals having fluctuating frequency and/or amplitude.

III. FARC OF GRID-CONNECTED INVERTER

Fig. 3 represents a three-phase grid-connected inverter, which is used to feed currents into/out of the utility grid.

To achieve the required results, the overall control scheme which combines the outer-loop PI voltage controller and an inner-loop conventional feedback controller plus a plug-in frequency adaptive repetitive controller for ac current control is developed. The dynamics of the VSI with linear resistive load in Fig. 3 are given as [14-15]:

$$\begin{pmatrix} \dot{i}_a \\ \dot{i}_b \\ \dot{i}_c \end{pmatrix} = \begin{pmatrix} -\frac{R_n}{L_n} & 0 & 0 \\ 0 & -\frac{R_n}{L_n} & 0 \\ 0 & 0 & -\frac{R_n}{L_n} \end{pmatrix} \begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix} + \begin{pmatrix} \frac{v_{sa} - v_a}{L} \\ \frac{v_{sb} - v_b}{L} \\ \frac{v_{sc} - v_c}{L} \end{pmatrix} \quad (6)$$

$$\dot{U}_{dc} = -\frac{1}{C_n R_n} U_{dc} + \frac{1}{C_n} i_o + \frac{1}{C_n R_n} E_f$$

where v_{sa}, v_{sb} and v_{sc} are the grid phase voltages; i_a, i_b and i_c are the feeding in/out phase currents; U_{dc} is the dc bus voltage; $v_a = u_a U_{dc}/2, v_b = u_b U_{dc}/2$ and $v_c = u_c U_{dc}/2$ are the PWM modulated voltages with u_a, u_b, u_c is the normalized control outputs of the controller; L_n, R_n and C_n are the nominal values of ac-side inductor L , ac-side resistance R and the dc bus capacitor C respectively; R_l is the load resistance and E_f is the *emf* of the load.

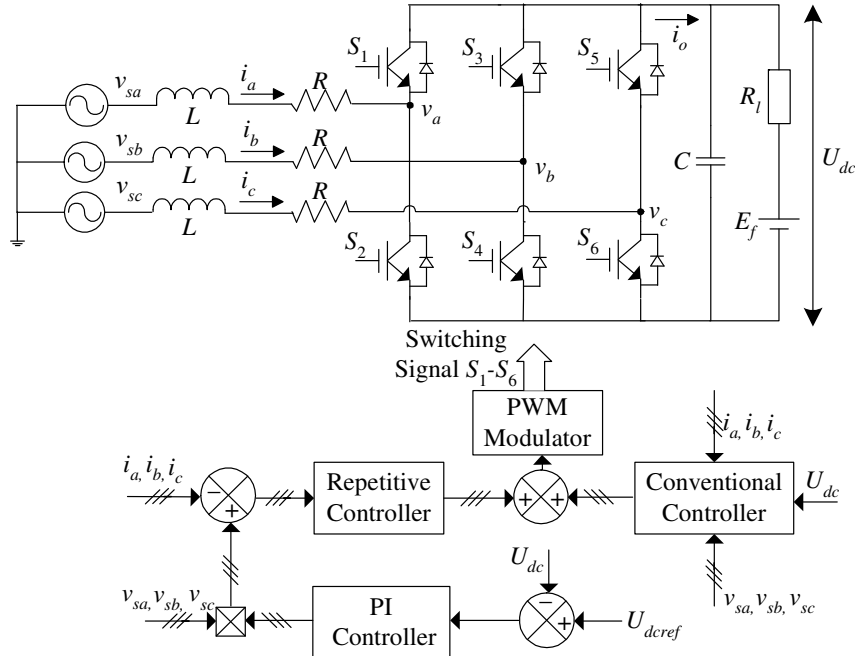


Fig. 3: Grid-connected three-phase PWM inverter

The objective of this controller for the inverter is to achieve power factor ≈ 1 , low harmonic distortion sinusoidal feeding current and constant ripple free dc bus voltage U_{dc} .

A linear system that can be represented by $\dot{x} = Ax + Bu$, can be sampled by $x(k+1) = e^{AT}x(k) + \int_0^T e^{A(T-\tau)}Bu(\tau)d\tau$. The corresponding sampled-data model of the Eq. (6) can be written as:

$$i_j(k+1) = \frac{b_1 - b_2}{b_1} i_j(k) + \frac{v_{sj}(k)}{b_1} - \frac{u_j(k)}{b_1} \frac{v_d(k)}{2} \quad (7)$$

where

the subscript $j = a, b, c$

$T =$ sampling time

$b_1 = L_n/T$

$b_2 = R_n$

If the current controller for the plant (7) is chosen as:

$$u_j(k) = \frac{2}{v_d(k)} [v_{sj}(k) - b_1 i_{jref}(k) + (b_1 - b_2) i_j(k)] \quad (8)$$

Then $i_j(k+1) = i_{jref}(k)$ is obtained, *i.e.* a deadbeat controller for current is obtained. However, the deadbeat controller for current is built on very precise nominal parameters of the inverter [16-18]. In practice, there are some uncertainties in parameter and load disturbances for the converter. As shown in Fig. 3 a FARC controller of (5) is added to always ensure high accuracy current tracking, where $G_f(z) = z^{-1}$ is a linear phase lead filter.

To achieve a constant dc bus voltage, a PI controller is employed. The PI controller has a transfer function as:

$$G_v(z) = k_1 + k_2 \frac{T}{z} \quad (9)$$

where gains k_1 and k_2 are designed to ensure a stable outer voltage loop with satisfactory dynamic and steady-state response.

IV. SIMULATION

A three-phase GCI with proposed FARC controller shown in Fig. 3 has been implemented on Matlab/Simulink platform.

TABLE I: System parameters

Nominal inductance	$L_n = 5$ mH
Actual inductance	$L = 3$ mH
Nominal load resistance	$R_n = 50$ Ω
Load resistance	$R = 40$ Ω
Dc bus capacitance	$C_n = 2200$ μ F
Reference dc bus voltage	$V_{dcref} = 80$ V
Sampling frequency	5 kHz
Load <i>emf</i>	$E_f = 0$ V
Repetitive control gain	$k_r = 0.1$
PI gains	$k_1 = 0.9, k_2 = 40$
Nominal ac voltage $v_{sj}(j=a,b,c)$	50 Hz, 30 V(peak)

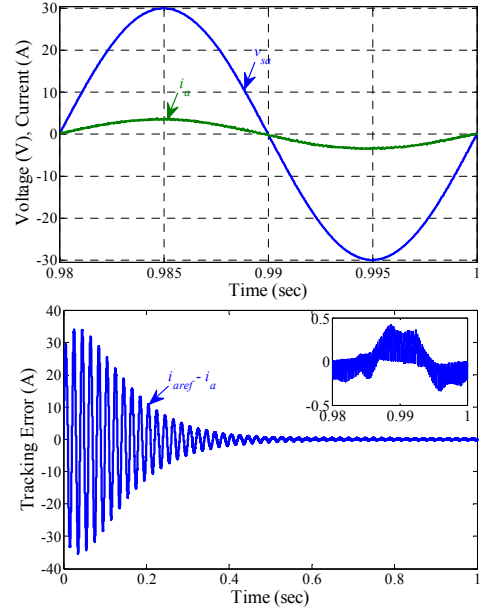


Fig. 4: CRC controlled GCI response with grid frequency $f=50$ Hz.

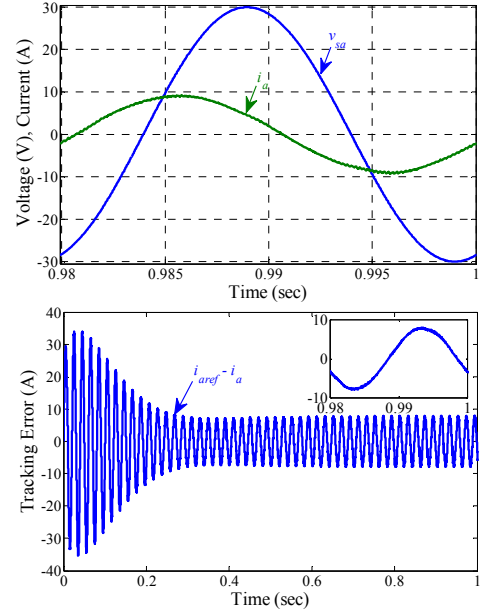


Fig. 5: CRC controlled GCI response with grid frequency $f=49.8$ Hz

The values of simulated system parameters are tabulated in the Table I. Fig. 4 shows the response of CRC controlled GCI with grid frequency of 50 Hz (*i.e.* $N=100$) in steady-state, where power factor is unity and steady-state current tracking error is less than 0.4 A.

Fig. 5 shows the steady-state response of CRC controlled GCI with grid frequency of 49.8 Hz (*i.e.* $N=100.40$), where power factor is about 0.56 ($\approx \cos 56^\circ$) and the steady-state current tracking peak error is $\approx \pm 8$ A. As the power factor is significantly far from unity and the current tracking error is very high so the CRC fails to provide a satisfactory performance when the order N of the controller is non-integer.

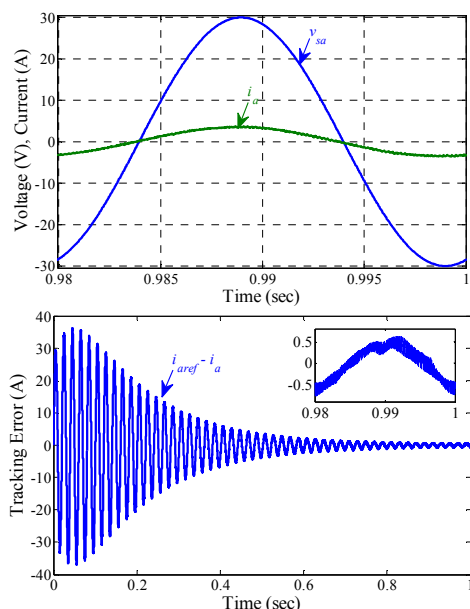


Fig. 6: FARC controlled GCI response with grid frequency $f=49.8$ Hz.

Fig. 6 shows the steady-state response of first order FD filter based FARC controlled GCI with grid frequency of 49.8 Hz (*i.e.* $N=100.40$, $N_f=100$ and $F=0.4$), where the power factor is unity and current tracking peak error is ≈ 0.6 A. Tracking error can be further reduced by employing a higher order FD filter based FARC controller. Usually a third order FD filter based FARC controller provides sufficient bandwidth for the controllers to compensate harmonic distortion.

Obviously the FARC control can always ensure high tracking accuracy in the presence of grid frequency variations (fractional order); while the CRC fails to provide satisfactory regulation capability.

I. CONCLUSION

This research paper proposes a frequency adaptive RC scheme with fixed sampling rate. This control scheme is able to achieve or eliminate any periodic signal with changing or fluctuating frequency. The FD filters that are designed using the Lagrange interpolation are used to closely approximate the fraction part in repetitive control, this FARC control scheme enables the quick on-line modification of new small delay and the quick change in the coefficients. Thus it provides simple and significant improvement in performance of the actual-time control solution to modern high performance converters. A case study of FARC based three-phase GCI is conducted. Experiment results clearly depict the effectiveness and validity of the proposed FARC.

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