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THE OPTIMAL USE BY FARMERS OF THE INCOME EQUALISATION SCHEME

by

A.T.G. McArthur

Technical Paper No. 17

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PREFACE

While the Income Equalisation Scheme has not been as widely used as some imagined it would, nevertheless there is sufficient scope for its use to justify this examination by Mr A. T. G. McArthur. He has used a dynamic programming procedure to establish rules for optimum strategies under conditions of fluctuating farm incomes. These strategies are presented in a form which will be of value particularly to farm accountants and farm management advisers.

J. D. Stewart
Director

December 1971

THE OPTIMAL USE BY FARMERS OF THE INCOME EQUALISATION SCHEME

INTRODUCTION

A progressive income tax penalises those taxpayers with a fluctuating income (for example, farmers), as compared with those on a stable income with the same average. However there are various methods of smoothing taxable income and hence reducing average tax payments.

One such scheme is the Income Equalisation Scheme which was proposed by the Taxation Working Party of the Agricultural Development Conference in 1965 and was subsequently adopted by the Government. Under this scheme a farmer can deposit up to a quarter of his income from one year in the Income Equalisation Fund. He must withdraw a deposit within five years, adding the withdrawal to his income for that year.

However, using the Income Equalisation Scheme has an opportunity cost, an opportunity foregone elsewhere. The funds deposited with the Government earn no interest. A thousand dollars deposited in the Fund for a year could have reduced a farmer' overdraft with his bank by that amount, saving him about $75. In deciding how best to use the Income Equalisation Scheme to smooth taxable incomes, the tax saving gain from a smoother income must be balanced against the opportunity cost of storing the income in the Equalisation Fund.

This paper describes a method for farmers and their advisers for making optimal use of the Income Equalisation Scheme. Optimal is defined as the maximisation of the present value of post-tax incomes. However readers should be aware that the scheme is of little value in reducing tax payments unless the farmer's income is highly variable.

In presenting the method which involves dynamic programming, the mathematics have been put in appendices so that the paper can be followed by those not skilled in mathematical techniques. The paper is divided into four sections. Firstly, a method of estimating farm income variability is given. Secondly, a method is presented for estimating the extra tax paid because of a fluctuating income. Thirdly, the results of using the Income Equalisation Scheme on historical incomes from Lincoln College's Ashley Dene farm are discussed. Finally, the rules for making optimal use of the Income Equalisation Scheme under realistic circumstances are presented.

1. ESTIMATING VARIABILITY

Understanding how to measure income variability is the first step in coming to grips with the implications of taxation for farmers whose income fluctuates. It is usual to use the standard deviation to measure variability. The larger the standard deviation the bigger the variation. If the standard deviation of income is zero, then future income can be predicted with certainty.

Standard deviation is calculated using the deviation of each figure from the average. These deviations are squared as part of the calculation. Appendix A gives the details of the method of calculating the standard deviation from a series of past incomes.

The standard deviation of historical financial and technical data may be of only limited value for estimating the situation for the future. This applies to means (averages) as well as standard deviations. For instance, taking the average wool price for the last 20 years and using this to estimate wool prices for the next 5 years is unlikely to be classed as 'realistic'. Likewise if a farm has been improved over recent years, historical lambing percentages may only be a partial guide to future lambing percentages. An informed guess of a 'likely figure to work on' will often be a better guide than an historical average because this historical average may reflect conditions which may not apply in the future. A method of estimating expected average income over the next few years ahead and its standard deviation is given below.

(a) Pick an extremely optimistic income. This income would assume a wool boom like the one in the mid sixties, coupled with high wool weights and a high lambing percentage. Call this OPT. There should only be a very small chance of such an optimistic income - one would bet something less than 1 chance in 100 of such a high income occurring in any one year.

(b) Pick an extremely pessimistic income - even lower wool and lamb prices than today, together with say effects of the worst drought in living memory. Call this <u>PESS.</u> There should be only one chance in 100 of such an extremely pessimistic figure occurring in any year.

(c) Now work out the most likely income - the figure used in normal budgeting. Call this <u>LIKE.</u>

The standard deviation for future income can then be calculated by taking one-sixth of the difference between optimistic and pessimistic.

$$\underline{\text{Standard devision of income}} = \frac{(\text{OPT} - \text{PESS})}{6} \quad \text{(approx.)}$$

<u>Expected income</u> (average future income) is calculated by:

$$\text{Expected income} = \frac{(\text{OPT} + 4 \times \text{LIKE} + \text{PESS})}{6} \quad \text{(approx.)}$$

Thus, supposing we have estimated that

OPT (most optimistic income)	=	\$ 20,000
LIKE (most likely income)	=	\$ 5,000
PESS (most pessimistic income)	=	\$ -5,000

then the standard deviation of income is:

$$\text{Standard deviation} = \frac{(20,000 - (-5,000))}{6} = \$\ 4,166$$

or roughly \$4,000; and the expected income is:

$$\text{Expected} = \frac{20,000 + 4 \times 5,000 + (-5,000)}{6} = \$\ 5,833$$

or roughly \$6,000.

The estimation of variability through the use of the standard deviation is foreign to nearly all farmers and most advisers. Yet this measure is almost essential for rational planning for the risky and variable conditions under which most farmers have to operate. Its estimation is also essential if optimal use of the Income Equalisation Scheme is to be made.

2. ESTIMATING EXTRA TAX BECAUSE OF A FLUCTUATING INCOME

Appendix B shows the derivation of the formula below for calculating the extra tax payable resulting from a fluctuating income with a certain standard deviation. Standard deviation of income is represented by the Greek symbol σ (sigma).

$$\text{Extra tax annually} = 1.42\ \sigma^2/100000 \quad \text{(approx.)}$$

Table 1 shows the expected extra tax caused by income variation.

TABLE 1

Expected Extra Annual Tax Payments
(Approximate)

Standard Deviation of Pre-Tax Income	Expected Extra Tax Annually
$	$
1000	14
2000	57
3000	128
4000	227

With a low standard deviation of pre-tax income of $1000 such as might be faced by a dairy farmer, the extra tax annual payments of $14 are small.

However, a run-holder with a standard deviation of income of $4000 could have a legitimate case for complaint on the grounds of an inequitable tax burden. It is only when the standard deviation of income is high that significant gains can be made from the optimal use of the Income Equalisation Scheme.

3. INCOME EQUALISATION SCHEME APPLIED TO HISTORICAL DATA

The use of the Income Equalisation Scheme allows the farmer to delay the arrival of income in his taxable account in return for a smaller tax payment. For instance a farmer with incomes of $9000, $3000 and $1000 in three succeeding years might decide to use the Income Equalisation Fund to smooth out his income to $5000 a year by holding $4000 of the first year's income in the Fund to build up the income in the third year. By comparing the present value[1] of the post-tax incomes under use and non-use of the Scheme, the farmer can make a rational decision as to whether or not he should smooth his income by the use of the Income Equalisation Scheme. Table 2 shows such a comparison, assuming an interest rate of 5 per cent and taxation exemption of $1000.

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[1] The present value equivalent of future incomes provides a valid way of comparing income streams with different distribution patterns. The present value of post-tax incomes is that amount of money which, if invested at a specified rate of interest, could build the same stream of post-tax incomes.

TABLE 2

Comparison of non-use versus use of Income Equalisation Scheme

	Year 1	Year 2	Year 3
Discount factor (5%)	0.952	0.907	0.864
Non-Use of Equalisation Scheme	$	$	$
Pre-tax Income	9,000	5,000	1,000
Taxable Income	8,000	4,000	0
Tax	2,760	990	0
Post-tax Income	6,240	4,010	1,000
Discounted post-tax Income	5,940	3,637	864
Present Value	5,940 + 3,637 + 864 = 10,441		
Use of Equalisation Scheme			
Post-tax Income	4,010	4,010	4,010
Discounted Post-tax Income	3,817	3,637	3,465
Present Value	3,817 + 3,637 + 3,465 = 10,919		
Extra Present Value	10,919 - 10,441 = $478		

The present value of the post-tax income when income is smoothed using the Income Equalisation Scheme amounts to $10,919, which exceeds the present value of the post-tax income when the scheme is not used by $478. This difference of $478 can be expressed as an equivalent annual gain by multiplying the sum of $478 by the amortisation factor. This is the compound interest formula for finding an equal set of cash flows over 3 years which has the same present value as the $478. The amortisation factor for 3 years at a 5% rate of interest is 0.368 which when multiplied by $478 gives an equivalent annual gain of $176.

Putting $4,000 in the Income Equalisation Scheme and holding it there for two years to bolster income in the third year is a better policy than not using the scheme at all, but it is not the optimal policy. The optimal policy can be determined by dynamic programming. Appendix C gives the mathematical basis of the dynamic programming solution for finding the optimal policy. The optimal policy is defined as that which maximises the present value of post-tax income. Meantime it is assumed that future income is known. The optimal policy for the case above is shown in Table 3.

TABLE 3

Optimal Policy for Use of Equalisation Scheme

Year	Income	Deposit (plus) Withdrawal (minus)	Fund
1	$9,000	+ $3,500	$3,500
2	$5,000	0	$3,500
3	$1,000	- $3,500	0

Present value of discounted post-tax income = $10,930

Extra present = $489

Equivalent annual gain = $180

The equivalent annual gain of $180 shown in Table 3, brought about by using the optimal policy is $4 a year ahead of smoothing income to $5,000 a year as shown in Table 2.

As an example, dynamic programming has been used to find what would have been the optimal policy for the use of the Income Equalisation Scheme on the incomes from Lincoln College's Ashley Dene farm. This farm runs fat-lamb producing ewes on stony soils. It exhibits a wide variation in pre-tax income because of the sensitivity of production to droughts and the variability of the prices of wool and lamb. Over the last 13 years the standard deviation of income on this farm has been $3,975 with a mean of $4,358. Assuming annual pre-tax incomes over the period were known, Table 4 shows the optimum use of the equalisation fund on the Ashley Dene farm. The interest rate used for discounting was $7\frac{1}{2}$ per cent. It is true that the assumption of certainty about future incomes is unrealistic but it illustrates the general method of approach to be used in the next section when the certainty assumption is dropped.

TABLE 4

Optimum Use of the Income Equalisation Scheme on the Ashley Dene Property

Pre-Tax Income	Fund Deposit-Withdrawal	Fund
- 1310	0	0
5520	1000	1000
- 728	-1000	0
1114	0	0
4200	0	0
6878	0	0
7688	0	0
6150	0	0
5616	0	0
12168	3000	3000
4134	0	3000
6509	1600	4600
- 1285	-4600	0

The present value of post-tax income with an optimal fund use was $26,243 as compared with $25,440 without its use. The difference of $803 is equivalent to an annual return over the 13 years of $99. Reference to Table 1 indicates that with a standard deviation of $4,000 the expected extra tax annually is $227. Hence the optimum use of the fund removes less than half the disadvantage of a variable income in this case.

4. THE OPTIMAL POLICY UNDER UNCERTAINTY

The Ashley Dene example requires perfect foreknowledge of future income. In practice farmers are able to recognise boom and slump years and can estimate roughly the expected income of the years ahead together with the range within which income is likely to fall. Intuitively a farmer might think it is worthwhile to put something into the Equalisation Fund after an excellent year and withdraw from the Fund in a bad year. Appendix C shows how dynamic programming can be used to derive optimal rules ("optimal" is defined as before) for planning the use of the Income Equalisation Scheme under these conditions of uncertainty.

The setting for the application of these rules is as follows. It is assumed that the farmer is going to deposit or withdraw income just before the close of the financial year and that he can estimate accurately the income for the current year. Moreover it is necessary for the farmer to estimate his expected income and its standard deviation on an annual basis for the next three or four years.

Tables 5a, 5b and 5c give the optimal rules derived by dynamic programming. The tables are for 5%, 7½% and 10% rates of interest respectively. If an overdraft is costing 7½% in interest a year, then Table 5b is the appropriate table for finding the optimum amount of cash to have in the fund at the end of the year.

An example is the best way of showing how to use Table 5b. Suppose income is expected to average \$6,000 over the next few years and that the standard deviation should be about \$4,000. These are the estimates made previously. Now suppose also that there is already \$2,000 in the Income Equalisation Fund and this year's income will amount to \$9,000, giving a "pre-tax income plus deposit in fund" of \$11,000.

TABLE 5(a)

RULES FOR OPTIMUM USE OF INCOME EQUALISATION FUND

VALUES IN TABLE ARE OPTIMUM AMOUNTS TO BE IN THE FUND AT THE END OF THE YEAR GIVEN THE PRE-TAX INCOME PLUS DEPOSIT IN THE FUND AT THE BEGINNING OF THE YEAR AND AN INTEREST RATE OF 5%

Pre-Tax Income & Deposit in Fund	Average Income = $3000			Average Income = $4000			Average Income = $5000			Average Income = $6000			Average Income = $7000		
	σ =$2000	σ =$3000	σ =$4000	σ =$2000	σ =$3000	σ=$4000	σ=$2000	σ=$3000	σ=$4000	σ=$2000	σ=$3000	σ=$4000	σ=$2000	σ=$3000	σ=$4000
3,000	0	200	200	0	0	0	0	0	0	0	0	0	0	0	0
3,400	400	400	600	0	0	0	0	0	0	0	0	0	0	0	0
3,800	400	800	800	0	200	400	0	0	0	0	0	0	0	0	0
4,200	800	1,000	1,200	200	200	600	0	0	200	0	0	0	0	0	0
4,600	1,000	1,200	1,400	400	600	800	0	200	400	0	0	0	0	0	0
5,000	1,000	1,400	1,600	600	1,000	1,000	0	400	600	0	0	0	0	0	0
5,400	1,400	1,800	2,000	800	1,200	1,400	200	600	1,000	0	0	400	0	0	0
5,800	1,800	1,800	2,200	1,200	1,400	1,800	400	800	1,200	0	200	600	0	0	0
6,200	1,800	2,200	2,400	1,200	1,600	2,000	600	1,200	1,400	200	400	800	0	0	200
6,600	1,800	2,600	2,600	1,600	2,000	2,200	800	1,200	1,600	200	600	1,000	0	0	600
7,000	1,800	2,800	3,000	1,800	2,000	2,400	1,000	1,400	2,000	400	1,000	1,400	0	400	800
7,400	1,800	2,800	3,400	1,800	2,400	2,800	1,400	1,800	2,200	800	1,000	1,400	0	400	1,000
7,800	1,800	2,800	3,400	1,800	2,600	3,000	1,600	1,800	2,400	800	1,400	1,800	0	800	1,200
8,200	1,800	2,800	3,800	1,800	2,800	3,200	1,800	2,200	2,600	1,200	1,600	2,200	200	1,000	1,600
8,600	1,800	2,800	3,800	1,800	2,800	3,600	1,800	2,600	3,000	1,400	1,800	2,200	600	1,000	1,600
9,000	1,800	2,800	3,800	1,800	2,800	3,600	1,800	2,600	3,000	1,400	2,000	2,600	600	1,000	2,000
9,400		2,800	3,800	1,800	2,800	3,800	1,800	2,800	3,400	1,400	2,400	2,800	600	1,400	2,200
9,800		2,800	3,800	1,800	2,800	3,800	1,800	2,800	3,800	1,800	2,400	3,200	800	1,600	2,200
10,200		2,800	3,800		2,800	3,800	1,800	2,800	3,800	1,800	2,400	3,200	1,000	1,600	2,200
10,600		2,800	3,800		2,800	3,800	1,800	2,800	3,800	1,800	2,600	3,600	1,000	1,600	2,600
11,000		2,800	3,800		2,800	3,800	1,800	2,800	3,800	1,800	2,800	3,800	1,000	2,000	2,800
11,400		2,800	3,800		2,800	3,800		2,800	3,800	1,800	2,800	3,800	1,000	2,200	2,800
11,800		2,800	3,800		2,800	3,800		2,800	3,800	1,800	2,800	3,800	1,000	2,200	2,800
12,200			3,800		2,800	3,800		2,800	3,800		2,800	3,800	1,200	2,200	3,200
12,600			3,800		2,800	3,800		2,800	3,800		2,800	3,800	1,600	2,200	3,600
13,000			3,800		2,800	3,800		2,800	3,800		2,800	3,800	1,800	2,200	3,600
13,400			3,800			3,800		2,800	3,800		2,800	3,800		2,400	3,600
13,000			3,800			3,800		2,800	3,800		2,800	3,800		2,800	3,600
14,200			3,800			3,800			3,800		2,800	3,800		2,800	3,600
14,600			3,800			3,800			3,800		2,800	3,800		2,800	3,800
15,000			3,800			3,800			3,800		2,800	3,800		2,800	3,800
15,400						3,800			3,800			3,800		2,800	3,800
15,800						3,800			3,800			3,800		2,800	3,800
16,200									3,800			3,800			3,800
16,600									3,800			3,800			3,800
17,000									3,800			3,800			3,800

TABLE 5(b)

RULES FOR OPTIMUM USE OF INCOME EQUALISATION FUND

VALUES IN TABLE ARE OPTIMUM AMOUNTS TO BE IN THE FUND AT THE END OF THE YEAR GIVEN THE PRE-TAX INCOME PLUS DEPOSIT IN THE FUND AT THE BEGINNING OF THE YEAR AND AN INTEREST RATE OF 7½%

Pre-Tax Income & Deposit in Fund	Average Income = $3000			Average Income = $4000			Average Income = $5000			Average Income = $6000			Average Income = $7000		
	σ=$2000	σ=$3000	σ=$4000	σ=$2000	σ=$3000	σ=$4000	σ=$2000	σ=$3000	σ=$4000	σ=$2000	σ=$3000	σ=$4000	σ=$2000	σ=$3000	σ=$4000
3,000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
3,400	0	400	400	0	0	0	0	0	0	0	0	0	0	0	
3,800	400	400	800	0	0	200	0	0	0	0	0	0	0	0	
4,200	600	800	800	0	200	200	0	0	0	0	0	0	0	0	
4,600	600	1,000	1,200	200	600	600	0	0	200	0	0	0	0	0	
5,000	1,000	1,200	1,400	400	600	1,000	0	0	400	0	0	0	0	0	
5,400	1,400	1,400	1,600	600	1,000	1,200	0	400	800	0	0	0	0	0	
5,800	1,400	1,800	1,800	800	1,200	1,400	200	600	800	0	0	400	0	0	
6,200	1,800	2,000	2,200	1,200	1,400	1,600	400	800	1,200	0	200	600	0	0	
6,600	1,800	2,200	2,600	1,200	1,600	2,000	600	1,000	1,400	0	400	600	0	0	200
7,000	1,800	2,400	2,800	1,400	2,000	2,200	800	1,200	1,600	200	600	1,000	0	0	400
7,400	1,800	2,800	3,000	1,800	2,000	2,400	1,000	1,400	1,800	400	800	1,400	0	200	800
7,800	1,800	2,800	3,200	1,800	2,200	2,800	1,200	1,800	2,000	600	1,000	1,400	0	200	800
8,200	1,800	2,800	3,600	1,800	2,600	2,800	1,600	1,800	2,200	600	1,200	1,800	0	200	1,200
8,600	1,800	2,800	3,800	1,800	2,600	3,200	1,600	2,200	2,600	600	1,600	2,000	0	600	1,200
9,000	1,800	2,800	3,800	1,800	2,800	3,400	1,800	2,400	2,800	1,000	1,600	2,200	0	600	1,200
9,400		2,800	3,800	1,800	2,800	3,600	1,800	2,600	3,000	1,000	1,600	2,400	400	600	1,400
9,800		2,800	3,800	1,800	2,800	3,800	1,800	2,800	3,400	1,000	1,800	2,800	400	800	1,000
10,200		2,800	3,800		2,800	3,800	1,800	2,800	3,600	1,200	2,000	2,800	400	1,200	1,000
10,600		2,800	3,800		2,800	3,800	1,800	2,800	3,800	1,600	2,000	2,800	400	1,200	1,800
11,000		2,800	3,800		2,800	3,800	1,800	2,800	3,800	1,600	2,000	3,000	400	1,200	2,000
11,400		2,800	3,800		2,800	3,800		2,800	3,800	1,600	2,400	3,200	400	1,200	2,400
11,800		2,800	3,800		2,800	3,800		2,800	3,800	1,600	2,600	3,200	800	1,200	2,400
12,200			3,800		2,800	3,800		2,800	3,800	1,600	2,600	3,200	800	1,200	2,400
12,600			3,800		2,800	3,800		2,800	3,800	1,600	2,600	3,600		1,600	2,400
13,000			3,800		2,800	3,800		2,800	3,800		2,600	3,800		1,800	2,400
13,400			3,800			3,800		2,800	3,800		2,600	3,800		1,800	2,600
13,800			3,800			3,800		2,800	3,800		2,800	3,800		1,800	2,800
14,200			3,800			3,800			3,800		2,800	3,800		1,800	3,000
14,600			3,800			3,800			3,800		2,800	3,800		1,800	3,000
15,000			3,800			3,800			3,800		2,800	3,800		2,000	3,000
15,400						3,800			3,800			3,800		2,400	3,000
15,800									3,800			3,800		2,600	3,000
16,200									3,800			3,800		2,600	3,200
16,600									3,800			3,800			3,600
17,000									3,800			3,800			3,800

TABLE 5(c)

RULES FOR OPTIMUM USE OF INCOME EQUALISATION FUND

VALUES IN TABLE ARE OPTIMUM AMOUNTS TO BE IN THE FUND AT THE END OF THE YEAR GIVEN THE PRE-TAX INCOME PLUS DEPOSIT IN THE FUND AT THE BEGINNING OF THE YEAR AND AN INTEREST RATE OF 10%

Pre-Tax Income & Deposit in Fund	Average Income = $3000			Average Income = $4000			Average Income = $5000			Average Income = $6000			Average Income = $7000		
	σ=$2000	σ=$3000	σ=$4000	σ=$2000	σ=$3000	σ=$4000	σ=$2000	σ=$3000	σ=$4000	σ=$2000	σ=$3000	σ=$4000	σ=$2000	σ=$3000	σ=$4000
3,000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3,400	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3,800	200	200	400	0	0	0	0	0	0	0	0	0	0	0	0
4,200	200	600	600	0	200	200	0	0	0	0	0	0	0	0	0
4,600	600	600	1,000	0	200	600	0	0	0	0	0	0	0	0	0
5,000	800	1,000	1,000	200	400	600	0	0	0	0	0	0	0	0	0
5,400	1,000	1,400	1,400	400	800	1,000	0	0	0	0	0	0	0	0	0
5,800	1,200	1,400	1,800	600	800	1,200	0	400	400	0	0	0	0	0	0
6,200	1,600	1,800	2,000	800	1,200	1,400	200	600	600	0	0	200	0	0	0
6,600	1,600	2,000	2,200	1,000	1,400	1,600	400	600	600	0	200	600	0	0	0
7,000	1,800	2,200	2,400	1,200	1,600	2,000	600	1,000	1,000	0	400	800	0	0	200
7,400	1,800	2,400	2,800	1,400	1,800	2,200	800	1,200	1,200	200	600	1,000	0	0	400
7,800	1,800	2,800	3,000	1,800	2,000	2,400	1,000	1,400	1,400	200	800	1,200	0	0	600
8,200	1,800	2,800	3,200	1,800	2,200	2,600	1,200	1,600	1,600	200	800	1,400	0	200	600
8,600	1,800	2,800	3,600	1,800	2,600	2,800	1,400	1,800	1,800	400	800	1,600	0	200	600
9,000	1,800	2,800	3,600	1,800	2,600	3,000	1,400	2,000	2,000	400	1,000	1,800	0	200	1,000
9,400		2,800	3,800	1,800	2,800	3,400	1,400	2,200	2,200	400	1,200	1,800	0	400	1,000
9,800		2,800	3,800	1,800	2,800	3,600	1,800	2,200	2,200	800	1,200	1,800	0	600	1,000
10,200		2,800	3,800		2,800	3,800	1,800	2,200	2,200	800	1,200	2,200	0	600	1,200
10,600		2,800	3,800		2,800	3,800	1,800	2,600	3,400	800	1,600	2,400	0	600	1,400
11,000		2,800	3,800		2,800	3,800	1,800	2,600	2,600	800	1,600	2,400	0	600	1,400
11,400		2,800	3,800		2,800	3,800		2,600	2,600	800	1,600	2,400	200	600	1,400
11,800		2,800	3,800		2,800	3,800		2,800	2,800	1,000	1,600	2,800	200	800	1,400
12,200			3,800		2,800	3,800		2,800	2,800		1,600	2,800	200	1,000	1,400
12,600			3,800		2,800	3,800		2,800	2,800		1,600	2,800	200	1,000	1,600
13,000			3,800		2,800	3,800		2,800	2,800		2,000	2,800	200	1,000	2,000
13,400			3,800			3,800		2,800	2,800		2,200	2,800		1,000	2,000
13,800			3,800			3,800		2,800	2,800		2,200	2,800		1,000	2,000
14,200			3,800			3,800					2,200	3,200		1,200	2,000
14,600			3,800			3,800					2,200	3,400		1,600	2,000
15,000			3,800			3,800					2,200	3,400		1,600	2,000
15,400						3,800						3,400		1,600	2,400
15,800						3,800						3,400		1,600	2,600
16,200												3,400		1,600	2,600
16,600												3,600			2,600
17,000												3,800			2,600

Find the value \$11,000 in the left hand column and read across. Under "Average income = \$6,000", and " σ = \$4,000", will be found the figure \$3,000. This is the amount which should be in the Equalisation Fund at the end of the financial year. This means that the Equalisation Fund should be increased by \$1,000[1] to bring it up to \$3,000. This will reduce pre-tax income to \$5,000 for the year.

Table 6 shows the result of applying these rules, and gives the expected equivalent annual gain from using the Income Equalisation Fund optimally. In parenthesis is the expected extra tax paid because of a fluctuating income as compared with a completely stable one.

TABLE 6

Expected equivalent annual gain from using the Income Equalisation Scheme optimally - with expected extra Tax in Parenthesis

	Standard Deviation					
Pre-tax Income \$	\$ 2000		\$ 3000		\$ 4000	
3,000	20	(119)	54	(247)	98	(392)
4,000	14	(99)	43	(216)	84	(357)
5,000	8	(76)	30	(174)	66	(299)
6,000	3	(58)	15	(132)	42	(238)
7,000	0	(37)	4	(89)	21	(173)

[1] This set of rules was worked out assuming that taxation exemptions amount to \$1,000. If exemptions are vastly different from this then modify Table 2 by reducing the "Average Income" by \$1,000 and labelling the columns "Taxable Income". Then estimate taxable income rather than average income and look up the optimal policy.

The pre-tax income of $4, 000 with a standard deviation of $4, 000, gives an expected equivalent annual gain from optimal fund use of $84. This situation is close to the historical Ashley Dene situation. (Mean income was $4, 358 and standard deviation was $3, 975.) The equivalent annual gain from using the Income Equalisation Scheme optimally with perfect foreknowledge was $99.) Thus even without perfect foreknowledge one can do reasonably well by using the rules in Table 5.

The expected extra tax from a fluctuating income (in parenthesis gives the potential for gain from income smoothing. Only a fraction of this can in fact be realised by using the Income Equalisation Fund optimally under conditions of uncertainty.

SUMMARY AND CONCLUSION

This paper has shown that the extra tax paid because of a fluctuating income is proportional to the square of the standard deviation. The extra cost is negligible with incomes which have a low standard deviation, but those with highly variable incomes pay appreciably more tax.

This disadvantage can only be partially overcome by the use of the Income Equalisation Scheme when used optimally. The Scheme is not worth using unless incomes are highly variabl e.

The paper presents tables for use by farmers and their advisers in making optimal decisions for planning the use of the Income Equalisation Scheme.

APPENDIX A

CALCULATING THE STANDARD DEVIATION

The standard deviation is defined as the square root of the average of "deviations-from-the-mean squared".

Taking the incomes in Table A1 below as an example their mean is \$5000. [(5500 + 4500 + 6000 + 5000 + 4000) /5].

The deviations from this mean of \$5000 are + 500, - 500, + 1000, 0, and - 1000.

Squaring the deviations automatically turns the negative deviations into positive numbers. Minus 500 squared becomes plus 250, 000. Minus 1000 squared becomes 1, 000, 000.

The squares are added up and averaged to give the variance. The square root of the variance is found. This is the standard deviation.

TABLE A1

An Example of a Standard Deviation Calculation

Pre-Tax Income	Deviation	Deviation Squared
\$	\$	\$
5500	+ 500	250, 000
4500	+ 500	250, 000
6000	+ 1000	1, 000, 000
5000	0	0
4000	- 1000	1, 000, 000
Mean= 5000		Sum= 2, 500, 000

Variance = 2, 500, 000/4 = \$625, 000

Standard Deviation = $\sqrt{\text{Variance}} = \sqrt{625,000}$ = \$791

In the calculation the division of the "sum of deviations squared" by 4 rather than 5 is the statistically correct procedure. The divisor is always one less the number of observations.

APPENDIX B

EXTRA TAX DUE TO INCOME VARIATION

An approximate method of calculating the expected extra tax paid by a farmer whose income has a variance as compared with a taxpayer who has a stable income with the same mean, can be readily derived. This is done by approximating the actual tax schedule by a quadratic function.

The schedule of taxation rates as set out in the 1970 budget for operation in the years from 1971-72 onwards is a function of pre-tax income and exemptions. Taxable income in the t^{th} year (Q_t) is pre-tax income (I_t) less taxation exemptions (E).

(1) $$Q_t = I_t - E$$

Within the range of taxable income between zero and \$12,000 a quadratic function (Equation 2), fits the tax schedule reasonably well.

(2) $$T_t = aQ_t^2 + bQ_t + c$$

T_t is tax due in the t^{th} year, and a, b and c are constants.

Using values of taxable income at \$500 intervals over the range \$0 to \$12,000, estimates for a, b and c were found using least squares regression.

These were

$$a = 1.42 \times 10^{-5}$$

$$b = 0.24$$

$$c = -139.$$

Thus the quadratic function which replaces the tax function is

(3) $$T_t = 1.42 \times 10^{-5} Q_t^2 + 0.24 Q_t - 139.$$

Table B1 shows the comparison between actual tax and estimated tax using this function.

TABLE B1

A Comparison between Actual Tax and Estimated Tax

Taxable Income	Actual Tax	Estimated Tax	Residual
$	$	$	$
0	0	- 139	139
1000	124	116	8
2000	345	399	54
3000	635	711	76
4000	990	1052	- 62
5000	1390	1420	- 30
9000	3240	3180	60
12000	4700	4797	- 97

Equation 3 fits the actual tax schedule quite well as shown by the reasonably small residuals in Table B1.

If it is assumed that exemptions are constant, then the standard deviation of pre-tax income (σ) is also the standard deviation of taxable income.

The expected tax E(T) is

$$(4) \quad E(T) = E(a Q^2 + b Q + c)$$

$$= a E(Q^2) + b E(Q) + c$$

The annual tax payment of a taxpayer on a stable income averaging E(Q) is symbolised by $\overline{T}$,

$$(5) \quad \overline{T} = a [E(Q)]^2 + b E(Q) + c .$$

The expected extra tax paid by the taxpayer with a fluctuating income can be found by subtracting equation 5 from equation 4.

$$(6) \quad E(T) - \overline{T} = a\,E(Q^2) - a\,[E(Q)]^2$$

$$= a\sigma$$

APPENDIX C

DYNAMIC PROGRAMMING METHODS

1. UNDER CERTAINTY

There are three underlying assumptions needed to apply dynamic programming to the problem of finding the optimum use of the Income Equalisation Scheme under uncertainty.

1) That the farmer can foresee with certainty the incomes he is going to receive over future years.

2) That the farmer wishes to maximise the present value of his future stream of post-tax incomes. By using the present value as the objective function to be maximised, the opportunity loss of having to wait for income stored in the Income Equalisation Fund is included.

3) That the income tax schedule remains constant over the planning horizon. This also assumes that farmers have the same tax exemptions in each year, though this is not an essential assumption.

Let I_t be the pre-tax income in the t^{th} year, $t = 1, 2, \ldots, n$. n is the length of the planning horizon. (A list of symbols is given at the end of this appendix.)

Let X_t be the amount of income deposited in or withdrawn from the Income Equalisation Fund in the t^{th} year. X_t is negative if income is withdrawn. The value which X_t takes on is the key decision which the farmer must make in order to get the most out of the Income Equalisation Scheme.

Let $g(I_t - X_t - E)$ be income tax. This is a function of taxable income. The taxable income in any one year inside the parenthesis is the income in that year (I_t) less deposits (or plus withdrawals) made into the Income Equalisation Fund (X_t) less exemptions (E).

Let F_t be the level of income in the Fund at the beginning of the t^{th} year.

Let r be the interest rate which is used to discount post-tax-income in future years and represents the opportunity loss of delay in receiving income.

The objective is to maximise the present value (P) of a stream of incomes over the horizon of n years.

$$(1) \quad P = \sum_{t=1}^{n} [I_t - X_t - g(I_t - X_t - E)] \ (1 + r)^{-t}$$

The maximisation is achieved by the usual dynamic programming stage-wise procedure. This is done by finding best values in each year for the decision variable (X_t) - the amount to be deposited in or withdrawn from the Fund in each year, given all possible levels of F_t.

The Fund at the beginning of a year is the previous year's Fund plus the amount deposited or withdrawn the year before.

$$(2) \quad F_t = F_{t-1} + X_{t-1}$$

However it is assumed that all funds are withdrawn in the n^{th} year.

$$(3) \quad F_n - X_n = 0$$

and that the initial state of the Fund in the first year is zero.

$$(4) \quad F_1 = 0$$

The amount of money which the farmer is allowed to deposit in the Fund is restricted to \$100 units and there is a limit on how much can be deposited, this being less than 25 per cent of pre-tax income. Further, withdrawal from the Fund must occur within 5 years of the deposit being made.

For a positive set of pre-tax-incomes, X_t could be as small as

$$(5) \qquad X_t(\text{smallest}) = \sum_{j=1}^{5} 0.25\, I_{t-j} \quad \text{(nearest \$100)}$$

Representing the withdrawal of 25 per cent of the pre-tax-income earned over the last five years, X_t could be as high as

$$(6) \qquad X_t(\text{biggest}) = 0.25\, I_t \qquad \text{(nearest \$100)}$$

The procedure for solving the dynamic programming problem is simple but laborious. It consists of starting in the n^{th} year and finding the best decision given every possible value of the variable F_n - the amount of money in the Fund at the beginning of the year. The value of this best decision is written

$$f_n(F_n)$$

In the n^{th} year the best decision is very simple to find, as it is predetermined by the rule that everything in the Fund must be withdrawn in the last year (see Equation (3) - X_n must equal F_n.

Hence

$$(7) \qquad f_n(F_n) = I_n + F_n - g(I_n + F_n - E)$$

It is necessary to store these best values ($f_n(F_n)$ for all possible levels of F_n.

Now move back to the $(n-1)^{th}$ year and search for the best amount to deposit or withdraw from the Fund given each possible level of the Fund at the beginning of the year, taking into consideration the discounted value of any money left in the Fund at the end of the year, when used optimally in subsequent years. This statement can be written in the form of the dynamic recursive relationship. Writing the dynamic recursive relationship for the t^{th} year (which is the same as for the $(n-1)^{th}$ year), then

$$(8) \qquad f_t(F_t) = \underset{X_t}{\text{Maximum}} \left[I_t - X_t - g(I_t - X_t - E) + (1+r)^{-1} f_{t+1}(F_t + X_t) \right].$$

Equation 8 should be read as follows. $f_t(F_t)$ means the value of the best decision in the t^{th} year given a particular level of the Fund at the beginning of the year, F_t.

Maximum followed by the brackets means "Search for the best value of X_t by evaluating the functional relationships within the brackets for values of X_t within the limits set by equations (5) and (6)." $I_t - X_t - g(I_t - X_t - E)$ represents post-tax-income in this year.

$f_{t+1}(F_t + X_t)$ is the value of the Fund policy at the beginning of next year if it is used in the best way. This set of values has already been determined for $f_n(F_{n-1} + X_{n-1})$ (Equation (7)). As the calculation moves successively through the years there will always be stored values for $f_{t+1}(F_t + X_t)$.

This last expression is multiplied by $(1+r)^{-1}$. This discounts the value using the interest rate r and hence allows for the opportunity loss of delay of one year in the benefits of all income coming to hand.

In solving the dynamic programming problem the recursion given by equation (8) is applied in each year until the evaluation of $f_1(F_1)$ (the value of the best decision in the first year) is found. F_1 can only take on the value of zero in the first year (equation (4)) so that this is a less laborious task.

Having found the value of following the best decision in the first year, given $F_1=0$ (symbolised by X_1^*) then move forward through the years finding X_2^*, X_3^*, ... X_n^* which maximise equation (7) - the objective function. This set of values is the solution to the problem.

2. UNDER UNCERTAINTY

In this section rules are developed for operating an Income Equalisation Scheme under conditions of uncertainty.

Under conditions of uncertainty the following is assumed:

(1) The farmer knows his pre-tax income for the current year with certainty and that he has a note of the amount (if any) deposited in his income equalisation account.

(2) The farmer can estimate his mean pre-tax income over the years ahead and its standard deviation.

(3) Income variation between successive years is independent.

(4) The probability distribution of income is normal.

In deriving these rules the continuous normal distribution has been simplified to a discrete equivalent form with incomes in \$200 intervals. The distribution has also been truncated to lie within two standard deviations of the mean. Thus with a mean of \$5,000 and a standard deviation \$400, the 'sample space' for the probability distribution is \$4,200, \$4,400, \$4,600, \$4,800, \$5,000, \$5,200, \$5,400, \$5,600 and \$5,800. The lowest possible income of \$4,200 lies two standard deviations below the mean and the incomes rise by \$200 intervals to \$5,800 which is two standard deviations above the mean.

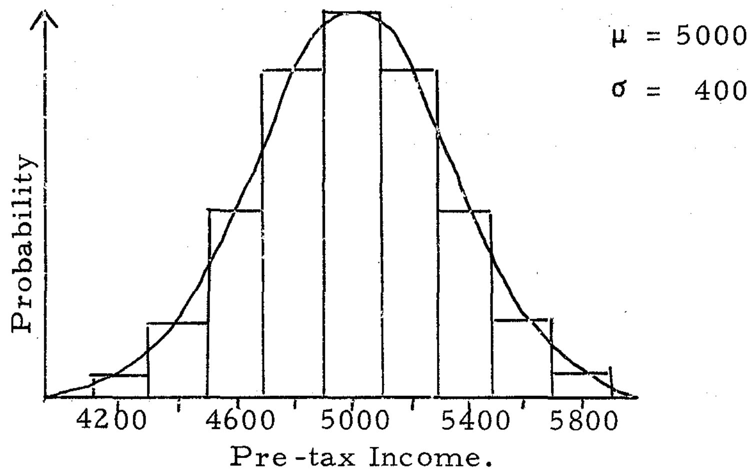
Figure 1 shows the truncated discrete approximation to the normal distribution.

If N is the number of possible incomes and σ is the standard deviation of pre-tax income,

(9) $N = (4\sigma/200) + 1$

which amounts to 9 possible incomes in the example above.

FIGURE 1 Truncated Discrete Approximation of the Normal Distribution



The expected pre-tax income $(\overline{I})$ is

$$(10) \qquad \overline{I} = \sum_{j=1}^{N} I_j P_j$$

where I_j is the j^{th} possible income and P_j is the probability of its occurrence.

For the discrete approximation of the normal distribution, P_j is calculated by

$$(11) \qquad Z_j = (\overline{I} - I_j)/\sigma$$

$$(12) \qquad P_j = e^{-z_j^2/2} \Big/ \sum_{j=1}^{N} e^{-z_j^2/2}$$

Substituting α for $(1+r)^{-1}$, the dynamic programming recursion can be written as follows.

$$(13) \quad f_t(F_t + Y) = \underset{F_{t+1}}{MAX}\left[F_t + Y - F_{t+1} - g(F_t + Y - F_{t+1} - E) + \alpha \sum_j^N f_{t+1}(F_{t+1} + I_j)P_j \right]$$

where Y is the known pre-tax income in the current year.

The decision variable in this recursion is F_{t+1}, the fund at the beginning of the subsequent year. Computationally it is easier to find the optimum by varying F_{t+1} than by introducing X_t, the amount deposited or withdrawn from the fund.

Equation (13) can be read as follows:

$f_t(F_t + Y)$ is the value of the optimal policy in the t^{th} year given various levels of $F_t + Y$. Two hundred dollar intervals of $F_t + Y$ were used which rise as high as $\bar{I} + 3\sigma$ as this was found to be sufficient.

Within the bracket for the maximisation, $I_t + Y - I_{t+1}$ is the pre-tax income which is to be available for consumption. It has to bear the tax of $g(F_t + Y - F_{t+1} - E)$.

The last term $\alpha \sum_j^N f_{t+1}(F_{t+1} + I_j)P_j$ is the discounted expected value of F_{t+1} when the optimal policy is used in subsequent years under conditions of uncertainty.

Using the recursion (given by equation (13)) the optimum amount to have in the fund at the end of the year, given a certain level at the beginning, plus income earned during the year, reaches stability after 5 or 6 years. That is, the set of rules for operating the fund becomes independent of the year after a few years, thus giving a set of rules for an infinite period time horizon. These rules are also optimal for a shorter time horizon.

LIST OF SYMBOLS USED

I_t	Pre-tax income in the t^{th} year
E	Taxation exemptions
σ	Standard deviation of pre-tax income
n	The length of the planning horizon
X_t	Amount deposited on or withdrawn from the income equalisations fund in the t^{th} year
$g(I_t - X_t - E)$	Income tax due in the t^{th} year
P	Present value
F_t	Fund level at the beginning of the t^{th} year
r	Rate of interest as a proportion
$f_t(F_t)$	The value of the best decision in the t^{th} year given a beginning-of-the-year fund of F_t
$X_1^*, X_2^*, \ldots X_n^*$	The best deposits or withdrawals to make
N	Number of possible discrete incomes
P_j	The probability of the j^{th} pre-tax income
$\bar{I}$	The expected pre-tax income
Z_j	An income deviation expressed in standard deviation units
α	The discount factor = $1/(1+r)$
Y	Known pre-tax income

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