“You have to find a way to glue it in your brain”: children’s views on learning multiplication facts

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Abstract

While there has been research on development of multiplicative reasoning, and how to teach multiplication facts, there is little research on how children consider they learn these. This study explores the children's learning as they consider how they commit their multiplication facts to memory, discover calculation strategies and develop multiplicative thinking. A group of eleven Year 4 children (8 years old) participated in a series of 13 lessons where they became co-researchers in the exploration of their learning. A contextually based thematic approach was provided through 'Crocodilian Studies'. The mixed-method approach to this study included formal assessment, participant observation, individual interviews, the children's written ideas, and individual case studies.

The most significant finding of this study was the powerful influence of peer learning. The children enriched and directed each other's learning as they shared ideas and reflected on their own mathematical learning as they observed and critiqued the thinking of peers. As the children were involved in thinking about how they learn they were able to identify gaps and construct their own learning pathways. A significant finding was that children can develop their multiplicative strategies while they commit their multiplication facts to memory, in a relatively short time provided that the learning process facilitates strategy development and understanding. By exposing the children to multiplication facts in sequenced clusters provided them with a manageable number of facts to be learnt at one time. Another finding related to how children develop calculation strategies through lesson activities rather than being explicitly taught them. The children considered practice important for memorisation. Parental support was significant in enriching the children's learning.

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Chapter 1: Introduction

1.1 Introduction

The teaching and learning of multiplication tables has traditionally been one of the areas of mathematics most consistently debated by educational researchers, curriculum writers, politicians, teachers and parents. For over one hundred years a goal for primary teachers has been to teach children multiplication facts, yet debate on the approach to teaching continues to flourish (Issacs & Carroll, 1999). Many parents deemed that being able to recite tables was a sign of mathematical ability and this belief has carried on through generations. Parents see the knowledge of multiplication facts as crucial and often value them more highly than the basic addition and subtraction facts. Anthony and Knight (1999) alert teachers to the fact that children who do not have instant recall of their multiplication facts feel they know very little about mathematics. By learning multiplication facts, children develop a sense of security and confidence. For the purpose of this study I have defined a *multiplication table* as referring to all facts connected with a specific table (e.g. 1 x 2, 2 x 2 up to 9 x 2). *Multiplication facts* are facts from the multiplication tables but isolated from the table (referred to 0's, 1's, 2's…).

The value of learning to instantly recall all the multiplication facts has been questioned over the past five decades by researchers and educators alike. For many parents, learning these facts is an inter-generational ritual and a rite of passage for all children. The introduction of the 'new maths' in the 1970s advocated that mathematics teaching place a greater emphasis on developing children's understanding of mathematical ideas rather than on imparting skills. The earlier teaching approaches had included the imparting of methods (formulae) for computation and the chanting and rote learning of multiplication tables. Theorists who advocated the move towards the development of children's understanding may argue that children will become able to recall multiplication facts automatically as they use these facts to solve problems and, therefore, the necessity to rote learn multiplication tables is eliminated (Anthony & Knight, 1999).

The view of the 'New Maths' theorists did not gain general acceptance amongst parents or amongst many teachers. Although teachers accepted the value of children understanding their mathematics, many considered that without rote practice, children would not master instant recall of their multiplication facts. These teachers and many parents are likely to agree with the 'back to basics' lobby of the 1990s, which criticised the 'New Maths' educational theorists and emphasised the value of being able to instantly recall multiplication facts (Anthony & Knight, 1999).

A further argument against concentrated practice of multiplication facts has been the belief that in the age of the calculator it is sufficient for children to have a conceptual understanding of multiplication (Anghileri, 1999). That is, it is sufficient for children to be able to select the appropriate operation, and press the correct buttons on the calculator to solve real problems. In this view, it is not necessary for children to have instant recall of multiplication facts. A difficulty with this argument, to which teachers are often alerted, is that if a child cannot estimate the approximate answer to a problem, he or she has no way of judging whether the correct keys have been pressed on the calculator (Fuson, 2003).

The above developments have left the teaching of multiplication facts in a state of flux. Many teachers have oscillated between the two opposing views, resulting in confusion over how to teach the multiplication tables. Although many teachers accept that instant recall of these facts is significant in developing mathematical ideas, successful teaching methods have not been widely shared.

1.2 Multiplication facts in the New Zealand Curriculum

Mathematics in the New Zealand Curriculum (Ministry of Education, 1992) emphasises the importance of children learning multiplication facts. The curriculum states that children around the ages of eight to nine years (Level 2) should be able to demonstrate the ability to use multiplication facts (pp.36-37), and from around the ages of nine to ten years (Level 3) they should be able to recall these facts quickly (pp.41-42). The New Zealand Curriculum states that there is an expectation for teachers to ensure their students have instant recall of multiplication facts by the time they leave primary school. It is, therefore, as important as ever to consider how this can be achieved successfully.

The New Zealand Curriculum reflects research recognising that a knowledge and understanding of multiplication tables enhances number sense and helps develop an ability to make mathematical estimations which then enables children to recognise mistakes when using a calculator. This is consistent with recent developments in Britain (Tanner & Jones, 2000), Australia (Mulligan, Bobis & Francis, 2000) and the U.S.A (Association for Supervision and Curriculum Development, 1999). In Japan requirements for teaching multiplication contrast with those of the New Zealand curriculum: there, children are introduced to their multiplication tables in their first year at school. By the end of their second year Japanese children have been introduced to the 2x to 9x tables, with the 0x being taught in their third year at school (Mousley, 2000).

During the 1990s the *Third International Mathematics and Science Survey (TIMSS)* indicated that New Zealand children were performing below expectations in number (Thomas & Ward, 2005). The results of the mathematics section of the *National Education Monitoring Programme* (NEMP**)** reinforced the results of the TIMMS report (Crooks & Flockton, 2002). The NEMP report indicated that Year 8 children did not consistently know their multiplication facts, with the greatest difficulty being in the 7x and 9x tables. In response to this research on children's number achievement, the Mathematics and Science Taskforce developed the strategic focus of the New Zealand Numeracy Project. The project was introduced to raise teachers' pedagogical knowledge, which in turn would raise children's mathematical achievement. This project is a school-based initiative of the Ministry of Education and was officially introduced in 2001 (Thomas & Ward, 2001). A further NEMP report (Flockton, Crooks, Smith & Smith, 2006) emphasises that although there are many areas of improvement in mathematics, in knowledge of multiplication facts there is a decline in performance from 2001 to 2005.

The Numeracy Project (Ministry of Education, 2005a) suggests that at the Advanced Additive/Early Multiplicative stage (stage 6), children are developing strategies to derive multiplication facts from known facts whilst they develop instant recall of all multiplication facts. At the end of stage 6 students require knowledge of all multiplication facts from 0 x 0 to 10 x 10 before they can approach the Advanced Multiplicative stage (stage 7). At this stage students will develop an ability to choose appropriately from a large range of part-whole strategies to solve multiplication and division problems. No guidance for teachers is given in the Numeracy Project on how learning multiplication facts can be integrated whilst teaching these strategies.

1.3 My interest in the teaching of multiplication tables

When I was seven years old I rote learnt my multiplication tables. However, I have no recollection of having any help to do this at home. I do recall the school punishments for providing incorrect answers. I also recall having to recite a table to reach an answer to the multiplication fact I was seeking. I remember vividly, around age ten, the day I discovered the nature of multiplication and the value of being able to instantly recall these tables. As a teacher I wanted to make this experience pleasurable and meaningful for the children I taught so I decided to put time into studying how this may be achieved.

In 1995, as part of the requirements for a Diploma of Mathematics Education through the University of Auckland, I undertook a research project. This study aimed to determine whether the understanding and the retention of multiplication facts was made easier for children when they were taught using a teaching sequence consistent with a constructivist approach, together with a set of mathematical principles and ideas designed to simplify and shorten the time required to learn the multiplication tables. The teaching sequence and approach used in this early study has been adapted and extended for teaching in this current research. As a result of this early study my teaching of multiplication facts, since 1995, has employed the principles and strategies advocated by Thornton, Tucker, Dossey and Bazik (1983). I deviated from this approach in that I encouraged the children to invent their own strategies (Baker & Baker, 1991). Prior to commencing a number topic in mathematics I individually interviewed each child to ascertain his or her current mathematical thinking and to establish the intuitive methods he or she was using (Mulligan & Mitchelmore, 1997). After interviewing for multiplication it was possible to encourage children to build on their current knowledge of addition and subtraction to develop understanding and strategies for multiplication and division facts (Steffe, 1994). In 2002 I was exposed to the New Zealand Numeracy Project, after which I incorporated such aspects as the teaching model (Ministry of Education, 2005c) into my teaching of multiplication facts when I considered it appropriate.

Although in the early study and in subsequent teaching sessions I found that both children’s understanding of multiplication and their knowledge of the multiplication facts improved markedly during the teaching sequences, it was beyond the design and scope of that study to establish the reasons for this improvement. This early study, therefore, raised unanswered questions about how individual children do move from thinking additively to understand multiplicative thinking and how they actually commit multiplication facts to memory. Although that study indicated that the use of strategies did aid understanding of multiplication, it did not answer the question of whether there was an appropriate age/stage when children should begin to work on understanding and then committing multiplication facts to memory. These unanswered questions have become the focus of my thesis.

Since the completion of this early study in 1995, and over the past four years as a numeracy facilitator, it is evident to me that many children do not have instant recall of multiplication tables and that this is hindering their development of more sophisticated mental strategies for multiplication. I have also observed that many teachers are unsure of how to incorporate the learning of multiplication facts into their teaching. Some children are still learning to chant their multiplication tables, and many are being taught by rote in their homes. I do not consider there is a place for rote learning of multiplication tables and current research supports this (Issacs & Carroll, 1999). However, I advocate that children who have discovered principles and strategies to understand multiplication facts need to then commit these facts to memory. Children who understand how multiplication facts ‘work’, I believe, develop a ‘feel for numbers’ (Anghileri, 2000) enabling them to use known facts to solve more complex problems.

I have been aware, as a teacher, that what is covered and how it is taught, in a topic such as multiplication tables, is the decision of one person. I have assumed, in the past, that as the children were often excited about their learning, the teaching approaches I chose were the ‘best’ for them. I often asked the children what they thought they had learned but not how they learnt it. I am now interested in how they think they learn their multiplication facts. Also, to inform my work as a numeracy facilitator, I want to find out how knowledge of multiplication tables and their interrelationships can be integrated with teaching the next strategies for children judged to be ready for the Advanced Additive/Early Multiplicative stage on the Numeracy Project.

1.4 Definitions of terms

For the purpose of this study I have defined *committing facts to memory* as the memorising multiplication facts while developing an understanding of multiplicative reasoning. It is important to differentiate between memorising by rote learning and memorising with understanding. *Rote learning* I have defined as the mechanical memorisation of a multiplication fact that occurs by repetitive practice and by chanting the multiplication tables to be able to instantly recall the facts. This is carried out without any consideration for meaning/understanding. The focus is on the correct product for each fact. In contrast, *memorising with understanding of the multiplication facts* I have defined as that of children committing the facts to memory whilst recognising the relationships and patterns within the facts. In any section where I refer to the same author/authors more than once I have not included the date every time.

1.5 Introduction to the study

My continued interest in the learning of multiplication facts and my observations as a numeracy facilitator has directed me back to the unanswered questions raised in my 1995 study. In this thesis I want to investigate how children think they memorise multiplication facts and what helps them to do this.

The key questions, which I ask, are:

1. How can children effectively develop multiplicative thinking and understanding?

2. How do children demonstrate they commit their multiplication facts to memory and what aids them as they develop understanding of multiplicative thinking?

3. How do children help each other increase their knowledge of, understanding of, and strategy-building techniques for multiplication facts?

1.6 Structure of the thesis

This research centres on a lesson sequence, carried out over 3 months, designed to teach multiplication tables to 11 children who were members of a Year 4 class. Past and current research and general literature on the teaching and learning of multiplication facts are reviewed in chapter 2. Chapter 3 describes the methodology undertaken for this study and sets out the procedures followed. It also describes comprehensively the data collection and data analysis methods used. Chapter 4 reports the results of the data collected. The findings are discussed in chapter 5, where they are synthesised with the findings of the research reviewed in earlier chapters. In chapter 6 the outcomes of the study are summarised and conclusions drawn. The strengths and limitations of this study are examined, and implications for children, teachers, school communities and curriculum writers are outlined. Recommendations for possible further research and the importance of this study are also discussed in this concluding chapter.

Chapter 2: Literature Review

2.1 Introduction

This chapter focuses on the literature about the teaching and learning of multiplication facts. In the first half (sections 2.2 to 2.4) there is a brief introduction to current theories of teaching and learning and understanding of memory. This section focuses, in more detail, on multiplicative thinking development, the debate between learning by rote or learning with understanding, the role of practice and the instant recall of multiplication facts. The second half (sections 2.5 to 2.10) of this chapter considers literature referring specifically to the teaching and learning of multiplication facts.

2.2 Theories of and approaches to teaching and learning

There is debate about how to approach the teaching and learning of many topics, including multiplication facts. Several theories are mentioned here. Multiple intelligence theory refers to the eight ‘intelligences’ identified by Gardner (1983, cited Adams, 2000), and recommends that teachers provide activities to use these types. Behaviourism, advocated by Skinner (Nye, 1996) focuses on environmental conditions and uses, and draws on stimulus reinforcement techniques. Constructivism, a theory of learning prominent over the last two decades, underpins the *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992) document and also guides this thesis. In assisting children to commit multiplication facts to memory a combination of eclectic approaches can be beneficial (Irwin & Irwin, 2000).

In defining constructivism Simon and Schiftler (1991) state: “Learners actively construct their understanding rather than passively absorb or copy understandings of others” (p.310). Von Glasersfeld (1992) extends this idea by arguing that knowledge cannot be transferred ready-made but has to be actively built by each learner in his or her own mind. Constructivist understandings are not new. Constructivist ideas have much in common with the ideas of the educational philosopher Dewey (1974) when he argued that no thought or idea can possibly be conveyed as an idea from one person to another. Piaget's (1973) work was clearly linked to constructivism when he claimed that by gaining understanding through free investigation and spontaneous effort children were able to retain their discoveries so this acquired knowledge was readily available for the rest of the child's life. Blais (1988) explains that a constructivist approach to learning is not an easy option for the teacher, as experiences for children to construct their own knowledge must be provided. He notes that a distinction is made by constructivists between information and knowledge. Through telling, information is readily transmitted and provides all that is necessary to achieve a correct performance while knowledge is something that learners must construct for themselves.

Vygotsky (cited Smith, 1998), who proposed the 'social development theory', has been a springboard for many other writers to focus on the importance of the social dimension of learning. These social constructivists argue that the constructivist teacher needs to allow opportunities for children to learn from each other. They take the view that learning is facilitated through social interaction. Yackel, Cobb, Wood and Merkel (1990) define social constructivism in this way:

> Although students construct their own mathematical understandings, they do not do so in isolation. Interactions with both other students and the teacher give rise to crucial opportunities (p.35).

Mayers and Britt (1995) explain the role of the teacher as that of a facilitator of learning. They claim that the teacher must provide situations where children can engage in collaborative mathematical problem solving in small groups, allowing opportunities to discuss, explain and justify their solutions. Whole class discussion of problems, interpretation and solutions also need to be facilitated by the teacher. In this model the teacher facilitates the learning, has comprehensive knowledge of the mathematics she is teaching but allows the children to take responsibility for their own learning. Children are encouraged to build on their current knowledge and to discuss their thinking, in order to clarify their ideas, as well as to listen to and question the thinking of others. According to Dengate (1998), teachers should pose problems or open-ended tasks that encourage children to develop mathematical thinking and build their strategy and skill knowledge.

Within the constructivist classroom Askew (2002) advocates an approach where the relationships among children, teacher and mathematics are paramount and all parties in the relationships are of equal importance. The teacher, with a broad knowledge of the principles of the number system, guides children to build on their own mental strategies by challenging them to think through problem solving, using classroom equipment, explaining their ideas and listening to others.

2.3 Multiplicative thinking

This section discusses theorists who distinguish between additive and multiplicative thinking and who argue that there is a progression from one to the other. These writers advise that, regardless of the approach to the teaching of multiplication, when considering how children can commit their multiplication tables to memory it is necessary to understand the development of multiplicative thinking. They identify differences between additive and multiplicative thinking and recognise the progressions that children go through to become multiplicative thinkers.

2.3.1 *The importance of multiplicative thinking*

Mulligan and Watson (1998) argue that multiplicative reasoning is essential to develop concepts and processes such as ratio and proportion, area and volume, probability and data analysis. Mulligan (2002) argues that if children in their early years of school do not develop multiplicative structures, their mathematical development is hindered when studying such strands as algebra and graphs at secondary school. Mulligan states that:

> Children need to develop and recognise underlying mathematical structure in order to understand how the number system is organised and ordered by grouping in tens, and how equal groups form the basis of multiplication and division concepts (p. 497).

The stages of multiplicative thinking through which children progress are described in many studies (Anghileri, 1989; Clark & Kamii, 1996; Jacob & Willis, 2003; Kouba, 1989; Ministry of Education, 2005d; Mulligan & Mitchelmore, 1997). Table 2.1 is my summary of these theorists' accounts of the progression from additive to multiplicative thinking.

Kouba 1989	**Anghileri 1989**	**Clark and Kamii 1996**	**Mulligan and Mitchelmore 1997**	**Jacob and Willis 2003**	**New Zealand Numeracy Project 2005**
Direct representation = unitary counting or counting on from the first set ↓ **Double counting** Used in division only ↓ **Transitional counting** = skip counting ↓ **Additive or subtractive** = repeated addition ↓	**Unitary counting** ↓ **Rhythmic counting** ↓ **Repeated addition** including use of number patterns ↓	**Children not yet numerical or additive** ↓ **Additive thinking with a numerical sequence** ↓ **Additive thinking** ↓	**Direct counting** = Unitary counting ↓ **Repeated addition** including rhythmic counting skip counting repeated addition additive doubling ↓	**One to one counting** = unitary counting ↓ **Additive composition** including skip counting and repeated addition with visual displays ↓ **Many to one counting** Without visual displays – numbers held in head ↓	**Counting all objects** = unitary counting ↓ **Rhythmic counting** ↓ **Skip counting** ↓ **Repeated addition** ↓
Recalled number facts	**Use of multiplication facts**	**Multiplicative thinking but not with immediate success** ↓ **Multiplicative thinking with immediate success**	**Multiplicative operation** including known facts and derived facts	**Multiplicative relations** ↓ **Operating on the operators**	**Derived multiplication** ↓ **Multiplicative operation**

Key: unitary counting: counting all objects, repeated addition: 3 + 3 + 3
rhythmic counting: 123, 456, 789… additive doubling: 3 + 3 = 6, 6 + 6 = 12
skip counting: counting in groups 5, 10, 15… derived multiplication – using known facts to find answers to unknown facts

Table 2.1.
Development of multiplicative thinking

In table 2.1 each theory describes the development of multiplicative thinking as passing through these broad stages in the same sequence. The additive thinking stage is described uniquely by these theorists. The multiplicative stage, which follows from the counting and additive stage, is where children are able to derive new facts from known facts, to recall facts instantly and use multiplicative knowledge to solve abstract problems. At the multiplicative stage, Clark and Kamii (1996), Jacob and Willis (2003) and the Numeracy Project (Ministry of Education, 2005d) separate instant recall of multiplication facts from known facts and derived facts. From this chart it appears that these theorists agree on a definite flow through the three broad stages. The Numeracy Project does not challenge the researchers' assumptions and the Numeracy Project indicators are guided by these researchers' ideas (Ministry of Education, 2005a). In section 2.3.3 I discuss a challenge to this apparently clear sequence made by Siegler (2000).

Clark and Kamii (1996) point out that multiplication is often presented in textbooks as a faster way of doing repeated addition. However, Frobisher, Monaghan, Orton, Orton, Roper and Threlfall (1999) argue that multiplication is more complex than repeated addition as it demands a more qualitative and meaningful modification of children's thinking than addition does. Similarly Steffe (1992, cited Clark & Kamii, 1996) suggests that children can be considered to be thinking multiplicatively when they can think simultaneously about units of one and about units of more than one. To be true multiplicative thinkers children must:

> …first be able to recognise multiplicative situations as involving three aspects: groups of equal size (multiplicand), numbers of groups (the multiplier) and the total amount (the product). When they can construct and co-ordinate these factors, in both multiplication and division problems, prior to carrying out the count, they are thinking multiplicatively (Jacob & Willis, 2001 p.397).

2.3.2 *The complexity of learning to think multiplicatively*

Multiplicative thinking is complex in the way it develops. Various researchers have explained the necessity to build on children's prior knowledge, the slowness of development, how there is not a definite flow between stages and the necessity to understand the difference between additive and multiplicative thinking. Steffe's (1994) investigation into the development of children's multiplying and dividing schemes (defined as cognitive proceedings), shows how three children progress from their construction of the number sequence through to concepts of multiplication. Steffe, through an extended effort of observation, interaction and further observation, researched how these children's conceptual raw material could be educationally directed to construct their developing multiplicative schemes. Children's initial mathematical schemes relating to advanced counting strategies for addition and subtraction were modified to construct schemes for multiplication and division. A much larger study of 336 children in grades 1 to 5 (Clark & Kamii, 1996) concluded that multiplicative thinking is clearly distinguishable from additive thinking and although it appears within the first years at school it develops very slowly. Mulligan and Mitchelmore (1997), in their study of 70 girls ranging in age from 6.5 to 7.5 years of age, identified intuitive models children use to solve multiplication and division problems. They found these children could solve a variety of multiplicative problems long before formal instruction on the operations of multiplication and division took place.

Mulligan (2002) asserts that multiplication and division knowledge and strategies develop with increasing sophistication. Seigler (2000), however, explains, through his 'overlapping waves theory', that children build a diverse bank of strategies and ways of thinking about addition and multiplication. As learning progresses these strategies and ways of thinking co-exist, are interwoven, and move in and out of a child's 'bank'. Therefore, children do not replace one strategy with another but oscillate between additive and multiplicative thinking which makes each child's pathway to multiplicative thinking unique. If understanding is valued in a classroom, explains Chambers (1996), children will rarely use a strategy they do not understand, regardless of its efficiency.

Steffe stresses the importance of teachers being able to recognise the difference between multiplicative and additive thinking to help children become multiplicative thinkers. It is apparent, claim Jacob and Willis (2001) that some children never use multiplicative thinking so it is imperative that teachers consciously develop ways to help them move beyond repeated addition. To develop multiplicative thinking, Steffe asserts that it is important for a teacher to wean children from counting and addition-based strategies at an appropriate time. Jacob and Willis reiterate this, explaining that although repeated addition can assist children in understanding multiplication, prolonged use can have a detrimental affect, as it does not give children the important multiplicative structures.

2.4 Memory and basic multiplication facts

2.4.1 *Memory*

In the teaching of multiplication facts it is important to consider theories related to the role of memory. Two approaches to understanding memory that helped me are those of Hiebert and Carpenter (1992) and Nuthall (2000). Nuthall argues that what children remember or forget is determined by the organisation of experiences in memory. He also noted that memorable learning activities enhance children's ability to recall or deduce what they learned during classroom experiences. He states that:

> Understanding how memory works is critical to understanding how children learn. Memory creates the bridges between successive experiences, making it possible to learn by connecting current experience with previous ones. Without memory, minds as we understand them would not exist (p.1).

Hiebert and Carpenter explain that in children's minds their mathematical knowledge is like internal networks of representations. The more structured and cohesive the network, the more understanding occurs.

The question arises, in reflecting on the ideas such as those of Nuthall, and Hiebert and Carpenter, as to how best to promote memory in the teaching of multiplication facts. Two approaches to the teaching and learning of multiplication facts, that of rote learning versus learning with understanding, are commonly debated. The following two sections examine the literature on advantages and disadvantages of these approaches.

2.4.2 *Rote learning versus learning with understanding*

Rote learning

Although children for generations have been required to commit their tables to memory (Anghileri, 2000), there is little current research that advocates the teaching of multiplication tables solely by rote. However, Henry (2001) notes that rote learning of times tables has made a comeback in the classroom in Britain since the introduction of the Numeracy Hour. He cites Steel and Funnell's (2001) research, which applauds the Numeracy Strategy's emphasis on mental calculation and a more explicit return to learning tables by rote. Henry also reported that some teachers were relieved to know they could now teach times tables without being seen as old fashioned and boring.

In contrast, according to other recent research, rote learning is considered to have many disadvantages. Serious disadvantages, warn Issacs and Carroll (1999) can occur if the approach of frequent drill and timed tests cause children to become anxious and the necessity for speed undermines understanding. Tanner and Jones (2000) argue that if the learning of multiplication facts is not handled sensitively it may alienate many children from mathematics. Booker (1998, cited Anthony & Knight, 1999) considers that continual practice causes children to lose the ability to be intuitive and to enjoy activities in mathematics. Another disadvantage caused by children experiencing only rote learning is that they do not develop more sophisticated strategies for addition or multiplication (Christensen, 1991).

Butterworth, Marchesini and Girelli (1999) studied 94 children ranging in ages from 8 to 11 years in an Italian school, where commutative pairs were taught by rote, to see if children stored both forms in their memories or the 'preferred' one. They also examined whether children reorganised their memory to favour problems with the larger operand in the first position regardless of the actual chronological order in which they were taught. They found that multiplication facts, in a child's memory, appear to be reorganised in a principled way that reflects a growing understanding of the properties of multiplication; in particular the commutative property. Butterworth *et al.* reported that there was indirect evidence that initial learning is more efficient when only one of the commutes is learned. Results also indicated that children were able to recall facts more quickly when the larger operand was given first.

Many researchers warn of the dangers of rote learning. Issacs and Carroll (1999) warn that individual differences are not catered for if rigid schedules of mastery are used and children are led to believe that mathematics is more about memorising than thinking. Burns (1994) warns that mediocrity is ensured if facts are learnt in a rote and an unthinking manner. Heege (1999) argues that mastery of individual facts is hindered if children can only recite the multiplication tables. Both Anthony and Knight (1999) and Chambers (1996) suggest rote learnt answers lead to increased associations with incorrect answers for students who find it difficult to generate an answer without solving a problem. Children who learn multiplication tables solely by rote, argues Askew (2002), will have a very different view of multiplication from that of the child who gains understanding through a range of activities that provide insight. Learning by rote without understanding, explain Tanner and Jones (2000), can be seen to achieve only short-term objectives.

Learning with understanding

Anghileri (2000) argues that understanding how tables 'work' enables children to use known facts to find answers to more difficult facts (derived facts) and to use these facts in more complex mathematical situations. Understanding, she explains, can be referred to as a 'feel for numbers' or as number sense that is built up over time, through exploring the number facts, visualising them and then knowing how to use them. Part of this number sense is estimation, which enables children to ascertain whether their answer to a multiplication fact is a reasonable one thus leading to flexibility in working with numbers. Anghileri argues that understanding integrates skill and knowledge and with understanding children will develop an awareness of the appropriate times to use various multiplication facts.

Wright, Martland, Stafford and Stranger (2002) assert that the term ‘understand’ is problematic because of the many levels of understanding in mathematical processes. Understanding as explained by Hiebert and Carpenter (1992) is generative: it promotes remembering, reduces the amount of information that needs to be remembered as it connects to a network, allows for the transfer of knowledge of previously learned strategies and influences children’s beliefs about mathematics. Anghileri (2001b), Hiebert and Carpenter and Hiebert and Wearne (1996) argue that understanding involves the relationships among numbers, operations, ideas and pieces of information. Carpenter, Franke, and Levi (2003) further argue that children need to understand how these relationships and ideas are used, know that mathematics makes sense and understand the importance of generalisation.

To build conceptual understanding, Hiebert and Carpenter explain that mathematical ideas need to be represented internally so that children can think about them in a way on which the mind can operate. Therefore, through discussion and external representations children can explain internal conceptual understanding. Later research of Hiebert and Wearne traced the emerging relations between understanding and skill in children’s mathematics, and investigated how instruction influences these relations. They concluded that conceptual understanding is important as it enables children to invent new procedures and modify old ones. They also concluded that understanding enables children to make sense of teacher introduced procedures and that it is advantageous to construct understandings early. Thornton *et al.* (1983) consider multiplication conceptualisation is complete when children can explain their answer using a physical model, verbally and with written symbols. Skemp (1989) sums up the necessity for learning with understanding by saying:

> … I would not think it sensible to make children learn to spell words they didn't understand the meaning of, and neither would I teach multiplication tables in that way (p.159).

Hiebert and Carpenter note that although merit in learning through understanding is widely acclaimed, it has been difficult to provide school learning environments that promote understanding. Sophian (1996) explains that mathematics is about understanding concepts and relations and is more complex than using mathematical procedures correctly, as children can forget procedures and how to interpret results appropriately. Anthony and Knight (1999) believe that active, intentional learning that involves reflection and communication is necessary to develop understanding. They consider that understanding is an active rather than passive experience that requires an attentive mind that is willing to identify objects and discriminate between them. To achieve understanding, they suggest that extensive, appropriate practice tasks are needed.

Memory and understanding are important in learning multiplication facts and I build on this notion throughout my thesis. It is also important to teach children to recall facts instantly in ways that foster their confidence and interest so I now consider it part of teaching practice to achieve this.

2.4.3 *The place of practice in learning multiplication facts*

Taylor (1976) suggests that practice, to understand and recall multiplication facts, is undeniably valuable. Children need to be given many and varied experiences directed towards abstraction of mathematical principles and their conceptual experiences need to be reinforced. However, he considers it a waste of time to give children more practice than they require, to maintain efficiency. He warns that practice, as in rote learning, will not bring about understanding but it can play a constructive part, once understanding is in place, in advancing children's learning by reinforcing a conceptual experience. Skemp (1989) also supports practice, to become fluent at multiplication facts, once children can distinguish between addition and multiplication and select the appropriate operations in practical situations. Skemp suggests that, once multiplication is understood, time is well spent in practising these multiplication facts until fluent recall is attained.

Even in the 1970s the role of practice was considered as being in a state of flux. Taylor claimed there were opposing opinions on the value of practice. Some teachers, he explains, condemned practice in an overt form as arid and unproductive, something that did not fit within the philosophy of modern educational needs. Adams and Hitch (1998) and Anderson, Reder and Simon (1996) believe it is essential to practise multiplication facts to acquire cognitive skill of any kind and while Anthony and Knight (1999) agree that this is a laudable claim, they consider practice remains a contentious issue.

Parents as tutors to assist practice

As part of their investigation into the teaching of multiplication facts, Wilson and Robinson (1997) involved parents beyond the classroom in this learning process. The methods that were used in this investigation, 'sequenced count-by', 'constant time delay' and multiplication fact recall, were considered uncomplicated and would be of minimum interruption to family routines. Results of this parental support and guidance were evident to teachers and parents: the children's knowledge of multiplication facts had increased in conjunction with their self-esteem and confidence. These changes were reflected in the children's improved class work, their willingness to attempt more difficult problems and their attitudes towards mathematics.

2.4.4 ***Understanding then committing tables to memory***

Skemp (1989) argues that skills such as knowing multiplication facts form a basis for other skills, so are worth developing to a high degree of fluency once children have an understanding of them. Kamii and Anderson (2003) note that some educators consider teaching for understanding of multiplication facts more important than teaching for speed but they do not support the distinction and express their own belief that children should have both understanding and instant recall of these facts. Hiebert and Carpenter (1992), Kouba and Franklin (1993) and Thornton *et al*. (1983) advise that committing these facts to memory for instant recall should occur once children have a 'sound and varied' conceptual basis of multiplication and Heege (1985) adds that to prevent children being at risk and getting stuck at an inefficient level of knowledge, speed and precision must occur late in the instructional sequence.

English and Halford (1995), when explaining their derived fact model, proposed that the associative memory component and meaningful fact learning is interwoven with the development of automaticity, so frequently used facts become automatic. They explain that if children are asked to practise facts before they have a conceptual understanding of them they risk:

> …losing their grip on any mental models they might have developed in the early years; they have little option but to turn to memorised rules and procedures. It is little wonder that they seldom consider whether their answers are sensible. Because they fail to recognise correspondences between the problems, they will not realise that they have produced two different answers for variations of the same problem (p.146).

Where English and Halford are concerned that children have a conceptual understanding of multiplication, Heege advocates developing children's positive attitude.

2.4.5 ***Instant recall of multiplication facts***

In his research into whether children know their multiplication facts 'by heart', Heege (1985) found that children must have a positive attitude to the activity and 'quite consciously' want to commit these facts to memory. He claims that it is a commonly held view that multiplication facts must be memorised but some children find this extremely difficult. In addition, children who can recite a multiplication table cannot be assumed to have memorised their multiplication facts as they may not know isolated multiplication facts.

Issacs and Carroll (1999) suggest that although most teachers consider the mastery of multiplication facts important they are not clear on how to assist children to achieve this. They consider teachers are in disagreement about what knowing multiplication facts means and when and if children should achieve mastery. Anghileri (2000) acknowledges that the teaching of mastery is difficult to monitor, and it is also difficult to predict progress as individual children progress at different rates. 'Knowing' multiplication basic facts is often referred to as mastery of facts. Mastery is defined by Hatfield, Edwards and Bitter (2000) as the automatic recall of a fact within a period of three seconds. Van de Walle and Watkins (1993) agree that three seconds is a benchmark to consider that a fact is memorised, but Thornton (1990) suggests that, from his work with children, a more appropriate retrieval time would be two seconds.

In this first section of the chapter I have referred to literature that underpins teaching and learning as it applies to multiplicative reasoning. The remainder of this literature review discusses the teaching and learning of multiplication facts, the main focus of this thesis.

2.5 The teaching of multiplication facts

In this research I do not address questions about the value of children learning their multiplication facts but focus on approaches to the teaching of these facts. The following section begins by discussing literature related to the order in which multiplication facts can be taught, considerations to be kept in mind when designing lessons to teach multiplication facts, progression in teaching strategies and the debate between taught and invented strategies.

Following this discussion, the components of a strategy lesson are examined, and finally, the question of how this teaching fits into a constructivist classroom climate is considered.

2.5.1 *Considering the order in which the multiplication facts should be taught*

Current research does not advocate that the multiplication tables are taught one after the other, in order of increasing difficulty, but that the multiplication facts are considered in clusters. Thornton *et al.* (1983) argue that the time that has traditionally been put into the task of learning the tables can be decreased if children are taught using carefully sequenced clusters.

Although there is no established order for teaching the multiplication tables, a number of researchers have asserted that multiplication facts can be learned more efficiently by sequencing facts in a hierarchical order and teaching mathematical principles and thinking strategies. Appendix 1 compares the preferred sequences of Frobisher *et al.* (1999), Thornton *et al*. (1983), Zevenbergen, Dole and Wright (2004), the Numeracy Project (Ministry of Education, 2005d) and lists the sequence that I intend to use. Frobisher *et al.*, Thornton *et al.*, Zevenbergen *et al.* are in agreement that the 2's 5's and 10's need to be studied first. Thornton *et al*. suggest the 0's and 1's should be taught before these tables but Zevenbergen *et al.* suggest they follow the 2's and the 5's. The 9's referred to as 'pattern facts' by Zevenbergen *et al*. are then taught. Thornton *et al.* suggest that skip counting, then 'easy parts' and finally 'twice as much' are addressed while Zevenbergen *et al.* suggest that square numbers and 'threes and sixes' follow this. Zevenbergen *et al.* advocate teaching the 4's as doubles of the 2's. Frobisher *et al*., and Thornton *et al*., Zevenbergen *et al*. agree that it is profitable to study the 2x and 5x tables first, as the pre-multiplicative strategies of rhythmic and skip counting assist the understanding of these facts. The 2x table also relates to 'doubles' (adding two digits of the same value) that children know by this stage (Ministry of Education, 2005a) and Butterworth *et al*. (1999) add that the 5x table appears easy for children to relate to when they understand the rule that any multiple of 5 can only end with 5 or 0.

Zevenbergen *et al*. argue that difficulties when multiplying by one or by zero often occur as children confuse multiplying with the adding of one or zero to a number. Thornton *et al.* recognise that children make ‘strange’ mistakes in multiplication and division problems that involve zero. Ashcraft (1992) supports the claim that ‘zero problems’ are not solved by straightforward retrieval but by understanding the rule of zero. Zevenbergen *et al.* suggest that manipulating materials can explain the principles of the 0x and 1x quite quickly. The importance of understanding multiplication by the power of ten is explained by Frobisher *et al*. when they alert teachers to the inappropriateness of explaining ‘just add a nought’ to the digit being multiplied by ten. They consider that it is important to understand that the “position of the digit being multiplied by ten moves from representing ones to representing tens… the ones position is vacated by the single digit and is filled by the zero” (p.200). They add that knowledge of this, with the associate property, allow children to derive answers based on multiplying by a power of ten (e.g. 2 x 3 = 6 so 2 x 30 = 60, 2 x 300 = 600 and so on).

Finding patterns, suggest Frobisher *et al*., in the 9x table can be considered an enjoyable experience for children as it gives them techniques to quickly derive the answers to this table. They explain that by using the distributive law of multiplication over subtraction the answers to the 9x table can quickly be derived from the 10x table. Other interesting patterns noted by Thornton *et al.*, are that the sum of the digits, of the answer to nine times any number, add up to nine. The use of ‘finger’ multiplication is another approach to working out this table. They explain that by using all fingers on both hands to multiply 3 times 9, bend the third finger. Two fingers, to the left, represent the tens and 7 fingers to the right of the bent finger represent the ones.

Zevenbergen *et al.* also argue that it is important to teach the square numbers as a group of special numbers as this is rewarded when children encounter them later in algebra and other number studies. They advise that early exploration of the square numbers provides children with the opportunity to discover the first ten square numbers and to identify that the square numbers received their name because they form the shape of a square when represented as an array.

Frobisher *et al.* accepts that there are 121 multiplication facts to learn but only 90 division facts, as division by zero is undefined. If the 10's facts are removed there are 100 multiplication facts. After the 2's, 5's, 10's, 0's, 1's and 9's are removed, and children understand the commutative property and the square numbers, both Thornton *et al.* and Frobisher *et al*. agree that there are only ten 'hard' facts to learn. Zevenbergen *et al.* mention six 'hard' facts as they remove the 4's before this (appendix 1). Included in appendix 1 are the 'hard ones' identified by Murray (1939, cited Frobisher *et al.*, 1999), Norem and Knight (1930 cited Frobisher *et al.*, 1999), Ruch (1932 cited Rathmell 1978) indicating similarities and that the 9's were considered difficult by these researchers.

2.5.2 *Considerations when teaching multiplication facts*

The following section reflects on research that influenced my approach to establishing a sequence of lessons to assist children to learn their multiplication facts. Steffe (1994) gives insight into the effectiveness of teachers' acknowledging children's current knowledge, building up models of children's informal initial approaches to multiplication and division and the need for them to find viable ways in which these spontaneous mathematical schemes can be educationally directed to construct new concepts. Anghileri (2000) argues that when teaching multiplication facts a powerful tool for making sense of a child's world is the use of a realistic context to allow them to learn from what they hear and see. She also warns that the traditional method of only teaching with easy numbers and then moving to more complex numbers and calculations does not reflect the way children learn. Frobisher *et al.* (1999) assert that ‘higher’ tables such as the 6’s, 7’s, 8’s and 9’s are more likely to be taught later in the sequence which can lead to children practising the ‘easier’ facts more often.

To balance the practising of the tables Thornton *et al.* (1983) suggest a sequence for practice: working with 6 to 8 facts at a time; practising frequently; moving from current cluster practice to cumulative review practice. Bobis (1996) warns, from her research on teaching, that teachers are inclined to teach tables in the ways they were taught. Many teachers learned mathematics as a set of disconnected rules, facts and procedures. Today, Russell (2000) advises, current research regards it as necessary for teachers to understand the mathematical principles and relationships underlying mathematical work so children can be given tasks that develop these.

Anghileri (1999) and Thornton *et al.* argue that, the consequences of the commutative, associative and distributive properties need to be explored during teaching, as these underlie many of the connections necessary for multiplication and division. Anghileri expands on this by saying that when children accept the commutative rule it marks progression from understanding multiplication as repeated addition to multiplicative approaches. She also asserts that children need to understand that division does not obey the commutative rule. Kennedy (1970) suggests that using a ‘100 multiplication facts’ chart aids the learning of the facts as it reinforces such ideas as the commutative property along with enabling the children to track their progress.

Anghileri (2000) advises teaching multiplication and division in tandem as it establishes the relationship within the number patterns that will be the ultimate key to successful calculating. When children recognise the number triple (for example 12,4,3) and identify these facts independently, efficient ways of calculating will be established. She also advises that by recording multiplication and division facts together, children will gain not only a sense of the number patterns but also of the relationships that exist between them.

For children to advance to developing a broad range of multiplicative mental strategies to solve two digit and higher whole number problems, in a wide range of contexts, Frobisher *et al.* suggest that the following six milestones need to have been covered: knowledge of the multiplication facts; understanding of place value; the ability to partition numbers; knowledge of the distributive property; knowledge of a number multiplied by ten; the ability to double numbers. The Numeracy Project (Ministry of Education, 2005a) uses ideas similar to Frobisher et al. and suggests that compensation, place value partitioning, reversibility, proportional adjustment (associativity), and finally teaching the written working forms (algorithms), make up the necessary mental strategies.

2.5.3 *Why teach strategies?*

Strategies can be defined as the mental processes children use to calculate answers and solve operational problems with numbers. The primary role of strategy development, argues Rathmell (1978), is to assist children to learn more sophisticated strategies to solve harder facts and this is an important factor in committing these facts to memory. In considering the advantages of a strategies based approach to the learning of multiplication facts, Issacs and Carroll (1999) report that "it works: children do learn their facts" (p.514). The advantages of a strategies approach are that it both assists children to remember and access facts, as they are able to organise them in a meaningful network and builds their understanding and confidence. Issacs and Carroll identified three disadvantages of a strategies approach: that children can learn strategies by rote; class discussions can "degenerate into the tedious recitation of every imaginable method, with little critical appraisal of the various approaches" (p.514); too much emphasis on multiple ways to solve problems can also cause children to think memorisation is not important. Although they consider that a sensitive and thoughtful teacher will avoid these problems they do not suggest how a teacher obtains this information.

Anghileri (2000) points out that with the early introduction of standard written algorithms and the perceived need to rote learn multiplication facts, mental strategies have not been explicitly taught. Bobis (1996), who advocates the teaching of mental strategies, considers they are important in everyday life as they assist the development of a sound understanding of number. Anghileri notes that current curriculum documents support this by identifying the value of mental strategies to solve problems and Pressley and Woloshyn (1995) add that children who 'tackle' challenging problems strategically become excellent problem solvers.

To reinforce the meaning of multiplication and to recognise an advantage of knowing multiplication facts, suggests Anghileri (1999), known facts should be extended to two and three digit numbers, e.g. 3 x 4 to 3 x 40. One way to establish understanding is to ask the children for more than one solution to a problem (Chambers, 1996).

2.5.4 ***The progression of strategies***

According to Treffers and Beishuizen (1999), in The Netherlands strategies are considered in different levels that range from 'informal context-bound' methods to 'high-level abbreviated' strategies. Some people will not reach the highest level of formal written standard algorithms but will have acquired a method, which they can utilise.

Mulligan and Mitchelmore (1997), in their study to identify intuitive models children were using to solve multiplication and division problems, identified twelve calculating strategies. They define an intuitive model as: "an internal mental structure corresponding to a class of calculation strategies" (p.1). The calculating strategies were then grouped, in order of sophistication, to infer underlying intuitive models: direct counting; repeated addition; multiplicative operation. Results from their study showed that children develop a "widening repertoire of increasingly efficient intuitive models" (p.12) and the structure of each model is derived from the previous one. Anghileri (1989) suggests that the increase in sophistication from counting strategies to repeated addition results from a deepening understanding of addition. Although Mulligan and Mitchelmore and Anghileri present the progression of sophistication of calculation strategies in a similar way, Anghileri singles out repeated addition as an extra step. The Numeracy Project (Ministry of Education, 2005a) identifies a sequence of eight 'global' stages within the addition/subtraction, multiplication/division, and proportions/ratios domains. The progression through these stages indicates "an expansion of knowledge in the range of strategies that students have available" (p.8). Children, it suggests, "build new strategies on their existing strategies and that these existing strategies are not subsumed" (p.1). In isolating the strategy progression to master multiplication facts (stages 2 – 6) the Numeracy Project progression reflects those of Mulligan and Mitchelmore, Anghileri and those used in The Netherlands.

In her investigation into young children's understanding of multiplication, Anghileri (1989) found that when children use processes, like rhythmic counting of groups and then skip counting, strategy development occurs along with number sense. As children become familiar with these number patterns, Anghileri also claims that there is less mental processing capacity required as an interim count of each group is no longer required. She considers there is a reduction in the memory capacity required even though each group has a more complex role. Anghileri also found that there was a correlation among thought processes, addition results and the number patterns. The addition results were being used as an 'explanation' for the solution to multiplication tasks. A new stage of understanding is produced when these 'counting on' procedures for addition and multiplication have generated independent schemas for adding two or more equal addends and the schema for producing a number pattern. Thompson (1999) and

Kouba and Franklin (1993) all encourage the teaching of skip counting from a very early age, as such patterns are of importance to later work in multiplication and division. Zevenbergen et al. (2004) support the above ideas and explain that early skip counting activities allow children to conceptualise a form of concrete reference to the size of numbers being counted.

Making connections between each fact and a whole lot of related others is an important aspect of learning multiplication (Anghileri, 2000). Askew (2002) argues that children who are thinking in a strategic way for multiplication have committed some ‘simpler’ multiplication facts to memory and have a number of strategies to draw on. Ashcraft (1992) adds that by inventing procedures based on their current knowledge, children can now use the facts that they know in order to derive facts that they cannot instantly recall.

2.5.5 ‘*Taught’ strategies versus child invented strategies*

While a number of writers, such as Thornton *et al.* (1983) advocate the direct teaching of principles and strategies to find the answers to the multiplication facts, other researchers such as Baker and Baker (1999) and Muthukrishna and Borokowski (1995) oppose this view and argue that children need to discover these strategies and principles for themselves. The Numeracy Project (Ministry of Education, 2005b) suggests that the strategy to be taught should be shared with the children as a learning intention, but that the lesson instructional path should be determined by the responses of the children, with all their suggested solution strategies accepted. Anghileri (2000) emphasises the value of allowing children to learn that there is flexibility and choice in solving problems, so they are able to see mathematics as a logical structure of connected processes and results. If children are not free to create these connections they will see mathematics as a sequence of standard procedures. She also points out that with this flexibility and choice, children are able to develop ownership when making decisions and deriving meanings for their actions. Their ownership, in turn, will develop confidence in their own thinking and allow them to become autonomous learners who will be driven by their fascination with numbers.

In their study of 241 children aged between 7 and 12 years, Steel and Funnell (2001) examined the development of multiplication skills. In their classrooms these children were taught by discovery methods. The study used computer based multiple-choice questions to track the development of strategies the children used. There was evidence of recall, recall plus calculation (derived facts), and counting methods. There was no evidence of children using the repeated addition strategy so it was assumed that this, normally the most common back-up strategy, was not utilised by the children, as they had not been taught it. Results indicated that instant recall was the fastest and least error-prone method while counting methods took the greatest amount of time and were less accurate. However, very few children used the most effective method of recall. There was no evidence that use of the recall strategy increased over the five years they were monitored and they found that when this strategy was used it was most likely to be only in facts with smaller operands. As the children were not taught retrieval strategy at school, and only a few children indicated they were supported to learn their facts at home, it was assumed that children taught themselves this method.

2.5.6 *Components of a strategy lesson*

Researchers such as Gervasoni (1999), Hatfield *et al.* (2000) and Mulligan and Mitchelmore (1996) argue that children, when learning a new concept, need to use manipulative aids to physically model the number ideas and that this stage needs to be pursued until children can develop a visual image. Children often use their fingers as tools for counting (Anghileri, 1997), and although the use of fingers is more complex for multiplication than addition and subtraction, they are often used in the place of other manipulative materials. Children often resort to using fingers and these along with other child invented teaching aids can be fascinating to watch.

Although more efficient strategies need to be developed, fingers provide an important link between practical and mental methods, enabling abstraction to develop with understanding. Children's drawings can also indicate a clear link between concrete material use and symbolic representations (Outhred, 1996). Drawings are an under-utilised source of information about both the children's understandings of a problem structure (e.g. multiplication facts) and the development of that structure.

In discussing the Numeracy Project teaching model (figure 2.1), which has been modified from the Pirie Kieren model, Hughes (2002) explains the three strategy lesson stages: materials, imaging and number properties.

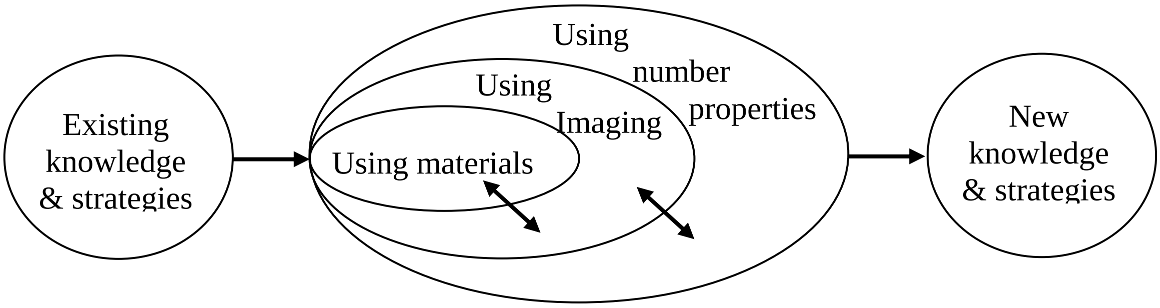

Figure 2.1
The Numeracy Project teaching model

Strategies are built on children's current knowledge. The Numeracy Project (Ministry of Education, 2005c) values children manipulating materials to understand strategies and suggests using materials that can be readily visualised when children move to the imaging stage. At the number properties stage, children abandon the use of materials and imaging, and problems are presented with higher numbers so that imaging is a burden and solutions are found through reasoning with abstract number properties. The Numeracy Project suggests the children oscillate between stages until they have acquired the new knowledge.

2.6 A constructivist classroom climate

To create a classroom in which children become mathematically powerful, researchers have identified as critical many aspects of constructivism including the emotional tone of the classroom, mutual trust and self-confidence, communication and collaborative learning. The following discusses research findings about these key ideas.

Emotional tone of the classroom

Cobb, Wood and Yackel (1991) claim that an appropriate climate is essential for each child to become mathematically powerful. In their opinion the most important aspect of a classroom is an emotional tone where children feel enthusiastic, are persistent, do not become frustrated and enjoy solving personally challenging problems. The teacher, claim Pressley and Woloshyn (1995) and Kamii and Anderson (2003), needs to be knowledgeable and excited about mathematics to enable children to become self-motivated to learn their multiplication facts.

Mutual trust and self-confidence

Muthukrishna and Borkowski (1995) believe it is essential that, within a 'sense making' atmosphere, children develop an excitement about searching for meaning and understanding, where they are encouraged to explore and trust their own intuitions. Cobb *et al.* (1991) see mutual trust, between teacher and child, as the most important feature of constructivist teaching while Lochhead (1991) directs the teacher to trust their children's minds and "give up the notion that they can do for students what, in practice, they can do for themselves" (p.86). Thompson (1999) emphasises the necessity of developing self-confidence in children so they will take risks and responsibility for their own learning while extending all the skills and knowledge they have. Myren (1996) adds that by trusting their own intelligence children are able to accept their mistakes as part of the learning process.

Communication

Lampert (1990) suggests that the construction of mathematical learning is all about communication. While Richards (1995) supports Lampert he adds that both the teacher and children must learn to talk and listen in a mathematically literate environment. Muthykrishna and Borkowski (1995) found that children actively participating in classroom discussions, where they are asking and answering questions, and sharing and explaining strategies used, are forced to a deeper level of processing as they explain, justify and defend their solutions. By discussing and evaluating each other's strategies children build their informal strategies and the skill of eliciting the most efficient strategies (Chambers, 1996). Hiebert and Carpenter (1992) assert that thinking and talking about similarities and differences can help children to construct relationships between procedures and strategies.

Collaborative learning

Caliandro (2000), when working with a group of children from her class, found that they often used inefficient methods to solve problems. However, she explains that even though an enormous amount of time was spent in thinking, learning and exploring for more efficient strategies, this time was of value as the children gained a deep understanding of multiplication and addition. As the children developed their own strategies and procedures that were meaningful to them, they did not forget them like memorised ones. Children were able to respond at their own pace.

Although each researcher mentioned above considers a different aspect of a constructivist classroom to be the most important, a combination of these is the aim of teachers who embrace the constructivist approach. Within a constructivist classroom, to enrich children's understanding and learning of multiplication facts, activities and resources can be considered an integral part of lessons along with appropriate practice activities as discussed in the following section.

2.7 Resources and activities in multiplication fact learning

Anghileri (1999) advises that although traditional textbooks have been an integral part of mathematics lessons for generations, they are not inclined to provide opportunities for making connections that match children's developing knowledge and strategies. She suggests that teacher generated activities and investigations, that are needs based, are more effective. The Numeracy Project (Ministry of Education, 2005c) emphasises the importance, following teaching sessions, of children being engaged in independent activities that are challenging and relevant to the strategies and concepts that are currently being taught. Suggestions for appropriate activities are included in the Project's 'material masters' and are designed to extend children's thinking. One suggestion is mathematical games.

2.7.1 *Games and speed sheets*

There is often debate about the value of children playing mathematical games. Kamii and Anderson (2003) argue that children should have an understanding of multiplication as well as develop speed in recalling multiplication facts but they warn against timed sheets without reflection on concepts being developed. Kamii and Anderson worked with advanced third grade children to establish the effectiveness of using games instead of worksheets and timed sheets, once children had developed the logic of multiplication. When selecting games they ensured that there were no coercion, timed tests or threat of a bad score in any of the chosen games but targeted the present needs of each child. These games "were fun and had a lot of variety" (p.7). Kamii and Anderson found that motivation to learn the multiplication facts came from the children as they built a desire to 'beat' the teacher and their peers. The teacher, they note, stimulated this motivation by the choice of the games, modelling the practice and the challenges she presented. They also noted that some children made flash cards with which to practise. As time progressed children selected appropriate partners and games, and chose to work with another child with similar skills to provide each with a chance to win. After several months of playing games, when tested, these children (except one who made two errors) were able to find correct answers to 100 multiplication facts in ten minutes. However, Rathmell (1978) argues that teachers may not find using interesting games and activities useful to aid the retention of multiplication facts because they will not necessarily increase speed of recall. Issacs and Carroll (1999) claim that timed tests are important to assess fact proficiency, and to indicate the importance of learning multiplication facts to parents and children, but they consider daily/weekly tests unnecessary as they work against a strategies approach.

2.8 Assessment

As they monitor the children's concept development, and assess their fact knowledge, teachers are constantly evaluating the appropriateness and usefulness of the activities and resources they employ in mathematics teaching. Issacs and Carroll (1999) explain that the aim of assessment is to identify how children understand underlying mathematical concepts and connections; the "assessment of children's fact knowledge should be balanced, based on multiple indicators, and aligned with instruction" (p.512). As Kouba and Franklin (1993) suggest, assessment must involve more than checking the correctness of children's answers, as the 'right' answer may be given without understanding procedures. Eliciting whether the child can explain his or her answers is a much more reliable check. Anghileri (1999) explains that describing their chosen method or ideas to others allows children to reflect on and communicate their understanding. By paying careful attention to children's actions and words, teachers are able to identify subtle differences that exist within the strategies they use (Anghileri, 2000).

Wright, Stewart, Mulligan and Bobis (1996) warn that, in gaining an adequate understanding of a child's current knowledge and in assessing their progress, a teacher meets the important challenge of ascertaining a child's most sophisticated strategy. They warn that children may not demonstrate the most sophisticated strategy of which they are capable. They recommend that crucial to understanding a child's strategy level is close observation and informed reflection during interviews. Although asking children to describe their strategy is useful, a child may unwittingly or unintentionally describe a strategy different from the one he or she used. By ongoing formative assessment teachers are able to pinpoint more accurately a child's level of development. A crucial part of assessment is feedback and feed forward to children; by working with small groups this assessment is made more viable (Miller & Mercer, 1997).

2.9 Exploring how children learn

In recognising how children develop concepts of multiplication and division, recent research has focused on strategies children use to calculate solutions to different problem types. Siegler (2000), however, applauds the move, within research, towards studying how children learn in contrast to studying their thinking and the strategies they use. He considers that understanding how children learn will create a more exciting field of cognitive development, covering the mechanisms that underlie the learning, along with assisting children to learn. He explains that by concentrating on children's thinking we currently know a lot about the strategies they are using but little about how they actually learn. Siegler advises that children's learning should be monitored to find out how they learn. It was previously thought that learning and development were basically different processes but Siegler supports the view that learning and development are both similar and inseparable. He considers that by increasing the focus on how children learn, teachers and researchers will gain a more comprehensive understanding of their development, leading to possible implications for future teaching. Questions about how children think they learn multiplication facts have been the major impetus for this thesis.

2.10 Summary

In summary, this chapter has discussed literature on theories of teaching and learning, multiplicative thinking, the arguments for and against rote learning, the understanding of multiplication and research into the teaching of multiplication facts along with children's learning of these. The chapter recognised children's understanding of multiplicative thinking and the value of children being able to instantly recall multiplication facts and the role of practice. A strategies approach to teaching was acknowledged within a classroom that promotes emotional tone to ensure all children become mathematically powerful. The chapter has identified that there is a need to investigate how children consider they learn multiplication facts and how their development of multiplicative thinking along with calculation strategies can be incorporated into this learning. The next chapter sets out the methodology and methods I used in this study.

Chapter 3: Methodology

3.1 Introduction

This chapter describes the mixed-method research methodology, and the rationale for selecting this approach to explore the research questions. It describes the quantitative and qualitative methodology used, and how this sits within an action research framework and is incorporated in an interpretative paradigm. It explains the rationale for using case studies to examine in-depth one child or a group's progress and reaction to a teaching method. The various methods of data collection are explained and described. These include quantitative data collection, and qualitative data collection through individual interviews, participant observation, field notes and journals. Ethical considerations are outlined. The research design includes a description of the participants and the procedures used to teach multiplication facts. The data gathering methods are described, followed by a description of the changes made to the research design. The chapter concludes with an examination of the data analysis method, where analysis themes are explained.

3.2 Research Approach

This research is of mixed-method design, is approached from an action research perspective, and is positioned within an interpretive paradigm.

3.2.1 *Methodology*

A mixed-method approach incorporates both quantitative and qualitative methods. In defining qualitative and quantitative research assumptions it is appropriate to make comparisons between them. Davidson and Tolich (1999) explain that qualitative research aims to identify qualities that can be used to interpret and explain behaviour while in contrast quantitative research translates predetermined variables into numbers. Neuman (2000) further explains that quantitative research considers issues of design, measurement and sampling prior to data collection while qualitative research considers analysis during and after the data has been collected. Robinson and Lai (2006) recognise that the distinction between the qualitative and quantitative methods is a fluid one but consider that once data has been counted it becomes quantitative while qualitative data is narrative. Neuman adds that despite distinct differences the two approaches can complement each other because researchers systematically collect and analyse empirical data to explain the social aspect of people's lives.

The advantage of using a mixed-methodology design is that it provides a more complete picture of the study and allows the inclusion of quantitative data in a predominantly qualitative research design (Greene & Caracelli, 1997). In a mixed-methodology design the methods and the results of the qualitative phase are recorded separately from the methods and results of the quantitative phase (Cresswell, 1994). In this study it is hoped that the two phases will elaborate, enhance and illustrate the results of the dominant qualitative design and extend the breadth of the inquiry. The study is predominately qualitative to provide in-depth data on how the children thought they committed their multiplication facts to memory. The quantitative data collection provided statistical data to monitor any shift in children's learning. The intent of the design was to triangulate or converge findings allowing for both themes and statistical analysis to be presented (Cresswell).

Qualitatively, this study is holistic and flexible (Janesick, 2000) with the data being analysed inductively (Davidson & Tolich) to allow for examination of emerging patterns. Exploration of these patterns was to effect deeper understanding of the possible implications of the study. Abstractions were built as the data was gathered and grouped together; theory was derived from the analysis of patterns, themes and common categories discovered in the data, together with interviews and the children's written and oral work (Bogdan & Biklen). The data was rich in description of the children's thinking, conversations and impressions (Neuman). The study had a natural setting, a teaching context, as the direct source of data and I was the key instrument for analysis as I was both the researcher and the teacher (Bogdan & Biklen) in the lessons relevant to the study – for other lessons, the children had a classroom teacher.

Davidson and Tolich consider that the strength of qualitative research lies in its validity; here, therefore, although results may not be generated which apply to other locations, the results presented reflect the words and actions of the children in the study. Triangulation of methods, as described by Patton (2001) was utilised in this study as it provided cross-data checking for consistency of findings generated from the data collection methods. Bloor (2001), however, argues that triangulation is only 'relevant' to the issue of validity as it only minimizes the elimination of biases.

Working within the interpretive paradigm, as explained by McNiff and Whitehead (2006), I focused in this study on the children's understanding of how they interacted with one another, in a classroom setting, to improve their learning of multiplication facts. The aim was to understand what was happening for these children and as they discussed their own understandings of their learning and the changes they made. Their ways of working were initiated by them and not imposed by me as the teacher. An action research approach was employed to guide this study.

3.2.2 *The Research Approach*

Action research as described by Hill and Capper (1999) is a type of applied research where researchers attempt to solve specific problems; action research should do more than understand the world, it should help change it. The inquiry is done, never to or on participants, but by or with participants within an organisation or community and is value laden (Herr & Anderson, 2005). Action research provides many advantages for teacher/researchers as they can generate theories about learning and practice to enhance and reinforce their own educational values (McNiff & Whitehead, 2006). The research findings allow them to change their teaching based on the data they gather (Sagor, 2005). Action research carried out by teacher/researchers can improve educational practices and advance knowledge and theory about how teaching and learning can be carried out and why. However, it is not suitable if the aim of the study is to draw comparisons or establish cause and effect relationships (McNiff & Whitehead). A narrative style of writing is often used in action research to allow the researcher to reflect on both the research process and the research findings (Herr & Anderson).

Action research was originally linked to social change for social justice and has been in existence for over 70 years. In the 1940s Kurt Lewin made an influential contribution to action research, as he believed that people would be more motivated to work if they became more involved in decision-making. Lewin produced a cycle of steps; planning, acting, observing and reflecting (McNiff & Whitehead). Other researchers, such as McNiff, have expanded on Lewin's cycle of steps. I favour the theory of McNiff (2003, cited McNiff & Whitehead, 2006)) who uses Lewin's steps as an ongoing spiral because when a provisional point is reached, more questions are raised and the cycle begins again. Action research provides, during the study process, the freedom to deal with complications and multiple issues while still maintaining focus on a core issue (McNiff, 2002). Action research allowed me to work with the children to consider what they perceived as successful ways to commit multiplication facts to memory and what aided them to achieve this, in a manner likely to benefit my own teaching theory and practice.

Action research has been used in mathematics education. Two significant studies are those of Lampert (1999) and Ball (1993) who researched approaches to overcome dilemmas/tensions in the role of teacher/researcher. Lampert found that by embracing pedagogical dilemmas rather than using the traditional problem solving role, she was able to shape the course and outcomes of her work with children. She was aware that a teacher, unlike a researcher, has to live with any consequences of decisions made. Rather than avoiding making a choice that only partially resolved a problem, she strove for solutions that appeased both her and the children she was working with. Lampert searched for ways to act with integrity to engage in, and utilise conflict rather than trying to solve it. Ball investigated how she could broach pedagogical complexities, which underlay current educational visions, in her mathematics teaching. In her study she faced and found solutions to dilemmas surrounding: content that built bridges between the experiences of the children and the knowledge of 'mathematical experts'; opportunities for children to experiment and invent mathematical ideas while sharing their thinking; creating a learning community. Ball found that researching her own teaching allowed her to think about what counts as evidence for believing or doing something in teaching.

Within my study I aimed to confront possible dilemmas that would arise between the classroom teacher and myself. By using the advice of Lampert to confront the tension, I could maintain respect for both the children's and my needs and identify solutions that would suit both. I utilised the advice of Lampert when the classroom teacher wanted to terminate my study to enable her to teach fractions and decimals to the whole class. By discussing her concerns and needs we were able to make compromises that placated both parties. I expected to overcome dilemmas centred on the pedagogy and classroom management, as advised by Ball, by developing a learning community where the children worked together to discover multiplicative ideas. I aimed to have the children owning their study by becoming involved in any decision-making. Like Ball, I found that creating a learning environment where children worked collaboratively and had a common purpose within a context that related to their world, assisted to maximise their learning.

3.2.3 *Case Studies*

This study is an in-depth examination of a single instance of social phenomenon, how children commit their multiplication facts to memory (Babbie, 2002). A case study approach, as described by Hopkins (2002), has the advantage of being a simple way of plotting the progress of one child or a group's reaction to a teaching method. From the data collected by many methods in a case study, information can give an accurate and representative picture of a case. However, disadvantages include the amount of data needed, and the time required, to make a case study of value. The amount of time the teacher has to wait for feedback is also considered a disadvantage (Hopkins). As guided by Stake (2000), I concentrated my own case study on how individual children gain understanding of multiplication facts and then memorise them. The case is the group of children in one particular setting, at one particular time (Bouma, 1996) with each child also constituting an individual case study. The children will be discussed as a group, their learning pathways will be outlined and, to identify the diversity of these pathways the data from three children will be considered and compared in more detail.

3.3 Data Collection

3.3.1 *Quantitative data collection*

Cresswell (2002) explains that quantitative research methods are concerned with collecting data that is specific, narrow and capable of being measured. Bouma (1996) describes pre- and post-assessments as valid methods of data collection to measure whether lessons, over time, have had any effect on children's knowledge. The quantitative component of this study is the collection of data, in the form of pre- and post-assessments to show the progress children make in 'knowing' more multiplication facts, their movement from additive to multiplicative thinking and the development of the strategies the children are employing.

Qualitative research, in contrast to quantitative research, uses participant observation, semi-structured in-depth interviews and non-statistical methods of analysing and reporting (Dooley, 2001). This study employed a number of qualitative procedures: interviews, participant observations, field notes, video recordings and children's written work.

3.3.2 *Individual interviews*

Interviewing children is a powerful way to tap into their mathematical knowledge (Aubrey, 1993; Hiebert & Weane, 1996; Kouba & Franklin, 1993; Mulligan & Mitchelmore, 1997). Graeber and Tannehaus (1993) support this belief when they state "The precise strategy a student uses to solve problems can generally be determined only during an interview" (p.109). Glesne (1999) defines topic interviewing as a search for perceptions, opinions and attitudes to learning and the interviewer participates in the interviews with a clear mind without letting personal experiences and ideas take control. Collins (1998) considers that even the most unstructured interview is structured. Open-ended questions should be used whenever possible (Patton, 1990, cited Glesne, 1999). To guide the selection of questions to be asked it is necessary to presuppose that the participant has something to say about the content (Glesne). Bourdieu's study (1997, cited Collins, 1998) points out that it is possible for an interviewee to extract from the interviewer what is expected to happen. For example children could be tempted to give the answer they think the interviewer is probing for, rather than what they are actually thinking or doing. Cresswell (1994) notes among the advantages of interviewing that it allows the researcher to have control over the questions that are asked and to obtain unobservable data. However, he considers disadvantages to include the fact that interviews do not take place in a natural field setting and that it provides 'indirect' information filtered through the view of the interviewees. The purpose of the interviews in this study was to gain understanding of the children's knowledge of multiplication, identify the calculation strategies they were using and allow them to verbalise how they thought they were memorising multiplication facts. Pre- and post-interviews were utilised as well as interviews during the teaching phase.

3.3.3 *Participant Observation*

According to Neuman (1997), when carrying out observations it is necessary for the researcher to become a research instrument to absorb all sources of information by paying attention, listening and watching. Robinson and Lai (2006) explain that an observer who wants to explore what is happening during a lesson knows the information he or she requires but does not predetermine the details of the observation. They also consider it necessary, when recording an observation, to use clear unbiased language and to eliminate evaluative words. Robinson and Lai suggest that to provide high quality information it is important to make a clear distinction between the observer's record of what is happening and his or her inferences about what it might mean. By making this distinction, important insights to motivate change can be identified. Dooley (1984) recognises the genuine social interactions that occur between the observer and the participants during direct observation and considers these observations have the characteristics of flexibility, spontaneity and open-endedness. Although these characteristics can be seen as a threat to reliability and validity in quantitative research, Dooley sees them as strengths in qualitative research because the observer is part of but does not invalidate the natural setting and is able to record accurate and unbiased data. The purpose of the observations, in this study, was to discern how the children shared the strategies and knowledge they had about multiplication facts, and the procedures they used to understand and then memorise their multiplication facts.

3.3.4 *Field Notes*

Gathering field notes has many advantages. Raw data can be recorded to describe people, events and conversations as well as the observer's actions, feelings and thoughts. Field notes attempt to record everything that can possibly be recalled about an observation or event to reflect on when analysing data at a later date (Taylor & Bogden, 1998). As field notes are being written they can alert the researcher to events that need a fuller explanation, interviewing that requires improvement, or new questions needing to be asked to clarify an event or conversation (Davidson & Tolich, 1999). Disadvantages of field notes include the failure to capture a conversation or event in detail sufficient to be useful during data analysis. Another disadvantage is the time that is necessary to record the most complete and comprehensive field notes possible (Taylor & Bogden). I was guided in recording my field notes by the suggestions of Patton (2001). He considers it necessary to ensure that the jottings recorded in field notes are concrete, detailed, without bias, and contain thick, deep and rich descriptions. Field notes, in this study, were methodically kept to record observations and the children's responses during teaching sessions (Taylor & Bogdan). They also recorded notes on how the children worked together and explained their thinking to each other.

As a back-up to the field notes, video recordings can be made. Using a video camera suggests McNiff and Whitehead (2006), enables the researcher to go beyond written accounts and capture actions as they happen. They also recognise video recording as a powerful tool to generate evidence to verify claims made in the research.

3.3.5 *Journals*

Berg (2004) describes a journal as a form of personal document that is created to record an individual's experiences and can be used by a researcher as a contribution to the data collection of a study. He considers that journals remain an under-utilised element in research as they provide a means for the writer to freely express feelings, opinions and understandings fully. In a recent small-scale study Darr and Fisher (2005) worked with a class of Year 7 children to give these children opportunities to self-regulate their learning in mathematics. Self-regulated learning, they explain, enables children to examine their own thinking, and observe, critique and emulate the thinking of others. One of the ways to provide children with the tools to develop self-regulated learning, they suggest, is by using reflective journaling. Darr and Fisher consider there are many advantages in children's written recordings. From journal writing children are able to judge their progress by reflecting on their performance and their learning behaviours. Neill (2005) also values journaling to develop self-regulated learning and claims that children recognise the value of journals to help them acquire better mathematical skills and understandings. Journals help children reflect, organise and remember their ideas about mathematics while teachers gain insight into children's learning by reading the journals. Neill recommends free choice writing in journals so children can express their views without limitations while Darr and Fisher offered a prompt to stimulate the writing. As the teachers, Darr and Fisher gave the children written feedback. A disadvantage in keeping journals is its time consuming nature and they questioned whether the activity would be possible to sustain in a busy classroom. Robinson and Lai (2006) suggest that another disadvantage is that journals are susceptible to the problem of reactivity. They suggest that a way to overcome this is to involve children in learning conversations where the importance of accurate answers can be discussed. In this study journals are referred to as diaries/diary sheets. The use of children's diaries/diary sheets was introduced in such a way that the children thought it was their idea and they gave instructions on how they were to be made. It was anticipated that children would be motivated to record their thoughts as the diaries/diary sheets were given a special name by the owner. These diaries were so hungry for information/research-discoveries that they had to be fed with words.

A modelling book (hereafter referred to as a 'shared communal journal') in the form of a large scrapbook or sheets of paper is recommended by the Numeracy Project (Ministry of Education 2005c) to take the place of a whiteboard when teaching. The shared communal journal is used to state the learning intentions, write problems and then their solutions, as well as record children's inventions and ideas. The shared communal journal also enables the teacher to keep a record of what has been studied and provides a reference for the children. Caliandro (2000) supports shared communal journals and suggests that the procedures and discoveries contributed by children should be referenced with their names. In this study both reflective journals and a shared communal journal were used to assist learning and as methods of data collection.

3.4 Data Collection Methods

3.4.1 *Quantitative data*

The quantitative data was gathered from three pre-/post-assessments. Firstly the *100 Multiplication Facts Test* (appendix 2) was administered. The same test was repeated as a post-test. This written test compared the number of multiplication facts that children knew at the beginning of the study with those they knew at the end of the study. Secondly, the *Multiplication Facts Verbal Pre-Test (*appendix 3*)* was administered to establish those facts that children are able to instantly recall (within three seconds) and to identify strategies children are using to solve the facts they cannot instantly recall. This test was repeated as a post-test and compared with the pre-test. Thirty-five multiplication facts were selected and included examples from each of the multiplication tables. These facts included the 21 facts to be explored in the teaching sessions, except for 9 x 9 and 9 x 6. Using all the 9x was considered not necessary, as four examples were already included. The facts were visible on the testing sheet for the children to read. Each fact was asked in turn and I recorded the children's instant response or their solution strategies. I asked the children to explain their solution strategies if their response took longer than three seconds or it was evident that they were using a strategy. Thirdly, the *Multiplication Knowledge and Strategy Interview Pre- assessment (*appendix 4*)* was used to identify strategies children were using to solve multiplication facts. Part A of this assessment consisted of four questions using the theme of crocodiles to induce a fun aspect. These whole number problems were considered difficult enough that most children would need to use a strategy to solve them. The problems/recording sheets allowed the children to read the problems and to record their solution strategies, with drawing to be a possible option. The children were informed that they were only expected to try answering the problems and it was acceptable if a solution was not found. No feedback was given to the correctness of an answer. I asked questions such as "How did you work that out?" or "Tell me what went on in your head when you were working out your answer" and I wrote down their responses. Part A pre- and post-assessment were compared at the end of the study. Part B of the *Multiplication Knowledge and Strategy Interview* assessment used the multiplication section of the Numeracy Project Diagnostic Test (Ministry of Education, 2005b) that is aligned to the Number Framework (Ministry of Education, 2005a). The diagnostic tool is designed to give information about the knowledge and mental strategies a child has acquired. I used it to ascertain whether the children had the known strategies and knowledge children need to move to the Advanced Additive/Early Multiplicative stage of the Numeracy Project. The results of the pre-assessment were compared with those on the school's records. Part B pre- and post-assessments were also compared at the completion of the study.

3.4.2 *Qualitative data*

Data of children's written records/thinking was collected using a variety of methods. When the children wrote in their diaries/diary sheets they recorded a reflection on their ideas, self-assessment of their progress, and a record of their mathematical/research discoveries. The recorded ideas were often shared during group discussions. When they were working in pairs/small groups on a problem, children recorded their thoughts and ideas and then shared with the group as a whole. Ideas were then recorded in the shared communal journal. The shared communal journal allowed additional opportunities, by providing a format for the children to reflect on the ideas of others and compare these with their own ideas.

Data I collected and recorded as field notes included lesson progressions, observations, individual interviews, discussions and ideas written in the shared communal journal. Notes were jotted down during teaching sessions and the sequence of events recorded at the end of a teaching session. Verbal exchanges were also recorded. The children's responses to each other, the activities used along with the progressions of learning were noted with implied and inferred connections between and amongst them also contributed to the field notes. The field notes were extended and typed up as soon as possible and compared with the video recordings, where appropriate. The data from these field notes was used to inform future teaching, give children feedback on their work and guide them forward.

When possible the teaching sessions were videoed to record responses, interactions and discussions. Initially I had organised the teaching sessions to be videoed by the assistant principal. Although some of the early lessons were videoed, not all lessons were as the assistant principal was often called away. The task of videoing was then given to a group of Year 6 children. As these children practised with the video, the quality of the videoing did improve. I viewed the tapes to corroborate my field notes, and clarify points and notes taken during observations of the children.

3.4.3 *Summary of data collection methods*

The following table summarises the types of data collection methods.

Method	**Purpose**	**Type of data Collected**	**Time in sequence of teaching unit**	**Number involved**
100 multiplication facts test	To establish known facts	Written by the children	Pre- and post-teaching	11
Multiplication fact Verbal Test	To establish which principles and facts are known To assess use of instant recall	Notes taken by teacher/researcher	Pre- and post-teaching	11
Individual multiplication knowledge and strategy interview	To establish what strategies children are using	Written by children with children's explanations recorded by the teacher/researcher	Pre- and post-teaching	11
Children's written records **Journals**	To reflect on the children's progress and their perceived learning (diaries, diary sheets) To keep a record of methods used to solve problems and strategies being used (pair recordings)	Written by the children	During teaching sessions	11
Field notes	To record teacher/researcher's observations of children's conversations, their use of strategies and interactions during an activity, individual interview responses and jottings from the shared communal journal	Written by the teacher/researcher	During teaching sessions with individual interviews in the playground	21 visits to the school over 3 months as explained in table 3.2
Video of lessons	To confirm ongoing formative assessment of my teaching and the children's responses and interactions	Videotaped	During teaching sessions	7 occasions

Table 3.1
Summary of data collection methods

3.5 Ethical considerations

For a research study there are key aspects of ethics that need to be considered. Robinson and Lai (2006) explain that like any professional activity, teacher research involves commitment to treating the participants in an ethical way. They consider that ethical consideration should be seen as a problem solving activity to identify the multiple constraints. Knowledge of the school setting and the existing power relations need to be considered prior to asking the participants for their free and informed consent, and distinction between institutional and individual consent must be maintained. McNiff and Whitehead (2006) alert teachers and researchers to the importance of obtaining informed consent when they explain that, in our current climate of sensitivity to abuse, this is a matter not just of courtesy but also of the law. They point out that it is important to assure the participants that you are monitoring not their practice but your own. McNiff and Whitehead and Robinson and Lai emphasise the importance of ensuring that informed consent is obtained not only from the parents (guardians) but also from the children if they are involved in the research. The researcher can read and then discuss issues with the children involved. Robinson and Lai also emphasise the importance of anonymity to protect both the school and participants being recognised. Schools and participants need to be protected from any harm that may arise from the research. By discussing the use of pseudonyms, anonymity can be agreed and the issue of confidentiality addressed.

It was necessary to address the common principles of ethical considerations as described in Davidson and Tolich (1999); those involved in this study were given assurance that: they would not be harmed in any way; their participation would be voluntary; their anonymity and confidentiality would be preserved; any deceit would be avoided; data would be reported and analysed faithfully.

Informed consent was sought from the school (principal), teacher and parents of the participating children. The letters *(*appendices 5,6,7*)* outlined the intended progression of the study and relevant ethical issues. The letters also included assurance of safety of the participants. The study was explained, in depth, to the principal, assistant principal and the classroom teacher during a meeting. It was considered that this study did not pose a risk to the school, staff or the children participating. Benefits to the children participating were discussed along with expected outcomes for these children. I explained that the children I would work with would gain knowledge of multiplication facts and the calculation strategies as suggested in the Numeracy Project for this stage. I assured both the principal and the classroom teacher that they would be kept informed of progress. My role was also defined in contrast to my previous role in the school, as a numeracy facilitator. Signed consent forms were returned prior to the commencement of the study. Informed consent was also sought from each participating child in a more informal consent form than a letter (Snook, 1999). The main points were described in the children's language and written on a crocodile (appendix 8). To ensure the children understood each point on their participant consent form, I read and explained each point when the group first met. The children expressed their understanding, answered the questions, completed and signed the form.

All parties were assured that every effort would be made to respect confidentiality and that data would only be available to themselves and my supervisors. For both the school and the participants, anonymity was guaranteed by using pseudonyms and I clarified that any work published would not allow the school or individuals to be identified. All parties were also reassured that participation was voluntary with the school or children free to withdraw from the study at any time. Prior to my beginning this study, research approval and ethical clearance was obtained in accordance with the requirements of the Christchurch College of Education consent regulations.

3.6 Research Design

3.6.1 *Participants*

A state primary school, in Auckland, was selected for this study. I was a personal friend of the assistant principal. While the principal and school knew me as a facilitator of Numeracy workshops in 2003, I had not met the classroom teacher. The principal and assistant principal selected the classroom teacher. The school follows the *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992) document through the use of the Numeracy Project guidelines. At the time of this study the school had been working on the Numeracy Project for three years. I considered the school was suitable as they had requested I work there and were supportive of the study.

A purposive sample of twelve children, in one Year 4 class, was selected by the classroom teacher for this study. I considered that twelve children would provide a cohesive social group that could work together, generate discussion and share ideas in pairs, small groups and the group as a whole. Twelve children were expected to also provide diversity of opinions, skills and understandings. Their ages ranged from seven years eleven months to eight years ten months at the commencement of this study. Of the twelve children selected seven were boys and five girls. Eleven children completed the study. One child went overseas on holiday after lesson 9 so it was decided not to include this child in the study, as it was not possible to give her the post-assessments. For the purpose of this study up to eight children were to be identified by their teacher as ready to move from the Early Additive-Part Whole stage (stage 5) to the Advanced Additive/Early Multiplicative stage (stage 6) of the Numeracy Project Framework (Ministry of Education, 2005a). The other children were to be randomly selected from those who had not yet reached this achievement stage of the Project but had worked within the previous stage (Advanced Counting, stage 4).

3.6.2 *Procedure*

This research was carried out over a period of three months, from late March to the end of June 2005, with a two week break for the school holidays and another three week break when the children were required to work on a fractions and decimals unit with the classroom teacher. The classroom teacher administered the *100 Multiplication Facts Pre-test* prior to the teaching sequence. The test was administered in the classroom in conditions where the children worked on their own without a time limit. Children were able to use their fingers but no other materials were supplied although they were free to write on the test sheet. The results of this test guided the classroom teacher's selection of children for the study.

When the classroom teacher had selected the children I met with them to outline the study. I explained my role as a researcher and how I wanted to help children learn their multiplication facts. I discussed with the children how they thought I could do this. The children decided that I needed their help in collecting data and by them helping I could follow how they learned their multiplication facts. Believing it was their idea, the children agreed to become 'co-researchers' and so a criterion (chapter 4.3.2) for this role was established with them. They were encouraged to identify what helped them gain understanding and what assisted them in committing multiplication facts to memory.

'Official/important looking' research folders, with the College emblem and identifying the children as co-researchers, were given to the children to file their 'research' (appendix 9). At this first meeting the children also considered why it was important to learn multiplication tables and prior to the teaching the children set learning goals, which they recorded on a diary sheet. Also at the first meeting I administered both the *Multiplication Facts Verbal Pre-test* and *Multiplication Knowledge and Strategy Interview* individually to each child. The children commented on the crocodile pictures on the testing sheets and during a group discussion it was decided our studies would be renamed as 'Crocodilian Studies'.

I planned two revision lessons and seven teaching lessons (appendix 10*)* following a progression through the 100 multiplication facts and related strategies, with the activities for Advanced Additive/Early Multiplicative stage of the Numeracy Project (Ministry of Education, 2004d) being incorporated as appropriate. The flexible lesson progression, to develop more sophisticated strategies, was determined by children's prior knowledge, my perception of their needs, their requests, and the results of the three pre-assessments. Ongoing formative assessment, throughout the teaching phase, also gave direction to these lessons. A lesson sequence was planned but, prior to the teaching, it was anticipated that it might be necessary to deviate from this plan. The timing and the frequency of the lessons was guided by the classroom teacher's programme. As I was unable to work in the classroom, the teaching took place in the staff room.

Although the lessons were developed to incorporate the strategies suggested by the Numeracy Project the main emphasis of this study was on the children's understanding of multiplication facts and then their committing them to memory. I designed a sequence of lessons to expose the multiplication facts. Of the one hundred multiplication facts to be committed to memory, the 0x, 1x, 2x, 5x constitute 64. The Numeracy Project suggests that these multiplication tables are learnt during the Early Additive stage (stage 5) of the Project (Ministry of Education, 2005d). After these facts have been committed to memory I considered that the number of facts needing to be learnt was reduced to 21, once the understanding of the commutative property had occurred.

Therefore, there were two teaching phases: the revision lessons of the 2's, 5's, 0's and 1's (lessons 1 to 6), and the main phase to teach the 21 remaining facts (lessons 7 to 13). I chose to teach the multiplication facts in this order, as the 2's can be considered easy once children know their doubles (eg 3 + 3 = 6) and they are able to skip count in twos. Learning the 5's is helped once the patterns within this table are known. The 0's and the 1's are made easy, at this stage, by the principles they follow. Once these tables are learnt children are able to derive answers for the remaining facts. The 21 facts were isolated to eliminate repetition in learning facts that have already been covered.

Lesson structure

Each of the lessons I designed had a common structure and began with a focus question. The focus questions were open-ended to provide stimulation for children of different conceptual levels and varying experiences to feel personally challenged. The children worked in pairs, without interruption, to share their thinking and consider how the problem could be solved. They were encouraged to interpret the question at their conceptual level of thinking and to find more than one solution to the problem. The children recorded their discoveries/thoughts/solutions on paper, from lessons 1, 2, and 6 and lessons 8, 10 and 12, so they could share and discuss these with the group. After a group discussion the children or myself then recorded the pairs' findings in the shared communal journal. Although lessons 3, 4, 5, and lesson 7 followed the above structure, the ideas and solutions to problems explored were only recorded in the shared communal journal. Lesson 13 became a lesson to evaluate the children's learning.

The Numeracy Project lessons (lessons 9 and 11) followed the structure suggested in the Numeracy Project guidelines (Ministry of Education, 2005d). During these lessons children were put in groups of four to solve problems and share strategies used. Solutions and preferred strategies were discussed and also recorded in the shared communal journal. In all lessons a large chart was displayed showing the one hundred multiplication facts to be learned (0x to 9x). As facts were explored during the lessons they were added to the chart so children could observe the progress they were making.

At the completion of each lesson the multiplication facts explored were noted and the children were encouraged to practise them. No specific instruction on how to learn them was given. The teaching sessions were to be enriched by children working in class on follow-up activities that I made available. As part of the children's class homework requirements the children were to practise these facts at home. Also for homework, children were encouraged to make games to share with other ‘crocodilians’.

Peer and group discussions

Wood, Cobb and Yackel (1995) emphasise that group discussions and small groups/pair collaborations are valuable for learners. Small groups provide opportunities to explain and justify solutions to problems and make sense of others’ explanations by listening and questioning. These discussions also allow opportunities to indicate agreement, disagreement or failure to understand the explanation of others. It is important to provide opportunities for children to take on the responsibility of their learning through productive engagement in groups without constant teacher monitoring (Cobb, Wood, Yackel, Nicholls, Wheatley, Trigatti, Perlwitz, 1991a). Burns (1990) supports these ideas and notes the value of small groups in providing children with opportunities to speak more often, as well as speculate, question and explore various approaches. The teacher-child-teacher-child… interaction is replaced by increased child involvement. These discussions allow children to discover relationships and patterns, leading them to become confident in using numbers to solve problems.

Guided by the work of Burns, the Numeracy Project (Ministry of Education 2005c) advocates working with groups of four children within a teaching group. It also suggests that these groups be made up of children at the same strategy stage of the Project. This is a Numeracy Project initiative, and although the Project claims that working in strategy-levelled groups appear to be successful, there is lack of literature or work recorded so far by researchers to support this.

Materials

The applicable Numeracy Project materials were used for the Numeracy Project lessons 9 and 11 taught (Ministry of Education, 2005d). For all other lessons I designed, a variety of manipulative equipment was available for children to use to explain ideas or solve problems if needed. I made resources, such as games, to support and practise strategies, and to assist memorisation of facts. Additional activities recommended by the Numeracy Project were added. Children were encouraged to request activities or resources they considered would help them with their task of committing their multiplication facts to memory. Through discussion with, and requests from the children, additional games and puzzles were made**.** To keep up motivation, after lesson 7, children's crocodile drawings were used as templates for cards and games. The children made an unanimous request for speed sheets with timers. I therefore created speed sheets for them. Fifteen of the facts currently being learnt were tested at a time. Time was recorded in seconds and the children marked each other's work. The children competed with themselves to better their number correct and time taken. These speed sheets were used during the teaching time and children were able to take copies home. Flashcards were also requested and made and new cards were added as more multiplication facts were explored.

At the end of the teaching sequence I administered the *100 Multiplication Facts Post-test,* the *Multiplication Facts Verbal Post-test* and the *Multiplication Knowledge and Strategy Interview* to eleven children under the same conditions as the pre-test (appendices 2,3,4).

3.6.3 *Changes made to the research design*

I made changes to my planned sequence due to a number of factors. Firstly, I found that the children did not know their 2's, 5's, 0's and 1's multiplication tables. This knowledge is considered a pre-requisite to move the children to the Advanced Additive/Early Multiplicative stage of the Numeracy Project. I found that six lessons, instead of the planned two, were necessary for all children to revise these tables. The length of each lesson was to be one hour (as is expected practice for the Numeracy Project) but often the teacher had to cut these lessons short, notifying me either just before the lesson or during the lesson. On occasion children were substantially late for teaching sessions, as they had to participate in other curriculum activities. I addressed this constraint by changing the order of the lessons or spreading them over two days.

Table 3.2 below indicates the times that data was collected

Visit	Data collected
1	*100 Facts Multiplication Pre-test* (administered by classroom teacher)
2	Initial discussion with children – introduction of co-researchers Began *Multiplication Facts Verbal Pre-test* and *Multiplication Knowledge and Strategy Interview (pre-assessment)*
3	Pre-assessments completed
4 - 11	Lessons 1 to 6 (revision of 2's, 0's, 1's and 5's)
12	Session with children to recap learning after a 5 week break
13 - 20	Lessons 7 to 13 (teaching of remaining 21 multiplication facts – see appendix 10)
21	Post- assessments of the *100 Facts Multiplication Pre-test, Multiplication Facts Verbal Pre-test, Multiplication Knowledge and Strategy Interview (pre-assessment)*

Table 3.2
Summary of visits to the school when data was collected

Initially it was planned to include all the lessons suggested for the Advanced Additive/Early Multiplicative stage of the Numeracy Project, but many of the strategies were 'discovered' thus eliminating the need to teach the Numeracy Project lessons. The shortened times impacted especially on the Numeracy Project lessons where the number abstraction stage was cut short; for example, where children were doing such thinking as using 6 x 7 to solve 60 x 7 or the extension of division problems, suggested in the Project, needed to be eliminated. Time did not allow for these lessons to be revisited. The lessons were changed to the order listed below.

	Lesson 7	**Lesson 8**	**Lesson 9**	**Lesson10**	**Lesson 11/12**	**Lesson 13 Evaluation lesson**
Facts to be covered	3 x 4 4 x 4 7 x 4 8 x 4	6 x 4 8 x 3 6 x 8	3 x 7 3 x 6 6 x 6 7 x 7 8 x 8 3 x 3	6 x 7	3 x 9 4 x 9 6 x 9 7 x 9 8 x 9 9 x 9	7 x 8

The related commutative fact was considered an integral part of each fact.

Table 3.3
Sequence of multiplication facts taught

The children's research folders were initially intended to provide a link between home and school. To prevent loss of data, I collected these folders after each session. Homework notebooks were supplied (displaying crocodiles) to replace these but were quickly abandoned as they were lost or not brought back to the group lessons.

Time to write up children's diaries encroached on the teaching time so, after four entries, the diaries were exchanged for diary sheets where I chose the questions. The time constraint also affected the writing of these diary sheets, and after group consensus, in their place I implemented individual diary-interviews, with set questions. I carried out the diary-interviews either before school or during the children's morning break. The continuity of the lessons was interrupted at the end of the revision lessons. When the classroom teacher was teaching fractions and decimals to her whole class I delayed the main lessons (7-13) for three weeks. I considered that working in two domains would be difficult for the children in the study.

Originally I made one copy of each game for the children's classroom group maths practice box. These boxes were to be changed, when appropriate, to support current multiplication facts being learnt. However, the children were not able to practise the games in class time so copies for each child were made with the idea of children playing them with their families. The inclusion of practice of multiplication facts could not be added to the children's homework schedule as syndicate-set homework was given. The school homework, on occasion, conflicted with methods I was advocating. For example, the children were asked to learn the 4x table, so they could recite it. None of the children indicated that they had achieved this.

3.7 Data Analysis

The following table indicates the data collected in this study.

Data collected – pre- and post- tests	**Recorded by**	**Per child**	**Total (11 children)**
100 multiplication facts test (pre- + post-)	teacher/researcher	2	22
Multiplication facts verbal test (pre- + post-)	teacher/researcher	2	22
Individual multiplication knowledge and strategy interview (pre- + post-)	teacher/researcher	2	22
			66
Data Collected during the teaching phase		**Per child**	**Total No.**
Children's diary sheets	each child	**7**	**77**
Children's entries in individual diaries	each child	4	44
Individual diary-interviews	teacher/researcher	6	66
Children's individual speed sheets	each child	7	77
Children's worksheet at final teaching session	each child	1	11
Children's recordings done in pairs	pairs of children	7	42
Video (days videoing occurred)	video		8
Shared communal journal (number of entries)	children and t/r		17
Field notes (entries/number of days)	teacher/researcher		24 days
			366
Total number collected			**366 + 66 = 432**

Table 3.4
Record of data collected

The quantitative data was analysed in the following ways. The *100 Multiplication Facts Pre- and Post-tests* results were compared and each child's increase in correct answers to the multiplication facts was recorded. Children's responses from the pre- and post-assessments, the *Multiplication Facts Verbal Test* and the *Multiplication Strategy Interview* (part A), were examined to find the calculation strategy used and then recorded under the corresponding intuitive model. The intuitive models, and their related calculation strategies identified by Mulligan and Mitchelmore (1997) provided the criteria. Mulligan and Mitchelmore define an intuitive model as "an internal mental structure corresponding to a class of calculation strategies" (p.319). The intuitive models identified by Mulligan and Mitchelmore for multiplication are direct counting, repeated addition and multiplicative operations. I adapted the model by separating out the multiplicative operations into derived facts, known facts, and instantly recalled facts. The intuitive models identified by Mulligan and Mitchelmore, and the adaptations I made are listed in the table below.

As identified by Mulligan and Mitchelmore (1997)		As adapted by V. Morrison for the purpose of this study	
Intuitive model	**Calculation strategy**	**Intuitive model**	**Calculation strategy**
1. Direct counting	Unitary counting	**1. Direct counting**	Unitary counting
2. Repeated addition	Rhythmic counting forward Skip counting forward Repeated adding Additive doubling (a)	**2. Repeated addition**	Rhythmic counting forward Skip counting forward Repeated adding Additive doubling (a)
3. Multiplicative operation	Known multiplication fact Derived multiplicative fact	**3. Multiplicative operation**	Derived multiplicative fact *(uses a known fact to find another fact)* Known multiplication fact *(recall a fact from memory in more than three seconds)* Instant recall (recalls a fact within 3 seconds)

(a) For example, “3 and 3 is 6, 6 and 6 makes 12”

Table 3.5
Intuitive models for multiplication

These assessments were also examined to identify the development in children’s multiplicative thinking by noting the number of answers using an additive strategy and the number of times a multiplicative strategy was used. The *Multiplication Knowledge and Strategy Interview* (part B) pre- and post-assessments were compared to note the increase in stages of the Numeracy Project at which the children were working.

During the study I wrote up the field notes, and transposed data, word for word, from the children’s written recordings and the individual interviews on to spreadsheets as soon as was possible. This enabled me to identify any gaps in the data and respond by asking the children for clarification, or adding additional questions in the diary sheets or individual interviews (Davidson & Tolich, 1999).

I began the qualitative data analysis by reading and becoming acquainted with the raw data collected in the field notes, the children’s recordings and the individual interviews (Baptiste, 2001). When quoting children's notes from their written work I corrected spelling errors as, at times, ideas/thoughts could be difficult for a reader to decipher. After scrutinising the data, to enable me to rigorously review what the data was saying, I retrieved the most meaningful pieces that would help answer the research questions, for example, a child's explanation of how he or she knew a multiplication fact was in his or her head. I selected the child's explanation that most coherently conveyed his or her thoughts on this process.

I assigned labels to this data, using words and phrases based on the content of the research questions, to create a coding system. Examples of labels were phrases such as 'times practised' or 'homework - practice choices'. During this phase of the coding I was able to link the main groups together to form categories such as 'hard and easy facts' or 'value of games'. The categories formed the basis of the data analysis (Coffey & Atkinson, 1996). Large pieces of paper were used to record all data relating to specific categories. To give a holistic picture, mind mapping or chronological order was used to organise the data within the categories. This enabled triangulation within and among the data to be identified. Data reduction removed unrelated categories (Janesick, 2000). Coding differentiated the field notes, observation data, interview data, the children's written recordings and the video data (Baptiste, 2001).

It was necessary to make judgments when coding the data, assigning data to categories and then combining these categories into themes as the data overlapped. The comments from children could fit into more than one category. Where overlapping occurred, I considered the big picture of the study and decisions were made on the basis of the criteria. An example occurred when assigning data to either committing facts to memory or to the role of practice categories. When a child stated that they said their fact "over and over and over…" a decision had to be made as to whether this was memorisation or practice. Where it seemed appropriate, I labelled children's ideas relating to the effect that practice had on memorising facts to the *memorisation* category while I assigned comments referring to how and what children practised to the *practice* category. Both these categories also overlapped with the category of resources and activities that influenced learning.

Initially there were 31 categories, but relationships amongst the categories emerged enabling them to be regrouped to create analysis themes. An example is: categories such as games, activities, resources, speed sheets, flash cards and homework could be combined to create the original *practice* theme (Coffey and Atkinson, 1996). Re-examination further reduced the analysis themes to three main ones (Janesick, 2000). These were the *progression of learning*, *memorisation*, and *practice*.

The *progression of learning* theme included data related to

the teaching and learning sequence

the progression in calculation strategies children discovered

the development and understanding of children's multiplicative thinking.

The *memorisation* theme included data related to

children's explanations on how they think they commit multiplication facts to memory

the role of games including speed sheets in memorisation

the choices children made in their learning

how children gave advice on memorisation to each other

difficulties they experienced in learning their multiplication facts

how they recognised they knew a multiplication fact

reasons why children thought they could not recall some facts.

Practice theme included data related to methods children used to practise

what children practised

role of resources/activities/games in practice

role of parents and other people.

Each child constituted a case study. Again large pieces of paper were used to collate data relating to individual children. The typed sheets recording data from the children's diaries, diary sheets, individual interviews and pre- and post-assessments were cut up, sorted and pasted to the relevant case. Data from the field notes, shared communal journal and video clippings were arranged to give a chronological picture. Each case was recorded in an identical way to enable similarities, comparisons and changes in results and patterns to be traced (Creswell, 1994).

3.8 Chapter Summary

The focus of this chapter has been to outline the mixed-method approach that frames this study, the research design, and the rationale for selecting this approach. I also explained the motive for undertaking action research within the setting of an interpretative paradigm. The rationale for the data collection methods was described. Ethical considerations were also outlined. Implementation of the teaching unit and the data gathering was reported on, including the changes made to research design. The chapter concluded with the data analysis methods being explained and the emergence of three analysis themes outlined. The following chapter will report on the data collected and the analysis of the data.

Chapter 4: Results

4.1 Introduction

This chapter outlines the results from the data analysis undertaken in the study. Firstly the data collected prior to the teaching sequence, from the pre-assessments, is examined. The data collected during the teaching unit is then described. The context of ‘Crocodilian Studies’, the collaborative role the children played as co-researchers and the children’s pre-study ideas are recorded. Following this is a report on the analysis of the data according to three analysis themes, the *progression of learning*, the children’s thoughts on *memorisation* and the role *practice* played in learning multiplication facts. Three case studies are then outlined. Finally, the results of the post-teaching assessments are examined and compared with the pre-assessments.

There were 11 children in the sample, now referred to as the group. The boys were Otis, Finn, Simon, William, Kyle, Jacob and Aaron, and the girls were Madison, Fleur, Lucy and Molly (all pseudonyms). There were 13 teaching sessions and I met with the children on a further six occasions. All children were present for all the teaching sessions except for Jacob (absent for seven sessions), Aaron (for three), and Finn, Fleur and Otis (each for one session). The teaching sessions ranged in length from 45 minutes to one hour and 20 minutes.

4.2 Data prior to teaching

Prior to the teaching phase of this study three assessments were carried out. These were the *100 Multiplication Facts Pre-test*, the *Multiplication Facts Verbal Pre-test*, and the *Multiplication Knowledge and Strategy Interview* (Part A and B). The classroom teacher administered the *100 Multiplication Facts Pre-test* to the whole Year 4 class to establish how many of the 100 facts each child could either recall or derive an answer to. She used the pre-test results (appendix 11) to assist her in selecting the children for the study. If children gave an incorrect response for a multiplication fact and the commutative fact, then both of these were recorded. The facts that the children gave the most incorrect answers to were 7 x 6, 7 x 8, 8 x 6, 8 x 7, 8 x 8, 8 x 9 and 9 x 9. From the data in the *100 Multiplication Facts Pre-test* I concluded that the zeros needed to be retaught, and although the 1's and the 2's were reasonably well known they would need to be revised and then committed to memory. I considered it was essential to teach all multiplication facts from the 3's to the 9's.

The *Multiplication Facts Verbal Pre-test*, using thirty-five questions, probed which multiplication facts children had committed to memory and the strategies they were using. If answers were not instant I asked the children to explain the strategy. The children's verbal explanation identified their calculation strategy, enabling these calculation strategies to be grouped to infer an intuitive model. An adaptation of the intuitive models as identified by Mulligan and Mitchelmore (1997), was used to group the responses (chapter 3.7, table 3.5). I recorded the facts as 'too hard' if an answer was not offered or incorrect answers were given. Appendix 12 records the number of additive and multiplicative calculation strategies each child used. Results from this pre-test show that children considered 38% of the questions were 'too hard'. For a further 20% of the answers, the children used an additive strategy while 42% were answered using a multiplicative strategy. All children indicated that they were able to use multiplicative strategies. Incorrect answers combined with additive strategies accounted for 58% of the calculation results. This pre-test indicated that although children had knowledge of some strategies they did not have instant recall of their multiplication facts.

In part A of the *Multiplication Knowledge and Strategy Interview*, word problems were used to ascertain what strategies children were using and whether they could instantly recall any of the four facts asked. The children's responses, in the form of drawings or the use of materials, along with their verbal explanations to the four multiplication problems, were examined. The children's calculation strategies were identified, further examined and grouped to infer an intuitive model (appendix13). The children's drawings identified the semantic structure of the problems but the intuitive model the children used was not identified until I listened to further explanations of the calculation strategy used. If a child used a strategy but gave an incorrect answer then the calculation strategy was noted.

Results of *Multiplication Knowledge and Strategy Interview* (Part A) were summarised to indicate the overall distribution of the intuitive models employed in responses to the multiplication word problems. All four intuitive models were utilised by the children to solve the problems in the pre-assessment. In this pre-assessment children showed a preference for direct counting 54% of the time, for repeated addition 23% of the time utilised derived facts 18% of the time and used known facts 5% of the time. There were two children using the multiplicative strategy, deriving facts, in the pre-assessment and not using any additive strategies. One of these children took an extended amount of time to answer the problems in the pre-assessment so it was uncertain exactly how he derived his answers. All children except two, who both used only direct counting, used at least two additive strategies. Question three (8 x 6) could be considered the most difficult question as the number fact involved higher numbers. Most of the responses classified as incorrect in this interview were related to question three.

Part B of the *Multiplication Knowledge and Strategy Interview* used the Numeracy Project Diagnostic Assessment, questions 1 to 3 of the multiplication section (Ministry of Education, 2005b, p.26), to establish the strategy stage of the children.

	Numeracy Project strategy stage - school records March 2006	Numeracy Project strategy stage pre-test March 2005
Lucy	5	5
Kyle	5	4
Otis	5	2/3
Molly	5	4
Madison	5	5
William	5	4
Simon	5	4
Fleur	4	4
Aaron	4	4
Jacob	4	4
Finn	4	2/3

Table 4.1
Results of *Multiplication Knowledge and Strategy Interview* Part B:
Numeracy Project strategy stages in school records and at pre-assessment

Results from the pre-assessment indicated that two children were at stage 5 (Lucy and Madison), seven were at stage 4 (Kyle, William, Aaron, Jacob, Simon, Fleur and Molly) and two were below this at stage 2/3 (Otis and Finn). In contrast, the strategy stages on the school records deviated from my results for six of the eleven children.

4.3 Data collected during the teaching unit

4.3.1 *The context of 'Crocodilian Studies'*

Crocodiles, chosen by the children, provided a theme that the children could relate to at their age and their stage of learning, and came to provide a context for every aspect of this study. After the theme was introduced in the first lesson, I noted the children gave all components of the study 'crocodilian' names and even referred to me as Mrs Croc.

In the staff room, the designated teaching space, a working area was created and defined as a pond for crocodiles. All work had to occur in or around the crocodile pond. These boundaries moderated the initial overexcitement of being in a large staff room. It was in this pond that the group was able to collate and discuss data gathered. The 'croc' research folders, in which all the children's work was filed, were considered important and valued documents and identified the children as co-researchers. These folders snapped like crocodiles as they consumed a new multiplication fact or valuable data for the study. The ‘croc’ diaries/diary sheets were given crocodilian names and treated as special friends who needed to be fed lots of interesting words and ideas. There was a rush, on arrival in the teaching space, for children to get their diaries to see if their ‘croc’ friend (I) had written back to them. By the incorporation of the children’s drawings of crocodiles on the last four ‘croc’ diary sheets, children were re-enthused to write about their learning. These crocodiles used speech bubbles to insinuate they were now asking the questions (example, appendix 14). The ‘crocofacts’ chart displaying the 100 facts (0x to 9x) that need to be memorised, recorded each fact as it was explored. The chart encouraged the children to keep practising: the 100 facts seemed less daunting as they were crossed off at a quick rate. The whole group took pride in this shared ceremony.

When all researchers (children and I) were gathered together, ideas were recorded in the ‘Big Green Croc’, a shared communal journal. Recording occurred at varying times during almost every lesson. The ‘Big Green Croc’ became a record of proceedings of each lesson, class discussions and thoughts that the ‘co-researchers’ considered important. The children often voluntarily referred to the ‘Big Green Croc’ if they had forgotten something, or used it to confirm an idea. As children recorded their ideas in this journal, others often challenged ideas for clarification or to identify a mistake. Examples of combined thoughts entered in the ‘Big Green Croc’, after a discussion of the 5x table in pairs, are recorded in appendix 15. As I listened to the discussions prior to entries being made in the ‘Big Green Croc’, I collected data about the children’s progress, understanding and their learning of multiplication facts. Otis identified the ‘Big Green Croc’ as one of the things that helped him most with his learning. He verified this by saying that the ‘Big Green Croc’ “helps you remember, it is like a memory card.” Other children supported the value of the ‘Big Green Croc” by saying

> It is…like a big brain that you keep feeding (Aaron)
> It is like storing it in your mind (Jacob)

People think of ideas and we share (Lucy)
It is like a computer…by telling us lots of ways to learn our 5x (Fleur)
It is like a computer …when you say something you write it down and then you can look back if you want to remember something… It is like a floppy disc (William).

Respect for the 'Big Green Croc' was shown by way the children competed with each other to write in it or carry it up the stairs to our teaching space. On two occasions when asked to comment on whether they felt 'crocodilian studies' was 'yum' or yuk', the children all gave a very positive 'yum' response. The children's enthusiasm for attending 'crocodilian studies' was verbally confirmed by the classroom teacher on two occasions.

4.3.2 *The children as co-researchers*

Before beginning the teaching sessions I asked the children to identify what they felt constituted a good researcher and their responses were recorded on a chart that was visible throughout the study. The following is a record of the children's ideas.

A good researcher:

works with others in the group (helps other researchers)
shares ideas
is not scared of saying something
is kind when someone says something they don't agree with
works hard
plans what they want to do
writes about what they discover.

The children's responses show that they perceived a co-researcher would work in a collaborative way and be responsible for regulating their learning. Both the children and myself often referred to the criterion of a successful researcher during group time. The children referred to the chart if they needed to remind each other to keep practising or to remember to 'collect' data to confirm their discoveries. The children would regularly mention helping a co-researcher or remind each other to remember what was happening for them as they were learning.

'Helping another researcher' was a phrase often used. Children helped each other when they were working in pairs by explaining their thinking and solution strategies to their partner if something was not understood. Explanations also occurred during whole group sessions. When a discovery was made, such as the commutative property, this was shared with the group. When Aaron, Jacob and Finn were falling behind, or had been away for sessions, their co-researchers expressed the intention of playing games and helping them in the classroom, but it appeared that this happened on only one occasion. In another instance a co-researcher helped another to play a game she did not understand as she had missed a teaching session.

Throughout the study the children shared their successes and difficulties. Molly, who took her role as co-researcher seriously, often called the researchers together to give advice as she diligently worked on memorising facts: "You just have to tell yourself no fingers – just say you want to do it"; on speed sheets she advised them to say "I want to do this so I should hurry up and get a good time". As the children considered themselves co-researchers who needed to collect data, they referred to any new learning or ideas as discoveries. In this chapter 'discovery' is used to refer to strategies children identified, patterns in multiplication tables or other new ideas.

The results reported in the remainder of the chapter come from the following data sources: written data from the children including the 'croc' diaries, the 'croc' diary sheets, the work they recorded during paired discussions and the evaluation 'croc' diary sheet. Teacher/researcher-recorded data included data from individual diary-interviews and observational field notes, which included notes of incidental discussions I had with the children and a summary of the video data. The data from the children's quotes was particularly rich as each child's path to memorising their multiplication facts could be followed along with the methods they were using to do this. The children's explanations also indicated progressions in their developing multiplicative thinking and strategy development.

4.3.3 *Children's learning goals*

During my first meeting with the children they recorded their goals, how they thought they learnt their multiplication tables and the reasons why they considered it important to learn multiplication facts. The children's personal goals were written in their own way; for example William's goal, which was "to learn how I get my multiplication and how others get their X tables". After setting personal goals, an overall group goal was set, with wording suggested by Lucy, to "get tables into our heads and keep them there".

The children considered they learnt their tables by

> skip counting (Finn)
> plussing quickly but I forget the hard ones (Aaron)
> writing answers to facts (Kyle, William, Otis, Jacob, Lucy)
> doing your homework and practising (Simon, Madison, Molly Fleur).

I also asked the children why it was necessary to learn multiplication tables. Some responses were

> You might have to count stuff at work (Finn)
> When you're older if you had lots of money it would take too long to count it (Kyle)
> You might work at a shop and someone might want 5kg of bananas and you don't know the answer (Otis)
> You need to know how many as high school is hard (Madison).

The children, when setting their goals, indicated that knowing multiplication facts would be useful in their lives. Their responses to how they learnt their multiplication tables identified strategies they could use but did not give ideas about how to commit multiplication facts to memory. Children's reasons for why it is necessary to learn these facts were centred on future careers and schooling, and reflected experiences within their lives. At the completion of the study the children were asked to comment on whether they had achieved their goal. William thought, "I've improved but not all my facts are in my head" while Simon knew he had achieved his goal as "Lots of them are in my head". Lucy recognised her improvement also when she said, "Got lots but not all in my head … know them heaps more".

4.4 Analysis of the data

The following section reports on the results from the analysed qualitative data, with reference to the three analysis themes, *progression of learning, memorisation* and *practice.*

4.4.1 *Progression of learning*

The first analysis theme was the *progression of learning*. This theme incorporated the teaching and learning sequence including the progression in calculation strategies that children discovered along with their development and understanding of multiplicative thinking. Firstly, this section reports on key observations from the revision lessons (lessons 1-6). Data related to the first facts learnt, the 2's, is set out followed by data from the revision of the 0's and 1's and then the 5's. Strategies, as they were exposed, are recorded. Secondly, data is reported from the remaining lessons (7 to 13), which explored the remaining 36 facts.

The first phase of the teaching, the revision phase, began by revising the additive strategies of forwards and backwards counting in 2's, 5's and 10's. After revising the 2's the children explained skip counting and how doubles (eg 6 + 6) could be used to find answers. Skip counting became the predominant strategy used with some children incorporating 'fingers'. Appendix 16 gives an example of recordings by William and Kyle when discussing the 2's. They demonstrated their understanding of strategies such as repeated sets, doubling numbers, skip counting, repeated addition and the ability to use a strategy and push it into higher numbers. William later told his diary "Since I have been taught I have been better at my 2 times tables, I can go faster at my two times table, I really like it". During a lesson to re-familiarise the children with the 0x and 1x principles, the group shared ideas. To explain the 0x principle, the children used explanations such as, "Even if the two numbers are the same (0 x 0) the answer is still 0" and "The highest number in the world times 0 equals 0". The 1x principle was recorded as "the answer is always the number you are multiplying one by".

Kyle discovered the commutative property and recorded his ideas in his homework book. Kyle was awarded the distinction of having made the discovery. After demonstrating their understanding with materials to the group, all children chose to explain it to their diaries. Jacob wrote "Today I proved if you switch a times table it will be the same answer 3 times 5 and 5 times 3" while Kyle reinforced his discovery and told his diary that "…today I proved it". Discoveries of strategies and patterns were shared as the 5's were explored and practised in revision lesson 3. Children were now using strategies such as halving the number being multiplied by 5 (eg 8 x 5 = 4 x 10), although at this point most children did not recognise the doubling and halving consequence. Using this method the children discovered that if it is an odd number you just add on a 5, for example, 7 x 5 is odd so (6 x 5) + 5 = 35. Discovering the patterns in each table was now seen as a motivating challenge. William explained to his diary friend

> …And always 10 times four = 40ty ty means ten you can count 5, 10, 15, 20, 25, 30, 35…But it is better to go 5,10 (10). 15,20 (20) …(to 100). If the number is multiplied is odd (1,3,5) the answer is an odd answer, is even (2,4,6) the answer is even. from William to Crocobro

Deriving facts from known facts was beginning to emerge; for example, Lucy explained that 4 x 5 = 20 and 20 + 5 = 25 so 5 x 5 = 25. She also explained that division is the inverse operation to multiplication: "If you know your division you can find out 25 ÷ 5 = 5 because 5 x 5 = 25. Repeated addition was now readily discussed and used occasionally for explanations to solve a problem.

By the end of this revision phase (revision of the 2's, 0's, 1's and 5's) seven children had the required knowledge and strategies to begin learning the remaining multiplication facts. Simon, Jacob, Aaron and Finn could not instantly recall the facts taught so far, but Aaron was the only child who still used his fingers to skip count for most facts. Simon recognised he was not able to recall these facts and after practising his sheet "… because I had nothing to do", realised he had made progress. He became the fastest at calling out an answer to a flash card. The 'crocofacts' chart now indicated that the children had been exposed to 64 of their 100 multiplication facts and eight of the eleven children had committed them to memory. Although Simon, Jacob, Aaron and Finn had not committed all their 2's, 5's, 0's and 1's to memory, lessons to teach the remaining 36 facts began.

When the children, in lesson 7, 'discovered' the idea of doubling the 2's to find the answers to the 4's. For example, Fleur said "If you want to work out 8 x 4 go 8 x 2 = 16 and 8 x 2 = 16, add them together which makes the answer 32". The idea of doubling and halving facts also emerged without my input but was not initially embraced. Children used materials, such as counters, to explain their thoughts to others, indicating an understanding of the associative property. In lesson 9 the Numeracy Project Lesson 'Fun with Fives' (Ministry of Education, 2005d, p.12) was used to demonstrate the distributive property. The children progressed though this lesson but when asked to operate just using numbers, they wanted to choose a strategy to use rather than use the one being taught. For example, when asked to work out 8 x 8, William insisted on bypassing the taught strategy by doubling 8 x 4 and when questioned he pointed out that he liked his way. At this point Aaron, Jacob and Finn realised that they were experiencing difficulty understanding more complex strategies, as they had not memorised the 5's. These children approached me and we decided that they would still work alongside the other children but concentrate on their 2's and 5's. Understanding of the distributive property by the other eight children was evident when they explained, in lesson 12, how they would work out 6 x 7 if they did not know the answer. Lucy and Madison, when working together, recorded 15 variations of deriving facts, for example, they showed their understanding of the 2's and the 5's when they found the answer to 6 x 7 by working out 6 x 5 and 6 x 2. I accepted deriving facts as an expression of understanding.

The strategy of compensation was discovered as the children worked with the suggested materials for the Numeracy Project Lesson ‘A Little Bit More A Little Bit Less’ (Ministry of Education, 2005d, p.15). The children used the 10’s to derive answers to the 9’s. Finn could not complete this strategy as he did not know his basic facts to ten, which hindered him working out 60 – 9. Finn’s group spent time with the materials to help him understand this strategy. Aaron and Jacob needed a lot more practice at the materials stage also. These examples are indications of how not being able to recall the 5’s hindered Aaron, Jacob and Finn when developing strategies in the two Numeracy Project lessons. Having to use strategies like skip counting before deriving the answers to the multiplication facts in which we were now working, caused confusion for these children. Children who were able to work with number properties in the Numeracy Project lessons allowed me to gauge their understanding of strategies such as the distributive property and compensation.

When the group considered ‘What do we know about the 9 times table?’ the children discovered the patterns of the 9’s with enthusiasm. For example, Fleur explained that the numbers in the answers always add up to 9 and then exclaimed, “The 9’s are cool”. An approach was explained by William, “If the question is 6 times 9 you know that the answer will be in the 50’s as it is not 10x, so 5 plus something equals 9 therefore, it is 54”. The children paired with Finn, Jacob or Aaron spent time explaining the concepts to them. Although I knew Aaron could find answers to the 9x table by using the ‘finger method’ (of putting down the finger which represents the number you are multiplying by and reading your fingers to the left as tens and the fingers to the right as the ones) I purposely did not let the idea emerge. I considered, given the time schedule, the finger method would hinder memorisation to the children’s disadvantage.

The final lesson of 7 x 8 deviated from its original purpose of reinforcing deriving facts to answer unknown facts, and was instead used as a lesson to assess each child's understanding of multiplicative thinking. The children were enthusiastic when Simon discovered 5678 is 56 = 7 x 8. Children were individually asked to record how to work out an answer to a fact they did not know, give advice on how to memorise multiplication facts, use drawings to illustrate the problem and use 7 x 8 to find answers to 'harder' facts (example, appendix 17). Results indicated that all children except Jacob were able to derive facts, including Aaron and Finn who had previously been unable to do this. Finn wrote "7 x 9 = 63 and 63 – 7 = 56 which is 7 x 8". Lucy wrote "If you are doing 7 times 8, you could go 5 times 8 equals 40, and 2 times 8 equals 16, add together and it makes 56". All the children except Simon, Kyle, Aaron and Finn could use 7 x 8 to solve problems such as 8 x 70 = 560. When asked what 7 x 8 would look like all children drew ponds and used symbols to represent the crocodiles. Jacob, who could not keep on task, claimed "it's too hard as I have been away so often". The children were asked to comment on how they would tell a friend to commit their multiplication facts to memory. Eight children suggested that practice is the most effective way of getting facts into your head and the other two showed a way to derive facts. One child (Jacob) was unable to give an explanation. During the teaching sessions Lucy, Kyle, Otis, William and Madison all demonstrated an understanding of the reversibility strategy (division) in their records.

Results from the analysis with respect to the *progression of learning* theme indicate that the children who were able to commit their 2's, 0's, 1's and 5's to memory were able to develop and use more sophisticated strategies than those who were not. The group discovered multiplicative strategies, without being directly taught and the progression in which these were exposed followed closely the progression as suggested by the Numeracy Project (Ministry of Education, 2005d, p.5 and p.12). As the children developed their multiplicative thinking and strategies they were committing their multiplication facts to memory.

4.4.2 *Memorisation*

The second analysis theme was *memorisation* which included categories of children's explanations of how they think they commit multiplication facts to memory, the role of games and speed sheets, the choices children made in their learning, how children gave advice on memorisation, difficulties they experienced in learning facts, how they recognised they knew a multiplication fact and why they could not recall some facts.

Over the course of the study I asked a question relating to memorisation such as "What do you think helped you put facts into your head?" on many occasions. This was done in a variety of ways: four times during discussion times (recorded in the field notes), eight times through the diary sheets and four times during diary-interviews. In order to eliminate the possibility of children giving answers that they thought I wanted to hear I slightly varied the wording or phrasing of the question each time. The children provided explanations of how they thought they committed their multiplication facts to memory and some of the questions and responses follow. When the children were asked, "What do you do when you can't remember a fact?" their initial responses included comments such as

> Do 'croc' speed sheets (Molly)
> Play games (Madison)
> You have to find a way to glue it in your brain (Aaron)
> Count on my fingers (Molly).

By the end of the study when asked "What would you tell a friend to do to get a fact into their head?" their suggestions reflected the use of a strategy, such as deriving facts. Practising speed sheets or saying the fact over and over was suggested by 10 of the children. Fleur encouraged her fellow researchers by saying "If you learn your tables in your head you will be great at your tables forever" and to recall facts she 'has done' she keeps thinking "remember, remember,

remember and I get the answer". Molly considered it necessary to just practise, and tell yourself "no fingers and just say you want to do it". Recommended times to practise included while riding your bike around (Otis), when skipping (Madison) or while you are waiting for 'play station' to load (William).

Games were considered helpful to memorise multiplication facts. When the children were asked, "How do you know games help?" five children made comments similar to Lucy's: "Because they are fun I don't realise I am learning". Other comments included

> Because when you play you can feel the sums flying into your head (Fleur)
> Because I get faster and I remember more after I have played them (William)
> I don't know (Finn)
> Because when I do them I get better 'cause Mum tells me the answer and I learn, I remember the answers (Kyle).

To assist the children in committing the multiplication facts to memory, speed sheets were introduced at the request of the children, as was the use of a timer. The children were asked "How do speed sheets help you learn your multiplication facts?" A variety of affirming comments were made, such as

> They help you by making you go fast (William)
> Cause they make me do them fast and I remember them. I find the ones I don't know (Lucy)
> Because you have to time yourself and every time you get better and better (Madison)
> Keep trying and you remember them quicker and quicker (Kyle)
> If you do them lots you remember (Fleur)
> Cause you want to get a good time, so you say I need to get this done … I practise my 'croc' speed sheets because I love them (Molly)
> Because it is a challenge – makes you get them in your head (Finn).

In their diaries many of children commented on their speed improvement, and how they would practise. An example directly quoted is "Dear Crock, When I got timed in maths yesterday I got 39 and today I got 20 because I'm PRACTISING. I think the games are fun because you learn maths Love Molly." Lucy wrote in her diary

> Dear Mira I have played my games that big Bear has made for me. I have improved my 2x and 5x. I've got fast on the speed test. I've realised too it's (ty) for ten because if it was fifty its got ty on the end. fif(ty) I have helped a researcher by making them

> understand what ty is. 5x goes odd even odd even odd even. It's a cool way of doing 5x Mrs Morrison.

Children's views about which activities/resources helped them memorise their multiplication facts changed over time. Initially there was a strong preference for games but as the study progressed the children chose tasks with a speed element as they "Make me go faster – I just make myself" (Simon). Appendix 18 indicates how the preference moved towards memorising by repeating facts, with speed sheets and games considered less effective. The group supported Lucy when she explained that an effective way to memorise facts was to keep "saying (them) over, over and over … you know they are there when you say them quick". Group discussions, however, continued to give support to speed sheets as being of equal value.

Children enjoyed helping another researcher and often gave each other advice on memorisation. During lesson 8 when some children were having difficulty in instantly recalling their 5's, Molly identified that prolonged use of a strategy was not helpful to memorisation when she informed her fellow co-researchers that "you just have to decide no skip counting and that helps you make yourself remember them… keep trying and don't worry if you get them – try again". Molly also advised that it is necessary to "make yourself get quicker". William declared, "skip counting is BAD" as it prevents facts "getting into your head" and "fingers take too long too". Another example of advice came from Otis, "you have to learn the tough answers like 6 x 8 by keeping doing it". Madison suggested it was not sensible to panic, just think of a way to work it out and then practise it. Children took seriously the advice of others and this was particularly evident when all children tried to stop using their fingers and skip counting. William told his diary that other researchers really helped him learn his multiplication facts.

At intervals throughout the study children were asked what they found hard about learning multiplication facts and which ones were the hardest to learn. All children, except Simon, considered that either working with ‘big numbers’ or remembering the facts was the hardest thing about multiplication tables. Simon thought that the hardest thing for him was “putting numbers into groups”. Initially children mentioned the multiplication facts within the 7, 8, and 9 times tables were the hardest to remember. Towards the end of the study the group supported Madison when she mentioned that when she found facts hard it was usually because she had not learnt them. William, however, admitted that he reverted to using his fingers when he had ‘a block’ and then he just kept repeating the fact. Easy facts, on each occasion asked, were considered the 0’s, 1’s, 2’s and 10’s.

The children considered how they knew they had committed a fact to memory. Ten of the children made comments such as “I think of them and then the answer pops out” (Kyle) and “It is in my head when I do it fast” (Simon). Some of the other comments were

> I feel them pop into my head (Lucy)
> Mum reads them (flash cards) and you can feel them go into your head (Simon)
> Being able to ‘answer it quick’ tells me I know them (Fleur).

Children identified the facts they had not committed to memory and were able to give reasons such as “I just haven’t learnt them” (Madison) or that they were just hard to get into their heads (Aaron). Other comments when they had improved included

> Because I’ve learnt ways to work them out - 4’s and 9’s especially (Kyle)
> Heaps because I didn’t know many, now I do (Fleur)
> Yep but hard to explain as I keep getting them in my head (Madison).

Results from analysing the data with reference to the memorisation theme indicate that the children considered that both games with a speed factor and speed sheets assisted them in committing their multiplication facts to memory. They also identified practice as a key element in memorising these facts and noted how lack of practice hindered the process. The children recognised the support that they gave each other and considered it necessary, when recalling facts, to discard strategies that prevented memorisation. The memorisation theme does overlap with the practice theme but can be considered separately, as practice deals with what kind of and how much practice children chose to do.

4.4.3 *Practice*

The third analysis theme, practice, included methods children used to practice, what they practised, the role of games/activities and the role that parents played in assisting children to learn multiplication facts.

Initially the children indicated that they had practised, but they admitted it was only for a few minutes. However, by the end of term one (14 April 2005) as they experienced success, most of the children wanted to work on committing their facts to memory. I assumed that the children had practised when they increased speed in completing speed sheets and when I asked them multiplication facts individually or when using flash cards.

The practice children did was self-motivated. They chose whether and what to practise at home. William, who did not practise over one week in June, became extremely angry with himself when he was unable to complete his speed sheet. When I asked the following day why he was so much better at his 9's, he claimed he had "practised all night". Another example of William's commitment to practice was recorded, early in the study, in his 'croc' friend. William wrote

> Dear Crocobro
> I have been better at my 5 and twos because all of the games and worksheets. I was helping another researcher today because I helped them in a game. I practised at home because I knew I should for my homework and to get 100% on my maths. My mum helped me because she would like me to be very smart when I am older and when I'm young
> Love from William

The children were asked on numerous occasions which activities they had practised, since the previous time we met, at home. Their responses could indicate that they practised on more than one occasion. Below is a summary of their responses.

Date asked	**Finn**	**Kyle**	**William**	**Otis**	**Aaron**	**Jacob**	**Simon**	**Lucy**	**Fleur**	**Madison**	**Molly**	**No. of chn**
30/3			games					games	games		games	**4**
31/3	games		games		games	speed		games		games	games speed	**7**
5/4					games		games	games		speed		**4**
6/4		games	games speed	games			games speed	games speed	games speed			**6**
12/4		speed	games speed	games speed			games speed		games	games speed		**6**
14/4		√	√	√			√			√	√	**6**
5/5		games speed	games speed	games speed	games	games	games speed	games speed	games		games	**9**
24/5		√	√	√		√	√	√	√	√	√	**9**
26/5		√	√	√			√		√	√	√	**7**
31/5			games	games	games		games	games		games	games	**7**
9/6		games	games	games speed				games		games	games	**6**
13/6			games	games speed				games		games speed	games speed	**5**
16/6		games	games	√			games	games	games speed	speed	games speed	**8**
23/6		speed		games speed			games	games	games speed	speed	speed	**7**
24/6			games speed	games						games speed	games speed	**4**
Total games	1	4	10	8	4	1	7	10	6	6	8	
Total speed		3	4	5		1	3	2	3	6	5	
Total unknown		3	3	4		1	3	1	2	3	3	
Total days practiced	1	9	13	12	4	3	10	11	8	12	12	95

Ticks indicate children were not asked what they practised. Speed = speed sheets. Games includes flash cards.

Table 4.2
Record of times that children did practice at home and the activity they used

Eight of the children indicate that they practised at least 8 of the 15 times they were questioned. Finn, Jacob and Aaron, who struggled to get past the 2's, 5's, 0's and 1's, are the three children who practised the least.

There was a wide range of evidence of practice over the five-week break (between lessons 6 and 7). When I returned Kyle was elated, as he no longer needed to use his fingers to work out the 2's and 5's. Kyle's real breakthrough came during this time when he took his multiplication fact games on holiday to Australia. His mother became involved in his learning and Kyle explained, "On the holiday I have done my speed test. I have improved because I have done my games with my mum". In contrast, some children agreed with Lucy when she said she didn't practise. "Because you didn't come I forgot to practise." Madison commented,

> I haven't learnt my homework but heaps from my teacher. I definitely know I have improved since my last holiday. I hope I keep it up, in fact I think I definitely am going to keep it up. I absolutely love this."

The impetus to practise their multiplication facts was reinstated with the games, using their crocodile drawings as templates. Fleur commented that her crocodile was helping her learn.

The children were not asked at every teaching session if someone had helped them practise. The table below indicates their responses on the occasions on which they were asked.

Date asked	**Finn**	**Kyle**	**William**	**Otis**	**Aaron**	**Jacob**	**Simon**	**Lucy**	**Fleur**	**Madison**	**Molly**
30/3			Mum Brothr Sister		*Mum* *Dad*	*Dad* *Mum* *Brothr*					Dad
31/3					Mum Dad Brothr	*Dad* *Mum* *Brothr*					
6/4			Mum	Mum				Mum Dad Group			
14/4	Dad		Mum	Mum Dad			Dad	Mum Dad		Mum Dad	
5/5		Mum									
26/5			Mum	Mum Dad			Mum Sister				Dad
9/6		Brothr	Sister	Dad				Mum Brothr		Mum	Mum
13/6			Sister	Dad Mum						Mum Dad	
16/6		Mum	Brothr	Dad		Dad					Dad
23/6		Mum		Mum			Mum	Brothr			Mum
	1	**4**	**7**	**7**	**1**	**1**	**3**	**4**	**0**	**3**	**5**

Table 4.3
Responses from children when asked
"Who helped you learn your multiplication facts?"

Aaron and Jacob did not practise, using 'crocodilian' resources, on the days written in italics. Although they both insisted they had worked with a parent they contradicted this when they were asked what they practised. Finn claimed that he practised on all occasions but he revised this idea and said he only practised once.

It was evident when parents helped their children practise. For example, during class flash card sessions and when working on speed sheets, both Kyle and Simon made rapid progress once they began to receive parental help. Kyle claimed, “If you are on a speed sheet and I am trying to do a hard one but I can’t – I tell my mum and she tells me and then I write it in. I then keep doing them”. An example of parental encouragement was expressed when Molly declared “Mum thinks I have improved as before I didn’t even know 2 x 7!” and Fleur reassured me of progress being made when reporting “My mum says that ‘croc’ group is really amazing, you know your tables better than me … and I said I love it.” After these comments each child was asked if their parents thought they were improving and seven claimed their parents had commented on their improvement. Aaron, and Finn did not receive much help from home and they are two of the children who had difficulties in moving past the 2’s and 5’s.

Jacob did receive help from home but he explained when he doesn’t know a fact he “works it out in his head with his dad”. When trying to work out a fact in class he complained regularly as he wanted someone to give him the answers like his dad does. He explained that when he is away from school (at least two days per week) he does not play games, or learn crocodilian facts but “I learn my 6x table as my dad tells me to”. Jacob further explained that he did not get help with his ‘crocodilian studies’ as his brother and sister have heaps of homework and his parents work lots.

Likewise, Fleur did not receive parental help but explained that she had help from her soft toys. Fleur claimed, in her final interview, that playing ‘teachers’ helped her learn her facts as she tested her toys and played the games with them. Fleur thought her progress to instantly recall facts was hindered by “Mum having to help Reuben with his reading”. Lucy and Madison, during discussions, stated that they did not need help as they practised by themselves. Molly told us that she and her dad practised "heaps".

At the completion of the study, although all the children admitted that they had improved, nine children suggested they could have played their games or practised more, three children commented that their parents were too busy working or helping siblings with homework to help them, time was an issue for three children and Jacob commented that he found it hard as he was away from school so often. Results from analysing the data with reference to the *practice* theme indicate that the children were able to identify resources/activities they considered most effective to practise. They recognised the benefits of activities with a speed element and that games and speed sheets assisted their learning. The role parents played in assisting children was also recognised by the children and they were able to comment on the positive effect practice had for their learning. The following case studies highlight different approaches children took to their learning.

4.5 Three case studies

Each of the eleven children constituted a case study, as each child's pathway and his or her experiences toward developing skill in memorising multiplication facts was unique. I have chosen the case studies of three children for the purpose of illustrating shifts in understanding, emerging knowledge of multiplication facts, use of strategies to solve problems and views on what helps them commit multiplication facts to memory. The three case study children were Otis, Finn and Madison, who represent the group's range of mathematical skills and the diversity of the group.

4.5.1 *Otis*

Otis is a representative case of a child who began with little interest in learning his multiplication facts but when his mother began helping him with memorisation, shifts in his attitude and learning occurred. The initial Numeracy Project diagnostic test indicated that Otis was at stage 2/3, as he counted all the objects from one to find answers to multiplication facts. He got 47 facts correct in the *100 Multiplication Facts Pre-test* but did not complete the 100 facts. He methodically answered facts in order but only answered 51 over an extended amount of time, which could indicate he was using counting strategies to find the answers. The classroom teacher reported that Otis did use counting strategies. During the *Multiplication Facts Verbal Pre-test* Otis would only admit to five facts being too hard. He insisted he persevere to get an answer, often taking minutes, and refusing to use equipment or draw a picture. He was unable to instantly recall his 2's and 5's. A confusing picture of his strategy stage and understanding of multiplication facts also occurs when comparing the Numeracy Project diagnostic test with the *Multiplication Knowledge and Strategy Interview Pre-assessment* where Otis used a combination of derived facts with counting on or back. The same determination and time span occurred during this pre-assessment, as in the other pre-assessments, again suggesting he was working at the lower stage than school records indicated.

Initially Otis showed little interest in participating in this study. He was inattentive and did not want to work with another researcher. After lesson five Otis recorded in his diary "Yesterday I got my highest score of 25 seconds because I practised a 100 times. Yesterday I did my homework for the first time because my mum didn't have to work." His homework pattern was now set and he reported practising on 12 of the 15 occasions he was asked. Evidence of his continual practice was shown by the diminishing time he took to complete a row of a speed sheet. Otis claimed practice was the only way to memorise his multiplication facts.

Otis's understanding of the strategies explored during the study and his multiplicative thinking was evident through the recordings done in pairs, the diary sheets, the final 7 x 8 worksheet and contributions to group discussions. His recordings also indicated an understanding of division as the inverse operation to multiplication. In the *100 Multiplication Facts Post-test* Otis was able to answer 99 multiplication facts correctly with 8 x 7 as the only incorrect answer. The *Multiplication Verbal Post-test* emphasised Otis's multiplicative thinking when he answered 33 of the 35 facts instantly with a short delay on the other two, 8 x 6 and 9 x 3. All answers were correct. The *Multiplication Knowledge and Strategy Interview Post-assessment* showed he derived 8 x 6 from 7 x 6 but instantly answered the other three questions.

At the completion of the study, Otis's ability to use the appropriate strategies, his knowledge of all the multiplication facts from 0 x 0 to 10 x 10 and related division facts showed he had completed the Advanced Additive/Early Multiplicative stage (stage 6) and was ready to move to the Advanced Multiplicative stage (stage 7) of the Numeracy Project. Otis considered he had reached his goal of wanting to get "multiplication sums into my head", as he "practised and practised – Mum helped".

4.5.2 *Finn*

Finn is a representative case of a child who found learning his multiplication facts difficult and was unable to receive support from home. In the *100 Multiplication Facts Pre-test* Finn was able to answer 41 of the facts correctly, all of which were in the 0's, 1's, 2's and a few 5's. In the *Multiplication Verbal Pre-test* Finn was unable to explain 'lots of' and he showed confusion between the addition and multiplication signs. He was unable to instantly recall his 2's and 5's. When asked word problems in the *Multiplication Knowledge and Strategy Interview Pre-assessment* Finn used direct counting to solve the four problems, answering one correctly. Finn was working at stage 2/3 of the Numeracy Project at the beginning of the study. The school records suggested stage 4.

After six (revision) teaching sessions Finn would do anything to stay off task. There was no evidence that Finn was practising but in his diary/diary sheets/interviews he claimed he had with his dad, and sometimes his sister. Finn made errors when skip counting, which is the strategy the Numeracy Project suggests follows direct counting (stage 4). By lesson 4, Finn's progress was hindered by not being able to instantly recall his 2's and 5's so he decided not to move on to the advanced clusters of facts but to concentrate on these multiplication facts. He decided not to time himself until he regained his confidence. Finn's speed sheet times started to decrease indicating he was making progress with the 2's and 5's from lesson five. By the final lesson, the evaluation sheet on 7 x 8, Finn began to show understanding of multiplication but he had not acquired all the strategies that had been explored. With help from other children Finn was now able to participate in group or pair discussions. Children who worked with Finn took time to explain strategies we were working on either verbally or by using materials, such as blocks.

In the *100 Multiplication Facts Post-test* Finn increased his number correct from 41 to 70. In the *Multiplication Facts Verbal Post-test* Finn was able to instantly recall his 0's. 1's', 2's and 5's and was starting to use multiplicative strategies to solve unknown facts. In the *Multiplication Knowledge and Strategy Interview Post-assessment* he was using a combination of direct counting and repeated addition to find answers. The Numeracy Project diagnostic post-assessment confirmed that Finn was at stage 5 and ready to move to the Advanced Additive/Early Multiplicative stage (stage 6) of the Project. Finn knows he has a fact in his head when he "sees it and remembers it quick".

During the final individual interview Finn admitted he had only practised once when his dad played Snakes and Chances with him. He went on to say "We don't have any time… I can't find the games – may have been thrown out (mum). Dad listens to my wee brother read at night and I have other homework to do." Finn did not think he had reached his goal "because I have not got many into my head" and wished he had practised.

4.5.3 *Madison*

Madison is a representative case of a child who was self-motivated to practise and subsequently memorised all her multiplication facts. In the *100 Multiplication Facts Pre-test* Madison gave correct answers to 60 facts. There was not a pattern to the multiplication facts she knew. Madison's completion of stage 5 status was challenged in the *Multiplication Verbal Pre-test* as she was unable to instantly recall the 2's and 5's. Madison took an extended amount of time to work out the word problems of the *Multiplication Knowledge and Strategy Interview Pre-assessment.* She was determined to work out the answers without using counters or using a pencil. She knew 3 x 4 and she used facts she knew and then counting on for the other three facts. One answer was incorrect. The Numeracy Project Diagnostic test confirmed the school records that noted Madison was working within stage 5 of the Numeracy Project.

Although always conscientiously on task in class, Madison did not initially show any interest in practising her multiplication facts, as "I do my other homework". By revision lesson 5, Madison practised as she became more proficient at completing her speed sheets. Madison commented that she felt timing herself helped her increase her speed in recalling her multiplication facts. She indicated her understanding of strategies and multiplicative thinking through the pairs sheets, group discussions, the diary sheets, the final 7 x 8 worksheets. Madison's recordings showed she had an understanding that division is the inverse operation to multiplication.

Madison was the only child to complete the *100 Multiplication Facts Post-test* in less than five minutes. Her time indicated that she instantly recalled all her facts in less than three seconds each. Madison answered 97 of these facts correctly, faltering on 8 x 3, 6 x 4, and 6 x 7 although the commutative facts were correct. The *Multiplication Facts Verbal Post-test* indicated that she was not recalling all facts instantly as, of the 35 facts asked, she derived the answers to 8 x 3, 6 x 4, 8 x 6 and 8 x 4. Other than 8 x 4 the incorrect answers are the same as in the other two tests. The *Multiplication Knowledge and Strategy Interview Post-assessment* also challenged her ability to instantly recall all multiplication facts. Of the four facts asked, Madison answered two instantly, hesitated on 8 x 6 and derived the answer to 6 x 4, a fact she had answered incorrectly previously. Madison knows when facts are in her head because they just “pop in”.

Madison can be said to have completed the Advanced Additive/Early Multiplicative stage (stage 6) of the Numeracy Project and she is ready to move to the Advanced Multiplicative stage (stage 7) as she has the required multiplicative knowledge and strategies. Madison felt she reached her goal of ‘learning how to do my multiplication and division by all the teaching and my practice”.

I have included these three case studies because, regardless of the stage at which each child began this study and his or her learning pathway, the data highlights that they all made shifts in their understanding of multiplication and in the number of instantly recalled multiplication facts. Although Otis and Finn started at stage 2/3 and Madison at stage 5 of the Numeracy Project both Otis and Madison reached the end of stage 6, indicating they knew their multiplication facts. In contrast, Finn reached stage 5 and he could instantly recall his 2’s, 5’s, 0’s and 1’s. The data also confirms that all three of these children view practice as the main ingredient to instantly recalling multiplication facts and the motivation to practise occurs once success is experienced. The data also indicates that practice did make a difference for these three children. Otis received parental help, Madison was self-motivated to practise while Finn began to progress once he started to practise his 2’s and 5’s.

4.6 Post-teaching assessments

The results of the post-assessments are now reported and compared with the pre-assessment data. The *100 Multiplication Facts Post-test* results (appendix 11) indicate that nine children either knew or were able to derive at least 94% of the 100 facts they were asked. Aaron was able to answer 89% while Finn answered 70% correctly. Results from the *Multiplication Facts Verbal Post-test* (appendix 12) indicate all children progressed towards multiplicative thinking. In the post-test 93.53% facts were solved using a multiplicative strategy, 5.7% of the answers were incorrect or considered 'too hard', while less than one percent of the answers were solved using additive strategies. Finn indicated difficulty with some facts but he was able to instantly recall all the facts related to the 0's, 1's, 2's and 5's. Otis knew all the asked facts and recalled thirty-three instantly with a short delay on two facts. There was no evidence of Jacob using strategies in the post-test. Aaron used multiplicative strategies but he was still making mistakes. Both William and Molly used skip counting for one of their facts.

In part A of the *Multiplication Knowledge and Strategy Interview Post-assessment* (appendix 13), children used multiplicative strategies 91% of the time. Using these multiplicative strategies, 30% of the answers were derived, 2% known and 59% were instantly recalled. There were three recorded incorrect answers, Finn (2) and Fleur (1), and two of these incorrect answers were from question three (8 x 6). Ten children were using multiplicative strategies: Finn was the only child who continued to use direct counting. Jacob and Molly were the two children who answered all four questions instantly.

In Part B (Numeracy Project Diagnostic Interview) of this post-assessment (appendix 13), all children except Jacob and Finn, were working at stage 6 (Advanced Additive/Early Multiplicative). Stage 6 is one stage above the stage expected of a Year Four child working on the Numeracy Project. Finn showed the skills of stage 5 whereas Jacob is considered at stage 4 because he used skip counting to obtain an answer.

4.6.1 *Comparison of pre- and post-assessments*

All assessments indicate growth in knowledge and/or strategies on the part of all the children. Appendix 11 shows that the number of incorrect answers is reduced from that found in the pre-test. When comparing the results of the *100 Multiplication Facts Pre- and Post-tests* all children showed a percentage gain in the number of facts they could correctly answer. Jacob made the greatest percentage gain, from 22% to 94% correct. The majority of incorrect answers were those within the 7x, 8x and 9x tables.

Comparing the *Multiplication Facts Verbal Pre- and Post-tests* indicates the children's movement from using additive to using multiplicative thinking to find answers to multiplication facts (appendix 12). In the pre-test 38% of the answers given were considered 'too hard' or were incorrect but this was reduced to 5.7% in the post-test. 20% of multiplication facts were solved using additive strategies, with a reduction to less than one percent in the post-test. Multiplicative strategies were used 42% of the time in the pre-test and 93.53% of the time in the post-test. All children showed an increase in being able to instantly recall multiplication facts. Finn, Aaron and Jacob were the only children with incorrect answers or who found some facts too hard. Finn, William and Molly each used an additive strategy on one occasion while all other answers used a multiplicative strategy.

Part A of the Multiplication Knowledge and Strategy Interview also showed the children's movement from additive thinking to multiplicative thinking by the increase from 18% of the answers considered multiplicative in the pre-test to 91% in the post-test (appendix 13). The following table 4.3 indicates stages children were recorded at both at the beginning and the end of the study.

	Numeracy Project strategy stage - school records March 2006	Numeracy Project strategy stage pre-test March 2005	Numeracy Project strategy stage post-test June 2005
Lucy	5	5	6
Kyle	5	4	6
Otis	5	2/3	6
Molly	5	4	6
Madison	5	5	6
William	5	4	6
Simon	5	4	6
Fleur	4	4	6
Aaron	4	4	6
Jacob	4	4	4
Finn	4	2/3	5

Table 4.4
Results of *Multiplication Knowledge and Strategy Interview* Part B:
Numeracy Project strategy stages in school records and at pre- and post-assessments

In Part B of the Multiplication Knowledge and Strategy Interview, using the pre-assessment results of the Numeracy Project Diagnostic Assessment strategy stage as a benchmark, results show one child did not increase his strategy stage, two children made an improvement of one stage, seven children showed a two stage improvement, and one child a three stage improvement. These results indicate that nine of the children are working above the expected level for Year 4 children (stage 5), one child is at the expected stage and one below (stage 4).

The post-assessments found that all children except Jacob were using multiplicative strategies. There is no evidence in any of the post-tests that Jacob was using strategies to find answers to multiplication facts. All the answers he gave were either known/instantly recalled facts or he gave incorrect answers to the facts. Jacob missed seven of the teaching sessions and told me his dad makes him learn his tables one at a time.

There were some discrepancies in data from the three assessments. In the Numeracy Project Diagnostic Pre-assessment Otis indicated that he was unable to use additive or multiplicative strategies but in two other tests he indicated that he could derive some facts if given an extended amount of time. Although Jacob showed improvement in his ability to instantly recall facts, when tested on the Numeracy Project Assessment, his results indicated his strategy stage was stage 4 (skip counting). In other assessments there was no indication that he used this strategy to solve any problems. Finn used direct counting (additive thinking) to solve problems in the Multiplication Knowledge and Strategy Interview Post-assessment but showed multiplicative thinking in the Multiplication Facts Verbal Post-test. Although Madison was able to answer all her 100 Multiplication Facts Post-test in less than five minutes, the Multiplication Facts Verbal Post-Test indicates that she is still using strategies for some of her facts.

Results from the pre- and post-assessments indicate that all children increased their ability to recall multiplication facts and increase their understanding of multiplication. All children, except one, showed that they increased their knowledge of multiplication strategies. Although children made shifts in these areas they also made a shift in their confidence in their mathematical ability.

4.7 Children's increased confidence

There was evidence that children's confidence in their ability to achieve at maths grew during this study. Kyle is an example. The classroom teacher explained that Kyle became part of the group "as a treat". He was not expected to achieve. Kyle's dependence on constantly asking for clarification of tasks and for reassurance diminished after a few lessons. At the end of the study he was able to explain strategies with confidence, including explaining the distributive strategy to the assistant principal in detail, and he reported his achievements by saying "yep – I remember most of them" (the 100 facts). Children's increase in confidence was also shown when they made reference to their 'successes' during discussions and in the fact that they were able to design their on-going learning tasks.

4.8 Chapter summary

This chapter set out to report on the data, collected and analysed, of how eleven children think they commit their multiplication facts to memory while developing their multiplicative thinking. Rather than reporting the study in chronological order, the results have been analysed according to three themes to produce a cohesive picture of the study. The progression of learning theme reported on the teaching and learning sequence, children's development of calculation strategies and multiplicative thinking. The memorisation theme examined how children consider they commit their multiplication facts to memory and the resources and activities that assist them. The practice theme reported on the ways children practised, the resources they used and the people who helped them. Three case studies set out to record the different pathways children took in their learning. Pre- and post-assessments were reported and compared. Key findings based on these results will be discussed in the following chapter.

Chapter 5: Discussion

5.1 Introduction

This chapter discusses the results and key findings of the study and the manner in which these findings relate to the literature. My role as a researcher will be critiqued. How the context of 'Crocodilian Studies' was conducive to learning will then be discussed. The three analysis themes will be examined in the light of the results and discussed in turn: the progression of learning; memorising multiplication facts and the impact of children's practice; and the roles both parents and peers play in the learning process. Finally, my role as the teacher and its impact upon the study will be discussed.

5.2 The role of the researcher

In this section I will discuss my roles as both teacher and researcher, the decisions made during the study and the approaches taken to dilemmas that occurred. There were some tensions related to my role as teacher/researcher, the collection and analysis of data, and the procedures and content of teaching sessions. Firstly, at times the roles of teacher and researcher presented conflicting purposes. As the researcher I was aware of the importance of collecting data yet as the teacher I wanted to be responsive to the children's individual needs and allow these to direct the teaching. An example of such tension arose when, at late notice, the time available for lessons was shortened to fit in with the classroom programme. I chose to change the order of the planned lessons while, as researcher, I wanted to carry out the study as planned. The change in lesson sequence meant that the 9's were taught later than planned. This may have influenced the number of multiplication facts children committed to memory, as the strategies for the 9's can be considered 'easier'. In addition, I often had to refocus myself as the researcher because, while engrossed with a child, I could easily overlook or miss opportunities to record children's thoughts about their learning.

Although, I took every care to report on the data in an accurate way there are limitations to the data I have collected. It was not possible to record every statement a child made and rich discussions or events may have occurred that I did not observe or which were not recorded on video. When the children were working in pairs, there were multiple and simultaneous conversations related to their learning. Unless I overheard part of these conversations, and encouraged the children to discuss their findings with the group, the opportunity to record their thoughts was lost. Although I attempted to record the children's ideas verbatim, from the child uttering a statement to the recording of it, inaccuracies may have occurred. I ensured that I participated in interviews and conversations with a clear mind to prevent my personal experiences and biases taking control. However, although every care was taken, I accept that my interpretation of conversations or observations I made, may not reflect the exact meaning a child was intending. An example of this is when I asked the children whether games help with their learning, Lucy claimed that as games were fun she did not realise she was learning. I assumed she meant that the games helped her memorise her multiplication facts but I did not check this meaning with her.

Some of the richness of the data collected was compromised when the children's personalised choice of recordings were eliminated (the change from diary writing to researcher-generated questions on diary sheets and then notes during individual interviews). Children recorded in their diaries thoughts such as how they were feeling about the progress they were making or the difficulties they were experiencing with practising at home. By introducing diary sheets and then interviewing the children, I gave them less choice about what they recorded. Perhaps my asking the children early on if they had practised their multiplication facts influenced their belief, too soon, that practice was important. As practising multiplication facts was not part of the children's homework schedule, I considered it important to monitor those children who were choosing to work at home. If I had not highly praised those who practised, would practice have become such a prominent part of the children's learning?

The quotes and examples that I have used in the study are those that I considered the most appropriate to reflect the data. Some children's ideas have been referenced more than those of other children. I have used children's comments to express the importance they put on their ideas at a particular time in the teaching sequence. Some ideas became less important over time and I have included the quotes and examples that, in my view, best conveyed these changes. For example, Molly directed her fellow researchers away from using fingers to skip count, which reflected the current strategy stage the children were working at.

I consider that there were some aspects of my research design that may have compromised my role as the teacher. I could not set up systems to influence learning as a classroom teacher can. For example, I was unable to provide reinforcing activities for children during class time or include work on multiplication facts as part of their homework schedule. Originally a sentence stating I would provide these activities was included in the classroom teacher's ethical approval letter but I removed it, as I believed this would happen naturally. I now question my action because multiplication fact activities might have been incorporated in the class mathematics programme and become part of their homework requirements, but this did not occur.

I also consider that restrictions were imposed on me as the teacher, because I was not the classroom teacher. If I had administered the initial testing of the *100 Multiplication Facts Test* I would have been able to observe how children were working out the answers. For example, I might have observed whether Otis was actually using counting strategies to obtain answers. For the three children who were not working on their facts at home I was not, as researcher, able to speak to their parents. Access to their parents might have altered the help the children were given and influenced the number of multiplication facts they committed to memory.

5.3 Context for Learning: 'Crocodilian Studies'

A key finding of this study was the significant effect the context of 'Crocodilian Studies' had on the children's learning. The context was created to capture interest in learning multiplication facts and to infuse an element of fun into a topic often considered 'boring'. The children chose the context of crocodiles after they observed a crocodile on their first assessment sheet. Immediately the children indicated their support for 'Crocodilian Studies' and they decided that their 'research' folders snapped like crocodiles when fed 'data'. Another example of the context filtering into all aspects of the unit of work was when the children asked that green skin (paper) become the colour we used whenever possible. The shared communal journal was given the name of the 'Big Green Croc'.

'Crocodilian Studies' was the base from which the cohesiveness of this group was developed. Crocodiles provided a sense of belonging to a purposeful group that related to their imaginative world. The children demonstrated their collective purpose when they became 'crocs' and named me 'Mrs Croc', treating me as a group member of equal standing in my role as a co-researcher.

They also requested that crocodile elements be included in the activities and resources. The children insisted the logo of crocodiles be added to all of their resources and eventually their drawings of crocodiles were used as templates for the resources. Their crocodiles were used on the 'croc' diary sheets, with speech bubbles, so that their crocodiles appeared to be eliciting the 'data' from the children. Children also turned their diaries into 'croc' diaries who became named friends and delighted in being fed lots of interesting words and ideas.

The context of 'Crocodilian Studies' was a successful innovation that provided a stimulating, motivating and fun dimension. The way the children responded to 'Crocodilian Studies' supports the view of Cobb et al. (1991) who advise that an important part of a learning environment is the emotional tone. These children relaxed into an environment where they were enthusiastic, imaginative and persistent and they thoroughly enjoyed solving personally challenging crocodilian problems. This study supports the view of Anghileri (2000) that a powerful tool for making sense of a child's world, when teaching mathematics, is the use of a realistic context. Although 'Crocodilian Studies' cannot be defined as realistic it does fit into the context of a child's imaginative world.

5.4 The progression of learning

This section discusses key components of the teaching sequence that are significant for children's learning; the sequencing and clustering of multiplication facts, the children's discovery of strategies, and the children's strategy development and multiplicative understanding.

5.4.1 *Lesson sequence*

The sequence in which multiplication facts were taught supported new learning to occur in manageable clusters. By learning the 2's and the 5's first, and then establishing the properties of the 0's and 1's, the children were then able to use these multiplication facts to discover emerging strategies with understanding. The need to commit these facts to memory was verified when Aaron, Finn and Jacob were not able to instantly recall their 2's and 5's to develop strategies such as using the distributive property. The three children chose to concentrate on these facts before moving on to more difficult clusters. Madison and Lucy showed their understanding of the 2's and 5's when they worked out the answer to 6 x 7 by using (6 x 5) + (6 x 2). Clustering the number of facts presented manageable numbers of facts for children to learn at one time.

This sequence of teaching the multiplication facts is aligned to that of Frobisher *et al.* (1999), Thornton *et al.* (1983) and Zevenbergen *et al.* (2004) who advocate the teaching of the 2's first as this table is easier because children can relate it to addition with doubles and skip counting. An example of children relating to the 2's strategies is shown in the transcript of Kyle and William in lesson one (appendix 16 and chapter 4.4.1). By learning the remaining facts in clusters the children avoided unnecessary practice and the time needed to commit their facts to memory was shortened. For example, William was able to memorise his 9's overnight (6 facts) when he realised his friends had learnt theirs. In planning, I considered the warning of Frobisher *et al.*, that by teaching the 'higher' facts last, the 'easier' facts receive more practice, but overcame this by adopting the advice of Thornton *et al.* to work on only six to eight facts at a time.

A significant finding was that children reached a similar stage of multiplicative thinking even when starting with varying prior knowledge. For example, although Otis was at stage 2/3 and Madison at stage 5 of the Numeracy Project initially, both children reached stage 6 at the end of the study (chapter 4.5.4). Finn, who did not commit his 2's and 5's to memory at the same time as the majority of the group, asked to work alongside the group and just to practise his 2's and 5's with Jacob and Aaron. Finn knew his 2's and 5's at the end of the study and he also was able to derive 7 x 8 from 7 x 9. It was clear from the results that regardless of the child's initial knowledge and understanding, all children who followed the learning process progressed to a similar achievement level. They moved out of a prescriptive process by following an individualised learning progression that brought each of them to a similar stage. This finding has significance because it may mean that current practices, which require a prescriptive progression, may need to be re-examined. This will be further discussed in Chapter 6.

5.4.2 *Strategy development and multiplicative understanding*

The children 'discovering' strategies were also a significant process. Making 'discoveries' was one of the criteria, selected by the children, for a co-researcher. The children considered this a prestigious event and during lessons discovered all the strategies considered, by the Numeracy Project, appropriate to develop an understanding of multiplication facts. As children publicly shared their discoveries with the group, other children built on their current knowledge. When Kyle discovered the commutative property, his co-researchers all showed their approval of this strategy by checking it with materials, such as blocks, and by recording the event in their 'croc' diaries. On occasions, when a child was sharing a strategy, the reactions of other children varied because not all children accepted a new strategy at the same time. When Lucy presented her discovery of deriving facts to the group, very early in the study, some children used this idea immediately but most listened and did not utilise it until later. When Lucy shared her discovery of doubling the 2's to work out the 4's, William mentioned the doubling and halving strategy (without naming it) but the children did not use this until later lessons. The children did not replace one strategy with another immediately but oscillated between them (Seigler, 2000) and they did not use strategies they did not understand (Dengate, 1998).

Discovering strategies allowed new learning to emerge and was more accepted by the children than when I taught a strategy. For example, when I used a Numeracy Project lesson to teach the distributive property, William used the doubling strategy and claimed he preferred to do it his way (chapter 4.4.1). William's action supports the view of Baker and Baker (1991) and Muthukrishna and Borokowskil (1995) that imposing principles and strategies on children will prevent them from effectively using multiplication facts. This study also supports the view of Dengate (1998) that children who are presented with open-ended tasks are able to build on their current knowledge to develop their understanding of multiplicative thinking. When children were solving a problem, they were all able to obtain an answer by using strategies they had discovered at their current level of understanding. Where Finn would use skip counting to find an answer, Lucy would derive the answer by using her known facts (Chapter 4.4.1). Although the Numeracy Project (Ministry of Education, 2005d) and writers such as Thornton *et al.* (1983) advocate the direct teaching of principles and strategies, results of this study support the view of Baker and Baker (1991) and Muthukrishna and Borokowskil (1995) that children need to discover these strategies for themselves rather than be taught them.

The children found the discovery of patterns in the multiplication tables was significant and they considered this an exciting and motivating challenge. Simon explained that you have to think in groups when you are doing multiplication, but the patterns in each of the 2's, 5's and 9's aroused the most enthusiasm. The discovery of a pattern, when the children were working in pairs, was greeted with a squeal of delight and eight of the children wrote in their diaries about the patterns they found in the 5's (example William, chapter 4.4.1). Fleur expressed her delight in the patterns in the 9's and announced to the group that she thinks the 9's are cool as "if it was 4 x 9 = 36 then add 3 + 6 = 9" because the digits of the answers add up to 9. This finding supports the view of Frobisher et al. (1999) that discovering patterns in multiplication tables can be an enjoyable experience for children as it gives them techniques to quickly derive answers to multiplication tables such as the 9's. Similarly, when children discovered patterns in the 5's, they realised the answer to any 5's fact will end in zero or 5 (Butterworth et al., 1999).

Another significant finding was that the children who developed a bank of strategies to work out answers to facts they did not know, gained an understanding of multiplicative thinking. I considered that such an understanding was indicated by the children's explanations when they used materials, their verbal explanations and written recording. Jacob, who was regularly absent from school, could only give an answer to a fact if he had learnt the related multiplication table with his father. I considered this to be rote learning because he was unable to explain or use strategies to work out any multiplication facts he did not know. This finding is consistent with the view of Burns (1994) who warns that mediocrity is ensured if facts are learnt in a rote and unthinking manner and the view of Heege (1985) who adds that mastery of individual facts is hindered if children can only recite multiplication tables. Jacob's view of multiplication contrasts with children who were able to use multiplication facts to derive facts in more complex mathematical situations (Anghileri, 2001a). The results of my study indicate that all the children, except Jacob, developed understanding of multiplication facts. I considered multiplication conceptualisation was complete when the children explained their answers by a physical model, verbally or with written symbols (Thornton et al., 1983).

5.5 Memorisation and practice

The way in which children considered they commit their multiplication facts to memory is explored within this section. The analysis themes of *memorisation* and *practice* are discussed.

5.5.1 *Memorisation*

Findings on memorisation were drawn from data categories of children's explanations and included children's explanations of how they think they commit multiplication facts to memory, the role of games and speed sheets, the choices children made in their learning, how they gave advice on memorisation, difficulties they experienced in learning facts, how they recognised they knew a multiplication fact, and why they could not recall some facts.

A key finding was that the children in this study found that it was possible to know when a fact was memorised and, consequently, they could identify the ones that still needed to be learnt. Fleur explained she knew she had memorised a fact when she was able to answer facts quickly while Lucy could feel the facts pop into her head. The children also gave reasons why they had not memorised a fact. Madison explained that it was only because she had not learnt it and when identifying hard facts Simon claimed that facts are only hard until you learn them. Lucy also explained that when she made mistakes on speed sheets she then knew which facts she had to practise. Therefore, it is important that the children become aware of their learning and do not spend time working on facts that they have already committed to memory (Taylor, 1976).

Another significant finding was that when the children had difficulty recalling a fact, they found they had ways to assist themselves. Initially children would, like Molly, suggest you use your fingers, do a speed sheet, or play a game but by the end of the study children suggested that a strategy such as saying facts until you remember is more efficient. At the end of the study the children were suggesting that if a friend doesn't know a fact you would show them how to work it out (derive). The children who recalled multiplication facts instantly were found to use these facts to solve more difficult problems (7 x 80), as they did not have to work out the multiplication fact first. This supports the view of Frobisher (1999) who explains that knowledge of place value and the associative property allows children to derive answers based on multiplying by the power of ten. By explaining derived facts to others illustrates that children who understand how tables 'work' can use known facts to find answers to more difficult facts (derived facts) and use these facts in more complex mathematical situations (Anghileri, 2000).

Some significant findings related to how children memorised facts are discussed further in the '*role of peers'* section. The selection of resources children chose to help them memorise facts overlaps with the practice the children did using these resources, so these resources are discussed within the *practice* theme.

5.5.2 *Practice*

The ways in which practice assisted children to commit multiplication facts to memory is explored in this section. Similarly, the ways that parents contributed to their child's learning is examined. In this study, *practice* included the methods children used to practise, what they practised, the role of resources/activities/games in practice, and the role of parents and other people.

A key finding was that children recognised what they needed to practise, and were able to select the activities/resources that most assisted them in memorising. Individual children made choices about what they practised and would request, for example, extra speed sheets or a game to take home. Initially the children played their games at home, especially if they included templates of their crocodile drawings. Lucy explained that they were so much fun she did not realise she was learning while Molly thought that games were fun because you learn maths. William considered he got faster at recalling multiplication facts as he remembered more after he had been playing his games. It is clear from these findings that children are able to recognise what they need to practise, and be able to select the activities/resources that most assist them in memorising. The comments and achievements of the children related to practice activities that they enjoyed support the claim of Nuthall (2000) that meaningful activities enhance children’s ability to recall and deduce what they are learning through experiences.

As the study progressed, the children recognised that the activities with a speed element most aided their memorisation. The children’s choice to introduce speed sheets provided them with relatively quick feedback and identified their successes. The practice was private to each individual child and their achievements motivated more practice. Molly practised speed sheets because she “loved” them and she wanted to get a good time, while Kyle kept trying so he could remember them more quickly. Children also chose to introduce a timer to monitor their improvement on their speed sheets, but they made individual choices about when they would use the timer. An example of this flexibility was that on days when Finn did not feel confident of having learnt his current cluster of facts, he chose not to use a timer. The value the children placed upon speed sheets contrasts with Kamii and Anderson (2003) who considered that initially children are motivated to learn multiplication facts by playing non-competitive games and once a speed element is included then games are more effective than speed sheets.

The children considered practice essential for 'getting facts into their heads'. At the conclusion of the study, insufficient practice was the reason given by all children who considered they had not reached their goal to instantly recall their multiplication facts. For example, William reported that he was so angry with himself when he could not do his speed sheet that he "practised all night" to achieve much improved results the next day. Each child's approach to practise was unique as they chose activities based on their perceived need. This study agrees with Taylor (1976) that practice is undeniably valuable if children have varied experiences directed towards abstraction of mathematical principles and to reinforce conceptual experiences. Practice does not need to be prolonged, but only used until efficiency is maintained. Although practice will not bring about understanding, it can play a constructive part, once understood, in advancing children committing their multiplication facts to memory (Hiebert & Carpenter, 1992; Kamii & Anderson, 2003; Kouba & Franklin, 1993; Taylor, 1976; Thornton *et al.*, 1983).

The role of the parents was significant. It was apparent almost immediately that parental involvement had a significant impact on their children's learning of multiplication facts. Kyle explained he was not able to practise with his mother until he went on holiday to Australia. He took his flash cards so his mother would play with him, indicating a wish to keep up his learning. When Kyle returned to a new term he was the group's fastest when we used flash cards. Fleur reported that her mother told her that 'croc' studies were amazing and that she was better at her tables than her. Fleur was unable to receive help from home but she overcame this by playing teachers so she could test her soft toys and play games with them. It is interesting to note that the three children, who did not progress as quickly as the other children, were not assisted to practise the games or use the speed sheets at home. The support and encouragement given by parents also instilled the desire, in the children, to want to practise their multiplication facts. This finding supports the research of Wilson and Robinson (1997) who found that help from home assisted children to build their self-esteem and confidence, enhance their class work and their attitudes to mathematics.

5.6 Role of peers

This section discusses the findings related to the collaborative ways in which children worked as co-researchers, and how they set their learning direction through a group goal and individual goals. The approach to learning is discussed and includes collaborative decision-making, listening, questioning and sharing ideas, children's journal recordings and how they evaluated and planned for future learning.

5.6.1 *Children as co-researchers*

A key element in building an atmosphere of collegiality was introduced when the children decided, during our first meeting, to participate in the study as co-researchers. The shared co-researcher guidelines were the need to share ideas, to work together, and to write about 'discoveries' (chapter 4.3.2). The co-researcher guidelines affirmed the value children placed on each other's views by showing pride in others' discoveries and accepting ideas and calculation solutions as suggestions they could emulate. For example, when William advised that skip counting was bad if you were going to put facts into your head, the other children listened intently and immediately tried to adhere to his advice. During the study the children referred back to these co-researcher guidelines to refocus each other. They would remind each other to collect data so they could remember how they memorised another multiplication fact.

5.6.2 *Group and individual goal setting*

By the setting of both personal goals and a group goal, maintaining these goals and then evaluating them, gave purpose to learning and allowed children to direct their own learning. William wrote that his goal was “to learn how I get my multiplication and how others get their X tables”. When the children discussed their personal goals they found that there was one main goal so the establishment of a group goal laid the foundation for the children to work collaboratively. Lucy reflected the group goal when she explained the aim was to “get tables into our heads and keep them there”. The group goal was kept alive, throughout the study, by a chart displaying the 100 multiplication facts to be learned. The ‘100 Crocofacts Chart’, as suggested by Kennedy (1970), provided a public record of the multiplication facts explored. The ritual of adding each explored fact to the chart became an important part of each lesson. Adding facts in clusters signalled that to learn 100 facts was not so daunting. In the shared communal journal, a countdown from 36 facts was recorded and the children kept a personal tally on their ‘croc’ diary sheets. Children were able to monitor their progress and expressed their pride as more facts were added to their known facts. At the end of the study those children who considered they had not reached their goal were able to explain why. Fleur explained she knew most of her facts but she could not recall them instantly. She believed she would have reached her goal if she had played her games and practised her facts more often.

A significant moment occurred when the children identified a purpose for learning multiplication facts. At the start of ‘Crocodilian Studies’ each child considered why multiplication facts would be useful in his or her life, recorded his or her view, and then discussed it with the group. Otis explained that “you might work in a shop and someone might want 5kg of bananas and you don’t know the answer”. Multiplication facts, the group agreed, would be useful in their future work, echoing the views of Wilson and Robinson (1997) who consider knowing multiplication facts to be of value in everyday life for simple tasks like handling money. The children also identified multiplication facts as being useful at secondary school, reflecting the views of Issacs and Carroll (1999) who consider multiplication facts essential for further mathematical learning.

5.6.3 ***Collaborative decision-making***

Another significant finding concerning the role of peers was the importance of collaborative decision-making. As the children were encouraged to make suggestions for changes, spontaneous discussion times often resulted in the lessons being guided in a different direction. An example of this is when the time taken to write up the diaries encroached on learning time. By collaboratively deciding to use diary sheets instead of diaries the children believed it was their idea and supported it. Another example of collaborative decision-making occurred when the children decided that speed sheets should be introduced along with a timer. The speed sheets became a favourite activity through which to practise multiplication facts. The children considered the decision to become co-researchers was theirs, and fully supported this also.

5.6.4 ***Collaborative learning: sharing discoveries, successes and advice***

One of the most powerful aids to the children's learning and memorisation was the way they shared their successes/discoveries with each other and gave advice to enrich each other's learning. Molly often called the group together to suggest ideas, such as you need to "want to do this" (memorise facts) and "stop using your fingers and say you can do it" (put facts into your head). Towards the end of the study Madison concluded that it is not sensible to panic if you don't know a fact, just think of a way to work it out and then practise it so you can 'glue' it into your head. After advice had been actioned children would feed back their successes or concerns during a group discussion, such as when Madison reported, in the initial stages of the study after skip counting had been discovered, that she knew her 2's quite well as skip counting had helped her.

Social interaction was a significant influence on the children's learning. The opportunities that were presented for children to articulate, discuss and reflect on their thinking as they shared their ideas enabled them to analyse their thinking by explaining and clarifying calculation strategies/ideas to a partner. Children re-examined their ideas during discussion and then recorded them in the shared communal journal. During discussions the children showed respect for each other by listening to a peer's explanation/idea and then questioning the presenter for clarification or to point out that they suspected a solution was incorrect. The expectation that children understood each other's ideas encouraged them to listen and to ask questions. The children developed their ideas as they delved for clarification and understanding of another's thoughts, while the presenter clarified his or her thinking through explanations. Questioning also enabled children to identify any mistakes they had made, as mistakes were seen as part of the learning process. For example, Aaron was challenged when he explained his idea of using the 2's to work out a fact in the five times table. Aaron was able to clarify his thinking and declared his idea was "silly". Listening and questioning each other also enabled the children to reflect on how they might emulate another researcher's idea. Madison commented that she liked listening to other children so she could use their ideas next time. This finding supports the views of Lampert (1990) when she writes that the construction of mathematical learning is about communicating. Children, by actively participating in classroom discussions, asking and answering questions, and sharing ideas were assisted to a deeper level of processing as they explained, justified and defended their ideas (Muthykrishna and Borkowski, 1995). The children created the role of co-researchers, discussed why it is important to learn multiplication facts, defined both individual and group goals and were involved in collaborative decision-making; their processes and self-observations support the views of social constructivists who believe opportunities need to be presented for children to learn through social interaction and that children construct their own mathematical understandings but not in isolation (Yackel *et al.*, 1990).

Another key finding was the children's recognition of the importance of the recordings in the shared communal journal. Entries were made by both the children and myself, and became a record of children's ideas and learning where each child had contributed. This shared journal was a cumulative record of lesson content and progression, group discussions, thoughts and the unpacking of ideas. Within the first half of this study the children identified this journal as one of the most important aids to their learning. Many reasons were identified (chapter 4.3.1) such as, it is like a memory card that you keep feeding, and it records other people's ideas for later reference. The children elevated this shared journal to 'friend' status by incorporating it into the context of the study and naming it the 'Big Green Croc'. The children felt proud of their contributions, as their ideas were given status by being discussed, challenged and recorded in the shared communal journal. Entries in this shared journal confirmed that the learning of multiplication facts was a class challenge and provided the children with feedback on their contributions whereby they could self-evaluate their ideas as they were challenged or accepted. The children's responses also gave me an insight into the understanding, strategy development and the conceptual growth of each child. This finding supports other studies of shared journals. Children feel proud of their own and others' ideas as they are recorded in a shared journal. Although it was time consuming to allow children to share, listen, search for and record more efficient strategies, much was gained in understanding of multiplication and division (Caliandro, 2000). The Numeracy Project (Ministry of Education 2005c) suggests that calculation strategies, as well as children's inventions and ideas are recorded in a shared communal journal to enable the teacher to keep a record of what has been studied and provide a reference for children. My study supports the benefit of also recording children's collaborative decisions, and group ideas about learning.

5.6.5 *Individual learning: self-reflection and analysis*

A further significant finding was the importance of providing rich opportunities for children to reflect on their thinking and reinforce their learning. This occurred as they wrote in their journals ('croc' diaries) and diary sheets or discussed their ideas during the diary-interviews. William and Kyle gave examples when explaining to their diary-friends about their 'discovery' of the commutative property and how it works. Feedback from me, in their diaries, in the form of replies from their 'croc' friends also supported reflection. The diaries, being friends, provided the impetus to keep up the often-tedious task of recording progress, as the children loved feeding their 'friends' so they would not be hungry (for words) and their friends could understand what they had learnt. 'Free' writing was found to be more beneficial (Neill, 2005), in contrast to the opinions of Darr and Fisher (2005) who prefer to offer a prompt for writing. This study considers journals are an under-utilised element in learning as they provide an invaluable opportunity for the writer to freely express feelings, opinions and understandings (Berg, 2004). Journals also enable children to judge their progress (Darr and Fisher) and provide opportunities to reflect, organise and remember their ideas (Neill).

Children were found to evaluate their progress and plan for their future learning. They were honest about what they were able to do and delighted in informing the group that another multiplication fact had 'popped' into their heads. They also loved to report successes in their diaries. Molly told 'Croc' how she knew she had learned more multiplication facts as she had reduced her time in doing a speed sheet, and she explained that her success resulted from practising and playing her games. The children were able to plan for future learning by being selective about the activities they would practise, make requests for resources that would enrich their current learning, such as specific games and speed sheets, and ask for timers to help them decrease the time taken in recalling a multiplication fact.

I consider the main finding about children's learning in this study can be attributed to the role of peers. The children welcomed opportunities to structure and reflect on their own mathematical thinking as well as observe and critique the thinking of their peers. Successes and discoveries were shared, and advice given to each other. By being able to articulate, discuss and feed back ideas to the group, each child had opportunities to listen, question and ask for clarification of the ideas of his or her peers. Sharing ideas also enabled them to build on their own thinking and emulate the ideas of others. The approach described above and the role I played as the teacher not only relate to theory of constructivist learning but also to the definition of self-regulated learning as described by Darr and Fisher (2005). Self-regulated learners become masters of their own learning processes by being actively involved in maximising their opportunities and abilities to learn.

5.7 The role of the teacher

The key role of the teacher contributed to shifts in children's learning. Establishing myself as a co-researcher and entering the children's world of crocodiles, as Mrs Croc, appeared to give me equal status and I was included in the excitement of the environment. I was able to keep the fun element alive with new crocodilian ideas. For example, Molly practised her crocodile speed sheets because she loved them. My releasing the control of the activities and the decisions to the children ensured their commitment to learning their multiplication facts and encouraged them to monitor their progress. For example, Lucy thought that the way we did the 5x was cool (chapter 4.4.2). As part of the group I was still able to encourage and excite the children as well as highlight important ideas as the facilitator of learning. The children's learning was recognised by the parents and after receiving encouragement from her mother Fleur assured her mother she loved 'Crocodilian Studies' (chapter 4.4.3). The success of the fun environment was affirmed when the children were asked whether they thought 'Crocodilian Studies' was yum or yuk and a

100% yum answer was consistent with the statement from the classroom teacher that the children loved attending.

The role of the teacher changed over time as the children identified and utilised their own learning pathways. Initially my input in content and process was significant as I directed the lessons and chose the activities the children would use to practise. Once the format of the lessons and discussion procedures were established, a mutual trust between the children and myself developed and the role of the teacher changed. Increasingly children interacted with each other to direct their learning by giving advice and by sharing their discoveries. Children would often ask a question of a co-researcher rather than myself.

It was important for me, as the teacher, to have a broad knowledge of how children move from additive to multiplicative thinking, to monitor that all strategies were exposed, and to direct learning. By listening to the children's solution strategies it was possible to gauge children's learning shifts and therefore, ensure that each child was building a network of ideas. I presented problems that allowed for multiple solution strategies, and children were able to present answers at their own level of thinking. The children accepted the personal challenge and believed in their ability to find the answer to any multiplication fact being explored. Evidence of the children responding to personal challenges appeared as they persevered, with enthusiasm, until they had 'discovered' a solution strategy.

My role as the teacher can be explained through the views of Mayers and Britt (1994): I became a facilitator of learning and I provided situations where the children engaged in collaborative mathematical problem solving in small groups which allowed opportunities to discuss, explain and justify their solutions. It was necessary for me to have a comprehensive knowledge of multiplication facts (Dengate, 1998), be excited about maths (Kamii & Anderson, 2003), and be able to release the control of the learning to allow the children to take on this responsibility. Children were provided with experiences to construct their own knowledge (Blais, 1988) and they developed a trust in each other and trusted their own intuitions (Mathukrishna & Borowski, 1995). Mutual trust was developed between the children and myself (Cobb *et al.*, 1991) yet I did not do anything for the children that they could do for themselves (Lockhead, 1995).

5.8 Chapter summary

This chapter has discussed the key findings of the study and how these related to the literature. Some tensions between teacher and researcher roles were discussed such as those occurring in the collection and analysis of data, and the procedures and content of teaching sessions. A significant finding was that the context of 'Crocodilian Studies' captured interest in learning multiplication facts and was a successful innovation, providing a stimulating, motivating and fun dimension. The progression of learning was examined in detail and success of this learning process was judged in the light of the results. Key findings around children's memorisation and practice were discussed. The significance of findings on memorisation and practice drawn from data categories of children's explanations was discussed in some depth. Arguably, the most significant findings reflected the role peers played, including the role of co-researcher, in how children increased their knowledge, understanding, and strategy-building techniques regarding multiplication facts. Lastly, my role as the teacher and its impact on the study was discussed. The implications of the findings are now discussed in more depth in the final chapter of this thesis.

Chapter 6: Conclusion

6.1 Introduction

In this concluding chapter I summarise the key findings of this study and discuss their relationship to the research questions. In particular, I focus on how children can effectively develop multiplicative thinking in a learning process that facilitates strategy development and understanding. I also point to what the study revealed about how children consider they commit their multiplication facts to memory and what resources aid them to do this. A section on the roles of peers focuses on the ways children can help each other increase their knowledge and understanding of multiplication facts while developing multiplicative strategies. I draw conclusions around the importance of the findings and consider the strengths and limitations of this study. Next I discuss the implications this study presents for learners, teachers, school communities and curriculum support writers. The potential for further research is acknowledged and the chapter concludes by considering the overall importance of this study.

6.2 The development of multiplicative thinking in a learning process

The first question was

How can children effectively develop multiplicative thinking and understanding?

The findings of this study suggest that children can effectively develop multiplicative thinking in a learning process that facilitates strategy development and understanding of multiplication facts, provided that: opportunities are presented for them to take ownership of their own learning; they are presented with tasks that encourage them to explore multiplication facts so they consider they 'discover' strategies rather than are taught them; multiplication facts are taught in sequenced clusters; and the learning takes place within a peer learning community.

This study illustrates how children can commit their multiplication facts to memory in a relatively short time, regardless of their initial knowledge. A significant finding was that by memorising the 2's, 5's, 1's and 0's first, children can use these to support memorisation of the remaining 36 facts easier. These 36 facts can then be learned in up to six lessons if they are exposed in small clusters and the teaching incorporates the understanding of multiplicative thinking and strategy development within a peer learning community. Using this approach, many children at different strategy stages can progress to the stage where they can recall, or derive, all their multiplication facts and use corresponding strategies.

This study's findings suggest that children's strategy development and multiplicative thinking is enhanced when they are given a focus question to explore which leads them to 'discover' strategies for themselves. As these children explored each focus question and shared ideas, their learning of multiplication facts developed simultaneously with their multiplicative thinking and strategy development. As the children's calculation strategies became meaningful to them and the learning community valued their ideas, they were able to take ownership of their learning.

6.3 Memorisation and practice

The second research question was

How do children demonstrate they commit their multiplication facts to memory and what aids them as they develop understanding of multiplicative thinking?

Each child in this study had a unique way to explain how he or she knew a fact was memorised. Once the children were able to recall facts instantly, they recognised that these facts no longer required focused practice and they could therefore concentrate their attention on unknown facts. When the children had difficulties in recalling a fact, they selected a strategy to assist them find an answer. By using equipment or giving a verbal explanation of their choice of strategy, they demonstrated their developing multiplicative thinking. The one child who rote learnt his tables with his father's help was not able to demonstrate multiplicative thinking or use a strategy to find an answer. This could suggest that he had less understanding of multiplication compared with the other children.

The overwhelming consensus amongst the children in this study was that practice of multiplication facts was essential for memorisation. Individual children made choices about when, how and what they would practise. Practice was self-motivated and carried out occasionally when we met as a group but mainly at home. Success in memorisation of a few facts prompted further dedication to practice. The study showed the children had ideas about resources and activities that would best aid their current learning needs. They justified their selection of activities and resources and requested appropriate additional activities to fill the gaps they identified in their learning. The children valued multiplication games, especially those based around their drawings, but as time progressed they chose games and activities that would most increase their speed of recall. Their request for speed sheets with timers proved a very successful learning strategy for speedy recall. Because the children chose their own practice activities from the wide variety given, they had further ownership of their learning. The children identified the invaluable resource of parental support in both assisting with practising facts and encouraging their learning

To summarise, the children in this study were able to direct their own learning to memorise their multiplication facts and develop their multiplicative thinking. They could explain how they knew a multiplication fact was memorised and they could identify the facts they had not learnt. They built a bank of calculation strategies to find answers to unknown facts. The children identified practice as a key factor for memorisation and they could make choices about how and what they practised. They also recognised the importance of parental help and encouragement.

The significance of memorisation and practice will be expanded upon when implications are discussed.

6.4 The role of peers

The third research question was

How do children help each other increase their knowledge, understanding, and strategy-building techniques for multiplication facts?

While individualised learning played an important role in successful development of multiplicative thinking, memorisation and practice, this study found that shared learning experiences had by far the greatest impact.

This study demonstrated how the children built an atmosphere of collegiality to enrich each other's learning. Guidelines for their role as co-researchers gave them licence to help each other and share their ideas, successes and difficulties. They valued each other's views as they questioned, advised and directed each other, to construct their concepts and understandings. By working collaboratively to set common goals and identify purposes for learning multiplication facts they directed their learning process and a new synergy was created. Because the children made decisions collaboratively they faced and overcame problems and dilemmas that arose together. As they supported each other, they gained the confidence to take ownership of their learning pathways.

The role of peers, in this study, emphasised the importance of shared learning and how this was significantly influenced by the particular social interaction of the group. Within their learning community the children supported each other as they worked in pairs/groups to share ideas and to 'discover' multiplicative strategies to find solutions to problems. Their mutual support provided an atmosphere where they confidently articulated, discussed and reflected on their contributions. By questioning each other's solution strategy or asking for clarification of ideas they helped each other to analyse, elucidate and refine their own thinking and the thinking of others.

The children in this study identified the use of the shared communal journal as a cumulative record of mathematical thinking and decisions as a significant aid to support each other's learning. By enthusiastically acknowledging each other's ideas and contributions to the journal, the children generated pride and gave status to peers' ideas. The journal entries gave them concrete evidence of their progress towards learning their multiplication facts, and when they revisited their entries they further cemented their thinking and found examples to emulate. The children's individual diaries were equally important in providing opportunities for reflection, analysis and the evaluation of their progress. Writing in these diaries and later sharing their thoughts with other children allowed them to further think about, organise and remember their ideas. Together with the shared communal journal and the teaching sequence, the diary writing process (diaries, diary sheets, diary-interviews) helped the children to develop their own learning pathways and plan for further learning. By openly evaluating their learning and sharing their ideas the children helped each other recognise which strategies were hindering memorisation and needed discarding and which ones were the most useful and helpful.

In conclusion the findings in this study support the argument that children can help each other increase their knowledge, understanding, and strategy-building techniques with regard to multiplication facts in a collaborative learning community.

6.5 Evaluation: strengths and limitations of the study

The strengths and limitations of this study relate to the sample of participants, the role of the teacher and the tensions created by the combination of teacher/researcher roles, the collection of data, and the learning environment. These are discussed in the following sections.

6.5.1 *The sample of participants*

The sample of participants provided both strengths and limitations to this study. A strength is that the sample was representative of children working at different stages of the Numeracy Project Framework (Ministry of Education, 2005a). A range of strategy stages provided the opportunity to work with children with differing prior knowledge. The sample of eleven children included both genders and provided a sizeable enough group to obtain data and form a cohesive social group. The group could generate discussion, share a range of ideas and skills while working in small groups as well as pairs. The participant sample is also a limitation as the children were from a specific socio-economic group and were not representative of the ethnic diversity within New Zealand's population. Findings may differ with a differing sample size, a single gender group, and participants from different socio-economic or ethnic backgrounds. This limitation impacted on data collected in that it did not allow for generalisations to be made across groups.

6.5.2 *The role of teacher*

The role of the teacher brought both strengths and challenges to the study. My ability to guide mathematical learning was underpinned by my qualifications in numeracy teaching and a long period of study of children's multiplication learning over many years. The children built their own learning community as I facilitated the opportunity for them to feel empowered to think for themselves. A limitation was that I was not the classroom teacher. This presented time constraints and prevented me from incorporating aspects of multiplicative thinking, such as division, into the teaching sequence. With more time the teaching sequence could have been extended until all children had committed their 100 multiplication facts to memory. For those children who experienced difficulties, alternative guidance could have been provided and children could have been encouraged to work on their multiplication facts during other mathematics lessons. I also lacked prior knowledge of the children in their wider learning context, which limited me until I identified specific learning needs. Not being the classroom teacher did not allow me the opportunity to integrate, reinforce learning or make connections with other curriculum areas at other times of the day. Another limitation was that I was not with the children on a daily basis so I could not capture the impromptu references children make to their learning as they arrive at school or discuss their learning with each other. Duplication of this study by classroom teachers could result in more success providing they have knowledge of how multiplicative thinking develops and the children are involved in both selecting the learning activities and in decision-making.

6.5.3 *The teacher/researcher role*

The combined roles of the teacher and researcher provided both strengths and limitations, too. A strength of this study was that as a teacher/researcher using an action research approach I could analyse teaching practice and reflect upon its effectiveness, whilst using pedagogical knowledge to adapt or change my planning for teaching to meet learning needs. A researcher without teacher pedagogical knowledge may not have adapted to the teaching process and similarly a teacher without the reflective skills of an action research approach may not have identified underlying dilemmas. The tension between the two roles when they are combined can also be seen as a limitation. Difficulties occurred when, as the researcher, my aim was to obtain data about the children's learning, whereas as the teacher I instinctively wanted to ignore the data collection and focus upon each child's learning rather than the process. Other teacher/researchers may experience these tensions in a lesser or greater degree, which may affect their findings.

6.5.4 *Data collection*

The methods of data collection also had both strengths and limitations. The richness of the children's voices proved to be a strength as it provided insight into their learning process. Children shared their 'discoveries' of calculation strategies, talked about their movement towards using increasingly more sophisticated strategies and how they were practising their multiplication facts. These explanations illuminated the children's understandings. However, accuracy in teacher recording of verbalised comments was difficult in a busy classroom environment where multiple voices were vying for attention. The school records of children's stages differed from my assessment using the Numeracy Project diagnostic tool. These results, alone, gave insufficient information to indicate the strategy stage of each child. My assessment tools complemented the Numeracy Project but as my assessment was non-standardised it may have limitations. My interpretation of the children's comments could be considered a limitation, as I did not always have the time to probe to find out more of the thinking behind the comment. The large amount of data that I collected around the actual teaching process could be considered a limitation, as some was unrelated to my research questions.

6.5.6 *The learning environment*

The learning context was a significant aspect of this study in establishing the learning environment. When children are involved in a context that captures their imaginations at their age and stage of learning, they are motivated into learning a mathematics topic that is often considered tedious. The 'Crocodilian Studies' learning context was a strength of this study because it related to the children's imaginative world. The thematic approach assisted in the establishment of a cohesive group where learning occurred in a trusting, challenging and fun environment. The lack of a permanent physical setting for the teacher/researcher was a limitation, as it did not allow me to set up an ongoing creative, stimulating, thematic environment similar to a classroom setting.

6.5.7 *The children as co-researchers*

Another significant strength of this study was the fact that children were co-researchers. Accepting the roles and responsibilities of co-researchers gave the children purpose and a sense of importance. The children devised their own co-researcher guidelines, and worked within these parameters. This built an atmosphere of collegiality and drove the co-operative learning.

6.6 Implications

This study has a number of implications for children, teachers, school communities and curriculum support writers, which I will discuss in this following section.

6.6.1 *Implications for children*

One implication for children is the importance of self-regulated learning in an environment where children can create their own learning community. Therefore it is important that children become self-regulated learners in the process of learning their multiplication facts. When teachers use strategies that allow self-regulation to develop, children can be involved in their own learning processes, whilst developing sound understandings, skills and knowledge of multiplication. Children are able to take on the responsibility of committing their multiplication facts to memory as they explore their own learning pathway, delve for more efficient multiplicative strategies and select activities and resources that best suit their own learning needs. Success in their learning will provide the motivation for practice and for further learning. Personal journal writing provides children with the opportunity to freely express their feelings, and explore and evaluate their thinking and progress. Class/group discussions and recording ideas in a shared communal journal will further enable them to reflect on their own thinking, and observe and use the thinking of others. As children develop self-regulated learning skills, they can select facts that they need to commit to memory as they recognise the facts they can instantly recall. Similarly, the setting of a group goal supports a collaborative approach to learning while developing individual goals underpins self-regulated learning. However, it is important to note that self-regulation springs from a cooperative learning environment. It is through the process of group discussion and sharing that children develop self-regulation.

Peer learning along with self-regulated learning has serious implications for the way in which we encourage children to learn. A cooperative learning community enables children to enhance and direct each other's learning. It is important that children are involved in structuring and reflecting on their own mathematical learning. Opportunities for children to observe and critique the thinking of peers develops and extends their own thinking. It allows them to evaluate their progress and plan for future learning. It is important that children are involved in collaborative decision-making to encourage them to make suggestions for change and to ensure they support decisions made.

6.6.2 *Implications for teachers*

The learning sequence employed in this study was successful in facilitating the learning of multiplication facts and therefore presents implications for teachers. Assessment of the children's prior knowledge is essential in order to establish a learning direction. Teachers should be aware that if the Numeracy Project suggestion of using only one question is followed, insufficient information is obtained to ensure an understanding of children's thinking and the strategies they use. A quick individual interview of each child prior to teaching, using targeted questions, will direct teaching and ensure relevance of the lesson sequence. An interview also identifies strategies children are using (individually and collectively) and allows them to verbalise their multiplicative thinking.

An implication for teachers is the need to consider the learning environment. An important part of a learning environment is the creation of an emotional tone where children are relaxed and enthusiastic. An appropriate context for study chosen by the children will provide motivation and continued engagement for learning. A context that captures children's interest, relates to their world and has a fun element provides purpose and a sense of belonging and helps towards group cohesion. Guidelines for cooperative learning need to be established to create an atmosphere of collegiality. It is also important that children explore the importance of multiplication in their further learning of mathematics and in their everyday lives.

An important implication for teachers is the teaching and learning sequence of multiplication facts. When the learning sequence suggested in this study is followed, children can develop their multiplicative thinking as they commit their multiplication facts to memory in a manageable and fun way. Teachers can develop their multiplicative thinking programmes around a lesson sequence that enables the children to commit their 2's then 5's facts to memory first, followed by the 1's and 0's. Once children have committed these facts to memory they can then derive answers to 'harder' facts. Clustering the remaining multiplication facts provides children with a manageable number of facts to work on at one time. These facts should be explored randomly and not learnt as a multiplication table. Flexibility within the lesson structure is important so that children are able to 'discover' multiplicative strategies by working at their own level and pace. Without the rich conversations around their learning processes and the exploration of multiplication facts, children may be less likely to develop multiplicative thinking, or understand and retain their multiplication facts. Lessons should be spread over a period of time, dependent on children's needs, to allow for each cluster of facts to be committed to memory. The choice of whether to teach multiplication facts to a whole class or to a group of children also depends on the children's needs. When teachers listen to children's explanations of how children commit their facts to memory, rather than just asking them to relate the strategy used, greater understanding of the child's learning will be achieved and modifications to teaching can be made more successfully. However, teachers should remember that children at different stages will support each other to reach a common goal. This study has not addressed ways in which more advanced learning might be included in children's experiences.

A further implication for teachers is the importance of creating an environment where children can develop self-regulated learning skills and where peer learning is paramount. Facilitation of self-regulatory learning techniques, where children want to take responsibility for their own learning, will further enhance the learning process. The use of personal diaries is a powerful tool for child self-reflection. This writing could be incorporated into the literacy programme during the teaching of multiplication facts. Opportunities need to be provided for children to be involved in thinking about how they learn, so they identify their gaps and construct their own learning pathway. Within lessons, children need to have opportunities to work in pairs or small groups, to share strategies and ideas about their learning. They need the chance to discover for themselves the importance of practice to memorisation; such discovery brings buy-in and motivation. Provision of a wide range of resources is important so children can explore and select to meet their self-identified learning needs. Parental support is a valuable resource that children should be encouraged to recognise and utilise.

6.6.3 *Implications for school communities*

School communities also need to know that they can play a pivotal role in supporting children to practise their multiplication facts. Firstly, a home/school partnership programme to encourage parents to work with their children could be undertaken. Parents need to be made aware of how encouraging their children to practise and in assisting with practice has a profound influence on learning: it is reflected in children's increased desire to learn, growing children's confidence and self esteem and a more positive attitude to mathematics. Because some parents may be unable to help their children at home, schools should explore ways to overcome this. One such way might be developing a 'maths mileage' scheme similar to that of the community volunteers who assist with reading mileage. A multiplication facts fun club could also be trialled in schools.

6.6.4 *Implications for project developers and curriculum support writers*

Several issues that have risen as a result of this study will be of interest to the Numeracy Project developers, and to developers of similar projects. The current Numeracy Project support materials give guidelines to strategy teaching but it is important that teachers are also given direction on how to teach memorisation and practice of multiplication facts, as well as how to assess for prior knowledge. Further, although currently the Numeracy Project suggests children are grouped according to strategy stages, children of mixed-strategy stages can support each other to successfully learn their multiplication facts. This has implications for the Numeracy Project in that it may be advisable for the Project to recommend flexible delivery (regardless of strategy stage) for the learning of multiplication facts.

The successful use of a focus question for the children to explore at the beginning of each lesson, as used in this study, is in marked contrast to the Numeracy Project, which requires that children are given a learning intention and that the selected strategy be taught. Further, use of the modelling book can be extended to include teacher and child recording of rich discussions and discoveries of multiplicative thinking, and to encourage deeper reflection.

In the light of this study, numeracy project developers may like to consider a personalised learning process, one where children are encouraged to discover calculation strategies, rather than be guided through a teacher-directed process. Children's ownership of their learning is severely limited and opportunities for personalised learning are stifled when learning intentions are dictated by the teacher. A personalised learning process requires self-regulated learning strategies and incorporates the building of a learning community. Ways of working within the classroom will require flexibility if the successes of this study are to be incorporated into the Numeracy Project. Those involved in writing curriculum support documents need to know that teachers require guidance on how to incorporate multiplication facts into the teaching of multiplication.

6.7 Issues for further research

As different population samples (different ethnicities, socio-economic groups, single gender, age) may respond differently to the sample in this study, further action research studies would be useful. A study involving older children or children from diverse cultures could give further data and identify whether such children respond similarly to those in this study. This study could be carried out to investigate whether this method of teaching is equally effective when used with a whole class. Research into the needs of older children could give direction to approaches that would best suit them. It would also be interesting to see how a classroom teacher's replication of this study's methods, might differ from that of the teacher/researcher. Research could be carried out into what practical support, professional development and resources teachers require to successfully implement ideas from this study. An investigation into how the Numeracy Project could include ideas from this study and the effectiveness of such inclusions would provide valuable data. Another area of research could be to investigate whether children who only rote-learn their multiplication facts lack multiplicative understanding, as was found in this study. Although the classroom teacher commented that the children still recalled their multiplication facts four months later further investigation would confirm this or identify ideas for maintenance.

There was no opportunity, in this study, to explore the importance of the children becoming co-researchers. It would be interesting to further investigate this aspect and to consider variations to this approach to eliminate repetition.

Further research could examine the varying pace at which children understand and memorise multiplication facts. It would be interesting to discover why some children lag behind, and to investigate ways to further support these children. Gifted learners may reach a level of multiplicative thinking in advance of other children, and it would be valuable to explore this.

In recognising the highly significant role played by parents, another area of research could involve an investigation into how schools could provide support for children who are unable to receive help from home, or who are in families who do not have access to commercially-produced aids, to practise their multiplication facts. Research to investigate, or trials carried out on, practical support initiatives may highlight such suggestions as a fun tables club. Home/school partnership sessions where parents could be given suggestions and aids to work with their children. Sponsorship to set up such initiatives could be sought. Another investigation could consider how community volunteer schemes where adults listen to children read could be investigated to see if this could be expanded to include practising multiplication facts.

6.8 Importance of this study

In summary this study gives direction to how a range of children can develop their multiplicative strategies while they commit their multiplication facts to memory, in a relatively short time, provided that the learning process facilitates strategy development and understanding. For this to occur: children need to be allowed to take ownership of their own learning and discover calculation strategies rather than be taught them; multiplication facts need to be taught in sequenced clusters; and learning should take place within a stimulating peer-learning community.

This study affirms the idea that children can commit their multiplication facts to memory when they discover and direct their own learning. As children are involved in thinking about how they learn, they can identify gaps and construct their learning pathways. They discover the importance of practice for memorisation as they experience success. ’Buy-in’ occurs and further practice is self-motivated. Children best explore their thinking by having access to a wide range of resources, so they can select to meet their self-identified learning needs. Children should be encouraged to recognise and utilise parental support.

The findings of this study suggest that the most significant influence on the children’s learning is the role of peers. Through the sharing of ideas with peers, children begin to make sense of their own multiplicative thinking. Such sharing can enhance further knowledge, understanding, and strategy-building techniques of multiplication facts. Furthermore, in addition to a constructivist-learning environment, a self-regulatory learning process develops when the peer learning community has been developed.

Recent research has been directed towards children’s strategy development without researching how children learn. This study has taken the liberty of asking the children how they think they commit their multiplication facts to memory to provide implications for further learning. Acknowledging the ideas of these children into future teaching sequences will make the task of learning multiplication far less daunting for both teachers and the children. I conclude that in order to assist children in their learning it is imperative to identify what assistance is best for them.

“Just because we are little it doesn’t mean we are not smart” (William, aged 8)

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Appendices

Appendix	Title
1	Comparison of multiplication facts that are considered, by researchers, difficult to learn: comparison of research done to consider the order to teach multiplication facts
2	*100 Multiplication Facts Pre- and Post-test*
3	*Multiplication Facts Verbal Pre- and Post-test*
4	*Multiplication Knowledge and Strategy Interview*
5	Letter to principal to obtain ethical approval
6	Letter to classroom teacher to obtain ethical approval
7	Letter to parents to obtain ethical approval
8	Letter to children to obtain ethical approval
9	Cover page for children's folders
10	Lessons to assist children commit their tables to memory
11	Results of *100 Multiplication Facts Pre- and Post-test*
12	Children's Movement from additive to multiplicative thinking
13	Results *Multiplication Knowledge and Strategy Interview* Part A and Part B
14	Example of Children's Diary Sheet
15	Example of page from "Big Green Croc'
16	An example of ideas, when discussing the 2x table, recorded by two children
17	Recordings from Lesson 13. What do we know about 7 x 8 - evaluation?
18	Collation of children's ideas of 'What puts facts into your head'?

Appendix 1

Comparison Multiplication Facts that are considered, by researchers, difficult to learn
and
Comparison of Research done to consider the order to teach multiplication facts

Norem & Knight 1930	**Ruch 1932**	**Murray 1939**	**Thornton *et al.* 1983**	**Frobisher *et al.* 1999**	**Zevenbergen *et al* 2004**	**Teacher/Researcher's (Morrison) approach 2005**	**Numeracy Project 2005**
Facts considered hard	Facts considered hard	Facts considered hard	*Eliminate* 2's 5's 9's square nos 0's 1's and commutative property ***10 remaining facts***	Eliminate 0's 1's 2's 10's 5's 9's square nos ***20 remaining facts –*** *elimate commutative property* ***= 10 facts***	*Eliminate* 2's 5's 10's 1's 0's 4's 9's and sq nos and commutative property ***6 remaining facts***	*Eliminate* 2's 5's 1's 0's 9's and the commutative property **= 15 facts**	Teach the 2's 5's first
		1 x 1					No advice given
			3 x 4	3 x 4		3 x 4	
			3 x 6	3 x 6	3 x 6	3 x 6	
			3 x 7	3 x 7	3 x 7	3 x 7	
			3 x 8	3 x 8	3 x 8	3 x 8	
			4 x 6	4 x 6		4 x 6	
4 x 7	/ 7 x 4		4 x 7	4 x 7		4 x 7	
4 x 8			4 x 8	4 x 8		4 x 8	
4 x 9	/ 9 x 4						
6 x 7 / 7 x 6	6 x 7 / 7 x 6		6 x 7	6 x 7	6 x 7	6 x 7	
8 x 6	6 x 8 / 8 x 6	6 x 8	6 x 8	6 x 8	6 x 8	6 x 8	
6 x 9	6 x 9 / 9 x 6	6 x 9					
7 x 8	7 x 8 / 8 x 7	7 x 8	7x 8	7x 8	7 x 8	7x 8	
7 x 9	/ 9 x 7	7 x 9 / 9 x 7					
	/ 9 x 8	/ 9 x 8				3 x 3	
						4 x 4	
						6 x 6	
7 x 7	7 x 7	7 x 7				7 x 7	
		8 x 8				8 x 8	
		9 x 9					

Appendix 2
100 Multiplication Facts Test

To establish known Multiplication Facts — Pre and Post Test

Name_____________________________ **Date**__________________

1	9 x 0 =	26	4 x 5 =	51	9 x 5 =	76	4 x 1 =
2	8 x 1 =	27	3 x 7 =	52	8 x 6 =	77	3 x 2 =
3	2 x 5 =	28	7 x 8 =	53	2 x 6 =	78	7 x 4 =
4	6 x 7 =	29	1 x 0 =	54	5 x 0 =	79	1 x 9 =
5	5 x 7 =	30	0 x 8 =	55	6 x 2 =	80	0 x 4 =
6	4 x 3 =	31	8 x 4 =	56	4 x 8 =	81	9 x 2 =
7	3 x 9 =	32	2 x 2 =	57	3 x 4 =	82	8 x 9 =
8	7 x 7 =	33	6 x 1 =	58	7 x 3 =	83	2 x 8 =
9	1 x 7 =	34	5 x 5 =	59	1 x 4 =	84	6 x 5 =
10	0 x 1 =	35	4 x 6 =	60	0 x 6 =	85	5 x 4 =
11	9 x 9 =	36	9 x 7 =	61	9 x 4 =	86	4 x 0 =
12	8 x 2 =	37	3 x 6 =	62	8 x 7 =	87	3 x 1 =
13	2 x 9 =	38	7 x 9 =	63	2 x 7 =	88	7 x 5 =
14	6 x 8 =	39	1 x 5 =	64	6 x 3 =	89	1 x 3 =
15	5 x 8 =	40	0 x 7 =	65	5 x 2 =	90	9 x 1 =
16	4 x 4 =	41	9 x 6 =	66	4 x 9 =	91	8 x 0 =
17	3 x 8 =	42	8 x 5 =	67	3 x 3 =	92	6 x 6 =
18	7 x 1 =	43	2 x 3 =	68	7 x 0 =	93	2 x 0 =
19	1 x 8 =	44	6 x 0 =	69	1 x 1 =	94	0 x 3 =
20	0 x 9 =	45	5 x 1 =	70	0 x 5 =	95	5 x 6 =
21	9 x 8 =	46	4 x 7 =	71	9 x 3 =	96	4 x 2 =
22	8 x 3 =	47	3 x 5 =	72	8 x 8 =	97	3 x 0 =
23	2 x 1 =	48	7 x 2 =	73	2 x 4 =	98	1 x 2 =
24	6 x 9 =	49	1 x 6 =	74	6 x 4 =	99	7 x 6 =
25	5 x 9 =	50	0 x 0 =	75	5 x 3 =	100	0 x 2 =

Time ______ Number correct ______

Appendix 3
Multiplication Facts Verbal Test Pre/Post Test

To establish the multiplication facts that are instantly recalled (within 3 seconds)

Name______________________ Date__________________

	Incorrect or no attempt	Instantly Recalled	Delay	Comment
1 x 0				
0 x 4				
9 x 0				
0 x 100				
8 x 1				
1 x 7				
23 x 1				
1 x 17				
8 x 2				
2 x 7				
6 x 2				
2 x 9				
8 x 5				
5 x 9				
6 x 5				
5 x 7				
9 x 3				
8 x 9				
9 x 4				
7 x 9				
3 x 3				
4 x 3				
6 x 3				
3 x 7				
8 x 3				
4 x 4				
6 x 4				
4 x 7				
8 x 4				
6 x 6				
8 x 6				
7 x 6				
7 x 7				
8 x 7				
8 x 8				

Adapted from a test by Peter Hughes University of Auckland

Appendix 4
Multiplication Knowledge & Strategy Interview

To establish what strategies children are using to solve multiplication and division problems.

Part A. Questions 1 to 4 aim to establish what strategies children are currently using. The questions were written on cards/sheets. Children could use tally marks, draw or use equipment to work out the answers.

Children were asked to explain their answers – if they gave an incorrect answer they were asked to check it by drawing it.

Part B aims to ascertain the child's Numeracy Project Strategy stage for multiplication and division. These questions were carried out as suggested in the Numeracy Book 2 "The Diagnostic Test"

<table>
<tr><th></th><th>Purpose</th><th colspan="2">Questions</th></tr>
<tr><td rowspan="2">Part A
Equal Groups

For these 2 questions pen/paper and equipment will be available for use</td><td rowspan="2">To ascertain if children have an understanding of the concept of equal groups and to observe strategies and recording children use to solve multiplication facts
To go beyond the 2's and 5's to observe what strategies children use to solve multiplication facts.</td><td>(1) There are 6 ponds with 4 crocodiles in each pond. How many crocodiles are there altogether?</td><td>(2) There are 4 lines of crocodiles with 3 crocodiles in each line. How many crocodiles are there altogether?</td></tr>
<tr><td>(3) There are 8 ponds with 6 crocodiles in each pond. How many crocodiles are there altogether?</td><td>(4) There are 7 lines of crocodiles with 6 crocodiles in each line. How many crocodiles are there altogether?</td></tr>
<tr><td>Part B.
Numeracy Project Diagnostic Test Questions

Equipment Numeracy Project Diagnostic Test cards</td><td>To ascertain the child's Numeracy Project strategy stage for multiplication and division</td><td>P. 40 – Question 1 from The Diagnostic Interview Book 2 (Ministry of Education, 2005b)</td><td>P. 41-42 – Questions 4-7 from The Diagnostic Interview Book 2 (Ministry of Education, 2005b)</td></tr>
</table>

Appendix 5
Letter to Principal to obtain Ethical Approval

Teacher/researcher address

27 February 2005

Addressed to the Principal

Dear (name),

As we have previously discussed, I would like to work with a group of twelve children from a Year 4 class in your school as participants in a research thesis. This thesis is a requirement of a Master of Teaching and Learning degree at the Christchurch College of Education, which I am currently undertaking. I will be working under the supervision of Dr Jane McChesney, a Senior Lecturer at the Christchurch College of Education and Dr Kay Irwin, a Senior Lecturer in Education at the University of Auckland.

My project is called 'Tracking the Learning of Multiplication Tables and the Development of Multiplicative Strategies'. The aim of this research is to assist children to commit their multiplication tables to memory while they are working on related strategies in their current programme. I am also interested in what children think assists them in committing their tables to memory. Learning multiplication tables this way, I hope, will make working on multiplication tables more enjoyable and effective for children.

Children will be asked to participate in a 'Multiplication Tables Verbal Test', a 'Multiplication Knowledge Questionnaire' and a '100 Multiplication Facts Test' at both the beginning and end of the research. For the verbal test and the questionnaire I would like to test the children individually. There will not be any pressure put on them to complete these assessments.

I will then work with them to develop the strategies of the Advanced Additive/Early Multiplicative stage of the Numeracy Project. Multiplication tables will be integrated into the lessons. I will be the main teacher of these lessons although I will work alongside the classroom teacher. I would like to work with the children up to twice a week if the classroom teacher is agreeable. Timed daily speed tests will be given to the children when they feel ready to do this. These tests will be part of my lesson but it is hoped that the children will also be able to carry this out on the days I am not present. For these tests the children will be asked to compete against themselves to improve their time in recalling facts we have worked with. The children will also be asked to complete a diary after each teaching session during the term to record thoughts about their mathematical thinking and experiences.

I will video my lessons and audiotape record the Multiplication Knowledge Questionnaire. I assure you that these will only be seen or listened to by myself, and my supervisors if they feel it is necessary.

The tests and the questionnaire should take about 15 minutes each to complete and can be done at a convenient time to Tracey. The daily speed tests should take less than 5 minutes to be carried out. The diaries will take approximately 5 minutes for each entry and will be done at the end of a teaching session.

No findings that could identify any individual participant will be published. Since data must be stored for at least five years according to college regulations, anonymity of both the school and the participants is assured as I will systematically use pseudonyms to identify the school and the individuals.

Participation in the research project is, of course, entirely voluntary. Children who do not participate will not be penalised or disadvantaged in any way.

Children, whose parents agree to let them participate, can withdraw their child at any time.

The Christchurch College of Education Ethics Committee has reviewed and approved this study.

Complaints Procedure
The College requires that all participants be informed that if they have any complaint concerning the manner in which a research project is conducted, it may be given to the researcher, or, if an independent person is preferred, to:

The Chair
Ethical Clearance Committee
Christchurch College of Education
P O Box 31-065
Christchurch
Phone: (03) 348 2059

Please contact me if you have any other queries or concerns about the project or would like to be informed of the aggregate research finding. I can be reached by phone on: 0275 770 104 or by email: v.morrison@auckland.ac.nz

For further clarification you are welcome to contact my supervisor Dr Jane McChesney by e mail: jane.mcchesney@cce.ac.nz

I really forward to working in your school.
Thank you.

Kind regards

Vivienne Morrison

Declaration of Consent

I consent to my school participating in the project 'Tracking the Learning of Multiplication Tables and the Development of Multiplicative Strategies'.

I have read and understood the information provided to me concerning the research project and what will be required of the children and the teacher when we participate in the project.

I understand that the information we provide to the researcher will be treated as confidential and that no findings that could identify either the children or my school will be published.

I understand that our participation in the project is voluntary and that we may withdraw children from the project at any time without incurring any penalty.

Name: ____________________________________ Date: ______________

Signature: __________________________

Appendix 6
Letter to the Classroom Teacher to obtain Ethical Approval

Teacher/researcher address

27 February 2005

(Teacher's name and address)

Dear (name),

As we have previously discussed, I would like to work with a group of twelve children from your class as participants in a research thesis. This thesis is a requirement of a Master of Teaching and Learning degree at the Christchurch College of Education, which I am currently undertaking. I will be working under the supervision of Dr Jane McChesney, a Senior Lecturer at the Christchurch College of Education and Dr Kay Irwin, a Senior Lecturer in Education at the University of Auckland.

My project is called 'Tracking the Learning of Multiplication Tables and the Development of Multiplicative Strategies'. The aim of this research is to assist children to commit their multiplication tables to memory while they are working on related strategies in their current programme. I am also interested in what children think assists them in committing their tables to memory. Learning multiplication tables this way, I hope, will make working on multiplication tables more enjoyable and effective for children.

The children will be asked to participate in a 'Multiplication Tables Verbal Test', a 'Multiplication Knowledge Questionnaire' and a '100 Multiplication Facts Test' at both the beginning and end of the research. For the verbal test and the questionnaire I would like to test the children individually. There will not be any pressure put on them to complete these assessments.

I will then work with them to develop the strategies of the Advanced Additive/Early Multiplicative stage of the Numeracy Project. Multiplication tables will be integrated into the lessons. I will be the main teacher of these lessons although I look forward to working alongside you. I would like to work with the children up to twice a week if this is agreeable to you. Timed daily speed tests will be given to the children when they feel ready to do this. These tests will be part of my lesson but it is hoped that the children will also be able to carry this out on the days I am not present. For these tests the children will be asked to compete against themselves to improve their time in recalling facts we have worked with. The children will also be asked to complete a diary after each teaching session during the term to record thoughts about their mathematical thinking and experiences.

I will video my lessons and audiotape record the Multiplication Knowledge Questionnaire. I assure you that these will only be seen or listened to by myself, and my supervisors if they feel it is necessary.

The tests and the questionnaire should take about 15 minutes each to complete and can be done at a convenient time to you. The daily speed tests should take less than 5 minutes to be carried out. The diaries will take approximately 5 minutes for each entry and will be done at the end of a teaching session.

No findings that could identify any individual participant will be published. Since data must be stored for at least five years according to college regulations, anonymity of both the school and the participants is assured as I will systematically use pseudonyms to identify the school and the individuals.

Participation in the research project is, of course, entirely voluntary. Children who do not participate will not be penalised or disadvantaged in any way.

Children, whose parents agree to let them participate, can withdraw their child at any time.

The Christchurch College of Education Ethics Committee has reviewed and approved this study.

Complaints Procedure
The College requires that all participants be informed that if they have any complaint concerning the manner in which a research project is conducted, it may be given to the researcher, or, if an independent person is preferred, to:

The Chair
Ethical Clearance Committee
Christchurch College of Education
P O Box 31-065
Christchurch
Phone: (03) 348 2059

Please contact me if you have any other queries or concerns about the project or would like to be informed of the aggregate research finding. I can be reached by phone on: 0275 770 104 or by email: v.morrison@auckland.ac.nz

For further clarification you are welcome to contact my supervisor Dr Jane McChesney by e mail: jane.mcchesney@cce.ac.nz

I really forward to working with you.
Thank you.

Kind regards

Vivienne Morrison

Declaration of Consent

I consent to twelve children from my class participating in the project 'Tracking the Learning of Multiplication Tables and the Development of Multiplicative Strategies'.

I have read and understood the information provided to me concerning the research project and what will be required of the children and the teacher when we participate in the project.

I understand that the information I provide to the researcher will be treated as confidential and that no findings that could identify either the children or my school will be published.

I understand that our participation in the project is voluntary and that we may withdraw children from the project at any time without incurring any penalty.

Name: ______________________________________ Date: _______________

Signature: ___________________________

Appendix 7
Letter to Parents to obtain Ethical Approval

Address of teacher/researcher

27 February 2005

Dear Parents/Guardians

I am working towards a Master of Teaching and Learning degree at the Christchurch College of Education. As part of my degree I am required to undertake a research project. I will be working under the supervision of Dr Jane McChesney, a senior lecturer in the School of Secondary Teacher Education at the Christchurch College of Education and Dr Kay Irwin a senior lecturer in Education at the University of Auckland.

My project is called: 'Tracking the Learning of Multiplication Tables and the Development of Multiplicative Strategies'

The aim of this research is to assist children to commit their multiplication tables to memory while they are working on related multiplication strategies in their current programme. Learning multiplication tables this way, I hope, will make working on tables more enjoyable and effective for children.

I will be working with a group of children in your child's Year 4 class. The children will be tested on their multiplication tables at the beginning and the end of the study. I will also ask them questions to establish their understanding of multiplication. They will participate in a series of lessons on multiplication, which incorporates assisting them in committing their multiplication tables to memory. I am also interested in what children think assists them to commit their tables to memory. Children will also be asked to write in a diary at the end of each teaching lesson.

The questionnaires should take about 15 minutes each to complete and will be done at a convenient time to their classroom teacher at the beginning and the end of the time I am in the school. The teaching will be done during the children's normal mathematics teaching time. The diaries will take approximately five minutes at the end of a lesson.

Anonymity for both the school and your child will be provided by using pseudonyms. Any findings that could identify any individual participant will not be published. I also assure you that all information will remain confidential and will only be seen by my supervisors and myself.

Participation in this study is entirely voluntary and if you agree to have your child take part, you can withdraw at any time by writing to the teacher. Your child may also choose not to answer some of the questions. Children who do not participate will not be penalised or disadvantaged in any way.

The Christchurch College of Education Ethics Committee has reviewed and approved this study.

Complaints Procedure

The College requires that all participants be informed that if they have any complaint concerning the manner in which a research project is conducted, it may be given to the researcher, or, if an independent person is preferred, to:

The Chair
Ethical Clearance Committee
Christchurch College of Education
P O Box 31-065
Christchurch
Phone: (03) 348 2059

Please contact me if you have any other queries or concerns about the project or would like to be informed of the aggregate research finding. I can be reached by phone on: 09 832 2822 or by email: v.morrison@ace.ac.nz.
Please fill the consent form below and return it to the school if you are happy for your child to participate in this research project.
Thank you.

Vivienne Morrison

Parent/Guardian

I give permission for ________________________________ to participate in the project, 'Tracking the Learning of Multiplication Tables and the Development of Multiplicative Strategies'.

I have read and understood the information provided to me concerning the research project and what will be required of the children.

I am satisfied that __________________________ understands what will be required of participants in the project.

I understand that the information the children provide to the researcher will be treated as confidential and that no findings that could identify either them or their school will be published.

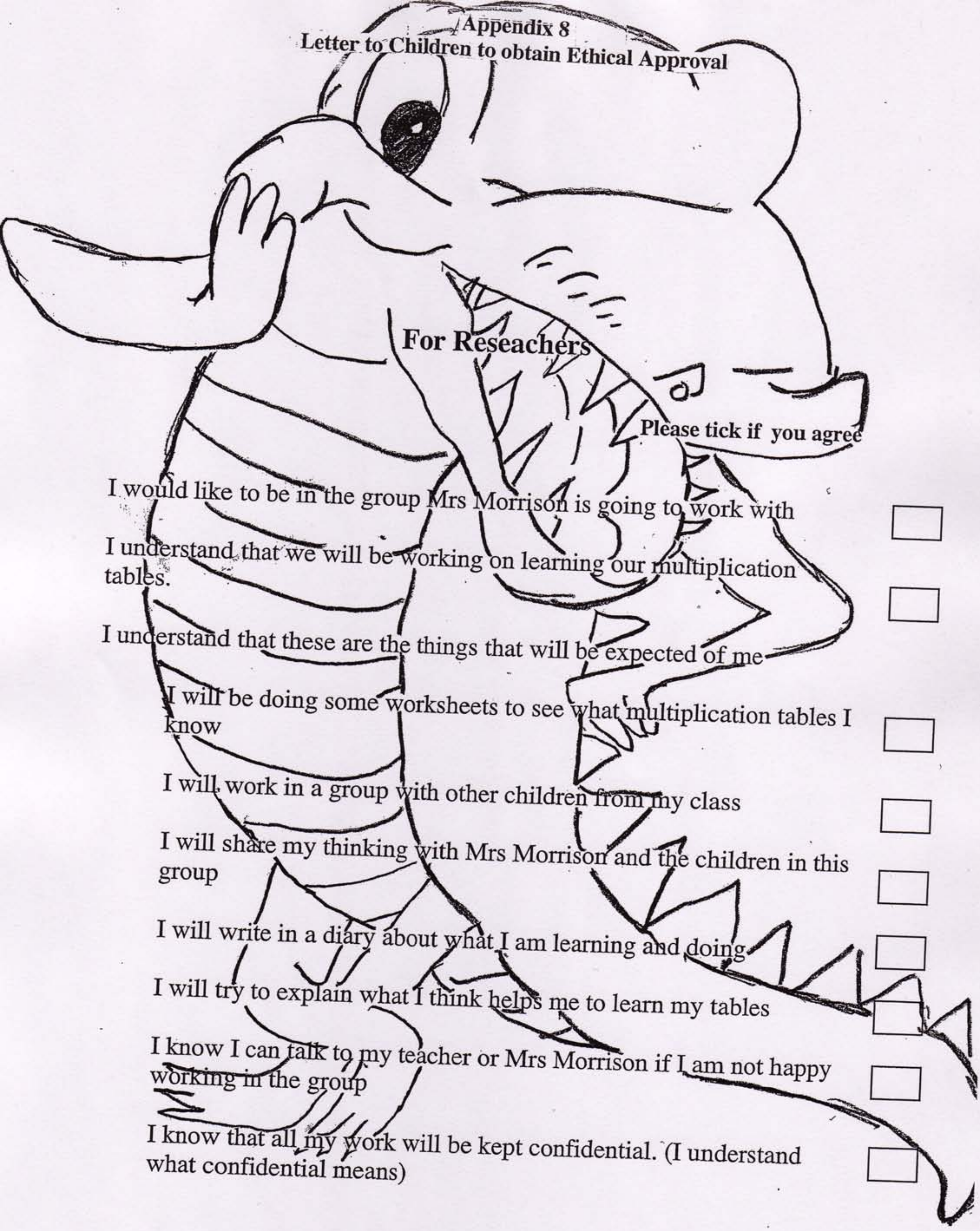
I understand that participation in the project is voluntary and that either I or my child may choose to withdraw from the project at any time without incurring any penalty.

Name: _______________________________________ Date: _______________

Signature: ________________________________

Appendix 8

Letter to Children to obtain Ethical Approval



For Reseachers

Please tick if you agree

I would like to be in the group Mrs Morrison is going to work with ☐

I understand that we will be working on learning our multiplication tables. ☐

I understand that these are the things that will be expected of me

- I will be doing some worksheets to see what multiplication tables I know ☐
- I will work in a group with other children from my class ☐
- I will share my thinking with Mrs Morrison and the children in this group ☐
- I will write in a diary about what I am learning and doing ☐
- I will try to explain what I think helps me to learn my tables ☐
- I know I can talk to my teacher or Mrs Morrison if I am not happy working in the group ☐

I know that all my work will be kept confidential. (I understand what confidential means) ☐

Signature____

Appendix 9
Cover Page for Children's Folders

Very Important Research for the

CHRISTCHURCH
COLLEGE *of* EDUCATION
Te Whare Whai Matauraka Ki Otautahi
DISCE UT DOCEAS

to

discover how children learn their multiplication tables

Researcher's name________________________________

This important document will contain the things that the researcher thinks are important for this study

Appendix 10

Lessons to Assist Children Commit their Tables to Memory 19.3.05

Stage: Moving Children from Early Additive-Part Whole to Advanced Additive/Early Multiplicative of the Numeracy Project – and incorporating the multiplication tables

Lessons and Learning Intentions	**Concept to be taught**	**Related activities in Numeracy Project Book 6**	**Tables to be committed to memory**	**Follow up for children**	**Support Activities + Tables used**
To allow children to be involved in self evaluation				Children begin to write up 'research journals' as part of the warm down. Beginning with where they think they are at	
Revision 1. To ascertain knowledge and understanding of multiplying by 0x, 1x, 2x and 5x multiplication tables. To revise the relevant principles and multiplication tables if necessary	Go over 0's, 1's, 2's, 5's What do we know about these tables? Revise: 3 Principles – Multiplying by 1 – Multiplying by 0 – Commutative Property-use lesson 2 here 2 tables - Doubles (eg 2 x 7) Multiplying by 5	"Twos, Fives and Tens" p.9 Bk6 - if needed	O's, 1's, 2's, 5's	Get smart at 2's and 5's Children may choose to challenge themselves to increase their speed at recalling the facts by using a timer and pre printed sheets (pairs) – when children feel ready. Journal Writing	Loopy Set A Croco Old Maid Snakes + chances Croco puzzle Basic Facts Cards Dice 2's 5'5 0/1/2/5's Bingo I have Who Has Times Out Bingo 2's and 5's Multidice 2's and 5's Flashcards *Minus 36 multiplication facts* *Minus 64 multiplication facts*
Revision 2. To ascertain understanding of the commutative property and develop if necessary	***Commutative Property*** Children will use equipment of their choice to explain to others the meaning of the commutative property – eg using Animal Strips from the Numeracy Project MM 5-2 *Give children 64 facts test to show them they already know at least 64 facts*	"Turn Abouts" P.17 – if needed		Four in a Row Multiplication" MM 6-6 Numeracy Project Journal writing Possibly - continue 'speed' tests	**Animal Arrays** Multiplication Grid Game

1. To use doubles to generate the 4's To assist children to derive unknown facts from known facts by doubling and halving.	Doubling the 2 x table = 4 x table. Use the doubles to generate the triples if appropriate ***Proportional Adjustment – Doubling and Halving*** 3 x 4 (6 x 2) 4 x 4 (8 x 2) or 7 x 4 (14 x 2) 8 x 4 (16 x 2)		3 x 4 4 x 3 4 x 4 oops one 7 x 4 4 x 7 8 x 4 4 x 8	Make up grids of tables showing doubling Journal writing Possibly – continue 'speed' tests	Crocoteeth Triangle Puzzle Multidice 4's Times Out Bingo 4's *Minus 71 multiplication facts*
2. To assist children work out 6's, 7's, and 8's table from knowledge they have of the 5's.	***Distributive Property*** Using the 5 x table to generate the 6's, 7's, 8 's eg 6 x 6 = (6 x 5) + (6 x 1) – begin with using the abacus	"Fun with Fives" P.12 if needed	3 x 7 7 x 3 3 x 6 6 x 3 6 x 6, 7 x 7 8 x 8 3 x 3	"Fly Flip Multiplication" MM 6-4 – Numeracy Project Possibly speed sheet practice with any new facts added Journal writing	Frog Young Maid Multidice 3's Times Out Bingo 3's Loopy Set B "Fun with Fives" worksheet *Minus 79 multiplication facts*
3. To develop and understanding of compensation by using the 10's to generate the 9's	***Compensation*** Eg (6 x 10) - 6	"A Little Bit More/A Little Bit Less" P.15 – if needed		"Four in a row multiplication" MM 6-6	Crafty Crocodiles (extension)
4. To explore ways to work out the answers to the 9x table	***Nine times table*** Brainstorm – What do we know about the nine times table?	"Goesintas" – division lesson p.20 Bk 6 "Long Jumps" - division lesson p.19 Bk 6	3 x 9 9 x 3 4 x 9 9 x 4 6 x 9 9 x 6 7 x 9 9 x 7 8 x 9 9 x 8 9 x 9	Possibly speed sheet practice with any new facts added Journal writing	Multidice 9's Times Out Bingo 9's Flashcards Monster Multiplication "A little Bit more, A little Bit Less" w/sheet Happy Croc Families "Goesintas" Worksheet "Long Jumps" Croc-minoes Crocminoes 2 *Minus 90 multiplication facts*

5. To consider different number sentences that give the answer 24 and then 48	What do we know about 24? 48?		6 x 4 4 x 6 8 x 3 3 x 8 6 x 8 8 x 6	Possibly speed sheet practice with any new facts added Journal writing	Crocodile Tag Crocodilian Young Maid *Minus 96 multiplication facts*
6. To use known multiplication facts to derive unknown facts	If I didn't know 6 x 7 how could I work it out? – brainstorm ideas		6 x 7 7 x 6	Possibly speed sheet practice with any new facts added Journal writing	Old Maid Bees x2 Multidice 6's Times Out Bingo 6's Flashcards *Minus 98 multiplication facts*
7. To use known multiplication facts to derive unknown facts	If I didn't know 7 x 8 how could I work it out?		7 x 8 8 x 7	Possibly speed sheet practice with any new facts added Journal writing	Slithering Snakes Multidice 7's 8's Times Out Bingo 7's 8's Loopy Set C I have Who Has (all facts) Crocodilian I Have Who Has Flashcards Dominoes Memory *Minus 100 multiplication facts*
Extra games To extend concepts that have been worked on in this lesson sequence	dice, x wheels number mat Buzz Multiplication Madness Flash Cards Bingos		 Snakes and Chances Bowl A Fact In and Out Loopy Dividing think about x first Number Boggle Monsters Playing Teachers	Material Masters "Connected" Website Figure It Out's Marie's sheets New MEI sheets Monster Multiplication 10x Happy Croc Families ABCD	Arithmefacts I have, who has Jigsaws Milk Container game Norma's Kiwi Game Chn make their own games

Lessons flexible – to adhere to needs of the children Set up a game making area A chart will show on going record of multiplication facts covered
Games to be added – for children to do when I am not in the classroom
Materials: abacus, fly flips, unifix cubes, beans and canisters, ice cream containers, animal strips, calculators, number line and pegs, Happy Hundreds arrays, masking cards, 100's board

Appendix 11
Results of '100 Multiplication Facts' Pre and Post Tests

Name	Number Correct /100 pre test	Number Correct /100 post test	Incorrect answer on the final 100 Multiplication Facts Test 24 June
Lucy	74	99	8 x 7 = 54
Kyle	67	99	7 x 7 = 48
Otis	47	99	8 x 7 = 54
Molly	76	97	8 x 3 = 16 3 x 7 = 28 9 x 7 = 48
Madison	60	97 (5 mins)	6 x 7 = 56 8 x 3 = 27 6 x 4 = 27
William	65	95	5 x 7 = 30 3 x 9 = 26 4 x 8 = 28 8 x 8 = 72 8 x 9 = 74
Simon	70	100	
Fleur	72	95	7 x 9 = 64 4 x 8 = 40 9 x 4 = 46 8 x 8 = 54 7 x 6 = 46
Aaron	48	89	6 x 7, 7 x 7, 6 x 8, 7 x 8, 8 x 6, 8 x 7, 8 x 8, 6 x 4, 7 x 4, 6 x 6, 7 x 6
Jacob	22	94	7 x 7, 7 x 8, 7 x 9, 8 x 6, 8 x 7 = 80, 8 x 8 = 76,
Finn	41	70	6 x 7, 3 x 9, 7 x 7 =55, 2 x 9=15 6 x 8, 4 x 4, 9 x 8=80, 2 x 1, 6 x 9, 7 x 8, 8 x 4, 4 x 6, 9 x 6, 4 x 7, 7 x 2, 8 x 6=5, 4 x 8, 3 x 4 =9, 9 x 4=26, 8 x 7 4 x 9, 3 x 3, 9 x 3, 8 x 8=57, 6 x 4, 7 x 4=25, 1 x 9=6, 8 x 9=56, 6 x 6, 7 x 6

Appendix 12
Children's Movement from Additive to Multiplicative Thinking

Analysed using an adaptation of the 'Intuitive Models for Multiplication of Mulligan and Mitchelmore' (1997)

	Too hard or incorrect		**Direct Counting**		**Repeated Addition**			**Derived Facts**		**Known Facts**		**Instantly recalled**	
	Pre	**Post**	**Pre**	**Post**	**Pre**	**Post**		**Pre**	**Post**	**Pre**	**Post**	**Pre**	**Post**
Lucy	17				1			6	7	2	1	7	27
Kyle	14				5			2	5	7	3	7	27
Otis	6				3			11		4	2	11	33
Molly	12				12	1		4	1	4	5	5	28
Madison	5				1			13	5	7	1	9	29
William	10		7		7	1		2	5	2	5	7	24
Simon	14				9 (2)			2	5		4	8	26
Fleur	12		7		2			1	9	5	1	8	25
Aaron	16	5			6 (5)				11	8	1		18
Jacob	20	6			4			1		8	6	2	23
Finn	20	14		1	5			4	4	5		1	16

The numbers in each category indicate the number out of 35 facts asked in this test.
Brackets = 9x table finger trick

Appendix 13
Results of *Multiplication Knowledge and Strategy Interview*
Initial Test March 2005 - Post Test 27 June 2005 (italics)

Analysed using Intuitive Models (Mulligan & Mitchelmore , 1997 (Table 3.5)

Name	**Question 1** 6 x 4	**Question 2** **3 x 4**	**Question 3** **8 x 6**	**Question** **7 x 4**
Lucy	Direct counting	Repeated Addtn	***Derived fact**	Derived fact
	Derived	*instant*	*instant*	*instant*
Kyle	Direct counting	Repeated Addtn	Direct counting	Direct counting
	instant	*instant*	*derived*	*instant*
Otis	Known Fact	Derived	***Derived**	Derived
	instant	*instant*	*derived*	*instant*
Molly	Repeated Addtn	Direct counting	***DirectCounting**	Direct counting
	instant	*instant*	*instant*	*instant*
Madison	Derived	Known Fact	***Derived**	Derived
	derived	*instant*	*known*	*instant*
William	Repeated Addtn	Direct counting	Direct counting	Repeated Addtn
	derived	*instant*	*derived*	*instant*
Simon	Repeated Addtn	Repeated Addtn	Repeated Addtn	Direct counting
	instant	*instant*	*derived*	*instant*
Fleur	Direct counting	Repeated Addtn	Direct counting	Direct counting
	derived	*instant*	** derived*	*derived*
Aaron	***Directcounting**	***Directcounting**	***Directcounting**	Direct counting
	derived	*instant*	*derived*	*derived*
Jacob	Direct counting	Direct counting	Direct counting	Direct counting
	instant	*instant*	*instant*	*instant*
Finn	Direct counting	*RepeatedAddtn	*Direct counting	*Direct counting
	direct counting	***direct counting***	**direct counting*	*direct counting*

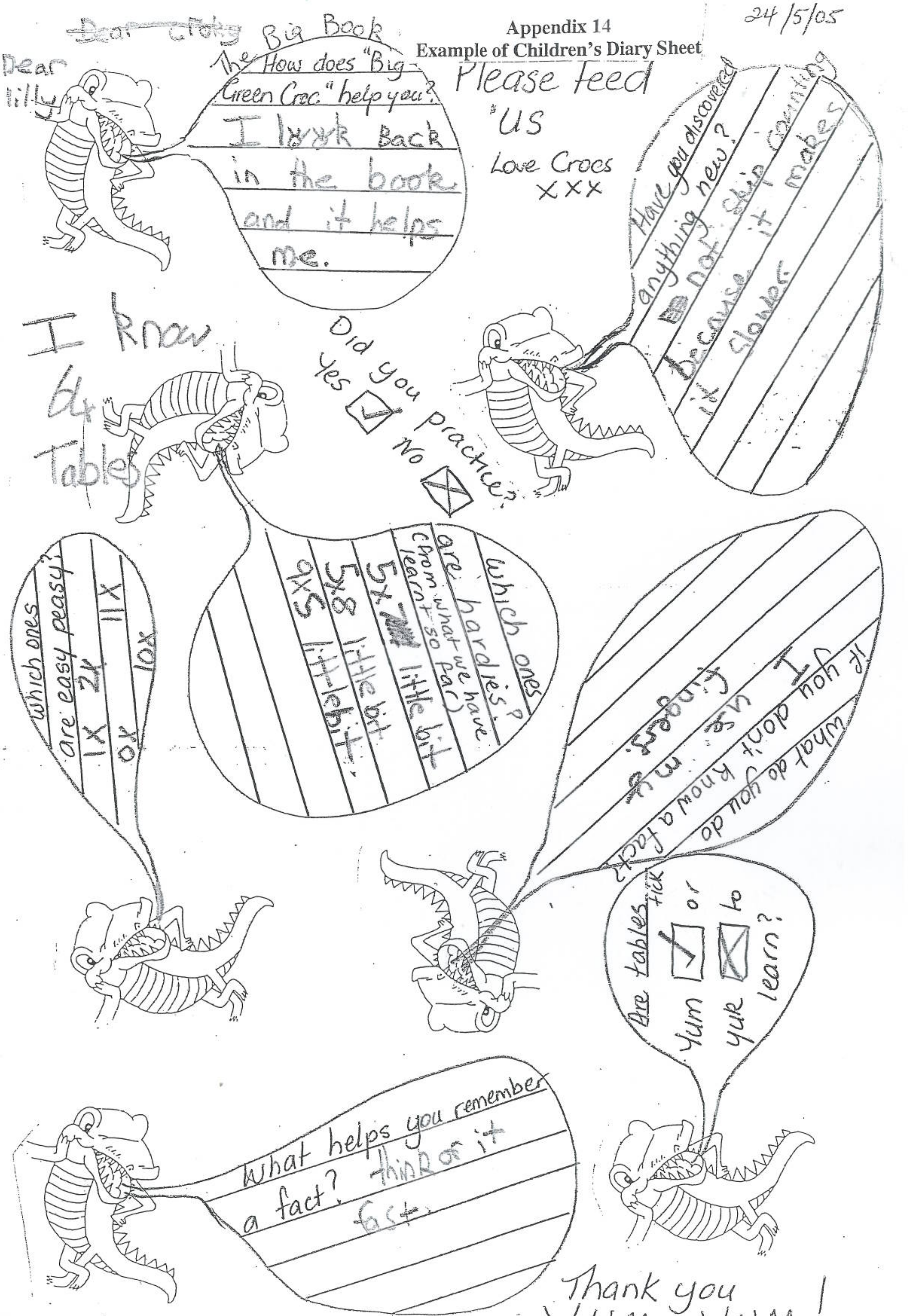
NB if incorrect but a strategy is used – the strategy category is used   Brackets = strategy stage

If a known fact is used and then the child has counted on = derived fact

***= Incorrect: Known = recalled from memory but not instant: Instant = instant recall**

Part B	**Diagnostic Test – Q 1**	**Diagnostic Test – Q 2**	**Diagnostic Test – Q3**	**Diagnostic TQ4**
Lucy	Known fact + addition (5)			
	EA (5) instant	*Derived fact (6)*	*Derived fact (6)*	
Kyle	Advanced Counting (4)			
	EA (5) instant	*Derived fact (6)*	*Derived fact (6)*	
Otis	Counting from One**I**(3)			
	EA (5) instant	*Derived fact (6)*	*Derived fact (6)*	
Molly	Advanced Counting (4)			
	EA (5) instant	*Derived fact (6)*	*Derived fact (6)*	
Madison	Known fact (5 - 6)			
	EA (5) instant	*Derived fact (6)*	*Derived fact (6)*	
William	Advanced Counting (4)			
	EA (5) instant	*Derived fact (6)*	*Derived fact (6)*	
Simon	Advanced Counting (4)			
	EA (5) instant	*Derived fact (6)*	*Derived fact (6)*	
Fleur	Advanced Counting (4)			
	EA (5) known	*Derived fact (6) slow*	*Derived fact (6) slow*	
Aaron	Advanced Counting (4)			
	EA (5) instant	*Derived but counted onF*	*Derived fact (6)*	
Jacob	Advanced Counting (4)			
	Advanced Counting (4)	**derived but incorrect*	**derived but incorrect*	
Finn	Advanced Counting (4)			
	Known	*Too hard*	*Too hard*	

Appendix 14
Example of Children's Diary Sheet



What do we know about the 5x table

0 x 5 = 0
1 x 5 = 5
2 x 5 = 10
3 x 5 = 15
4 x 5 = 20
5 x 5 = 25

(3 x 10 = 30)
6 x 5 = 30
7 x 5 = 35
8 x 5 = 40
9 x 5 = 45

10 10
20 20
30 30
40 40

If the number multiplied
...is odd (1, 3, 5...) the answer will be a odd number
... is even (2, 4, 6...) the answer is even

goes up in 5's
5, 10, 15 skip Count
We can do beter

you can count 5, 10, 15, 20, 25......
But it is beter to skipcounting
(eg? (10) (20) (30) (40)
(5, 10, 15, 20, 25, 30, 35, 40)!

~~evrey second~~
evrey second number is 5 and then 0...

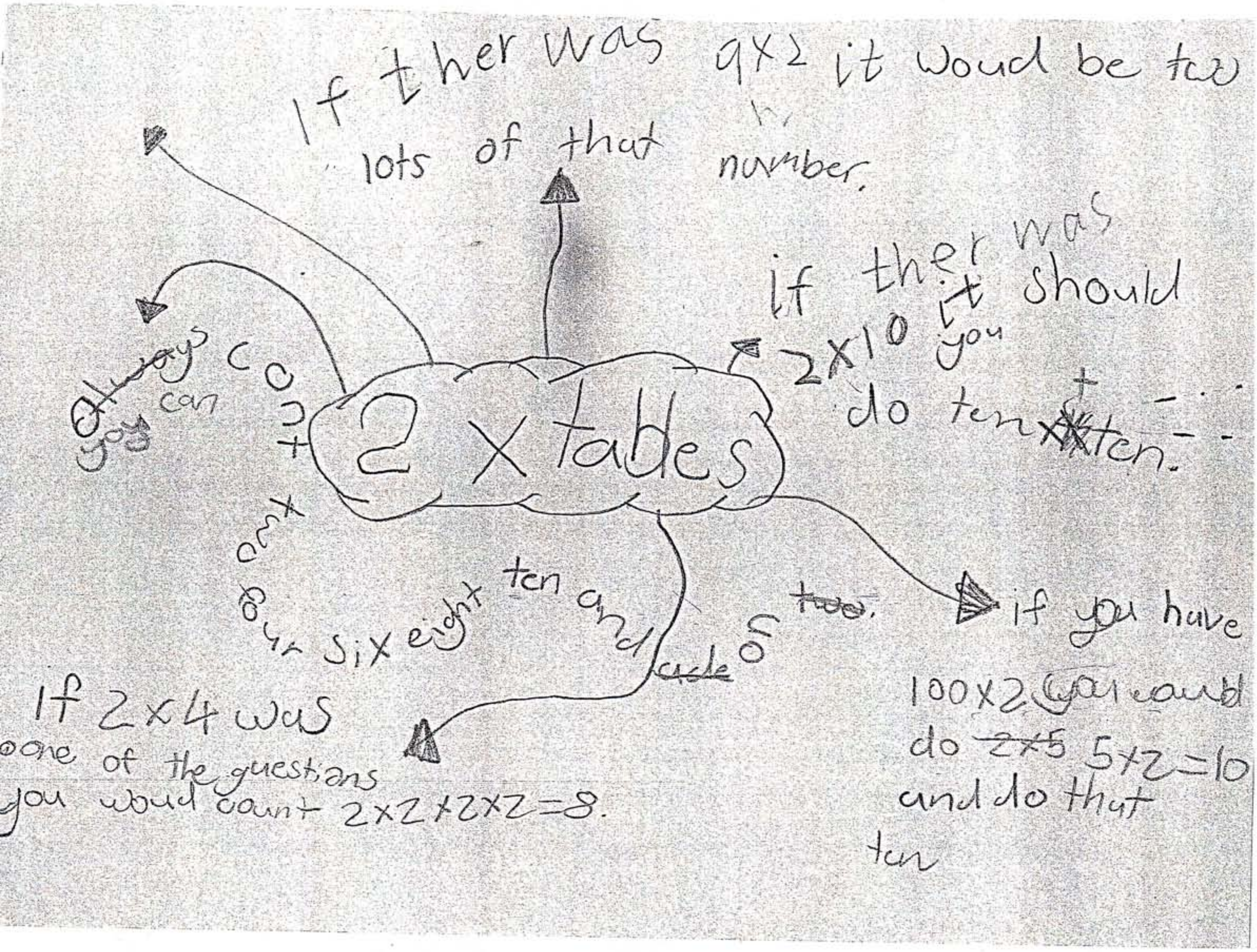
it goes odd even odd even

If you know your ÷ really well you can do 25 ÷ 5 = 5 you do it backwarks like 5 x 5 = 25

(Nichols) 2 x 5 = 10 + 10 = 4 x 5

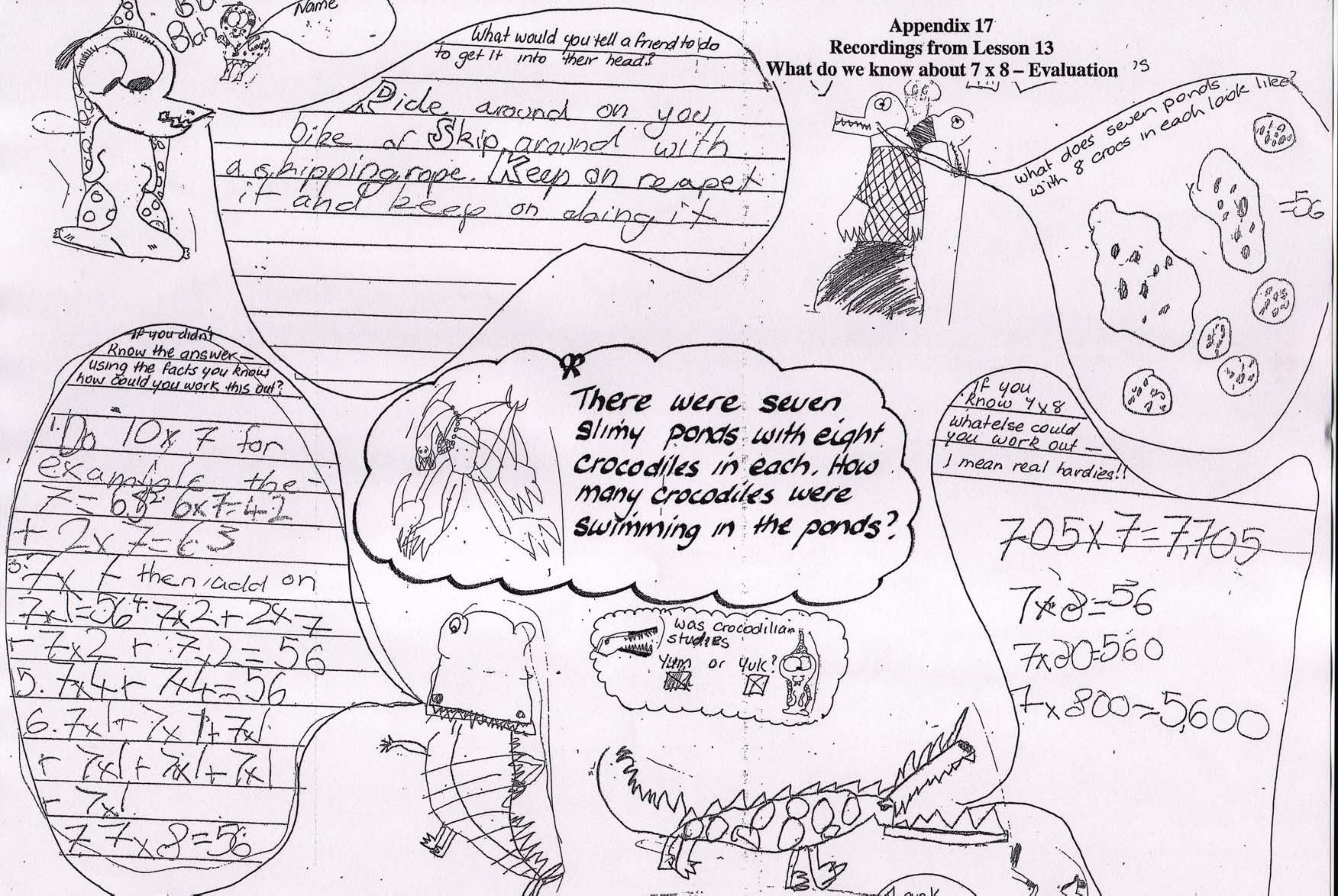
Appendix 16
An example of ideas, when discussing the 2x table, recorded by two children

(William and Kyle)



- if it was 9 x 2 it would be two lots of that number
- if there was 2 x 10 – it would be ten + ten
- if you have 100 x 2 do 5 x 2 = 10 and do that 10
- you can count two, four…ten and so on
- if it was 2 x 2 + 2 x 2 = 8

Appendix 17
Recordings from Lesson 13
What do we know about 7 x 8 – Evaluation



Appendix 18
Collation of Children's Ideas of 'What Puts Facts Into Your Head'

Date 2006	**Speed sheets**	**Practice saying the facts**	**"Think of fact until you find it"**	**Games**	**Big Green Croc**	**Flash cards**	**By timing yourself**	**Use a strategy**	**Tested by an adult**
22/3	1	4						3	4
14/4	6			7	5	2	1		
24/5	3	3	2	4	3			1	
26/5	2	6	1	1				1	
31/5	1	7		2		4		1	2
23/6	1	8		1		2		1	1
Total	**14**	**28**	**3**	**15**	**8**	**8**	**1**	**7**	**7**