Boundary Element Methods in Elastography - A First Explorative Study

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Introduction

In the field of preventative breast cancer screening, Digital Image Elasto-Tomography (DIET) is an emerging elastographic method intended for use as a low-cost breast cancer screening test. In the DIET-system, mechanical material parameters are reconstructed solely based on surface motion data that are obtained using digital cameras. Fig. 1 schematically shows the motion capture system, as currently employed for the application on gelatine phantoms.

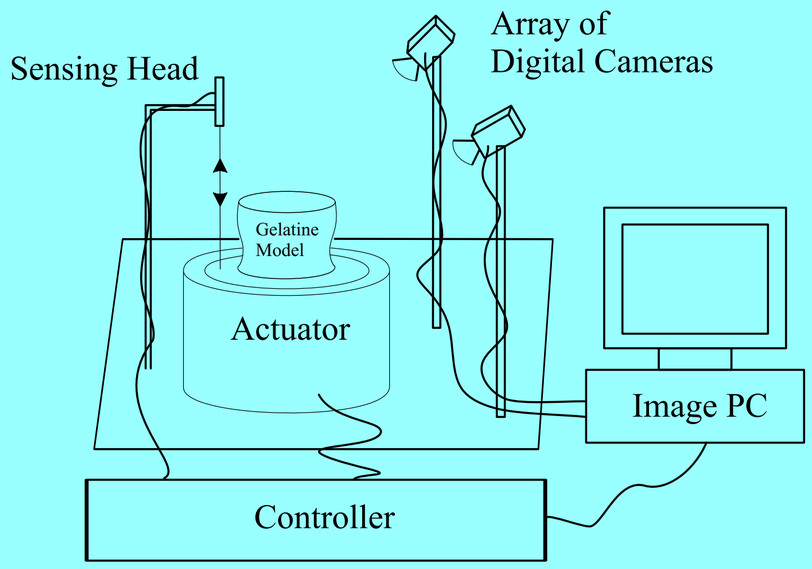


Figure 1. The DIET Motion Capture

The motion data obtained is now used in an optimization based inverse algorithm to reconstruct interior parameters, with an appropriate numerical forward solver embedded. Where traditionally Finite Element Methods that require the mesh of the full domain have been employed to solve the governing PDE on the given geometry, this study explores the use of the Boundary Element Method (BEM) that requires a surface mesh only.

Methods

The governing PDE for dynamic linear Elasticity is given by Navier's Equation in terms of displacements,

$$(\qquad)u_{j,ji} + \quad u_{i,jj} + b_i = \quad (d^2u/dt^2),$$

which can be transformed to it's corresponding Boundary Integral Equation,

$$c_{ij}(\mathbf{y})\, u_i(\mathbf{y}) + \quad T_{ij}(\mathbf{x},\mathbf{y})\, u_i(\mathbf{x})\, d\quad(\mathbf{x}) = \quad U_{ij}(\mathbf{x},\mathbf{y})\, t_i(\mathbf{x})\, d\quad(\mathbf{x})$$

using the fundamental solutions U_{ij} and T_{ij} for the static or harmonic case.

The BEM was implemented using a linear and quadratic boundary element discretization. Based on the displacement error between the measured and simulated surface displacements, the BEM was embedded into a non-linear Conjugate Gradient Method for the reconstruction of mechanical parameters. A square plate was used as a sample geometry (cmp. Fig.2), with the following conditions:

- Tensile test on a homogeneous plate (static)
  Reconstruction of Poissons Ratio
- Harmonic Actuation at its bottom side
  Reconstruction of Young's Modulus E
- Tensile test of inclusion problems of multiple size
  Reconstruction of Poisson's Ratio and
  Reconstruction of Elastic Contrast (main body to inclusion)

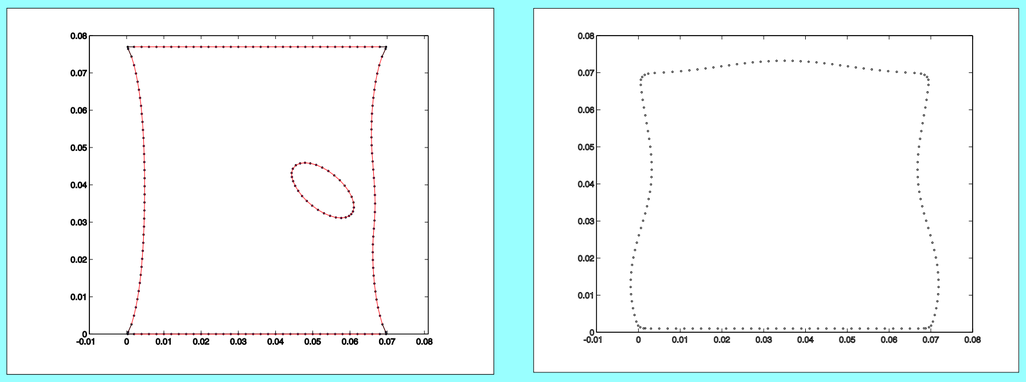
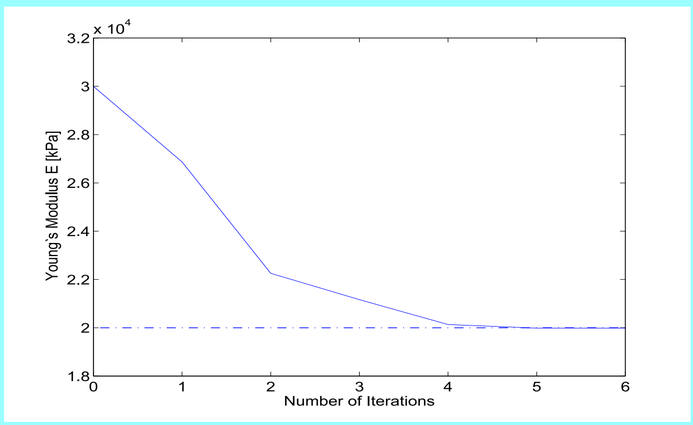


Figure 2. Static Inclusion and Dynamic Forward Simulation

Results

Figure 3. Convergence for the Reconstruction of Young's Modulus of a homogeneous plate from a harmonic BEM



Convergence behavior was observed to be good in all reconstruction problems with a small number of iterations required, but with sensitivity to start values when large amounts of noise was added. Fig. 3 shows the typical convergence behavior for any of the inverse prblems. Fig. 4 shows results for the reconstruction of Elasticity contrast for different inclusion sizes and noise levels.

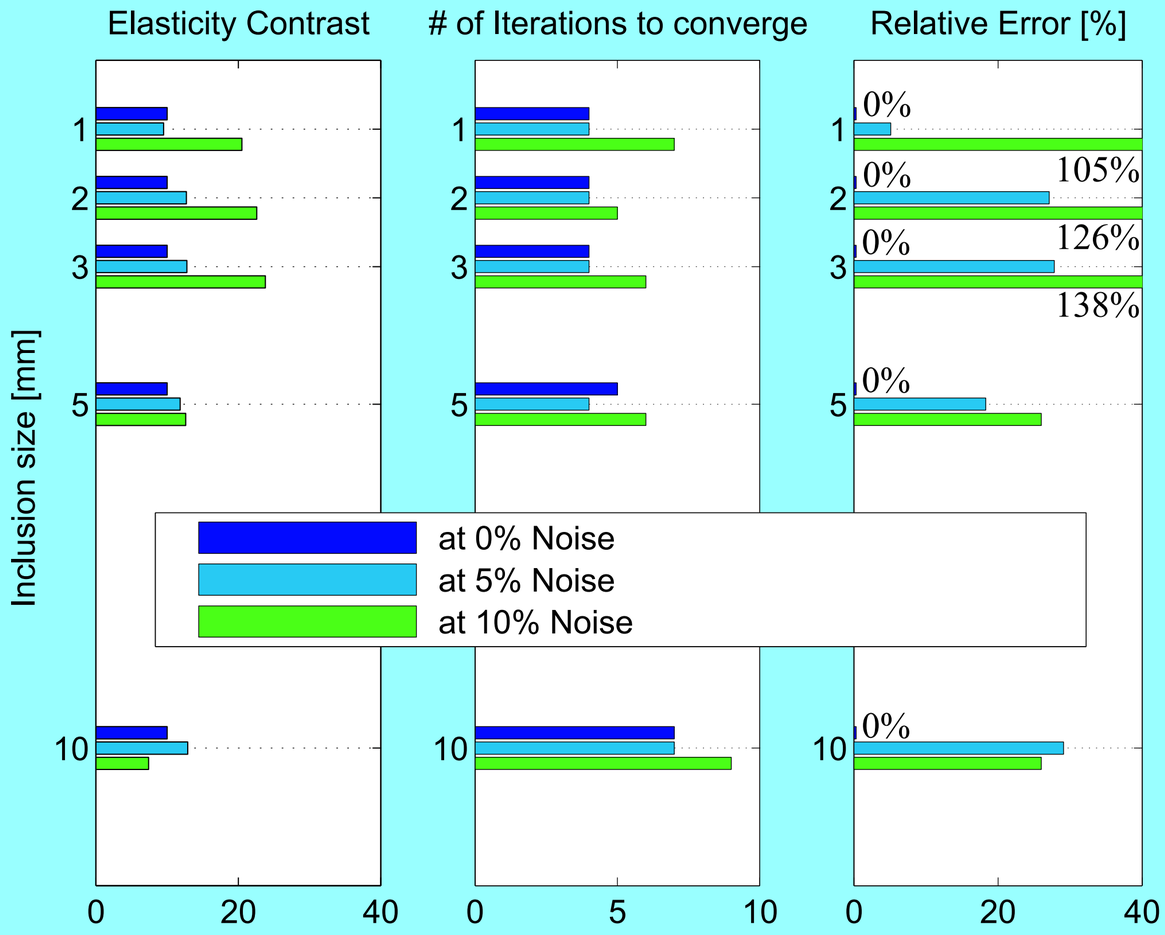


Figure 4. Results for the Reconstruction of Elasticity Contrast $= E_{II}/E_I$ for different inclusion sizes and noise levels

Conclusions

The use of the 2D BEM for the reconstruction of elastic parameters in the DIET-system has shown promising results. Elasticity information has been determined accurately and with fast convergence. Also, in the presence of low noise levels, acceptable results were achieved. Future work will focus on the extension of the method to a full 3D application and experimental testing on gelatine phantoms.