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Article (Accepted for Publication)
(Refereed)

Original Citation:
Proaño, Christian R. and Schoder, Christian and Semmler, Willi (2014) Financial stress, sovereign debt and economic activity in industrialized countries: Evidence from dynamic threshold regressions. *Journal of International Money and Finance*, 45. pp. 17-37. ISSN 02615606

Financial Stress, Sovereign Debt and Economic Activity in Industrialized Countries: Evidence from Dynamic Threshold Regressions

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February 13, 2014

Abstract

We analyze how the impact of a change in the sovereign debt-to-GDP ratio on economic growth depends on the level of debt, the stress level on the financial market and the membership in a monetary union. A dynamic growth model is put forward demonstrating that debt affects macroeconomic activity in a non-linear manner due to amplifications from the financial sector. Employing dynamic country-specific and dynamic panel threshold regression methods, we study the non-linear relation between the growth rate and the debt-to-GDP ratio using quarterly data for sixteen industrialized countries for the period 1981Q1-2013Q2. We find that the debt-to-GDP ratio has impaired economic growth primarily during times of high financial stress and only for countries of the European Monetary Union and not for the stand-alone countries in our sample. A high debt-to-GDP ratio by itself does not seem to necessarily negatively affect growth if financial markets are calm.

Keywords: financial stress, sovereign debt, economic growth, dynamic panel threshold regression

JEL Classification: E20, G15, H63

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1 Introduction

The 2007-08 global economic recession has been followed by a crisis in public finances, especially in the countries of the European Monetary Union (EMU). Naturally, a vast literature on the relationship between the fiscal stance and sovereign debt on the one hand and the macroeconomic performance on the other has emerged. While the previous work such as Blanchard and Perotti (2002) has typically used linear theoretical models and estimation techniques to study these relations, more recent contributions focus on the regime-dependence of the effects of fiscal policy and sovereign debt on economic activity.

An influential contribution is the study by Reinhart and Rogoff (2010) who identify the debt-to-GDP ratio as a relevant threshold variable in an empirical analysis of forty-four advanced and emerging economies. They argue that sovereign debt beyond a certain threshold, 90% of GDP according to their results, will reduce economic growth.[1] A second strand of research such as Mittnik and Semmler (2012*b*), Auerbach and Gorodnichenko (2012), Taylor et al. (2012) and Fazzari et al. (2013) investigates how the growth effect of the fiscal stance depends on the business-cycle. The core result is that fiscal multipliers are more pronounced during recessions then during booms. Finally, a third strand in the literature emphasizes *financial-market stress* as the crucial source for the non-linearity in the relationship between a country's fiscal stance and its macroeconomic performance. Afonso et al. (2011) and Mittnik and Semmler (2013), for instance, argue that the main factor determining the effectiveness of fiscal policy as well as the sustainability of fiscal debt is the state of financial markets, and not the extent of public indebtedness itself as postulated by Reinhart and Rogoff (2010).

The mechanism through which financial markets affect the relationship between the fiscal stance and sovereign debt on the one hand and economic activity on the other have been studied extensively in the theoretical literature (cf. Stein 2011, 2012, Brunnermeier and Sannikov 2012 and Mittnik and Semmler 2012*a*, 2013).[2] The common theme in this literature is that financial stress affects the

[1] As demonstrated by Herndon et al. (2013), however, coding errors and selective exclusion of observations biased the results of Reinhart and Rogoff (2010). Using the same data set, they argue that the threshold debt-to-GDP ratio of 90% is not significant and that growth effects of debt in high- and in low-debt regimes do not differ considerably.

[2] Stein (2011, 2012) emphasize over-leveraging of economic agents. Because of low interest rates, low credit spreads, rising capital gains and leveraging, economic booms may increase the vulnerability of the banking sector. Feedback loops between the financial sector and the macroeconomy may then give rise to a regime of low financial stress and a stable period of expansion, but may also generate destabilizing forces which trigger contractions and recessions when the financial sector starts to come under stress with risk premia rising and rapidly falling capital gains affecting aggregate demand and output. The models by Brunnermeier and Sannikov (2012) and Mittnik and Semmler (2012*a*) focus solely on the banking system which borrows to accumulate assets with returns, while there are preferences over payouts, serving as a consumption stream. When leveraging and payouts are less constrained, and financial stress and risk premia are high, the banking system is vulnerable and more prone to instability. With stronger restrictions, and low interest rates

relationship between debt and economic growth via its impact on risk premia, in particular bond spreads. For instance, Brunnermeier and Oehmke (2012) put forward the possibility of a *diabolic loop* according to which sovereign debt held by the banks can make the banking system unstable by exposing it to financial stress, forcing banks to cut down on loans, reducing economic growth and generating a downward spiral.

Given the crucial role of bond spreads for the effect of sovereign debt on economic growth, an additional aspect of the non-linear nexus between these two variables is the question whether a country is in a monetary union or not. As argued by De Grauwe and Ji (2013), this point is of particular relevance for the EMU since bond spreads may be more sensitive to investor sentiment in countries of a monetary union than in stand-alone countries. Their findings suggest that the explanatory power of the debt-to-GDP ratio for bond spreads in EMU countries is significantly higher during times of economic distress than during times of economic stability. Yet, for stand-alone countries, the relationship between the debt-to-GDP ratio and bond spreads is weak, independent of the state of the financial market. A similar result is obtained by Schoder (forthcoming) who additionally argues that investor sentiment is even more volatile for the EMU countries of the periphery than for the core EMU countries. According to these findings, the non-linearity in the sovereign debt-economic growth relation should be expected to be more pronounced in EMU countries, especially in the peripheral countries, than in stand-alone countries. The task taken up in this paper is to analyze empirically the non-linearities in the relationship between growth, sovereign debt and financial stress for core and peripheral EMU countries as well as stand-alone countries.

Our contribution to the literature is thus twofold: On the one hand, we discuss a theoretical framework along the lines of Mittnik and Semmler (2012*a*, 2013). In this model, debt affects in a non-linear manner the dynamics of the economy due to severe macroeconomic amplifications which arise when the financial sector comes under stress. We show that the macroeconomic amplifications crucially depend on whether there is low or high financial stress through the use of a finite horizon model. The theoretical model is solved numerically using a method known as Non-Linear Model Predictive Control (NMPC) which is a control technique used in industry since the 1980s and now introduced to economics by Grüne and Pannek (2011). It is characterized by the use of non-linear system models for prediction which are solved iteratively on a finite prediction horizon.

On the other hand, we estimate the effects of the sovereign debt-to-GDP ratio on economic growth

and low credit spreads there is a greater corridor of stability, creating a more stable environment for the banks. On the other hand, with less decision constraints and a banking system facing state-dependent risk premia and credit spreads which increases the cost of leveraging of banks, there may exist only a small corridor of stability, as banks may be more vulnerable to financial stress and crises. Schoder (2013) emphasizes asymmetric credit and profitability constraints on private investment which depend on the state of the business cycle.

through an econometric regime-dependent framework. We assume the regimes to depend on the level of the debt-to-GDP ratio and the level of the financial stress index (FSI). In particular, we analyze if the debt-to-GDP ratio's effect on economic growth is non-linear with respect to the debt-to-GDP ratio itself; if the debt-to-GDP ratio's effect on growth depends on financial stress; and finally, how the effect of debt on growth is related to both the level of debt and the level of financial stress. We investigate these relationships empirically for 16 OECD countries from 1981Q1 to 2013Q2, employing quarterly data and using dynamic country-specific and dynamic panel threshold regression analysis. We pay special attention to the differences between EMU and stand-alone countries as well as between northern EMU and southern EMU countries. Our sample includes Australia (AUS), Austria (AUT), Belgium (BEL), Canada (CAN), Denmark (DNK), Germany (DEU), Spain (ESP), France (FRA), the United Kingdom (GBR), Greece (GRC), Italy (ITA), Japan (JPN), the Netherlands (NLD), Portugal (PRT), Sweden (SWE), and the United States (USA).

Our estimation results suggest the following: First, in contrast to Reinhardt and Rogoff's (2010) results, we do not find evidence for a robust and significant threshold level of sovereign debt as a share of GDP beyond which a rise in debt reduces growth for any of our samples considered, if financial stress is ignored as a source of non-linearities in the debt-growth nexus. Neglecting the state of the financial markets, the debt-to-GDP ratio does not seem to be the relevant measure along which investor sentiment switches from growth-enhancing to growth-impairing. It seems that debt does not become a problem just because a country faces a high debt-to-GDP ratio. Second, consistent with our theoretical model, we identify financial stress as a crucial source of a non-linearity between debt and economic activity. Only at high levels of financial stress, the debt-to-GDP ratio may negatively affect growth, regardless of the level of debt. Yet, third, we find evidence that debt reduces growth during episodes of high financial stress only for the countries of our sample which are part of the European Monetary Union. Finally, our analysis suggests that the interaction of the level of debt and the level of financial stress is important for the debt-growth nexus. Especially in the southern EMU countries does a rise in debt impair growth at high levels of debt and financial stress.

Our work is mostly related to Afonso et al. (2011), in the sense that we investigate the interaction between public finances and economic growth from a non-linear perspective. However, in contrast to Afonso et al. (2011), who use country-specific threshold VAR models to investigate the interaction between GDP growth, fiscal variables, inflation, the interest rate and using the FSI as the threshold variable, we focus solely on how GDP growth is affected by more or less the same variables, but using a threshold dynamic panel approach to exploit the cross-sectional dimension of the data using not only the FSI, while also using the debt-to-GDP ratio in an alternative specification. Our work can thus be considered as complementary to Afonso et al. (2011) not only in terms of the choice of variables and countries, but also because our results widely support and extend their results on the basis of the estimation of different country subgroups in a dynamic panel context.

The remainder of the paper is organized as follows: In the next section we investigate the dynamics of a stylized dynamic macroeconomic model of a small open economy in low and high financial stress regimes. Section 3 discusses the data set used, followed by a discussion of the econometric methodology in section 4. Section 5 presents the estimation results and section 6 concludes the paper.

2 A Model of Economic Activity and Debt

In the following, we briefly discuss a model variant of Mittnik and Semmler (2012*a*) to describe the dynamic interaction of aggregate indebtedness and macroeconomic activity in low and high financial stress regimes. This will motivate our subsequent empirical analysis on sovereign debt, financial stress and GDP growth. Even though the current theoretical framework focuses on aggregate debt, its insights are applicable to the empirical analysis of the next section, which focuses on the interaction between sovereign debt and GDP growth, due to the high correlation between current account and fiscal imbalances (cf. Berger and Nitsch 2010).

Let us consider a stylized model economy in which a social planner chooses the levels of consumption c_t and the growth rate of the capital stock g_t such as to maximize inter-temporal utility. The problem can be stated as

$$V\left(K,B\right)=\max_{C_t,g_t,}\int_0^T e^{-rt}U(C_t)dt \tag{1}$$

subject to

$$\begin{aligned} dK_t &= (g_t-\delta)K_t dt && (2)\\ dB_t &= (r_tB_t-(Y_t-C_t-I_t-\varphi(g_tK_t)))dt && (3)\\ Y_t &= K_t^{\alpha} && (4)\end{aligned}$$

where K_t is the capital stock, $I_t=g_tK_t$ aggregate investment, δ the depreciation rate of capital, r_t the real interest rate, and B_t the economy's aggregate debt (private and public).[3] Eq.(2) describes the evolution of the economy's capital stock, and eq.(3) the aggregate debt's evolution, where $\varphi(\,\cdot\,)$ stands for the quadratic adjustment of costs of investment (cf. Blanchard and Fischer 1989, ch.2 and Stein 2012, ch.8).[4] Eq.(4) is the production function.

[3] We could also allow for a spread between interest and discount rates as done, for example, in models with two types of agents such as Eggertsson and Krugman (2011) without changing the results qualitatively. Further, even though our model is interpreted in the context of an open economy, it can also be interpreted as a closed economy with borrowers and lenders as in Eggertsson and Krugman (2011).

[4] Brunnermeier and Sannikov (2012) include a stochastic shock in a Brownian motion with volatility-dependent asset

According to eq.(1) the social planner maximizes the discounted utility of private consumption not over an infinite time horizon, as it is standard in the literature, but over a finite horizon given by T. We make this assumption to keep the link between the current state of the economy and the social planner's decisions. This also allow us to stay with the general notion of forward-looking, inter-temporal utility maximizing behavior. Yet, as is widely known, a recurrent problem of infinite horizon models is the volatility of macroeconomic variables being too low which results from the extreme consumption smoothing resulting precisely from the infinite horizon underlying the decision rules of the agents. In order to solve this forward-looking, finite horizon model in such a way that its solution may be consistent with the infinite horizon solution if $N \to \infty$, we apply a technique known as Non-Linear Model Predictive Control recently introduced in economics for solving inter-temporal optimization problems by Grüne and Pannek (2011). We briefly describe this methodology which, in a nutshell, can be characterized as the repeated solution of a finite-horizon open-loop optimal control problem subject to system dynamics and state constraints. Details are spelled out in the Appendix A.

In order to model explicitly the non-linear relationship between debt and economic activity, we allow the interest rate on debt to depend on the level of debt and on the state of the financial market. Hence, we focus on the bond yields as one transmission channel among others through which the state of the financial market may affect the nexus between debt and economic activity. In order to resemble the observed behavior of the Euro Area government bond yields since the outbreak of the debt crisis, we assume that the bond yields are a non-linear function of the economy's aggregate debt-to-capital ratio B_t/K_t. In the vein of De Grauwe and Ji (2012), we further assume that the level of financial stress f and the membership in a monetary union $m = \{0, 1\}$ affects the relationship between the debt-to-capital ratio and the interest rate. Hence, we suppose

$$r(B_t/K_t) = \bar{r} + \theta(f, m) \arctan(B_t/K_t), \tag{5}$$

with $\theta(0, 0) = 0$, $\partial\theta(f, 1)/\partial f > \partial\theta(f, 0)/\partial f > 0$ and $\theta(f, 1) > \theta(f, 0)$ for all f. $\bar{r}$ denotes the interest rate independent from debt. Since investor sentiment has been found to be more volatile with respect to the debt-to-GDP ratio in countries within a monetary union than in stand-alone countries, we assume that the state of the financial markets affects the impact of debt on the interest rate primarily when $m = 1$ (cf. De Grauwe and Ji 2012, 2013). For members of a monetary union this implies the following: When both the debt-to-capital ratio and the financial stress are low, the link between these variables and the bond yields is weak and the yield level itself is low too. In contrast, when either B_t/K_t and f_t or both are high the link between these variables and $r(\cdot)$ is tighter, and the yield level

prices, and formulate the debt dynamics as a net worth dynamics but in a closed economy framework. In our open economy framework, in contrast, we implicitly also allow for sovereign debt, and could also include stochastic shocks.

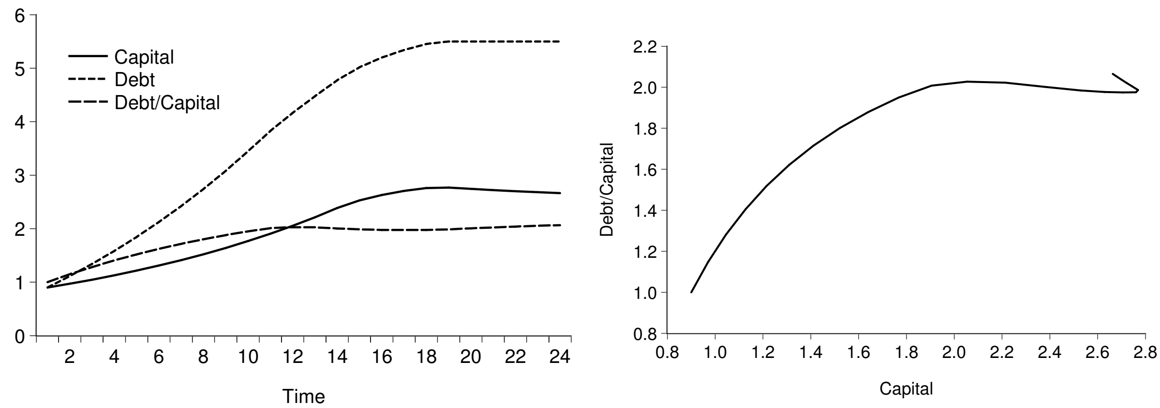

Figure 1: Convergent dynamic paths of sovereign debt for constant interest rate for initial conditions $K(0) = 0.9$, $B(0) = 0.9$, with $r = 0.02$.

is likely to be much higher too.

In order to highlight how the model's properties are affected by the economy's overall indebtedness and/or financial stress levels, in the following we study the dynamics of the system under a regime of low financial stress for a stand-alone country and a regime of high financial stress for a country within a monetary union. In the first model variant the interest on aggregate debt is held constant. This is scenario in the vein of Minsky where financial fragility may arise in a period of tranquility and thus low or zero risk premia can be observed.[5]

The left graph in Figure 1 shows the NMPC solution paths for capital, debt and the debt-to-capital ratio over time for $\theta(\cdot) = 0$ and $\bar{r} = 0.02$ such that $r = 0.02$ with the initial conditions $K(0) = 0.9$ and $B(0) = 0.9$. The right graph plots the debt-to-capital ratio and the capital against each other, with the debt-to-capital ratio on the vertical axis and the capital stock on the horizontal axis. As can be observed, if the interest rate remains constant at $r = 0.02$, while the path of aggregate debt and capital still feature a positive growth rate, the debt-to-capital ratio converges to a given value which describes the steady state of the economy (this is a property of the present model which applies for multiple initial values of K and B).

As this first simulation shows, as long as the financial stress is low (either through central bank intervention or as a result of the overall market expectations) and the bond yields remain by and large invariant at low levels, the debt-to-capital ratio will eventually stabilize about a finite ratio,

[5]Implicitly, in this case, on the asset side, as Stein (2012) shows, the present value of the assets will tend to become very large, because there is no correction through a risk premium, and capital gains help to service the debt. Alternatively, this is equivalent to the case of the central bank pursuing a low – or near zero – interest rate policy. By this, in fact it might attempt to keep the economy in a low financial stress regime, see Christiano et al. (2011), and Woodford (2011).

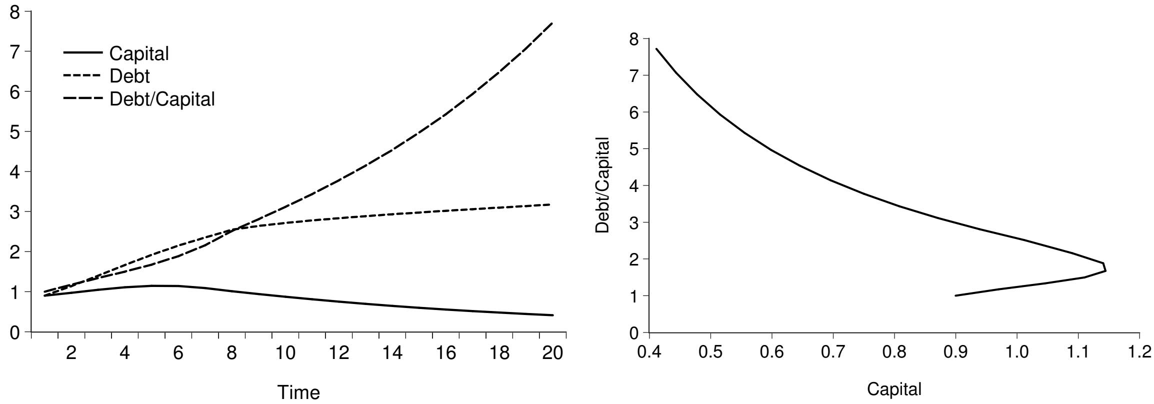

Figure 2: Debt dynamics with endogenous interest rate starting from the initial conditions $K(0) = 0.9$, $B(0) = 0.9$.

with capital growing at a positive rate. However, precisely in such a stable macroeconomic scenario an over-leveraging process may take place which may induce financial instability in the Minsky (1992) sense.

Figure 2 illustrates in contrast the dynamics of a scenario where the interest rate on debt is a non-linear function of the debt-to-capital ratio which is affected by the state of the financial markets and the membership in a monetary union as in eq.(5) starting from the same initial values as in the previous simulation and with $\theta(\cdot) = 0.1$. As can be clearly observed, a contraction of the capital stock and an accelerating increase both in the level of aggregate debt as well as in the debt-to-capital ratio takes place due to the activation of the non-linear negative effects of financial stress on the government debt yields and the subsequent dynamic feedback mechanisms of higher risk premia and yields, larger interest payments and lower consumption and investment.[6] This, of course, is an unsustainable scenario, as additional increases in the interest rate on debt lead to further increases in the interest payments, which will eventually become larger than the surplus generated to pay such debt service.[7]

Before turning our attention to the econometric analysis, we would like to stress again the fact that these simulated dynamics represent the optimal solution of a finite-horizon dynamic problem, and are thus consistent with a forward-looking, utility maximizing modeling approach. However, it

[6]Note that a strong contractionary effect could also occur if the creditors become unwilling to lend when a certain debt to GDP ratio is reached and new borrowing or rolling over of old debt will be discontinued. For a model including such a sudden rise in credit market constraint, see Ernst and Semmler (2013).

[7]The shift to a high financial stress regime could also be related to a situation in which the central bank either does not attempt or is not able to pursue an unconventional monetary policy to bring down credit spreads in order to calm down the financial markets. For an empirical contribution studying the sustainability of debt accumulation of European economies, see Schoder et al. (2013).

is precisely due to the finite planning horizon of the current framework that the relationship between the dynamics of debt and capital is much closer in the medium term than what it would be in an infinite horizon framework.

In the following we investigate empirically how sovereign debt and financial stress affect the relationship between debt and economic activity in a representative sample of 16 industrialized countries for the period 1981Q1 to 2013Q2 by means of dynamic country-specific and dynamic panel estimation techniques.

3 Data Description

For our empirical analysis, we use a quarterly and seasonally adjusted data set ranging from 1981Q1 to 2013Q2 for the following 16 countries: Australia (AUS), Austria (AUT), Belgium (BEL), Canada (CAN), Denmark (DNK), Germany (DEU), Spain (ESP), France (FRA), the United Kingdom (GBR), Greece (GRC), Italy (ITA), Japan (JPN), the Netherlands (NLD), Portugal (PRT), Sweden (SWE) and the United States (USA).[8] The following variables are used:

The growth rate of GDP at constant prices has been obtained from the OECD Economic Outlook (EO) 94 database (variable GDPV). The debt-to-GDP ratio is defined as nominal net financial liabilities of the general government over nominal GDP. For AUS, CAN, JPN and USA, the variable has been obtained from the OECD EO 94 database (variable GNFLQ). For the other countries in the sample, data on net financial liabilities have been interpolated and extrapolated, respectively, from annual data obtained from the OECD EO 94 database (variable GNFL) as well as from quarterly government net lending data and quarterly gross government debt data obtained from different sources following the procedure proposed by Chow and Lin (1971). The constructed net financial debt series is then normalized by GDP taken from the OECD EO 94 database (variable GDP). Appendix B spells out the details on how the net financial debt series has been constructed for each country. The financial stress index (FSI) has been obtained from the IMF data set as discussed in Danninger et al. (2009) and Cardarelli et al. (2009).[9] For GRC, the growth rate of the labor force is taken from the OECD Main Economic Indicators (MEI) database. For all other countries, it has been obtained from the OECD EO 94 database (variable LF). As additional variables, we use the interest rate on long-term government bonds minus the growth rate of the GDP deflater, the nominal effective exchange rate as

[8] For AUS, GRC and PRT the ranges of the data set including the variables used in the baseline specification discussed below are from 1987Q3-2012Q4, 1990Q2-2012Q4, 1995Q1-2013Q2 and 1984Q1-2013Q3, respectively.

[9] The FSI is a composite indicator comprising information on the *banking-sector beta*, stock market returns, time-varying stock market return volatility, sovereign debt spreads, and an exchange market pressure index. Afonso et al. (2011) use the same variable in their estimations of country-specific threshold vector auto-regressions.

well as the growth rate of the GDP deflater all taken from the OECD EO 94 database (variables IRL, PGDP and EXCHEB).

The debt-to-GDP ratio, the financial stress index and the growth rate of GDP in percent are plotted in figure 3 for all countries investigated. The net sovereign debt-to-GDP ratio differs considerably between countries, with SWE and DNK having the lowest ratios at the end of the sample considered, as they managed to reduce their net sovereign debt considerably since the mid 1990s. In contrast, the highest current debt-to-GDP ratio among the considered countries can be observed in JPN, ITA and GRC with levels beyond 100% of GDP. Further, apart from the level of the debt-to-GDP ratio its evolution over time is of interest. Countries such as BEL, CAN, DNK, ESP and NLD managed to reduce their debt-to-GDP ratio since the mid 1990s. Others, in particular AUT, FRA, GBR and ITA more or less stabilized the ratio in that time, whereas DEU and JPN experienced an increase in the debt ratio. In GRC, the haircuts in 2011 can be clearly seen in the data. In general, it is interesting to note that the troubled southern EMU countries do not face much higher debt-to-GDP ratios than other countries that are rather considered as financial safe havens such as the USA, JPN or DEU.

Concerning the dynamics and the interaction of the FSI and the GDP growth, as Fig. 3 clearly illustrates, there is a strong negative correlation between GDP growth and the FSI, not only during the recent financial crisis, where GDP growth slumped in the majority of countries around the world and the FSI spiked to unprecedented levels, but also in the previous periods. This is not surprising. Yet, due to the way the FSI is constructed, a drop in stock prices, for instance – a phenomenon which is closely linked to the occurrence of economic recessions – leads to an increase in the FSI. In this context, it is interesting to note that while the output contraction during the recent financial crisis was rather similar across all countries considered, the FSI reaction was, on the other hand, quite differentiated, with the FSI of countries such CAN, JPN, NLD and USA soaring about three times higher than in the remaining countries.

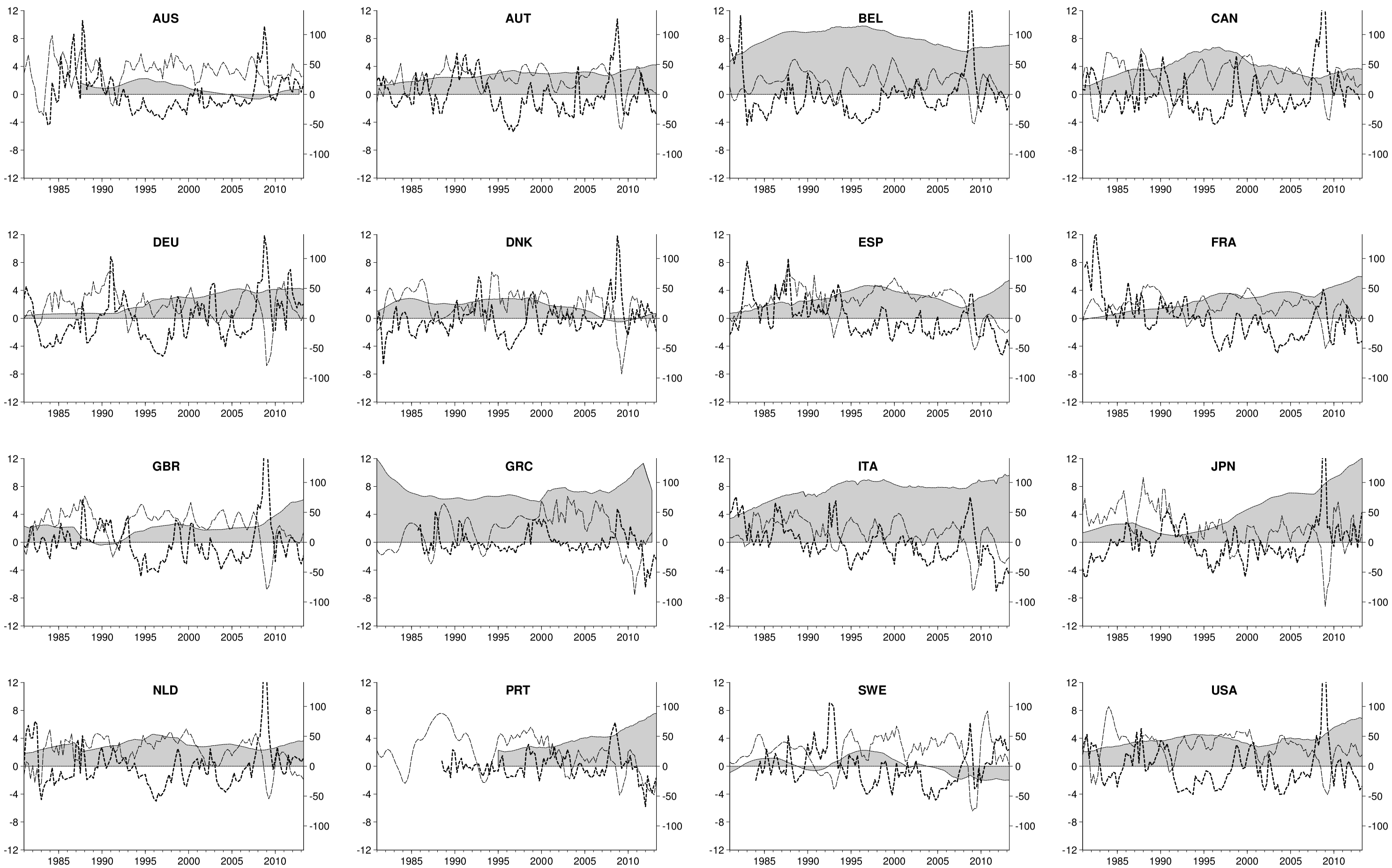

Figure 3: Net sovereign debt-to-GDP ratio (shaded area, left axis), year-to-year GDP growth rate (solid line, right axis) and financial stress index (dashed line, right axis) from 1981:1 to 2010:4

4 Econometric Methodology

In order to analyze how the level of debt as well as the level of financial stress affect the impact of the debt-to-GDP ratio on the growth rate of GDP in EMU and non-EMU countries, we employ country-specific and panel threshold regressions. Threshold regressions allow us to distinguish low-debt/high-debt and low-stress/high-stress regimes, respectively. Panel regressions allow us to additionally exploit the information along the cross-section dimension to obtain more robust results, as well as to identify potential differences between particular country sub-samples. As sub-samples we consider EMU and non-EMU countries as well as northern and southern EMU countries. More specifically, we study first if the debt-to-GDP ratio's effect on economic growth is non-linear with respect to the debt-to-GDP ratio itself; second, how the debt-to-GDP ratio's effect on growth depends on financial stress; and finally, how the effect of debt on growth is related to both the level of debt and the level of financial stress.

To briefly discuss the estimation strategy, consider the following dynamic threshold regression model:

$$y_{it} = \mu_i + \mathbf{z}_{it}\boldsymbol{\alpha} + x_{it}\beta^L I(q_{it} \le \gamma) + (x_{it}\beta^H + \delta^H)I(q_{it} > \gamma) + \varepsilon_{it} \tag{6}$$

where $i = 1, \ldots, N$ denotes a country-index, $t = 1, \ldots, T$ the time index. μ_i is a country-specific fixed effect, ε_{it} an *i.i.d.* country-specific random disturbance with zero mean and a variance $\sigma^2_{\varepsilon i}$, and $\mathbf{z}_{it}$ a vector which contains the lagged dependent variable as well as other endogenous and exogenous explanatory regime-independent variables. $I(\,\cdot\,)$ is an indicator function which takes on the value of one if the condition in its argument holds and otherwise takes on a value of zero. q_{it} is the threshold variable and γ is a given threshold level. x_{it} is an exogenous variable and δ is the difference between the intercept in regime L, i.e. μ_i, and the intercept in regime H, i.e. $\mu_i + \delta$. The coefficients β^L and β^H represent the effect of the variable x_{it} on the dependent variable y_{it} for $q_{it} \le \gamma$ and $q_{it} > \gamma$, respectively.

Hansen (1999) proposed a strategy for estimation of and inference for non-dynamic panel threshold regression models with exogenous covariates. The threshold value is determined endogenously. It is the realization of the threshold variable which minimizes the residual sum of squares across specifications with different threshold values. For time series or cross sections, Caner and Hansen (2004) developed an IV-estimator for the threshold model to account for endogenous covariates. The lags of the endogenous regressors are used as instruments. Analyzing the threshold effect of inflation on economic growth, Kremer et al. (2013) extended Caner and Hansen's 2004 IV threshold estimation methodology for the case of panel data. We extend the methodology used in the latter contribution by considering up to two threshold variables and, hence, up to four regimes.

In order to use the estimation procedure proposed by Caner and Hansen (2004) in a dynamic panel framework, the country-specific fixed effects have to be eliminated without violating the asymptotics of the estimator. We follow Kremer et al. (2013) and apply the forward orthogonal deviations transformation to the variables: each observation has subtracted from it the mean of all future observations. As shown by Arellano and Bover (1995), this transformation eliminates the country-specific fixed effects while leaving the lags of the endogenous variables available as valid instruments.

In our study the following estimation strategy is employed: First, for the panel analysis, the forward orthogonal deviation transform is applied to all variables expect the threshold variable(s). For the country-specific regressions, the variables are not transformed. Second, the endogenous variables are regressed on the complete set of instruments. Then, for given threshold values, the endogenous variables are replaced by the fitted values in the threshold regression model. These steps are repeated for all realizations of the threshold variables, except the lowest and largest 5% in order to ensure that there is a minimum number of observations in each regime. The values which are associated with the smallest sum of squared residuals are then selected as the threshold values. Inference on the estimated threshold value can be conducted by noting that a likelihood ratio for a given threshold value can be constructed by using the relation between the residual sum of squares associated with this threshold value and the residual sum of squares associated with the threshold estimate (cf. Caner and Hansen, 2004). Third, the regression model is estimated again using the threshold value obtained in the previous step.

5 Estimation Results

We perform dynamic country-specific and dynamic panel threshold regressions. For the panel estimations, we consider the following groups of countries: all countries of the sample, EMU countries (AUT, BEL, DEU, ESP, FRA, GRC, ITA, NLD, PRT), non-EMU countries (AUS, CAN, DNK, GBR, JPN, SWE, USA), northern EMU countries (AUT, BEL, DEU, FRA, NLD) and southern EMU countries (ESP, GRC, ITA, PRT). We analyze, first, how debt affects the impact of debt on growth, second, how financial stress affects the impact of debt on growth and, third, how both debt and financial stress affect the impact of debt on growth.

5.1 Debt-to-GDP ratio as threshold variable and regime dependent regressor

Let us first consider a specification which is appropriate to address the research question raised by Reinhart and Rogoff (2010): Is there a threshold level of the debt-to-GDP ratio beyond which a rise in the debt-to-GDP ratio reduces the growth rate of the economy? Hence, the debt-to-GDP ratio is assumed to be the only threshold variable. The model is

$$y_{it} = \mu_i + \alpha_y y_{i,t-1} + \alpha_n n_{it} + \beta_b^L b_{i,t-1} I(b_{i,t-1} \leq \gamma_b) + (\beta_b^H b_{i,t-1} + \delta^H) I(b_{i,t-1} > \gamma_b) + \varepsilon_{it} \qquad (7)$$

where y_{it} is the quarter-to-quarter growth rate of real GDP, n_{it} is the growth rate of the labor force, and b_{it} is the net sovereign debt-to-GDP ratio. Since, the regressor $y_{i,t-1}$ is endogenous, its lags are used as instruments after applying the orthogonal forward transformation to the regressors.[10] The regime-independent regressor n_{it} as well as the regime-dependent regressor $b_{i,t-1}$ are assumed to be exogenous. We have also considered specifications including the real interest rate on long-term government bonds, the nominal effective exchange rate, the rate of inflation based on the GDP deflater and the financial stress index as additional control variables. Since the results do not change substantially by extending the baseline model, we only report the results of the more parsimonious specification. Yet, we indicate below when the results of the more general specifications differ substantially.[11]

Table 1 reports the results of the country-specific threshold regressions followed by the results of the panel threshold regressions. Since the estimated threshold values would be quite heterogeneous and often indicating break points at rather low debt-to-GDP ratios and since we are interested in how the growth effect of public debt changes at high levels of debt as a percentage of GDP as suggested by Reinhart and Rogoff (2010), we specify that, instead of 5%, the low-debt regime has to include at least 50% of all observations. Moreover, we restrict the intercept to be regime-independent to increase the robustness of the results to including additional control variables.

For the single countries, the estimated threshold values γ_b are quite heterogeneous ranging from 5.42% in Sweden to 112.50% in Belgium. Yet, this is mainly due to the different ranges of the sovereign debt-to-GDP ratio in the countries of our sample. Note that the estimated thresholds are not robust for most single countries as well as groups of countries. For the groups of EMU, non-EMU, northern EMU and southern EMU countries, figure 4 plots the likelihood ratio for every γ_b in order to test the null hypothesis that the true threshold value is equal to γ_b, given the estimate $\hat{\gamma}_b$, as well as the critical value for the 95% confidence level. The set of γ_b's for which the null cannot be rejected is the domain with likelihood ratios below the line indicating the critical value. For the panels of EMU and southern EMU countries, no feasible threshold value can be excluded from this set. For no single feasible threshold value can we reject the null that this threshold value is the true one at a reasonable level of significance. For the non-EMU countries, the plot of the likelihood ratios suggests that the true threshold value of the debt-to-GDP ratio lies between 33% and 46%. For the northern EMU

[10]In all specifications to follow, we use the maximum number of instruments available for each endogenous regressor unless the instrument matrix is close to singular. In this case we keep reducing the number of instruments for each regressor by 10 and re-estimating the model until the instrument matrix is clearly full-rank.

[11]The estimation results for the extensions of the baseline model can be obtained from the authors upon request.

Table 1: Country-Specific and panel dynamic threshold GMM estimation results with the debt-to-GDP ratio as the threshold variable and the debt-to-GDP ratio as a regime dependent regressor (equation 7)

	γ_b	L	H	α_y	α_n	β_b^L	β_b^H
	COUNTRY-SPECIFIC THRESHOLD REGRESSIONS						
	Northern EMU countries						
AUT	42.13 [34.31,42.13]	119	10	0.432*** (0.069)	−0.048 (0.066)	0.011*** (0.002)	0.002 (0.001)
BEL	112.50 [94.39,112.50]	119	10	0.673*** (0.043)	−0.089 (0.060)	0.001*** (0.000)	0.004*** (0.001)
DEU	35.34 [32.36,48.87]	80	49	0.135** (0.065)	0.647*** (0.234)	0.013*** (0.003)	0.005*** (0.002)
FRA	42.81 [35.37,55.82]	100	28	0.603*** (0.053)	0.113** (0.052)	0.006*** (0.001)	0.002** (0.001)
NLD	43.80 [34.54,48.15]	108	21	0.151** (0.061)	0.488*** (0.181)	0.009*** (0.002)	0.011*** (0.002)
	Southern EMU countries						
ESP	33.72 [33.72,52.97]	67	62	0.088 (0.064)	0.742*** (0.096)	0.008*** (0.003)	0.004*** (0.001)
GRC	91.53 [83.53,108.43]	75	16	0.075 (0.073)	−0.128 (0.086)	0.009*** (0.001)	−0.003*** (0.001)
ITA	91.98 [91.98,103.70]	67	62	0.460*** (0.065)	−0.073 (0.100)	0.003*** (0.001)	0.001** (0.001)
PRT	49.99 [46.43,71.72]	52	21	0.228*** (0.074)	0.341 (0.207)	0.008*** (0.002)	−0.004*** (0.001)
	Non-EMU countries						
AUS	11.20 [10.42,22.53]	60	41	0.237*** (0.068)	0.624*** (0.103)	−0.003 (0.011)	0.023*** (0.005)
CAN	49.75 [42.56,72.80]	90	38	0.506*** (0.052)	0.467*** (0.125)	0.001 (0.002)	0.004*** (0.001)
DNK	27.42 [23.84,32.80]	85	44	−0.261*** (0.050)	0.200** (0.096)	0.014*** (0.004)	0.031*** (0.004)
GBR	25.90 [25.00,49.35]	82	47	0.553*** (0.051)	−0.079 (0.171)	0.018*** (0.003)	0.005*** (0.002)
JPN	32.53 [32.53,111.20]	67	62	0.082 (0.066)	0.042 (0.206)	0.033*** (0.005)	0.002** (0.001)
SWE	5.42 [1.56,23.89]	70	47	0.357*** (0.055)	0.433*** (0.115)	−0.012*** (0.004)	0.025*** (0.005)
USA	59.06 [44.37,67.74]	115	14	0.416*** (0.069)	0.128 (0.168)	0.009*** (0.001)	0.005*** (0.001)
	PANEL THRESHOLD REGRESSIONS						
All	42.17 [34.60,99.70]	1189	739	0.360*** (0.027)	0.177*** (0.045)	0.003 (0.002)	−0.001 (0.001)
EMU	54.18 [41.67,103.74]	690	376	0.443*** (0.031)	0.121** (0.051)	0.002 (0.002)	−0.001 (0.001)
Non-EMU	42.19 [25.89,75.09]	649	213	0.233*** (0.038)	0.275*** (0.073)	0.007*** (0.002)	0.000 (0.001)
North EMU	71.96 [36.15,104.28]	525	119	0.380*** (0.035)	0.112* (0.063)	−0.002 (0.002)	0.003* (0.002)
South EMU	91.82 [68.24,102.68]	341	81	0.424*** (0.042)	0.158** (0.065)	−0.001 (0.002)	−0.004** (0.002)

Notes: Standard errors are in parenthesis. ***, ** and * denote the level of significance at 0.01%, 0.05% and 0.1%, respectively. The interval of feasible threshold values is in brackets.

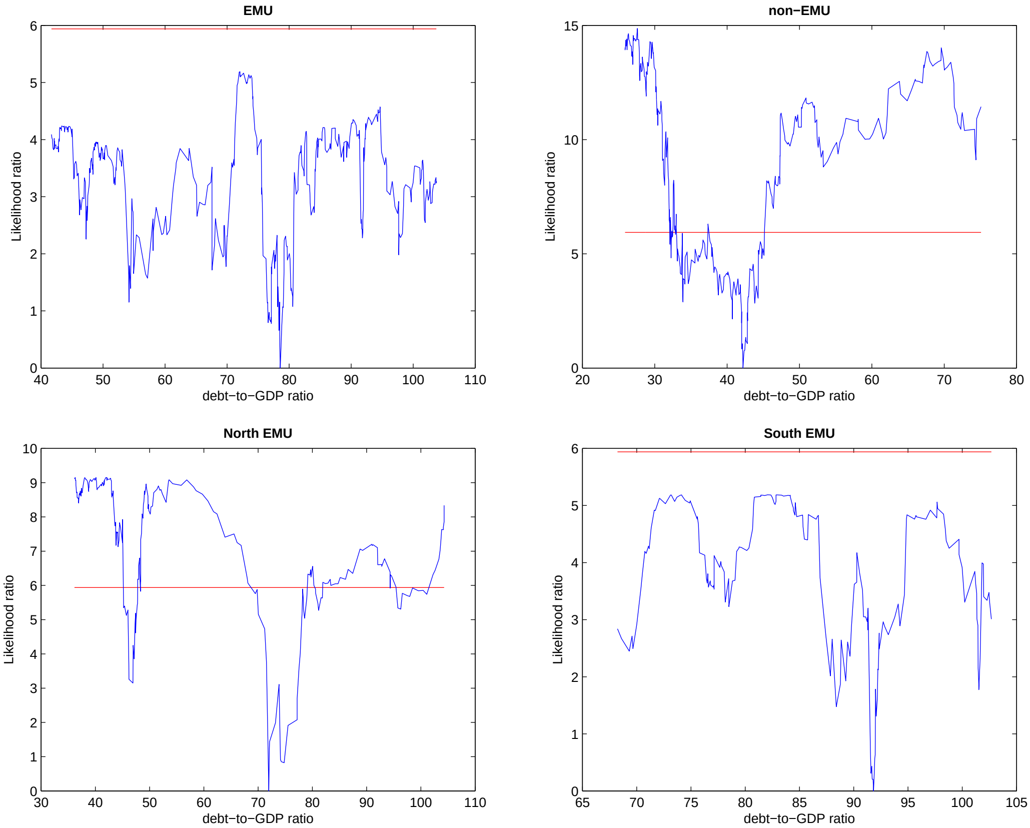

Figure 4: The likelihood ratio for different threshold values

countries, the plot indicates that the threshold is either around 47%, between 70% and 80% or around 97%. Note that these results are not very robust to including additional control variables.

Coming back to table 1, note that the coefficient of the lagged endogenous variable is positive and typically significant for all countries except Denmark and for all groups of countries. Further, the effect of a one unit change in the growth rate of the labor force on real GDP growth is mostly positive and statistically significant. For some countries, the point estimates are negative but, in this case, not significant.

We are mainly interested in the coefficient of the regime dependent variable, i.e. the debt-to-GDP ratio. Since the country-specific estimates of the threshold values are typically not very robust to including additional control variables to the regression model, the regime dependent estimates of the debt-to-GDP ratio's coefficients are not robust either. One robust finding is that, the high-debt regime is associated with a negative and significant effect of debt on growth only in Greece and Portugal. For most countries, debt seems to have a positive effect on growth in both regimes. The estimations results obtained for the panels of countries suggest that a rise of debt reduces growth primarily in the southern EMU countries, while debt does not seem to affect growth at high levels relative to GPD in non-EMU as well as northern EMU countries. At low levels of sovereign debt, a rise in debt increases growth in non-EMU countries. These results are robust to including additional control variables.

Overall, our results suggest that the impact of sovereign debt on growth may not necessarily be negative. They qualify Reinhardt and Rogoff's 2010 suggestion that gross debt reduces economic growth beyond 90% of GDP, in general. Large debt does not necessarily increase the debt's absolute effect on growth nor does it necessarily negatively affect growth. If the relationship between sovereign debt and economic activity is driven by investor sentiment and expectations, these results imply that the debt-to-GDP ratio is not necessarily the relevant variable which causes the trend reversal in the market's beliefs. Only in the southern EMU countries sovereign debt seems to reduce growth at high debt levels.

5.2 FSI as threshold variable and debt-to-GDP ratio as regime dependent regressor

Next, we analyze the non-linear effect of debt on economic activity using the FSI as the threshold variable. As argued in the theoretical section, the impact of sovereign debt on economic growth may be negative primarily during times of high financial stress since rising risk premia and credit spreads deteriorate the balance sheet of the banking sector which holds part of the sovereign debt. Since credit spreads may respond more sensitively to changes in debt in member countries of a monetary union than in stand-alone countries (cf. De Grauwe and Ji 2013), this non-linearity in the debt-growth relationship can be expected to be more pronounced in the countries of the EMU than in the other countries of our sample. We estimate the following specification:

$$y_{it} = \mu_i + \alpha_y y_{i,t-1} + \alpha_n n_{it} + \beta_b^L b_{i,t-1} I(f_{i,t-1} \leq \gamma_f) + (\beta_b^H b_{i,t-1} + \delta^H) I(f_{i,t-1} > \gamma_f) + \varepsilon_{it} \qquad (8)$$

where f_{it} is the financial stress index and the other parameters and variables are as before.

Table 2 reports the estimation results of the regression model described by eq.(8) with the government debt-to-GDP ratio as the only regime dependent variable apart from the constant, using the first lag of the FSI as the threshold variable.[12]

In contrast to the previous specification, the threshold estimates for the FSI are rather homogeneous especially across groups of countries. Typically the regime change is indicated at large values of the FSI relative to the feasible values which exclude the lowest and highest 5% of the FSI. For almost all countries and for all groups, the vast majority of observations lies in the low-stress regime. Moreover, the threshold estimates are typically highly significant which is illustrated in Figure 5 plotting the

[12] Again, we report only the regression output for the baseline specification since adding control variables does not change the main results.

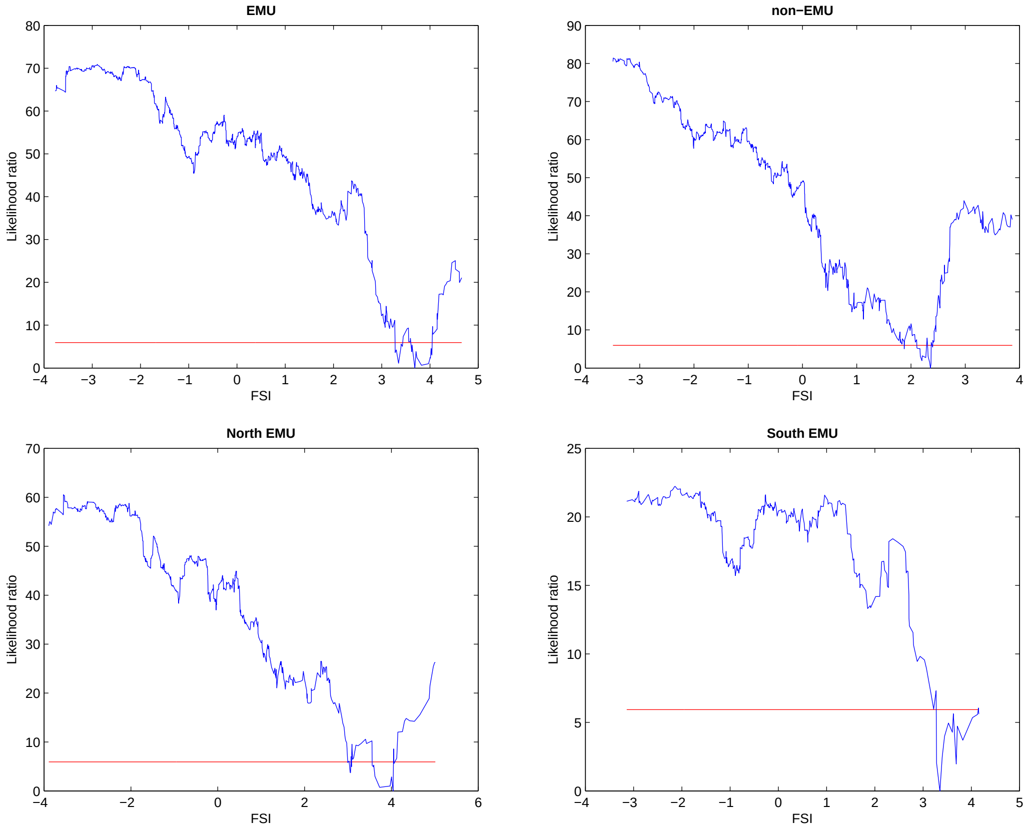

Figure 5: The likelihood ratio for different threshold values

likelihood ratios for different threshold values for four the same groups of countries as before. As can be clearly seen, the global minima can easily be narrowed down to a small range and are typically located at high levels of financial stress. Further, it is interesting to note that, compared to the previous analysis, the value of the threshold variable beyond which a regime change occurs is much higher in the EMU than in the group of stand-alone countries (3.69 vs. 2.36). Hence, in the EMU, the change in the sovereign debt effect on growth occurs only at extremely high values of financial stress. This finding is highly consistent with the argument put forward by De Grauwe and Ji (2012) that countries within a monetary union are more sensitive to changes in investor sentiment then stand-alone countries. Because of overly optimistic expectations, high financial stress starts affecting growth only at very high levels.

The parameter estimates for the lagged endogenous variable as well as the growth rate of the labor force are very similar to the results obtained in the previous section and, usually, have the correct signs.

The country-specific results for the regime dependent coefficient suggest that the effect of debt on growth is low during times of high financial stress as compared to times of financial turmoil. In average the coefficients in the high stress-regimes are lower in EMU countries than in non-EMU countries. For the panels of countries, we find that an increase of the debt-to-GDP ratio during times of high financial stress reduces growth in countries of the EMU, but not significantly in stand-alone countries. Within the EMU it seems that a rise in debt affects growth primarily in the southern EMU countries.

These results, especially the finding that sovereign debt has a stronger negative effect on economic growth during times of high financial stress in EMU countries rather than in non-EMU countries reinforces the conclusion by De Grauwe and Ji (2013) that countries within a monetary union are more sensitive to changes in investor sentiment than stand-alone countries.

5.3 Debt-to-GDP ratio and FSI as threshold variables and debt-to-GDP ratio as regime dependent regressor

In section 5.1, we have studied how the level of debt affects the impact of debt on growth concluding that there is not a significant threshold value separating two distinct regimes. In the subsequent section 5.2, we have analyzed how the FSI influences this relationship and we have found evidence for a threshold value beyond which a rise in debt reduces growth significantly especially in the EMU countries. This negative growth effect of debt, however, does not depend on the level of debt. Hence, a one-unit increase in the debt-to-GDP ratio is assumed to have an effect on growth independent of the level of debt. Economic theory and the model outlined above, however, suggest that a rise in debt may impair economic activity when both debt and financial stress are high. Therefore, we finally analyze how the effect of the debt-to-GDP ratio on economic growth depends on the level of the debt-to-GDP ratio itself as well as the level of the FSI. We include two threshold variables to the regression model giving rise to four regimes: low-debt and low-stress (LL), low-debt and high-stress (LH), high-debt and low-stress (HL) as well as high-debt and high-stress (HH). We estimate the following specification:

$$
\begin{aligned}
y_{it} = {} & \mu_i + \alpha_y y_{i,t-1} + \alpha_n n_{it} + \\
& + \beta_b^{LL} b_{i,t-1} I(b_{i,t-1} \leq \gamma_b, f_{i,t-1} \leq \gamma_f) \\
& + \beta_b^{HL} (b_{i,t-1} + \delta^{HL}) I(b_{i,t-1} > \gamma_b, f_{i,t-1} \leq \gamma_f) \\
& + \beta_b^{LH} (b_{i,t-1} + \delta^{LH}) I(b_{i,t-1} \leq \gamma_b, f_{i,t-1} > \gamma_f) \\
& + \beta_b^{HH} (b_{i,t-1} + \delta^{HH}) I(b_{i,t-1} > \gamma_b, f_{i,t-1} > \gamma_f) + \varepsilon_{it}
\end{aligned} \tag{9}
$$

where the variables and parameters are as before.

Table 3 reports the estimation results for the four-regime threshold regression. When interpreting the results, note the following: First, we only consider the results for the panels of countries since the

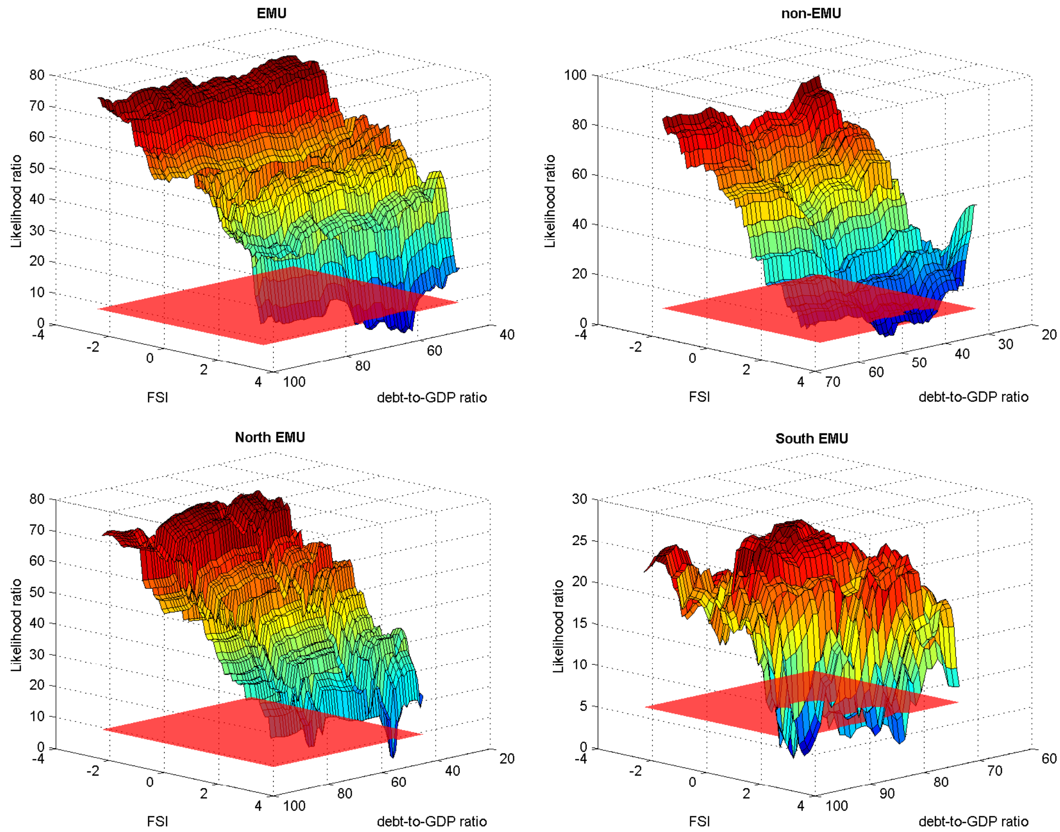

Figure 6: The likelihood ratio for different threshold values

country-specific estimates are not very reliable due to a low number of observations in each of the four regimes. Second, we present only the results of the baseline specification and refer to deviations in the extensions as we discuss the estimation results. Third, to reduce the computational effort for identifying the threshold values, we round the debt-to-GDP ratio to full integers and the FSI to one decimal digit for search. Fourth, we specified that at least 50% of the observations have to be in one of the low-debt regimes. To ensure enough observations in all regimes, we also specified that instead of 5% there have to be at least 10% of the observations in one of the high-debt as well as in one of the high-stress regimes. Fifth, we assume the intercepts to be regime dependent only along the dimension of the FSI. A regime-switch between low-debt and high-debt does not change the intercept.

The threshold estimate for the debt-to-GDP ratio is not as robust and significant as the estimate for the FSI threshold. Yet, it is more robust as in the specification with only one threshold variable discussed in section 5.1. This can be observed in Figure 6 which plots the likelihood ratios for different threshold values of the debt-to-GDP ratio and the FSI. Note that the perspective on the surface is from below. The transparent plain indicates the critical values. Along the axis of the debt-to-GDP ratio, the likelihood ratio is rather flat for any given FSI threshold. In contrast to that, the likelihood typically increases substantially as we move away from the minimum of the likelihood ratio along the axis of the FSI at any given threshold of the debt-to-GDP ratio. For the set of combinations of debt-to-GDP values and FSI values with likelihood ratios below the plain of the critical values, the null hypothesis that the true threshold values are equal to a given combination of the threshold variables cannot be rejected. Note that, compared to the confidence sets estimated in the specification without the FSI as a threshold variable in section 5.1, the range of debt-to-GDP ratios which cannot be rejected to be the true thresholds is rather narrow. This suggests that the debt-to-GDP threshold is only of relevance in combination with financial stress. As expected the high-debt and high-stress regime features the lowest number of observations.

As before, the coefficients of the lagged growth rate of GDP and the growth rate of the labor force have the expected positive sign and are statistically significant for all country groups.

The following results for the regime-dependent growth effects of debt which hold for the baseline specification as well as for the extensions including additional control variables are worth to note: First, for the EMU countries the negative growth effect of debt turns considerable more pronounced moving from the low-stress to the high-stress regimes rather than moving from the low-debt to the hight-debt regimes. Yet, for the non-EMU countries the coefficients decrease when moving from the low-debt to the high-debt regimes. For this group of countries, however, the negative coefficients are typically not significant. This reinforces the previous result that EMU countries are more vulnerable to a rising debt-to-GDP ratio during times of financial turmoil than non-EMU countries. Second, for both northern and southern EMU countries the negative impact of debt on growth is more pronounced in the high-stress regimes. Interestingly, it is the southern EMU countries rather then the northern EMU countries for which we find a strong negative coefficient during times of high debt and high financial stress. This suggests that rising sovereign debt at high levels of debt during times of financial distress is primarily a problem for the southern EMU countries.

As in the previous section, the autonomous growth rate of GDP is lower in the high-stress regime than in the low stress regime, as indicated in the last column of Table 3.

6 Concluding Remarks

How does the debt-to-GDP ratio affect economic activity? Reinhart and Rogoff (2010) have argued that gross sovereign debt beyond 90% of GDP tends to reduce growth. We have argued theoretically and empirically that the answer depends not primarily on the debt level but on the state of the financial market and on whether the central banks is willing or able to reduce financial market stress by some monetary policy intervention. On theoretical terms we proposed a stylized dynamic macroeconomic model based on inter-temporal optimization in a finite horizon framework which illustrated how changing financial market conditions may affect the interaction of growth and debt. By means of numerical simulations, we illustrated the destabilizing effect that rising financial market stress (assumed to be a function of the degree of aggregate indebtedness in the economy) can have for the actual paths of capital and debt over time, and how a reduction of the former can bring about macroeconomic stability.

On empirical grounds, our econometric analysis based on dynamic country-specific and dynamic panel threshold regression techniques to investigate non-linearities in the relationship between economic growth, the sovereign debt-to-GDP ratio and financial market stress in industrialized economies delivered a variety of findings relevant for the current discussion on the role of the debt-to-GDP ratio as the main variable driving macroeconomic stability in advanced economies. Summing up, we found the following empirical results: First, we do not find evidence for a robust and significant threshold sovereign debt-to-GDP ratio beyond which a rise in debt reduces growth, if financial stress is ignored as a source of non-linearities in the debt-growth nexus. Neglecting the state of the financial markets, the debt-to-GDP ratio does not seem to be the relevant measure along which investor sentiment switches from growth-enhancing to growth-impairing. This is in contrast to the influential contribution by Reinhardt and Rogoff (2010) who argued that the relevant threshold level of gross debt is 90% of GDP. It seems that a rise in debt does not reduce growth just because a country faces a high debt-to-GDP ratio. Second, we have identified the financial stress as a crucial source of a non-linearity between debt and economic activity. Only at high levels of financial stress, the debt-to-GDP ratio may negatively affect growth, regardless of the level of debt. Third, we find evidence that debt reduces growth during episodes of high financial stress primarily for the countries of our sample which are part of the European Monetary Union where the central bank had some difficulty to bring down financial stress. Finally, our analysis suggests that the interaction of the level of debt and the level of financial stress is important for the debt-growth nexus. In particular, this is visible in the southern EMU countries where contractionary fiscal policy increased financial market stress and monetary policy did not turn out to be reducing stress.

Table 2: Country-specific and panel dynamic threshold GMM estimation results with the financial stress index as the threshold variable and the debt-to-GDP ratio as a regime dependent regressor (equation 8)

	γ_f	L	H	α_y	α_n	β_b^L	β_b^H	δ^H
	COUNTRY-SPECIFIC THRESHOLD REGRESSIONS							
	Northern EMU countries							
AUT	2.97	113	16	0.416***	0.038	0.011***	−0.001	−0.069
	[−3.23,4.06]			(0.063)	(0.075)	(0.002)	(0.020)	(0.643)
BEL	3.56	118	11	0.608***	−0.140**	0.002***	−0.025**	1.546**
	[−3.49,3.59]			(0.040)	(0.066)	(0.000)	(0.012)	(0.726)
DEU	1.56	96	33	0.096	0.632***	0.011***	0.011**	−0.576***
	[−4.09,5.01]			(0.058)	(0.227)	(0.002)	(0.005)	(0.194)
FRA	1.32	96	32	0.567***	0.107**	0.005***	−0.005	0.126
	[−3.79,3.62]			(0.052)	(0.050)	(0.001)	(0.004)	(0.080)
NLD	3.73	116	13	0.034	0.475***	0.013***	−0.069	1.229
	[−3.35,4.05]			(0.071)	(0.163)	(0.002)	(0.125)	(3.219)
	Southern EMU countries							
ESP	−1.50	46	83	0.155**	0.669***	0.002*	0.008***	−0.004
	[−2.95,4.13]			(0.064)	(0.090)	(0.001)	(0.003)	(0.118)
GRC	0.96	72	19	0.521***	−0.029	0.002***	0.004	0.128
	[−1.27,2.72]			(0.073)	(0.067)	(0.001)	(0.013)	(1.008)
ITA	3.69	113	16	0.369***	−0.036	0.003***	−0.012**	0.657*
	[−3.40,4.61]			(0.064)	(0.097)	(0.001)	(0.006)	(0.376)
PRT	−1.40	17	56	0.275***	0.884***	0.003	−0.002	0.053
	[−2.63,2.20]			(0.072)	(0.206)	(0.003)	(0.005)	(0.283)
	Non-EMU countries							
AUS	2.36	89	12	0.233***	0.786***	0.020***	0.001	−0.289*
	[−2.90,2.49]			(0.067)	(0.105)	(0.005)	(0.013)	(0.154)
CAN	0.58	89	39	0.457***	0.456***	0.004***	0.015***	−0.594***
	[−3.16,3.53]			(0.050)	(0.108)	(0.001)	(0.004)	(0.192)
DNK	−1.64	32	97	−0.284***	0.146	0.033***	0.026***	−0.138
	[−2.87,2.62]			(0.055)	(0.111)	(0.004)	(0.006)	(0.109)
GBR	1.29	99	30	0.599***	−0.138	0.012***	−0.001	−0.021
	[−3.18,3.30]			(0.049)	(0.157)	(0.002)	(0.004)	(0.097)
JPN	2.46	111	18	0.102	0.468**	0.006***	−0.010***	0.230
	[−3.36,2.81]			(0.080)	(0.209)	(0.001)	(0.004)	(0.246)
SWE	0.53	76	41	0.561***	0.323**	0.004	0.010**	0.004
	[−3.97,3.55]			(0.047)	(0.126)	(0.004)	(0.004)	(0.058)
USA	1.74	98	31	0.438***	0.399**	0.007***	0.028*	−1.111*
	[−3.52,3.59]			(0.069)	(0.163)	(0.001)	(0.014)	(0.610)
	PANEL THRESHOLD REGRESSIONS							
All	2.27	1605	323	0.299***	0.164***	−0.004***	0.000	−0.602***
	[−3.72,4.53]			(0.027)	(0.047)	(0.001)	(0.002)	(0.077)
EMU	3.69	971	95	0.396***	0.113**	−0.003**	−0.011***	−0.179
	[−3.77,4.66]			(0.032)	(0.048)	(0.001)	(0.003)	(0.143)
Non-EMU	2.36	721	141	0.189***	0.217***	−0.002	−0.002	−0.658***
	[−3.49,3.87]			(0.036)	(0.072)	(0.001)	(0.002)	(0.093)
North EMU	4.04	588	56	0.314***	0.086	−0.001	−0.010**	−0.418***
	[−3.89,5.01]			(0.037)	(0.060)	(0.002)	(0.005)	(0.178)
South EMU	3.35	382	40	0.393***	0.140**	−0.007***	−0.012***	−0.186
	[−3.14,4.15]			(0.045)	(0.070)	(0.002)	(0.003)	(0.208)

Notes: Standard errors are in parenthesis. ***, ** and * denote the level of significance at 0.01%, 0.05% and 0.1%, respectively. The interval of feasible threshold values is in brackets.

Table 3: Dynamic panel threshold GMM estimation results with the debt-to-GDP ratio and the FSI as the threshold variables and the debt-to-GDP ratio as a regime dependent regressor (equation 9)

	γ_b	γ_f	LL	HL	LH	HH	α_y	α_n	β_b^{LL}	β_b^{HL}	β_b^{LH}	β_b^{HH}	δ^H
	PANEL THRESHOLD REGRESSIONS												
All	83	2.2	1383	215	307	23	0.297*** (0.028)	0.162*** (0.047)	−0.004*** (0.001)	−0.003*** (0.001)	0.003 (0.002)	−0.003 (0.002)	−0.658*** (0.082)
EMU	57	3.1	611	336	90	29	0.385*** (0.033)	0.104** (0.049)	0.002 (0.002)	−0.004*** (0.001)	−0.013*** (0.004)	−0.011*** (0.003)	−0.011 (0.145)
Non-EMU	42	2.4	537	188	107	30	0.149*** (0.038)	0.198*** (0.072)	0.007*** (0.002)	−0.001 (0.001)	0.006 (0.004)	−0.004 (0.002)	−0.607*** (0.100)
North EMU	46	3.0	390	173	60	21	0.327*** (0.037)	0.098* (0.059)	−0.002 (0.003)	−0.001 (0.002)	−0.019*** (0.005)	−0.005 (0.003)	−0.203 (0.133)
South EMU	84	0.4	177	91	115	39	0.377*** (0.043)	0.183*** (0.066)	−0.009*** (0.003)	−0.005** (0.002)	0.006* (0.003)	−0.004* (0.002)	−0.692*** (0.183)

Notes: Standard errors are in parenthesis. ***, ** and * denote the level of significance at 0.01%, 0.05% and 0.1%, respectively.

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A Non Linear Model Predictive Control

The NMPC methodology can be summarized as follows: Consider a general dynamic decision problem described by

$$\max \quad \int_0^T e^{-\rho t}\ell(x(t),u(t))dt, \tag{10}$$

where the evolution of the state variable $x(t)$ is given by $\dot{x}(t) = f(x(t),u(t))$, with $x(0) = x_0$ and where $u(t)$ represents the control variables of the problem. A discrete-time approximation of such a problem is given by

$$\max \quad \sum_{i=0}^{T} \beta^i \ell(x_i,u_i)dt, \tag{11}$$

where the maximization is now performed over a sequence u_i of decision values and the sequence x_i satisfies $x_{i+1} = \Phi(h,x_i,u_i)$, Here $h > 0$ is the discretization time step, $\beta = e^{-\rho h}$ and Φ is a numerical scheme approximating the solution of $\dot{x}(t) = f(x(t),u(t))$ on the time interval $[ih,(i+1)h]$. For details and references about the discretization procedure see Grüne et al. (2013). The essence of NMPC lies in replacing the maximization of the above large horizon functional, where we could have $T \to \infty$, by the iterative maximization of finite horizon functionals

$$\max \quad \sum_{k=0}^{N} \beta^i \ell(x_{k,i},u_{k,i})dt, \tag{12}$$

for a truncated finite horizon $N \in \mathbb{N}$ with $x_{k+1,i} = \Phi(h,x_{k,i},u_{k,i})$ and the index i indicating the number of the iteration. As discussed in Grüne and Pannek (2011), it can be shown that under appropriate assumptions about the problem – such as the existence of an equilibrium for the infinite horizon problem – the solution (x_i,u_i) (which depends on the choice of N above) converges to the correct, infinite horizon solution of eq.(11) as $N \to \infty$.

B Interpolating and Extrapolating the Net Financial Liabilities Series

To compute the debt-to-GDP ratio some series have to be interpolated from annual data as well as extrapolated. We employ the Chow and Lin (1971) procedure to compute quarterly series. We suppose that there exists a relationship between a $4n \times 1$ vector of quarterly observations of a variable Y and a $4n \times 3$ matrix, X, comprising explanatory variables of the form

$$Y = X\beta + u, \tag{13}$$

where u is a random vector with mean zero and covariance matrix V. Using a dot to indicate annual data, we have

$$Y. = CY = CX\beta + Cu = X.\beta + u., \tag{14}$$

with C being a $n \times 4n$ transformation matrix with the $[i, 4i-3]$-th element being 1 for $i = 1, \ldots, n$ and the others being zero for an interpolation problem. Assuming that the quarterly residuals follow a first-order auto-regressive process with coefficient a, disturbances ϵ and variance-covariance matrix $E(\epsilon_t \epsilon_s = \delta_{ts}\sigma^2)$, Chow and Lin (1971) show that the best linear unbiased predictor $\hat{B}_z$ of B_z, which is a $(n+m) \times 1$ vector with m denoting the number of quarters to extrapolate, is

$$\hat{Y}_z = X_z(X.'V.^{-1}X.)^{-1}X.'V.^{-1}Y. + (V_z.V.^{-1})\hat{u}. - [Y. - X.(X.'V.^{-1}X.)^{-1}X.'V.^{-1}Y.] \qquad (15)$$

where

$$V = \begin{bmatrix} 1 & a & a^2 & \ldots & a^{4n-1} \\ a & 1 & a & \ldots & a^{4n-2} \\ a^2 & a & 1 & \ldots & a^{4n-3} \\ & & & \ldots & \\ a^{4n-1} & \ldots & a^2 & a & 1 \end{bmatrix} \frac{\sigma^2}{1-a^2} \qquad (16)$$

where a is estimated by an iterative procedure. Presupposing an autocorrelation coefficient of the annual residuals, q, one computes a as the 4-th root of q in an interpolation problem and uses this value to generate V and the new annual residuals whose autocorrelation coefficient is taken as the q for the next iteration.

For AUT, BEL, DNK, DEU, ESP, FRA, GBR, GRC, ITA, NLD, PRT and SWE, the quarterly series of net financial liabilities of the general government has been interpolated from annual data and a quarterly proxy for the net financial liabilities following the Chow and Lin (1971)-procedure. For reasons of data availability, this proxy has been constructed differently for different countries.

For GBR, the annual stock of net financial liabilities at the end of 2005 is taken from the OECD EO 94 as the reference value for 2005Q4 of the proxy. We then added to and subtracted from it subsequently the cumulative net lending of the general government taken from the OECD EO 94 (variable NLG) in order to get the values for the proxy before and after 2005Q4, respectively. Note that all OECD data are seasonally adjusted.

For AUT, DEU, FRA and ITA, we also took the annual stock of net financial liabilities in 2005 as the reference value for 2005Q4. Then, we used discontinued data on net financial liabilities from the OECD EO 79 and adjusted it to match the respective value for 2005Q4 in order to get observations for the period before 2005Q4. For the period thereafter, each observation is obtained by subtracting from the previous observation the seasonally adjusted net lending of the general government taken from Eurostat.

For GRC, NLD and SWE, the value for 2005Q4 is also taken from the annual series. The preceding values are computed by adding to it subsequently the cumulative net lending of the general government which has been obtained from the OECD EO 79 and is discontinued. As for the previous countries, the values for the period after the reference quarter are generated by subtracting from the previous value the seasonally adjusted net lending of the general government taken from Eurostat.

For BEL, DNK, ESP and PRT, the reference values are 1990Q4, 1998Q4, 1994Q4 and 1998Q4, respectively. The values of the proxy preceding the reference date are computed from data on government net lending taken from a discontinued IMF database (for ESP and PRT) and primary surplus as estimated by Schoder (forthcoming) (for BEL and DNK), respectively. Note that these series have been adjusted to match the first observations overlapping with the series on government net lending provided by Eurostat. The values after the reference quarter are computed using the cumulated net lending data obtained from Eurostat.

Applying Chow and Lin (1971) to the annual debt series using a quarterly the proxies for net financial liabilities as discussed above yields the interpolated net debt series as plotted in Figure 7 together with the annual net debt.

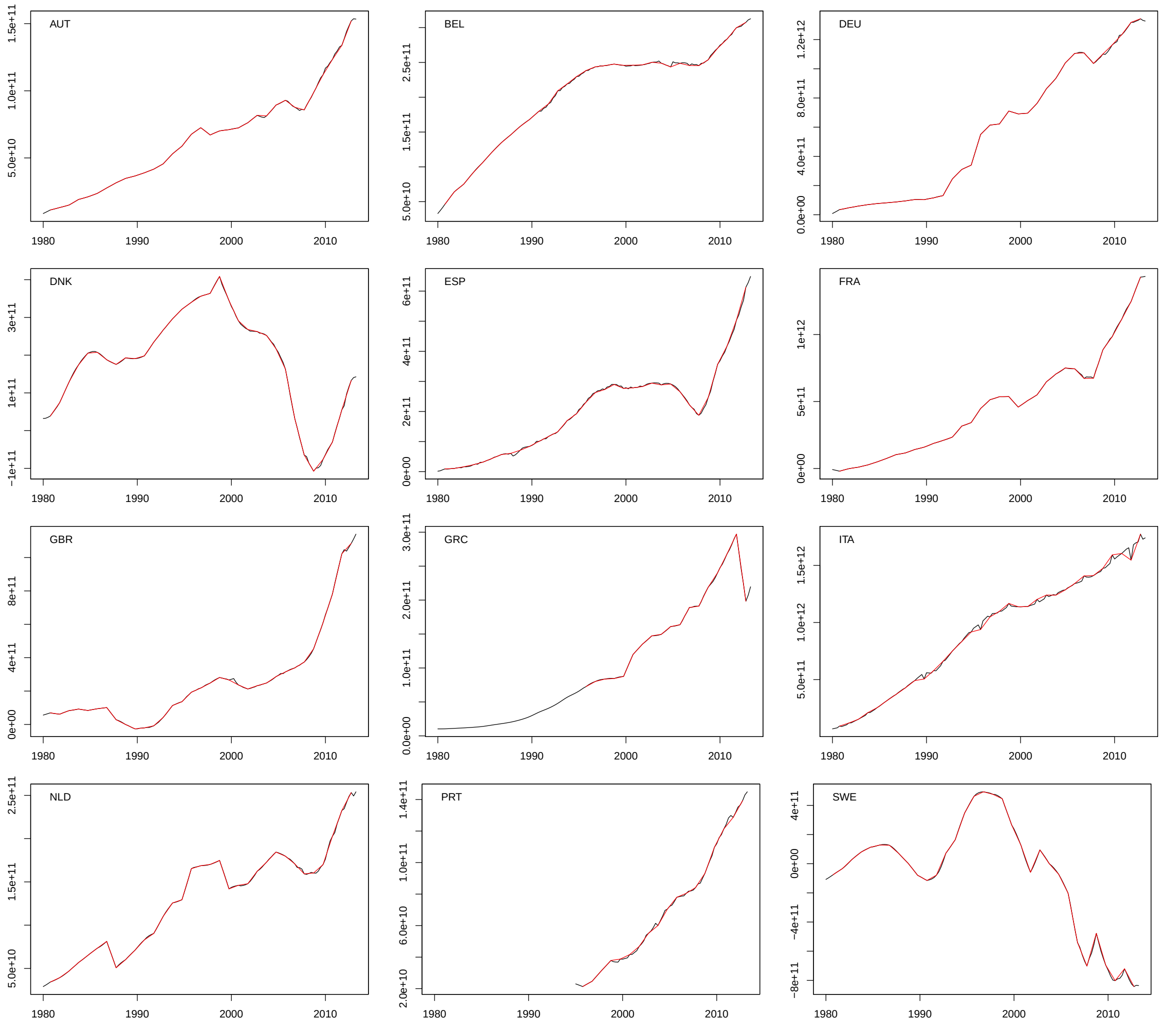

Figure 7: Annual net sovereign debt (dashed line) and interpolated quarterly net sovereign debt (solid line) from 1980:1 to 2010:4