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EFFECTIVENESS DETERMINATION OF HIGHER EDUCATION USING LINEAR PROGRAMMING

Tatjana Atanasova-Pacemska[1], Riste Timovski[2]
[1]Associate professor, Faculty of Computer Sciences, Goce Delcev University in Stip, Republic of Macedonia, tatjana.pacemska@ugd.edu.mk
[2]Head of E-Index department, Rector's cabinet, Goce Delcev University in Stip, Republic of Macedonia, riste.timovski@ugd.edu.mk

Abstract: *Higher education presents main engine of society's overall progress and development. The producers of the knowledge and future progressive force in all sectors of today's living are personified in the Universities. As generators of academic individuals, it is important to achieve high level of educated and qualified students/future employees that will be able to fit in the real sector and contribute to the collective/personal live. To accomplish that, resources investments are necessary to be made by the Universities. The question is how to organize the resources in order of efficient production of high qualified future academic individuals. Moreover, in cases of inefficiency detection, the question is what is necessary to be done for improvement. This paper shows concrete linear programming technique application called DEA (Data Envelopment Analysis), used for measuring the efficiency of the University study program, who through its real implementation represents a basic generator of knowledge.*
Keywords*: Linear Programming, Data Envelopment Analysis, Decision Making Units, Production levels, Composite units*

1. INTRODUCTION

One of the main issues that companies' and institutions' management deals with is permanent optimization and improvement of its main process or production. Given the fact that solving of one problem in many cases can be done in more ways, it is important to find and implement the optimal between them. According to the literature that deals with the system optimization in terms of overall efficiency improvement, especially complex for analysis are service oriented institutions/organizations. Considering the fact that Universities in its basis aim to offer a service called quality education to the consumers personified in students, they can be analyzed precisely in this constellation.

Mathematical programming is special mathematical approach of selecting the best/optimal from the set of possible alternatives towards solving the problem. It assumes that it is possible to represent the problem as a mathematical model that reflects certain production function (more precisely, mathematical function). In this direction, resolution of the problem of optimization converges to determination of the optimal (minimum or maximum, depending on the nature of the model) value of the production function, having in mind all the limitations of the model/environment, represented in the limitation equations. In general, mathematical optimization implies that the production function and the accompanying limitation equations can be linear or nonlinear equations and/or inequalities. If the production function and all limitation equalities are represented in linear form, the mathematical optimization takes form of linear programming modelling and solvation. Nevertheless, some mathematical models have high complexity level and cannot be solved through known mathematical optimization models. Then, not optimal, but "good enough" solution is the subject, using orientation and heuristic models [1].

In context of the previously stated, the starting point for this work is how to build a model that portrays the knowledge and skills delivery very real, taking into account the fact that Universities are the knowledge and skills generators, in order to detect its pros and cons and certain, to propose concrete ways of improvement of this process. For this purpose, DEA (Data Envelopment Analysis) technique is used, starting from the fact that it's nature of application offers very good possibility of model definition approach, as well as its mathematical processing and further discussion. Actually, this technique developed a new approach to the traditional cost / profit analysis and methods for efficiency measurement that enables policy-making through "learning from the best" and inducing theory from practice [2].

1.1. DEA (Data Envelopment Analysis)

There are several phases in LP appliance that have to be conducted:

- Definition of the problem and its mathematical formulation;

- Construction of the mathematical model;
- Finding the optimal solution of the model;
- Analysis of the results and sensitivity of the model, and
- Evaluation of the model and its implementation.

DEA as frontier analysis is designed for organization quality measurement and performance improvement, as main intention of the management. It is nonparametric non-statistical multi-criteria method that allows handling heterogeneous data. Based on LP, it operates with the term technological efficiency, defined as [3]:

$$\theta = \frac{Output}{Input} \tag{1}$$

where input is presented as sum of the resources invested and output is presented as sum of the outcomes of the entity which efficiency is measured. The efficiency is noted as pareto efficiency if allocation of the resources (input and output) is such that better performances are not possible for the entity analyzed. Pareto efficient subject is the subject (entity) with best possible allocation of the resources. Here, it is impossible to improve the output without worsening the input.

DEA declares as effective those entities that produce a certain or more output parameters with fixed inputs or use the same or a smaller quantity of inputs to produce a certain output, compared with the other subjects within the same group being analyzed. The set that is subject of measurement is consisted of entities called DMUs (Decision Making Units) and they have to be homogenous. All DMUs have same inputs and outputs in different quantities. DMU can be institution, bank, human, production line, vehicle, part of vehicle etc. In this work, the subject/course of a concrete University's study program is qualified as DMU.

The mathematical model of DEA, starting from the definition of the efficiency of the k-DMU, in set of n DMUs with m inputs and s outputs given by [4]:

$$\theta_k = \frac{u_1 y_{1k} + u_2 y_{2k} + \cdots + u_s y_{sk}}{v_1 x_{1k} + v_2 x_{2k} + \cdots + v_m x_{mk}} \tag{2}$$

Is defined as:

- Goal:

$$max\,(\theta_k = \frac{u_1 y_{1k} + u_2 y_{2k} + \cdots + u_s y_{sk}}{v_1 x_{1k} + v_2 x_{2k} + \cdots + v_m x_{mk}}) \tag{3}$$

- Given:

$$\frac{u_1 y_{11} + u_2 y_{21} + \cdots + u_s y_{s1}}{v_1 x_{11} + v_2 x_{21} + \cdots + v_m x_{m1}} = \frac{\sum_{i=1}^{s} u_i y_{i1}}{\sum_{j=1}^{m} v_j x_{j1}} \leq 1$$

$$\cdots$$

$$\frac{u_1 y_{1k} + u_2 y_{2k} + \cdots + u_s y_{sk}}{v_1 x_{1k} + v_2 x_{2k} + \cdots + v_m x_{mk}} = \frac{\sum_{i=1}^{s} u_i y_{ik}}{\sum_{j=1}^{m} v_j x_{jk}} \leq 1$$

$$\cdots$$

$$\frac{u_1 y_{1n} + u_2 y_{2n} + \cdots + u_s y_{sn}}{v_1 x_{1n} + v_2 x_{2n} + \cdots + v_m x_{mn}} = \frac{\sum_{i=1}^{s} u_i y_{in}}{\sum_{j=1}^{m} v_j x_{jn}} \leq 1 \tag{4}$$

$$v_1, \ldots, v_m \geq 0, u_1, \ldots, u_s \geq 0; \tag{5}$$

$$x_{jk} \geq 0, y_{ij} \geq 0; j = 1, \ldots, m; i = 1, \ldots, s; k = 1, \ldots, n \tag{6}$$

where x_{jk} is j-th input in k-th DMU, y_{ik} is i-th output of the k-th DMU, $v_1, \ldots, v_m$ are weights of the m inputs and $u_1, \ldots, u_s$ are weights of the s outputs, both of the k-th DMU. DEA sets weights in such way – best way, to maximize the efficiency of every single DMU. So, the frontier is consisted of efficient DMUs with efficiency equals to 1. Every other DMU with efficiency below 1 is considered to be inefficient. Often, as in this paper, the dual DEA CCR model is used. It is represented with following equations:

- Find $min\theta$;
- Having limitations:

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{i0}, \qquad i = 1, \ldots, m \tag{7}$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{r0}, \qquad r = 1, \ldots, s \tag{8}$$

$$\lambda_j \geq 0, \qquad j = 1, \ldots, n \tag{9}$$

where index 0 represents each DMU that equations are solved for (in order to maximize its efficiency – observed DMU), lambdas are weighted coefficients that are used to represent the so-called composite DMU for each real DMU that will be located as inefficient. The composite DMU for each inefficient real DMU is consisted as sum of the ERS (efficiency reference set – efficient DMUs used for interpretation of the composite DMU for the observed real DMU) multiplied with its lambda coefficients. If A and B are efficient DMUs (m inputs, s outputs) and define the ERS set of observed inefficient C DMU, the composite DMU of C → C' can be interpreted as:

$$\lambda_A \begin{bmatrix} y_{1A} \\ \dots \\ y_{sA} \\ x_{1A} \\ \dots \\ x_{mA} \end{bmatrix} + \lambda_B \begin{bmatrix} y_{1B} \\ \dots \\ y_{sB} \\ x_{1B} \\ \dots \\ x_{mB} \end{bmatrix} = \begin{bmatrix} y_{Composite} \\ \dots \\ y_{Composite} \\ x_{Composite} \\ \dots \\ x_{Composite} \end{bmatrix} = C' \tag{10}$$

2. MATHEMATICAL MODEL

The model built is based on the fact that the delivery of knowledge, skills and competencies of students is conducted through practical implementation of curricula or course components during the study. DEA is particularly applied to service-oriented institutions and requires existence of a basic entity whose efficiency will be measured, with defined constellation of input resources invested and output outcome. Universities can be understood as subjects that deal with knowledge, skills and abilities delivery (basic function). Bearing this in mind, a mathematical model is conducted based on the need of measuring the efficiency level of the financial and other structural resources allocation by the University, with consideration of the achieved results in the process of knowledge, skills and abilities delivery to the students through the courses within a specific academic program. The resources are invested for each course and each course produces knowledge, skills and abilities.

So, course / subject is the entity whose efficiency is measured, i.e. course as decision making unit (DMU):

- All inputs to the DMU are constellated so that their increase can reduce the efficiency of each study course individually. In this manner, all output are considered to represent the outcome of the study program or each course practical implementation so that their increase will influence the increase of the efficiency of each study program individually.
- Inputs of a single DMU are represented with:
    - Expenses and financial structure within each subject consumed on behalf of gross salaries for professors and assistants that were engaged in the study program processing during years. Expenses are calculated in terms of gross salaries / expenses, with consideration of three months length (12 working weeks) of the semester (period of knowledge delivery) and the percentage of load in terms of number of students of the observed course and all students that the concrete person (teacher or assistant) covered during those three months. All increases and salary / expenses fluctuations are considered also. This data is gathered using the finance and human resources modules in ERP platform of the Goce Delcev University in Stip;
    - Expenses and financial structure within each subject consumed on behalf of the equipment and inventory used for the concrete study program. This is done by calculation of the degree of utilization, i.e. the ratio of annual depreciation of computer equipment and inventory used, in accordance with their gross purchase price and the legislation. Percentage of load / utilization in terms of number of students of the observed course and all students that used the same equipment and inventory is considered, and
    - Number of classes held, having in mind that classes load, forming study groups, classes and schedule organization is very important part in the process of overall course organization;
- Outputs of a single DMU are represented with:
    - Index of the level of contribution of each course in skills, competencies and knowledge delivery, prescribed with the accreditation elaborate of the study program (IP-KKV). Massive survey was realized at representative sample of 28 (of total of 88 students graduated at the study program), that generated indexes' values, and
    - Index of the quality of skills, competencies and knowledge delivered (IPKS-KKV) through the study program. This parameter, represented by the average grade of each course is calculated using the reporting module of the student information system of Goce Delcev University, fully automated for students' e-administration.

The model is shown in figure 1:

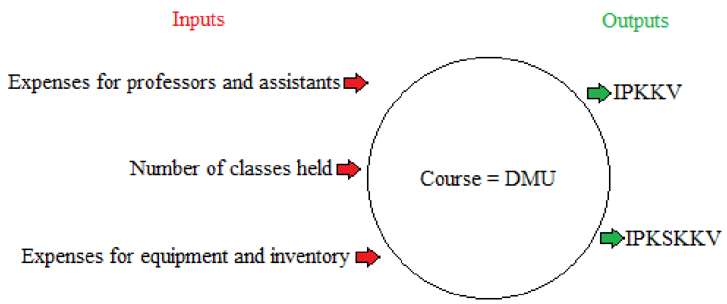

Figure 1. DEA Model: Course as DMU / Input = resources invested / Output = knowledge produced

3. CALCULATION, RESULTS AND DISCUSSION

Considering the above elaborated model and approach, table 1 shows the numerical side of it:

Table 1: Input / Output data

Number of classes held	Expenses for equipment and inventory	Expenses for professors and assistants	DMUs / Courses	IPKKV	IKPSKKV
24	28737,71	83066,86	English language 1	3,571	9,022
168	31190,88	122253,1	Electrical engineering	3,910	7,818
252	28737,71	87609,05	Math 1	3,928	7,170
168	31190,88	105634,6	Programing	4,267	6,852
24	25154,8	79832,87	English language 2	3,571	8,816
252	24766,98	101473,2	Linear algebra	3,875	7,556
252	27093,87	101473,2	Math 2	3,982	7,079
168	27093,87	107276,6	Objective programing	4,267	7
108	31979,06	36434,47	Probability and statistics	3,785	6,943
96	31979,06	78218,35	Digital logic	4,285	7,113
60	218481,7	51455,4	Operational systems	3,875	7,704
96	218481,7	131314	Software processes	3,964	7,295
96	218481,7	117407,6	Data structures and algorithms	4,25	6,954
96	154104,2	187273,8	Computer architecture	3,678	7,147
96	227342,5	62070,34	Data bases	4,321	7,318
72	118954,8	65492	Internet programing	4,339	7,238
72	12381,86	62527,75	Microprocessors	3,571	6,795
72	13413,68	64872,67	Software analysis and modeling	3,928	7,125
48	128867,7	65273,13	Graphics and visualization	4,160	8,170
72	128867,7	95668,31	Multimedia	3,982	7,693
72	87132,91	64428,28	Visual programing	4,339	7,056
72	7203,188	90306,62	Intelligent systems	4,142	7,693
72	7536,731	67004,76	Distant learning systems	3,196	7,602
72	7536,731	57935,99	Software projects management	3,821	7,011

The model has been applied on 24 subjects of the study program of Informatics at the Faculty of Computer Sciences, generation of students 2007/2008. The numerical model is processed in Open Source DEA software application (http://www.opensourcedea.org/index.php?title=Open_Source_DEA). The results of the subjects' efficiency measurement are given in table 2:

Table 2: Courses efficiency

DMU / Subject	Efficiency	DMU / Subject	Efficiency
English language 1	1	Data bases	0,856
English language 2	1 **(ERS=6)**	Digital logic	0,82
Probability and statistics	1 **(ERS=12)**	Multimedia	0,684
Operating systems	1 **(ERS=2)**	Math 1	0,59
Graphics and visualization	1 **(ERS=10)**	Linear algebra	0,566
Intelligent systems	1	Data structures and algorithms	0,558
Distant learning systems	1	Objective programming	0,551
Software projects management	1 **(ERS=16)**	Programming	0,548
Visual programming	0,98	Math 2	0,538
Software analysis and modeling	0,958	Electrical engineering	0,494
Internet programming	0,942	Software processes	0,487
Microprocessors	0,893	Computer architecture	0,375

Courses with efficiency = 1 are noted as relativly efficient courses and are set of representative courses for the courses with efficiency bellow 1, noted as inefficient courses. Most efficient courses are used in most of the cases of composite courses. In general, composite unit (in this case, the course) is the unit that DEA propose for each inefficient unit towards its optimization. Each inefficient unit has its own composite unit, as its example of best input / output allocation. The composite units are generated as the sum of the detected and specific (diferent for each inefficient DMU) ERS units and lambda corresponding coefficients, known as production levels. Courses that are bold in table 2 are noted as entities consisting Efficiency Reference Set – ERS, used in creating composite courses. ERS = X notes the number of times this course is used in creation of composite units for the inefficient detected courses. This defines **Graphics and visualization** and **Probability and statistics** as best efficient subjects, and **Software processes** and **Computer architecture** as subjects with worst resource invested and outcome results constellation.
Table 3 defines all inefficient courses in terms of definition of their composite DMUs (that are "on the frontier" and are what they "should be like").

Table 3. Production levels of inefficient DMUs

COMPOSITES	EFFICIENCY REFERENCE SET				
	English language 2	Probability and statistics	Operating systems	Graphics and visualization	Software processes management
INEFFICIENT COURSES	**LAMBDAS / PRODUCTION LEVELS**				
Electrical engineering	0,086514754	0,230296305	0	0	0,778229001
Math 1	0	0,375590319	0	0	0,655957254
Programming	0	0,325830999	0	0,009357358	0,783848429
Linear algebra	0	0,240252428	0	0	0,839884745
Math 2	0	0,2745388	0	0	0,770090534
Objective programming	0	0,265867267	0	0	0,853439903
Digital Logic	0	0,05593835	0	0,135779195	0,918245408
Software processes	0,080624363	0	0	0,804743829	0,085840258

Data structures and algorithms	0	0,04764483	0	0,93064462	0,051678035
Computer architecture	0,532210613	0	0	0,33862454	0,096534391
Data bases	0	0,308539831	0,813776755	0	0
Internet programming	0	0,273756379	0,010198233	0,784337473	0
Microprocessors	0,105563595	0,04925189	0	0,007417073	0,779054684
Software analysis and modeling	0,097614486	0	0	0,027600157	0,906758254
Multimedia	0,142915191	0	0	0,644193784	0,207101916
Visual programming	0	0,189735113	0	0,598011357	0,296446308

Having this in mind, in order of inefficient DMUs to become efficient, they have to become as their efficient or composite DMUs. For example, most inefficient subject is **Computer architecture**. The composite course can be calculated as:

$$CA_{composite}\begin{bmatrix} ExpensesTeachers \\ ExpensesEquInv \\ NumberOfClasses \\ IPKKV \\ IKPSKKV \end{bmatrix} = 0.5322106 * EJ2\begin{bmatrix} 79832,87 \\ 25154,8 \\ 24 \\ 3,571428572 \\ 8,8160 \end{bmatrix} + 0,33862454 * GV\begin{bmatrix} 65273,13 \\ 128867,7 \\ 48 \\ 4,160714286 \\ 8,1704 \end{bmatrix} + 0,096534391 * SPM\begin{bmatrix} 57935,99 \\ 7536,731 \\ 72 \\ 3,821428571 \\ 7,0113 \end{bmatrix} = \begin{bmatrix} 70183,79877 \\ 57752,97057 \\ 35,977508 \\ 3,678571429 \\ 7,1477 \end{bmatrix}$$

$$CA_{real}\begin{bmatrix} 187273,8 \\ 154104,2 \\ 96 \\ 3,678571429 \\ 7,1477 \end{bmatrix} : CA_{composite}\begin{bmatrix} 70183,79877 \\ 57752,97057 \\ 35,977508 \\ 3,678571429 \\ 7,1477 \end{bmatrix} = Deviation\begin{bmatrix} +166,83\% \\ +166,83\% \\ +166,83\% \\ +0\% \\ +0\% \end{bmatrix}$$

Discussion: The cost in terms of the teaching staff in terms of inventory and depreciation of computer equipment are 166.83 % higher than the potentially optimal constellation of parameters. This directly alludes to the fact that equally good results in the output, ie the same level of knowledge of the skills and competencies of the students with the same quality can be achieved mainly by reducing the cost of entry, and that means that the cost for teaching / associate staff and equipment costs . In addition, possible optimization may be required and reducing the number of classes that are realized in practice.

4. CONCLUSION

The purpose of this work is to review the system of delivery of knowledge, skills and abilities in a specific study program through the elaboration of the results achieved, as a result of the resources invested. The specific skills, knowledge and abilities are strictly defined in the official acts for study program accreditation. For its real processing, the University has to make efficient resource allocation (material and non-material), in order to deliver skills, knowledge and abilities at the expected level. Having this in mind, input oriented DEA model is used because it is assumed that the result (level of skills, knowledge and abilities, conducted through complex survey) is invariant and ask what can be done in order to decrease the resources investment for efficiency improvement.

The detection of inefficient courses in terms of inadequate allocation of input resources and / or achieved poor results proposes specific measures to optimize inefficient courses:

- Reduce the cost of engagement and collaboration of the teaching staff
    - Increasing the workload of the teacher
    - Reduction of gross salary

- Option for freelance engagement of staff with less financial burden
- Reduce the cost of equipment and inventory
  - Purchase of equipment and inventory at a lower purchase price
  - Increasing utilization of the unit (in terms of number of students)
  - Increasing the rate of depreciation
- Reducing the number of realized lectures and tutorials
  - Reduction and convergence of matter in fewer lectures
  - Pipe classes for lectures and tutorials
  - Reducing the number of students' groups
  - Intervention within the study programs / change relevant parameters

For more accurate investigation, it is possible to do an extension of the model by increasing the parameters that are not taken into account in this case, according to the perception of the management of the University. DEA allows that in order of geenration even more appropriate and realistic model.

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