Essays in New Economic Geography

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As customary, and as it is right, the usual disclaimer applies.

Introduction

The recent Nobel Prize assigned to Paul Krugman "for his analysis of trade patterns and location of economic activity" witnesses the important role that the scientific community gives to the insights of the so-called New Economic Geography (NEG) literature. This field of economic analysis has always been particularly appealing to policy makers, given the direct link between its results and regional policy rules. For the same reason it is useful to deepen the analysis of its most important outputs by testing the theoretical robustness of some of its more relevant statements. This thesis tries to offer a contribution in this direction by focusing on a particular sub-field of NEG literature, the so-called New Economic Geography and Growth (NEGG) literature, having in Baldwin and Martin (2004) and Baldwin *et. al* (2004) the most important theoretical syntheses. These two surveys collect and present in an unified framework the works by Baldwin, Martin and Ottaviano (2001), where capital is immobile and spillovers are localized, Martin and Ottaviano (1999) where spillovers are global and capital is mobile. Other related papers are Baldwin (1999) which introduces forward looking expectations in the so-called Footloose capital model developed by Martin and Rogers (1995); Baldwin and Forslid (1999) which introduces endogenous growth by means of a q-theory approach; Baldwin and Forslid (2000) where spillovers are localized, capital is immobile and migration is allowed. Some more recent developments in the NEGG literature can be distinguished in two main strands. One takes into consideration factor price differences in order to discuss the possibility of a monotonic relation between agglomeration and integration (Bellone and Maupertuis (2003) and Andres (2007)). The other one assumes firms heterogeneity in productivity (first introduced by Eaton and Kortum (2002) and Melitz (2003)) in order to analyse the relationship between growth and the spatial selection effect leading the most productive firms to move to larger markets (see Baldwin and Okubo (2006) and Baldwin and Robert-Nicoud

(2008). These recent developments are related to our work in introducing some relevant departures from the standard model. Indeed this thesis develops and extends the theoretical framework of New Economic Geography theory along several routes.

In the third chapter of the thesis we develop a New Economic Geography and Growth model which, by using a CES utility function in the second-stage optimization problem, allows for expenditure shares in industrial goods to be endogenously determined. The implications of our generalization are quite relevant. In particular, we obtain the following novel results: 1) catastrophic agglomeration may always take place, whatever the degree of market integration, provided that the traditional and the industrial goods are sufficiently good substitutes; 2) the regional rate of growth is affected by the interregional allocation of economic activities even in the absence of localized spillovers, so that geography always matters for growth and 3) the regional rate of growth is affected by the degree of market openness: in particular, depending on whether the traditional and the industrial goods are good or poor substitutes, economic integration may be respectively growth-enhancing or growth-detrimental.

In the fourth chapter of the thesis we build a New Economic Geography and Growth model based on Baldwin, Martin and Ottaviano (2001) with an additional sector producing Non-tradable goods (services). By assuming intersectoral and localized knowledge spillovers from the innovation sector to the service sector, we show that firms' allocation affects regional real growth. More precisely we assume that the unit labour requirements (and thereby the prices) in the service production are a negative function of the output of innovation, i.e. the stock of knowledge capital. Due to this new specification, real growth rates in the two regions always diverge when the firms allocation pattern differs from the symmetric one. This result is a novelty in the standard theoretical NEGG literature where regional gap in real growth rate is always zero. Moreover, this result has strong policy implications because it suggests that concentrating industries in only one region may also bring a dynamic loss for the periphery. By analyzing the trade-off between the dynamic gains of agglomeration (due to localized intertemporal spillovers) and the dynamic loss of agglomeration (due to localized intersectoral spillovers), we also discuss different notions of optimal level of agglomeration.

The thesis will proceed as follows: in the chapters one and two we describe the state of the art in New Economic Geography and its further developments such as the New Economic Geography and Growth, the possibility of a monotonic relation between agglomeration and integration, and finally the firms heterogeneity in New Economic Geography models. Instead in chapters three and four we present our original contribution to the theory, i.e. the analysis of endogenous expenditure shares and intersectoral knowledge spillovers on the agglomeration patterns and economic growth.

Part I

State of the Art

Chapter 1

Overview of the New Economy Geography Models

We will start the literature reviewby describing the three fundamental New Economic Geography models: the core-periphery, the Footloose capital and the Footloose entrepreneurs models as presented in Baldwin *et al.* (2004). As it is widely know, the first version of the core-periphery model was elaborated by Krugman (1991). Instead the footloose capital and entrepreneurs models were developed respectively by Martin and Rogers (1995) and independently by Ottaviano (1996) and Forslid (1999).

1.1 The Core-Periphery Model

The core-periphery model (Krugman, 1991) aims at explaining why regions with similar underlying features develop in a very different way from the economic point of view, as well as the mechanisms according to which the spatial distribution of the economic activities changes as the integration between regions goes further. Despite it's limited tractability, the core-periphery model is able to shed some light on the so-called agglomeration economies, defined as the tendency of a spatial concentration of economic activity to create economic conditions that foster the spatial concentration of economic activity.

The mechanics of the core-periphery model is led by three distinct effects. The first is the *market access effect* due to the fact that firms tend to locate in bigger markets. The second is the *cost of living effect*, that is given by the fact that in the region where more firms are located the price of industrial varieties is cheaper due to transport costs. The last one is the *market crowding effect*, consisting in the fact that the higher the number of firms in a region, the fiercer will be the competition. The first two effects foster agglomeration of economic activities, while the latter encourages their dispersion. Indeed when trade costs are high the *market crowding effect* is stronger than the *market access* and the *cost of living effects*. On the contrary when trade costs become to fall the strenght of the *market crowding effect* weakens faster than the *market access* and the *cost of living effects* thereby leading to agglomeration following a mechanism of circular cumulative causality. Let's see how it works.

In the standard core-periphery model with capital immobility we have that the reward of the mobile factor (i.e. workers' wage) is spent locally, thereby migration leads to expenditure shiftings, that in turn foster further production shiftings because in the region where the expenditure is higher firms gain more operating profits: this is the so-called *demand-linked circular causality*. Moreover production shiftings lead to expenditure shiftings because if more firms are present in one region, there will be a lower price for the consumers located in this region due to trade costs. Hence more workers will be attracted in the region where the cost of the industrial varieties is lower, so the cost shiftings will drive further production shiftings: this is the so-called cost-linked circular causality.

1.1.1 Structure of the Model

In this model we have two production factors (the industrial workers H and the agricultural workers L), and two sectors (industry M and agriculture A). There are 2 regions (north and south) with equal preferences, technologies, transport costs and initial endowments. The industrial sector works in Dixit-Stiglitz monopolistic competition and every firm in it employs only industrial workers as to produce its own output with constant return to scale. In particular the production of a single variety requires a fixed input consisting in F units of industrial workers, and a variable input consisting in a_M units of H for unit of output produced. So the cost function is $w\,(F + a_M x)$, where w is the wage of the industrial workers and x is the output of a firm. Instead the A sector produces an homogeneous good in perfect competition under constant returns to scale employing only agricultural workers (L). More precisely the sector uses a_A units of L for producing a unit of product. The representative consumer in each region has a utility function divided in two parts. The first part determines the division of the expenditure between the agricultural good and the industrial varieties. The second part describes the preferences of the consumer on the different industrial varieties. The specific functional form of the first part is a Cobb-Douglas where the share of expenditure in the industrial and agricultural sector is constant and is equal to μ and $1-\mu$ respectively. The functional form of the second part is a CES, with constant elasticity of substitution $1/\sigma$.

1.1.2 Law of Motion of Workers

The world distribution of workers L^w is symmetric, so the initial endowment of each region is $L^w/2$. Also the initial distribution of industrial workers H^w at the world level is symmetric , but while the agricultural work is immobile, the industrial work can migrate between regions, so its distribution is endogenous. The migration follows the law:

$$\dot{s}_H = (\omega - \omega^*)\, s_H \,(1 - s_H)\,, \qquad s_H \equiv \frac{H}{H^w}, \qquad \omega = \frac{w}{P}, \qquad \omega^* = \frac{w^*}{P^*} \quad (1.1)$$

where s_H is the share of the industrial workers located in the north, H is the total quantity of workers in the north, w and w^* are the wages of the industrial workers in both regions, while ω and ω^* are the corresponding real wages.

The wages are a measure of the utility of workers, so the workers migrate in the region that gives them a higher utility level.

1.1.3 Mechanism of Agglomeration

The first important force in the model leading agglomeration is the *demand-linked circular causality* that derives from the *market-access effect*. The following equations that we are going to present show how the forces determined by the *market-access effect* reinforce themselves. The first expression describes the reward for a firm. Given that we are in monopolistic competition (Dixit and Stiglitz, 1977), the equilibrium operating profits are given by the value of the product sold multiplied by σ. Moreover, for starting to produce a new variety is necessary one unit of capital, so $n^w = K^w$. Then we can express the profits like this:

$$\pi = \frac{\mu}{\sigma}\frac{E^w}{K^w}\left(\frac{w^{1-\sigma}s_E}{s_n w^{1-\sigma} + (1-s_n)\phi\,(w^*)^{1-\sigma}} + \frac{\phi(1-s_E)w^{1-\sigma}}{s_n\phi w^{1-\sigma} + (1-s_n)\,(w^*)^{1-\sigma}}\right) \tag{1.2}$$

in which $s_E = \frac{E}{E^w}$ is the share of expenditure in the north, while $(1-s_E) = \frac{E^*}{E^w}$ is the share of expenditure in the south. At the same time $s_n = \frac{n}{n^w}$ is the share of firms possessed by the north, while $(1-s_n) = \frac{n^*}{n^w}$ is the one possessed by the south. Finally we have ϕ that represents the *freeness of trade* , that is the inverse of the transportation costs. If ϕ is equal to 1 we have full freeness of trade so transportation costs equal zero, if ϕ equals 0 there is no trade. The other important equation in our model is the one representing the northern expenditure share:

$$s_E = (1-\mu)\left(s_L + \frac{wH^w}{w_L L^w}s_H\right) \tag{1.3}$$

here $s_L = \frac{L}{L^w} = 1/2$ in the symmetric equilibrium. In this equation s_L is the share of agricultural workers in the north and w_L is the wage in the agricultural sector in the north. This expression tells us that the share of expenditure in the north is an average of L and H. In fact if starting in the symmetric case a small migration from the north to the south determines an increase in s_E and a decrease in $(1-s_E)$, because the wage is spent where is earned. So northern market grows while southern market decreases. In presence of transport costs the firms would prefer to locate in the bigger market (*market-access effect*), because an higher expenditure means more profits, so the increase in expenditure determined by migration will induce an higher level of production.

This mechanism is self-reinforcing because when the firms move to north they will also bring a small number of workers, whose wage will be spent in the new region, so an increase in production leads to an increase in expenditure. The point is that an expenditure increase in a region determines a relocation of firms more than proportional in order to keep valid the zero-profit condition: this is the Home-Market Effect (Krugman, 1980). Given the more than proportional increase in the number of workers, we have that more production leads to even more expenditure.

The second agglomeration force considered is the cost-linked circular causality. To describe this force we specify in a better way the definition of real wage. Let's see the first:

$$\omega = \frac{w}{P}, \qquad P \equiv p_A^{1-\mu} \left(\Delta n^w\right)^{-a}, \qquad \Delta \equiv \frac{\int_{i=0}^{n^w} p_i^{1-\sigma} di}{n^w}, \qquad a \equiv \frac{\mu}{\sigma - 1} \tag{1.4}$$

where w is the real wage and P is the price index. Observing the definition of real wage and the law of motion of workers (0.1) we have the explanation of the *circular causality* mechanism. In case of symmetry a small migration of workers from south to north gives an increase in H and a decrease in H^*, determining a higher share of firms in the north s_n. If these firms sell locally the varieties produced they do not incur in transport costs, so if n increases the price index decreases in the north and increases in the south (*cost of living effect*). But a lower price index in the north means a higher real wage, leading other people to move to the north thereby causing a new increase in s_n.

The last force considered is dispersion force: the *market-crowding dispersion force*. Let's consider once again the expression for the profits of a firm:

$$\pi = \frac{\mu}{\sigma} \frac{E^w}{K^w} \left(\frac{w^{1-\sigma} s_E}{s_n w^{1-\sigma} + (1 - s_n)\phi \left(w^*\right)^{1-\sigma}} + \frac{\phi(1 - s_E)w^{1-\sigma}}{s_n \phi w^{1-\sigma} + (1 - s_n) \left(w^*\right)^{1-\sigma}} \right) \tag{1.5}$$

A migration of workers from south to the north gives an increase in s_n. This will lead to more competition in the local market so less profits, thereby the wage paid by the firms to the workers will be lower, driving back the workers to the other region.

1.1.4 Local Stability Analysis

The core equation in the core periphery model is the following:

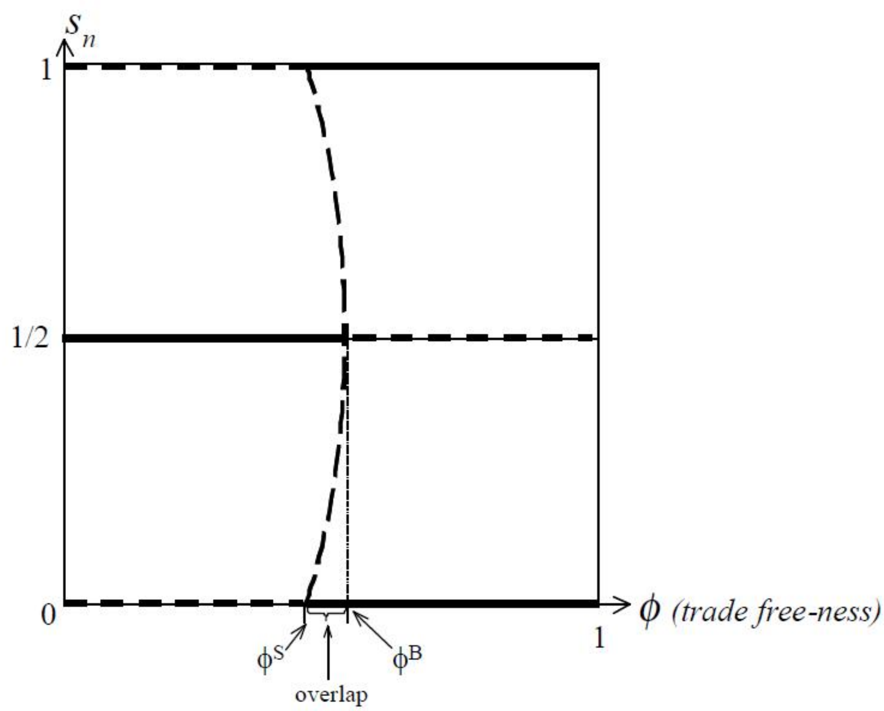
$$\dot{s}_H = s_H \left(1 - s_H\right) \Omega \left[s_H\right], \Omega = \omega - \omega^*$$

where $\Omega\left[s_H\right]$ describes the relation between s_H and the real wage gap. If we linearize this equation around the steady-state s_H°, we can check the coefficient of s_H. If is negative, the system is locally stable, while if is positive, the system is locally unstable. The linearization yields:

$$s_H = \left(s_H^\circ \left(1 - s_H^\circ\right) \frac{d\Omega\left[s_H^\circ\right]}{ds_H} + \left(1 - 2s_H^\circ\right) \Omega\left[s_H^\circ\right] \left(s_H - s_H^\circ\right) \right)$$

the system is clearly stable at the symmetric outcome if $d(\omega - \omega^*)/ds_H < 0$. Then qualitative features and the long run equilibria of the core periphery model can be observed in the so-called tomahawk diagram (figure 1) that plots the share of laborers in the north s_H against the freeness of trade ϕ.

Figure 1.1: Tomahawk Diagram for the core periphery Model



The solid lines represent the stable equilibria, while the dashed lines represent the unstable equilibria. As we can see the symmetric equilibrium looses its stability when the level of freeness of trade is beyond a level called break point ϕ^B, while the CP equilibrium becomes stable after another level of freeness of trade: the sustain point ϕ^S. It is possible to show that under a *no-black-hole-condition* $1 > a\sigma$ the break point comes before the sustain point. There are at most five equilibria: two CP outcomes (stable), one symmetric outcome (stable) and two interior asymmetric equilibria (unstable).

Moreover, in the range of freeness of trade between the break and the sustain point there are three stable equilibria that overlap.

1.1.5 Global Stability Analysis

For checking the global stability of these model the authors use the standard Liapunov method. Let's see the logic: if we have a steady-state (x^*, y^*) of a planar system of differential equations like

$$\dot{x} = x\left(A - By\right)$$

$$\dot{y} = y\left(Dx - C\right)$$

let F be a function of two variables which has a strict local minimum at (x^*, y^*). Considering the derivative:

$$\left(\dot{F}\left(x,y\right) = \frac{\vartheta F}{\vartheta x}\left(x,y\right) \cdot f\left(x,y\right) + \frac{\vartheta F}{\vartheta y}\left(x,y\right) \cdot g\left(x,y\right)\right)$$

if $\dot{F}\left(x,y\right) < 0$ then (x^*, y^*) is an asymptotically stable equilibrium, otherwise the system is unstable.

The stability of the system in the symmetric equilibrium's 'basin of attraction' can be checked choosing the function $\left(s_n - \frac{1}{2}\right)^2/2$. Using this function the system is clearly stable, in fact $F = \left(s_n - \frac{1}{2}\right)s_n < 0$.

Instead the stability of the system in the Core Periphery equilibrium's 'basin of attraction' can be checked defining the function $\left(s_n - 1\right)^2/2$. Also in this case the system is stable.

1.2 The Footloose Capital Model

The biggest problem of the core-periphery model is that it is not analytically solvable because the expressions of wages and prices involve powers non-integer hence cannot be solved as explicit functions of the spatial distribution of economic activities. This caveat led Martin and Rogers (1995) to introduce the so-called footloose capital model, where firms are assumed to migrate in search of the highest operating profits. However, despite it's tractability, the footloose capital model does not display the demand- and cost-linked circular causality present in the core-periphery model, hence possibility of agglomeration is ruled out. Indeed if capital mobility is assumed the reward of the mobile factor (in this case firms' profits) is spent not in the region where capital is employed, but in the region where the owners of capital live. Thus we have to distinguish the share of capital owned by residents of a given region (say north) $s_K = K/K^W$ from the share of the capital employed in the same region $s_n = n/n^w$. Assuming that profits are repatriated rules out *demand-linked* and *cost-linked circular causality* because capital movements lead to production shifting that are not followed by expenditure shifting (thus the demand-linkage is cut), and the price index is irrelevant with respect to the location of the capital (thus the cost-linkage is cut).

1.2.1 The Basic Structure

The basic structure of the footloose capital model is similar to the one of the core periphery model. In fact we have two regions: north and south; two sectors: manufacture and agriculture; two production factors: labor and capital. The two regions are perfectly symmetric in terms of tastes, preferences and endowment. In fact the industrial sector exhibits monopolistic competition, increasing returns and iceberg trade costs. On the contrary the agricultural sector produces an homogeneous good in perfect competition and constant returns to scale, and its output is shipped without any cost. Between the footloose capital and the core periphery model there are some differences. One of them is the assumption about the mobility of factors: in fact in the footloose capital model the two production factor are labor L and physical capital K, and its assumed that the capital can migrate between regions. Moreover is assumed that the reward of the mobile factor (in this case capital) is spent not in the region where capital is used, but in the region where the owners of capital live. Another difference between the two models concerns the technology of production of the industrial sector. In fact the cost function of an industrial firm is not homothetic in the sense that the factor intensity of the fixed differs from the factor intensity of the variable cost. Each industrial firm requires one unit of capital K as a fixed cost to start the production and a_m units of labor for producing a unit of output. So the cost function is

$$\pi + w_L a_m x$$

where π is the profit, w_L is the wage and x is the output produced.

1.2.2 Law of Motion of Capital

Also in this model the inter-regional factor flow is governed by an *ad hoc* equation:

$$\dot{s}_n = (\pi - \pi^*)(1 - s_n)s_n$$

Physical capital migrates in response to a change in the higher nominal reward instead of the higher real reward. This because the reward of capital is spent in the owner's region without taking into account where the capital is employed, so there is no influence of the *price index*.

1.2.3 Market Access Effect and Market-Crowding Effect

The first expression that we consider is the mobile factor reward: as we have shown above under Dixit-Stiglitz monopolistic competition the operating profits are given by the value of sales divided by σ, that is the elasticity of substitution between varieties. So we have $\pi = \frac{px}{\sigma}$. Hence, using the demand function and the *mill pricing* we can express the capital reward as:

$$\pi = \frac{\mu}{\sigma}\frac{E^*}{K^*}\left[\frac{s_E}{s_n + \phi(1 - s_n)} + \phi\left(\frac{s_E^*}{\phi s_n + 1 - s_n}\right)\right]$$

in which E^w is the world expenditure while s_E and s_E^* are the northern and southern share of it.

Another important expression is the one relating the expenditure share in the with the share of firms and capital in the north:

$$s_E = (1 - \frac{\mu}{\sigma})s_L + \frac{\mu}{\sigma}s_K$$

in which $s_L \equiv \frac{L}{L^w}$ and $s_K \equiv \frac{K}{K^w}$.

These two equations express the reward of capital in the two regions as functions of the spatial distribution of firms (s_n), workers (s_L), and capital owners.[1]

As shown by Ottaviano (2001) the profit differential is zero/positive/negative when the right hand side of the following expression is zero/positive/negative:

[1] It is assumed that each owner can possess only one unit of capital.

$$sgn(\pi - \pi^*) = (1-\phi)sgn\left\{(1+\phi)(s_E - \frac{1}{2}) - (1-\phi)(s_n - \frac{1}{2})\right\}$$

In case of total freeness of trade ($\phi = 1$) the profit of the firm in the two region is equal, so we do not have relocation. Instead if the trade is not perfectly free ($\phi < 1$) we have that the location decision of a firm is determined by the interaction of two opposite forces: the *market access effect* and the *market crowding effect*.

The first term of the equation in curly brackets is the *market access effect* that shows how the spatial distribution of expenditure affects the spatial distribution of firms. Given that ($\phi < 1$) we have that $(1+\phi)$, so its an advantage for the firms to locate in the larger market.

The second term in curly brackets is the market-crowding effect that shows the market disadvantage of being in the region with a larger number of firms, given that $-(1-\phi)$ is negative.

1.2.4 Local Stability Analysis

The footloose capital model is very simple from an analytical point of view, so is not difficult to find algebraically the break point, that is the level of transport costs at which the symmetric equilibrium becomes unstable, and the sustain point, that is the level of transport costs at which the core-periphery equilibrium becomes stable.

For checking the local stability we differentiate the gap in the profit for a firm in the two regions $\pi - \pi^*$ with respect to the symmetric equilibrium $s_n = s_n = \frac{1}{2}$:

$$d(\pi - \pi^*) = 4\frac{\mu}{\sigma}(\frac{1-\phi}{1+\phi})ds_E - 4\frac{\mu}{\sigma}(\frac{1-\phi}{1+\phi})^2 dn$$

where $ds_E = \vartheta s_E / \vartheta s_n dn$. Since capital owners are immobile and profits are repatriated, we have $\vartheta s_E / \vartheta s_n = 0$. Consequently the symmetric equilibrium is stable as long as the trade is not perfectly free. We do not have expenditure-shiftings but only market-crowding effect.

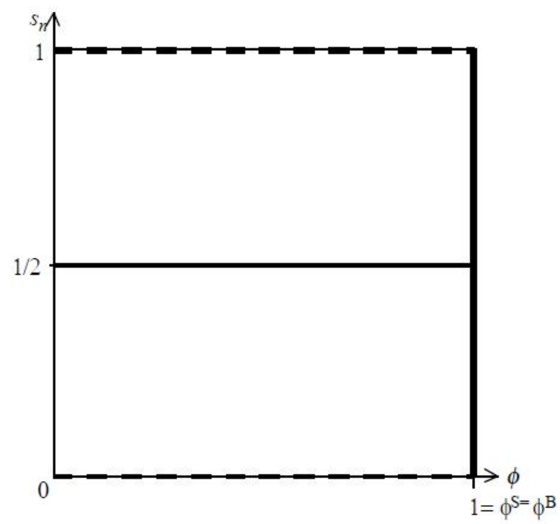
Now we investigate the stability of the northern core-periphery deriving the profit gap at $s_n = 1$:

$$\pi - \pi^* = -\frac{\mu}{\sigma}\frac{(1-\phi)^2}{2\phi}$$

is clear that the sustain point in the footloose capital model is $\phi = 1$. So there will be a tendency in the model to move from the core-periphery outcome to the symmetric one.

In fact, as shown in the tomahawk diagram (figure 2), the model has one interior symmetric equilibrium at $s_n = \frac{1}{2}$ and two core-periphery equilibria at $s_n = 0$ and at $s_n = 1$. Furthermore the system is always stable for all ϕ up to $\phi = 1$, and sustain and break point coincide. Moreover we have that the system does not have overlapping equilibria, so there is no room for issues like indeterminacy driven by rational expectation and self-fulfilling prophecies .

Figure 1.2: Tomahawk Diagram for the Footloose Capital Model



1.3 The Footloose Entrepreneur Model

The footloose entrepreneur model can be seen as a mixture between the core periphery model and the footloose capital model. Indeed the main assumption of the model is that to start the production a firm needs one unit of human capital that is the entrepreneur. Then when the firm relocates to another region, moves with its own entrepreneur. Alike the core periphery model we have migration driven by real wage differences, giving the possibility of a *demand-linked* and *cost-linked circular causality.* For instance when the reward of the mobile factor is spent where it is earned, we have expenditure shiftings that yields production shiftings that produce other expenditure shiftings. Likewise, since the mobile factor migrates in response of changes in the real wage, we have that production shiftings influences the cost-of-living via the price index inducing further migration. So the two forms of circular causality are the same as of the core periphery model, thus also the main features of the two models are analogous. But the footloose entrepreneur model resembles also the footloose capital model in its assumptions so in its tractability. In fact the footloose capital model is tractable because there is the possibility to express in closed form the equilibrium condition for reward to the mobile factor, because is assumed that the mobile factor itself is used only in the fixed part of the cost necessary to produce an industrial variety.

1.3.1 The Law of Motion of Entrepreneurs

The assumption of the footloose entrepreneur model are almost the same as the core periphery model, so we do not repeat them. We just specify that workers are not interregional mobile but are equally located between the two regions, then $L = L^* = L^W/2$. Instead the location of the mobile factor H, that are the entrepreneurs, is endogenous. The entrepreneurs migrate in response to the difference in the real wage, so in the indirect utility difference, according to the following law of migration:

$$\dot{s}_H = (\omega - \omega^*) s_H (1 - s_H)$$

where $s_H \equiv H/H^W$ is the share of entrepreneurs in the north, in which H is the northern stock of entrepreneurs and H^w is the world total stock. Moreover, ω and ω^* are the northern and southern real wages for H.

1.3.2 Local Stability Analysis

Let's take into account the indirect utility function for a typical northern entrepreneur that are ω and ω_L:

$$\omega = \frac{w}{P},\ \omega^* = \frac{w_L}{P},\ P \equiv p_A^{1-\mu}(\Delta n^w),\ \Delta \equiv \left(\int_{i=0}^{n^w} p_i^{1-\sigma} di\right)/n^w,\ a \equiv \frac{\mu}{\sigma - 1}$$

in which w is the northern wage for entrepreneur and w_L is its southern correspondent, while P is the well known *price index*. From here we can define our location condition as

$$\omega = \omega^*,\ 0 < s_n < 1$$

If we differentiate this condition with respect to s_n at the symmetric equilibrium $s_n = 1/2$ we have that the system looses its stability for values of freeness of trade beyond the break point:

$$\phi^B = \left(\frac{1-b}{1+b}\right)\left(\frac{1-a}{1+a}\right)$$

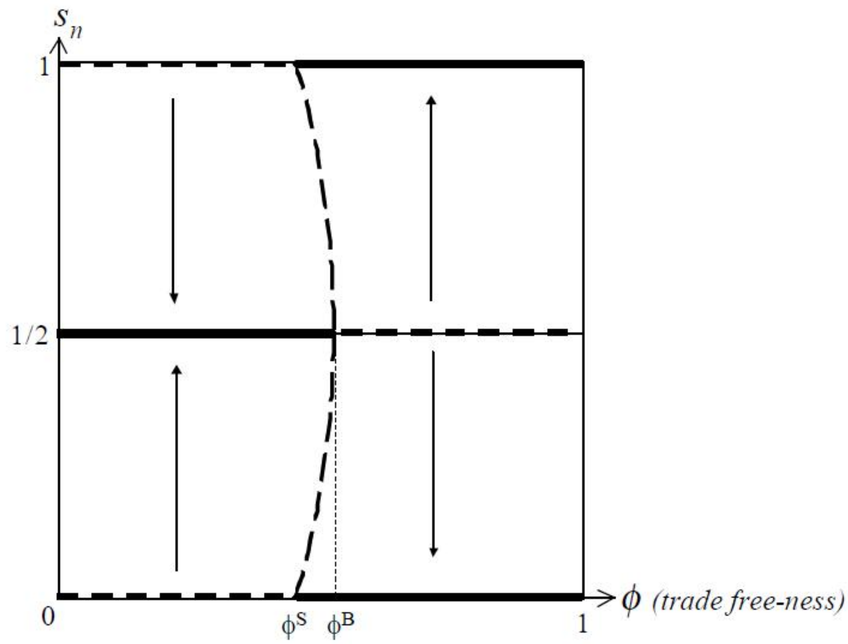
As we know the break point is decreasing in μ and increasing in σ, so the range of transport cost for the system for loosing its stability at the symmetric equilibrium strictly depends on the expenditure share in manufacture. Furthermore, a decreasing in σ has the opposite effect because it implies a lower markup so lower agglomeration forces.

For checking the stability of the core-periphery equilibrium we define the combination of s_E and s_n from which the log real wage gap is zero: $\Omega \equiv \ln(\omega/\omega^*) = 0$. Then we evaluate this for $s_n = 0$ or $s_n = 1$, then after many transformations we can implicitly define the sustain level of trade costs ϕ^S as the lowest root of:

$$1 = (\phi^S)^a(\frac{1+b}{2}\phi^S + \frac{1-b}{2\phi^S})$$

We can describe our results in the tomahawk diagram (figure 3):
As we can see, for levels of freeness of trade below the sustain point $\phi < \phi^S$, the only stable equilibrium is the symmetric one. Instead, for levels of freeness of trade beyond the break point $\phi > \phi^B$ we have three steady states: the symmetric one and the two core-periphery, even if only the two

Figure 1.3: Tomahawk Diagram for the footloose entrepreneur Model



core-periphery are stable. Finally with $\phi^S < \phi < \phi^B$ there are five steady states: the two core-periphery (stable), two interior asymmetric (unstable) and one symmetric unstable. So we have that the core periphery and the footloose entrepreneur model display the same behavior for what concerns the equilibrium and the local and global stability properties. Moreover these two models show the same implications for what concerns the indeterminacy of equilibria determined by forward-looking migration.

1.4 Conclusions

In this chapter we presented the three pivotal New Economic Geography: the core-periphery model (Krugman, 1991), the footloose capital model (Martin and Rogers, 1995) and the footloose entrepreneur model (Ottaviano, 1996 and Forslid, 1999). The three models diplay different assumptions in terms of factor mobility as well as different outcomes and degrees of analytical tractability. Concerning the assumptions about the mobility of factors we have that in the core periphery model workers are assumed to migrate in response to wage differences, while in the footloose capital model the mobile factor consists in firms looking for the higher operating profits, and finally in the footloose entrepreneur model firms shift region together with the entrepreneurs as each firm needs a unit of knowledge capital (embodied in the entrepreneur) in order to start producing a new variety. Regarding the outcomes of the model, the core periphery and the footloose entrepreneurs models display a circular cumulative causality mechanism leading to agglomeration, while in the footloose capital model demand- and cost-linked circular causality is ruled out by the assumption that the capital reward is repatriated. Indeed in the core periphery and in the footloose capital models the reward of the mobile factor is spent locally hence expenditure shiftings determine production shifting. Moreover the mobile factor shifts region responding to changes in the real wages, hence production shiftings foster migration that in turn determines further expenditure shiftings, thereby feedbacking the mechanism. Finally concerning the degrees of analytical tractability we have that the footloose capital and the footloose entrepreneur models allow for closed forms for the equilibrium conditions because in the footloose capital model the rewards of the mobile factor (firms profits) are assumed to be repatriated whilst in the footloose entrepreneur model it is assumed that the mobile factor itself is used only in the fixed part of the cost necessary to produce an industrial variety. By contrast in the core periphery model the expressions of wages and prices involve powers non integer hence cannot be solved as explicit functions of the spatial distribution of economic activities.

Chapter 2

Further Developments in New Economic Geography

In this chapter we will present some recent developments of the New Economic Geography literature. First we will describe the New Economic Geography and Growth approach (*inter alias*, Baldwin, Martin and Ottaviano (2001)), in which endogenous growth is added to a version of Krugman's celebrated core-periphery model (Krugman 1991). Second we will present the two more recent strands of the literature: one which takes into consideration factor price differences in order to discuss the possibility of a monotonic relation between agglomeration and integration (Bellone and Maupertuis 2003, Andres 2007). The other one which assumes firms heterogeneity in productivity (first introduced by Eaton and Kortum (2002) and Melitz (2003)) in order to analyse the relationship between growth, spatial selection and trade openness (Baldwin and Okubo 2006, Baldwin and Robert-Nicoud 2008).

2.1 New Economic Geography and Endogenous Growth

2.1.1 Geography and Growth Stages

The pivotal New Economic Geography and Growth model is the one developed by Baldwin, Martin and Ottaviano (2001). The most important feature of the model and the source of its most novel results is the introduction, in a core-periphery setting, of endogenous growth *à la* Romer (1990) taking the form of intertemporal localized knowledge spillovers. Thanks to this departure we have that the cost of innovation is minimized when the whole manufacturing sector is agglomerated. In this case in fact innovating firms have a higher incentive to invest in new units of knowledge capital with respect to a situation in which manufacture firms are scattered along the two regions. Thereby the rate of growth is maximized in the core-periphery configuration. Baldwin, Martin and Ottaviano (2001) give also a theoretical explanation of four industrial revolution stages of growth. In the first stage trade costs are high and the industry is internationally dispersed. In the second stage trade costs begin to fall and north industrializes and grows rapidly. In the third stage, with low trade costs high growth and global divergence become self-sustainable, while in the forth stage, when the trade cost of ideas falls, south converges.

The structure of the economy

In the analysis are assumed two regions symmetric in terms of technology, preferences, transport costs and initial endowments. Each region is endowed with two production factors: labor L and capital K. Three production sectors are active in each region: modern (manufacture) M, traditional (agriculture) T and a capital producing sector I. Labor is assumed to be immobile across regions but mobile across sectors within the same region. The Traditional good is freely traded between regions whilst manufacture is subject to iceberg trade costs[1] (Samuelson, 1954). For the sake of simplicity the authors focus on the northern region[2].

The manufactures are produced under Dixit-Stiglitz monopolistic competition (Dixit and Stiglitz, 1977) and enjoy increasing returns to scale: firms face a fixed cost in terms of knowledge capital. In fact it is assumed that for producing a variety is required a unit of knowledge interpreted as a blueprint, an idea, a new technology, a patent, or a machinery. Moreover firms face a variable cost a_M in terms of labor. Thereby the cost function is $\pi + wa_I x_i$, where π is the rental rate of capital, w is the wage rate and a_M are the unit of labor necessary to produce a unit of output x_i.

Each region's K is produced by its I-sector which produces one unit of K with a_I unit of labor. So the production and marginal cost function for

[1] It is assumed that a portion of the good traded melts in transit.

[2] Unless differently stated, the southern expressions are isomorphic

the I-sector are, respectively

$$\dot{K} = Q_K = \frac{L_I}{a_I} \tag{2.1}$$

$$F = wa_I \tag{2.2}$$

Note that this unit of capital in equilibrium is also the fixed cost F of the manufacturing sector. As one unit of capital is required to start a new variety, the number of varieties and of firms at the world level is simply equal to the capital stock at the world level: $K + K^* = K^w$. We denote n and n^* as the number of firms located in the north and south respectively. As one unit of capital is required per firm it is clear that: $n + n^* = n^w = K^w$. However, depending on the assumptions made on capital mobility, the stock of capital produced and owned by one region may or may not be equal to the number of firms producing in that region. In the case of capital mobility, the capital may be produced in one region but the firm that uses this capital unit may be operating in another region. Hence, when capital is mobile, the number of firms located in one region is generally different from the stock of capital owned by this region.

To individual I-firms, the innovation cost a_I is a parameter. However, following Romer (1990), endogenous and sustained growth is provided by assuming that the marginal cost of producing new capital declines (i.e., a_I falls) as the sector's cumulative output rises. In our specification, learning spillovers are assumed to be localized. The cost of innovation can be expressed as

$$a_I = \frac{1}{AK^w} \tag{2.3}$$

where $A \equiv \theta_K + \lambda (1 - \theta_K)$, $0 < \lambda < 1$ measures the degree of globalisation of learning spillovers and $\theta_K = K/K^w$ is the share of firms allocated in the north. The south's cost function is isomorphic, that is, $F^* = w^*/K^w A^*$ where $A^* = \lambda\theta_K + 1 - \theta_K$. For the sake of simplicity in the model version examined, capital depreciation is ignored[3].

Because the number of firms, varieties and capital units is equal, the growth rate of the number of varieties is therefore

$$g \equiv \frac{\dot{K}}{K}; g^* \equiv \frac{\dot{K}^*}{K^*} \tag{2.4}$$

[3]See Baldwin (2000) and Baldwin et al. (2004) for similar analysis with depreciation

Finally, the T-sector produces a homogenous good in perfect competition and constant returns to scale. By choice of units, one unit of T is made with one unit of L. The infinitely-live representative consumer's optimization is carried out in three stages. In the first stage the agent intertemporally allocates consumption between expenditure and savings. In the second stage she allocates expenditure between M- and T-goods, while in the last stage she allocates manufacture expenditure across varieties. The preferences structure of the infinitely-live representative agent are given by:

$$\begin{aligned} U_t &= \int_{t=0}^{\infty} e^{-\rho t} \ln Q_t dt \\ Q_t &= \ln \left(C_M^{\alpha} C_T^{1-\alpha} \right) \qquad (2.5) \\ C_M &= \left[\int_{i=0}^{K+K^*} c_i^{1-1/\sigma} di \right]^{\frac{1}{1-1/\sigma}} \qquad (2.6) \end{aligned}$$

As a result of the intertemporal optimization program, the path of consumption expenditure E across time is given by the standard Euler equation:

$$\frac{\dot{E}}{E} = r - \rho \qquad (2.7)$$

with the interest rate r satisfying the no-arbitrage-opportunity condition between investment in the safe asset and capital accumulation:

$$r = \frac{\pi}{F} + \frac{\dot{F}}{F} \qquad (2.8)$$

where π is the rental rate of capital and F its asset value which, due to perfect competition in the I-sector, is equal to its marginal cost of production.

In the second stage of the utility maximization the agent chooses how to allocate the expenditure between M- and the T- good according to the following optimization program:

$$\begin{aligned} \max_{C_M, C_T} Q_t &= \ln \left(C_M^{\alpha} C_T^{1-\alpha} \right) \qquad (2.9) \\ s.t. \ E &= P_M C_M + p_T C_T \end{aligned}$$

The objective function is:

$$L : \ln \left(C_M^{\alpha} C_T^{1-\alpha} \right) + \eta \left(E - P_M C_M - p_T C_T \right)$$

Yielding the following demand functions:

$$C_M = \alpha \frac{E}{P_M}$$

$$C_T = (1-\alpha)\frac{E}{p_T}$$

where p_T is the price of the Traditional good and $P_M = \left[\int_{i=0}^{K+K^*} {p_i}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$ is the Dixit-Stiglitz price index. It is clear that the shares of expenditure in the three types of goods are constant.

Finally, in the third stage, the amount of $M-$ goods expenditure αE is allocated across varieties according to the a CES demand function for a typical M variety $c_j = \frac{p_j^{-\sigma}}{P_M^{1-\sigma}}\alpha E$, where p_j is variety j's consumer price. Free trade in traditional good implies that its price is equal between regions. Moreover it is assumed no-specialization and perfect competition, hence wages are also equal between regions. Thus th T-good can be taken as a numeraire so wages and prices in both regions are tied to unity: $p_T = p_T^* = w = w^* = 1$.

Concerning the M-sector, since wages are uniform and all varieties' demands have the same constant elasticity σ, firms' profit maximization yields local and export prices that are identical for all varieties no matter where they are produced: $p = wa_M\frac{\sigma}{\sigma-1}$. Then, by imposing the normalization $a_M = \frac{\sigma-1}{\sigma}$:

$$p = w = 1$$

By iceberg import barriers, prices for markets abroad are higher:

$$p^* = \tau p;\ \tau \geq 1$$

and by CES demand function for variety:

$$\pi = B\left(\frac{\alpha E^w}{\sigma K^w}\right) = \frac{\alpha E^w}{\sigma K^w}\left[\frac{\theta_E}{\theta_K + \phi(1-\theta_K)} + \frac{\phi(1-\theta_E)}{\phi\theta_K + 1 - \theta_K}\right], \theta_E = \frac{E}{E^w}$$

where $E^w = E + E^*$ is world expenditure, θ_E is north's share of expenditure and $\phi = \tau^{1-\sigma}$ is the freeness of trade going from 0 (prohibitive cost) to 1 (costless trade).

The market clearing condition for the M-good implies the value of production $\alpha_M(L_M + L_M^*)$ to be equal to expenditure αE^w. Same thing for the

T-good where the expenditure is given by $(1-\alpha) E^w$ and the world supply is $(L_T + L_T^*)$. At world level labor market clearing condition implies $2L = (L_T + L_T^*) + (L_M + L_M^*) + (L_I + L_I^*)$, and by using M- and T-good market clearing condition:

$$E^w = \frac{\sigma}{\sigma - \alpha} (2L - L_I - L_I^*)$$

where labor employed in innovation is equal to income minus consumption (investment):

$$L_I = L + \pi K - E$$

The dynamic system describing the evolution of the economy is given by two Euler equations (one for each region) and a capital law of motion:

$$\frac{\dot{E}}{E} = \frac{\alpha}{\mu} E^w (AB - Bs_K - \lambda B^* (1 - s_K)) - L(1+\lambda) + (\lambda E^* + E) - \rho$$

$$\frac{\dot{E}^*}{E^*} = \frac{\alpha}{\mu} E^w (A^* B^* - \lambda B s_K - B^* (1 - s_K)) - L(1+\lambda) + (\lambda E^* + E) - \rho$$

$$\begin{aligned} \dot{s}_K &= s_K (1 - s_K) \left(\frac{L_I a_I}{s_K} - \frac{L_I^* a_I^*}{1 - s_K} \right) \\ &= \left((1 - s_K) \left(L + \frac{\alpha}{\sigma} E^w B s_K - E \right) A - s_K \left(L + \frac{\alpha}{\sigma} E^w B^* (1 - s_K) - E^* \right) A^* \right) \end{aligned}$$

The Long-Run Equilibrium

In the long run equilibium $\dot{E} = \dot{E}^* = \dot{\theta}_K = 0$. The capital law of motion implies only two kinds of steady states: either both regions innovate at the same rate $\bar{g}$ (interior outcome) or only one region does so (core-periphery outcome, i.e. $\theta_K \equiv 0, 1$). For what concerns the steady state expenditure level, labor income should be equal to the current value of the steady state wealth:

$$\bar{E} = L + \rho \frac{\bar{\theta}_K}{\bar{A}}$$

$$\bar{E}^* = L + \rho \frac{1 - \bar{\theta}_K}{\bar{A}^*}$$

The steady states values of θ_K are three:

$$\bar{\theta}_K = \frac{1}{2}, \bar{\theta}_K = \frac{1}{2}\left[1 \pm \sqrt{\left(\frac{1+\lambda}{1-\lambda}\right)\left(\frac{1+\lambda\Lambda}{1-\lambda\Lambda}\right)}\right]$$

$$\Lambda = \left\{1 - \frac{2\rho\phi\left(1-\lambda\phi\right)}{\left[\lambda\left(1+\phi^2\right)-2\phi\right]L}\right\}$$

The first one is an interior symmetric equilibrium while the other two are interior non-symmetric steady states. The threshold in trade costs for the symmetric equilibrium to loose its stability is given by:

$$\phi_B = \frac{\left[L\left(1+\lambda\right)+\rho\right] - \sqrt{\left(1-\lambda^2\right)\left[\left(1+\lambda\right)+\rho\right]^2 + \lambda^2\rho^2}}{\lambda\left[L\left(1+\lambda\right)+2\rho\right]}$$

For this level of trade costs the second and third solution converge to 1/2 from above. Instead for levels of trade costs above another critical value:

$$\phi_{CP} = \frac{2L+\rho-\sqrt{\left(2L+\rho\right)^2 - 4\lambda^2 L\left(L+\rho\right)}}{2\lambda\left(L+\rho\right)}$$

The second solution is imaginary and the third exceeds unity: hence for this level of trade costs the core-periphery equilibrium becomes stable. The steady state level of labor employed in innovation is:

$$\bar{L}_I = \frac{\bar{\theta}_K}{\bar{A}}\left\{\frac{\alpha}{\sigma}\left[2L+\rho\left(\frac{\bar{\theta}_K}{\bar{A}}+\frac{1-\bar{\theta}_K}{\bar{A}^*}\right)\right]\bar{A}\bar{B}-\rho\right\}$$

For the symmetric equilibrium:

$$\bar{L}_I = \bar{L}_I^* = \frac{\alpha\left(1+\lambda\right)L - \rho\left(\sigma-\alpha\right)}{\sigma\left(1+\lambda\right)}$$

Instead in core in the north equilibrium:

$$\bar{L}_I = \frac{\alpha 2L - \rho\left(\sigma-\alpha\right)}{\sigma},\ \bar{L}_I^* = 0$$

Finally the steady state growth rate of capital is:

$$\bar{g} = \frac{\bar{L}_I\left[\bar{\theta}_K + \lambda\left(1-\bar{\theta}_K\right)\right]}{\bar{\theta}_K}$$

The Logic of Catastrophic Agglomeration

By using the Tobin's q approach the equilibrium level of investment is given by the equality between the replacement cost of capital P_K and the stock market value of a unit of capital V, which given in the two regions by:

$$\bar{V} = \frac{\pi}{\rho + \bar{g}}, \ \bar{V}^* = \frac{\bar{\pi}^*}{\rho + \bar{g}}$$

The M-sector free entry condition implies $q = V/P_K = 1$, and the steady states q are given by:

$$\bar{q} = \frac{\bar{\pi}/(\rho + \bar{g})}{\bar{F}}, \ \bar{q}^* = \frac{\bar{\pi}^*/(\rho + \bar{g})}{\bar{F}^*}$$

In the model two kinds of circular causality emerge. A *demand-linked circular causality*, according to which production shiftings lead to expenditure shiftings through the permanent income hypothesis. This in turn fosters further production shiftings because in the region where the expenditure is higher there is more incentive to invest in new firms. A cost-linked circular causality, according to which production shiftings lead to expenditure shiftings because if more firms are present in one region, there will be a lower price for the consumers. Hence more investments will be attracted in the region where the cost of the industrial varieties is lower, so the cost shiftings will drive further production shiftings. The only force contrasting agglomeration is the *market crowding effect*, due to the fact that an increase in the share of firms in a region decreases the profits hence Tobin's q. We check the stability by investigating the impact of an increase of the share of firms on the regional $\bar{q}$'s ratio. The symmetric equilibrium is stable if $\partial\bar{q}/\partial\theta_K$ is negative, because in this case southern Tobin's q in the north falls while raising in the south. By contrast if $\partial\bar{q}/\partial\theta_K$ is positive, the symmetric equilibrium becomes instable. Differentiating Tobin's q ratio with respect to θ_K yields:

$$\left(\frac{\partial\bar{q}/\bar{q}^*}{\partial\theta_K}\right)_{|\bar{\theta}_K=1/2} = 2\left(\frac{1-\phi}{1+\phi}\right)\left(\frac{d\bar{\theta}_E}{d\theta_K}\right)_{|\bar{\theta}_K=1/2} + \frac{4}{1+\lambda}\frac{1+\phi^2}{(1+\phi)^2}\left[\frac{-(1-\phi)^2}{1+\phi^2} + 1 - \lambda\right]$$

where $d\bar{\theta}_E/d\theta_K = 2\rho\lambda/\left[L(1+\lambda)+\rho\right](1+\lambda)$. The first and third terms represent the destabilizing forces, the *demand-linked* and the *growth-linked circular causality*. The negative term is the *market crowding effect* and acts as stabilizing force. Clearly the system is unstable for sufficiently low trade costs, i.e. at some point the destabilizing forces are stronger than the stabilizing one.

Growth Stages

The CES price index for manufacturing the composite good C_M is given in the two regions by:

$$P_M = K^{w^{\frac{1}{1-\sigma}}} \left[\theta_K + \phi\left(1-\theta_K\right)\right]^{\frac{1}{1-\sigma}},\ P_M^* = K^{w^{\frac{1}{1-\sigma}}} \left[\phi\theta_K + 1 - \theta_K\right]^{\frac{1}{1-\sigma}}$$

In the symmetric steady state $\bar{\theta}_E = \bar{\theta}_K = 1/2$ and the growth rate in the two regions is:

$$Stage\ I : \bar{g} = \bar{g}^* = \frac{\alpha\left(1+\lambda\right)L - \rho\left(\sigma-\alpha\right)}{\sigma}$$

The real income is given the nominal one Y divided by the perfect consumption price index P, hence:

$$\bar{g}_{income} = \bar{g}^*_{income} = \frac{\alpha^2\left(1+\lambda\right)L - \rho\alpha\left(\sigma-\alpha\right)}{\sigma\left(\sigma-1\right)}$$

Clearly the growth rate raises with λ and α but falls ρ and σ. Finally the rate of investment in steady state is:

$$Stage\ I : \frac{\bar{L}_I}{\bar{Y}} = \frac{\alpha\left(1+\lambda\right)L - \rho\left(\sigma-\alpha\right)}{\left(\sigma+\alpha\right)\left(1+\lambda\right)L + \alpha\rho}$$

The growth rate in the second stage (north take off) cannot be analytically found, while in the third stage, when south does not innovate the growth rate is:

$$StageIII : \bar{g} = \frac{\alpha 2L - \rho\left(\sigma-\alpha\right)}{\sigma}$$

Growth in core-periphery does no longer depend on spillovers and is common to both regions. Moreover it is higher than the growth rate in the symmetric equilibrium, hence geography matters for growth. The third stage northern investment ratio is:

$$Stage\ III : \frac{\bar{L}_I}{\bar{Y}} = \frac{\alpha L - \rho\left(\sigma-\alpha\right)}{\left(\sigma+\alpha\right)L + \alpha\rho}$$

In the fourth stage of growth industry is fully agglomerated in the north ($\bar{\theta}_K = 1$). As λ starts to increase, it becomes profitable to innovate in the south. So at some point the steady state q^* :

$$\bar{q}^* = \lambda\frac{\left(1+\phi^2\right)L + \phi^2\rho}{\left(2L+\rho\right)\phi}$$

exceeds unity. The threshold in the cost of trading ideas for the core-periphery equilibrium to loose its stability is:

$$\lambda^{mir} = \frac{\phi\,(2L + \rho)}{L\,(1 + \phi^2) + \rho\phi^2}$$

that is clearly rising with the freeness of trade ϕ. Hence when the cost of trading ideas falls beyond a certain threshold industry starts to relocate to south.

2.2 Monotonicity Between Agglomeration and Integration

In a New Economic Geography framework there are two models displaying a non-monotonic relation between integration and agglomeration in case of labor immobility. One is presented by Puga (1999), who relying on the factor price difference introduced in Krugman and Venables (1995), shows that in the agglomerated region wages are higher than in the periphery, hence for low enough trade costs the industrial firms can be incited to move back. The other one is presented by Baldwin, Martin and Ottaviano (2001), who rule out factor prices difference while showing that a U-shaped convergence scenario may emerge due to fall in the cost of trading ideas.

2.2.1 Economic Integration and Regional Income Inequalities

Bellone and Maupertuis (2003) contrast the possibility of a non-monotonic relation between integration and agglomeration by removing the no-specialization condition, allowing unequal wage trajectories to emerge and the share of expenditure on manufacture to take all its range of values. Thanks to these departures Bellone and Maupertuis (2003) are able to show that even in case of wage gap industrial activities do not move back to periphery because if knowledge is imperfectly localized a wage advantage cannot overcome an innovative capabilities one.

The Basic Model

In this models there are two regions, A and B and three sectors: innovative I, industrial M and traditional T. Each region has identical consumer's preferences and is endowed with the same amount of labor force (L^A and L^B) that can move freely among the sectors. The modern good is produced under Dixit-Stiglitz (1977) monopolistic competition with increasing returns to scale: the fixed cost is represented by an unit of capital while the variable cost is represented by labor. The modern good is shipped with iceberg trade costs (Samuelson, 1954): it is assumed that a part of the good shipped melts in transit. The traditional good is homogeneous and is produced with constant returns to scale in perfect competition and with unit labor cost. Moreover the traditional good is shipped without incurring in any trade cost. The innovation sector works in perfect competition with endogenous growth *à la* Romer (1990). It produces new units of knowledge capital that are then used by industrial firms for setting new production activities. Due to intertemporal spillovers, the cost of producing new knowledge decreases with the stock of knowledge already created. Moreover the cost of producing new knowledge depends the localized spillovers.

Consumers maximize the following utility function:

$$U = \int_0^\infty e^{-\rho t} \ln\left(C_T^{1-\alpha} C_M^{\alpha}\right) dt$$

where ρ is the rate of time preferences and C_T and C_M are respectively the demand for the traditional good and a CES aggregate for the manufacture that can be expressed as:

$$C_M = \left(N^i x_i^{i\beta} + N^j x_j^{i\beta}\right)^{1/\beta}$$

where $x_{i,j}$ represents the consumption of a single variety belonging to $N^{i,j}$, that is the mass of varieties in the two regions . Finally β is the love for variety in manufacture goods. Consumers divide their nominal expenditure E traditional and modern according to the following relation:

$$(1-\alpha) E^i = p_T C_T$$

$$\alpha E^i = p_M C_M$$

By contrast, for what concerns the world market equilibrium of traditional good we have:

$$p_T\left(T^A+T^B\right)=(1-\alpha)\,E^w$$

The traditional sector works in constant returns to scale and perfect competition, hence it will be produced only in the region with the lowest production cost and it will be priced at the minimum cost. By unitary labor cost we have that:

$$\begin{aligned} p_T &= \min\left(w^A,w^B\right) \\ s_T^i &= 0 \text{ when } w^i>w^j \end{aligned}$$

where $s_T^i=T_i/(T^A+T^B)$ is the share of region i of traditional good production and w^i is $i's$ region wage. The share of world spending on industrial goods produced by region i is instead:

$$s_M^i=\frac{N^i\left(p_i^ix_i^i+p_i^jx_i^j\right)}{\alpha E^w} \text{ for } i,j=A,B \text{ and } j\neq i$$

where p_i^i is the price of a variety produced and sold in region i, while p_i^j is the price of a variety produced in region i and sold in region j. The demand functions coming from the minimization of the cost of attaining the manufacture composite C_M are respectively in the two regions:

$$x_i^i=\frac{\alpha E^i\left(p_i^i\right)^{-\sigma}}{N^i\left(p_i^i\right)^{1-\sigma}+N^j\left(p_j^i\right)^{1-\sigma}}$$

$$x_j^i=\frac{\alpha E^j\left(p_j^i\right)^{-\sigma}}{N^i\left(p_i^i\right)^{1-\sigma}+N^j\left(p_j^i\right)^{1-\sigma}}$$

where $\sigma=1/(1-\beta)$ is the elasticity of substitution among varieties. The monopolistic profits are therefore given by:

$$\pi^i=\frac{\alpha E^w}{\sigma N^w}\frac{s_M^i}{\theta_N^i}$$

$$s_M^i=\theta_N^i\left[\frac{\theta_E^i\left(w^i\right)^{1-\sigma}}{\theta_N^i\left(w^i\right)^{1-\sigma}+\left(1-\theta_N^i\right)\phi\left(w^i\right)^{1-\sigma}}+\frac{\theta_E^j\phi\left(w^i\right)^{1-\sigma}}{\left(1-\theta_N^i\right)\left(w^i\right)^{1-\sigma}+\theta_N^i\phi\left(w^i\right)^{1-\sigma}}\right]$$

where $\theta_N^i = N^i/N^w$ and $\theta_E^i = E^i/E^w$ are respectively the world share of firms and expenditure of region i and $\phi = \tau^{1-\sigma}$ represents the well known freeness of trade. Regarding the innovation sector, the fixed cost of starting the production of a new variety is given by the cost of research for creating a new unit of knowledge capital, that is decreasing with the stock of knowledge already created. So the cost of creating a new variety is $w^i/\left(N^i + \lambda N^j\right)$. Moreover by imperfect localized knowledge spillovers we have that $A^i = \theta_N^i + \lambda\left(1 - \theta_N^i\right)$, so the free entry condition becomes:

$$v^i \leq \frac{w^i}{N^w A^i} \text{ with strict inequality if } \dot{N}^i > 0$$

whilst the no-arbitrage condition is:

$$\pi^i + \dot{v}^i = r v^i$$

Being L_N^i, L_M^i and L_T^i the quantities of labor allocated respectively in the innovation, industrial and traditional sectors the labor market condition can be expressed as:

$$L^i = \frac{g^i \theta_N^i}{A^i} + \frac{\beta \alpha E^w s_M^i}{w^i} + \frac{\left(1-\alpha\right) E^w s_T^i}{w^i}$$

Equal-Wage Steady States

In this setting only two kinds of steady states may arise: one in which both regions innovate, and another one in which only one region does so. In this setting equal or unequal wage steady states may arise according to the share of global spending devoted to industrial goods and the regions' relative size. Indeed equal wages emerge as long as both regions produce some traditional good due to competitive pricing in the global market, intersectoral labor mobility and unit wage cost.

The first long-run equilibrium of this kind occurs when innovation and industrial activities are concentrated in the same region and both locations produce some traditional good. In this case, if agglomeration occurs in region i we have $g^i > g^j$ and $s_M^i = \theta_N^i = 1$, and the steady state is characterized by a no-arbitrage condition and two labor market conditions:

$$r + g^i = \frac{\alpha E^w}{\sigma w},\ i = A, B$$

$$L^i = g^i + \frac{\beta\alpha E^w}{w} + \frac{(1-\alpha)\, E^w s_T^i}{w},\ i = A, B$$

$$L^j = \frac{(1-\alpha)\, E^w s_T^j}{w},\ i = A, B \text{ and } j \neq i$$

For both region to produce some traditional good it should be $s_T^j \leq 1$, that implies:

$$\frac{L^j}{L^i + r} \leq \frac{1-\alpha}{\alpha}$$

i.e. if the region performing all the R&D is large with respect to the trade partner and if share of expenditure in traditional goods is high.

The second long-run equilibrium with equal wage rates occurs when both regions still performs R&D, industrial and traditional activities. In this case the no-arbitrage condition becomes:

$$r + g^k = \frac{s_M^k \alpha E^w}{\sigma w}$$

while the labor market equilibrium common to both regions is:

$$L^k = \frac{g^k \theta_N^k}{A^k} + \frac{\beta\alpha E^w s_M^k}{w} + \frac{(1-\alpha)\, E^w s_T^k}{w}$$

In this case obviously $s_M^i = s_M^j = 1/2$. This, together with $s_T^j \leq 1$ implies:

$$\frac{L^j - L^i}{L^i + r} \leq \frac{2\,(1-\alpha)}{\alpha}$$

i.e. both regions perform R&D, industrial and traditional activities if their size is relatively small.

Unequal-Wage Steady States

The necessary condition for having unequal wage steady state is that traditional good ends up being produced by only one region. The first of these equilbria is when one region (say i) performs all the R&D and industrial activities while the other region performs only the traditional activities. Such an equilibrium is described by a no-arbitrage and two labor market clearing equations:

$$r + g^i = \frac{\alpha E^w}{\sigma w^i},\ i = A, B$$

$$L^i = g^i + \frac{\beta\alpha E^w}{w^i},\ i = A, B$$

$$L^j = (1-\alpha)\, E^w,\ j = A, B \text{ and } j \neq i$$

This is the case when the share of expenditure in manufacture is high or when the region performing all the R&D is small relatively to its trade partner.

By contrast if region i performs only R&D and industrial activities while the other region performs all the three the equilibrium is described by the following three equations:

$$r + g = \frac{\alpha E^w}{\sigma w} \frac{s_M^k A^k}{\theta_N^k},\ k = i, j$$

$$L^i = \frac{g\theta_N^i}{A^i} + \frac{\beta\alpha E^w s_M^i}{w^i},\ i = A, B$$

$$L^i = \frac{g\theta_N^j}{A^j} + \beta\alpha E^w s_M^j + (1-\alpha)\, E^w s_T^j,\ j = A, B \text{ and } j \neq i$$

This case may arise if the size of the two regions is very different, or in case of equally sized regions if the share of world spending on manufacture is higher than 1/2 and R&D is concentrated only in one region, i.e. if:

$$\alpha \leq \frac{(L+r)}{(2L+r)}$$

The U-Shaped Convergence Scenario Revisited

Like in Baldwin, Martin and Ottaviano (2001) the growth rate depends on the size of labor force L, the degree of monopolistic power σ, the share of expenditure in manufacture α and the localized spillovers λ. Indeed the equation representing the rate of growth is:

$$g_S^i = \frac{\alpha\,(1+\lambda)\, L - \rho\,(\sigma - \alpha)}{\sigma}$$

The price index for consumption is $P = P_T^{1-\alpha} P_M^{\alpha}$ decreases at rate $g_S^i/(\sigma - 1)$. P_T is a numeraire, while the price index for manufacture is:

$$P_M^i = \left[\int_0^{N^i} \left(p_i^i\right)^{1-\sigma} di + \int_0^{N^j} \left(p_j^i\right)^{1-\sigma} dj\right]^{1/(1-\sigma)}$$

Then regional real incomes decreases at common rate $g^i_{YS} = \alpha g^i_S/(\sigma - 1)$ and the real wage is $1/P$.

When trade costs fall below a certain threshold a core-periphery outcome appears. In case of equal wages the growth rate in core-periphery is:

$$g^i_{CPe} = \frac{\alpha 2L - \rho\left(\sigma - \alpha\right)}{\sigma}$$

Instead in case of unequal wages:

$$g^i_{CPu} = \frac{L - \rho\left(\sigma - 1\right)}{\sigma}$$

Both regions gain from the steady decrease in industrial goods price, hence the real growth rate is the same in both regions in equal and unequal wage cases:

$$g^i_{YCPe} = g^{ji}_{YCPe} = \alpha g^i_{YCPe}/\left(\sigma - 1\right) \text{ with } \alpha < 1/2$$
$$g^i_{YCPu} = g^{ji}_{YCPu} = \alpha g^i_{YCPu}/\left(\sigma - 1\right) \text{ with } \alpha > 1/2$$

But the price index is higher in periphery due to trade costs hence also the real wage levels are different:

$$\omega^i = 1/P^i = \left(N^w\right)^{\frac{\alpha}{\sigma-1}} > 1/P^j = \left(\phi N^w\right)^{\frac{\alpha}{\sigma-1}} = \omega^j \text{ with } \alpha < 1/2$$

$$\omega^i = w^i/P^j = \left(\phi N^w\right)^{\frac{\alpha}{\sigma-1}} w_i^{-\alpha} >> 1/P^j = \left(\phi N^w\right)^{\frac{\alpha}{\sigma-1}} w_i^{-\alpha} = \omega^j \text{ with } \alpha > 1/2$$

An interesting result is that the growth rate is higher when the traditional good is produced only in the periphery, because in this case more resources can be devoted in the core. Another interesting feature of the model is that in case of unequal wages the growth rate in the core periphery outcome is higher than in the symmetric outcome only if:

$$\lambda < \frac{1 - \alpha}{\alpha}$$

This is a condition for existence of unequal wages in the long run. Indeed when agglomeration occurs, the core stops producing traditional good hence the continuous demand for labor in innovation and in manufacture drives the nominal wage to increase. At this point $w^i > w^j = 1$ and the cost of innovation is w^i/N^i, that is permanently lower than the cost of starting innovation again in periphery, i.e. $\lambda w^j/N^j$. Concluding we can say that in case of knowledge externalities localized enough and not excessively high share of expenditure in manufacture unequal wage steady state are likely to arise. Moreover, opposite to Puga (1999) and Baldwin, Martin and Ottaviano (2001), the relation between integration and agglomeration is monotonic.

2.2.2 Divergence, Wage-gap and Geography

Another model removing the no-specialization condition has been carried out by Andres (2007). The outcomes of the model are very similar to Bellone and Maupertuis (2003): if one removes the no-full specialization condition, stating that both country always produce a part of the agricultural good, wages are not anymore equalized between countries because the agricultural sector will vanish in the north and the manufacturing sector by increasingly demanding labor will make the northern wages raise independently of the trade costs. Hence the outcome of the model is different from Baldwin, Martin and Ottaviano (2001) because there is a further dispersion force and thereby the Core-Periphery outcome might never be reached, and is also different from Puga (1999) because the symmetric equilibrium does not become stable for low trade costs. Moreover the concentration of firms in a single can bring to a rate of growth lower than in symmetry.

The basic framework of the model is the same as in Bellone and Maupertuis (2003). The two models deviate when Andres (2007) uses the northern current account to study the switching from the equal wage to the unequal wage regimes.

According to the northern market clearing condition total labort is by the sum of the labor devoted to innovation L_I, manufacture L_M and agriculture L_T:

$$L = L_I + L_M + L_T = \frac{gs_K}{w} + \frac{b(\sigma-1)E^w s_K}{w}\left(\frac{s_E}{s_K + \phi s_K^* \left(\frac{w^*}{w}\right)^{1-\sigma}} + \frac{\phi(1-s_E)}{s_K^* \left(\frac{w^*}{w}\right)^{1-\sigma} + \phi s_K}\right) + \frac{(1-\alpha)E^w s_T}{w}$$

so if the intersectoral allocation of labor is constant in steady state then wages and shares are constant too.

The northern current account is:

$$\underbrace{Y_T - C_T}_{\text{Net supply of T-good}} = \underbrace{\tau K^* x_S^N}_{\text{Importations of M-good}} - \underbrace{\tau w K x_N^S}_{\text{Exportations of M-good}}$$

that states that the net supply of T-good (so production minus export) must be equal to exportations minus importations of the M-good in order to be in equilibrium. To find the frontier between the two regimes we solve this equation with y_T equal to 0 and $w = 1$.

Long-run Equilibrium

The dynamics of the model is described by four dynamic equations like in Baldwin, Martin and Ottaviano (2001): the northern Euler equation, the northern capital accumulation equation and their symmetric for the south. By adopting s_K instead of K and K^* as state variables the dimension of the dynamic system can be reduced:

$$\frac{\dot{E}}{E} = bE^w \left[\frac{B}{w} (1 - s_K) - \lambda \frac{A}{A^*} B^* (1 - s_K) \right] + \left(\frac{E}{w} + \lambda E^* \frac{A^*}{A} \right) - L \left(1 + \lambda \frac{A^*}{A} \right) + \frac{w}{w^*} - \rho$$

$$\frac{\dot{E}^*}{E^*} = bE^w \left[B^* (A^* - (1 - s_K)) - \lambda \frac{A}{A^*} B^* s_K \right] + \left(E^* + \lambda E \frac{A}{A^*} \right) - L \left(1 + \lambda \frac{A}{A^*} \right) - \rho$$

$$\dot{s}_K = \left((1 - s_K) \left(L + \frac{b s_K B (E + E^*) - E}{w} \right) - s_K A^* (L + b (1 - s_K) B^* (E + E^*) - E^*) \right)$$

Clearly in steady state the distribution of capital and the nominal spending in both regions is constant ($\dot{s}_K = \dot{E} = \dot{E}^* = 0$), hence by inspection of the law motion of capital we can see that there are only two kinds of steady states: either both regions innovate at the same rate, or only one does so. As we can see, so far the model behaves like a standard NEG one. The steady state value of a typical firms is:

$$\bar{v} = \frac{\bar{\pi}}{\rho + g}$$

While the steady state value cost of a new unit of capital is:

$$\bar{F} = \bar{w} \bar{s}_K / \bar{A}$$

And finally the steady state value of expenditure is respectively in the two regions:

$$\bar{E} = \bar{w} \left(L + \rho \frac{\bar{s}_K}{A} \right)$$

$$\bar{E}^* = \bar{w} \left(L + \rho \frac{(1 - \bar{s}_K)}{A^*} \right)$$

As we know the Tobin's q approach states that a firm will invest until the value of a new unit of capital will be equal to its cost, i.e. until $q = v/F = 1$. Hence the two kinds of equilibria can be characterized as:

$$
\begin{aligned}
0 &< \bar{s}_K < 1 \Leftrightarrow \bar{g} = g = g^* \\
\bar{s}_K &= 1 \Leftrightarrow \bar{q} = 1 \text{ and } \bar{q} < 1 \ \Leftrightarrow \bar{g} = g \text{ and } g^* = 0
\end{aligned}
$$

That implies:

$$
\bar{w} = \frac{\bar{A}\bar{\pi}}{\bar{A}^*\bar{\pi}^*}
$$

Meaning that for both regions to find profitable to invest, a higher cost of investment (lower A) has to be compensated by higher profits. Indeed a positive wage differential is an additional cost for the innovation sector that has to be compensated either by higher externalities in research (through A) or by higher profits.

Unequal wage trajectories

We will not describe the equal wage regime as it is analogous to Bellone and Maupertuis (2003). We will instead concentrate on the analysis of the unequal wage trajectories.

When allowing to complete specialization, there is no always an agricultural sector in both regions that pins down the wage to one, but the wage is endogenized as the three sectors compete for labor input. In this case the model is no longer analytically solvable because the dynamics of agglomeration depends on the wage rate through the profit function, where it appears in a non-linear fashion. Let's investigate the no-full specialization condition. We know that the agricultural production of south is $Y^* = L^*$ and that the world demand of T-good is $(1-\alpha)E^w$. In this case there is not full specialization if:

$$
(1-\alpha)E^w \geq L^*
$$

How to use it? We know that income should be equal to expenditure plus replacement of capital:

$$
Y = E + F\dot{K}
$$

and also that income is equal to labor income plus profits:

$$
Y = wL + \pi s_K K
$$

By combining this two last equations:

$$E + F\dot{K} = wL + \pi s_K K, E = wL + \pi s_K K - F\dot{K}$$

Finally, using the no-arbitrage condition:

$$\alpha \leq \frac{L + \rho}{L^w + \rho}$$

Hence if α is less than $1/2$ there is not complete specialization and wages will not diverge. Notice that the likelihood that $\alpha > 1/2$ increases as the concentration of the manufacture increases (so increases with L).

Looking for the Frontier

If the no-specialization condition is removed, there is the possibility of a regime with unequal wages. Let's find now the frontier representing the regime switching point. The northern current-account is:

$$\underbrace{Y_T - C_T}_{\text{Net supply of T-good}} = \underbrace{\tau K^* x_S^N}_{\text{Importations of M-good}} - \underbrace{\tau w K x_N^S}_{\text{Exportations of M-good}}$$

Notice that in the first regime both regions produce some traditional good hence $Y_T = L_T > 0$. By contrast in the second regime the north becomes a net exporter of M-good hence $Y_T = L_T > 0$ as the south produces all the T-good. The current-account can be written as:

$$L_T\left[s_E, s_K, w\right] = \frac{s_E}{1 - s_E}\left(1 - \frac{\alpha s_K}{s_K + \phi(1 - s_K)(\frac{w^*}{w})^{1-\sigma}}\right) - \alpha\left(1 - \frac{1 - s_K}{1 - s_K + \phi(\frac{w^*}{w})^{1-\sigma} s_K}\right)$$

The definitions of s_K and s_E are standard:

$$s_K = \frac{1}{2} + \frac{\left(s_E - 1/2\right)\left(1 - \phi^2\right)\left(A\left(1 - s_K\right) + A^* s_K\right)}{\lambda(1 + \phi^2) - 2\phi}$$

$$s_E = \frac{w(L + \rho(\frac{s_K}{A}))}{L\left(w + 1\right) + \rho\left[w(\frac{s_K}{A}) + \frac{1 - s_K}{A^*}\right]}$$

Using this fact and setting $w = 1$ and $L_T\left[s_E, s_K, 1\right] = 0$ the frontier of expenditure share can be expressed as function of parameters:

$$\alpha = \frac{\left(\phi(\phi\lambda - 2) + \lambda\right)\left[L\lambda\left(1 - \lambda^2\right)\left(1 - \phi^2\right) + \chi\right]}{\lambda(1 - \phi^2)\left[L\lambda\left((1 - \lambda^2) + \phi\left(2\lambda + \phi - \lambda^2\right)\right) - 2L\phi + \chi\right]}$$

where:

$$\chi = \sqrt{(1-\lambda^2)\left\{4\rho^2\phi^2(\phi\lambda-1)^2 + L^2\left(1-\lambda^2\right)\left(\phi(\phi\lambda-2)+\lambda\right) + 4L\rho\phi\left[2\phi-\lambda(1+3\phi^2)+\lambda^2\phi\left(1+\phi^2\right)\right]\right\}}$$

As we can see the implicit switch point ϕ^{SW} occurs for higher trade costs the higher the share of consumer's expenditure, the smaller the spillovers and the higher the rate of time preferences.

The Core-Periphery equilibrium in the second regime

Let's characterize now the core-periphery equilibrium in case of unequal wage trajectories. The current account is:

$$L_T\left[s_E, s_K, w\right] = \frac{s_E}{1-s_E}\left(1-\frac{\alpha s_K}{s_K+\phi(1-s_K)(\frac{w^*}{w})^{1-\sigma}}\right) - \alpha\left(1-\frac{1-s_K}{1-s_K+\phi(\frac{w^*}{w})^{1-\sigma}s_K}\right)$$

Evaluating this around $s_K = 1$:

$$L_T\left[s_E, s_K, w\right] = \frac{s_E}{1-s_E}\left(1-\alpha\right) - \alpha = 0$$

Hence:

$$s_E = \alpha$$

Now taking the definition of s_E:

$$s_E = \frac{w(L+\rho(\frac{s_K}{A}))}{L\left(w+1\right)+\rho\left[w(\frac{s_K}{A})+\frac{1-s_K}{A^*}\right]}$$

We find the core-periphery constant value of wage:

$$\bar{w} = \frac{\alpha L}{\left(1-\alpha\right)\left(L+\rho\right)}$$

Let's find now the sustain point. Knowing that:

$$\bar{q}^* < 1$$

$$q^* = V/F = \frac{\pi^*}{\rho+g}/\frac{w}{K^w A^*}$$

that can be written as:

$$q^* = \bar{w}^\sigma \lambda \left((1 - s_E)\, \phi^{-1} + \phi s_E \right)$$

We finally solve for $q = 1$ and $s_E = \alpha$:

$$\phi_{1,2} = \frac{1 \pm \sqrt{1 + 4Aw^{2\sigma}\alpha\lambda^2(\alpha - 1)}}{2w^\sigma \alpha\lambda}$$

The negative root is the economic relevant one and shows that when α is too high the core-periphery equilibrium no longer exists.

Growth Rate

In case of no complete specialization the growth rate is assessed using Tobin's q evaluated in core-periphery and it is equal to:

$$\bar{g}^{1st}|_{s_K=1} = bL^w - \frac{\rho}{\sigma}(\sigma - \alpha)$$

In the second regime case the authors just substitute the definition of steady state for w that is:

$$\bar{w} = \frac{\alpha L}{(1-\alpha)(L+\rho)}$$

Yielding:

$$\bar{g}^{2st}|_{s_K=1} = \frac{L - \rho(\sigma - 1)}{\sigma}$$

We see that $g^2 > g^1$: in the second regime the rate of growth is higher**.** This because north does not longer produce the traditional good hence labor is solely divided between manufacture and innovation. The growth rate in the symmetric equilibrium is instead:

$$g^{sym} = \frac{\alpha(1+\lambda)L - \rho(\sigma - \alpha)}{\sigma}$$

Thus the growth rate in core periphery is higher than the symmetric one if and only if:

$$\lambda < \frac{(L+\rho)(1-\alpha)}{\alpha L}$$

Hence the rate of growth might be lower in core periphery than in symmetry if spillovers are not too localized and the expenditure share on manufacture is not too high. This outcome clearly clashes with the standard NEG literature stating that agglomeration is always beneficial for both countries.

2.3 Firms' Heterogeneity in New Economic Geography Models

The most tricky assumption of the NEG framework is that of homogeneous firms. Indeed several empirical studies show that firms vary both in terms of size (Cabral and Mata 2003) and in terms of productivity levels (Helpman *et al.,* 2004). This assumption is relaxed in Baldwin and Okubo (2006), that introduce a Melitz (2003) style monopolistic competition with heterogeneous firms in a NEG setting. This deviation has a spatial selection effect leading the most productive firms to move to large markets. Why is it so? As we know, agglomeration forces are led by backward and forward linkages while the dispersion force consists in a *market crowding effect*. The most productive firms are the ones having lower marginal costs, thereby likely to sell more. Thus backward and forward linkages operating in bigger markets are more attractive to them. On the other hand the most productive firms are less harmed by the high degree of local competition present in the big markets. Hence the most productive firms will move to bigger markets first. This spatial selection effect has two main implications. On one side there will be a bias in the measurement of agglomeration economies: firms that move to agglomerated regions have above average firm-level productivity independently of any agglomeration economies. On the other side subsidies aimed at increasing the share of industry in periphery regions will attract only less productive firms that have the least opportunity cost in leaving the agglomerated region. The homogeneous hypothesis firms is relaxed also in Baldwin and Robert-Nicoud (2008), who embedd a heterogeneous-firms trade model in a series of product-innovation endogenous growth models (Grossman and Helpman 1989, 1991, Romer 1986, 1990, Rivera-Batiz and Romer 1991a, b, Coe and Helpman 1995). The main findings of the Baldwin and Robert-Nicoud (2008) model is that openness may slow down or boost growth depending on the impact of openness on the marginal cost of innovating. Indeed the Melitz-type selection effect by increasing the expected cost of introducing a new variety has detrimental effects upon the growth rate. By contrast freer trade has a positive impact on the marginal cost of innovating thereby having pro-growth effects. However, despite the tension between the dynamic and the static welfare effects when greater openness slows growth, the overall impact on welfare is unambiguously positive. The main difference between the Baldwin and Okubo (2006) and the Baldwin and Robert-Nicoud

(2008) models are the hypothesis regarding the mobility of capital. Indeed Baldwin and Okubo (2006) is a footloose capital model, while in Baldwin and Robert-Nicoud (2008) capital and firms are not mobile.

2.3.1 Firms' Heterogeneity and Spatial Selection

The model presented by Baldwin and Okubo (2005) can be seen as a marriage between the footloose NEG capital model conceived by Martin and Rogers (1995) extended for allowing heterogeneity in firms' marginal cost (Melitz, 2003). The basic set up is the same as the footlose capital model already presented. The only extensions concern the already mentioned firms heterogeneity and the quadratic adjustment costs faced by firms when switching regions.

The Basic Model

The heterogeneity is modeled by assuming firms to have different unit input coefficients (different a). Each firms needs a unit of knowledge to start production, hence the source of heterogeneity can be assigned to knowledge capital. It is then assumed that each unit of capital in each region is associated with a particular level of productive efficiency measured by the unit labor requirements a which are Pareto distributed:

$$G(a) = \left(\frac{a^\rho}{a_0^\rho}\right), 1 \equiv a_0 \geq a \geq 0, \rho \geq 1$$

where $a_0 < \infty$ is the highest possible marginal cost (normalized to unity) and ρ is the shape parameter.

The second deviation from the footloose capital model stems from the fact that relocation is subject to quadratic adjustment costs. Indeed the cost of switching regions is χ units of labor per firms:

$$\chi = \gamma m$$

where m is the flow of migrating firms.

Short Run Equilibrium

Dixit-Stiglitz monopolistic competition maximization yields the following northern and southern operating profits as function of productivity level:

$$\pi(a) = a^{1-\sigma}\left(\frac{s_E}{\Delta} + \frac{\phi\left(1-s_E\right)}{\Delta^*}\right)\frac{E^w}{K^w\sigma}$$

$$\pi^*(a) = a^{1-\sigma}\left(\frac{\phi s_E}{\Delta} + \frac{1-s_E}{\Delta^*}\right)\frac{E^w}{K^w\sigma}$$

where:

$$\Delta = \lambda\left(s_n + \phi\left(1-s_n\right)\right), \Delta^* = \lambda\left(\phi s_n + 1 - s_n\right), \lambda = \frac{\rho}{1-\sigma+\rho} > 0$$

The deltas are a measure of the degree of competition in the market, while $a^{1-\sigma}$ can be seen as the competitiveness of a firm with marginal cost a. Combining these two facts we have that a firm's market share $a^{1-\sigma}/\Delta K^w$ depends upon its relative competitiveness.

Now, according to the Home Market Effect the big market (in the case at hand the north) will a more than proportional share of industry. Let's see now which firm moves first. The change in operating profits from a single firm moving from south to north, considering an initial situation where no firms have moved ($s_n = s_K$) and using the symmetry of region's relative factor endowments ($s_E = s_K > 1/2$) is:

$$\pi(a) - \pi^*(a) = a^{1-\sigma}\left(\frac{\left(1-\phi\right)E^w}{\lambda\sigma K^w}\right)\frac{2\phi\left(s-\frac{1}{2}\right)}{\left(\left(1-\phi\right)s+\phi\right)\left(1-s+\phi s\right)}$$

where s is the north share of E and K. From this equation we can see that southern firms would move to north, that no northern firms would gain from moving to south and that most efficient southern firms would gain more by relocating.

It is pretty obvious that the most efficient firms, gaining most from delocation, are the ones willing to pay the quadratic delocation costs. But of course there is a feedback from migration and the *market crowding effect* through Δ :

$$\Delta = \lambda\left(s + \left(1-s\right)a_R^{1-\sigma+\rho} + \phi\left(1-s\right)\left(1-a_R^{1-\sigma+\rho}\right)\right)$$

$$\Delta^* = \lambda\left(\phi s + \phi\left(1-s\right)a_R^{1-\sigma+\rho} + \left(1-s\right)\left(1-a_R^{1-\sigma+\rho}\right)\right)$$

where a_R is the threshold level for marginal costs migration. Using this expression it can be displayed the value of delocation of any southern firm as a function of its own marginal cost and the range of firms already moved:

$$v(a,a_R) = \pi(a,a_R) - \pi^*(a,a_R) = a^{1-\sigma}\left(\frac{s_E}{\Delta(a_R)} + \frac{\phi(1-s_E)}{\Delta^*(a_R)}\right)\frac{E^w}{K^w\sigma} - a^{1-\sigma}\left(\frac{\phi s_E}{\Delta(a_R)} + \frac{1-s_E}{\Delta^*(a_R)}\right)\frac{E^w}{K^w\sigma}$$

Given that the southern firms in north is $K^* a_R^{\rho}$ the cost of moving becomes:

$$\chi = \gamma K^* \rho a_R^{\rho-1} \dot{a}_R$$

Firms will move until the benefits of doing so will be greater or equal to the costs. The marginal cost a will be pinned down by the equality between benefits and costs of migrating, so the value of the marginal firms of migration will be:

$$v(a_R) = \gamma K^* \rho a_R^{\rho-1} \dot{a}_R$$

This function is declining in a_R, hence we have that most efficient firms will move first.

The Long Run Equilibrium

In the long run equilibrium delocation does no longer take place, hence the marginal adjustment costs and the location condition $v(a_R)$ are zero. Solving for the cut-off level of marginal costs a_R :

$$a_R^{1-\sigma+\rho} = \frac{2\phi\left(s-\frac{1}{2}\right)}{(1-\phi)(1-s)}; s_n = s + (1-s)\, a_R^{\rho}$$

We can see that if trade gets freer more inefficient firms will delocate. The threshold in trade costs for complete delocation is the same ad in the standard footloose capital model:

$$\phi^{CP} = \frac{1-s}{s}$$

Summarizing all the stability results we have that heterogeneity leads to a spatial selection effect according to which most efficient firms are the first to move to the bigger market. This implies a Home Market Magnification Effect: the big market attract more than its usual firms' share.

Selection Bias

For testing the agglomeration the average productivity of a region must be related to the amount of industry in the region itself . The simplest framework is:

$$\ln(lprod_r) = c + \alpha \ln(s_{nr}) + \varepsilon$$

where $lprod_r$ is the labor productivity in region r and s_{nr} is the share of industry in region r. In this case α would measure the impact of an increase in the share of industry in one region on the labor productivity in this same region. Performing a similar test for the footloose capital model we have that north's labor productivity is given by the ratio between the real value of manufacturing output (i.e. northern manufacturing revenue) $np^{1-\sigma}(E/\Delta + \phi E^*/\Delta^*)$ and the total labor input $ap^{-\sigma}(E/\Delta + \phi E^*/\Delta^*)$. Then due to mill pricing the north's labor productivity will be $1/(1-1/\sigma)$. Converting to real terms by dividing for the consumer price index $(np^{1-\sigma})^{1/(1-\sigma)}$ the labor productivity measure becomes:

$$\ln(lprod_r) = \ln\left(\frac{s_n^{\frac{1}{\sigma-1}}}{a(1-1/\sigma)}\right)$$

As we can see the labor productivity increases with the share on industry in the north: this is the measure of agglomeration economies. By adding heterogeneity we have that the value of output is $\int p(i)^{1-\sigma}(E/\Delta + \phi E^*/\Delta^*)$ and the total labor input $\int a(i)\, p(i)^{-\sigma}(E/\Delta + \phi E^*/\Delta^*)$. Then the labor productivity measure becomes:

$$\ln(lprod_r)_{het} = \ln\left(\frac{\left(\lambda K + \lambda K^* a_R^{1-\sigma+\rho}\right)^{\frac{1}{\sigma-1}}}{a(1-1/\sigma)}\right)$$

Clearly the estimate of agglomeration economies would be overestimated because firms relocated in the north would have systematically higher than average productivity. So there would be a bias in the standard econometric tests for agglomeration.

Regional Policy Implications

Starting form a core-periphery situation assume a regional policy paying firms a subsidy S for moving from the large to the small region. The change in the operating profits for a firm moving from north to south (not including subsidies) would be:

$$a^{1-\sigma}\frac{(1-\phi)\,E^w}{\lambda\sigma}\left(\frac{1-s}{\phi}-s\right)<0$$

It is clear that the loss of relocation is decreasing with the firm's marginal cost a. So the first firms to move to the small region in response to a subsidy would be the less efficient ones. Now let's see the precise relation between the subsidy and the cut-off marginal cost. If all firms with marginal cost higher than a_S would move to the south:

$$\Delta=\lambda\left(a_S^{1-\sigma+\rho}+\phi\left(1-a_S^{1-\sigma+\rho}\right)\right)$$

$$\Delta^*=\lambda\left(\phi a_R^{1-\sigma+\rho}+\left(1-a_R^{1-\sigma+\rho}\right)\right)$$

Thus the change in the operating profits for a firm moving from north to south including subsidies becomes:

$$a_S^{1-\sigma}\frac{S\sigma}{E^w}=(1-\phi)\left(\frac{s}{\Delta}-\frac{1-s}{\Delta^*}\right)$$

The left hand side of the expression is increasing in a_S. By contrast the right hand side is always decreasing in a_S because the competition in north falls as a_S rises. Hence there is only one solution for a_S. It is clear that an increase in subsidies raise the left hand side (the right is not affected) hence there is a decrease in the cut-off level of efficiency. On the contrary a decrease in trade costs would lower the right hand side not affecting the left hand one, so subsidies become more effective ad trade gets freer.

Summarizing it is clear that the subsidy is more effective in promoting relocation the large is the subsidy and the freer is trade, but the relocating firms are the less efficient because are the ones gaining with the lower opportunity cost of leaving the big region.

2.3.2 Firms' Heterogenity and Market Selection

Baldwin and Robert-Nicoud (2008) present a new-new trade model related to Helpman, Melitz and Yeaple (2004) and a portmanteau product-innovation growth model including as special cases Grossman and Helpman's product innovation model (Grossman and Helpman 1991, chapter 4), and the Rivera-Batiz and Romer (1991) lab-quipment model. All these models are based on Helpman and Krugman (1985) and Krugman (1980) with three additions. First the model displays endogenous growth by allowing intertemporal knowledge spillovers in variety creation: it is assumed that the cost of producing a new blueprint falls at the same rate as the value of introducing a new variety. Second in the model is present heterogeneity in marginal production cost using the Hopenhayn-Melitz variety generation/selection set up where firms are assigned a marginal cost after paying the start-up cost. Finally in the model are present sunk market-entry costs: the sunk costs and the firm heterogeneity discriminate between firms selling only locally and firms also exporting.

The Basic Model

The basis of the model is the Helpman-Krugman monopolistic competition with a single factor L, two regions and a Dixit-Stiglitz manufacture sector. Each firm pays a sunk start-up cost F_I (I stays for innovation) and constant marginal production cost. The typical firm's a is drawn from a density function $G\left[a\right]$ with support $0 \leq a \leq a_0$. There are three fixed cost: a start-up cost F_I and the two beachhead costs F_D and F_X for selling the product within borders and abroad. The star-up cost requires κ_I units of knowledge, and producing in local and export markets requires κ_D and κ_X units of knowledge:

$$F_I = \kappa_I P_K,\ F_D = \kappa_D P_K,\ F_X = \kappa_X P_K$$

where P_K is the cost of one unit of knowledge. The technology of the innovation sector I is:

$$Q_K = \frac{S}{P_K},\ P_K = c\left[w, \vec{a}, n\right]$$

where Q_K is the output and S is the total expenditure on new knowledge. Competition implies P_K to be equal to the marginal cost depending on the price of labour w and the unit-input coefficient vector $\vec{a}$.

The Equilibrium

Firms have to take two types of decisions: the start-up and the two market-entry decisions. The two thresholds given by the two beachhead costs (F_X and F_D) define three types of firms: those more productive ($a < a_X$) sell both in the local and in the export market; the last efficient ($a > a_D$) do not produce; firms with intermediate efficiency ($a_X < a < a_D$) will only sell within borders. Taking labour as numeraire and by Dixit-Stiglitz optimization firm-j's price is in the local market is $a_j/(1-1/\sigma)$ and in the export market is $\tau a_j/(1-1/\sigma)$. A firm's market share as a function of its marginal selling cost is:

$$s[m] = \left(\frac{1}{n}\right)\frac{m^{1-\sigma}}{\bar{m}},\ \bar{m} = \int_0^{a_D} a^{1-\sigma} dG[a|a_D] + \phi \int_0^{a_X} a^{1-\sigma} dG[a|a_D],\ 0 \leq \phi \equiv \tau^{1-\sigma} \leq 1$$

where m and $\bar{m}$ are the firm's marginal selling cost and the weighted average of firm's marginal selling cost, and $G[a|a_D]$ is the conditional density function for the a's. Notice that $\frac{m^{1-\sigma}}{\bar{m}}$ is a measure of firm's market specific competitiveness and that the firm's average marginal selling cost $\bar{m}$ depends on two cut-off marginal costs, a_D and a_X, and their distribution. The benefit of the market entry is given by the present value of operating profits, that given the discount rate γ are respectively equal for the local and export market to $s[a_D]E/\sigma\gamma$ and to $\phi s[a_X]E/\sigma\gamma$. Hence the market-entry cut-off conditions are

$$s[a_D]\frac{E}{\sigma\gamma} = P_K\kappa_D,\ \phi s[a_X]\frac{E}{\sigma\gamma} = P_K\kappa_X$$

The expected operating profits of a winner must match the average profits of market, thereby:

$$\text{Expected value of a winner} = \frac{E}{\sigma n\gamma}$$

The expected fixed cost of developing a D or a X patent (i.e. being a winner) is:

$$\bar{F} \equiv P_K\bar{\kappa},\ \bar{\kappa} \equiv \kappa_D + \kappa_X\frac{G[a_X]}{G[a_D]} + \kappa_I\frac{1}{G[a_D]}$$

where $\bar{\kappa}$ represents the fixed costs for sales in the three markets. Free entry drives operating profits to zero, hence the free entry condition for variety introduction is:

$$\frac{E/\sigma}{n\gamma} = P_K\bar{\kappa}$$

which corresponds to Tobin's q. Using an intensive form of P_K, the cut-off and free entry conditions can be written as:

$$\frac{a_D^{1-\sigma}E}{\bar{m}\sigma\gamma} = p_K\kappa_D,\ \frac{a_X^{1-\sigma}E}{\bar{m}\sigma\gamma} = p_K\kappa_X,\ \frac{E}{\sigma\gamma} = p_K\bar{\kappa},\ p_K \equiv nP_K = c\left[w, \vec{a}, n\right]$$

Saving, investment and growth

The individual intertemporal preferences are:

$$U = \int_0^{\infty} e^{-\rho t}\ln D_t dt;\ D = \left(\int_{i\in\Theta} d_i^{1-1/\sigma} di\right)^{1/(1-1/\sigma)},\ Y = E+S;\ Y = L+\frac{E}{\sigma}$$

where D is the CES consumption composite and Θ *is* the set of consumed varieties. The Euler equation for intertemporal division of consumption is:

$$\dot{E}/E = r - \rho$$

The expenditure and the labor in innovation definitions are:

$$E = \frac{L - L_I}{1 - 1/\sigma} \Longleftrightarrow L_I = L - E\left(1 - 1/\sigma\right)$$

While the growth rate is:

$$\dot{n} = \frac{Q_K}{\bar{\kappa}} \Longleftrightarrow g = \frac{L_I}{p_K\bar{\kappa}},\ g \equiv \frac{\dot{n}}{n}$$

Utility maximizing expenditure equals permanent income:

$$E = L + \rho p_K\bar{\kappa}$$

By using this and $\gamma = \rho + g$ the cut-off and free entry conditions are respectively:

$$\frac{a_D^{1-\sigma}\left(L + \rho p_K\bar{\kappa}\right)}{\bar{m}\sigma\left(\rho + g\right)} = p_K\kappa_D,\ \frac{\phi a_X^{1-\sigma}\left(L + \rho p_K\bar{\kappa}\right)}{\bar{m}\sigma\left(\rho + g\right)} = p_K\kappa_X,\ \frac{L + \rho p_K\bar{\kappa}}{\sigma\left(\rho + g\right)} = p_K\bar{\kappa}$$

Growth Effects of Market Opening

We can express growth in the following way:

$$g = \frac{L/\sigma}{p_K \bar{\kappa}} - \frac{\rho(\sigma - 1)}{\sigma}$$

Thus we have that closer economic integration is pro-growth only if it lowers the expected sunk cost of developing a new variety $(p_K \bar{\kappa})$. Hence there are two growth effect channels: one through p_K and the other one through $\bar{\kappa}$. The $\bar{\kappa}-$channel is anti-growth because market openness increases competition both in the local and in the export market hence raising the fixed knowledge-requirement of new varieties conditional on entry. By contrast the p_K channel is pro-growth because of knowledge spillovers and intermediate inputs that lower the cost of innovation.

Assuming a Pareto distribution for $G[a]$ we have:

$$G[a] = (a/a_0)^k, \; 0 \leq a \leq a_0 \equiv 1$$

Using this fact leads to:

$$g = \frac{L/\sigma}{p_K \bar{\kappa}} - \frac{\rho(\sigma - 1)}{\sigma}, \; a_D = \left(\frac{(\beta - 1)\kappa_I}{(1+\Omega)\kappa_D}\right)^{1/k}, \; a_X = \left(\frac{\Omega(\beta - 1)\kappa_I}{(1+\Omega)\kappa_X}\right)^{1/k}$$

Microfoundation for the I-sector Marginal Cost Function

From the model may arise five different cases according to the hypothesis upon the microfoundation of the I-sector marginal cost function $c[w, \vec{a}, n]$.

Assuming a learning curve where the marginal cost of creating knowledge falls as the innovation output rises we have:

$$c[w, \vec{a}, n] = \frac{w}{n + \lambda n^*} \Longrightarrow p_k \bar{\kappa} = \frac{\beta \kappa_D (1+\Omega)}{(\beta - 1)(1+\lambda)}$$

In this case the model boils down to yhe Grossman-Helpman case. Instead if equating λ to the fraction of imported varieties $(a_X/a_D)^k$ we have:

$$\tilde{\lambda} \equiv (a_X/a_D)^k \Longrightarrow \frac{\beta \kappa_D (1+\Omega)}{(\beta - 1)(1+\Omega/T)}$$

In this case the model boils down to the Coe-Helpman case. Another possibility is to consider efficiency-linked knowledge spillovers by assuming that the extent of spillovers is proportional to the average efficiency of produced varieties:

$$c[w, \vec{a}, n] = \frac{w}{(n\bar{m} + \lambda n^* \bar{m}^*)/2} \Longrightarrow p_k \bar{\kappa} = \frac{\{(\beta - 1)\kappa_I\}^\beta}{(1+\Omega)^\beta \kappa_D^{\beta-1}}$$

One could also consider the reverse engineering case, presuming that the I-sector only learns from varieties that it can actually purchase:

$$c[w, \vec{a}, n] = \frac{w}{n\bar{\kappa} + \tilde{\lambda} n^* \bar{\kappa}_X} \Longrightarrow p_k \bar{\kappa} = \left(1 + \frac{(\beta - 1)\bar{\kappa}_X \Omega}{\beta T (1+\Omega)}\right)^{-1}$$

If finally following Rivera-Batiz and Romer (1991) it is supposed that knowledge is produced using the final good CES composite we have:

$$c[w, \vec{a}, n] = P^\alpha,\ P = (n\bar{m})^{\frac{1}{1-\sigma}},\ \alpha = \sigma - 1 \Longrightarrow p_K \bar{\kappa} = \kappa_D \left(\frac{(\beta - 1)\kappa_I}{(1+\Omega)\kappa_D}\right)^\beta$$

In this case there is a link between the productivity of innovation and the cost of goods.

The Growth Effect of Freer Trade in Goods and Ideas

In all five cases the $\bar{\kappa}$ channel is anti-growth, hence we have an overall positive effect if the p_K-channel is sufficiently strong. Reducing iceberg trade costs slows growth in two cases and speeds in three. In knowledge-creating technology (Grossman-Helpman) trade costs decrease do not have an impact on p_K so the overall impact is anti-growth. The same thing in the Coe-Helpman case, trade costs decrease have a positive impact of p_K but not large enough to offset the negative $\bar{\kappa}-$channel effect. By contrast in efficiency-linked, reverse engineering and lab-equipment cases the effect of lowering iceberg trade costs on the p_K-channel is positive and strong enough to overcome the anti-growth $\bar{\kappa}$ channel. Concerning the lowering of fixed trade-costs, if λ is exogenous the qualitative effect is the same as a change in iceberg trade costs. Otherwise if λ depends on the fraction of foreign varieties that are traded, lowering the fixed trade-costs is unambiguously pro-growth.

Indeed in this case higher knowledge spillovers lower the expected cost of getting a winner without any impact on the reward of getting a winner.

Welfare

The utility function can be written in the following way:

$$U_0 = \left(\frac{\sigma}{p_K \bar{\kappa}}\right) \left(\beta \left(\frac{1+\Omega}{\beta - 1}\right)^{1+\beta} \left(\frac{\kappa_D}{\kappa_I}\right)^{\beta}\right)^{1/(\sigma-1)}$$

The first and the second terms in parenthesis capture respectively the dynamic and the static effects. The static welfare effect of market opening is positive, while the dynamic welfare effect is positive if and only if greater openness raises the growth rate because laissez-faire growth is sub-optimal from the social-welfare point of view. Hence market openness is welfare increasing if it lowers the expected cost of innovation $p_K \bar{\kappa}$. Concluding we can say that market openness can have ambiguous welfare effects in some technologies, e.g. in the Grossman-Helpman case:

$$U_0 = \left(\frac{\sigma}{p_K \bar{\kappa}}\right) \left(\beta \left(\frac{1+\Omega}{\beta - 1}\right)^{2-\sigma+\frac{k}{\sigma-1}} \left(\frac{\kappa_D}{\kappa_I}\right)^{\frac{k}{\sigma-1}}\right)^{1/(\sigma-1)}$$

2.4 Conclusions

In this chapter we have presented several recents research strands of the New Economic Geography. The first one is the New Economic Geography and Growth, where Baldwin, Martin and Ottaviano (2001) introduce endogenous growth *à la* Romer (1990) and localized knowledge spillovers in a core periphery setting. These depertures from the standard theory have several implications in terms of growth equilibrium and policy implications. Indeed Baldwin, Martin and Ottaviano (2001) are able to show that growth and geography, meant as manufacture firms location, are related. This because with intertemporal localized knowledge spillovers the higher is the concentration of industrial firms, the higher will be the incentive to invest in new units of capital and therefore the growth rate. Hence policy makers should not prevent economies from agglomerating in order to maximize growth, because in this way the global dynamic gains in growth would overcome the static losses due to trade costs sustained by the south.

In the second section of the chapter we have introduced two models discussing the possiblity of a monotonic relation between agglomeration and integration. These two models, namely Bellone and Maupertuis (2003) and Andres (2007), remove the no-specialization condition and allow wages to differ between regions. In this way industry do not move back to the periphery even in case of very low trade costs because in case of localized knowledge spillovers and avantage in innovation capabilities is always higher than the disadvantage due to higher wages. These two models contrast with Puga (1999) and Krugman and Venables (1995), showing that when wages are higher in the agglomerated region for low enough trade costs manufacture firms return to periphery, and Baldwin, Martin and Ottaviano (2001), showing that when the cost of trading ideas falls beyond a certain threshold industry starts to relocate to south.

In the last section of the chapter we have presented two models introducing firm's heterogeneity in productivity in New Economic Geography. The first model, Baldwin and Okubo (2006), introduce a Melitz (2003) monopolistic competition with heterogeneous firms in a New Economic Geography framework showing that the most productive firms settle in larger markets because they suffer lesser local competition and enjoy more backward and forward linkages than the least productive firms. This fact implies that the productivity of the firms settled in the core will be above average independently of any agglomeration economies. Moreover we have that regional subsidies aiming to attract manufacture firms to periphery will attract only the least productive firms. The second model we have taken into consideration has been developed by Baldwin and Robert-Nicoud (2008), who introduce heterogeneity in firms marginal cost (Melitz, 2003) in the footloose capital model conceived by Martin and Rogers (1995). Thanks to this extension, they show that from one side opening trade increases the expected cost of introducing a new variety thereby slows down growth. On the other side they show that freer trade has a positive impact on the marginal cost of innovation therefore boosts growth. Despite the ambiguity of the trade openness effects, the overall impact on welfare is unambiguously positive.

Part II

Original Extensions to the Theory

Chapter 3

Agglomeration and Growth with Endogenous Expenditure Shares

3.1 Introduction

In this chapter, we develop a NEGG model which deviates from the standard approach in two respects: 1) we explicitly consider the love of variety parameter separating it from the *intrasectoral* elasticity of substitution between the different varieties within the industrial sector; 2) we use a more general Constant Elasticity Function (henceforth CES) instead of a Cobb-Douglas utility function in the second-stage optimization problem, thereby allowing the elasticity of substitution between manufacture and traditional good (*intersectoral* elasticity henceforth) to diverge from the unit value.

The main effect of these departures is that the share of expenditure on manufactures is no longer exogenously fixed (as in the Cobb-Douglas approach) but it is endogenously determined via agents' optimization. By endogenising the expenditure shares in manufacturing goods, we are able to test the robustness of several well-established results in the NEGG literature and we show that the validity of such results, and of the associated policy implications, crucially depends on the particular Cobb-Douglas functional form used by this class of models.

Our generalizations of the standard NEGG literature act at two different levels: a) the dynamic pattern of equilibrium allocation of economic activities and b) the equilibrium growth prospect. As for the first level, the main result of our analysis is the emergence of a new force, which we dub as the *substitution effect*. This force, which is a direct consequence of the dependence of the expenditure shares on the allocation of economic activities, is neutralized in the standard NEGG model by the unitary intersectoral elasticity of substitution. Our model "activates" this force and the associated new economic mechanism opens the door to a series of novel results. First we show that this substitution effect acts as an agglomeration or a dispersion force according to whether the traditional and the differentiated goods are respectively good or poor substitutes. Then we analyze the implications of this new force and we show that, unlike the standard model, catastrophic agglomeration may always take place whatever the degree of market integration may be if the substitution effect is strong enough. This result, which is a novelty in the NEGG literature, has important implications in two respects: first, policy makers should be aware of the fact that policies affecting the degree of market integration are able to affect the equilibrium location of economic activities only for a restricted set of values for the parameters describing the economy. Second, the emergence of the substitution effect suggests that the intersectoral elasticity of substitution has a crucial role in shaping the agglomeration or the dispersion process of economic activities.

As for the equilibrium growth prospect, results are even more striking. We show that, thanks to the variable expenditure shares: 1) the regional rate of growth is affected by the interregional allocation of economic activities even in the absence of localized spillovers, so that geography *always* matters for growth and 2) the regional rate of growth is affected by the degree of market openness: in particular, according to whether the intersectoral elasticity of substitution is larger or smaller than unity, economic integration may be respectively growth-enhancing or growth-detrimental. These results are novel with respect to the standard NEGG literature according to which geography matters for growth only when knowledge spillovers are localized and, moreover, trade costs never affect the growth rate. This second set of results is characterized by even more important policy implications: first, our results suggests that interregional allocation of economic activity can always be considered as an instrument able to affect the rate of growth of the economy. In particular, when the average interregional expenditure share on industrial goods are higher in the symmetric equilibrium than in the core-periphery one, then each policy aiming at equalizing the relative size of the industrial sector in the two regions will be good for growth, and vice-versa.

Second, each policy affecting economic integration will also affect the rate of growth and such influence is crucially linked to the value of the intersectoral elasticity of substitution.

A first attempt to introduce endogenous expenditure shares in a NEGG model has been carried out by Cerina and Pigliaru (2007), who focused on the effects on the balanced growth path of introducing such assumption. In the present chapter we carry out an extension of Cerina and Pigliaru (2007), considering that we deepen the analysis of the implications of endogenous expenditure shares by fully assessing the dynamics of the model, the mechanisms of agglomeration and the equilibria growth rate.

We believe that the results obtained in this chapter are important from at least three different perspectives: 1) a purely theoretical one: a tractable endogenous expenditure share approach, being more general than an exogenous one, represents a theoretical progress in the NEG literature and enables us to consider the standard NEGG models as a special case of the one developed here; 2) a descriptive perspective: the endogenous expenditure share approach, by introducing some new economic mechanisms, might be empirically tested and it can be extended to several other NEG models in order to assess their robustness; 3) a policy perspective: our work suggests that policy makers should not trust too much on implications drawn from standard NEGG models because of their limited robustness.

The rest of the chapter is structured as follows: section 2 presents the analytical framework, section 3 deals with the equilibrium allocation of economic activities, section 4 develops the analysis of the growth rate and section 5 concludes.

3.2 The Analytical Framework

3.2.1 The Structure of the Economy

The model structure is closely related to Baldwin, Martin and Ottaviano (2001). The world is made of 2 regions, north and south, both endowed with 2 factors: labour L and capital K. 3 sectors are active in both regions: manufacturing M, traditional good T and a capital producing sector I. Regions are symmetric in terms of: preferences, technology, trade costs and labour endowment. Labour is assumed to be immobile across regions but mobile across sectors within the same region. The traditional good is freely traded between regions whilst manufacture is subject to iceberg trade costs following Samuelson (1954). For the sake of simplicity we will focus on the northern region[1].

Manufactures are produced under Dixit-Stiglitz monopolistic competition (Dixit and Stiglitz, 1975, 1977) and enjoy increasing returns to scale: firms face a fixed cost in terms of knowledge capital[2] and a variable cost a_M in terms of labor. Thereby the cost function is $\pi + wa_M x_i$, where π is the rental rate of capital, w is the wage rate and a_M are the unit of labor necessary to produce a unit of output x_i.

Each region's K is produced by its I-sector which produces one unit of K with a_I unit of labour. So the production and marginal cost function for the I-sector are, respectively:

$$\dot{K} = Q_K = \frac{L_I}{a_I} \tag{3.1}$$

$$F = wa_I \tag{3.2}$$

Note that this unit of capital in equilibrium is also the fixed cost F of the manufacturing sector. As one unit of capital is required to start a new variety, the number of varieties and of firms at the world level is simply equal to the capital stock at the world level: $K + K^* = K^w$. We denote n and n^* as the number of firms located in the north and south respectively. As one unit of capital is required per firm we also know that: $n + n^* = n^w = K^w$. As in Baldwin, Martin and Ottaviano (2001), we assume capital immobility, so that each firm operates, and spends their profits, in the region where the capital's owner lives. In this case, we also have that $n = K$ and $n^* = K^*$. Then, by defining $s_n = \frac{n}{n^w}$ and $s_K = \frac{K}{K^w}$, we also have $s_n = s_K$: the share of firms located in one region is equal to the share of capital owned by the same region[3].

To individual I-firms, the innovation cost a_I is a parameter. However, following Romer (1990), endogenous and sustained growth is provided by assuming that the marginal cost of producing new capital declines (i.e. a_I falls) as the sector's cumulative output rises. In the most general form,

[1]Unless differently stated, the southern expressions are isomorphic.

[2]It is assumed that producing a variety requires a unit of knowledge interpreted as a blueprint, an idea, a new technology, a patent, or a machinery.

[3]We highlight that our results on the equilibrium growth rate holds even in the case of capital mobility.

learning spillovers are assumed to be localized. The cost of innovation can be expressed as:

$$a_I = \frac{1}{AK^w} \tag{3.3}$$

where $A \equiv s_K + \lambda(1 - s_K)$, $0 < \lambda < 1$ measures the degree of globalization of learning spillovers and $s_K = n/n^w$ is share of firms allocated in the north. The south's cost function is isomorphic, that is, $F^* = w^*/K^w A^*$ where $A^* = \lambda s_K + 1 - s_K$. However, for the sake of simplicity, we focus on the case of global spillovers, i.e. $\lambda = 1$ and $A = A^* = 1$[4]. Moreover, in the model version we examine, capital depreciation is ignored[5].

Because the number of firms, varieties and capital units is equal, the growth rate of the number of varieties, on which we focus, is therefore:

$$g \equiv \frac{\dot{K}}{K}; g^* \equiv \frac{\dot{K}^*}{K^*}$$

Finally, traditional goods, which are assumed to be homogenous, are produced by the T-sector under conditions of perfect competition and constant returns. By choice of units, one unit of T is made with one unit of L.

3.2.2 Preferences and consumers' behavior

The preferences structure of the infinitely-live representative agent is given by:

$$\begin{aligned} U_t &= \int_{t=0}^{\infty} e^{-\rho t} \ln Q_t dt; && (3.4) \\ Q_t &= \left[\delta\left(n^{w^{v+\frac{1}{1-\sigma}}} C_M\right)^{\alpha} + (1-\delta) C_T{}^{\alpha}\right]^{\frac{1}{\alpha}}, \alpha \leq 1, v > 0 && (3.5) \\ C_M &= \left[\int_{i=0}^{n+n^*} c_i^{1-1/\sigma} di\right]^{\frac{1}{1-1/\sigma}}, \sigma > 1 \end{aligned}$$

where v is the degree of love for variety parameter, α is the elasticity parameter related to the elasticity of substitution between manufacture and traditional goods and σ is the elasticity of substitution across varieties.

[4] Analysing the localised spillover case is possible, but it will not significantly enrich the results and it will obscure the object of our analysis.

[5] See Baldwin (1999) and Baldwin et al. (2004) for similar analysis with depreciation

Following Cerina and Pigliaru (2007), we deviate from the standard NEGG framework in two respects:

1) As in Benassy (1996) and Smulders and Van de Klundert (2003), the degree of love for variety parameter is explicitly considered. In the canonical NEGG framework the love for variety parameter takes the form $\frac{1}{\sigma-1}$, being tied to the elasticity of substitution across varieties σ (*intrasectoral* elasticity henceforth). By contrast, in the present model v is not linked to σ but it is independently assessed[6].

2) We use a more general Constant Elasticity Function (henceforth CES) instead of a Cobb-Douglas, thereby allowing the elasticity of substitution between manufacture and traditional good (*intersectoral* elasticity henceforth) to diverge from the unit value: indeed the intersectoral elasticity is equal to $\frac{1}{1-\alpha}$ which might be higher or lower than unity (albeit constant) according to whether α is respectively negative or positive. The main effect of this modification is that the share of expenditure on manufacture is no longer constant but it is affected by changes in the price indices of manufacture. This consequence is the source of most of the results presented in this chapter.

Allowing for a larger-than-unity intersectoral elasticity of substitution, requires the introduction of a natural restriction on its value relative to the one of the intrasectoral elasticity of substitution. The introduction of two distinct sectors would in fact be useless if substituting goods from the traditional to the manufacturing sector (and vice-versa) would be easier than substituting goods within the differentiated industrial sectors. In other words, in order

[6]Take an utility function $U\left(C_T.C_M\right)$ where $C_M = V_n\left(c_1,...,c_n\right)$ is homogeneous of degree one, with n being the number of varieties. By adopting the natural normalization $V_1\left(q_1\right) = q_1$, we can define the following function:

$$\gamma(n) = \frac{V_n(c,...,c)}{V_1(nc)} = \frac{V_n(1,...,1)}{n}$$

with $\gamma(n)$ representing the gain in utility derived from spreading a certain amount of expenditure across n varieties instead of concentrating it on a single one. The degree of love for variety v is just the elasticity of the $\gamma(n)$ function:

$$v(n) = \frac{n\gamma'(n)}{\gamma(n)}$$

In the standard NEGG framework $C_M = \left(\int_0^n c_i^{\frac{\sigma-1}{\sigma}} di\right)^{\frac{\sigma}{\sigma-1}}$ hence $\gamma(n) = \frac{1}{\sigma-1}$.

for the representation in terms of two distinct sectors to be meaningful, we need goods belonging to different sectors to be poorer substitutes than varieties coming from the same differentiated sector. The formal expression of this idea requires that the intersectoral elasticity of substitution $\frac{1}{1-\alpha}$ is lower than the intrasectoral elasticity of substitution σ:

$$\frac{1}{1-\alpha} < \sigma$$

This assumption, which will be maintained for the rest of the chapter, states that α cannot not be too high. It is worth to note that this assumption is automatically satisfied in the standard cobb-douglas approach where $\frac{1}{1-\alpha} = 1$ and $\sigma > 1$.

The infinitely-live representative consumer's optimization is carried out in three stages. In the first stage the agent intertemporally allocates consumption between expenditure and savings. In the second stage she allocates expenditure between manufacture and traditional goods, while in the last stage she allocates manufacture expenditure across varieties. As a result of the intertemporal optimization program, the path of consumption expenditure E across time is given by the standard Euler equation:

$$\frac{\dot{E}}{E} = r - \rho \tag{3.6}$$

with the interest rate r satisfying the no-arbitrage-opportunity condition between investment in the safe asset and capital accumulation:

$$r = \frac{\pi}{F} + \frac{\dot{F}}{F} \tag{3.7}$$

where π is the rental rate of capital and F its asset value which, due to perfect competition in the I-sector, is equal to its marginal cost of production.

In the second stage the agent chooses how to allocate the expenditure between manufacture and the traditional good according to the following optimization program:

$$\begin{aligned} \max_{C_M, CT} Q_t &= \ln \left[\delta \left(n^{w^{v+\frac{1}{1-\sigma}}} C_M \right)^{\alpha} + (1-\delta) {C_T}^{\alpha} \right]^{\frac{1}{\alpha}} \\ s.t. P_M C_M + p_T C_T &= E \end{aligned} \tag{3.8}$$

By setting $v = \frac{1}{\sigma-1}$ and $\alpha = 0$ the maximization program boils down to the canonical CD case. As a result of the maximization we obtain the following demand for the manufactured and the traditional goods:

$$P_M C_M = \mu(n^w, P_M)E \tag{3.9}$$

$$p_T C_T = \left(1 - \mu(n^w, P_M)\right) E \tag{3.10}$$

where p_T is the price of the traditional good, $P_M = \left[\int_{i=0}^{K+K^*} p_i{}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$ is the Dixit-Stiglitz perfect price index and $\mu(n^w, P_M)$ is the share of expenditure in manufacture which, unlike the CD case, is not exogenously fixed but it is endogenously determined via the optimization process and it is a function of the total number of varieties (through v) and of the $M-$ goods price index (through α). This feature is crucial to our analysis.

The northern share of expenditure in manufacture is given by:

$$\mu(n^w, P_M) = \left(\frac{1}{1 + P_M^{\frac{\alpha}{1-\alpha}} \left(\frac{1-\delta}{\delta}\right)^{\frac{1}{1-\alpha}} \left(n^{w\,\frac{1}{1-\sigma}-v}\right)^{\frac{\alpha}{1-\alpha}}}\right) \tag{3.11}$$

while the symmetric expression for the south is:

$$\mu(n^w, P_M^*) = \left(\frac{1}{1 + P_M^{*\frac{\alpha}{1-\alpha}} \left(\frac{1-\delta}{\delta}\right)^{\frac{1}{1-\alpha}} \left(n^{w\,\frac{1}{1-\sigma}-v}\right)^{\frac{\alpha}{1-\alpha}}}\right) \tag{3.12}$$

so that northern and southern expenditure shares only differ because of the difference between northern and southern manufacture price index, P_M and P_M^* respectively.

Finally, in the third stage, the amount of $M-$ goods expenditure $\mu(n^w, P_M)E$ is allocated across varieties according to the a CES demand function for a typical M-variety $c_j = \frac{p_j^{-\sigma}}{P_M^{1-\sigma}}\mu(n^w, P_M)E$, where p_j is variety j's consumer price. southern optimization conditions are isomorphic.

3.2.3 Specialization Patterns, Love for Variety and Non-Unitary Elasticity of Substitution

Due to perfect competition in the T-sector, the price of the agricultural good must be equal to the wage of the traditional sector's workers: $p_T = w_T$.

Moreover, *as long as both regions produce some T*, the assumption of free trade in T implies that not only price, but also wages are equalized across regions. It is therefore convenient to choose home labour as numeraire so that:

$$p_T = p_T^* = w_T = w_T^* = 1$$

As it is well-known, it's not always the case that both regions produce some T. An assumption is actually needed in order to avoid complete specialization: a single country's labour endowment must be insufficient to meet global demand. Formally, the CES approach version of this condition is the following:

$$L = L^* < \left(\left[1 - \mu(n^w, P_M)\right] s_E + \left[1 - \mu(n^w, P_M^*)\right] (1 - s_E) \right) E^w \tag{3.13}$$

where $s_E = \frac{E}{E^w}$ is northern expenditure share and $E^w = E + E^*$.

In the standard CD approach, where $\mu(n^w, P_M) = \mu(n^w, P_M^*) = \mu$, this condition collapses to:

$$L = L^* < (1 - \mu) E^w.$$

The purpose of making this assumption, which is standard in most NEGG models[7], is to maintain the M-sector and the I-sector wages fixed at the unit value: since labour is mobile across sector, as long as the T - sector is present in both regions, a simple arbitrage condition suggests that wages of the three sectors cannot differ. Hence, $M-$ sector and I-sector wages are tied to T -sector wages which, in turn, remain fixed at the level of the unit price of a traditional good. Therefore:

$$w_M = w_M^* = w_T = w_T = w = 1 \tag{3.14}$$

Finally, since wages are uniform and all varieties' demand have the same constant elasticity σ, firms' profit maximization yields local and export prices that are identical for all varieties no matter where they are produced: $p = w a_M \frac{\sigma}{\sigma - 1}$. Then, imposing the normalization $a_M = \frac{\sigma-1}{\sigma}$ and (4.13), we finally obtain:

$$p = w = 1 \tag{3.15}$$

As usual, since trade in the $M-$good is impeded by iceberg import barriers, prices for markets abroad are higher:

$$p^* = \tau p; \;\; \tau \geq 1$$

[7]See Bellone and Maupertuis (2003) and Andrés (2007) for an analysis of the implications of removing this assumption.

By labeling as p_M^{ij} the price of a particular variety produced in region i and sold in region j (so that $p^{ij} = \tau p^{ii}$) and by imposing $p = 1$, the $M-$goods price indexes might be expressed as follows:

$$P_M = \left[\int_0^n (p_M^{NN})^{1-\sigma} di + \int_0^{n^*} (p_M^{SN})^{1-\sigma} di\right]^{\frac{1}{1-\sigma}} = (s_K + (1 - s_K)\,\phi)^{\frac{1}{1-\sigma}}\, n^{w\frac{1}{1-\sigma}} \tag{3.16}$$

$$P_M^* = \left[\int_0^n (p_M^{NS})^{1-\sigma} di + \int_0^{n^*} (p_M^{SS})^{1-\sigma} di\right]^{\frac{1}{1-\sigma}} = (\phi s_K + 1 - s_K)^{\frac{1}{1-\sigma}}\, n^{w\frac{1}{1-\sigma}} \tag{3.17}$$

where $\phi = \tau^{1-\sigma}$ is the so called "phi-ness of trade" which ranges from 0 (prohibitive trade) to 1 (costless trade).

A quick inspection of condition (3.13) and expressions (4.15) and (4.16) reveals that the introduction of the no-specialization assumption in our model is sensibly more problematic than in the standard CD case and these difficulties are crucially linked to the role of the love for variety parameter v. In order to see this in detail, we need to get a little bit deeper on the role of the expenditure share and of the love for variety parameter. The removal of the no-specialization assumption opens the room to a complete specialization of the two regions. Indeed there will be a core performing all the $R\&D$ and manufacture activity, and a periphery producing exclusively the traditional good. According to the sign of the intersectoral substitution parameter α the agglomeration of innovation and industry will take place in the north (if α is positive) or in the south (if α is negative). The rationale of this result is the following: if $\alpha > 0$ a shift in s_n would determine an increase in the share of expenditure in manufacture in the north and a decrease of the share of expenditure in manufacture in the south. As agglomeration goes further, the share of expenditure in northern manufacture will increase until reaching the unity, while in the south it will be driven down to zero. As we know workers can be occupied either in the innovation sector, in manufacture or in the traditional sector. By labour market clearing condition we have that the total expenditure in a given good should be equal to the value of production, given by the units of labour necessary to produce one unit of output times the wage rate. Hence as agglomeration proceeds in the north more and more workers will be drained into the industrial sector, while in the south more and more workers will be drained into the traditional sector. At some point the southern traditional sector will be able to satisfy the global demand, hence the two regions will specialize in a core producing innovation and industrial goods, and a periphery producing agriculture. It is worthwhile to stress the fact that complete specialization will also determine a difference in the regional wage rates, that were tied down to unity due to perfect competition and costless trade in the traditional goods, together with intersectoral labour mobility. Indeed as soon as agricultural production vanishes at north, northern wage will not anymore be equal to the southern unitary wage, but moreover will increase because of the labour demand tensions in innovation and manufacture.

Love of variety and expenditure shares

Substituting the new expressions for the $M-$goods price indexes in the northern and southern $M-$goods expenditure shares, yields:

$$\mu(n^w, s_K, \phi) = \left(\frac{1}{1 + \left(\frac{1-\delta}{\delta}\right)^{\frac{1}{1-\alpha}} \left(n^{w^{-\frac{v\alpha}{1-\alpha}}} \left(s_K + (1 - s_K)\,\phi\right)^{\frac{\alpha}{(1-\sigma)(1-\alpha)}} \right)} \right) \tag{3.18}$$

$$\mu^*(n^w, s_K, \phi) = \left(\frac{1}{1 + \left(\frac{1-\delta}{\delta}\right)^{\frac{1}{1-\alpha}} \left(n^{w^{-\frac{v\alpha}{1-\alpha}}} \left(\phi s_K + 1 - s_K\right)^{\frac{\alpha}{(1-\sigma)(1-\alpha)}} \right)} \right). \tag{3.19}$$

As we can see the shares of expenditure in manufactures now depends on the localization of firms s_K, the parameter ϕ and the overall number of firms in the economy n^w.

We can make a number of important observations from analyzing these two expressions.

First, when the elasticity of substitution between the two goods is different to 1, (i.e. $\alpha \neq 0$), north and south expenditure shares differ ($\mu(n^w, s_K, \phi) \neq \mu^*(n^w, s_K, \phi)$) in correspondence to any geographical allocation of the manufacturing industry except for $s_K = 1/2$ (symmetric equilibrium). In particular, we find that[8]:

$$\alpha > (<)\, 0 \Leftrightarrow \frac{\partial \mu}{\partial s_K} = \frac{\alpha\,(1-\phi)\,\mu\,(1-\mu)}{(1-\alpha)\,(\sigma-1)\,((s_K+(1-s_K)\phi))} > (<)\, 0 \quad (3.20)$$

$$\alpha > (<)\, 0 \Leftrightarrow \frac{\partial \mu^*}{\partial s_K} = \frac{\alpha\,(\phi-1)\,\mu^*\,(1-\mu^*)}{(1-\alpha)\,(\sigma-1)\,((s_K+(1-s_K)\phi))} < (>)\, 0 \quad (3.21)$$

Hence, when $\alpha > 0$, production shifting in the north ($\partial s_K > 0$) leads to a relative increase in the southern price index for the M goods because southern consumers have to buy a larger fraction of M goods from the north, which are more expensive because of trade costs. Unlike the CD case, where this phenomenon had no consequences on the expenditure shares for manufactures which remained constant across time and space, in the CES case expenditure shares on M goods are influenced by the geographical allocation of industries because they depend on relative prices and relative prices change with s_K.

Secondly, the impact of trade costs are the following:

$$\alpha > (<)\, 0 \Rightarrow \frac{\partial \mu}{\partial \phi} = \frac{\alpha(1-s_K)\mu\,(1-\mu)}{(1-\alpha)\,(\sigma-1)\,((s_K+(1-s_K)\phi))} > (<)\, 0 \quad (3.22)$$

$$\alpha > (<)\, 0 \Rightarrow \frac{\partial \mu^*}{\partial \phi} = \frac{\alpha s_K \mu^*\,(1-\mu^*)}{(1-\alpha)\,(\sigma-1)\,((s_K+(1-s_K)\phi))} > (<)\, 0 \quad (3.23)$$

so that, when the two kinds of goods are good substitutes ($\alpha > 0$) economic integration gives rise to an increase in the expenditure share for manufactured goods in both regions: manufactures are now cheaper in both regions and since they are good substitutes of the traditional goods, agents in both regions will not only increase their total consumption, but also their shares of expenditure. Obviously, the smaller the share of manufacturing firms already present in the north (south), the larger the increase in expenditure share for the M good in the north (south). The opposite happens when the two kinds of goods are poor substitutes: in this case, even if manufactures are cheaper, agents cannot easily shift consumption from the traditional to the differentiated good. In this case, even if total consumption on manufactures may increase, the share of expenditure will be reduced.

[8]For simplicity's sake we omit the arguments of the functions μ and μ^*.

Finally, the impact of the number of varieties is the following:

$$\begin{aligned}\alpha &> (<)\,0 \Rightarrow \frac{\partial\mu}{\partial n^w} = \frac{\alpha v}{1-\alpha}\frac{(1-\mu)\,\mu}{n^w} \geq (\leq 0) \\ \alpha &> (<)\,0 \Rightarrow \frac{\partial\mu^*}{\partial n^w} = \frac{\alpha v}{1-\alpha}\frac{(1-\mu^*)\,\mu^*}{n^w} \geq (\leq 0)\end{aligned}$$

Therefore, when goods are good (poor) substitutes ($\alpha > 0$), and individuals love variety $(v > 0)$, the expenditure share for the $M-$ goods in both regions *is an increasing (decreasing) function of the total number of varieties.* In the analytical context of the NEGG models, this result (which is a feature of the CES utility function we have chosen) has highly unwelcome effects from the viewpoint of the formal dynamic of the model. In particular, since along the balanced growth path the number of total varieties is increasing $(\frac{\dot{n}^w}{n^w} = g \geq 0)$, the expenditure shares μ and μ^* will asymptotically approach to 1 or 0 according to whether α is positive or negative. This result is a consequence of the interplay between non-unitary intersectoral elasticity of substitution and love for variety. Consider the case when α is positive: when agents love variety, an increase in the number of total variety is sufficient to let their perceived price index for the manufactured goods decrease. As a consequence, because of the elasticity of substitution larger then 1, they will devote a larger share of total expenditure to the $M-$goods. Since the role of the M-goods expenditure shares is crucial in the NEGG models, their non-constancy has a series of important and correlated consequences. Some of them are the following:

1. The real growth rate of the two regions never reach a constant value in a finite time and might differ as the agglomeration process takes place.

2. Since when $\alpha > 0$, μ and μ^* goes to 1, the no-specialization condition cannot hold forever: there comes a (finite) time when the expenditure shares for the traditional good becomes so small that a single country will be able to produce everything is necessary to meet the global demand.

The first result is particularly relevant by the point of view of the policy implications. However, we will not focus on it: for a detailed analysis of this issue please refer to Cerina and Pigliaru (2007). The second result also triggers some new important mechanisms involving the role of wage differentials but makes the analysis highly intractable.

Since the aim of this chapter is to focus on the effects that "geography" (i.e.: interregional firms' allocation and trade costs) has on a NEGG model when a CES second-stage utility function is considered, we henceforth abstract from the two previous consequences by imposing $v = 0$, that is, we assume *no love for variety.* From now on, the second stage maximization program is then:

$$\begin{aligned} \max_{C_M, CT} Q_t &= \ln \left[\delta \left({n^w}^{\frac{1}{1-\sigma}} C_M \right)^{\alpha} + (1-\delta) {C_T}^{\alpha} \right]^{\frac{1}{\alpha}} \\ s.t. P_M C_M + p_T C_T &= E \end{aligned} \tag{3.24}$$

giving rise to the following expressions for the northern and southern expenditure shares:

$$\mu(s_K, \phi) = \left(\frac{1}{1 + \left(\frac{1-\delta}{\delta}\right)^{\frac{1}{1-\alpha}} \left(\left(s_K + (1-s_K)\,\phi\right)^{\frac{\alpha}{(1-\sigma)(1-\alpha)}} \right)} \right) \tag{3.25}$$

$$\mu^*(s_K, \phi) = \left(\frac{1}{1 + \left(\frac{1-\delta}{\delta}\right)^{\frac{1}{1-\alpha}} \left(\left(\phi s_K + 1 - s_K\right)^{\frac{\alpha}{(1-\sigma)(1-\alpha)}} \right)} \right) \tag{3.26}$$

where the influence of the argument n^w has been neutralized by the condition $v = 0$ so that μ and μ^* are only affected by firms' allocation (s_K) and by the freeness of trade (ϕ). Since the latters are constant along the balanced growth path, μ and μ^* are constant too.

What are the drawbacks of eliminating the love for variety on the descriptive relevance of our model? We believe they are not so important for several reasons.

First, from the theoretical point of view, the assumption according to which $v = 0$ is just as general as the standard NEGG assumption according to which $v = \frac{1}{\sigma-1}$.

Second, from an empirical perspective, there are several empirical analysis assessing a value for the v parameter lower than what assumed in standard NEGG models (see for instance Ardelean 2007). In this case the impact of the product variety on economic growth and industrial agglomeration is smaller than what typically assumed and, therefore, closer to 0.

Third, several other NEGG studies abstract from the love for variety. Murata (2008) for instance uses a similar but more restrictive (because $\alpha = 0$) function utility to investigate the relation between agglomeration and structural change. This assumption can also be found in the "new Keynesian economics" literature (see Blanchard and Kiyotaki (1987, p.649), for example), which is another strand of literature based on the model of monopolistic competition by Dixit and Stiglitz (1977).

Fourth, from the point of view of the generality of our model, the introduction of the restriction according to which $v = 0$ is compensated by the introduction of the parameter α which, unlike the standard NEGG models, allows the elasticity of substitution to deviate from the unit value.

The analytical gains of introducing this restrictions are by contrast very relevant.

First, By eliminating the love for variety we are able to maintain a version of the typical assumption in NEGG models which states that a single country's labour endowment must be insufficient to meet global demand. We are entitled to do this because, when $v = 0$, both μ and μ^* cannot reach the unit value. The no-specialization condition should be modified as follows:

$$L < \left(\left[1 - \mu(s_K, \phi)\right] s_E + \left[1 - \mu^*(s_K, \phi)\right](1 - s_E)\right) E^w, \ \forall \, (s_K, \phi) \in (0, 1) \subset \mathbb{R}^2. \tag{3.27}$$

Since s_E has to be constant by definition and even[9]:

$$E^w(s_E, s_K, \phi) = \frac{(2L - L_I - L_I^*)\,\sigma}{s_E\,(\sigma - \mu(s_K, \phi)) + (1 - s_E)\,(\sigma - \mu^*(s_K, \phi))} \tag{3.28}$$

is constant in steady state, (3.27) can be accepted without any particular loss of generality. Our analysis can be developed even without the no-specialisation assumption.

Second, by imposing $v = 0$, we are able to focus on the effect that a non-unitary value of the intersectoral elasticity of substitution has on the equilibrium outcomes of the model. By allowing for this elasticity parameter to deviate from the unit value, we obtain some novel results on the agglomeration and growth prospects of the model. In the next two session we will extensively describe these results.

[9] The expression for E^w can be found by using an appropriate labour market-clearing condition.

3.3 Equilibrium and stability analysis

This section analyses the effects of our departures from the standard NEGG literature on the equilibrium dynamics of the allocation of northern and southern firms.

Following Baldwin, Martin and Ottaviano (2001), we assume that capital is immobile. Indeed, capital mobility can be seen as a special case of capital immobility (a case where $\frac{\partial s_E}{\partial s_K} = 0$). Moreover, as we shall see, capital mobility does not provide any significant departure from the standard model from the point of view of the location equilibria (the symmetric equilibrium is always stable). However, it should be clear that our analysis can be carried on even in the case of capital mobility. In particular, the results of the growth analysis developed in section 4 holds whatever the assumption on the mobility of capital.

In models with capital immobility the reward of the accumulable factor (in this case firms' profits) is spent locally. Thereby an increase in the share of firms (production shiftings) leads to expenditure shiftings through the permanent income hypothesis. Expenditure shiftings in turn foster further production shiftings because, due to increasing returns, the incentive to invest in new firms is higher in the region where expenditure is higher. This is the so-called *demand-linked circular causality.*

This agglomeration force is counterbalanced by a dispersion force, the so-called market-crowding force, according to which, thanks to the unperfect substitutability between varieties, an increase in the number of firms located in one region will decrease firms' profits and then will give an incentive for firms to move to the other region. The interplay between these two opposite forces will shape the pattern of the equilibrium location of firms as a function of the trade costs. Such pattern is well established in NEGG models (Baldwin, Martin and Ottaviano 2001, Baldwin at al. 2004, Baldwin and Martin 2004): in the absence of localized spillovers, since the symmetric equilibrium is stable when trade costs are high and unstable when trade costs are low, catastrophic agglomeration always occur when trade between the two countries is easy enough. That happens because, even though both forces decreases as trade costs become lower, the *demand-linked force* is lower than the *market crowding force* (in absolute value) when trade costs are low, while the opposite happens when trade costs are high.

By adopting the CES approach we are able to question the robustness of such conclusions. In particular our model displays a new force, that we call **substitution effect**. This force fosters agglomeration or dispersion according to whether the T and the $M-$commodities are respectively good or poor substitutes. By introducing this new force, which acts through the northern and southern M-goods expenditure shares, we show that, according to different values of the intersectoral elasticity of substitution, the symmetric equilibrium might unstable for *every* value of trade costs. These results have several implications. First, when the intersectoral elasticity of substitution is allowed to vary from the unit value, the location patterns of firms may not be affected by the market integration process: catastrophic agglomeration will or will not occur unregarding the trade costs so that policy-makers should not be concerned with the effect of market integration. Second, the intersectoral elasticity of substitution becomes a crucial parameter in the analysis of the location pattern of firms and, hence, on the relative welfare of northern and southern agents. We will now explore such implications in detail.

3.3.1 Tobin's q and Steady-state Allocations

Before analysing the equilibrium dynamics of firms' allocation, it is worth to review the analytical approach according to which such analysis will be carried on. As in standard NEGG models, we will make use of the Tobin q approach (Baldwin and Forslid 1999 and 2000). We know that the equilibrium level of investment (production in the I sector) is characterized by the equality of the stock market value of a unit of capital (denoted with the symbol V) and the replacement cost of capital, F. With E and E^* constant in steady state, the Euler equation gives us $r = r^* = \rho$. Moreover, in steady state, the growth rate of the world capital stock K^w (or of the number of varieties) will be constant and will either be common ($g = g^*$ in the interior case) or north's g (in the core-periphery case)[10]. In either case, the steady-state

[10] By time-differentiating $s_K = \frac{K}{K^w}$, we obtain the dynamics of the share of manufacturing firms allocated in the north is

$$\dot{s}_K = s_K \left(1 - s_K\right) \left(\frac{\dot{K}}{K} - \frac{\dot{K}^*}{K^*}\right)$$

so that only two kinds of steady state ($\dot{s}_K = 0$) are possible: 1) one in which the rate of growth of capital is equalized across countries ($g = g^*$), 2) one in which the manufacturing industries are allocated and grow in only one region ($s_K = 0$ or $s_K = 1$).

values of investing in new units of K are:

$$V_t = \frac{\pi_t}{\rho + g}; V_t^* = \frac{\pi_t^*}{\rho + g}.$$

Firms' profit maximization and iceberg trade-costs lead to the following expression for northern and southern firms' profits:

$$\pi = \frac{E^w}{\sigma K^w}\left[\frac{s_E}{s_K + (1 - s_K)\,\phi}\mu(s_K, \phi) + \frac{\phi\,(1 - s_E)}{\phi s_K + (1 - s_K)}\mu^*(s_K, \phi)\right] = B(s_E, s_K, \phi)\frac{E^w}{\sigma K^w} \tag{3.29}$$

$$\pi^* = \frac{E^w}{\sigma K^w}\left[\frac{s_E\phi}{s_K + (1 - s_K)\,\phi}\mu(s_K, \phi) + \frac{1 - s_E}{\phi s_K + (1 - s_K)}\mu^*(s_K, \phi)\right] = B^*(s_E, s_K, \phi)\frac{E^w}{\sigma K^w} \tag{3.30}$$

where this expression differs from the standard NEGG in only one respect: it relies on endogenous $M-$good expenditure shares which now depend on s_E, s_K and ϕ.

By using (4.2), the labour market condition and the expression for northern and southern profits, we obtain the following expression for the northern and southern Tobin's q:

$$q = \frac{V_t}{F_t} = B(s_E, s_K, \phi)\frac{E^w}{(\rho + g)\,\sigma} \tag{3.31}$$

$$q^* = \frac{V_t}{F_t} = B^*(s_E, s_K, \phi)\frac{E^w}{(\rho + g)\,\sigma} \tag{3.32}$$

Where investment in K will take place? Firms will decide to invest in the most-profitable region, i.e. in the region where Tobin's q is higher. Since firms are free to move and to born in the north or in the south (even though, with capital immobility, firm's owners are forced to spend their profits in the region where their firm is located), a first condition characterizing *any interior equilibria* ($g = g^*$) is the following:

$$q = q^* = 1 \tag{3.33}$$

The first equality (*no-arbitrage condition*) tells us that, in any interior equilibrium, there will be no incentive for any firm to move to another region. While the second (*optimal investment condition*) tells us that, in equilibrium, firms will decide to invest up to a level such that the expected discounted value of the firm itself is equal to the replacement cost of capital. The latter is crucial in order to find the expression for the rate of growth but it will not help us in finding the steady state level of s_K. Hence, we focus on the former. By using (4.20), (4.21), (3.31) and (3.32) in (3.33) we find the steady-state relation between northern expenditure share s_E and northern firms' share s_K which can be written as:

$$s_E = \frac{1}{2} + \frac{1}{2}\left(\frac{\mu^*(s_K,\phi)\left(s_K + (1-s_K)\,\phi\right) - \mu(s_K,\phi)\left(\phi s_K + (1-s_K)\right)}{\mu(s_K,\phi)\left(\phi s_K + (1-s_K)\right) + \mu^*(s_K,\phi)\left(s_K + (1-s_K)\,\phi\right)}\right)$$

The other relevant equilibrium condition is given by the definition of s_E when labour markets clear. This condition, also called *permanent income condition*, gives us a relation between northern market size s_E and the share of firms owned by northern entrepreneurs s_K:

$$s_E = \frac{E}{E^w} = \frac{L + \rho s_K}{2L + \rho} = \frac{1}{2} + \frac{\rho\,(2s_K - 1)}{2\,(2L + \rho)} \tag{3.34}$$

By equating the right hand side of these two equation we are able to find the relation between s_K and s_K that has to hold in every interior steady state:

$$\frac{\mu^*(s_K,\phi)\left(s_K + (1-s_K)\,\phi\right) - \mu(s_K,\phi)\left(\phi s_K + (1-s_K)\right)}{\mu(s_K,\phi)\left(\phi s_K + (1-s_K)\right) + \mu^*(s_K,\phi)\left(s_K + (1-s_K)\,\phi\right)} = \frac{\rho\,(2s_K - 1)}{(2L + \rho)} \tag{3.35}$$

so that the steady state level of s_K is one which satisfies the last condition.

It is easy to see that the symmetric allocation, $s_K = \frac{1}{2}$, is always an interior equilibrium. In this case, in fact, the latter condition becomes an identity. In the appendix, we also show that the assumption according to which $\frac{1}{1-\alpha} < \sigma$, assures that the symmetric equilibrium is also the *unique* interior equilibrium.

As for the core-periphery equilibria, things are much simpler. We know that $\dot{s}_K = 0$ also when $s_K = 0$ or $s_K = 1$. For simplicity, we focus on the latter case keeping in mind that the other is perfectly symmetric. The core-periphery outcome is an equilibrium if firms in the north set the investment to the optimal level ($q = 1$) while firms in the south have no incentive to invest ($q^* < 1$).

To sum up, as in the standard CD approach with global spillovers, we only have three possible equilibria: a symmetric equilibrium ($s_K = \frac{1}{2}$) and two core-periphery equilibria ($s_K = 0$ or $s_K = 1$). We will now study the stability properties of such equilibria.

3.3.2 Stability Analysis of the symmetric equilibrium

Following Baldwin and Martin (2004) we consider the ratio of northern and southern Tobin's q:

$$\frac{q}{q^*}=\frac{B(s_E,s_K,\phi)}{B^*(s_E,s_K,\phi)}=\frac{\left[\frac{s_E}{s_K+(1-s_K)\phi}\mu(s_K,\phi)+\frac{\phi(1-s_E)}{\phi s_K+(1-s_K)}\mu^*(s_K,\phi)\right]}{\left[\frac{s_E\phi}{s_K+(1-s_K)\phi}\mu(s_K,\phi)+\frac{1-s_E}{\phi s_K+(1-s_K)}\mu^*(s_K,\phi)\right]}=\gamma\left(s_E,s_K,\phi\right) \tag{3.36}$$

Starting from an interior (and then symmetric) steady-state allocation where $\gamma(s_E,s_K,\phi)=1$, any increase (decrease) in $\gamma(s_E,s_K,\phi)$ will make investments in the north (south) more profitable and thus will lead to a production shifting in the north (south). Hence the symmetric equilibrium will be stable (and hence catastrophic agglomeration will not occur) if a production shifting, say, in the north ($\partial s_K>0$) will reduce $\gamma(s_E,s_K,\phi)$. By contrast, if $\gamma(s_E,s_K,\phi)$ will increase following an increase in s_K, then an equilibrium is unstable and catastrophic agglomeration becomes a possible outcome.

We remind that this method is the same employed by standard NEGG models. The only and crucial difference is that, in our framework, the northern and southern expenditure shares $\mu(s_K,\phi)$ and $\mu^*(s_K,\phi)$ play a crucial role because their value is not fixed but it depends on geography (i.e.: firms' allocation and trade costs). The key variable to look at is then the derivative of $\gamma(s_E,s_K,\phi)$ with respect to s_K.evaluated at $s_K=\frac{1}{2}$. This derivative can be written as:

$$\left.\frac{\partial\gamma\left(s_E,s_K,\phi\right)}{\partial s_K}\right|_{s_K=s_E=\frac{1}{2}}=\frac{(1-\phi)}{(1+\phi)}\frac{1}{\mu\left(\frac{1}{2},\phi\right)}\left(\frac{\partial\mu}{\partial s_K}-\frac{\partial\mu^*}{\partial s_K}\right)-4\left(\frac{1-\phi}{1+\phi}\right)^2+4\frac{\partial s_E}{\partial s_K}\frac{(1-\phi)}{(1+\phi)} \tag{3.37}$$

The stability of the symmetric equilibrium is then determined by the interplay of the the three forces given by:

- $\frac{(1-\phi)}{(1+\phi)}\frac{1}{\mu\left(\frac{1}{2},\phi\right)}\left(\frac{\partial\mu}{\partial s_K}-\frac{\partial\mu^*}{\partial s_K}\right)$
- $4\left(\frac{1-\phi}{1+\phi}\right)^2$
- $4\frac{\partial s_E}{\partial s_K}\frac{(1-\phi)}{(1+\phi)}$

The last two forces are the same we encounter in the standard NEGG model and they are the formal representation of, respectively, the *market-crowding effect* and the *demand-linked effect*. In the standard model, the stability of the equilibrium is the result of the relative strength of these only two forces. The first force represents the novelty of our model. In the standard case, where $\mu^*(s_K, \phi) = \mu(s_K, \phi) = \mu$ and then $\frac{\partial \mu}{\partial s_K} = \frac{\partial \mu^*}{\partial s_K} = 0$, this force just doesn't exist. We dub this force as the **substitution effect** in order to highlight the link between the existence of this force and a non-unitary value of the intersectoral elasticity of substitution. As we will see in detail below, the substitution effect might be a stabilizing (when negative) or an destabilizing one (when positive) according to whether the manufactured and the traditional good are respectively poor ($\alpha < 0$) or good ($\alpha > 0$) substitutes.

But what is the economic intuition behind this force? Imagine a firm moving from south to north ($\partial s_K \geq 0$). For a given value of ϕ, this production shifting, via the home-market effect, reduces the manufactured good price index in the north and increases the one in the south. In the standard case, where the manufactured and the traditional goods are neither good nor poor substitutes, this relative change in the price levels has no effect on the respective expenditure shares. By contrast when the intersectoral elasticity of substitution is allowed to vary from the unitary value, the expenditure shares change with the $M-$price index and hence with s_K. In particular, when the manufactured and the traditional goods are good substitutes ($\alpha > 0$), a reduction in the relative price level in the north leads to and increase $\left(\frac{\partial \mu}{\partial s_K} \geq 0\right)$ in the northern expenditure shares and a decrease $\left(\frac{\partial \mu^*}{\partial s_K} \leq 0\right)$ in the southern expenditure shares, then increasing the relative market size in the north and then providing an (additional) incentive to the southern firms to relocate in the north. The opposite ($\frac{\partial \mu}{\partial s_K} \leq 0$ and $\frac{\partial \mu^*}{\partial s_K} \geq 0$) happens when the manufactured and the traditional goods are poor substitutes ($\alpha < 0$): in this case, southern relative market size increases and this gives and incentive for the moving firm to come back home. This is why, when the M and the T goods are good substitutes the substitution effect acts as an destabilizing force, while the opposite happens when the M and the T goods are poor substitutes.

We can re-write (3.37) by using (3.20) and (3.21) which reveals that $\frac{\partial \mu}{\partial s_K} = -\frac{\partial \mu^*}{\partial s_K}$. Hence:

$$\left.\frac{\partial\gamma\left(s_E,s_K,\phi\right)}{\partial s_K}\right|_{s_K=s_E=1/2}=\underbrace{4\left(\frac{1-\phi}{1+\phi}\right)^2\frac{\alpha\left(1-\mu\left(1/2,\phi\right)\right)}{\left(\sigma-1\right)\left(1-\alpha\right)}}_{\text{Substitution effect}}\underbrace{-4\left(\frac{1-\phi}{1+\phi}\right)^2}_{\text{Market-crowding effect}}\underbrace{+4\frac{\left(1-\phi\right)}{\left(1+\phi\right)}\frac{\partial s_E}{\partial s_K}}_{\text{Demand-linked effect}}$$

Where, using the permanent income condition (3.34), we have $\frac{\partial s_E}{\partial s_K}=\frac{\rho}{2L+\rho}$. The symmetric equilibrium will be stable or unstable according to whether the previous expression is positive or negative. Again, the only difference with respect to the standard case is the presence of the first term in the left-hand side, the substitution effect, which is in fact zero when $\alpha=0$.

It is easy to show that, since $\frac{1}{1-\alpha}<\sigma$, we always have:

$$\underbrace{4\left(\frac{1-\phi}{1+\phi}\right)^2\frac{\alpha\left(1-\mu\left(1/2,\phi\right)\right)}{\left(\sigma-1\right)\left(1-\alpha\right)}}_{\text{Substitution effect}}\underbrace{-4\left(\frac{1-\phi}{1+\phi}\right)^2}_{\text{Market-crowding effect}}\leq 0 \text{ for any } \phi\in[0,1] \tag{3.38}$$

so that the substitution effect will never offset the market-crowding effect[11]. Moreover, analogously to the other two forces, the substitution effect will be decreasing in the freeness of trade. Hence, the dynamic behavior of the agglomeration process as a function of ϕ will not be qualitatively different from the standard case: as ϕ decreases, each forces will reduce their intensity (in absolute value) but the decrease of the Demand-linked effect will be slower.

Nevertheless, the presence of our additional force will introduce the possibility of an additional outcome which was excluded from the standard CD case. In order to do that, we remind the notion of *break-point*, that is the value of ϕ above which the stability of the interior equilibria is broken and then an infinitesimal production shifting in the north (south) will trigger a self-reinforcing mechanism which will lead to a core-periphery outcome. In the standard CD case, since $\alpha=0$, we have that:

$$\left.\frac{\partial\gamma\left(s_E,s_K,\phi\right)}{\partial s_K}\right|_{s_K=s_E=1/2}\geq 0\Leftrightarrow\phi\geq\phi_B^{CD}$$

where $\phi_B^{CD}=\frac{L}{L+\rho}$ is the break-point level of the trade costs. Since $\phi_B^{CD}\in(0,1)$, there is always a feasible value of the trade costs above which the

[11]From this result, we can derive a corollary for the capital mobility case. In this case, s_n should not equal s_K and, above all, there is no permanent income condition so that $\frac{\partial s_E}{\partial s_n}=0$. Hence the stability condition reduces to (3.38) and, just as in the standard case, the symmetric steady-state is always stable when capital is mobile.

interior equilibrium becomes from stable to unstable and then agglomeration will take place.

In our model, it is not possible to calculate an explicit value for the break-point. That's because ϕ enters the expression for $\mu(1/2, \phi)$ as a non-integer power. Nonetheless, we can draw several implications from the existence of the substitution effect. Let's re-write the condition according to which the symmetric equilibrium is unstable $\left(\left. \frac{\partial \gamma(s_E, s_K, \phi)}{\partial s_K} \right|_{s_K = s_E = 1/2} \geq 0 \right)$ in this way:

$$\frac{\alpha \left(1 - \mu \left(1/2, \phi\right)\right)}{\left(\sigma - 1\right)\left(1 - \alpha\right)} \geq \frac{2\left(L + \rho\right)}{\left(1 - \phi\right)\left(2L + \rho\right)} \left(\phi_B^{CD} - \phi\right) \tag{3.39}$$

We can notice that the sign of the left-hand side, being $\frac{2(L+\rho)}{(1-\phi)(2L+\rho)}$ always non-negative, is completely determined by $\left(\phi_B^{CD} - \phi\right)$. First, note that in the standard case, when $\alpha = 0$, this condition reduces to:

$$\phi > \phi_B^{CD}$$

so that, by definition, the equilibrium is unstable when the freeness of trade is larger than the break-point level.

Secondly, note that in our model the right-hand side of (3.39) might be strictly positive or negative according to whether α is positive or negative. That means that the break-point in our model (call it ϕ_B^{CES}) might be higher or lower than ϕ_B^{CD} according to whether the intersectoral elasticity of substitution is larger or smaller than 1. Formally:

$$\begin{array}{rcl} \phi_B^{CES} & > & \phi_B^{CD} \Leftrightarrow \alpha > 0 \\ \phi_B^{CES} & < & \phi_B^{CD} \Leftrightarrow \alpha < 0 \end{array}$$

In other words, and quite intuitively, the presence of an additional agglomeration force (the substitution effect when $\alpha > 0$), shifts the break-point to a lower level so that catastrophic agglomeration is a more likely and it occurs for a larger set of values of ϕ. By contrast, when the substitution effect acts as a dispersion force ($\alpha < 0$), the break-point shifts to an upper level so that catastrophic agglomeration is less likely as it occurs for a smaller set of values of ϕ.

Thirdly, and most importantly, there is a set of parameters such that $\phi_B^{CES} < 0$ and so catastrophic agglomeration may always occur for any value of ϕ. To see this we should find two set of parameters such that the symmetric equilibrium is unstable even for $\phi = 0$. That is, by rewriting (3.39) for $\phi = 0$ and using the fact that $\phi_B^{CD} = \frac{L}{L+\rho}$, this condition reduces to:

$$\frac{\alpha\left(1-\mu\left(1/2,0\right)\right)}{\left(\sigma-1\right)\left(1-\alpha\right)} > \frac{2L}{2L+\rho}$$

which, expressing it in terms of α, becomes:

$$\alpha > \frac{2L\left(\sigma-1\right)}{\left(1-\mu\left(1/2,0\right)\right)\left(2L+\rho\right)+2L\left(\sigma-1\right)} > 0$$

which is a possible outcome, provided that $\frac{2L(\sigma-1)}{(1-\mu(1/2,0))(2L+\rho)+2L(\sigma-1)} < \frac{\sigma-1}{\sigma}$ and so:

$$\mu\left(1/2,0\right) < \frac{\rho}{2L+\rho} = \frac{\partial s_K}{\partial s_E}$$

In this case, therefore, the symmetric equilibrium is unstable and then catastrophic agglomeration always takes place for any feasible value of ϕ.

For the sake of precision, note that we are not able to show the opposite, i.e. that when α is negative enough the break-point level of ϕ is larger than 1 and then agglomeration may never take place. To see this, just notice that the substitution effect ($4\left(\frac{1-\phi}{1+\phi}\right)^2\frac{\alpha(1-\mu(1/2,\phi))}{(\sigma-1)(1-\alpha)}$) decreases with ϕ at the same speed of the marked-crowding effect. Hence, for ϕ close enough to 1, the agglomeration force ($4\frac{\rho}{2L+\rho}\frac{(1-\phi)}{(1+\phi)}$) will always be larger in absolute value than the sum of the two dispersion forces because it decreases at a lower speed[12].

3.3.3 Stability analysis of the Core-Periphery Equilibrium

The share of expenditure in industrial goods is constant ($\dot{s}_K = 0$) even when $s_K = 1$ or $s_K = 0$. Since the two core-periphery equilibria are perfectly

[12]For catastrophic agglomeration never to occur, we need that condition (3.39) never hold even when ϕ is very close to 1. Formally, this condition can be written as

$$\lim_{\phi\to 1}\frac{\alpha\left(1-\mu\left(1/2,\phi\right)\right)}{\left(\sigma-1\right)\left(1-\alpha\right)} < \lim_{\phi\to 1} -\frac{2\rho}{2L+\rho}\frac{1}{1-\phi} = -\infty$$

Which can never be true for any negative finite value of α because the left-hand side always take a finite value.

symmetric, we just focus on the first where the north gets the core. By following Baldwin and Martin (2004), for $s_K = 1$ to be an equilibrium, it must be that $q = v/F = 1$ and $q^* = V^*/F^* < 1$ for this distribution of capital ownership: continuous accumulation is profitable in the north since $v = F$, but $V^* < F$ so no southern agent would choose to setup a new firm. Defining the core-periphery equilibrium this way, it implies that it is stable whenever it exists. By using (3.34) and (3.32) we conclude that, at $s_K = 1$ implies:

$$
\begin{aligned}
q\left(s_K, s_E, \phi\right)\big|_{s_K=1} &= \frac{\left[(L+\rho)\,\mu(1,\phi) + L\mu^*(1,\phi)\right]}{(\rho+g)\,\sigma} = 1 \\
q^*\left(s_K, s_E, \phi\right)\big|_{s_K=1} &= \frac{\left[(L+\rho)\,\phi^2\mu(1,\phi) + L\mu^*(1,\phi)\right]}{\phi\sigma\,(\rho+g)} < 1
\end{aligned}
$$

From the first we can derive the value of the growth rate of world capital in the core-periphery outcome g_{CP} which, in this case, coincides with north's capital:

$$
g_{CP} = \frac{L\left(\mu(1,\phi) + \mu^*(1,\phi)\right) - \rho\left(\sigma - \mu(1,\phi)\right)}{\sigma} \tag{3.40}
$$

By substituting in $q^* < 1$, this condition collapses to:

$$
\phi > \frac{L}{L+\rho}\frac{\mu^*(1,\phi)}{\mu\,(1,\phi)} \tag{3.41}
$$

The solution of this inequality yields the sustain point of our model (call it ϕ_S), i.e., the value of the freeness of trade ϕ above which the core-periphery equilibrium exists and it is stable. Even though this inequality cannot be solved explicitly, yet we can draw several useful observation by analysing it.

First notice that, when $\alpha = 0$, we have that $\mu^*(1,\phi) = \mu\,(1,\phi) = \mu$ and this condition reduces to:

$$
\phi > \phi_S^{CD} = \frac{L}{L+\rho}
$$

as in the standard case (where $\phi_S^{CD} = \phi_B^{CD} = \frac{L}{L+\rho}$).

Secondly, if we allow α to be different from 0, we conclude that:

$$
\begin{aligned}
\alpha &> 0 \Leftrightarrow \phi_S < \phi_S^{CD} = \frac{L}{L+\rho} \\
\alpha &< 0 \Leftrightarrow \phi_S > \phi_S^{CD} = \frac{L}{L+\rho}
\end{aligned}
$$

These results represents a confirmation of our previous intuitions related to the symmetric equilibrium. When the substitution effect behaves as an additional agglomeration force ($\alpha > 0$), then catastrophic agglomeration is more likely and the core-periphery equilibrium becomes stable for lower values of the freeness of trade. By contrast, when the substitution effect behaves as an additional dispersion force, then catastrophic agglomeration is less likely to occur and the core-periphery equilibrium becomes stable for higher values of the freeness of trade.

As in the symmetric case, we are not able to find a set of parameters' value such that the core-periphery is never reached for every level of trade costs , i.e. such that $\phi_S^{CES} > 1$. *Indeed:*

$$\phi_S^{CES} = \phi_S^{CD} \frac{\mu^*(1,\phi)}{\mu(1,\phi)} > 1 \Longrightarrow \mu^*(1,\phi) > \frac{L+\rho}{L}\mu(1,\phi)$$

which is never true for $\phi = 1$ *taken into account in our model. Instead the condition according to which the core-periphery is reached for any value of trade costs is:*

$$\phi_S^{CES} = \phi_S^{CD} \frac{\mu^*(1,\phi)}{\mu(1,\phi)} < 1 \Longrightarrow \mu^*(1,\phi) < \frac{L+\rho}{L}\mu(1,\phi)$$

that is reached for $\phi = 0$ *provided that* $\mu(1,0) > L/\left(L+\rho\right).$

Summarizing, from the stability analysis we can draw many interesting conclusions. Our model displays a new force: the substitution effect, which fosters agglomeration (dispersion) if agriculture and manufacture are good (poor) substitutes. In case of good substitutability between the traditional and the modern good agglomeration is reached for lower levels, and for some values of parameters is reached for every level of trade costs. By contrast in case of poor substitutability between the traditional and the modern good agglomeration is reached only for higher levels of freeness of trade.

3.4 Geography and Integration *always* matter for Growth

A well-established result in the NEGG literature (Balwin Martin and Ottaviano 2001, Baldwin and Martin 2004, Baldwin et al. 2004) is that geography matters for growth *only* when spillovers are localized. In particular, with localized spillovers, the cost of innovation is minimized when the whole manufacturing sector is located in only one region. If this is the case, innovating firms have a higher incentive to invest in new units of knowledge capital with respect to a situation in which manufacturing firms are dispersed in the two regions. Thereby the rate of growth of new units of knowledge capital g is maximized in the core-periphery equilibrium and "agglomeration is good for growth". When spillovers are global, this is not the case: innovation costs are unaffected by the geographical allocation of firms and the aggregate rate of growth is identical in the two equilibria being common in the symmetric one ($g = g^*$) or north's g in the core-periphery one. Moreover, in the standard case, market integration have no direct influence on the rate of growth which is not dependent on ϕ. When spillovers are localized, trade costs may have an indirect influence on the rate of growth by affecting the geographical allocation of firms: when trade costs are reduced below the break point level, the symmetric equilibrium becomes unstable and the resulting agglomeration process, by lowering the innovation cost, is growth-enhancing. But even this indirect influence will not exist when spillovers are global.

In what follows, we will question these conclusions. We will show that in our more general context (i.e. when the intersectoral elasticity of substitution is not necessarily unitary), *geography and integration always matters for growth*, even in the case when spillovers are global. In particular we show that

1. *Market integration has always a direct effect on growth*: when the intersectoral elasticity of substitution is larger than 1, then market integration (by increasing the share of expenditures in manufactures) is always good for growth. Otherwise, when goods are poor substitutes, integration is bad for growth.

2. *The geographical allocation of firms always matters for growth*: the rate of growth in the symmetric equilibrium differs from the rate of growth in core-periphery one. In particular, growth is faster (slower) in symmetry if the share of global expenditure dedicated to manufactures is higher (lower) in symmetry than in the core-periphery. If this is the case, then agglomeration is bad (good) for growth

3.4.1 Growth and economic integration

We now look for the formal expression of the growth rate in both the symmetric and the core-periphery equilibrium. As we have seen, the expression for the growth rate can be found by making use of the optimal investment condition (3.33). By using (3.31), (3.32) we find that, in the interior equilibrium we should have:

$$g_S = B(s_E, s_K, \phi)\frac{E^w}{\sigma} - \rho = B^*(s_E, s_K, \phi)\frac{E^w}{\sigma} - \rho$$

In the symmetric equilibrium we have $s_K = s_E = \frac{1}{2}$, so that, by using (4.20) and (4.21) we know that $B(\frac{1}{2}, \frac{1}{2}, \phi) = B^*(\frac{1}{2}, \frac{1}{2}, \phi) = \mu\left(\frac{1}{2}, \phi\right)$ and therefore:

$$g_S = \mu\left(\frac{1}{2}, \phi\right)\frac{E^w}{\sigma} - \rho$$

Finally, we know that $E^w = 2L + \rho$ so that we can write:

$$g_S = \frac{2L\mu\left(\frac{1}{2}, \phi\right) - \rho\left(\sigma - \mu\left(\frac{1}{2}, \phi\right)\right)}{\sigma} \tag{3.42}$$

It is easy to see that:

$$\frac{\partial g_S}{\partial \phi} = \frac{\partial \mu\left(\frac{1}{2}, \phi\right)}{\partial \phi}\frac{2L + \rho}{\sigma}$$

and by (3.22) and (3.23) we conclude that:

$$\begin{aligned}
\frac{\partial g_S}{\partial \phi} &> 0 \Leftrightarrow \alpha > 0 \\
\frac{\partial g_S}{\partial \phi} &< 0 \Leftrightarrow \alpha < 0 \\
\frac{\partial g_S}{\partial \phi} &= 0 \Leftrightarrow \alpha = 0
\end{aligned}$$

so that integration is good for growth if and only if the traditional and the manufacturing goods are good substitutes. In the standard approach, the special case when $\alpha = 0$, integration has no effect on growth.

In the core-periphery equilibrium, innovation takes place in only one region (say, the north) so that $s_K = 1$ and $g > g^* = 0$. The expression of the growth rate is the same we have encountered in the previous section (3.40):

$$g_{CP} = \frac{L\left(\mu(1,\phi) + \mu^*(1,\phi)\right) - \rho\left(\sigma - \mu(1,\phi)\right)}{\sigma}$$

The relation between growth and integration is not qualitatively different from the symmetric equilibrium. In particular we have:

$$\frac{\partial g_{CP}}{\partial \phi} = \frac{L}{\sigma}\left(\frac{\partial \mu(1,\phi)}{\partial \phi} + \frac{\partial \mu^*(1,\phi)}{\partial \phi}\right) + \frac{\rho}{\sigma}\frac{\partial \mu(1,\phi)}{\partial \phi} \tag{3.43}$$

so that, similarly to the symmetric case:

$$\begin{aligned} \frac{\partial g_{CP}}{\partial \phi} &> 0 \Leftrightarrow \alpha > 0 \\ \frac{\partial g_{CP}}{\partial \phi} &< 0 \Leftrightarrow \alpha < 0 \\ \frac{\partial g_{CP}}{\partial \phi} &= 0 \Leftrightarrow \alpha = 0 \end{aligned}$$

We conclude that the relation between growth and market integration is not qualitatively affected by the geographical location of firms: both in symmetry and in core-periphery this relation is only affected by the value of α. In both equilibria, when α is positive, so that the intersectoral elasticity of substitution is larger than unity, the policy maker should promote policies toward market integration in order to maximize the (common) growth rate. By contrast, if we accept that the two kinds of goods are poor substitutes, then policies favoring economic integration are growth-detrimental and if the policy-maker is growth-oriented then he should avoid them. In any case, the growth rate in the standard case (where growth is unaffected by ϕ and is identical in the symmetric and core-periphery equilibrium) is obtained as a special case ($\alpha = 0$).

What is the economic intuition behind this result? We should first consider that growth is positively affected by the total expenditure share in manufacturing goods at the world level: an increase in this variable would increase manufacturing profits, raising Tobin's q and then incentives to invest. As a result, growth will be higher. Then, any policy instrument able to increase total expenditure on manufacturing goods at the world level will fasten economic growth. The issue is then: what are the determinants of the total expenditure share on manufactures at the world level? From our previous analysis we know that, with CES intermediate utility function, northern and southern expenditure shares depend on the geographical location of firms (s_K) and on the degree of economic integration ϕ. We leave the first determinant aside for a moment and we concentrate on the second. A reduction in the cost of trade will always bring to a reduction in the price index for the manufacturing goods in both regions. However, this reduction will have opposite effect on $\mu(\cdot)$ and $\mu^*(\cdot)$ according to whether the intersectoral elasticity of substitution is larger or smaller than 1. In the first case, since the traditional good (which is now relatively more expensive) can be easily replaced by the industrial goods, the expenditure shares on the latters will increase in both regions, and this will also increase the growth rate. By contrast, when the traditional good cannot be easily replaced by the industrial goods, a reduction in the price index of industrial goods may increase total expenditure but it will decrease its *share* of expenditure in both regions. As a result, any integration-oriented policy will also reduce growth.

3.4.2 Growth and firms' location

Since the only two possible kinds of equilibria are the symmetric and the core-periphery allocation, in order to find the relation between geographical location of firms and growth, we just need to compare (3.42) with (3.43). We then have:

$$g_S > g_{CP} \Leftrightarrow \mu\left(\frac{1}{2}, \phi\right) > s_E^{cp}\mu(1, \phi) + (1 - s_E^{cp})\,\mu^*(1, \phi) \tag{3.44}$$

where $s_E^{cp} = \frac{L+\rho}{2L+\rho}$ is the market size of the north when the whole industry is concentrated in this region ($s_K = 1$). In other words, growth in the symmetric equilibrium will be faster than in the core-periphery equilibrium if and only if the industrial-goods' expenditure share in manufactures in the symmetric equilibrium (which is common in the two regions), is larger than a weighted average of the industrial goods' expenditure share in the core-periphery equilibrium in the two regions, where the weights are given by the reciprocal regional market sizes. What is important in this case is then the relative importance of the industrial goods in the consumption bundle at the world level. If at the world level the industrial good is relatively more important in the symmetric equilibrium than in the core-periphery one, then agglomeration is bad for growth and a growth-oriented policy-maker should promote policies which favor dispersion of economic activities. It is worth to note that this condition is not trivial at all since we have:

$$\begin{aligned}
\alpha &> 0 \Leftrightarrow \mu\left(1,\phi\right) > \mu\left(\frac{1}{2},\phi\right) > \mu^{*}\left(1,\phi\right) \\
\alpha &< 0 \Leftrightarrow \mu^{*}\left(1,\phi\right) > \mu\left(\frac{1}{2},\phi\right) > \mu\left(1,\phi\right)
\end{aligned}$$

A further analysis of condition (3.44), e.g. by using (3.25) and (3.26), will not provide any significant insight. The validity of condition (3.44) is highly dependent on the curvature of $\mu\left(\cdot\right)$ and $\mu^{*}\left(\cdot\right)$ with respect to s_K and its analysis does not provide any relevant economic intuition.

3.5 Conclusions

In this chapter we make a first attempt to introduce endogenously determined expenditure shares in a New Economic Geography and Growth model. We do this by allowing the intersectoral elasticity of substitution to be different from the unit value and we show how this slight change in the model assumptions leads to different outcomes in terms of the dynamic of equilibrium allocation of economic activities, the equilibrium growth prospect and the policy insights.

Concerning the dynamic of allocation of economic activities, our model displays three main results: 1) when the modern and the traditional goods are poor substitutes, the substitution effect acts a dispersion force hence the agglomeration outcome can be reached for level of trade openness which are higher with respect to the standard case; 2) when the traditional and the industrial goods are good substitutes, the substitution effects acts an agglomeration force hence agglomeration is reached for lower degree of market openness; 3) there are values of parameters such that the degree of integration is irrelevant because agglomeration can be reached for whatever level of trade costs.

From the growth perspective, results are even more relevant: 1) unlike the standard NEGG models, the growth rate is influenced by the allocation of economic activities even in absence of localized knowledge spillovers and 2) the degree of economic integration always affects the rate of growth, being growth-detrimental if the intersectoral elasticity of substitution is lower than unity and being growth-enhancing in the opposite case. We are then able to provide a rationale for the rather counterintuitive conclusion according to which an integration-oriented policy rule is bad for growth.

The policy implications of our analysis are relevant. A first message of our model is that policy makers should not blindly rely on standard NEGG models' suggestions because some of their main results are highly dependent on the underlying assumptions. A typical example is the well-established result stating that policy makers should not try to avoid the agglomeration of economic activities because the concentration of the innovative and the increasing returns sectors will increase growth at a global level when spillovers are localized. This conclusion does not take into account the fact that the incentive to invest in new units of capital (and thereby the growth rate) depends on the Dixit-Stiglitz operating profits of manufacturing firms, that in our model is influenced by the share of expenditure in the modern goods. If the average regional expenditure share in this sector is higher in the symmetric equilibrium than in case of agglomeration, then firms' profits are higher when the economic activities are dispersed among the two regions and concentrating them in only one region will reduce economic growth.

A second message we carry out is that policies should take into account the crucial role of the intersectoral elasticity of substitution. To our knowledge, there are no empirical studies assessing the value of this parameter in the context of a NEG model. An empirical analysis of the intersectoral elasticity of substitution would be an expected follow-up of our analysis and would be highly needed in order to assess the relative empirical relevance of the theoretical results we have obtained.

3.6 Appendix

The dynamic of our model is described by three differential equations. Indeed we have two Euler equations (one for each region) representing the evolution of expenditure along time and another equation representing the evolution of capital location:

$$\frac{\dot{E}}{E} = \frac{1}{\sigma}\left[\frac{\mu E\left(1-\phi\right)\left(1-s_K\right)}{s_K+\left(1-s_K\right)\phi} + \frac{\mu^* E^*\left(\phi-1\right)\left(1-s_K\right)}{\phi s_K+\left(1-s_K\right)}\right] - 2L + E^* + E - \rho$$

$$\frac{\dot{E}^*}{E^*} = \frac{1}{\sigma}\left[\frac{\mu E\left(\phi-1\right)s_K}{s_K+\left(1-s_K\right)\phi} + \frac{\mu^* E^*\left(1-\phi\right)s_K}{\phi s_K+\left(1-s_K\right)}\right] - 2L + E^* + E - \rho$$

$$\dot{s}_K = L\left(1-2s_K\right)+\frac{1}{\sigma}\left[\frac{\left(1-\phi\right)\mu E s_K\left(1-s_K\right)}{s_K+\left(1-s_K\right)\phi} + \frac{\left(\phi-1\right)\mu^* E^* s_K\left(1-s_K\right)}{\phi s_K+\left(1-s_K\right)}\right]-E+s_K\left(E^*+E\right)$$

The steady state of the system is given by the values $(\bar{E},\ \bar{E}^*,\ \bar{s}_K)$ such that $\dot{E},\ \dot{E}^*,\ \dot{s}_K = 0$. The steady state values of the state variables are:

$$\begin{aligned}
\bar{E} &= L + \rho s_K \\
\bar{E} &= L + \rho\left(1-s_K\right) \\
\bar{s}_{Ksym} &= \frac{1}{2} \\
\bar{s}_{Kcp} &= 1
\end{aligned}$$

Let's now assess the local stability of the system[13]. By applying the linearization theorem of Hartman & Grobman (1959, 1960a) around the symmetric steady state we obtain the following Jacobian matrix:

$$J=\begin{bmatrix} \left(L+\frac{\rho}{2}\right)\left(1-\frac{(-1+\phi)}{\Lambda(1+\phi)\sigma}\right) & \left(L+\frac{\rho}{2}\right)\left(1+\frac{(-1+\phi)}{\Lambda(1+\phi)\sigma}\right) & -\frac{(-1+\phi)^2(2L+\rho)^2(-\Lambda(-1+\sigma)+\alpha\Delta)}{(-1+\alpha)\Lambda^2(-1+\sigma)\sigma(1+\phi)^2} \\ \left(L+\frac{\rho}{2}\right)\left(1+\frac{(-1+\phi)}{\Lambda(1+\phi)\sigma}\right) & \left(L+\frac{\rho}{2}\right)\left(1-\frac{(-1+\phi)}{\Lambda(1+\phi)\sigma}\right) & \frac{(-1+\phi)^2(2L+\rho)^2(-\Lambda(-1+\sigma)+\alpha\Delta)}{(-1+\alpha)\Lambda^2(-1+\sigma)\sigma(1+\phi)^2} \\ \frac{1}{4}\left(-2-\frac{2(-1+\phi)}{\Lambda(1+\phi)\sigma}\right) & \frac{1}{4}\left(2+\frac{2(-1+\phi)}{\Lambda(1+\phi)\sigma}\right) & \rho-\frac{(-1+\phi)^2(2L+\rho)(-\Lambda(-1+\sigma)+\alpha\Delta)}{(-1+\alpha)\Lambda^2(-1+\sigma)\sigma(1+\phi)^2} \end{bmatrix}$$

Where we have defined for the sake of simplicity:

$$\begin{aligned} \Upsilon &= \left(\left(\frac{1-\delta}{\delta}\right)^{\frac{1}{1-\alpha}}\left(\frac{1+\phi}{2}\right)^{\frac{\alpha}{(1-\alpha)(1-\sigma)}}\right) \\ \Lambda &= (1+\Upsilon) \\ \Delta &= (-1+\sigma+\Upsilon\sigma) \end{aligned}$$

For a three dimensional system the basic form of the characteristic polynomial is:

$$\det(J)-C\Theta+tr(J)\Theta^2-\Theta^3$$

where det and tr are respectively the trace and the determinant of the Jacobian matrix, and C is the sum of its principal minors. By solving the characteristic polynomial we find the eigenvalues that we dub as $\Theta_{1,}$ Θ_2 and Θ_3.

The first eigenvalue Θ_1 is equal to $2L+\rho$ as in Baldwin, Martin and Ottaviano (2001) with global spillovers, and is positive for all values of trade costs. The second and third eigenvalues Θ_2 and Θ_3 switch sign from negative to positive (leading the symmetric equilibrium from stable to unstable) as the according to the following condition:

$$\Lambda>(<)\frac{\alpha\left(-1+\phi\right)\left[2L+\rho\right]}{\left(1+\phi\right)\left(-1+\sigma\right)\left(1+\alpha\right)+\left(\phi-1\right)\left(2L+\rho\right)\left(\alpha\sigma+1-\sigma\right)}\Longleftrightarrow\Theta_2,\Theta_3>(<)0$$

Hence after that trade costs have overcome a certain threshold, the symmetric equilibrium looses its stability and the system moves to the other

[13]The whole numerical analysis was carried out with Mathematica and is available upon request

steady-state equilibrium, the core-periphery one (indeed we have shown that there are no others interior equilibria). Thus we have a supercritical pitchfork bifurcation. We are able to show that the switching condition for the system to loose stability is analogous to the intuitive approach presented in section (3.2). Indeed let's start from the ratio of northern and southern Tobin's q evaluated at the symmetric steady state:

$$\left.\frac{\partial \gamma\left(s_E, s_K, \phi\right)}{\partial s_K}\right|_{s_\Upsilon=s_E=1/2} = 4\left(\frac{1-\phi}{1+\phi}\right)^2 \underbrace{\frac{\alpha\left(1-\mu\left(1/2,\phi\right)\right)}{\left(\sigma-1\right)\left(1-\alpha\right)}}_{\text{Substitution}} \underbrace{-4\left(\frac{1-\phi}{1+\phi}\right)^2}_{\text{Market-crowding}} \underbrace{+4\frac{\left(1-\phi\right)}{\left(1+\phi\right)}\frac{\rho}{\left(2L+\rho\right)}}_{\text{Demand-linked}}$$

As we know the symmetric equilibrium will be stable or unstable according to whether the previous expression is positive or negative, i.e. if $\left(\left.\frac{\partial\gamma(s_E,s_K,\phi)}{\partial s_K}\right|_{s_K=s_E=1/2} > (<) 0\right)$. Knowing that:

$$\mu\left(1/2,\phi\right) = \frac{1}{\left(1+\left(\frac{1-\delta}{\delta}\right)^{\frac{1}{1-\alpha}}\left(\frac{1+\phi}{2}\right)^{\frac{\alpha}{(1-\alpha)(1-\sigma)}}\right)} = \frac{1}{1+\Upsilon} = \frac{1}{\Lambda}$$

the ratio of northern and southern Tobin's q can be written in the following way:

$$\left.\frac{\partial \gamma\left(s_E, s_K, \phi\right)}{\partial s_K}\right|_{s_\Upsilon=s_E=1/2} = -\frac{\left(-1+\phi\right)\alpha\left(\Lambda-1\right)}{\left(-1+\sigma\right)\left(1-\alpha\right)\Lambda\left(1+\phi\right)} + \frac{\left(-1+\phi\right)}{\left(1+\phi\right)} + \frac{\rho}{\left(2L+\rho\right)}$$

By rearranging the terms we find that the symmetric equilibrium switches from stable to unstable according to the following condition:

$$\Lambda > (<) \frac{\alpha\left(-1+\phi\right)\left[2L+\rho\right]}{\left(1+\phi\right)\left(-1+\sigma\right)\left(1+\alpha\right)+\left(\phi-1\right)\left(2L+\rho\right)\left(\alpha\sigma+1-\sigma\right)} \Longleftrightarrow \left.\frac{\partial \gamma\left(s_E, s_K, \phi\right)}{\partial s_K}\right|_{s_K=s_E=1/2} > (<) 0$$

Thereby the formal and the intuitive approaches are equivalent.

Chapter 4

Intersectoral Spillovers and Real Income Growth

4.1 Introduction

The most sounding policy implication of the introduction of the endogenous growth in an economic geography framework is that economic policies aimed at bringing to a more equal distribution of economic activities, such as new infrastructures or direct money flows directed toward the less developed regions, may determine a decrease in the spatial concentration of economic activities as well as a lower real growth rate at aggregate level. This is because in presence of intertemporal localized knowledge spillovers in R&D the spatial concentration of innovation activities brings to a lower cost of producing new commodities and then to a higher real growth rate. Thus there exists a trade-off between equity (meant as a spread out distribution of economic activities) and growth at aggregate level. According to this conclusion it is preferable a more specialized and agglomerated economy, even if the immobile workers of the periphery could be harmed by the higher transport costs they have to pay on the imported innovative varieties. Indeed workers remaining in the region where there is no longer innovative sector face two opposite welfare effects. The first one is a static negative effect that is determined by the fact that the same workers have to pay transport costs on the varieties imported. The second one is a positive dynamic effect implied by the fact that if the innovative sector is concentrated, in case of intertemporal localized knowledge spillovers the growth rate will be higher.

Hence the increase in economic growth led by agglomeration can be the welfare counterpart of the static loss determined by an increase in transport costs. We question this optimistic conclusion by building a NEGG model based on Baldwin, Martin and Ottaviano (2001) with an additional sector producing Non-tradable goods (services). In our work the mechanisms of agglomeration and the nominal growth rate analysis are equal to the benchmark case.When we instead turn to the analysis of the real growth rate, we deviate from the standard NEGG framework. The central and innovative element in our analysis is the introduction of the Non-tradable sector, the services, which are produced under perfect competition and constant returns to scale, and are not interregionally traded [1]. Firms' profits optimization in perfect competition implies that the price level depends upon the wage rate, that is driven to unit due to intersectoral mobility of workers, and the labor units requirements, that is a proxy for productivity. We specify intersectoral spillovers from innovation to services in the following way: the productivity parameter in the labor units requirements is a negative function of the output of innovation. Hence the higher the stock of knowledge capital in one region, the more productive are services because their labor requirement for production is lower. In fact real growth rates in the two regions are allowed to diverge when the innovation allocation pattern differs from the symmetric one. This result is a novelty in the standard theoretical NEGG literature where regional gap in real growth rate is always zero. Indeed in the standard NEGG models real GDP and consumption growth rate is the same in the two regions both in the interior equilibrium (where both are innovating) and in the core-periphery equilibria (where only one is doing so). This is due to the fact that real growth is determined by the constant decrease in the price index that is driven by a continuously widening range of varieties which is common to the two regions. Instead in our work the real growth is not only due to the decrease in the price index of varieties, but also to the decrease of the price index in the services that depends on the rate of innovation. In the symmetric equilibrium the innovative sector is present in both regions hence the rate of decrease in the service price index is the same. On the contrary in case of agglomeration the innovation sector is present only in one region hence the rate of decrease in the services price index in the periphery is zero.

[1] It might be objected that in the current stage of globalization also services start to be interregionally and internationally traded, but we think that the traded portion is so small that can be neglected

Thus by assuming intersectoral and localized knowledge spillovers from the innovation sector to the service sector, we show that innovation (and thereby manufacture) allocation affects regional real growth. Our innovative result has strong policy implications because it suggests that concentrating industries in only one region determines for the periphery not only the aforementioned static loss due to transport costs, but also a dynamic loss in terms of lower real growth rate. Moreover, by analyzing the trade-off between the dynamic gains of agglomeration (due to localized intertemporal spillovers) and the dynamic loss of agglomeration (due to localized intersectoral spillovers), we also discuss different notions of optimal level of agglomeration.

The rest of the chapter is structured as follows: section 2 presents the analytical framework, section 3 deals with the mechanisms of agglomeration, section 4 develops the analysis of the growth rate, section 5 discusses different notions of optimal level of agglomeration and section 6 concludes.

4.2 The Analytical Framework

4.2.1 The structure of the economy

In our analysis we assume two regions symmetric in terms of technology, preferences, transport costs and initial endowments. Each region is endowed with two production factors: labour L and capital K. Four production sectors are active in each region: Modern (manufacture) M, Traditional (agriculture) T, capital producing sector I and a services producing sector S. Labour is assumed to be immobile across regions but mobile across sectors within the same region. The Traditional good is freely traded between regions whilst manufacture is subject to iceberg trade costs[2] (Samuelson, 1954). For the sake of simplicity we will focus on the northern region[3].

The manufactures are produced under Dixit-Stiglitz monopolistic competition (Dixit and Stiglitz, 1977) and enjoy increasing returns to scale: firms face a fixed cost in terms of knowledge capital. In fact it is assumed that for producing a variety is required a unit of knowledge interpreted as a blueprint, an idea, a new technology, a patent, or a machinery. Moreover firms face a variable cost a_M in terms of labor. Thereby the cost function is $\pi + wa_I x_i$,

[2]It is assumed that a portion of the good traded melts in transit.

[3]Unless differently stated, the southern expressions are isomorphic

where π is the rental rate of capital, w is the wage rate and a_M are the unit of labor necessary to produce a unit of output x_i.

Each region's K is produced by its I-sector which produces one unit of K with a_I unit of labour. So the production and marginal cost function for the I-sector are, respectively

$$\dot{K} = Q_K = \frac{L_I}{a_I} \tag{4.1}$$

$$F = wa_I \tag{4.2}$$

Note that this unit of capital in equilibrium is also the fixed cost F of the manufacturing sector. As one unit of capital is required to start a new variety, the number of varieties and of firms at the world level is simply equal to the capital stock at the world level: $K + K^* = K^w$. We denote n and n^* as the number of firms located in the north and south respectively. As one unit of capital is required per firm we also know that: $n + n^* = n^w = K^w$. However, depending on the assumptions we make on capital mobility, the stock of capital produced and owned by one region may or may not be equal to the number of firms producing in that region. In the case of capital mobility, the capital may be produced in one region but the firm that uses this capital unit may be operating in another region. Hence, when capital is mobile, the number of firms located in one region is generally different from the stock of capital owned by this region.

To individual I-firms, the innovation cost a_I is a parameter. However, following Romer (1990), endogenous and sustained growth is provided by assuming that the marginal cost of producing new capital declines (i.e., a_I falls) as the sector's cumulative output rises. In our specification, learning spillovers are assumed to be localised. The cost of innovation can be expressed as

$$a_I = \frac{1}{AK^w} \tag{4.3}$$

where $A \equiv s_n + \lambda(1 - s_n)$, $0 < \lambda < 1$ measures the degree of globalization of learning spillovers and $s_n = n/n^w$ is the share of firms allocated in the north. The south's cost function is isomorphic, that is, $F^* = w^*/K^w A^*$ where $A^* = \lambda s_n + 1 - s_n$. For the sake of simplicity in the model version we examine, capital depreciation is ignored[4].

[4]See Baldwin (2000) and Baldwin et al. (2004) for similar analysis with depreciation

Because the number of firms, varieties and capital units is equal, the growth rate of the number of varieties, on which we focus, is therefore

$$g \equiv \frac{\dot{K}}{K}; g^{*} \equiv \frac{\dot{K}^{*}}{K^{*}} \tag{4.4}$$

Finally, the T-sector produces a homogenous good in perfect competition and constant returns to scale. By choice of units, one unit of T is made with one unit of L.

4.2.2 The Services Sector

What are the reasons for introducing a new sector and for its underlying assumptions? We introduce the Services because we want to show that in a NEGG framework the introduction of an intersectoral spillover between innovation and a Non-tradable sector would lead to a different real growth rate in the two regions. Moreover the new sector has to be non tradable because otherwise the price benefits determined by the intersectoral spillovers would be transmitted between the regions through commerce.

The $S-$sector works in perfect completion and constant returns to scale, with $a_S(\cdot)$ units of labour necessary to produce one unit of output. The $S-$sector production function is:

$$S = \frac{L_S}{a_S(\cdot)} \tag{4.5}$$

where S is the quantity of services produced at north, L_S is the share of labor devoted to Services production and $a_S(\cdot)$ are the labor units requirements. The perfect competition profits are given by:

$$\begin{aligned} \pi_S &= p_S S - w_S L_S \\ \pi_S &= p_S \frac{L_S}{a_S(\cdot)} - w_S L_S \end{aligned}$$

Hence firm's optimization implies the following pricing rule:

$$p_S = w_S a_S(\cdot) \tag{4.6}$$

As we can see the price level depends upon the wage rate and the labor units requirements, that is a proxy for the productivity of the firm. We assume knowledge intersectoral spillovers from innovation to services: the productivity parameter is a negative function of the output of innovation: i.e. the higher is K (or K^* in the south), the more productive are Services because their labor requirement is lower. Hence we have for the north and the south respectively:

$$\frac{\partial a_S(K)}{\partial K} < 0$$

$$\frac{\partial a_S^*(K^*)}{\partial K^*} < 0$$

4.2.3 Preferences and consumers' behavior

The infinitely-live representative consumer's optimization is carried out in three stages. In the first stage the agent intertemporally allocates consumption between expenditure and savings. In the second stage she allocates expenditure between M- and T-goods, while in the last stage she allocates manufacture expenditure across varieties. The preferences structure of the infinitely-live representative agent are given by:

$$U_t = \int_{t=0}^{\infty} e^{-\rho t} \ln Q_t dt \tag{4.7}$$

$$Q_t = \ln \left(C_M^{\alpha} C_T^{\beta} C_S^{\gamma} \right) \tag{4.8}$$

$$C_M = \left[\int_{i=0}^{K+K^*} c_i^{1-1/\sigma} di \right]^{\frac{1}{1-1/\sigma}}$$

$$\alpha + \beta + \gamma = 1 \tag{4.9}$$

As a result of the intertemporal optimization program, the path of consumption expenditure E across time is given by the standard Euler equation:

$$\frac{\dot{E}}{E} = r - \rho \tag{4.10}$$

with the interest rate r satisfying the no-arbitrage-opportunity condition between investment in the safe asset and capital accumulation:

$$r = \frac{\pi}{F} + \frac{\dot{F}}{F} \tag{4.11}$$

where π is the rental rate of capital and F its asset value which, due to perfect competition in the I-sector, is equal to its marginal cost of production.

In the second stage of the utility maximization the agent chooses how to allocate the expenditure between M-, S- and the T- good according to the following optimization program:

$$\begin{aligned} \max_{C_M, C_T, C_S} Q_t &= \ln\left(C_M^{\alpha} C_T^{\beta} C_S^{\gamma}\right) \\ s.t.\ \ E &= P_M C_M + p_T C_T + p_S C_S \end{aligned} \tag{4.12}$$

The objective function is:

$$L : \ln\left(C_M^{\alpha} C_T^{\beta} C_S^{\gamma}\right) + \eta \left(E - P_M C_M - p_T C_T - p_S C_S\right)$$

Yielding the following demand functions:

$$C_M = \alpha \frac{E}{P_M}$$

$$C_T = \beta \frac{E}{p_T}$$

$$C_S = \gamma \frac{E}{p_S}$$

where p_T is the price of the Traditional good, p_S is the price of services, and $P_M = \left[\int_{i=0}^{K+K^*} p_i{}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$ is the Dixit-Stiglitz price index. It is clear that the shares of expenditure in the three types of goods are constant.

Finally, in the third stage, the amount of $M-$ goods expenditure αE is allocated across varieties according to the a CES demand function for a typical M variety $c_j = \frac{p_j^{-\sigma}}{P_M^{1-\sigma}} \alpha E$, where p_j is variety j's consumer price.

4.2.4 The no-specialization condition

Due to perfect competition in the T-sector, the price of the agricultural good must be equal to the wage of the T-sector's workers: $p_T = w_T$. Moreover, *as long as both regions produce some* T, the assumption of free trade in T implies that not only price, but also wages are equalized across regions. It is therefore convenient to choose home labour as numeraire so that

$$p_T = p_T^* = w_T = w_T^* = 1$$

Unfortunately it's not always the case that both regions produce some T. Hence for avoiding complete specialization we have to assume that a single country's labour endowment must be insufficient to meet global demand. Formally:

$$L = L^* < \beta E^w$$

where $s_E = \frac{E}{E^w}$ is the northern expenditure share and $E^w = E + E^*$.

The purpose of making this assumption, which is standard in most NEGG models[5], is to maintain M-sector and T-sector wages fixed at the unit value: since labour is mobile across sector, as long as the T - sector is present in both regions, a simple arbitrage condition would suggest that wages of the two sectors cannot differ. Hence, $M-$ and $S-$sector wages are tied to T -sector wages which, in turn, remain fixed at the level of the unit price of the T-good. Therefore:

$$w_M = w_M^* = w_T = w_T^* = w_S = w_S^* = w = 1 \tag{4.13}$$

Finally, since wages are uniform and all varieties' demands have the same constant elasticity σ, firms' profit maximization yields local and export prices that are identical for all varieties no matter where they are produced: $p = wa_M \frac{\sigma}{\sigma-1}$. Then, by imposing the normalization $a_M = \frac{\sigma-1}{\sigma}$ and equation (4.13), we finally have;

$$p = w = 1 \tag{4.14}$$

As usual, since trade in M is impeded by iceberg import barriers, prices for markets abroad are higher:

$$p^* = \tau p; \ \tau \geq 1$$

By labeling as p_M^{ij} the price of a particular variety produced in region i and sold in region j (so that $p^{ij} = \tau p^{ii}$) and by imposing $p = 1$, the $M-$goods price indexes might be expressed as follows

$$P_M = \left[\int_0^n (p_M^{NN})^{1-\sigma} di + \int_0^{n^*} (p_M^{SN})^{1-\sigma} di \right]^{\frac{1}{1-\sigma}} = (s_n + (1 - s_n)\,\phi)^{\frac{1}{1-\sigma}}\, n^{w\,\frac{1}{1-\sigma}} \tag{4.15}$$

[5]See Bellone and Maupertuis (2003) for an analysis of the implications of removing this assumption.

$$P_M^* = \left[\int_0^n (p_M^{NS})^{1-\sigma} di + \int_0^{n^*} (p_M^{SS})^{1-\sigma} di \right]^{\frac{1}{1-\sigma}} = (\phi s_n + 1 - s_n)^{\frac{1}{1-\sigma}} n^{w \frac{1}{1-\sigma}} \tag{4.16}$$

where $\phi = \tau^{1-\sigma}$ is the so called "phi-ness of trade" which ranges from 0 (prohibitive trade) to 1 (costless trade).

4.3 Mechanisms of Agglomeration

The mechanisms of agglomeration are the same of the standard model. Indeed the introduction of a Non-tradable sector and of intersectoral spillovers between innovation and services do not affect the dynamics of the system. As in the standard NEGG with local spillovers agglomeration in our model is determined by the interaction between two destabilizing forces, *market access effect* and *localised spillovers effect*, and one stabilizing force, the *market crowding effect*. The *market access effect* is due to the fact that industrialized regions are more attractive because their market is larger while the *localised spillovers effect* is determined by the fact that innovation activity is more productive in the region owning a higher capital share. By contrast the dispersion force called *market crowding effect* is given by the fact that in the more industrialized regions there is higher competition. When transport costs are high the *market crowding effect* is stronger than the *market access* and the *localised spillovers effects*. On the contrary when trade costs become to fall the strength of the *market crowding effect* weakens faster than the *market access effect* and the *localised spillovers effects* (which is not affected by trade costs) thereby leading to agglomeration. The *market access effect* is led by a mechanism called *demand-linked circular causality*: production shiftings lead to expenditure shiftings through the permanent income hypothesis. This in turn fosters further production shiftings because in the region where the expenditure is higher there is more incentive to invest in new firms. Instead the *localised spillovers effect* gives rise to a cost-linked circular causality: an increase in the share of capital in one region makes the innovators more productive and boosts the investments thereby feedbacking the mechanism. Alike the standard model, there is agglomeration only in case of capital immobility because the reward of the mobile factor is spent locally. By contrast in capital mobility case it is assumed that the reward of the mobile factor (in this case firms' profits) is spent not in the region where capital is employed, but in the region where the owners of capital live. Assuming that profits are repatriated rules out the *demand-linked circular causality* because capital movements lead to production shiftings that are not followed by expenditure shiftings, thus the demand-linkage is cut[6]. Moreover the decision to accumulate capital will be identical in both regions so that the initial shares of capital are permanent. The equilibrium dynamics of the system is assessed by the mean of the Tobin' q approach (Baldwin and Forslid, 1999 and 2000). According to the microfoundation of investment theory a firm invests in an additional unit of capital only if its own cost is at least equal to the discounted value of the future stream of profits:

$$q = V/F \geq 1$$

The cost of a unit of capital (replacement cost) is:

$$F = a_I = \frac{1}{K^w A}$$

Instead the value of a unit of capital is given by operating profits discounted for the rate of intertemporal preferences and the rate of growth. Concerning the rate of intertemporal preferences, given that expenditure in both regions is constant in steady state ($\dot{E} = \dot{E}^* = 0$) by Euler equation $r = r^* = \rho$. Instead for what concerns the nominal growth rate, in steady state it will be constant and will either be common ($g = g^*$ in the interior symmetric case) or north's g (in the core-periphery case). This because the control variable of the system is the share of manufacturing firms allocated in the north s_K. Differencing its definition $s_K = K/K^w$ with respect to time we obtain:

$$\dot{s}_K = s_K \left(1 - s_K\right) \left(g - g^*\right) \tag{4.17}$$

As we can see two kinds of steady state are possible: one in which the rate of growth of capital is equalized across countries; the other in which the manufacturing industries are allocated and grow in only one region. In any case the value of a unit of capital is:

$$v = \int_{t=0}^{\infty} \pi_i e^{-t(r+g)} dt = -\frac{\pi_i}{r+g} e^{-t(r+g)} \Bigg|_0^\infty = \frac{\pi}{\rho + g}$$

[6]The cost-linked circular causality in case of mobile capital does not boosts investments but only firms' relocation

in the north and:

$$v^* = \frac{\pi^*}{\rho + g}$$

in the south. Hence Tobin's q is respectively in the two regions:

$$q = \frac{v}{F} = \frac{AK^w \pi}{\rho + g} \tag{4.18}$$

$$q^* = \frac{v^*}{F} = \frac{A^* K^w \pi^*}{\rho + g} \tag{4.19}$$

By monopolistic competition equilibrium operating profits are given by the revenues divided by σ. Moreover profits are entirely absorbed by the fixed cost of production (the rental rate of capital) due to free entry. Thus firms' profit maximization leads to the following expressions for north and south firms' profits (regardless the hypothesis on capital mobility):

$$\pi = \frac{\alpha E^w}{\sigma K^w} \left[\frac{s_E}{s_n + (1 - s_n)\,\phi} + \frac{\phi\,(1 - s_E)}{\phi s_n + (1 - s_n)} \right] = \frac{E^w}{\sigma K^w} B(s_E, s_n, \phi) \tag{4.20}$$

$$\pi^* = \frac{\alpha E^w}{\sigma K^w} \left[\frac{s_E \phi}{s_n + (1 - s_n)\,\phi} + \frac{1 - s_E}{\phi s_n + (1 - s_n)} \right] = \frac{E^w}{\sigma K^w} B^*(s_E, s_n, \phi) \tag{4.21}$$

where we have that:

$$B(s_E, s_n, \phi) = \left[\frac{s_E}{s_n + (1 - s_n)\,\phi} + \frac{\phi\,(1 - s_E)}{\phi s_n + (1 - s_n)} \right]$$

$$B^*(s_E, s_n, \phi) = \left[\frac{s_E \phi}{s_n + (1 - s_n)\,\phi} + \frac{1 - s_E}{\phi s_n + (1 - s_n)} \right]$$

The expression for profits, similarly to the standard NEGG, depends on s_n and ϕ.

Let's now assess the global stability of the system by the mean of an informal approach[7]. Let's take into consideration the expressions for the Tobin's q derived by substituting for the profits:

$$\begin{aligned} q &= B(s_E, s_n, \phi) \frac{AE^w}{(\rho + g)\,\sigma} \\ q^* &= B^*(s_E, s_n, \phi) \frac{A^* E^w}{(\rho + g)\,\sigma} \end{aligned}$$

[7]A full formal stability analysis of the model will be proposed in the appendix

Firms will decide to invest in the most-profitable region, i.e. in the region where Tobin's q is higher. Following Baldwin and Martin (2004) we then consider the ratio of northern and southern Tobin's q:

$$\frac{q}{q^*} = \frac{AB(s_E, s_n, \phi)}{A^*B^*(s_E, s_n, \phi)} = \frac{A\left[\frac{s_E}{s_n+(1-s_n)\phi} + \frac{\phi(1-s_E)}{\phi s_n+(1-s_n)}\right]}{A^*\left[\frac{s_E\phi}{s_n+(1-s_n)\phi} + \frac{1-s_E}{\phi s_n+(1-s_n)}\right]} = \Omega\left(s_E, s_n, \phi\right)$$

Starting from an equilibrium situation where $\Omega\left(s_E, s_n\right) = 1$, an increase (decrease) in $\Omega\left(s_E, s_n\right)$ will make investments in the north more profitable and thus will lead to a production shifting in the north (south). Hence, the symmetric equilibrium will be (globally) stable (and hence catastrophic agglomeration will not occur) if a production shifting, say, in the north ($\partial s_n > 0$) will reduce $\Omega\left(s_E, s_n\right)$. By contrast, if $\Omega\left(s_E, s_n\right)$ will increase following an increase in s_n, then the symmetric equilibrium is (globally) unstable and catastrophic agglomeration is a possible outcome. Summing up, we have that

$$\begin{aligned}
\left.\frac{\partial\Omega\left(s_E, s_n, \phi\right)}{\partial s_n}\right|_{s_n=s_E=1/2} &< 0 \Rightarrow \text{the symmetric equilibrium is stable} \\
\left.\frac{\partial\Omega\left(s_E, s_n, \phi\right)}{\partial s_n}\right|_{s_n=s_E=1/2} &> 0 \Rightarrow \text{the symmetric equilibrium is unstable}
\end{aligned}$$

4.3.1 Capital Mobility Case

Let's now describe the impact of a firms relocation in case of capital mobility:

$$\left.\frac{\partial\Omega\left(s_E, s_n, \phi\right)}{\partial s_n}\right|_{s_n=s_E=1/2} = \frac{4}{(1+\lambda)}\frac{(1+\phi^2)}{(1+\phi)^2}\left[1-\lambda-\frac{(1-\phi)^2}{(1+\phi^2)}\right] < 0 \quad (4.22)$$

The negative term in the right hand side of the equation is the *market crowding effect* and is a stabilizing effect. By contrast the positive term in the right hand side is the *localized spillovers effect* and is a destabilizing effect. Is it clear that the *market crowding effect* is stronger than the *localized spillovers effect* for every value of freeness of trade, hence like in the benchmark model there is no agglomeration in case of capital mobility.

4.3.2 Capital Immobility Case

On the contrary in case of capital immobility:

$$\left.\frac{\partial\Omega\left(s_E, s_K, \phi\right)}{\partial s_K}\right|_{s_n=s_E=1/2} = 2\left(\frac{1-\phi}{1+\phi}\right)\left(\frac{2\lambda\rho}{[(1+\lambda)(L(1+\lambda)+\rho)]}\right)+\frac{4}{(1+\lambda)}\frac{\left(1+\phi^2\right)}{\left(1+\phi\right)^2}\left[1-\lambda-\frac{\left(1-\phi\right)^2}{\left(1+\phi^2\right)}\right] \tag{4.23}$$

The positive terms represent the *demand-linked* and the *localized spillovers effects* (destabilizing forces). Instead the negative term is the *market crowding effect* (stabilizing effect). For a certain threshold in trade costs the destabilizing effects are stronger than the stabilizing ones (i.e. equation (24) turns positive) hence like in the benchmark there is agglomeration. This threshold in transport costs is called break point and is equal to:

$$\phi_{bp} = \frac{L\left(1+\lambda\right)+\rho-\sqrt{\left(1-\lambda^2\right)\left[L\left(1+\lambda\right)+\rho\right]^2+\lambda^2\rho^2}}{\lambda\left[L\left(1+\lambda\right)+2\rho\right]}$$

4.4 Growth

The section on growth is undoubtedly the core of our analysis. In the first subsession we will assess the nominal growth rate of our model showing that the nominal growth rate is the same in the two regions in both equilibria and as in the standard NEGG framework. On the contrary, in the subsession assessing the real growth rate, it will be shown how the adoption of intersectoral knowledge spillovers between innovation and services allows the real growth rates of the two regions to deviate in case of Core-Periphery equilibrium.

4.4.1 Nominal Growth

Let's start by defining the labor market equilibrium. Workers can be occupied either in the innovation sector, in manufacture, in services or in the T-sector. In manufacture we have that the total expenditure in manufactural goods should be equal to the value of production.

The total expenditure devoted to manufacture is αE^w while the value of production is given by the quantity of labor L_M times the units of labor per unit of output $1-1/\sigma$ times the wage rate that is normalized to 1. The total work devoted to the Modern sector hence becomes:

$$L_M = \frac{\sigma-1}{\sigma}\alpha E^w \tag{4.24}$$

Similarly the total expenditure in the T-good βE^w should be equal to the value of production given by the units of labor required to produce a unit of product multiplied by the wage rate (both normalized to 1) and by the quantity of labor L_T. Hence the total work devoted to the Traditional sector is:

$$L_T = \beta E^w \tag{4.25}$$

By contrast in the Non-tradable sector we have two market clearing conditions, one for each region. Hence the expenditure services in north and south (respectively γE and γE^*) should be equal to the value of production given by the units of labor required to produce a unit of product multiplied by the wage rate (both normalized to 1) and by the respective quantity of labor (L_S and L_S^*). So the market clearing conditions are respectively in north and south:

$$\gamma E = L_S$$

$$\gamma E^* = L_S^*$$

At the aggregate level:

$$L_S^w = L_S + L_S^* = \gamma E^w \tag{4.26}$$

Finally the total work devoted to Innovation is:

$$L_I = F\dot{K}^w = \frac{g}{A} \tag{4.27}$$

where $F = a_I$ are the units of labor per units of output and $\dot{K}^w$ are the new units of capital (i.e. new varieties) created.

Thus by summing up equations (24) to (27) we have the labor market clearing condition:

$$2L = \frac{g}{A} + E^w \left(\frac{\sigma - \alpha}{\sigma} \right) \tag{4.28}$$

Let's now assess the growth rate. Optimizing consumers set their expenditure at the permanent income hypothesis level in steady state. Hence they gain a labor income plus ρ times their steady state wealth. Thus we have respectively in the north and in the south:

$$E = L + FK = L + \frac{\rho s_K}{A} \tag{4.29}$$

$$E^{*} = L + FK^{*} = L + \frac{\rho\,(1 - s_K)}{A} \tag{4.30}$$

Hence the world level expenditure becomes:

$$E^{w} = E + E^{*} = 2L + \frac{\rho}{A} \tag{4.31}$$

By substituting equation (31) in equation (28) we find the growth rate in the interior equilibria:

$$g = \frac{2AL\alpha - \rho\,(\sigma - \alpha)}{\sigma} \tag{4.32}$$

As we can appreciate in case of localized knowledge spillovers the rate of nominal growth depends on the location of firms. Indeed the growth rate changes between the symmetric and the Core-Periphery equilibrium. In the symmetric equilibrium we have:

$$g_{sym} = \frac{(1 + \lambda)\,L\alpha - \rho\,(\sigma - \alpha)}{\sigma} \tag{4.33}$$

By contrast in the core-periphery case:

$$g_{cp} = \frac{2L\alpha - \rho\,(\sigma - \alpha)}{\sigma} \tag{4.34}$$

It is clear that the nominal growth rate is higher for both regions in case of agglomeration. Hence the NEGG policy recommendation is that economic integration is always desiderable.

4.4.2 Real Growth

In our model the nominal income level is analogous to the standard NEGG one:

$$Y = L + \pi s_K K^{w} = L + \frac{\alpha E^{w} A}{\sigma}\left[\frac{s_E}{(s_K + (1 - s_K)\phi)} + \frac{\phi(1 - s_E)}{(\phi s_K + 1 - s_K)}\right] \tag{4.35}$$

$$Y^{*} = L + \pi^{*}(1 - s_K)K^{w} = L + \frac{\alpha E^{w} A^{*}}{\sigma}\left[\frac{\phi s_E}{(s_K + (1 - s_K)\phi)} + \frac{1 - s_E}{(\phi s_K + 1 - s_K)}\right] \tag{4.36}$$

Similarly the growth rates of nominal income are constant in steady state:

$$\frac{\dot{Y}}{Y} = \frac{\dot{Y}^{*}}{Y^{*}} = 0$$

And also the difference in the nominal income level in the two regions in case of agglomeration (e.g. in the north) is similar:

$$Y = L + \frac{\alpha}{\sigma} E^w \tag{4.37}$$

$$Y^* = L \tag{4.38}$$

But let us now show that with intersectoral spillovers in core-periphery the real growth rate diverges between the two regions. For doing that let's consider the price index of M-, T- and S-goods. The price index for the Modern good (equations (15) and (16)) is:

$$P_M = (s_K + (1 - s_K)\phi)^{\frac{1}{1-\sigma}} \, K^{w\frac{1}{1-\sigma}}$$

$$P_M^* = (\phi s_K + (1 - s_K))^{\frac{1}{1-\sigma}} \, K^{w\frac{1}{1-\sigma}}$$

Instead the optimizing price for Services in north and south is:

$$p_S = a_S(K); p_S^* = a_S^*(K^*) \tag{4.39}$$

where $a_S(K)$ and $a_S^*(K^*)$, the labor units requirements for northern and southern firms, are negative function of the output of innovation. Finally we have the normalized price for the Traditional good:

$$p_T = p_T^* = 1$$

Taking into account the agricultural and the services sector, the perfect price index associated to the second stage Cobb-Douglas utility function is respectively in north and south:

$$P = P_M^{\alpha} p_T^{\beta} p_S^{\gamma}; P^* = P_M^{*\alpha} p_T^{*\beta} p_S^{*\gamma} \tag{4.40}$$

The growth rate of prices for the Modern good is:

$$\frac{\dot{P}_M}{P_M} = \frac{\dot{P}_M^*}{P_M^*} = \frac{1}{1-\sigma} \frac{\dot{K}^w}{K^w} = -\frac{g}{\sigma - 1}$$

The growth rate of prices for the Traditional good is:

$$\frac{\dot{p}_T}{p_T} = \frac{\dot{p}_T^*}{p_T^*} = 0 \tag{4.41}$$

that is nil because the T-good is taken as numeraire. By contrast the growth rate of prices for the Non-tradable good is:

$$\frac{\dot{p}_S}{p_S} = \frac{\dot{a}_S(K)}{a_S(K)} = \frac{\partial a_S(K)/\partial K}{a_S(K)}\dot{K} = \theta(K)\frac{\dot{K}}{K} \tag{4.42}$$

$$\frac{\dot{p}^*_S}{p^*_S} = \frac{\dot{a}^*_S(K^*)}{a^*_S(K^*)} = \frac{\partial a^*_S(K^*)/\partial K^*}{a^*_S(K^*)}\dot{K}^* = \theta^*(K^*)\frac{\dot{K}^*}{K^*} \tag{4.43}$$

As we can see the growth rate of prices for the Non-tradable good is given by the growth rate of capital ($\dot{K}/K$ in the north and $\dot{K}^*/K^*$ in the south) times the elasticity of the productivity parameter (the labor units requirements) with respect to the output of innovation in the two regions. Those elasticities, $\theta(K)$ and $\theta^*(K^*)$, are negative as:

$$\frac{\partial a_S(K)}{\partial K} < 0$$

$$\frac{\partial a^*_S(K^*)}{\partial K^*} < 0$$

Hence an increase in the output of innovation (knowledge capital) determines a decrease in the price growth rate of the Non-tradable goods. Finally, by substituting the growth rates of agriculture (that is nil), manufacture and services in the perfect price index associated to the second stage Cobb-Douglas the growth rate of prices in the two regions becomes:

$$\frac{\dot{P}}{P} = -\frac{\alpha g}{\sigma - 1} + \gamma\theta(K)\frac{\dot{K}}{K} \tag{4.44}$$

$$\frac{\dot{P}^*}{P^*} = -\frac{\alpha g}{\sigma - 1} + \gamma\theta^*(K^*)\frac{\dot{K}^*}{K^*} \tag{4.45}$$

Hence the real income growth rate is:

$$\varphi(K) = \frac{\dot{Y}}{Y} - \frac{\dot{P}}{P} = \frac{\alpha g}{\sigma - 1} - \gamma\theta(K)\frac{\dot{K}}{K} \tag{4.46}$$

$$\varphi^*(K^*) = \frac{\dot{Y}^*}{Y^*} - \frac{\dot{P}^*}{P^*} = \frac{\alpha g}{\sigma - 1} - \gamma\theta^*(K^*)\frac{\dot{K}^*}{K^*} \tag{4.47}$$

The gap in the real growth rate between the two regions is given by the difference in the price growth rate of the services sector and the growth rate of innovation:

$$\varphi(K) - \varphi^*(K^*) \Longleftrightarrow \gamma\theta^*(K^*)\frac{\dot{K}^*}{K^*} - \gamma\theta(K)\frac{\dot{K}}{K} \tag{4.48}$$

Thus we have a positive steady state real growth differential if:

$$\theta^*(K^*)\frac{\dot{K}^*}{K^*} > \theta(K)\frac{\dot{K}}{K} \Rightarrow \varphi(K) > \varphi^*(K^*) \tag{4.49}$$

Let's see now what happens in different equilibria. In interior equilibrium both regions invest ($\frac{\dot{K}^*}{K^*} = \frac{\dot{K}}{K} = g$) hence the real income growth is the same:

$$\theta^*(K^*)g = \theta(K)g \Rightarrow \varphi(K) = \varphi^*(K^*) \tag{4.50}$$

Instead in case of agglomeration in the north we have:

$$\theta^*(K^*)\frac{\dot{K}^*}{K^*} = 0; \theta(K)\frac{\dot{K}}{K} = \theta(K)g < 0 \Rightarrow \varphi(K) > \varphi^*(K^*) \tag{4.51}$$

As we can see K^* does no longer grow because innovation is completely vanished in the south thus the price growth of services is nil. By contrast in north, where innovation is clustered, the price growth rate of services is $\theta(K)g$. Due to this fact, the real income growth diverges between the two regions. In particular the real income growth diverges as soon as innovation and manufacture are unequally distribuited (i.e. $s_n \neq 1/2$).

4.5 Optimal Agglomeration Levels

In this section we will discuss the optimal level of agglomeration at aggregate and at regional level. We will first assess the effect of agglomeration on the real aggregate growth rate, afterward we will show the real growth rate at regional level. By equations (32) and (46) we can explicit the real growth rate in the north:

$$\varphi\left(s_n, K\right) = \frac{\left(2\alpha L\left(s_n + \lambda\left(1 - s_n\right)\right) - \rho\left(\sigma - \alpha\right)\right)\left(\alpha - \gamma\theta(K)\left(\sigma - 1\right)\right)}{\sigma\left(\sigma - 1\right)} \tag{4.52}$$

In the same way, by symmetry and equation (47) we have for the south:

$$\varphi^*\left(s_n, K^*\right) = \frac{\left(2\alpha L\left(\lambda s_n + 1 - s_n\right) - \rho\left(\sigma - \alpha\right)\right)\left(\alpha - \gamma\theta^*(K^*)\left(\sigma - 1\right)\right)}{\sigma\left(\sigma - 1\right)} \tag{4.53}$$

The aggregate growth is just the average of the growth rate in the two regions:

$$\Theta(s_n, K, K^*) = \frac{\left(2\alpha L\left(s_n + \lambda\left(1 - s_n\right)\right) - \rho\left(\sigma - \alpha\right)\right)\left(2\alpha - \gamma\theta(K)\left(\sigma - 1\right) - \left(\sigma - 1\right)\gamma\theta^*(K^*)\right)}{2\sigma\left(\sigma - 1\right)} \tag{4.54}$$

In the symmetric equilibrium $K = K^* = K_{sym}$ hence $s_n = 1/2$, i.e. the initial distribution of knowledge capital across the regions is equal. Moreover $g = g^*$, hence the knowledge capital is growing in both regions ($\theta(K) = \theta^*(K^*) = \theta(K_{sym})$). So the aggregate real growth rate is:

$$\Theta\left(1/2, K_{sym}\right) = \frac{\left(\alpha L\left(1 + \lambda\right) - \rho\left(\sigma - \alpha\right)\right)\left(2\alpha - \gamma\theta(K)\left(\sigma - 1\right) - \left(\sigma - 1\right)\gamma\theta^*(K^*)\right)}{2\sigma\left(\sigma - 1\right)}$$

By contrast in the core-periphery equilibrium we have that $K = K_{cp}$ and $K^* = \bar{K}_{sym}$ hence $s_n = 1$. Moreover there is innovation only in the north ($\theta(K_{cp}) = \theta(K); \theta^*(K^*) = 0$). Then the aggregate real growth rate is:

$$\Theta(1, K_{cp}) = \frac{\left(2\alpha L - \rho\left(\sigma - \alpha\right)\right)\left(2\alpha - \gamma\theta(K_{cp})\left(\sigma - 1\right)\right)}{2\sigma\left(\sigma - 1\right)}$$

Agglomeration is growth enhancing at aggregate level if:

$$|\theta(K_{cp})| \geq |\theta(K) + \theta^*(K^*)| \Longrightarrow \Theta(1, K_{cp}) > \Theta\left(1/2, \bar{K}_{sym}\right) \tag{4.55}$$

i.e. if the dynamic loss of periphery due in the service sector price index decrease has been at least counterbalanced by the dynamic gains of the core.

So, at aggregate level both regions gain from agglomeration, but at regional level it may not be the case. The north has two dynamical gains from agglomeration. The northern real growth rate in symmetry is:

$$\varphi\left(1/2, K_{sym}\right) = \frac{\left(\alpha L\left(1+\lambda\right) - \rho\left(\sigma-\alpha\right)\right)\left(\alpha - \gamma\theta(K)\left(\sigma-1\right)\right)}{\sigma\left(\sigma-1\right)}$$

Instead in core-periphery the northern real growth rate is:

$$\varphi(1, K^{w}) = \frac{\left(2\alpha L - \rho\left(\sigma-\alpha\right)\right)\left(\alpha - \gamma\theta(K^{w})\left(\sigma-1\right)\right)}{\sigma\left(\sigma-1\right)}$$

Clearly the real growth rate of the north is higher in core-periphery:

$$\varphi(1, K^{w}) > \varphi\left(1/2, K_{sym}\right) \tag{4.56}$$

On the contrary the periphery may loose from agglomeration. In fact the southern real growth rate in symmetry is:

$$\varphi^{*}\left(1/2, K_{sym}\right) = \frac{\left(\alpha L\left(1+\lambda\right) - \rho\left(\sigma-\alpha\right)\right)\left(\alpha - \left(\sigma-1\right)\gamma\theta^{*}(K^{*})\right)}{\sigma\left(\sigma-1\right)}$$

Whilst in core periphery the real growth rate becomes:

$$\varphi^{*}(1, \bar{K}_{sym}) = \frac{\left(2\alpha L - \rho\left(\sigma-\alpha\right)\right)\alpha}{\sigma\left(\sigma-1\right)}$$

Hence agglomeration is welfare harming for the south if:

$$\lambda > \frac{\alpha^{2}L + \left(\alpha L - \rho\left(\sigma-\alpha\right)\right)\left(\sigma-1\right)\gamma\theta^{*}(K_{sym})}{\alpha L\left(\alpha - \left(\sigma-1\right)\gamma\theta^{*}(K_{sym})\right)} \Longrightarrow \varphi^{*}\left(1/2, K_{sym}\right) > \varphi^{*}\left(1, \bar{K}_{sym}\right) \tag{4.57}$$

That is inside ranges (remember that must be $0 < \lambda < 1$) if:

$$L < \frac{\rho\left(\sigma-\alpha\right)}{2\alpha}$$

The result shown in equation (57) is due to the fact that if spillovers are less localized (λ higher) the firms' location is less important therefore the difference in growth between core-periphery and symmetry (i.e. the dynamic gain from agglomeration) is lower.

For what concerns the interior asymmetric equilibria (i.e. where $s_n \neq 1/2$), the growth rate of manufacture, that unfortunately is not possible to assess analytically, is the same in both regions. Remember indeed that in our model the only possible equilibria are $s_n = 1$, $s_n = 0$ or $g = g^*$. However, we can argue that in our framework while the nominal rate of growth must be equal across regions, this is not the case for the real growth rate. Indeed each asymmetric distribution of capital will lead the growth price of services to diverge across the two regions. Hence the agglomerating region will enjoy a higher real growth rate as soon as $s_n \neq 1/2$.

4.6 Conclusions

In presence of intertemporal localized knowledge spillovers in innovation, according to standard NEGG policy makers face a trade off between equity and efficiency. If the industrial pattern is scattered, the equity is satisfied but the nominal and real growth is minimized. By contrast if industry and innovation are clustered the nominal and real growth rate are maximized, benefiting both the core and the periphery. This dynamic gain counterbalances (and being dynamic, at some point overcomes) the static loss suffered by the south due to the trade costs on varieties that have to be imported from the north. Hence agglomeration is undoubtedly welfare enhancing for both regions. In this chapter, by introducing intersectoral localized knowledge spillovers between innovation and the newly added services sector, we show that the outcome is more puzzling. Assuming a core-periphery in the north pattern, we have different effects for the two regions. The north would be better off enjoying two dynamic gains: the first one is an increase in the nominal growth rate of manufacture due to the localized knowledge spillovers; the second one is a decrease in the growth rate of services price due to the intersectoral knowledge spillovers. Instead the south would experience two opposite effects: a dynamical gain given by the increase in the nominal growth rate of manufacture (that is imported from the north) due to the localized knowledge spillovers; and a dynamical loss due to the fact that southern capital does not grow anymore hence the services price is fixed (no intersectoral spillovers). Assuming that the dynamical loss of the south is counterbalanced by the dynamical gain in the services for the north we have that agglomeration is welfare enhancing at aggregate level. But if we consider the south alone we have that the real growth rate in the symmetric equilibrium can be higher than in the core-periphery equilibrium if the dynamic loss in the services sector overcomes the dynamic gain in manufacture. This conclusion clashes with the standard NEGG stating that in agglomeration both core and periphery enjoy the same dynamical gains.

4.7 Appendix A

Let's compute the law of motion for expenditure in the north. We start from the expression for the capital replacement cost in the north:

$$F = wa_I = \frac{1}{AK^w} = \frac{1}{(K + \lambda K^*)}$$

By time differentiation we have:

$$\dot{F} = -\frac{\dot{K} + \lambda \dot{K}^*}{(K + \lambda K^*)^2}$$

Now, using equations (1) and (4):

$$\dot{K} = \frac{L_I A}{s_K} K$$

$$\dot{K}^* = \frac{L_I^* A^*}{1 - s_K} K^*$$

Substituting in the expression for $\dot{F}$ we obtain:

$$\dot{F} = -\frac{1}{(K + \lambda K^*)^2} \left(\frac{L_I A}{s_K} K + \frac{\lambda L_I^* A^*}{1 - s_K} K^* \right) = -\frac{K^w}{(K + \lambda K^*)^2} \left(L_I A + \lambda L_I^* A^* \right)$$

As we know labour in the I-sector is equal to the value of investments (i.e. income minus expenditure) so it is given respectively in each region by:

$$L_I A = LA + \pi K A - EA = LA + \frac{E^w}{\sigma} B s_K A - EA$$

$$L_I^* A^* = LA^* + \frac{E^w}{\sigma} B^* \left(1 - s_K\right) A^* - E^* A^*$$

Moreover we know that:

$$A = s_K + \lambda \left(1 - s_K\right) = \frac{K + \lambda K^*}{K^w}$$

$$A^* = \lambda s_K + \left(1 - s_K\right) = \frac{\lambda K + K^*}{K^w}$$

Thus we can write:

$$\frac{\dot{F}}{F}=-L\left(1+\lambda\right)\frac{\left(\lambda K+K^{*}\right)}{\left(K+\lambda K^{*}\right)}+\lambda E^{*}\frac{\left(\lambda K+K^{*}\right)}{\left(K+\lambda K^{*}\right)}+E-\frac{E^{w}}{\sigma}\left(Bs_{K}-\lambda B^{*}\left(1-s_{K}\right)\frac{\left(\lambda K+K^{*}\right)}{\left(K+\lambda K^{*}\right)}\right)$$

By substituting this last expression first in the no-arbitrage condition (equation (11)) and then in the Euler equation (equation (10))we finally have:

$$\frac{\dot{E}}{E}=\frac{E^{w}}{\sigma}\left(AB-Bs_{K}-\lambda B^{*}\left(1-s_{K}\right)\frac{\left(\lambda K+K^{*}\right)}{\left(K+\lambda K^{*}\right)}\right)-L\left(1+\lambda\frac{\left(\lambda K+K^{*}\right)}{\left(K+\lambda K^{*}\right)}\right)+\lambda E^{*}\frac{\left(\lambda K+K^{*}\right)}{\left(K+\lambda K^{*}\right)}+E-\rho$$

The expression for the south is symmetric:

$$\frac{\dot{E}^{*}}{E^{*}}=\frac{E^{w}}{\sigma}\left(A^{*}B^{*}-\lambda B^{*}\left(1-s_{K}\right)-Bs_{K}\frac{\left(\lambda K+K^{*}\right)}{\left(K+\lambda K^{*}\right)}\right)-L\left(1+\lambda\frac{\left(\lambda K+K^{*}\right)}{\left(K+\lambda K^{*}\right)}\right)+\lambda E^{*}\frac{\left(\lambda K+K^{*}\right)}{\left(K+\lambda K^{*}\right)}+E-\rho$$

Concerning the law of motion of the capital location, from equation (17):

$$\dot{s}_{K}=s_{K}\left(1-s_{K}\right)\left(g-g^{*}\right)$$

We then substitute equations (1) to (4) in order to find:

$$\dot{s}_{K}=s_{K}\left(1-s_{K}\right)\left(\frac{L_{I}a_{I}}{s_{K}}-\frac{L_{I}^{*}a_{I}^{*}}{1-s_{K}}\right)$$

Given the expressions for labour in the I-sector:

$$L_{I}=L+\pi K-E=L+\frac{E^{w}}{\sigma}Bs_{K}-E$$

$$L_{I}^{*}=LA^{*}+\frac{E^{w}}{\sigma}B^{*}\left(1-s_{K}\right)-E^{*}$$

We finally find:

$$\dot{s}_{K}=\left(\left(1-s_{K}\right)\left(L+\frac{E^{w}}{\sigma}Bs_{K}-E\right)A-s_{K}\left(L+\frac{E^{w}}{\sigma}B^{*}\left(1-s_{K}\right)-E^{*}\right)A^{*}\right)$$

4.8 Appendix B

The dynamic of our model is described by three differential equations. Indeed we have two Euler equations (one for each region) representing the evolution of expenditure and another equation representing the evolution of capital location:

$$\frac{\dot{E}}{E}=\frac{E^w}{\sigma}\left(AB-Bs_K-\lambda B^*\left(1-s_K\right)\frac{(\lambda K+K^*)}{(K+\lambda K^*)}\right)-L\left(1+\lambda\frac{(\lambda K+K^*)}{(K+\lambda K^*)}\right)+\lambda E^*\frac{(\lambda K+K^*)}{(K+\lambda K^*)}+E-\rho$$

$$\frac{\dot{E}^*}{E^*}=\frac{E^w}{\sigma}\left(A^*B^*-\lambda B^*\left(1-s_K\right)-Bs_K\frac{(\lambda K+K^*)}{(K+\lambda K^*)}\right)-L\left(1+\lambda\frac{(\lambda K+K^*)}{(K+\lambda K^*)}\right)+\lambda E^*\frac{(\lambda K+K^*)}{(K+\lambda K^*)}+E-\rho$$

$$\dot{s}_K=\left(\left(1-s_K\right)\left(L+\frac{E^w}{\sigma}Bs_K-E\right)A-s_K\left(L+\frac{E^w}{\sigma}B^*\left(1-s_K\right)-E^*\right)A^*\right)$$

Let's now assess the local stability of the system. By applying the Hartman-Grobman theorem we linearize the system around the two steady states (symmetric and core-periphery equilibria). The Jacobian matrix for the symmetric equilibrium is:

$$J_{sym}=\begin{bmatrix}\delta\lambda\Lambda+1 & \lambda-\delta\lambda\Lambda & 4\lambda E\delta\left(\Pi-\Lambda^2\right)\\ 1-\delta\lambda\Lambda & \lambda+\delta\lambda\Lambda & 4\lambda E\delta\left(\Lambda^2-\Pi\right)\\ \left(\delta\Lambda-1\right)\left(\frac{1+\lambda}{4}\right) & \left(1-\delta\Lambda\right)\left(\frac{1+\lambda}{4}\right) & 2E\delta\xi+\lambda\rho\end{bmatrix}$$

The Jacobian matrix for the core-periphery equilibrium is:

$$J_{cp}=\begin{bmatrix}1 & 1 & \delta\Psi\\ \delta\left(\phi-1\right)+1 & \delta\left(\frac{1-\phi}{\phi}\right)+1 & -\delta\Psi\\ 0 & 1 & \delta\Psi+\rho\end{bmatrix}$$

Where for the sake of simplicity we have defined:

$$\Psi=\left[E\left(\phi-1\right)+\frac{E^*\left(1-\phi\right)}{\phi}\right]$$

$$\xi=\left[\frac{2\phi-\lambda-\lambda\phi^2}{\left(1+\phi\right)^2}\right]$$

$$\Lambda=\frac{(1-\phi)}{(1+\phi)}$$

$$\Pi=\frac{(1-\lambda)}{(1+\lambda)}$$

$$\delta=\frac{\alpha}{\sigma}$$

Conclusions

The aim of this work is to deepen the analysis and to test the theoretical robustness of some of the more relevant statements of the New Economic Geography and Growth literature. We do that in the third chapter by introducing a CES utility function in the second-stage optimization problem, hence allowing for the expenditure shares in industrial goods to be endogenously determined; and in the fourth chapter, by assuming intersectoral and localized knowledge spillovers from the innovation sector to the service sector, hence showing that firms allocation affects regional real growth. These analytical departures from the standard NEG theory have several important implications from the dynamic pattern of equilibrium allocation of economic activities, the equilibrium growth perspectives and the policy recommendation.

From the equilibrium allocation of economic activities point of view, the introduction of endogenous expenditure shares in manifacture sheds light on a new force that we name as substitution effect. This force fosters agglomeration if the traditional and the modern commodities are good substitutes. In this case the core-periphery pattern is reached for lower degrees of market integration, and for some values of parameters it is reached whatever the level of trade costs. By contrast if the traditional and the modern commodities are poor substitutes agglomeration is reached for higher degrees of market integration.

From the equilibrium growth point of view the introduction of endogenous expenditure shares brings to the fact that even in absence of localised spillovers growth depends on the location of industry and on the degree of economic integration. In particular if manufacture and agriculture are good substitutes integration is growth enhancing. Otherwise if manufacture and agriculture are poor substitutes integration is growth harming: in this case, unlike the benchmark model, integration is welfare decreasing.

Also the introduction of intersectoral spillovers has important consequences on the equilibrium growth point of view. Indeed with intersectoral spillovers is possible to have different real growth rates between the two regions. In this case the workers remaining in the region where there is no longer manufacture/innovation face two negative welfare effects: a static loss due to the increase in trade costs paid on the manufacture varieties that have to be imported (present even in the standard case); a negative dynamic effect due to the fact that the real income growth is lower. Hence the growth differential led by agglomeration worsens the static loss suffered by the loosing region thus from a welfare point of view integration might be detrimental.

Turning to the policy implication of our work, a first recommendation we give is that policy makers should not blindly rely on standard NEGG models considered their intrinsecal analytical weakness. One example is the result stating that in case of localized intertemporal spillovers in innovation agglomeration is growth enhancing at a global level. As we have shown, in case of poor substitutability between the traditional and the modernd good or intersectoral spillovers between innovation and sector producing a non-tradable good agglomeration can be growth detrimental. A second recommendation we give is that policy makers should take into consideration the roles of the elasticity of substitution between the traditional and the innovative goods as well as the presence of intersectoral localized knowledge spillovers from the innovation sector to non-tradable sectors settled in the same region. At the best of our knowledge, these crucial elements have been neglected both from a theoretical and an empirical point of view.

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