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ABSTRACT

Local Social Capital and Geographical Mobility: A Theory*

In this paper, we attempt to understand the determinants of mobility by introducing the concept of *local* social capital. Investing in local ties is rational when workers anticipate that they will not move to another region. Reciprocally, once local social capital is accumulated, incentives to move are reduced. Our model illustrates several types of complementarity leading to multiple equilibria (a world of local social capital and low mobility vs. a world of low social capital and high propensity to move). It also shows that local social capital is systematically negative for mobility, and can be negative for employment, but some other types of social capital can actually raise employment.

JEL Classification: J2, J61, Z1

Keywords: European unemployment, geographical mobility, social capital

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Geographical mobility is one of the signs of a well-performing economy: it reveals the ability to cope with change and to reallocate factors of production where they are more efficient. Residential mobility differs widely across countries. The fraction of the 0-99 years old population having moved to the current residence within a year is small in Europe, around 5% according to estimates from the European Community Household Panel (ECHP hereafter). Further, it varies across countries: residential mobility is lowest in Southern European countries (2.8% in Spain, 2.7% in Portugal, 2.1% in Italy, 1.9% in Greece) and in countries such as Ireland and Austria (1.9 and 2.3 respectively) and highest in Scandinavian countries (7.0% in Sweden, 9% in Finland, 6.6% in Denmark) and Germany (6.8%). Regional mobility is also weak in Europe: while in the US, about 30% of individuals were born in a different state, in Europe, this number is typically around 20% for countries where regions have similar size as US states. In particular, it is 19.2% in Belgium, 12.7 in Portugal, 16.8 in Austria. It is slightly higher in Spain (23.5%) but Spain has smaller regions. On average in these 4 countries, the rate is 18.1%[1].

In this paper, we provide an explanation for low mobility in Europe, and why it differs across European countries, in particular why it is lower in the South and higher in the North of Europe. The explanation lies on geographical mobility costs. Our contribution is to associate mobility cost to a concept of *local* social capital, which affects the cost of moving. Local social capital will characterize the ties of agents to their region/area of origin and is therefore partly or fully depreciated upon mobility, leading to a decline in the welfare of movers.

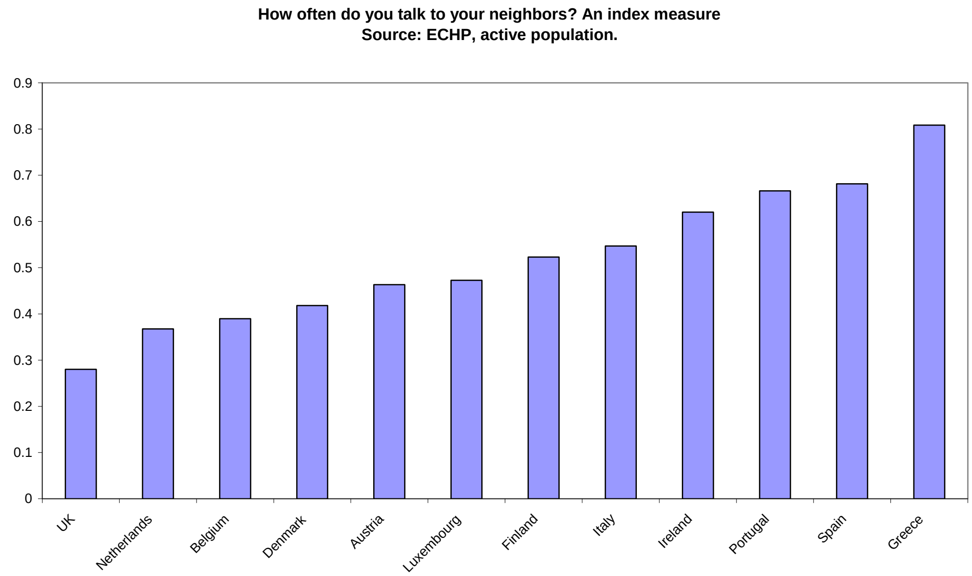
In the European Community Household Panel, social capital measures are derived from the three following questions:

- Are you a member of any club, such as a sport or entertainment club, a local or neighborhood group, a party etc...?
- How often do you talk to any of your neighbors?
- How often do you meet friends or relatives not living with you, whether here at home or elsewhere?

The answer to the first question is yes/no (and is attributed the value 1 or 0). The answer to the last two questions defines a frequency on a discrete support, as follows: 1. On most days;

[1] Here we reported only countries with large regions, thus excluding notably the UK, which has 46 regions, as opposed for instance to Belgium which has three regions. See Wasmer et al. 2005 for regional mobility figures.

Figure 1: Social capital in Europe: frequency of contacts with neighbors.



2. Once or twice a week; 3. Once or twice a month; 4. Less often than once a month; 5. Never. In order to simplify the exposition of the results, we build an index measure as follows:

$$Z_{i,t} = I\left[X_{i,t} = 1\right] + I\left[X_{i,t} = 2\right].\frac{2}{7} + I\left[X_{i,t} = 3\right].\frac{2}{30} + I\left[X_{i,t} = 4\right].\frac{1}{60} + I\left[X_{i,t} = 5\right].0,$$

where $Z_{i,t}$ is the index value for individual i at time t and $X_{i,t}$ the answer to the question. $I[.]$ is an indicator function that takes value 1 if the expression in brackets is true and 0 if it is wrong.

Inspection of Figures 1 to 3 shows that there is indeed a North-South divide in the *nature* of social capital: in the South of Europe (and in Ireland too), social capital seems to be more associated with family ties and having friends, and less so with clubs and association membership. The opposite holds in the North of Europe.

One way to summarize these differences is to argue that social capital is more local in the South, that is, more associated with a particular location. Presumably, the cost of moving to another region is higher for individuals having strong family and friendly ties. In the North of Europe, being part of clubs is instead much more frequent. To the extent that being a member of a club (such as a Scrabble or a chess league) is geographically general because club members can build new ties in another club in the new city, this helps to cope with mobility.

We have thus here an interesting explanation for mobility differences across European countries. In the companion paper (David et al. 2008), we proceed to a formal empirical analysis

Figure 2: Social capital in Europe: frequency of contacts with friends and relatives.

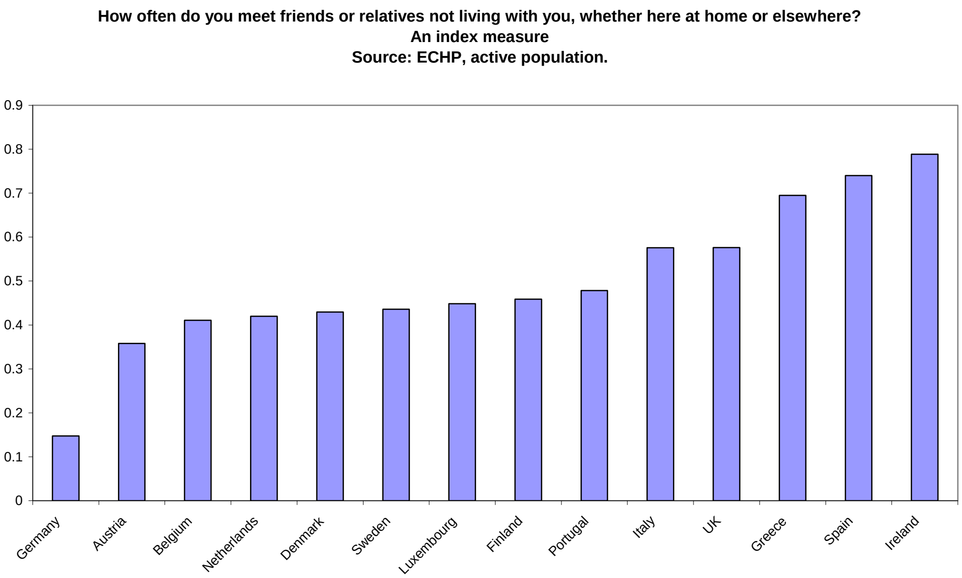
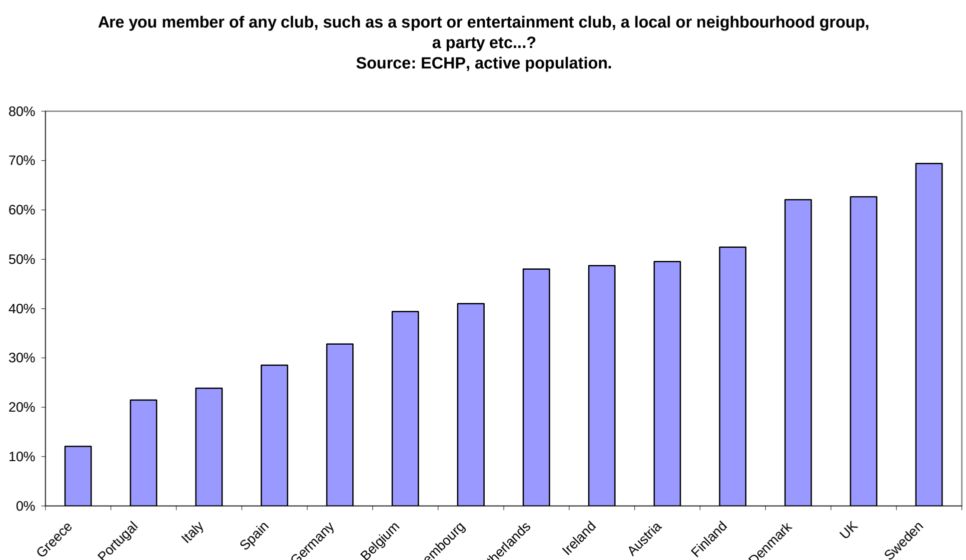


Figure 3: Social capital in Europe: club membership.



establishing a few stable relations in the data, notably

1. Individuals endowed with more local social capital as described by the variables "friend", "neighbor" or "club" are less likely to move to another region in the short-run.

2. Individuals endowed with more local social capital such as described by the variables friends or neighbors are more likely to become unemployed in the short-run.

3. In contrast, individuals who are members of a club are less likely to become unemployed in the short-run.

4. Workers in a region different from their birth region have a lower stock of social capital in all three dimensions measured ("friend", "neighbor" and "club").

Given 1 and 4, causality between mobility and social capital goes both ways: we have thus in principle both a channel for low mobility (the existence of local social capital) and a potential theory of multiple equilibria. These are possibilities that we need to formally explore. Further, their impact of social capital on unemployment is potentially ambiguous, as one could expect social capital to have a positive impact on employability. We need to determine when the positive effect dominate over the negative one.

This is the aim of this paper. We base our modelling strategy on recent works on social capital, surveyed in the next section. The closest paper is by Glaeser et al. (2002), who introduce an explicit theory of endogenous social capital characterized, in their work, as association membership. They notably argue that "*social capital declines with expected mobility*" and confirm the prediction with an expected probability score based on demographics. In a somewhat different context, Spilimbergo and Ubeda (2004a and b) argue that US Blacks workers are less mobile than Whites because of family ties (2004b) and successfully test this using the PSID survey.

As a matter of fact, our model emphasizes that the relation between local social capital and mobility has two causalities. On the one hand, the anticipation of mobility affects social capital investments, as in Glaeser et al. (2002). If agents perceive themselves as being strongly attached to a village, a township or a region, they will invest in local social capital, because the returns of those local ties are high. On the other hand, highly local social capital raises the cost of mobility and in turn reduces mobility. Finally, if individuals expect their friends to remain in the neighborhood as well, the returns to creating social ties are larger. This social externality creates multiple equilibria and thus potentially reinforces the low mobility of some regions or demographic groups. As a result of these two *self-reinforcing* causalities and this externality, within the European countries and regions of low mobility, local social capital is a binding factor: even strong economic incentives to migrate, such as regional unemployment differentials, are insufficient and individuals prefer to live on welfare and enjoy local social capital.

Our model illustrates the several types of complementarity leading to multiple equilibria (a world of local social capital and low mobility vs. a world of low social capital and high propensity to move). It also shows that local social capital is systematically negative for mobility, and can be negative for employment, but some other types of social capital can actually raise employment.

Needless to say, our paper is not the first one to involve geographical mobility among the determinants of unemployment. Another very close paper is the influential work by Bertola and Ichino (1996), among the very first to document the inability of European workers to move to more dynamic regions. This arose according to the authors, because of wage and income compression, lowering the returns from mobility. To understand the determinants and implications of low mobility, we do not focus on the returns of mobility, as in Bertola and Ichino (1996), even though this dimension will be present, but, in a dual perspective, on the costs of mobility. The novelty of our approach is first to give an explicit content to mobility costs, namely social capital, and to make it endongeous, while in many theoretical analyses including large portions of urban and regional economics, mobility costs are exogenous.

In **Section 1**, we will first review the ample literature on social capital and emphasize its implicit or explicit geographical dimensions, notably what we call localness. It is interesting to note that most works surveyed emphasize the positive role of social capital on labor market performance, while we tend to emphasize some negative channels. A counterexample is Bentolila et al. (2004), who focus more specifically on European countries, and emphasize the potential negative links between social capital and labor markets - in particular, the fact that jobs obtained through social networks tend to have a wage discount, distorting choices towards inefficiency.

In **Section 2**, we develop a theory embedding the mechanisms found in the data. We will characterize how various types of social capital have different impacts on mobility and unemployment rates. We start by defining social capital and notably assume it has two dimensions: a local one, i.e. a fraction is depreciated when an agent moves to another region, and a professional one, i.e. a fraction is depreciated when the agent is unemployed. Both depreciation rates matter in the mobility/job acceptance decisions. We first illustrate one side of the bilateral causality: local social capital reduces geographical mobility within a simple, tractable model of wage offers. We also find that social capital raises unemployment if the local dimension of social capital dominates over its professional dimensions, that is, *if depreciation of social capital is greater after a geographical move than after job loss*. The fact that the relative depreciation rates matters for the sign of the impact of social capital on unemployment justifies why we consider the two dimensions of social capital, since it helps to rationalize the empirical results.

Then, in **Section 3**, we explore the determinants of social capital. We find the existence of two local maxima in the net returns of local social capital, implying that observationally close individuals may behave very differently: some will not invest much in local social capital and will thus be more mobile and better employed, while others will invest more in local social capital and prefer to remain at the margin more locally unemployed. This is a first complementarity between immobility and social capital. We then allow agents to choose the type of social capital (local or professional) under constraint and find another complementarity: when the ex-ante probability of being unemployed is larger than the ex-ante probability of moving to another region, agents invest more in local social capital.

Section 4 presents an extension: employment protection is shown to induce local social investments and therefore to reduce mobility. The Appendix presents a second extension: the presence of a social externality is natural in this context. The more likely the friends of an individual are to remain in a geographical area, the higher the returns to investing in social skills. This leads to multiple equilibria and thus a third type of complementarity: a low mobility/high local social capital coexists with a high mobility/low social capital.

Our work is a first step to clarify the concept of local social capital and derive some important implications. It points out robustly to the fact that unemployment and low mobility are linked and that if low mobility is partly due to economic factors, it is also in a large part due to social factors over which policy makers have little control.

1 Social capital: a selective survey

Surveying the concept of social capital is beyond the scope of this work: definitions of social capital are numerous. The aim of this section is instead to briefly review the existing literature on social capital under the concept of "localness" and argue that the geographical dimension of social capital is often implicit. This will serve as a basis for the main assumption made in this paper. In the second part of the section, we also survey the use of social capital (local or not) in the labor literature.

1.1 Localness of social capital

In their review of the literature on social capital, Durlauf and Fafchamps (2004) point to its origin with the seminal works of Loury (1977) and later Coleman (1988). They mainly distinguish two different definitions of social capital. First, there are the "outcome-oriented" definitions (see Coleman (1990), Putnam *et al.*(1993), Fukuyama (1995)). These definitions insist on the importance of group externalities caused by the existence of social capital. They are more concerned with the consequences of the existence of social capital than with its nature. Second, there are definitions focussing on the nature of the relations and the interdependence of individuals embodied in social capital (see Putnam (2000), Bowles and Gintis (2002) and Lin (2001)). Among different authors, Durlauf and Fafchamps identify two main ideas for our purpose: first, social capital has positive exernalities to group members ; second, the externalities are associated with "*shared trust, norms and values*"[2].

In what follows, we do not necessarily want to follow the distinction "function vs. nature", as the localness of social capital is clearly one aspect of its nature, but at the same time the consequences of social capital notably in terms of externality and spillover are often local too.

In the "nature category of works", and even before the term "social capital" was introduced, one can find studies on related issues. For instance, Jacobs' (1961) work on large U.S. cities underlined how implicit rules matter in neighborhoods. The knowledge of those implicit rules allows for building trust. She notably showed that social ties are stronger in older neighborhoods. This work is one of the earliest in which the geographical dimension of social capital is underlined: social ties as defined here cannot be moved from one place to another.

Coleman (1990) identifies different forms of social capital: 1) obligations and expectations that depend on the trustworthiness of the social environment; 2) capability of information; and 3) norms accompanied by effective actions. Some have a local dimension, others do not (think, for instance, of norms that inhibit crime in a particular neighborhood or norms in a community that support and provide rewards for high achievement in school, etc.). On the destruction side, Coleman (1990, p. 321) states that social capital can depreciate if there is no investment to

[2] Durlauf and Fafchamps (2004) p. 5

renew it. "*Social relationships die out if not maintained; expectations and obligations wither over time; and norms depend on regular communication*". Although there is no explicit spatial dimension here, a simple cost-benefit analysis suggests that being further away (geographically) raises maintenance cost of social capital and is associated with lower stock in equilibreium.

Our model will be in part inspired by the "economic approach" to social capital of Glaeser et al. (2002). They propose a theoretical framework in which they treat social capital as an individual rather than a community characteristic. The utility flows are determined by the individual amount of social capital and the aggregate level. Their main findings related to our concerns are the following: 1) Investment in social capital, like other investments, changes over the life cycle. It tends first to increase and then to decrease with age. 2) In order to test the effect of social capital on mobility, they build an expected mobility measure and find a strong negative correlation between social capital and this measure. 3) Home-ownership increases the investment in social capital. In particular, they predict more investment in local social capital. On the contrary, Winters et al. (2001), (pp. 181-182.), analyzing the effect of networks in the choice of migration from Mexico to the United States, find that there could be a positive link between social capital and migration, since networks provide information on where to move.

Kumar and Matsusoka (2004) introduce an interesting distinction between two types of social capital: village capital and market capital. The former "*consists of social networks, especially kinship, patron-client relations, and informal agreements within small groups of people that are enforced by reciprocity and social sanctions*" which we would call "local" in our labor market context. They provide several examples to explain why localized economies accumulate this kind of social capital and why it improves efficiency.[3]

Finally, it is worth noting, however, that social capital is not exclusively local, and can instead be built in order to promote mobility. A very good example is the development of Rotary Clubs in the beginning of the 20th century in the US, which was orginally designed to reproduce the social environment of professionals having moved from one place to the other, as a substitute to local social capital precisely. [4]

[3] The other concept, "*Market capital, consists of knowledge that facilitates transactions between potential strangers and parties who are unlikely to transact again in the future*". Village capital is best when economic activity is primarily local and market capital is essential for transactions between strangers and for the development of the economy.

[4] We thank Robert Putnam for this relevant example. On the web page of the Rotary Club, it is indede stated that "*The world's first service club, the Rotary Club of Chicago, Illinois, USA, was formed on 23 February 1905 by Paul P. Harris, an attorney who wished to recapture in a professional club the same friendly spirit he had felt*

1.2 Social capital in labor markets

In the labor market, social capital is usually considered to improve economic performance. The main channel in the literature is that social capital conveys information and leads to an improvement of the quality of the match between employers and employees, leading authors to find a positive effect on productivity and wages. For instance, Granovetter (1995) studies how workers find their jobs. In his study, social capital improves overall welfare through the creation of an efficient network made of social ties that allows for better expectations.

The idea that social capital improves individuals' perspectives on the labor market has been investigated more recently. See, for instance, Calvó-Armengol (2004) and Calvó-Armengol and Jackson (2004, 2006). In these works, there is a distinction between weak and strong ties, following Granovetter's view. Calvó-Armengol et Jackson (2004) propose a theoretical framework where they assume that the probability of finding a new job (for an unemployed agent) or to find a better job (for already employed agents) depends on the social network of the agent. This model explains both the lower rate of employment and the lower wages of individuals endowed with a weaker social network. In Calvó-Armengol (2004), network structures are endogenously created by workers' strategic actions. He shows that networks with different structures may induce different aggregate unemployment levels.

Montgomery (1991) studies the importance of the referrals to outcomes on the labor market. He assumes that there are both low- and high-ability workers and that ability is not observable by firms before the match. Ties are more likely to link different types of workers among themselves (high- with high-ability workers and "low with low"). Before accepting a job, a worker compares the various job offers and accepts the highest-paying job. This model highlights the importance of referrals and helps to explain why well-connected workers get higher wages and why firms hiring workers through referrals might earn higher profits.

In their survey analyzing the network effects on labor-market outcomes and inequality, Ioannides and Loury (2004) emphasize seven stylized facts based on both, the recent sociological and economic literature. First, there is a widespread use of friends, relatives and acquaintances during job searches. Second, this use varies by location and demographic characteristics. Third, it is generally productive to use one's network to get a job. The fourth and fifth stylized facts

in the small towns of his youth. The name "Rotary" derived from the early practice of rotating meetings among members' offices."

concern the productivity of the job search, which happens to depend on demographic characteristics. The sixth fact relates to the increasing use of the Internet. Finally, they observe differences across countries in the use of personal contacts by firms and workers.

Most of these works emphasize the positive links between social capital and economic performance, while our work emphasizes potentially negative channels. In that, we follow Bentolila et al. (2004), who investigate the possible detrimental effects social capital may have on labor-market outcomes. Using ECHP data, as in our paper, they focus on the impact of social capital on occupational choices. They argue (page 2) that "*on average, jobs found through social contacts are obtained more quickly but also pay lower wages, since at least some of them are filled by workers who sacrificed their productive advantage in order to get a job more easily*". They find a wage discount of 3% to 5 % in Europe and the US for jobs obtained through social contacts.

Finally, Belot and Ermisch (2006) share similar ideas to ours. They show from the British Household Panel Study that geographical proximity of friends matters for mobility decisions. Compared to our data, their dataset has the interesting characteristic that it allows to explore two dimensions of the strength of social ties: location of the closest friends and frequency of contacts. Their results emphasize the importance of the first factor. While we are also interested in the link between local ties and mobility decisions, both our theoretical model and empirical findings in Quentin et al. (2008) highlight several complementarities between the two with common consequences for unemployment and contribute to the debate in Europe.

1.3 Our concept of social capital

At this stage of the paper, there are many definitions of social capital, and we need to define it more precisely to serve our purpose: to link it with geographical mobility and employment decisions. The key concept here is the *localness* of social capital. For that, let us think of an individual living in a region, say A. Assume she is endowed with S units of social capital. Once she leaves region A, her social capital is depreciated: she only retains a fraction of it. Let us denote by δ_λ the depreciation rate, which describes the degree of localness of social capital. We may think for instance that by leaving the native region, she looses δ_λ of her friends, or meets her relative less frequently.

As argued in introduction, social capital is also to some extent *professional*, which is a second dimension of social capital useful to consider. Indeed, this dimension must have a first-order impact on job acceptance decisions, in the sense that, upon loosing one's job, one may loose a few social connections too. Let us denote by $1 - \delta_\pi$ the fraction retained by an individual after job loss, and thus δ_π is the associated depreciation rate.

The set of parameters $(\delta_\lambda, \delta_\pi)$ allows us to describe various types of social capital, as follows:

- $\delta_\lambda = 1$ and $\delta_\pi = 1$ would correspond to being a member of a local and professional association (e.g. the association of textile engineers in a given region, e.g. the North of France)

- $\delta_\lambda = 1$ and $\delta_\pi = 0$ would correspond to being a member of a local sport club (e.g. a local soccer club) or having friends in one's neighborhood.

- $\delta_\lambda = 0$ and $\delta_\pi = 0$ would correspond to being a member of a country-wide association (e.g. Scrabble, chess)

- $\delta_\lambda = 0$ and $\delta_\pi = 1$ would correspond to being a member of a country-wide economic association (such as the American Economic Association).

Although it may not be immediately clear why we focus on these two dimensions of social capital, we will see in the theory part that this is a necessary distinction to rationalize the empirical results, notably the effect of social capital on the unemployment probability.[5]

Definition. *Social capital is said to be local if $\delta_\lambda > \delta_\pi$, that is if more is lost from a regional move than from job loss.*

Our distinction between local and professional capital can easily be compared to that of weak and strong ties. In Calvó-Armengól et al. (2007), strong ties is seen as linking "*members of the same family or very close friends*" and weak ties as "*a transitory social encounter between two persons*". However here we will have a definition exclusively based on the depreciation of social capital, to simplify our analysis.

[5] We could have simplified the analysis and set right away $\delta_\pi = 0$, but we decided to keep the general case, first to rationalize some empirical findinds, second because employment decisions are clearly affected by the gain or loss of social capital: deciding to reject a job offer and remain long-term unemployed can lead to such a social capital depreciation that it becomes in principle a key element of the decision. Hence, we believe that the general case where δ_π and δ_λ take any arbitrary value between 0 and 1 is useful. In addition, Section 3.2 presents an extension where individuals choose not only S, but also the fractions δ_λ and δ_π given their expectation about wage offers and mobility.

2 Model

2.1 Setup

To simplify the setup, we will first think of social capital in a reduced form approach as *simply raising utility of individuals*. Indeed, there are several channels through which more social capital raises ex-ante utility, such as insurance, information flows or complementarity with the consumption of leisure. Having a fully developed model along these dimensions is beyond the task of the theory part here, since we are already focussing on other dimensions, like the localness of social capital and mobility decisions.

We consider a typical worker living two periods. Jobs last one period to simplify the exposition, and this assumption is relaxed in the extension Section. There are two regions. Without loss of generality, we assume the worker is born in region A, lives and works there in period 1. She may eventually leave region A to go to region B in second period or stay in region A. Period 2 is discounted with a factor $\beta < 1$.

At the end of period 1, the worker is endowed with S units of social capital. S will be made endogenous later on but at this stage, it is useful to consider it as given. Social capital is partly local, in the sense that a mover to region B would enjoy only a fraction $(1-\delta_\lambda)S$ of social capital. Similarly, a fraction $0 \leq 1-\delta_\pi \leq 1$ of social capital is retained if the worker is laid off. We assume that social capital increases utility only in second period, as a reduced form. Let Ω_2 be the income of the individual in second period. To simplify, we assume that utility in second period U_2 is

$$U_2 = \begin{cases} \Omega_2 + S & \text{if the worker is employed in region A} \\ \Omega_2 + (1-\delta_\pi)S & \text{if the worker is non-employed in region A} \\ \Omega_2 + (1-\delta_\lambda)S & \text{if the worker is employed in region B} \\ \Omega_2 + (1-\delta_\lambda)(1-\delta_\pi)S & \text{if the worker is non-employed in region B} \end{cases}.$$

The labor market is a standard partial equilibrium search set-up. All jobs last only one period, so that all individuals start period 2 in having to prospect for a job. We relax this assumption in Appendix and investigate the role of more stable employment relationships (lasting more than one period) and notably the role of employment protection.

In the beginning of the second period, workers receive one job offer with a wage w from a cumulated distribution F in region A and one job offer with a wage w^* from a cumulated distribution G in region B (f and g are the associated densities). The random draws are uncorrelated. We denote by $\overline{w}$ the upper support of those distributions.[6] In a world where all regions are symmetric and have the same labor market conditions, one may think that $G > F$ (first order stochastic dominance) to reflect that workers have more local contacts and thus receive better local offers. For instance, an interesting rationalization is that workers receive multiple independent offers in quantity n and p with $n > p$ from a common distribution F_0: one can easily show, in such a case, that the expected value of the wage is for instance $\int_0^{\overline{w}} wd(F_0^n(w))$ or alternatively that $F = F_0^n$ and $G = F_0^p$.[7] However, we have also in mind to reproduce the intuition that some regions are depressed and other regions are booming, in which case we expect the opposite: many good offers in region B, hence G small for a large part of the support of the wage distribution, and few good offers in region A, hence F large on the main part of the support.

Finally, if non-employed, we assume that workers receive an income $\Omega_2 = b$ interpreted as unemployment benefits or leisure independent of social capital.

2.2 Workers' program

In second period, there are four possible choices: staying in the home region and remaining unemployed ; moving and remaining unemployed ; staying and accepting the local wage offer ; moving and accepting the foreign job offer. We can discard the second possibility, given that $U_2 = b+(1-\delta_\lambda)(1-\delta_\pi)S$ is always lower than $b+(1-\delta_\pi)S$ if the individual remains unemployed in the home region. The decision set is thus summarized by

$$U_2(S) = Max\left\{b + (1 - \delta_\pi)S, w + S, w^* + (1 - \delta_\lambda)S\right\}, \tag{1}$$

where the max operator reflects the optimal mobility/job acceptance decisions, which are the joint decision explored next Section. Offers in and out the region occur simultaneously and so are decisions by the agent to move or to stay and to accept a job or remain unemployed. See notably the Appendix for the decision tree of the agent: the worker compares her (best) local offer w, her best foreign offer w^* and her outside option b, as indicated in equation (1).

At this stage, there are two useful notations we can introduce: the reservation wage for an offer in region A is defined as

$$w^r = b - \delta_\pi S. \tag{2}$$

[6] If they are different, we simply extend the c.d.f beyond its support. We also assume that the lower bound of the support is 0 in both case.

[7] See Lemma A1, Theorem A2 and Corollary A3 in Theory Appendix.

This is the local wage making the agent indifferent between accepting a job or rejecting the offer. This is increasing in b and decreasing in social capital, unless social capital is totally non-professional ($\delta_\pi = 0$): the worker has more to loose in rejecting a job offer if this reduces its utility by the loss of social capital $\delta_\pi S$. A higher S raises the acceptance rate in region A except in the singular case $\delta_\pi = 0$. This offers the possibility of a positive impact of S on employment.

Symmetrically, the reservation wage for an offer in region B would be

$$w^{*r} = b + (\delta_\lambda - \delta_\pi)S. \tag{3}$$

One can notably see that when social capital is local (recall that it was defined as local if $\delta_\lambda > \delta_\pi$), more social capital raises w^{*r} and thus reduces the acceptance rate of offers and consequently it reduces geographical mobility. Again the idea is simple, in accepting a job in region B and living one's friends, the individual faces a depreciation $\delta_\lambda S$, compensated by the no-depreciation of $\delta_\pi S$. Hence, the trade-off. We have here a mechanism for either a positive or a negative impact of S on unemployment, depending on localness of social capital.

In words, to the extent that social capital is local, workers are potentially marginally more immobile. To the extent that most offers comes from other regions than where workers currently live, as has been the case in several high unemployment regions (Bertola and Ichino, 1996), we have here a channel for the persistence of high unemployment.

Note here, that the decisions to accept and to move are simultaneous, these intuitions, albeit correct, must be studied in the more complex setup where all offers arrive simultaneously. This is defined in the next few lines, where the notations w^r and w^{r*} remain useful.

2.3 Geographical mobility and social capital

The ex-ante probability of moving is denoted by P_m. It depends on the value of w^* but also on the value of w. Remember that the draws in F and G are not correlated, for simplicity. Formally, the Appendix determines that

$$P_m = \int_{w^{r*}}^{\overline{w}} F(z - \delta_\lambda S) g(z)\, dz. \tag{4}$$

For a worker to be mobile, it requires a wage offer in region B above its reservatation wage w^{r*} (hence the integral between w^{r*} and the upper support of wage offers) and a local wage offer sufficiently low compared to the current offer z net of depreciated social capital if the worker moves (hence the term $F(z - \delta_\lambda S)$ representing the fraction of such low local offers).

It is then informative to examine how this probability varies with S. We obtain:

$$\frac{dP_m}{dS} = (\delta_\pi - \delta_\lambda)F(w^r)g(w^{*r}) - \delta_\lambda \int_{w^{*r}}^{\overline{w}} f(z - \delta_\lambda S)g(z)dz. \tag{5}$$

The interpretation is easy, and is the sum of two effects conveniently corresponding to the two terms in equation (5). The second term is the easiest to interpret. Except in the extreme case $\delta_\lambda = 0$ where social capital has no local dimension, it is always negative: a higher S means a higher loss of social capital in case of geographical mobility and thus reduces the number of acceptable offers in region B.

The first effect is more subtle. To understand it, imagine a marginal worker receiving a local offer below w^r in region A and a marginal offer w^{*r}in region B. She is indifferent between different options (moving or remaining unemployed. We know that she looses δ_π of social capital if she rejects both offers, and δ_λ of social capital if she accepts the offer in region B. So, giving her one more unit of social capital makes her more likely at the margin to remain in region A if the loss δ_λ is larger than the loss δ_π, e.g. $\delta_\pi - \delta_\lambda < 0$.

Proposition 1. *Effect of social capital on the mobility rate.*

i) A sufficient condition for mobility to decline *with S is that $\delta_\lambda > \delta_\pi$ i.e. in the case of (relatively more) local social capital; ii) When $\delta_\lambda > \delta_\pi$, w^{*r} increases to $\overline{w}$ (possibly equal to $+\infty$) as $S \to +\infty$ and thus the mobility rate goes to zero ; iii) A sufficient condition for mobility to* increase *with S is that $\delta_\lambda = 0$ and $\delta_\pi > 0$ (non local but professional social capital).*

The first part of the proposition corresponds to the case where social capital is more depreciated when the worker moves than when she is non-employed, which we believe characterizes well social capital such as friendship or neighborhood relations. As social capital becomes larger, incentives to move disappear. The last part of the proposition corresponds to the case when social capital is not local and is to some extent a professional one.

2.4 Employment, unemployment and social capital

The model also suggests various other relations between the employment status and social capital. We have, notably the probability of being unemployed is

$$P_u = F(w^r)G(w^{*r}). \tag{6}$$

The interpretation of (6) is easy: workers are unemployed if they receive two offers below their reservation wage. The impact of social capital is thus straightforward: we have (see also Appendix)

$$\frac{dP_u}{dS} = -\delta_\pi f(w^r)G(w^{*r}) + F(w^r)g(w^{*r})(\delta_\lambda - \delta_\pi). \tag{7}$$

We thus have:

Proposition 2. *Effect of S on unemployment.*

i) *A sufficient condition for social capital to raise unemployment is $\delta_\pi = 0$; ii) Another condition is that* G is small and F is large at the thresholds w^{*r} and w^r and that social capital is local $\delta_\lambda > \delta_\pi$; iii) *When $\delta_\pi > 0$, $w^r \to 0$ when $S \to +\infty$ and thus the unemployment rate goes to zero ; iv) A sufficient condition for social capital to reduce unemployment instead is $\delta_\pi > \delta_\lambda$, i.e. when social capital is more professional than local; v) In the general case, the effect is ambiguous.*

The first part states that, as argued above, social capital moderates wage claims if it depreciates upon unemployment. When $\delta_\pi = 0$, the only impact of social capital is that it reduces mobility due to localness. When G is large and F is small around the thresholds, this means that there are few good offers in region B and many good offers in region A: conditions are gathered for the localness effect to dominate over the effect of professional depreciation of social capital. The other parts of the proposition are derived from the same logic.

Finally, the probability of finding a job in the local region is

$$P_w = \int_{w^r}^{\overline{w}} G(z + \delta_\lambda S) f(z)\, dz. \tag{8}$$

The interpretation is similar to that of the probability of moving: for a worker to find a local job, the wage must be above the local reservation wage (hence the integral between w^r and the upper support of the distribution of wages) and the wage offers in region B must be low compared to the local wage offer given the depreciation of local social capital in case of a move to B (hence the term $G(z + \delta_\lambda S)$ representing the fraction of such low offers). In addition, we have:

Proposition 3. *The local employment probability is always increased by social capital except if $\delta_\lambda = \delta_\pi = 0$ in which case the probability is unaffected by S.*

Indeed,

$$\frac{dP_w}{dS} = \delta_\pi G(w^{*r})f(w^r) + \delta_\lambda \int_{w^r}^{\overline{w}} g(w + \delta_\lambda S)f(w)dw.$$

As before, the interpretation is easy: the first term represents the effect of one additional unit of social capital for a worker receiving an offer w^r and with an offer w^* below w^{*r}: she accepts the local offer all the more than her social capital is depreciated. The second term is zero if $\delta_\lambda = 0$ and positive otherwise: it reflects the supplementary gain from accepting a local offer when being away in region B depreciates social capital. When $\delta_\pi = \delta_\lambda = 0$, S is just scaling up utility but does not affect the arbitrage of worker between the different options.

2.5 Partial conclusion

We have provided a relatively simple and flexible model leading to the predictions summarized in Table 1. Columns 2 and 3 correspond to the case of local social capital. In that case, social capital reduces mobility with either ambiguous or positive effects on unemployment. Local employment is raised in each case, as workers accept fewer outside offers. One would thus expect, *ceteris paribus*, that, when social capital is exogenous, the unemployment rate is higher in regions with more "local social capital", and that the marginal effect of social capital is larger if there are few good offers locally. It is also interesting to note that the effect of social capital on unemployment is potentially both positive or negative, depending on the nature of social capital: when social capital is mostly professional, unemployment probability is decreased: the effect works through a reduction in the reservation wage, as individuals have more to loose to turn down a job offer.

Let us now make S endogenous and explore its determinants.

3 Optimal social capital

Thanks to the assumption that jobs last one period, the decision to invest in social capital in first period is independent from the activity status (employed, non-employed) in first period. We can thus describe the decisions recursively, in two steps. In the second period, workers take S as predetermined and, after collecting offers, decide whether to accept local offers or foreign offers. In first period, they anticipate their decisions in second period and decide accordingly how much to invest in social capital. Before period 1, workers are assumed to be attached to region A and thus immobile: the decisions at this stage are not relevant for the next steps and we ignore them.

In first period, workers maximize U_1 defined as ex-ante first period utility, which is given by:

$$U_1 = \max_S \{\Omega_1 - C(S) + \beta EU_2(S)\}, \tag{9}$$

where β is a discount factor and the cost of investing in social capital S is $C(S)$ with $C'(S) > 0$, $C''(S) > 0$. The key issue is thus to determine the quantity

$$EU_2 = \int_0^{\bar{w}} \int_0^{\bar{w}} \max\{b + 1 - \delta_\pi S, w + S, w^* + (1 - \delta_\lambda)S\} \, dF(w)dG(w^*). \tag{10}$$

It is a relatively complex derivation but it can be simplified after a few variable changes and integration by parts. Indeed, we show in Appendix that the expected utility of agents given optimal choices is given by

$$EU_2 = \bar{w} + S - \int_{w^r}^{\bar{w}} G(z + \delta_\lambda S)F(z)dz. \tag{11}$$

In the above formula, the impact of social capital on the expected utility is threefold. There is a positive direct effect on utility. The second and third effects, through the integral term, are actually negative (recall that w^r is decreasing in S whenever $\delta_\pi > 0$). As we will show, these two last effects arise from the fact that social capital reduces mobility and job acceptance: we can link the marginal effect of S to the various probabilities calculated above. This is done in the next Sub-Section.

3.1 Choice of the level of S

Let us first make the assumption that social capital is mostly local, i.e. depreciates more after a regional move than after job loss. This is from now on the benchmark case. In equations:

Assumption 1. $\delta_\lambda > \delta_\pi > 0$.

This yields some useful properties of $\frac{dEU_2}{dS}$.

Lemma 1. *Properties of* dEU_2/dS.

i) $\frac{dEU_2}{dS} = 1 - \delta_\pi P_u - \delta_\lambda P_m > 0$; *ii) under Assumption 1, we have* $dEU_2/dS \to 1$ *when* $S \to +\infty$ *; iii)* d^2EU_2/dS^2 *is strictly positive so that* dEU_2/dS *strictly increases, except if either* $\delta_\pi = \delta_\lambda = 0$ *or* $f = g = 0$. *In these two cases, the second derivative is zero.*

From equation (11), we have the marginal effect of S which can conveniently be rewritten, using (8) and (6). The return to social capital is therefore always strictly positive. A marginal increase in S raise utility by 1, minus the probability of moving P_m (in which case δ_λ is depreciated) minus the probability of remaining unemployed locally (in which case a fraction δ_π of social capital is depreciated). In the neutral case" $\delta_\pi = \delta_\lambda = 0$, the marginal return to social capital is constant, equal to 1.

The second point comes from the limits of P_m and P_u in Propositions 1 and 2. The last point is shown in Appendix. The interpretation is simple: except in the case of "neutral" social capital or with degenerate distribution of wage offers, utility is convex in social capital. Convexity arises when distributions are not degenerate because, by raising social capital, the agent can afford to reject more offers and thus optimize its mobility/acceptance strategy (in other words, she is better off because she has greater outside options).[8]

Denote by $\widehat{S}$ a level of S satisfying the first-order condition. We have

$$C'(\widehat{S}) = \beta(1 - \delta_\pi P_u - \delta_\lambda P_m), \tag{12}$$

where P_u and P_m depend on $\widehat{S}$ too. Equation (12) may be satisfied for more than one value of $\widehat{S}$. To see this, one can draw the left hand side of equation (12) which is an increasing function of S and the right hand side which is convex. The two curves can intersect several times - or not at all. We only know that for large values of S, the right-hand side converges to 1, while, with a quadratic cost function, the left hand side, the marginal cost, goes to infinity, so that utility decreases after the last intersection which is thus a minimum. We represent utility in Figure 4 in one of the "multiple intersections" case.

In such a case, there is usually a well defined global maximum (either the first or the second maximum) and the agent optimally chooses one or the other. The point we want to make is that a small deviation between two individuals, say, because they marginally differ in their cost functions, may lead them to behave observationally very differently. In Figure 4, the agent would choose a low degree of local social capital and hence would be ex-ante relatively mobile. Imagine now that the marginal cost of investing is decreased by a tiny amount: then, the bimodal curve changes, it is like a counter-clockwise rotation (due to $C(S)$ being relatively more reduced

[8] The quantity d^3EU_2/dS^3 is calculated in Appendix. As a special case, when both f' and g' are uniformally negative on their support, which is a widely used property in contract theory and known as the CRDC (concavity of the distribution function condition), it is possible to sign the four terms summing up to d^3EU_2/dS^3 but three are positive and one is negative, so we cannot sign this quantity in general.

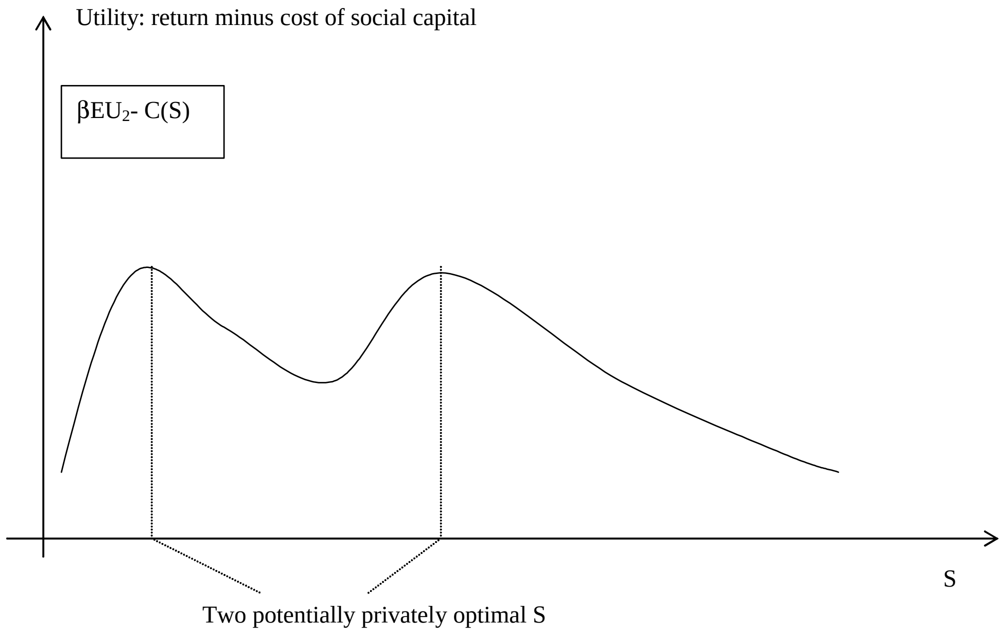


Figure 4: Net utility as a function of social capital: case of multiple extrema

for larger values of S) and thus the second local maximum becomes a global maximum. This individual is thus more likely to be immobile and invest a lot more in social capital.

Hence, we have a first complementarity between local social capital and mobility.

Complementarity property 1. *When the localness of social capital is exogenous to workers choice (that is, δ_π and δ_λ are given), local social capital is associated, in the cross section of workers, with low mobility rate, with large differences across individuals even though they have similar preferences.*

Now, in a maximum of utility, we have an additional property: the convexity of costs C with respect to S implies that the investment in social capital will be larger when the right hand side of (12) is higher, i.e. when both risks of depreciation P_u and P_m (i.e. unemployment and mobility) are lower and when the rates of depreciation is lower (that is, δ_π and δ_λ are lower). Again, if $\delta_\pi = 0$, the risk of unemployment plays no role on the choice of S, while the closer δ_λ to 0, the lower the impact of P_m on the choice of S.

Complementarity property 2. *For a given optimal choice of S and under assumption that $\delta_\lambda > \delta_\pi$, if social capital is higher, the expected mobility rate is lower. In turn, any exogenous* *decrease in expected mobility increases the optimal level of social capital.*

3.2 Choice of the composition of S

A natural extension is to consider how agents could *choose* their type of social capital: local or professional or a combination of both. We will see that this reinforces, if anything, the complementarity property 1. Indeed, assume now that agents can trade-off the two types of social capital in choosing δ_π and δ_λ given a constraint, say

$$\delta_\pi + \delta_\lambda \geq 1 + a, \tag{13}$$

where $0 < a < 1$ is a technologically-constrained parameter. They could for instance choose $\delta_\pi = a$ and $\delta_\lambda = 1$ (mostly local social capital) or $\delta_\pi = 1$ and $\delta_\lambda = a$ (mostly professional capital), or any interior combination. Simultaneously, they chose the total amount of S. In other words, they have a control, albeit limited by equation (13), on the type of social capital. The constraint of equation (13) reflects the fact that there is no free-lunch: social capital must depreciate in one dimension or the other.[9] In this case, the program of the agents rewrites:

$$\max_{S,\ \delta_\pi, \delta_\lambda} \{\Omega_1 - C(S, \delta_\pi, \delta_\lambda) + \beta E U_2(S, \delta_\pi, \delta_\lambda)\}$$
$$s.t. \text{ equation (13).}$$

where the cost function now explicitly depends on the chosen depreciation rates: a lower depreciation rate comes at a positive marginal cost, or

$$\partial C / \partial \delta_i < 0 \text{ for } i = \lambda, \pi.$$

Using equations (11), (2) and substituting δ_π by $1 + a - \delta_\lambda$, we then have the following program:

$$\max_{S,\ \delta_\lambda} \left\{ -C(S, 1 + b - \delta_\lambda, \delta_\lambda) + \beta \left(S - \int_{b-(1+a-\delta_\lambda)S}^{\bar{w}} G(z + \delta_\lambda S) F(z) dz \right) \right\}.$$

[9] An alternative modelling choice would be to fix $\delta_\lambda > 0$ and $\delta_\pi > 0$ as exogenous parameters, i.e. technologically given, but dissociate two types of social capital S_l (local social capital) and S_p (professional social capital) and let the agent invest optimally in these two dimensions. The program of the agent would then be $\underset{S_l, S_p}{\text{Max}} \{\Omega_1 - C(S_l, S_p) + \beta E U_2(S_l, S_p)\}$. We don't learn much more from this exercise compared to the current modelling choice so we do not push this possibility any further.

The first order condition on S is the same as in (12), while now, the optimal choice of δ_λ implies, after a variable change:

$$\frac{dC}{d\delta_\lambda} = \frac{\partial C}{\partial \delta_\pi}\frac{\partial \delta_\pi}{\partial \delta_\lambda} + \frac{\partial C}{\partial \delta_\lambda} = \beta S(P_u - P_m).$$

where $\frac{\partial \delta_\pi}{\partial \delta_\lambda} = -1$ along the constraint (13). A simple cost function would be $C(S, \delta_\pi + \delta_\lambda)$ with therefore perfect substituability between the choice of depreciation rates. Thus, the left-hand side $\frac{-\partial C}{\partial \delta_\lambda} + \frac{\partial C}{\partial \delta_\pi} \equiv 0$. This leads to particularly simple solutions: an interior solution and two corner solutions:

- in an interior solution, the agent chooses δ_λ at an optimal point where $P_m = P_u$
- if there is no interior solution, i.e. for instance $P_m > P_u$, the agents chooses $\delta_\lambda = a$ and $\delta_\pi = 1$
- if there is no interior solution, i.e. for instance $P_u > P_m$, the agents chooses $\delta_\lambda = 1$ and $\delta_\pi = a$

Let us first discuss the interior solution. The interpretation is as follows: the probability to be unemployed is the probability to loose a marginal unit of professional (π) social capital ; it has to be equal to the probability to move, that is to loose a marginal unit of local (λ) social capital. Note however that the agent may not always be able to implement such a solution, because it depends in large part on the external distributions of offers. If for instance, there are excellent offers in region B, the agent will move with high probability and remain unemployed with low probability. In other words, the agent has only a limited control on P_u and P_m.

When the agent selects one of the two corner solutions, the intuition is clear too. For instance, the second corner solution occurs in a low mobility/high unemployment world where $P_u > P_m$: since expected mobility is low, the agents will maximize the localness of their social capital ($\delta_\lambda = 1$) in order to reduce the depreciation of professional social capital. This will thus further reinforce immobility. Hence, we uncover a new complementarity property:

Complementarity property 3. *When agents can partly control the localness of social capital, they choose preferentially local capital when they expect a low mobility rate compared to the unemployment rate ; the opposite occurs when agents anticipate high mobility compared to the unemployment rate: they try to minimize the depreciation of local social capital.*

4 Extension: employment protection

An interesting extension, in the European context, is the effect of employment protection legislation. By raising the expected duration of jobs, it induces investments in all sort of specific capital, such as job specific skills, sector specific skills, housing and in our more specific case, local social capital. We will thus explore this mechanism here.

Assume that workers in first period may remain employed at the end of period 1 with probability τ. The previous analysis was thus simply the case $\tau = 0$. Here, τ can be thought as an index of employment protection. How does the previous analysis carry through? Well, the employment status of employees now matter, and there are two cases to consider.

Unemployed workers in period 1 are not affected, and make the same optimal choice $\widehat{S}$ as the one determined before in the first order condition (12). Consider now an employee with wage w_1 in first period. In the beginning of the second period, she may loose her job with probability $1-\tau$ and then face the same choice as before: draw a set of wage offers w, w^* and then maximize over the mobility/job acceptance decisions:

$$U_2(S) = \max\left\{b + (1-\delta_\pi)S, w + S, w^* + (1-\delta_\lambda)S\right\}. \tag{14}$$

Alternatively, she may have the option to keep her initial job with wage w_1, and face the following alternative with probability τ:

$$U_2(S) = \max\left\{w_1 + S, w + S, w^* + (1-\delta_\lambda)S\right\}. \tag{15}$$

In other words, denoting by $U_2(S, b, \delta_\pi, \delta_\lambda)$ the utility in case of a layoff, and $U_2(S, w_1, 0, \delta_\lambda)$ the utility in case of no-layoff (b is replaced by w_1 and δ_π by 0), the program in first period is now:

$$\max_S \; - C(S) + (1-\tau)U_2(S, b, \delta_\pi, \delta_\lambda) + \tau U_2(S, w_1, 0, \delta_\lambda).$$

The first order condition will be, using (12):

$$\begin{aligned} C'(S) &= \tau\left[1 - \delta_\lambda P_m(b, \delta_\pi, \delta_\lambda) - \delta_\pi P_u(b, \delta_\pi, \delta_\lambda)\right] + (1-\tau)\left[1 - P_m(w_1, 0, \delta_\lambda)\right] \\ &= (1-\tau)\left[1 - \delta_\lambda \int_{w^r}^{\bar{w}} g(z+S)F(z)dz - \delta_\pi G(w^{*r})F(w^r)\right] \\ &\quad +\tau\left[1 - \int_{w_1}^{\bar{w}} g(z+S)F(z)dz\right]. \end{aligned} \tag{16}$$

It is easy to verify that the quantity in the right hand side increases with τ, featuring that the returns to local social capital are higher, the higher the likelihood to remain employed in the same local job. As a consequence, as τ increases, $C'(S)$ increases, meaning that the optimal level of social capital $\widehat{S}$ invested is higher. In the same vein, the expression above is increasing in w_1: the higher the initial wage, the higher $\widehat{S}$. Finally, the interaction between the two is also positive: $d^2\widehat{S}/Sw_1 d\tau > 0$.

We thus have the following implications.

Proposition 4: *Employment protection raises the investment in local social capital. Higher local wages (relative to wages in region B) also raises local social capital, as workers are more likely to stay in region A. Finally, the two effects interact complementarily: the marginal effect of employment protection on social capital is higher, the higher local wages.*

5 Implications and conclusion

We have discussed here how low mobility could be the outcome of self-reinforcing factors. In the present case, investments in local social capital are induced by low mobility and are in turn themselves a factor of immobility. We have found several examples of complementarity between high local social capital and low mobility rate. In Appendix, we also discuss social externalities, that are important but not central to our main point here. Note that in Quentin et al. (2008), we explored empirically the relations between social capital and labor markets and found, in line with this paper, that social capital is strong factor of immobility. It is also a fairly large factor of unemployment when social capital is clearly local, while other types of social capital are found to have a positive effect on employability. We also find evidence of the reciprocal causality, that is, individuals born in another region have accumulated less local social capital.

This has several implications for the debates on unemployment. First, unemployment has increasingly been perceived in the last decades as the result of various market imperfections impeding mobility and therefore raising the reservation wage of the unemployed. For example, generous unemployment compensation raises the relative returns to staying in a local depressed area. Strong employment protection raises incentives to invest in local skills as job duration is anticipated to be much higher and thus reduces mobility ; it raises the incentives to invest in job-specific skills and thus reduces job-to-job mobility ; good market imperfections and notably obstacles to job creations in booming regions/sectors reduce the returns to mobility from the depressed regions ; and wage compression reduces the returns to moving in the booming regions.

These alternative or complementary explanations can be found in Hassler et al. (2000, 2005), Ljunqvist and Sargent (1998, 2002), Bertola and Ichino (1996), Wasmer (2006) and Bertola and Rogerson (1997).

Our theory is not necessarily a new tentative explanation for high unemployment: local social capital is simply complements to other explanations. However, we have a potential theory for its *high persistence*. Local social capital may indeed be a bottleneck that prevents mobility. A natural implication is that attempts to treat unemployment by changing the exogenous factors raising mobility may fail if there is the type of vicious circle between immobility and high local social capital. If local social capital is a bottleneck to higher mobility, deregulating labor markets may simply raise inequality and the share of the informal economy, but will not ncessarily raise mobility much. Said otherwise, if mobility is self-reinforced, it may not be enough to remove the immobility-friendly institutions. An efficient reform of the labor markets should instead combine traditional reforms and develop incentives towards mobility.

Appendix

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Appendix

A Extension: social externalities

In the model, we have exhibited a tendency in the model towards multiple *local* optima in social capital choices of individuals, which is different from multiple equibria, in the sense that observationally close workers may differ dramatically in their choices. However, each individual has a clear *global* optimum, and thus except in a degenerate case, there is no multiplicity of equilibria. We can now offer a discussion of the existence of multiple equilibria based on social externalities. For that, as we noted in the introduction, there is a straightforward rationalization: in a world of low mobility, it may be even more difficult, at an individual level, to leaves one's friends to move to another region: if none of my friends have moved to the city, I won't find any "old friends" there. The opposite occurs in a high mobility world: I am more likely to find these old friends in the new place.[10]

The introduction of social externalities will, quite expectedly, reinforce the complementarity between immobility and localness of social capital discussed above. Here, this will act through a standard mechanism: the fact that the choice of an individual is affected by others' choices.

There are several modelling choices here, because both the costs and the benefits of investing in local social capital and maintaining local connections depend heavily on other's mobility rates: if everyone leaves a neighborhood every other year, the returns to investing in local social capital is clearly lower. In a reduced form, this type of externality can be modelled either through the returns to social capital or through its cost, both depending on aggregate social capital.

We find it slightly more convenient analytically to introduce that externality on the cost side, without deep consequences. This development is coherent with the work of Glaeser et al. (2002) even if they take the externality into account through the returns to social capital instead of the cost. Assume that the level of social capital in the economy, $\overline{S}$, decreases the cost of acquiring social capital for an individual.

For simplicity, we assume again that $1-\delta_\pi$ and $1-\delta_\lambda$ are technologically given, and now that $C(S)$ becomes $C(S,\overline{S})$ and multiplicative separability property such that: $C(S,\overline{S}) = C_0(S)\sigma(\overline{S})$. We still assume that $C_0(S)$ is increasing and convex and adds that $\sigma(\overline{S})$ is decreasing and convex:

$$dC(S,\overline{S})/dS = C_0^{'}(S)\sigma(\overline{S}) > 0 \text{ and } d^2C(S,\overline{S})/dS^2 = C_0^{''}(S)\sigma(\overline{S}) > 0,$$
$$dC(S,\overline{S})/d\overline{S} = C_0(S)\sigma^{'}(\overline{S}) < 0; \text{ and } d^2C(S,\overline{S})/d\overline{S}^2 = C_0(S)\sigma^{''}(\overline{S}) > 0,$$
$$d^2C(S,\overline{S})/dSd\overline{S} = d^2C(S,\overline{S})/d\overline{S}dS = C_0^{'}(S)\sigma^{'}(\overline{S}) < 0.$$

We will also assume that: $\lim\limits_{s\to+\infty} C_0^{''}(S) > 0, \lim\limits_{\overline{s}\to+\infty} \sigma^{'}(\overline{S}) = 0$. Furthermore, it is useful to note that $\lim\limits_{s\to+\infty} \frac{d^2EU_2}{dS^2} = 0$ and $\lim\limits_{s\to+\infty} \frac{d^3EU_2}{dS^3} < 0$.

As for individuals, $\overline{S}$ is given. The first order condition for their investment in human capital becomes:

$$\frac{dC(S,\overline{S})}{dS} = C_0'(S)\sigma(\overline{S}) = \beta\frac{dEU_2}{dS}. \tag{A1}$$

This allows to rewrite a relationship between $\widehat{S}^*$, the privately optimal level of social capital in presence of externalities and $\overline{S}$:

$$\widehat{S}^* = \widehat{S}^*(\overline{S}).$$

Assuming that $\overline{S}$ is the average level of social capital and a symmetric equilibrium (all agents are identical and choose the same level of social capital at their optimum), the equilibrium value of S is

[10] One could also discuss the evolution of social norms with mobility. The idea is that, if mobility is low, "old friends" may be more important for individuals than new friends, and vice-versa in a high mobility world.

obtained by the intersection of the 45 degree line in the $(\overline{S}, S)-$*locus* with the curve $\widehat{S}^*(\overline{S})$. To characterize $\widehat{S}^*(\overline{S})$, let rewrite equation (A1) as:

$$\mathcal{F}(S, \overline{S}) = C_0'(S)\sigma(\overline{S}) - \beta \frac{dEU_2}{dS} = 0.$$

Using the implicit function theorem for $\frac{d\mathcal{F}}{dS} \neq 0$, we have:

$$\frac{d\widehat{S}^*(\overline{S})}{d\overline{S}} = -\frac{\frac{d\mathcal{F}(\widehat{S}^*,\overline{S})}{d\overline{S}}}{\frac{d\mathcal{F}(\widehat{S}^*,\overline{S})}{dS}} = -\frac{C_0'(S)\sigma'(\overline{S})}{C_0''(S)\sigma(\overline{S}) - \frac{d^2EU_2}{dS^2}} > 0, \tag{A2}$$

i.e. $\widehat{S}^*(\overline{S})$ has a positive slope.

Remark that the first order condition implies the equality between $C_0'(S)\sigma(\overline{S})$, the marginal cost of investing in social capital and $\beta\frac{dEU_2}{dS}$, its time-discounted marginal return. For this first order condition to be a maximum, the necessary condition is: $C_0''(S)\sigma(\overline{S}) > \beta\frac{d^2EU_2}{dS^2}$. Otherwise, this intersection would represent a minimum rather than a maximum. This implies that the denominator of equation (A2) is necessarily positive for any optimal choice of an individual excepted if the marginal cost is tangent to the marginal return. We do not treat this particular point as it is not our main concern.

To ease the analysis, we will assume again a quadratic functional form for $C_0(S)$ such that $C_0'(S) = S$ and $\sigma(\overline{S}) = \overline{S}^{-\gamma}$ where $\gamma > 0$. In this case, it is then possible to show that $\lim_{\overline{s}\to+\infty} \frac{d\widehat{S}^*(\overline{S})}{d\overline{S}} = 0$. This suggests that (at least for a minimal value of $\overline{S}$) $\widehat{S}^*(\overline{S})$ is concave (since it has a positive slope and an horizontal asymptote).

Since $\widehat{S}^*(\overline{S})$ is an increasing function in $\overline{S}$ and has an horizontal asymptote, there is necessarily one intersection at least between $\widehat{S}^*(\overline{S})$ and the 45° line in the $(\overline{S}, S) - $ *locus*. Unfortunately, it is difficult (without additional strong assumptions on the distribution function of the model) to determine the convexity of $\widehat{S}^*(\overline{S})$ (the expression of the implicit second order derivative is given in Appendix). Therefore, we prefer to present several possible shapes for $\widehat{S}^*(\overline{S})$ as represented in figures 5 and 6.

Proposition A-5. *There exists, at least, one stable equilibrium value of S in presence of externalities.*

Proof. The proof requires to treat several different cases.

- Let start by assuming that $\frac{d^3EU_2}{dS^3} < 0\ \forall S \in IR_0^+$. i.e. $\frac{dEU_2}{dS}$ is strictly concave. In this case, $\widehat{S}^*(\overline{S})$ is a function $\forall S \in IR_0^+$. Two cases have to be analyzed: First, suppose that $C_0'(S)\sigma(\overline{S}) > \frac{dEU_2}{dS}$ for $S \to 0$. In this case, we necessarily have a corner solution (corresponding to a local maximum) and we may have other interior solution(s). Note that the corner solution is unstable. Second, suppose that $C_0'(S)\sigma(\overline{S}) > \frac{dEU_2}{dS}$ for $S \to 0$. We do not have corner solution, but we must have an interior solution: Since $\lim_{\overline{s}\to 0}\widehat{S}^*(\overline{S}) > 0$ and $\lim_{\overline{s}\to+\infty} \frac{d\widehat{S}^*(\overline{S})}{d\overline{S}} = 0$, we must have, at least, one intersection between $\widehat{S}^*(\overline{S})$ and the 45 degree line.
- Suppose now that $\frac{dEU_2}{dS}$ is not strictly concave ($\frac{d^3EU_2}{dS^3} \geq 0$). In this case, $\widehat{S}^*(\overline{S})$ may be a correspondence rather than a function. As before, as long as $C_0'(S)\sigma(\overline{S}) > \frac{dEU_2}{dS}$ for $S \to 0$, we have (at least) a corner solution (unstable). We should also have another interior solution (stable) but we may have more. To better understand how this correspondence is shaped, it is instructive to look at Figure (5) and (6). The end of this proof is presented for $C_0'(S)\sigma(\overline{S}) < \frac{dEU_2}{dS}$ when $S \to 0$. In this situation, $\widehat{S}^*(\overline{S})$ may be a correspondence, but with a positive slope. Let $\overline{S}_1$ be the tangency point between $C_0'(S)\sigma(\overline{S})$ and $\frac{dEU_2}{dS}$. When $S \in (0, \overline{S}_1)$, if the correspondence crosses the 45 degree line, we have a stable equilibrium. If it does not cross it, let us consider the second part of this correspondence, that is for $S \in (0, \overline{S}_2)$, $\overline{S}_2$ being the tangency point between $C_0'(S)\sigma(\overline{S})$ and $\frac{dEU_2}{dS}$,

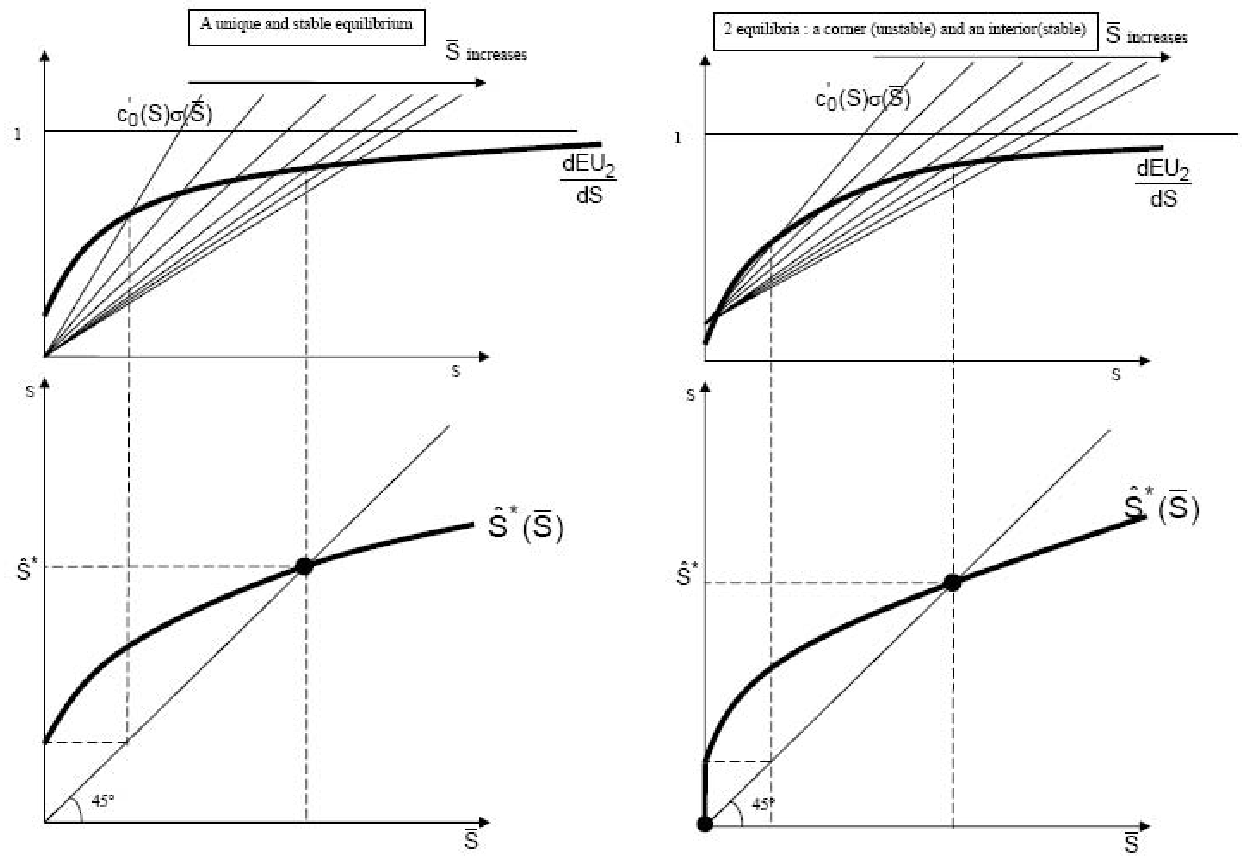

Figure 5: Case when $\widehat{S}^*(\overline{S})$ is a continuous function (unique extremum for S). Unique vs. multiple equilibria.

for the highest level of S. This second part is necessarily higher (in terms of the level of S) and $\overline{S}_2 < \overline{S}_1$. Since $\widehat{S}^*(\overline{S})$ has an horizontal asymptote, we necessarily have an intersection between with the 45 degree line when the first part of the correspondence does not have intersection.

■

This reinforces the type of complementarity already discussed in the benchmark model: indeed, we had *multiple maxima* for individuals agents: one with low mobility & high local social capital vs. another one with high mobility & low local social capital.[11]

Complementarity property 4. *There are multiple* aggregate *equilibria: an equilibrium with low aggregate local social capital implies a higher individual cost of investing in social capital, inducing higher aggregate mobility ; and an equilibrium with a high aggregate local social capital reducing the individual cost of investing in social capital, inducing aggregate immobility instead.*

[11] This multiplicity of equilibria was also investigated by Spilimbergo and Ubeda (2004a) in a study of migration and social environment.

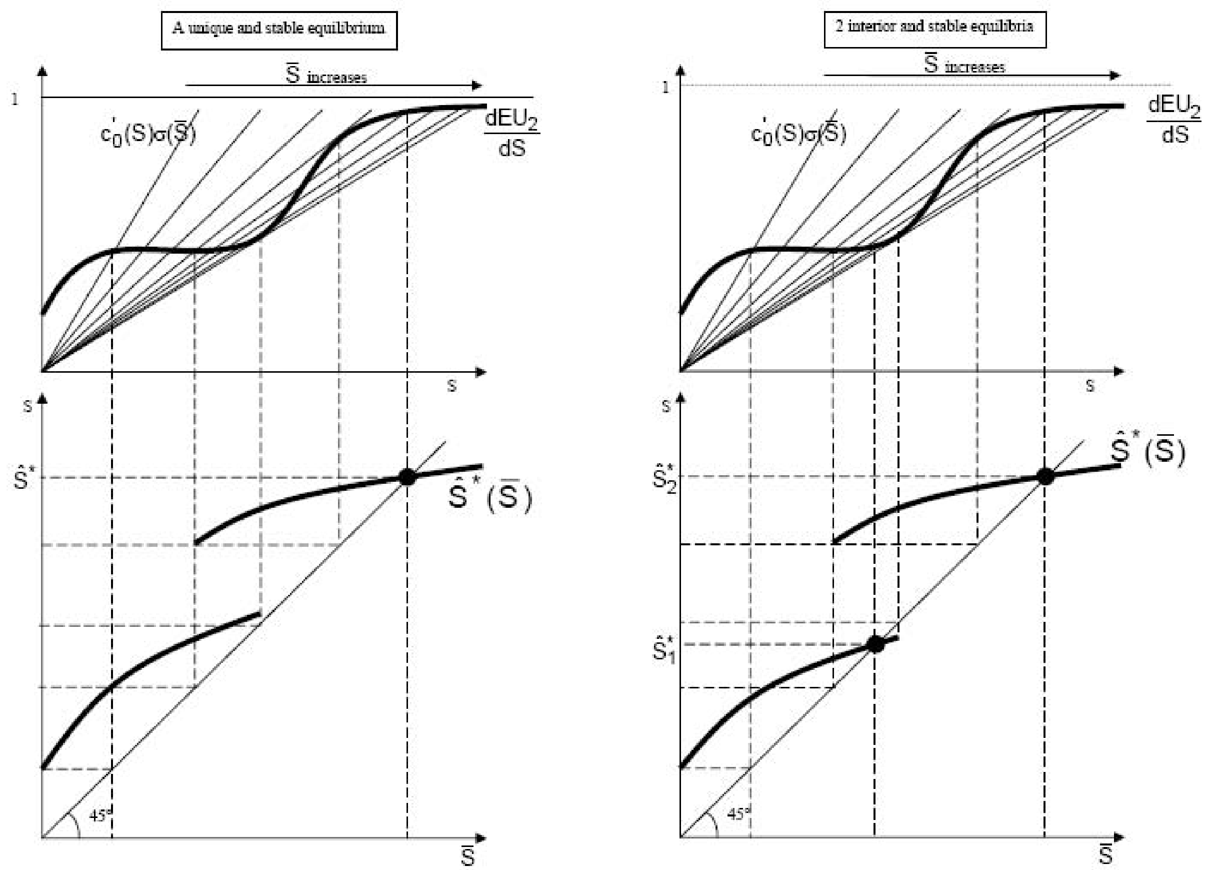

Figure 6: Case when $\widehat{S}^*(\overline{S})$ is a correspondence (multiple extrema for individual S). Unique vs. multiple equilibria

B Theory Appendix

B.1 Distribution of a Max

Lemma A1. *Let be two random variable X and Y such that $X \sim F$ and $Y \sim G$, with F and G two cumulative distribution functions defined over the support $[0\ B]$. Let be $Z = \max\{X, Y\}$. Then $Z \sim FG$.*

Proof. Let be H the cumulative distribution function of Z.

$$H(z) = \int_0^z \int_0^z \{I[x > y] + I[y > x]\} \, dF(x) \, dG(y),$$

$$H(z) = \int_0^z \int_0^z I[x > y] \, dF(x) \, dG(y) + \int_0^z \int_0^z I[y > x] \, dF(x) \, dG(y),$$

$$H(z) = \int_0^z \int_y^z f(x) \, dF(y) + \int_0^z \int_x^z g(y) \, dF(x),$$

$$H(z) = \int_0^z [F(z) - F(y)] \, g(y) dy + \int_0^z [G(z) - G(x)] \, f(x) dx,$$

$$H(z) = \int_0^z F(z) g(y) dy - \int_0^z F(y) g(y) dy + \int_0^z G(z) f(x) dx - \int_0^z G(x) f(x) dx,$$

$$H(z) = 2F(z)G(z) - \int_0^z \{F(w)g(w) + G(w)f(w)\} \, dw,$$

$$H(z) = 2F(z)G(z) - F(z)G(z),$$

$$H(z) = F(z)G(z).$$

■

Theorem A2: *Let be N random variables X_1, X_2, ..., X_N such that $X_1 \sim F_1$, $X_2 \sim F_2$, ..., $X_N \sim F_N$, with $F_1, F_2, ..., F_N$ N cumulative distribution functions defined over the support $[0\ B]$. Let be $Z = \max\{X_1, X_2, ..., X_N\}$. Then $Z \sim F_1 F_2 ... F_N$.*

Proof. The proof is recursive. For the special case where $N = 2$ the proof has been showed in the previous lemma. Suppose now the theorem is true for any $N = p$ and show it is true for $N = p + 1$. Let be $Y = \max\{X_1, X_2, ..., X_p\}$ and $Z = \max\{X_1, X_2, ..., X_{p+1}\}$. If the theorem is true for $N = p$, then $Y \sim F_1 F_2 ... F_p$.Moreover, $Z = \max\{X_1, X_2, ..., X_{p+1}\} = \max\{\max(X_1, X_2, ..., X_p), X_{p+1}\} = \max\{Y, X_{p+1}\}$.The previous lemma tell us then: $Z \sim (F_1 F_2 ... F_p) F_{p+1} \Leftrightarrow Z \sim F_1 F_2 ... F_p F_{p+1}$.Then if the theorem is true for $N = p$, it is true for $N = p + 1$. Since according to the previous it is true for $N = 2$, then, recursively, it is true for any N. ■

Corollary A3: *Let be N random variables X_1, X_2, ..., X_N such that $X_1 \sim F$, $X_2 \sim F$, ..., $X_N \sim F$, with F a cumulative distribution function defined over the support $[0\ B]$. Let be $Z = \max\{X_1, X_2, ..., X_N\}$. Then $Z \sim F^N$.*

Proof. The previous theorem shows that $Z \sim \underbrace{FF...F}_{N\ terms} \Leftrightarrow Z \sim F^N$. ■

B.2 Decisions of the agent at a given S and determination of P_m, P_u, P_w.

B.2.1 Decision tree

There are three main possible cases for an agent, in order: remaining non-employed, accepting a local offer, and finally accepting an offer in region B: we have thus

$$\begin{aligned} U_2 &= b+(1-\delta_\pi)S \\ \text{if } b+(1-\delta_\pi)S &> w+S > w^*+(1-\delta_\lambda)S \text{ or } b+(1-\delta_\pi)S > w^*+(1-\delta_\lambda)S > w+S \\ U_2 &= w+S \\ \text{if } w+S &> b+(1-\delta_\pi)S > w^*+(1-\delta_\lambda)S \text{ or } w+S > w^*+(1-\delta_\lambda)S > b+(1-\delta_\pi)S \\ U_2 &= w^*+(1-\delta_\lambda)S \\ \text{if } w^*+(1-\delta_\lambda)S &> w+S > b+(1-\delta_\pi)S \text{ or } w^*+(1-\delta_\lambda)S > b+(1-\delta_\pi)S > w+S. \end{aligned}$$

B.2.2 Determination of P_m

The probability of moving is formally

$$P_m = P\left[\{w^*+(1-\delta_\lambda)S > w+S\} \cap \{w^*+(1-\delta_\lambda)S > b+(1-\delta_\pi)S\}\right], \tag{B3}$$

and can be shown to be equal to

$$P_m = \int_0^{\overline{w}}\int_0^{\overline{w}} I\left[w^*+(1-\delta_\lambda)S > w+S\right] I\left[w^*+(1-\delta_\lambda)S > b+(1-\delta_\pi)S\right] dG\left(w^*\right) dF\left(w\right),$$

or

$$P_m = \int_{w^{r*}}^{\overline{w}} F(z-\delta_\lambda S) g\left(z\right) dz.$$

which gives equation (4).

B.2.3 Determination of P_u

P_u writes formally as

$$P_u = P\left[\{b+(1-\delta_\pi)S > w+S\} \cap \{b+(1-\delta_\pi)S > w^*+(1-\delta_\lambda)S\}\right]. \tag{B4}$$

We have

$$P_u = \int_0^{\overline{w}}\int_0^{\overline{w}} I\left[b+(1-\delta_\pi)S > w+S\right] I\left[b+(1-\delta_\pi)S > w^*+(1-\delta_\lambda)S\right] dG\left(w^*\right) dF\left(w\right),$$

or

$$P_u {=} \int_0^{\overline{w}} I\left[w^r > w\right] G(w^{*r}) dF\left(w\right) = F(w^r)G(w^{*r}).$$

Deriving, we have:

$$\frac{dP_u}{dS} = f(w^r)G(w^{*r})\frac{\partial w^r}{\partial S} + F(w^r)g(w^{*r})\frac{\partial w^{*r}}{\partial S},$$

which leads to equation (7) and thus to Proposition 2.

B.2.4 Determination of P_w

The local employment probability is formally

$$P_w = P\left[\{w + S > b + (1-\delta_\pi)S\} \cap \{w + S > w^* + (1-\delta_\lambda)S\}\right], \tag{B5}$$

and can be shown to be equal to

$$P_w = \int_0^{\overline{w}} \int_0^{\overline{w}} I\left[w + S > b + (1-\delta_\pi)S\right] I\left[w + S > w^* + (1-\delta_\lambda)S\right] dG\left(w^*\right) dF\left(w\right),$$

or

$$P_w = \int_0^{\overline{w}} I\left[w > w^r\right] G(w + \delta_\lambda S) dF\left(w\right),$$

which gives equation (8).

B.3 Proof of equation (11)

Proof. One derives now the value of EU_2 in detailing the different cases:

$$EU_2 = \int_0^{\bar{w}} \int_0^{\bar{w}} \left\{ \begin{array}{c} (b + (1-\delta_\pi)S)\, I\left[b - \delta_\pi S > w\right] I\left[b + (1-\delta_\pi)S > w^* + (1-\delta_\lambda)S\right] + \\ (w + S) I\left[w + \delta_\pi S) > b\right] I\left[w + S > w^* + (1-\delta_\lambda)S\right] + \\ (w^* + (1-\delta_\lambda)S) I\left[w^* + (1-\delta_\lambda)S > b + (1-\delta_\pi)S\right] I\left[w^* + (1-\delta_\lambda)S > w + S\right] \end{array} \right\} dF(w)dG(w^*).$$

Extending the cumulative distribution function above its support, i.e. $\forall w, w^* > \overline{w} \Rightarrow F(w) = G(w^*) = 1$ and $f(w) = g(w^*) = 0$, this expression rewrites as

$$\begin{aligned} EU_2 &= (b + (1-\delta_\pi)S)\, F(w^r) G(w^{*r}) \\ &\quad + \int_{w^r}^{\bar{w}} (w + S) G(w + \delta_\lambda S) dF(w) \\ &\quad + \int_{w^{*r}}^{\bar{w}} [w^* + (1-\delta_\lambda)S] F(w^* - \delta_\lambda S) dG(w^*). \end{aligned}$$

Pose $S' = \delta_\lambda S$ and note that $w^{*r} = w^r + S'$, we can rewrite it as

$$\begin{aligned} EU_2 &= (b + (1-\delta_\pi)S)\, F(w^r) G(w^{*r}) \\ &\quad + \int_{w^r}^{\bar{w}} (w + S) G(w + S') dF(w) \\ &\quad + \int_{w^{*r}}^{\bar{w}} [w^* + (1-\delta_\lambda)S] F(w^* - S') dG(w^*). \end{aligned}$$

A variable change is useful: pose $z = w^* - S'$, we have

$$\begin{aligned} EU_2 &= (b + (1-\delta_\pi)S)\, F(w^r) G(w^{*r}) \\ &\quad + \int_{w^r}^{\bar{w}} (w + S) G(w + S') f(w) dw \\ &\quad + \int_{w^r}^{\bar{w}-S'} (z + S) F(z) g(z + S') dz. \end{aligned} \tag{B6}$$

Note that

$$\int_{w^r}^{\bar{w}-S'} (z + S) F(z) g(z + S') dz = \int_{w^r}^{\bar{w}} (z + S) F(z) g(z + S') dz - \int_{\bar{w}-S'}^{\bar{w}} (z + S) F(z) g(z + S') dz,$$

and, in the second term, $z+S'>\bar{w}$ thus, $g(z+S')\equiv 0$. The integrals in (B6) can thus be simplified as

$$\begin{aligned} EU_2 &= (b+(1-\delta_\pi)S)\,F(w^r)G(w^{*r}) \\ &\quad + \int_{w^r}^{\bar{w}} (w+S)d[G(w+S')F(w)]dw. \end{aligned} \quad \text{(B7)}$$

Note also that $w^r+S=b+(1-\delta_\pi)S$, hence there is a simplification here. Let us integrate (B7) by part. We have

$$\int_{w^r}^{\bar{w}} (w+S)d[G(w+S')F(w)]dw = \bar{w}+S-(w^r+S)G(w^{*r})F(w^r) - \int_{w^r}^{\bar{w}} F(w)G(w+S')dw,$$

which immediately implies

$$EU_2 = \bar{w}+S-\int_{w^r}^{\bar{w}} F(w)G(w+S')dw,$$

or

$$EU_2 = \bar{w}+S-\int_{b-\delta_\pi S}^{\bar{w}} G(z+\delta_\lambda S)F(z)dz. \quad \text{(B8)}$$

Deriving with respect to S leads to

$$\frac{\partial EU_2}{\partial S} = 1-\delta_\pi G(w^{*r})F(w^r)-\delta_\lambda \int_{w^r}^{\bar{w}} g(z+\delta_\lambda S)F(z)dz.$$

■

B.4 Properties of $\mathbf{EU_2}$

B.4.1 First order derivative

Deriving equation (11), we have

$$\frac{dEU_2}{dS} = 1-\delta_\pi G(w^{*r})F(w^r)-\delta_\lambda \int_{w^r}^{\bar{w}} g(z+\delta_\lambda S)F(z)dz. \quad \text{(B9)}$$

Then, we can rewrite P_m after a change of variable:

$$\begin{aligned} P_m &= \int_{w^{r*}}^{\overline{w}} F(z-\delta_\lambda S)g(z)dz \\ &= \int_{w^{*r}-\delta_\lambda S}^{\overline{w}-\delta_\lambda S} F(z')g(z'+\delta_\lambda S)dz', \end{aligned}$$

with $z'=z-\delta_\lambda S$. Since $w^{*r}-\delta_\lambda S=w^r$ and $g(z'+\delta_\lambda S)\equiv 0$ for all $z'>g\overline{w}-\delta_\lambda S$, the last term of equation (B9) precisely corresponds to $\delta_\lambda P_m$. Hence, equation (12).

B.4.2 Second order derivative

Replacing dP_m/dS and dP_u/dS as calculated in Section 2 into (B9), we have:

$$\begin{aligned} d^2EU_2/dS^2 &= -\delta_\lambda(\delta_\pi-\delta_\lambda)F(w^r)g(w^{*r})+{\delta_\lambda}^2\int_{w^{*r}}^{\overline{w}} f(z-\delta_\lambda S)g(z)dz \\ &\quad -\delta_\pi[-\delta_\pi f(w^r)G(w^{*r})+F(w^r)g(w^{*r})(\delta_\lambda-\delta_\pi)]. \end{aligned}$$

Rearranging terms, we obtain

$$d^2EU_2/dS^2 = F(w^r)g(w^{*r})(\delta_\pi-\delta_\lambda)^2+\delta_\pi^2 f(w^r)G(w^{*r})+{\delta_\lambda}^2\int_{w^{*r}}^{\overline{w}} f(z-\delta_\lambda S)g(z)dz, \quad \text{(B10)}$$

which is strictly positive unless $\delta_\pi=\delta_\lambda=0$.

B.4.3 Third order derivative

Deriving (B10), we have then, after rearranging terms,

$$\begin{aligned} d^3EU_2/dS^3 &= g'(w^{*r})F(w^r)(\delta_\lambda-\delta_\pi)^3+(\delta_\pi-\delta_\lambda)^2\delta_\lambda f(w^r)g(w^{*r}) \\ &\quad -\delta_\lambda{}^3\int_{w^{*r}}^{\overline{w}} f'(z-\delta_\lambda S)g(z)dz-\delta_\pi{}^3 f'(w^r)G(w^{*r}). \end{aligned}$$

Again, this is equal to zero when $\delta_\pi=\delta_\lambda=0$. The sign is ambiguous.