SciencesPo. PARIS | Department of Economics

Discussion paper 2014-04

INFORMATION TRANSMISSION IN NESTED SENDER-RECEIVER GAMES

Ying Chen
Sidartha Gordon

Sciences Po Economics Discussion Papers

Information Transmission in Nested Sender-Receiver Games *

Ying Chen[†] Sidartha Gordon[‡]

February 24, 2014

Abstract

We introduce a "nestedness" relation for a general class of sender-receiver games and compare equilibrium properties, in particular the amount of information transmitted, across games that are nested. Roughly, game B is nested in game A if the players's optimal actions are closer in game B. We show that under some conditions, more information is transmitted in the nested game in the sense that the receiver's expected equilibrium payoff is higher. The results generalize the comparative statics and welfare comparisons with respect to preferences in the seminal paper of Crawford and Sobel (1982). We also derive new results with respect to changes in priors in addition to changes in preferences. We illustrate the usefulness of the results in three applications: (i) delegation to an intermediary with a different prior, (ii) the choice between centralization and delegation, and (iii) two-way communication with an informed principal.

Keywords: sender-receiver games, information transmission, nestedness, intermediary, delegation, informed principal.

JEL classification: D23, D82, D83.

*This paper supersedes Chen's (2010) "Optimism and Communication" and Gordon's (2011) "Welfare Properties of Cheap Talk Equilibria." We thank Oliver Board, Hector Chade, Alejandro Manelli, Edward Schlee, Joel Sobel and audiences at Arizona State University, Workshop on Language and Communication, Kellogg School of Management 2007, Midwest Theory Conference 2008, Canadian Economic Theory Conference 2009 and Society of Economic Design Conference 2009 for helpful suggestions and comments.

[†]Department of Economics, Johns Hopkins University and Department of Economics, University of Southampton. Email: ying.chen@jhu.edu.

[‡]Department of Economics, Sciences Po, 28 rue des Saints-Pères, 75007 Paris, France. Email: sidartha.gordon@sciences-po.org.

1 Introduction

In many situations of strategic communication, an agent who has information useful for a principal may have interests that are not perfectly aligned with the principal's. For example, a division manager who desires large investments in his division is motivated to overstate the profitability of an investment project to the company's CEO, and a financial analyst associated with the underwriting firm is under pressure to inflate claims about the value of a stock. Since the seminal work of Crawford and Sobel (1982) and Green and Stokey (2007), such situations have been successfully modeled as sender-receiver games in which a player (sender) who is privately informed about the state of the world sends a costless message to another player (receiver), who then chooses an action that determines both players' payoffs.

It is well established in the literature that the misalignment of interests between the players often makes it impossible for information to be fully conveyed from the sender to the receiver and only imperfect information transmission can take place in equilibrium. One central question is what determines the informativeness of communication in equilibrium. A partial answer was provided in Crawford and Sobel (1982): they show that under a regularity condition "M," more information can be transmitted in equilibrium when the sender's bias (captured by a preference parameter) is smaller. Although useful, the result is silent on how the amount of information transmitted in equilibrium relates to the players's prior beliefs or to the receiver's preference; moreover, it relies on the condition M.

In this paper, we introduce a "nestedness" relation between games which allows us to compare the amount of information transmitted across games (as well as across equilibria within games) more generally and in a unified way. Roughly speaking, game B is nested into game A if the players' optimal actions are closer in game B. In terms of the players' preferences and beliefs, a game is nested into another if the players' preferences are less extreme and/or the prior of the receiver is higher (in the sense of monotone likelihood ratio dominance).[1]

To describe our findings, recall that an equilibrium of a Crawford-Sobel game takes a

[1]We consider games in which the sender prefers a higher action than the receiver does. Analogous results can be obtained in games where the sender prefers a lower action. There, a game is nested into another if the prior of the receiver is lower.

partitional form, that is, the sender partitions his type space, a bounded interval on the real line, into a finite number of subintervals and conveys to the receiver what subinterval his type lies in. Without imposing condition M, we show in Theorem 1 that if game B is nested into game A, then the highest number of subintervals in an equilibrium partition is higher in game B; also, the cutoff types in the greatest equilibrium partition in game B are to the right of those in game A.

In Theorem 2, we show that the players in the nested game B have higher expected payoffs in the greatest equilibrium in game B than in the greatest equilibrium in game A. Moreover, we show in Theorem 3 that under some conditions (which roughly speaking guarantee that the cutoff types are not shifted "too far" to the right in equilibrium in game B), the receiver in game A also prefers the greatest equilibrium in game B than in the greatest equilibrium in game A. In this sense, more information can be transmitted in equilibrium in the nested game B.

Three special cases are of particular interest: (i) games that differ only in the sender's payoff; (ii) games that differ only in the receiver's payoff; (iii) games that differ only in the receiver's prior.[2] One advantage of our unified nestedness approach is that we are able to identify the underlying mechanics that govern how much information can be transmitted in a game.

To illustrate the usefulness of our results, we consider three applications.

Choosing an intermediary. The first application concerns the problem of a principal choosing a representative/intermediary to communicate with a privately informed agent and then make the decision on her behalf. Dessein (2002) shows that a principal may benefit from choosing an intermediary with a different preference from her own. We ask a related question: what intermediary should the principal choose if all potential intermediaries have the same preferences as her own, but may differ in their prior beliefs? Applying our result, we show that the principal may benefit from using an intermediary who has a different but more optimistic belief (in the sense of monotone likelihood ratio improvement) from her own.

Centralization versus delegation. The second application is about the allocation of authority in organizations, in particular, when should upper management centralize decision rights and when should it delegate them to lower management? We extend

[2]The sender's prior does not affect the set of equilibria.

the analysis in Dessein (2002) which assumes a uniform prior to a more general class of Beta distributions. Our main insight is to connect the value of communication to the principal's prior belief. Dessein (2002) shows that the principal prefers centralization to delegation only when the agent's bias is so large that communication is uninformative. We show that this result depends critically on the assumption of a uniform prior: when a principal has a more optimistic prior, there is a larger region of the agent's bias for which informative communication is possible and therefore the best organizational choice may be centralization with informative communication from the agent.

Two-way communication with an informed principal. In the third application, a principal privately observes a signal that is affiliated with an agent's type. We ask the following question: if the principal sends a message first to the agent before the agent reports, will the principal reveal her signal truthfully? We show that if the principal's message is verifiable, then she reveals her signal in equilibrium, but if her message is cheap talk, then she reveals her signal truthfully only when it is sufficiently informative.

In section 2, we describe the model and introduce the nestedness relation; in section 3, we compare equilibrium properties of games that satisfy the nested relation; in section 4, we compare the players' ex ante payoffs in equilibria across games and show that more information can be transmitted in a game that is nested into another; in section 5, we discuss applications; we conclude in section 6.

2 The Model

There are two players, the Sender and the Receiver.[3] Only the Sender has payoff-relevant private information, called his type. After observing his type, the Sender sends a message to the Receiver. The Receiver observes this message and then takes an action. The messages do not directly affect payoffs and are thus "cheap talk."

Let $T = [0, 1]$ be the Sender's set of types, with typical element t. Let $A = \mathbb{R}$ be a nonempty set of Receiver's possible actions, with typical element a. Let $M = \mathbb{R}$ be the message space. A pure strategy of the Sender is a mapping $T \to M$ and a pure strategy of the Receiver is a mapping $M \to A$. A profile of pure strategies for the Sender and the Receiver induces an outcome, the mapping $\alpha : T \to A$ obtained by composing the

[3] We use the pronoun "he" for the sender and "she" for the receiver.

strategies of the two players, and an information partition of T where each element in the partition is the set of all types in T that induce a particular action in A. Both the sender and the receiver are expected utility maximizers.

An **agent** i is characterized by a pair (u^i, f^i). The (Bernoulli) utility function u^i of agent i is a continuously differentiable mapping $A \times T \to \mathbb{R}$ that satisfies $u^i_{aa} < 0$ and $u^i_{at} > 0$. Thus $u^i(\cdot, t)$ is concave in a for each $t \in T$, and u^i is supermodular in (a, t). The prior $f^i(\cdot)$of player i is a positive probability density function.

A **game** is a triple $\Gamma = \left(u^S, u^R, f^R\right)$, where u^S is the payoff function of the Sender and $\left(u^R, f^R\right)$ characterizes the Receiver.[4]

For an agent i and for all t, t' such that $0 \leq t < t' \leq 1$, let

$$U^i(a, t, t') = \int_t^{t'} u^i(a, s) f^i(s) \, ds,$$

so that

$$\frac{U^i(a, t, t')}{\int_t^{t'} f^i(s) \, ds}$$

is agent i's expected payoff on $[t, t']$. Define agent i's optimal decision when his belief is on $[t, t']$ as

$$a^i(t, t') = \arg\max_a \begin{cases} \frac{U^i(a,t,t')}{\int_t^{t'} f^i(s) ds} & \text{if } t < t' \\ u^i(a, t) & \text{if } t = t'. \end{cases}$$

With some abuse of notations, let $a^i(t) = a^i(t, t)$. Concavity and supermodularity of u^i imply that $a^i(t, t')$ is increasing in (t, t'), in particular that $a^i(t)$ is increasing in t.

Given two functions $W : \mathbb{R} \to \mathbb{R}$, and $V : \mathbb{R} \to \mathbb{R}$, we say that W **weakly single-crossing dominates** V if for any $a' > a$, whenever $V(a') - V(a) > 0$, it is true that $W(a') - W(a) > 0$. We say that W **strictly single-crossing dominates** V if for any $a' > a$, whenever $V(a') - V(a) \geq 0$, it is true that $W(a') - W(a) > 0$. A standard monotone comparative statics result says that if W weakly single-crossing dominates V, and $\arg\max W$ and $\arg\max V$ are both singletons, then $\arg\max W > \arg\max V$.

[4] Although the model allows the case of heterogeneous priors, the prior of the Sender has no effect on the equilibrium strategies of the game and therefore we do not include it as a part of Γ. It does, however, affect the welfare comparisons for the Sender. When we present results on the Sender's welfare, we discuss the conditions on the Sender's prior.

Throughout the paper, we consider games $\Gamma = \left(u^S, u^R, f^R\right)$ in which u^S strictly single-crossing dominates u^R for any $t \in [0,1]$. This implies that $a^S(t) > a^R(t)$, i.e., the optimal action for the sender is higher than that for the receiver for every type t.

Recall that

$$\frac{U^i(a,t,t')}{\int_t^{t'} f^i(s)\,ds}$$

is agent i's expected payoff on $[t,t']$. We say that **the game** $\Gamma_B = \left(u^{S_B}, u^{R_B}, f^{R_B}\right)$ **is (weakly) nested into the game** $\Gamma_A = \left(u^{S_A}, u^{R_A}, f^{R_A}\right)$ if (i) u^{S_A} weakly single-crossing dominates u^{S_B} for all $t \in [0,1]$, and (ii) $U^{R_B}(a,t,t')$ weakly single-crossing dominates $U^{R_A}(a,t,t')$ for all t,t' such that $0 \leq t \leq t' \leq 1$. If at least one of the single-crossing dominance condition holds strictly, then we say that Γ_B **is strictly nested into** Γ_A. Condition (i) implies that $a^{S_A}(t) \geq a^{S_B}(t)$ for any $t \in [0,1]$. Condition (ii), a joint condition on the receiver's payoff function and prior, implies that $a^{R_B}(t,t') \geq a^{R_A}(t,t')$ on any interval $[t,t']$. Since the sender prefers a higher action than the receiver does, intuitively conditions (i) and (ii) say that the optimal actions for the sender and the optimal action for the receiver are closer in Γ_B than in Γ_A.

What conditions on payoff functions and beliefs result in nestedness? To illustrate, consider the class of games studied in the classic paper by Crawford and Sobel (1982). They assume that here exists a function $g(a,t,b)$ such that $u^S(a,t) = g(a,t,b_S)$, $u^R(a,t) = g(a,t,b_R)$ and the function g satisfies $g_{ab} > 0$ with $b_S > b_R$.[5] For this class of games, condition (i) for nestedness is satisfied if $b_{S_A} \geq b_{S_B}$. Condition (ii) depends on the Receivers' priors as well as their payoff functions. To describe the conditions on the Receivers' priors, let us introduce some terminology. Given two priors f^L and f^H, say that f^H MLR-dominates f^L if $\frac{f^H}{f^L}$ is weakly increasing on $[0,1]$, and that f^H strictly MLR-dominates f^L if $\frac{f^H}{f^L}$ is strictly increasing on $[0,1]$. (These relations are respectively denoted by $f^L \preceq_{MLR} f^H$ and $f^L \prec_{MLR} f^H$.) Condition (ii) is satisfied if $b_{R_B} \geq b_{R_A}$ and $f^{R_A} \preceq_{MLR} f^{R_B}$. When one game's interval $[b_R, b_S]$ is included in the other game's interval $[b_R, b_S]$, the divergence of interests between its players is unambiguously smaller. So, intuitively, Γ_B is nested into Γ_A if the players have a smaller divergence of interest in Γ_B and the receiver has a higher prior in Γ_B. Nestedness is sufficient, but not necessary

[5] One familiar example is the quadratic loss utility functions, commonly used in applications, where $u^S(a,t) = -(a-t-b)^2$ and $u^R(a,t) = -(a-t)^2$ and $b > 0$.

for a smaller divergence of interests.[6] At the end of section 4.1, we provide a notion of "smaller divergence of interest" which is both simpler and more general, but only applies to a more restricted class of games.

3 Equilibria

As Crawford and Sobel (1982) show, in any Bayesian Nash Equilibrium of a game in which u^S strictly single-crossing dominates u^R, the sender partitions the type space into a finite number $\kappa \geq 1$ of subintervals and (in effect) informs the receiver what subinterval his type belongs to. Let integer κ be the number of distinct actions that are induced in an equilibrium. The cutoff points $0 = x_0 < ... < x_\kappa = 1$ of such an equilibrium partition are determined by the arbitrage condition:

$$u^S\left(a^R\left(x_{l-1}, x_l\right), x_l\right) = u^S\left(a^R\left(x_l, x_{l+1}\right), x_l\right). \tag{1}$$

for each $l = 1, ..., \kappa - 1$.

For each $\kappa \geq 1$, let W_κ be the set of vectors $x \in T^{\kappa+1}$ such that $x_0 \leq ... \leq x_\kappa$, and let X_κ be the set of vectors in W_κ such that $x_0 = 0$ and $x_\kappa = 1$. Let a **κ-equilibrium partition** be a vector $x \in X_\kappa$ that satisfies the arbitrage condition (1), and let an **equilibrium partition** be a κ-equilibrium partition for some $\kappa \geq 1$. As Crawford and Sobel (1982) show, there is a positive integer $\overline{\kappa}$ that depends on $\left(u^R, u^S, f^R\right)$ such that all equilibrium partition of the game are in $X_1 \cup ... \cup X_{\overline{\kappa}}$, and for each $\kappa \in \{1, ..., \overline{\kappa}\}$, each X_κ contains at least one κ-equilibrium partition. We call κ the size of a κ-equilibrium partition.

To derive comparative statics and welfare comparisons, we it is useful to employ the technique introduced in Gordon (2010) that represents the κ-equilibria as the fixed-points in X_κ of a κ-equilibrium mapping.[7]

[6] The set of conditions we have presented are sufficient, but not necessary for nestedness. In section 4.1, we provide an example (Example 1) in which Γ_B is nested into Γ_A, but it does not fit into these conditions. More generally, using Theorem 2 in Quah and Strulovici (2012), we can show that there are weaker conditions on u^i and f^i that guarantee that U^{R_B} weakly single-crossing dominates U^{R_A}. Specifically, if (i) for all t,t' such that $0 \leq t < t' \leq 1$ and a, a' such that $a < a'$, if $u^{R_A}(a', t) - u^{R_A}(a, t) < 0$ and $u^{R_A}(a', t') - u^{R_A}(a, t') > 0$, then $\frac{u^{R_A}(a',t) - u^{R_A}(a,t)}{u^{R_A}(a',t') - u^{R_A}(a,t')} \frac{f^{R_A}(t)}{f^{R_A}(t')} \leq \frac{u^{R_B}(a',t) - u^{R_B}(a,t)}{u^{R_B}(a',t') - u^{R_B}(a,t')} \frac{f^{R_B}(t)}{f^{R_B}(t')}$, and (ii) $f^{R_A}_{MLR} \preceq_{MLR} f^{R_B}$, then U^{R_B} weakly single-crossing dominates U^{R_A}.

[7] Gordon uses this technique to study a broader class of games than the one considered in this paper,

Specifically, for each $\kappa \geq 1$, let D_κ be the subset of vectors $x \in W_\kappa$ such that

$$u^S\left(a^R\left(x_0, x_1\right), 0\right) \geq u^S\left(a^R\left(x_1, x_2\right), 0\right).$$

For each $\kappa \geq 2$, each $l \in \{1, ..., \kappa - 1\}$, and each $x \in D_\kappa$, let $\theta_l(x)$ be the unique element in $[0, 1]$ such that

$$u^S\left(a^R\left(x_{l-1}, x_l\right), \theta_l(x)\right) = u^S\left(a^R\left(x_l, x_{l+1}\right), \theta_l(x)\right).$$

Also, let $\theta_0(x) = x_0$ and $\theta_\kappa(x) = x_\kappa$. Let $\theta^\kappa(\cdot)$ be the function that maps each vector $x \in D_\kappa$ to the vector $\theta^\kappa(x) = (\theta_0(x), \ldots, \theta_\kappa(x)) \in T^{\kappa+1}$.[8] Note that $u^i_{aa} < 0$ and $u^i_{at} > 0$ imply that for all $x \in D_\kappa$, $\theta_1(x) \leq \ldots \leq \theta_{\kappa-1}(x)$. In addition, for all $x \in D_\kappa \cap X_\kappa$, we have $0 = \theta_0(x) \leq \theta_1(x)$ and $\theta_{\kappa-1}(x) \leq \theta_\kappa(x) = 1$. Thus, for all $x \in D_\kappa \cap X_\kappa$, we have $\theta^\kappa(x) \in X_\kappa$.

For each $X \subseteq T^{\kappa+1}$, the vector $x^* \in X$ is a **greatest element of the set** X if $x \leq x^*$ for all $x \in X$. We have the following result.

Lemma 1. *(i) The mapping $\theta^\kappa(\cdot)$ is increasing and the κ-equilibrium partitions are the fixed-points of $\theta^\kappa(\cdot)$. (ii) For each $\kappa \geq 1$, if the set of κ-equilibrium partitions is nonempty, it has a greatest element. Moreover, if x^* is the greatest element of κ-equilibrium partitions and y^* is the greatest element of $(\kappa+1)$-equilibrium partition, then $x^* \leq \left(y^*_1, ..., y^*_{\kappa+1}\right)$.*

To see why part (i) holds, note that $\theta^\kappa(\cdot)$ is a composite of the receiver's best reply and the sender's best reply in turn. Since they are both increasing, it follows that $\theta^\kappa(\cdot)$ is increasing. It also follows from the arbitrage condition that the κ-equilibrium partitions are the fixed-points of $\theta^\kappa(\cdot)$. From now on, we refer to the greatest element of κ-equilibrium partitions as the **greatest κ-equilibrium partition**.

We next compare equilibrium partitions in nested games.

Theorem 1. *Let Γ_A and Γ_B be two games such that Γ_B is nested into Γ_A. (i) If Γ_A has a κ-equilibrium, then Γ_B also has a κ-equilibrium. (ii) Let $\overline{x}^A$ and $\overline{x}^B$ be the respective greatest κ-equilibrium of Γ_A and Γ_B. Then $\overline{x}^A \leq \overline{x}^B$. If Γ_B is strictly nested into Γ_A, then $\overline{x}^A < \overline{x}^B$.*

where the sign of the bias of the sender depends on the state. A similar technique is used in Gordon (2011) to study the stability of the equilibria in cheap talk games.

[8]Following Gordon (2010), for all $l \in \{1, ..., \kappa - 1\}$, the type $\theta_l(x)$ is well-defined for any $x \in D_\kappa$, thus $\theta^\kappa(\cdot)$ is well-defined on D_κ.

Part (i) of Theorem 1 says that if a game has an equilibrium partition of a particular size, then any game that is nested into it must also have an equilibrium partition of that size. This immediately implies that the maximum size of an equilibrium partition is higher in a nested game.

To gain some intuition for why part (i) is true, consider the simple case of equilibrium partitions of size two. Suppose $(0, x_1^A, 1)$ is an equilibrium partition in Γ_A, and $a^{R_A}\left(0, x_1^A\right)$ and $a^{R_A}\left(x_1^A, 1\right)$ are the receiver's best responses in Γ_A. The sender of type x_1^A is indifferent between $a^{R_A}\left(0, x_1^A\right)$ and $a^{R_A}\left(x_1^A, 1\right)$ where $a^{R_A}\left(0, x_1^A\right)$ is lower than his ideal point and $a^{R_A}\left(x_1^A, 1\right)$ is higher than his ideal point. If we keep the partition but change the game to Γ_B, then, the sender of type x_1^A prefers the action associated with the lower interval $a^{R_B}\left(0, x_1^A\right)$ to the action associated with the higher interval $a^{R_B}\left(x_1^A, 1\right)$. Now consider another partition $(0, 1, 1)$. Since the sender of type 1 prefers the action associated with the (degenerate) interval $a^{R_B}(1, 1)$ to the action associated with the lower interval $a^{R_B}(0, 1)$, by continuity, in Γ_B there must exist a sender type $x_1^B \in (x_1^A, 1)$ such that type x_1^B is indifferent between the two actions $a^{R_B}\left(0, x_1^B\right)$ and $a^{R_B}\left(x_1^B, 1\right)$.

Part (ii) of Theorem 1 says that for a fixed equilibrium size, the cutoff points in the greatest κ-equilibrium partition in the nested game are to the right of those in the nesting game, coordinate by coordinate. This is an application of standard monotone comparative statics results (Milgrom and Roberts, 1994) to the greatest fixed points of two mappings, $\theta_A^\kappa(\cdot)$ (corresponding to Γ_A) and $\theta_B^\kappa(\cdot)$ (corresponding to Γ_B).

Note that Theorem 1 applies in particular to the following three cases. (We use S_A and R_A to denote the sender and the receiver in Γ_A and S_B and R_B to denote the sender and the receiver in Γ_B.)

(a) $u^{R_A} = u^{R_B}$, $f^{R_A} = f^{R_B}$ and for all $t \in [0, 1]$, u^{S_A} single-crossing dominates u^{S_B};

To describe case (b), we introduce a new notion.[9] Given two functions $W : \mathbb{R} \to \mathbb{R}$, and $V : \mathbb{R} \to \mathbb{R}$, we say that W **has greater difference than** V if for any $a' > a$, $W(a') - W(a) \geq V(a') - V(a)$.

(b) $u^{S_A} = u^{S_B}$, $f^{R_A} = f^{R_B}$ and for all $t \in [0, 1]$, u^{R_B} has greater difference than u^{R_A};

[9]The condition in case (b) that u^{R_B} has greater difference than u^{R_A} is stronger than the condition that u^{R_B} single-crossing dominates u^{R_A}. We use the stronger condition here because greater difference is preserved under integration whereas single-crossing dominance is not. There are conditions on u^{R_A} and u^{R_B} weaker than greater difference to ensure that U^{R_A} single-crossing dominates U^{R_A}, as discussed in footnote 6.

(c) $u^{S_A} = u^{S_B}$, $u^{R_A} = u^{R_B}$ and $f^{R_A} \preceq_{MLR} f^{R_B}$.

In each of these three cases, Γ_A and Γ_B differs in only one aspect: the sender's payoff function in (a), the receiver's payoff function in (b) and the receiver's prior in (c). It is important to note that these cases are independent, i.e., one cannot formulate any of them as a combination of the other two.[10]

In a recent paper, Szalay (2012) also compares equilibrium outcomes across sender-receiver games. He considers games in which payoff functions are identical for both players, but the receivers' priors are different. While we compare both equilibrium outcomes and welfare, Szalay (2012) focuses on comparing equilibrium outcomes. He independently establishes a result related to our Theorem 1. It says that for a given equilibrium size, the cutoff points in the equilibrium partition and the induced actions shift to the right when the prior of the sender's type in the new game MLR dominates the prior in the original game, similar to the case (c) discussed above. Szalay (2012) also provides additional comparisons for games that are symmetric with respect to the middle state and the middle action, which are not covered in this paper.[11] He establishes that if the receiver's prior in one game is a spread of the receiver's prior in the other game, in an MLR sense, then its equilibrium actions are also more dispersed.[12] One

[10] An interesting question is whether our results still hold under weaker notions of stochastic dominance than MLR dominance. We provide an example here which shows that Theorem 1 fails if f^{R_B} dominates f^{R_A} in the hazard rate order (which is stronger than first-order stochastic dominance), but not in the MLR order. Consider Γ_A and Γ_B in which $u^{S_A} = u^{S_B} = -(a - t - 0.05)^2$, $u^{R_A} = u^{R_B} = -(a - t)^2$. Also, suppose $f^{R_A}(t) = 1$ for $t \in [0, 1]$, $f^{R_B}(t) = 2/5$ for $t \in [0, 2/15)$, $f^{R_B}(t) = 1$ for $t \in [2/15, 6/15)$, $f^{R_B}(t) = 3/5$ for $t \in [6/15, 10/15)$, and $f^{R_B}(t) = 39/25$ for $t \in [10/15, 1]$. It is straightforward to verify that $(1 - F^{R_B}(t))/(1 - F^{R_A}(t))$ is increasing in t, i.e., f^{R_B} dominates f^{R_A} in the hazard rate order. Note however that f^{R_B} does not MLR dominate f^{R_A}. Calculation shows that the greatest equilibrium partition in Γ_A is $y^A = (0, 2/15, 7/15, 1)$ and the greatest equilibrium partition in Γ_B is $y^B = (0, 0.132, 0.495, 1)$. Since $0.132 < 2/15$, the cutoff points in y^B are not to the right of the cutoff points in y^A, coordinate by coordinate. So Theorem 1 fails under hazard rate dominance and first-order stochastic dominance. The reason for the failure is that under these weaker notions of stochastic dominance, it is no longer true that $a^{R_B}(t, t') \geq a^{R_A}(t, t')$ for all $t, t' \in [0, 1]$. In the example, for instance, $a^{R_A}(2/15, 7/15) = 0.3$ whereas $a^{R_B}(2/15, 7/15) = 199/690 < 0.3$. Without the receiver's best response being higher *for every interval* in the nested game, the comparative statics results fail.

[11] These games belong to a larger class studied in particular by Gordon (2010), where the function $a^S(t) - a^R(t)$ can take both positive and negative values. In this paper as in Crawford and Sobel (1982), this function is positive and bounded away from zero.

[12] As Szalay (2012) notes, this result is interesting because if these prior beliefs are interpreted as

interpretation is that if the receiver's prior in game B is an MLR spread of the receiver's prior in game A, then game B is nested into game A for high types whereas game A is nested into game B for low types. As suggested by Theorem 1, on the one hand, this leads high types to induce higher actions in game B than in game A; on the other hand, this also leads low types to induce lower actions in game B than in game A. Both effects lead to more dispersed equilibrium actions in game B than in game A.[13]

4 Comparing information transmission in nested games

Let a partition of size κ be a vector $y = (y_0, y_1, ..., y_\kappa) \in X_\kappa$. Recall that an agent i is characterized by a pair (u^i, f^i). Define the expected payoff of agent i under partition y when agent j makes the decision as follows:

$$E^{i,j}(y) = \sum_{h=0}^{\kappa-1} U^i \left(a^j(y_h, y_{h+1}), y_h, y_{h+1}\right).$$

Since we often consider the expected payoff of agent i under partition y when it is agent i himself who makes the decision, we simplify the notation $E^{i,j}(y)$ to be $E^i(y)$ when $i = j$.

Throughout our analysis, we assume that Γ_B is nested into Γ_A. To compare the amount of information transmitted in equilibrium in Γ_A and in Γ_B, we first consider the players in the nested game Γ_B. In particular, we compare their payoffs in the equilibrium partitions of the same size induced in Γ_A and Γ_B.

Theorem 2. *Suppose Γ_B is strictly nested into Γ_A. Let y be a κ-equilibrium partition of Γ_A. Then there is a κ-equilibrium partition y' in Γ_B such that (i) R_B prefers the partition y' to the partition y, i.e., $E^{R_B}(y) < E^{R_B}(y')$. (ii) If U^{S_B} weakly single-crossing dominates U^{R_B}, and $f^{R_B} \preceq_{MLR} f^{S_B}$, then S_B prefers partition y' with the decisions made by R_B to the partition y with the decisions made by R_A, i.e., $E^{S_B,R_A}(y) < E^{S_B,R_B}(y')$.*

posteriors formed after receiving certain signals of some underlying state, then the signal that results in the posterior that is an MLR spread of the other posterior is more informative, in a strong sense, than the other signal. Thus the result implies that a signal structure that is more informative than another in a certain sense, leads to more dispersed equilibrium actions.

[13] A recent paper by Lazzati (2013) also compares player's equilibrium actions and welfare, but she studies games of strategic complements and makes comparison across players for a fixed game rather than across games.

Part (i) compares the payoff of R_B, the receiver in the nested game. It says that R_B prefers the equilibrium partition y' in the nested game to the equilibrium partition y in the nesting game. Applying the result to different cases of nestedness, it has several interesting implications: (a) a receiver prefers to face a sender whose preference is closer to her own; (b) a receiver does not benefit if the sender believes that her preference is further away from the sender's than it really is; (c) a receiver does not benefit if the sender believes that the receiver hold a lower prior than her true prior.[14] Note that implication (a) generalizes Theorem 4 in Crawford and Sobel (1982), which says that under a regularity condition, for a given partition size, the receiver prefers the equilibrium partition associated with similar preferences (in their case, a smaller b).[15] Our result does not require this regularity condition.

Part (ii) compares the payoff of S_B, the sender in the nested game. The result says that S_B prefers to play against R_B, that is, a sender prefers to face a receiver whose preference is closer and he also prefers to face a receiver with a higher prior. Note that the condition $f^{R_B} \preceq_{MLR} f^{S_B}$ holds if $f^{R_B} = f^{S_B}$, i.e., if the players in Γ_B have the same prior, but this common prior assumption is not necessary for the comparison.

Does more information get transmitted in equilibrium in the nested game? If one uses the criterion that a signal is more informative if it is more valuable to *every* decision maker, then a more informative signal must be a sufficient statistic for a less informative one.[16] In the comparison of partitions, sufficiency amounts to refinement, and the equilibrium partitions induced in nested games typically do not satisfy this criterion.[17] But as we show in subsection 4.1, one can still establish useful results regarding the value of information contained in the equilibrium partitions induced in games that are nested. We have already established that the receiver in the nested game Γ_B prefers the equilibrium partition induced in Γ_B to the equilibrium partition induced under Γ_A. In the next subsection, we establish a similar result for the receiver in Γ_A under certain conditions. Together, these results imply that a receiver's payoff is higher when facing the partition

[14]For informal discussion like this, we use "closer" preference to mean single-crossing dominance in payoff functions, and "higher" prior to mean MLR dominance.

[15]The regularity is called condition M and its definition can be found in section 4.1.

[16]Here, a signal is generated by the sender's strategy.

[17]An exception is the comparison of a babbling equilibrium, which has only one partition element, and a non-babbling one. The partition in a babbling equilibrium is a strict coarsening of a partition in a non-babbling one.

induced in the nested game. In this sense, more information can be transmitted when the players preferences are closer or when the receiver's prior is higher.

4.1 Better information transmission in the nested game

To compare R_A's welfare across equilibrium partitions induced in Γ_A and Γ_B, we introduce a condition M_0, which is related to the regularity condition M, first introduced in Crawford and Sobel (1982).[18]

> *A game* $\left(u^S, u^R, f^R\right)$ *satisfies* **Condition** M *if for all* $\kappa > 1$ *and for all fixed-points* x *and* x' *of* θ^κ *in* D_κ*, such that* $x_0 = x'_0$ *and* $x_1 < x'_1$*, we have* $x_l < x'_l$ *for all* $l = 2, ..., \kappa$*.*

An agent (u^i, f^i) satisfies condition M_0 if the game (u^i, u^i, f^i) satisfies condition M. Note that in the game (u^i, u^i, f^i), both the sender and the receiver have the same preference as agent i and the receiver has the same belief as agent i. While M is a joint condition on both the sender and the receiver, M_0 is a condition only on an individual agent. (Related comparative statics results in Crawford and Sobel (1982) implicitly assume that M_0 holds and are special cases of our results.)

We provide a simple necessary and sufficient condition for M_0 in the following lemma, for which we need the following definition. For any subset X of $\mathbb{R}^k$ and any differentiable function φ that maps X to $\mathbb{R}$, a point x^* in the interior of X is *critical* for φ if $\frac{d\varphi}{dx} = 0$.

Lemma 2. *An agent* (u^i, f^i) *satisfies condition* M_0 *if and only if for any* $\kappa \geq 3$ *and any* $(x_1, x_{\kappa-1})$ *such that* $0 \leq x_1 \leq x_{\kappa-1} \leq 1$*, agent* i*'s payoff* $E^i(y)$ *has a unique critical point* $y^*(x_1, x_{\kappa-1})$ *in the interior of the set of partitions* $y \in X_\kappa$ *such that* $y_0 = 0$*,* $y_1 = x_1$*,* $y_{\kappa-1} = x_{\kappa-1}$ *and* $y_l = 1$*. Furthermore, if* M_0 *holds, this critical point* $y^*(x_1, x_{\kappa-1})$ *is an interior global maximum of* $E(y)$ *in this set.*

An immediate implication of this result is that when (u^i, f) satisfies M_0, then for all $\kappa \geq 1$, there exists a unique optimal partition in κ elements for agent i. The proof, in the Appendix, proceeds by showing that two distinct critical points exist if an only if two type vectors violating M_0 exist. We next show that if an agent (u^i, f^i) satisfies M_0,

[18] They show that under M, there exists at most one κ-equilibrium, for each $\kappa \geq 1$.

then $E^i(y)$, her expected payoff under partition y, is increasing in y for all y in a certain region $\mathcal{Z}_i$, which we introduce next.

For each $\kappa > 1$, let γ^κ be the κ-equilibrium mapping of the game (u^i, u^i, f^i). Let $Z_{i,\kappa} = \{z \in X_\kappa : z \leq \gamma^\kappa(z)\}$ and let

$$\mathcal{Z}_i = \bigcup_{\kappa=1}^{\infty} Z_{i,\kappa},$$

with the convention $Z_{i,1} = \{(0,1)\}$.[19] In words, $\mathcal{Z}_i$ contains all partitions in which at any cutoff point in the partition, the agent prefers the optimal action he would take in the immediately lower interval to that in the immediately higher interval. Note that this set contains all equilibrium partitions of all games in which agent (u^i, f^i) is the Receiver because the equilibrium mapping θ^κ of any such game is lower than γ^κ (i.e., $\theta^\kappa(z) \leq \gamma^\kappa(z)$ for any $z \in D^\kappa$) and for any equilibrium partition z, we have $z = \theta^\kappa(z)$.

Let $\leq^*$ be a partial order on $\mathcal{Z}_i$ defined as follows. For all $y' \in Z_{i,\kappa'} \cap X_{\kappa'}$ and $y'' \in Z_{i,\kappa''} \cap X_{\kappa''}$, we have $y' \leq^* y''$ if and only if $\kappa' \leq \kappa''$ and

$$\left(\underbrace{0, ..., 0}_{\kappa''-\kappa' \text{ times}}, y' \right) \leq y''.$$

Notice that when $\kappa' = \kappa''$, the partial order $\leq^*$ coincides with the partial order $\leq$. When $\kappa' < \kappa''$, the order $\leq^*$ requires that the last κ' coordinates in y'' are higher than the coordinates in y'. Let $<^*$ be the strict partial order associated with $\leq^*$. The next result shows that the expected payoff of agent i under partition y is increasing on $\mathcal{Z}_i$.

Lemma 3. *Suppose agent i satisfies M_0. Then $E^i(y)$ is increasing on $\mathcal{Z}_i$ for the partial order $\leq^*$.*

To gain some rough intuition as to why $E^i(y)$ is increasing on $\mathcal{Z}_i$ for $\leq^*$, note that for any y in $\mathcal{Z}_i$, at the cutoff points, the agent weakly prefers the optimal action he would take in the immediately lower interval to that in the immediately higher interval. Hence, as y increases on $\leq^*$ (i.e., the cutoff points shift to the right), the partition y becomes more "balanced," which reduces the average noise in the partition and makes the agent better off.

[19] When M_0 holds, one can show that for all x_0 and x_κ in $[0,1]$ such that $x_0 \leq x_\kappa$, the vector $y^*(x_0, x_\kappa)$ is the greatest element of the set $\{z \in Z_{i,\kappa} : z_0 = x_0 \text{ and } z_\kappa = x_\kappa\}$.

This lemma has a number of interesting implications. Fix a game $\Gamma = \left(u^S, u^R, f^R\right)$ and suppose R satisfies M_0. Recall that any equilibrium partition in Γ is in $\mathcal{Z}_R$. Let y' and y'' be two equilibrium partitions of Γ such that $y' <^* y''$. It follows from Theorem 3 that R's expected payoff is higher under y'' than under y', i.e., $E^R(y') < E^R(y'')$. Moreover, from Lemma 1, we know that the greatest equilibria of different sizes are ordered by $\leq^*$. Hence Lemma 3 directly implies the following result.

Corollary 1. *Fix a game $\left(u^S, u^R, f^R\right)$ in which the receiver satisfies M_0. If y^κ is the greatest κ-equilibrium partition for $\kappa = 1, ..., \overline{\kappa}$, then for any κ'-equilibrium partition y' such that $\kappa' \leq \kappa$ and $y' \neq y^\kappa$, we have $E^R(y') < E^R(y^\kappa)$.*

Note that Lemma 1 implies that the greatest κ-equilibrium partition of the highest size dominates all other equilibrium partitions for the order $\leq^*$. Call this equilibrium partition the **greatest equilibrium partition** of the game. Corollary 1 says that the greatest equilibrium gives the receiver the highest expected payoff among all equilibria of a given game. This generalizes a result in Crawford and Sobel (Theorem 3), which says that the receiver always prefers an equilibrium partition of a higher size. As shown in Proposition 1 in Chen, Kartik and Sobel (2008), the greatest equilibrium satisfies the selection criterion of "No Incentive To Separate" (NITS). As shown in Gordon (2011), it also uniquely satisfies the iterative stability criterion with respect to the best response dynamics.

In Theorem 3 below, we focus on the greatest equilibrium partition and compare the receivers' welfare in the equilibrium partitions induced under nested games. Suppose Γ_B is strictly nested into Γ_A. Theorem 3 says that under certain conditions, more information is transmitted in Γ_B in the sense that both the receiver in Γ_A and the receiver in Γ_B have higher expected payoffs under the greatest equilibrium partition induced in Γ_B than under the greatest equilibrium partition induced in Γ_A.

The crucial step in establishing this result is an immediate implication of Lemma 1, Theorem 1 and the definitions of the greatest equilibrium partition and the partial order $\leq^*$. It is the observation that the greatest equilibrium of a game nested into another one is greater in the sense of $\leq^*$ than the equilibrium of the nesting game.

Lemma 4. *Suppose Γ_B is strictly nested into Γ_A, y^B is the greatest equilibrium partition in Γ_B and y^A is the greatest equilibrium partition in Γ_A. Then $y^A <^* y^B$.*

As a direct implication of Lemma 3 and Lemma 4, we obtain the following welfare comparison.

Theorem 3. *Suppose* Γ_B *is strictly nested into* Γ_A, y^B *is the greatest equilibrium partition in* Γ_B *and* y^A *is the greatest equilibrium partition in* Γ_A. *(i) If* R_B *satisfies* M_0, *then* $E^{R_B}(y^A) < E^{R_B}(y^B)$. *(ii) If* R_A *satisfies* M_0 *and* $y^B \in \mathcal{Z}_{R_A}$, *then* $E^{R_A}(y^A) < E^{R_A}(y^B)$.

Part (i) says that R_B, the receiver in the nested game, prefers the greatest equilibrium partition in Γ_B. It complements part (i) of Theorem 2 since Theorem 2 compares equilibrium partition of the same size whereas the greatest equilibrium partition in Γ_B may have a higher size than that in Γ_A. In particular, it says that when we focus on the greatest equilibrium partitions, then a receiver prefers to face a sender with a closer preference.

Part (ii) says that if $y^B \in \mathcal{Z}_{R_A}$, then R_A, the receiver in the nesting game, also prefers the greatest equilibrium partition in Γ_B. As implied by Lemma 1 and Theorem 1, $y^A <^* y^B$. So part (ii) is a direct implication of Lemma 3. Intuitively, as long as y^B is not "too far" to the right, R_A is better off under partition y^B than under y^A.

In Remark 1 at the end of this section, we discuss a class of game pairs (Γ_A, Γ_B) for which the condition $y^B \in \mathcal{Z}_{R_A}$ is always satisfied. In other games, this condition can be violated. We illustrate this by the following example in which $y^B \notin \mathcal{Z}_{R_A}$ and R_A is better off under partition y^A than under y^B.

Example 1. *Suppose* $u^{S_A} = u^{S_B} = -(a-t-b)^2$ *for* $t \in [0,1]$ *where* $b > 0$, $u^{R_B} = -(a-t)^2$ *for* $t \in [0,1]$, $u^{R_A} = -10^5 (a-t)^2$ *for* $t \in [0, 0.0534]$, $u^{R_A} = -10^2 (a-t)^2$ *for* $t \in (0.0534, 0.6]$ *and* $u^{R_A} = -(a-t)^2$ *for* $t \in (0.6, 1]$. *Also, assume that* $f^{R_A} = f^{R_B} = 1$ *for all* $t \in [0,1]$.

It is straightforward to verify that $U^{R_B}(a,t,t')$ *weakly single-crossing dominates* $U^{R_A}(a,t,t')$ *for any* t, t' *and* Γ_B *is nested into* Γ_A.[20] *Intuitively, since both* R_A*'s and* R_B*'s optimal actions equal to weighted averages of the state and* R_A *places higher weights on lower states,* $a^{R_A}(t,t') \le a^{R_B}(t,t')$ *for any* t, t'.

Let $b = 0.125$. *Calculation shows that the greatest equilibrium partition in* Γ_B *is* $y^B = (0, 0.25, 1)$ *and the greatest equilibrium partition in* Γ_A *is* $y^A = (0, 0.0534, 1)$.

[20] One can use the result in Quah and Strulovici (2012), as described in footnote 6.

To see that $y^B \notin \mathcal{Z}_{R_A}$*, note that* $a^{R_A}(0, 0.25) = 0.02718$ *and* $a^{R_A}(0.25, 1) = 0.429$*. When* $t = 0.25$*,* $u^{R_A}(a, t) = -10^2 (a - t)^2$*, and* R_A *prefers the higher action* 0.429 *to the lower action* 0.02718 *at* $t = 0.25$ *and therefore* $y^B \notin \mathcal{Z}_{R_A}$*. Calculation shows that* $E^{R_A}(y^A) > E^{R_A}(y^B)$*. Intuitively, since* R_A *places much higher weight on what happens in states* $t \in [0, 0.0534]$ *than in other states, the information contained in the partition* $(0, 0.0534, 1)$ *is more valuable to* R_A *than the information contained in the partition* $(0, 0.25, 1)$*.*

Finally, we end this section by discussing a class of game pairs (Γ_A, Γ_B) for which some of our results still hold even when we relax the conditions of nestedness. Consider a class of game pairs (Γ_A, Γ_B) such that there exists a function $g(a, t)$ with $g_{aa} < 0$, $g_{at} > 0$, and $u^i(a, t) = g(a - b_i, t)$ for each $i \in \{R_A, R_B, S_A, S_B\}$. For example, the quadratic loss utility function, $g(a - b_i, t) = -(a - b_i - t)^2$, belongs to this class. For this class of game pairs, Γ_B is nested into Γ_A if and only if $b_{R_A} \leq b_{R_B} < b_{S_B} \leq b_{S_A}$ and f^{R_B} MLR-dominates f^{R_A}. Moreover, Γ_B is strictly nested into Γ_A if at least one of the weak inequalities holds strictly or if f^{R_B} strictly MLR-dominates f^{R_A}. We next show that under a weaker condition, which only restricts $b_{S_B} - b_{R_B}$ and $b_{S_A} - b_{R_A}$, some of our results still hold (Theorem 1, Theorem 2 part (i) and Theorem 3, but not Theorem 2 part (ii)).[21]

Theorem 4. *Suppose that* Γ_A *and* Γ_B *are such that* f^{R_B} *MLR-dominates* f^{R_A} *and there exists a function* $g(a, t)$ *such that* $g_{aa} < 0$ *and* $g_{at} > 0$ *with* $u^i(a, t) = g(a - b_i, t)$ *for each* $i \in \{R_A, R_B, S_A, S_B\}$*. Suppose further that* $b_{S_B} - b_{R_B} \leq b_{S_A} - b_{R_A}$*. Then the following statements hold.*

1. *(i) If* Γ_A *has a* κ*-equilibrium, then* Γ_B *also has a* κ*-equilibrium. (ii) Let* $\overline{x}^A$ *and* $\overline{x}^B$ *be the respective greatest* κ*-equilibrium of* Γ_A *and* Γ_B*. Then* $\overline{x}^A \leq \overline{x}^B$*. If* $b_{S_B} - b_{R_B} < b_{S_A} - b_{R_A}$ *or* f^{R_B} *strictly MLR-dominates* f^{R_A}*, then* $\overline{x}^A < \overline{x}^B$*.*

2. *Suppose that either* $b_{S_B} - b_{R_B} < b_{S_A} - b_{R_A}$ *or* f^{R_B} *strictly MLR-dominates* f^{R_A}*. Let* y *be a* κ*-equilibrium partition of* Γ_A*. Then there is a* κ*-equilibrium partition* y' *in* Γ_B *such that* R_B *prefers the partition* y' *to the partition* y*, i.e.,* $E^{R_B}(y) < E^{R_B}(y')$*.*

[21]Since we assume that u^S strictly single-crossing dominates u^R for any $t \in [0, 1]$, we have $b_{S_B} - b_{R_B} > 0$ and $b_{S_A} - b_{R_A} > 0$.

3. *Suppose that either* $b_{S_B} - b_{R_B} < b_{S_A} - b_{R_A}$ *or* f^{R_B} *strictly MLR-dominates* f^{R_A}, *and that* y^B *is the greatest equilibrium partition in* Γ_B *and* y^A *is the greatest equilibrium partition in* Γ_A. *(i) If* R_B *satisfies* M_0, *then* $E^{R_B}(y^A) < E^{R_B}(y^B)$. *(ii) If* R_A *satisfies* M_0 *and* $y^B \in \mathcal{Z}_{R_A}$, *then* $E^{R_A}(y^A) < E^{R_A}(y^B)$.

Part 1 is the counterpart of Theorem 1, part 2 is the counterpart of Theorem 2 part (i), and part 3 is the counterpart of Theorem 3. As for Theorem 2 part (ii), the counterpart does not hold because the sender S_B may prefer to play against the receiver R_A instead of the receiver R_B, if for example S_B has the same preferences as R_A. The idea of the proof of Theorem 4 is straightforward. For the class of games considered in the theorem, the equilibrium partitions only depend on the difference $b_S - b_R$ and the prior of the receiver. It follows that game Γ_B has the same equilibrium partitions as some game Γ'_B that is nested (or even strictly nested) into game Γ_A. Applying Theorem 1, Theorem 2 part (i) and Theorem 3 to Γ_A and Γ'_B, we obtain Theorem 4.

Remark 1. *In Theorem 4 part 3(ii), we require that* $y^B \in \mathcal{Z}_{R_A}$. *If* $f^{R_A} = f^{R_B}$, *the condition is always satisfied. This is because in this case we have* $\mathcal{Z}_{R_A} = \mathcal{Z}_{R_B}$. *Since* $y^B \in \mathcal{Z}_{R_B}$, *it follows that* $y^B \in \mathcal{Z}_{R_A}$.

5 Applications

We illustrate the usefulness of the comparative statics and welfare results derived in the previous section with the following three applications.

5.1 Choosing an intermediary

Instead of communicating with an informed agent and then making the decision herself, a principal sometimes may choose a representative (intermediary) to communicate with an agent and make decisions on her behalf. For example, diplomatic envoys are given the right to negotiate with a foreign country and make certain decisions on behalf of their government. Similarly, executives sometimes delegate decisions to external consultants.[22] What is the advantage of using an intermediary and what characteristics of an

[22]Note that the intermediary discussed here is different from the non-strategic mediator in Goltsman, Hörner, Pavlov and Squintani (2009) and the strategic intermediaries in Li (2010), Ambrus, Azevedo

intermediary make him attractive to the principal?

Earlier work by Dessein (2002) showed that a principal may benefit from giving authority to an intermediary whose preference is closer to the agent's because the agent communicates more information when facing such an intermediary. But another possibility, not discussed in Dessein (2002), is that intermediaries may have different *beliefs* from the principal, and we apply our result to this case here. To highlight the effect of prior beliefs on the choice of an intermediary, assume that the intermediary' payoff function is the same as the principal's. To be consistent with our main model, we maintain the assumption that the sender (i.e., the agent in this application) has an upward bias in his preference, i.e., his payoff function single-crossing dominates the payoff function of the principal and the intermediary.

Giving the communication and decision making right to an intermediary has two effects: (1) Since the agent now faces someone with a different prior, his communication incentives are different, which may result in either informational gain or loss, as will be seen precisely in the discussion that follows.[23] (2) Given the information conveyed by the agent, the intermediary's optimal choice of action is generally different from that of the principal's because of their differences in beliefs. Since this always results in a loss for the principal, the use of an intermediary makes a principal better off only when there is sufficient informational gain.

Since the agent wants to convince the principal that the state is higher than it is (for example, a division manager wants the executive of a corporation to believe that the investment prospect in his division is better than it is), we interpret a higher prior to be a more "optimistic" belief in this application. Let f^P be the principal's prior and f^I be the intermediary's prior.

Suppose $f^I \preceq_{MLR} f^P$, i.e., the intermediary is more pessimistic about the state of nature than the principal does. Then by Theorem 3, the principal strictly prefers communicating directly with the agent than having the intermediary communicating

and Kamada (2013) and Ivanov (2010). Here, the principal gives the communication and decision right to the intermediary whereas in the other papers, the mediator or intermediary only passes information from the agent to the principal and does not make any payoff-relevant decision himself. Moreover, the papers on strategic intermediaries focus on how difference in preference may impede or facilitate information transmission whereas the focus here is on the difference in beliefs.

[23]The intermediary's belief is assumed to be known to the agent.

with the agent. Using the intermediary results in informational loss and therefore the principal does not benefit from giving the decision making right to a more pessimistic intermediary.

Now suppose $f^P \preceq_{MLR} f^I$, i.e., the intermediary is more optimistic. In this case, as shown in Theorem 3, more information is transmitted when the agent communicates to the intermediary. This informational gain could be sufficiently high that the principal ultimately is made better off by giving authority to a more optimistic intermediary. The following example illustrates this point.

Example 2. *Suppose* $f^P(t) = 1$, $f^I(t) = \frac{1}{2} + t$, *the principal's (and also the intermediary's) utility function is* $-(a-t)^2$ *and the agent's utility function is* $-(a-t-0.2)^2$. *Since* $\frac{f^I(t)}{f^P(t)} = \frac{1}{2} + t$ *is increasing in* t, $f^P \preceq_{MLR} f^I$.

If the agent communicates directly with the principal, then the greatest equilibrium partition is $(0, 0.1, 1)$ *and the principal's expected payoff is* -0.061. *If the intermediary makes the decision, then the greatest equilibrium partition in the communication game between the agent and the intermediary is* $(0, 0.158, 1)$ *and the principal's expected payoff is* -0.031. *Hence the principal is better off giving the authority to the intermediary.*

More generally, we have the following result.

Result: *The principal never benefits from choosing a more pessimistic intermediary, but may benefit from choosing a more optimistic intermediary.*

The first part of the result follows from Theorem 2, which directly implies that a more pessimistic intermediary will obtain from the agent less information and the fact that given this information, the intermediary will take actions that are worse for the principal than the ones he would choose for himself. In other words the two effects we previously identified (amount of information and use of information) work in the same direction: they both cause a welfare loss for the principal.

Che and Kartik (2009) show that to motivate an advisor to acquire information, it is optimal for a decision maker to choose someone with a different prior than her own. Our result complements theirs, but there are a number of important differences. First, the decision maker chooses which advisor to communicate with in Che and Kartik (2009) whereas the principal chooses which intermediary to give the communication and decision-making right to here. Second, the information that an advisor possesses is endogenous in Che and Kartik (2009) whereas the agent's information is exogenously given here. Most importantly, Che and Kartik (2009) focus on the incentives created by the differences in opinion for *information acquisition*, but the focus here is on the advantage of optimism (a particular kind of difference in opinion) for *information transmission*.

5.2 Centralization versus delegation in organizations

When to delegate decisions to lower management and when to centralize authority is an important problem in organizations. The central trade-off that the upper management faces is that delegation makes better use of the lower management's local knowledge but typically results in distortions because of the different objectives that the lower management may have.

Whether centralized decision making (with communication from the informed agent) dominates delegation depends on how much information can be conveyed from the agent to the principal under centralization, which in turn depends on the principal's prior. Dessein (2002) provides a detailed characterization for uniform distributions and finds that under a uniform prior, delegation is better than centralization whenever the agent's bias is small enough that informative communication is feasible. This implies that whenever the principal chooses centralization over delegation, there is no information conveyed by the agent's report and the principal makes the decision based only on her prior. As one would expect, this result relies on the uniform assumption and Dessein (2002) provides more general results for symmetric priors, but it is still unclear what happens with asymmetric priors. Although a full characterization is hard to obtain, we take a step forward and provide some new insight into the value of centralization relative to delegation for a family of Beta distributions that includes the uniform distribution as a special case.

Suppose the principal's prior f_α is a Beta distribution on $[0,1]$ with $f_\alpha(x) = \alpha x^{\alpha-1}$. If $\alpha = 1$, then the distribution is uniform. Moreover, $f_{\alpha'} \preceq_{MLR} f_\alpha$ if and only if $\alpha' < \alpha$. Suppose the players's utility function take the commonly used quadratic form, i.e., the principal's utility function is $-(a-t)^2$ and the agent's utility function is $-(a-t-b)^2$.

If the principal delegates the decision to the agent, then the agent chooses the action $a = t + b$ and the principal's payoff is $-b^2$. If the principal keeps the authority and makes a decision based on her prior f_α, then, given the quadratic-loss utility function, her expected payoff is equal to $-\left(\frac{\alpha}{(1+\alpha)^2(\alpha+2)}\right)$, the negative of the variance of t. Moreover, if the principal keeps her authority but communicates with the agent, then an informative equilibrium exists when the agent's bias is sufficiently small, when $b < \frac{1}{2}\left(\frac{\alpha}{\alpha+1}\right)$. Since the principal's payoff in an informative equilibrium is higher than if she makes a decision based on only her prior, it immediately follows that if $-\left(\frac{\alpha}{(1+\alpha)^2(\alpha+2)}\right) \geq -b^2$ and $b < \frac{1}{2}\left(\frac{\alpha}{\alpha+1}\right)$, then informative communication is feasible under centralization and it dominates delegation. When are these conditions satisfied? In the case of uniform distribution ($\alpha = 1$), we have $\frac{1}{2}\left(\frac{\alpha}{1+\alpha}\right) = \frac{1}{4}$ and $-\left(\frac{\alpha}{(1+\alpha)^2(\alpha+2)}\right) = -\frac{1}{12}$. Since $-\frac{1}{12} < -\left(\frac{1}{4}\right)^2$, delegation dominates centralization whenever informative communication is possible, consistent with the finding in Dessein (2002). However, if $\sqrt{\frac{\alpha}{(1+\alpha)^2(\alpha+2)}} < \frac{1}{2}\left(\frac{\alpha}{1+\alpha}\right)$, i.e., $\alpha > \sqrt{5}-1$, there exists a range of biases such that informative communication is better than delegation. Note that for $\alpha > 1$, the function $\sqrt{\frac{\alpha}{(1+\alpha)^2(\alpha+2)}}$ is decreasing in α and the function $\frac{1}{2}\left(\frac{\alpha}{1+\alpha}\right)$ is increasing in α. So a principal with a higher prior is more likely to centralize authority as it is more likely for her to gain from communication.

5.3 Two-way communication with an informed principal

If the principal privately observes a signal s which is correlated with the agent's type t, then the principal's belief about t depends on the realization of the signal s. For simplicity, we consider the case in which the signal s has two realizations: s_H and s_L ($s_H > s_L$). Let f^H denote the principal's belief on t when observing $s = s_H$ and f^L denote her belief on t when observing $s = s_L$. Suppose the conditional probability distribution $p(s|t)$ satisfies the monotone likelihood ratio property, i.e., t and s are affiliated. Then $f^L \preceq_{MLR} f^H$. (The observation s_H is more "favorable" news than s_L in the sense of Milgrom, 1981.)

There are numerous applications in which the principal has private information correlated with the agent's type.[24] For example, managers ask their subordinates to evaluate workers to help with compensation and promotional decisions, but they may have their own assessment of workers from occasional interaction with them. In a setting like this, the principal often has an opportunity to communicate to the agent first, before the

[24]Only a few papers have modeled privately informed receivers in the literature. These include Seidmann (1990), Watson (1996), Olszewski (2004), Harris and Raviv (2005), Lai (2008), Ishida and Shimizu (2010) and Moreno de Barreda (2010), but these models have different assumptions on information structure and address different questions from ours.

agent reports. For example, a manager can discuss a worker's performance with the worker's supervisor before the supervisor submits his evaluation. In this game of two-way communication, does the principal reveal her signal to the agent in the first stage of communication?[25]

To see how the results from the previous section helps us answer this question, note that if the principal reveals her signal truthfully in the first stage, then she no longer has any private information in the second stage. The continuation game is a standard sender-receiver game with an uninformed receiver, with appropriately updated beliefs. In particular, since $f^L \preceq_{MLR} f^H$, the continuation game after the revelation that $s = s_H$ (call this Γ_H) is nested into the continuation game after the revelation that $s = s_L$ (call this Γ_L). In what follows, we consider two kinds of two-way communication game, differing in whether the principal's message is verifiable or not.[26] (In both cases we assume that the agent's message is cheap talk.)

Theorem 3 in the previous section implies that the principal with belief f^H prefers the greatest equilibrium partition in Γ_H to the greatest equilibrium partition in Γ_L, and therefore the principal who has observed s_H has no incentive to pretend to have observed s_L. This immediately implies the following result.

Result: *If the principal's signal s is verifiable, then there exists an equilibrium in which the principal truthfully reveals s in the two-way communication game.*

What happens if the principal's signal is not verifiable and she can only send a cheap-talk message in the two-way communication game? In Theorem 3, we see that under certain conditions the principal with belief f^L also prefers the greatest equilibrium partition in Γ_H to that in Γ_L. Under these conditions, the principal with the signal s_L has the incentive to pretend that she has observed s_H, making it impossible to have truthful revelation by the principal in equilibrium. Note that these conditions are satisfied if the equilibrium partition does not shift "too far" to the right when the principal's belief

[25]To be more precise about the game we analyze, consider the following extensive-form. At the beginning of the game, the agent privately observes t and the principal privately observes s. Then two-way communication takes place sequentially: in stage one, the principal sends a message to the agent; in stage two, the agent sends a message back to the principal. Then the principal chooses an action.

[26]Verifiability means that the messages available to the principal depends on the realization of s: she can truthfully reveal s or stay silent.

changes from f^L to f^H. How far the equilibrium partition shifts to the right in turn depends on the informativeness of the principal's signal.

To illustrates this, consider an example in which the principal's signal belongs to the following class of information structure. The conditional probability of s is given by: $p(s_H|t) = \pi q(t) + (1-\pi)c$ and $p(s_L|t) = \pi(1-q(t)) + (1-\pi)(1-c)$, where $q(\cdot)$ is a function of t. In other words, the principal observes a mixture between an informative and an uninformative experiment. With probability π, the experiment is successful and the signal is informative (the probability of observing s_H or s_L depends on t); with probability $(1-\pi)$, the experiment fails and the signal is uninformative (the probability of observing s_H or s_L does not depend on t). The principal knows the probability that the experiment is successful, but does not observe whether the experiment has succeeded or not.[27] Assume $q(t)$ is increasing in t, which implies that for a fixed π, the posterior of the principal after observing s_H MLR dominates her posterior after observing s_L. Moreover, a higher π means that the signal s is more informative (in the sense of Blackwell (1951)). So π is an informativeness parameter of the principal's signal. One useful characteristic of this class of information structure is that as the informativeness parameter varies, the resulting posteriors of the principal are ordered by MLR dominance. In particular, suppose $\tilde{s}$ is a signal with informativeness parameter $\tilde{\pi}$ and let $f^{\tilde{H}}$ denote the principal's posterior after observing that $\tilde{s} = s_H$ and $f^{\tilde{L}}$ denote her posterior after observing $\tilde{s} = s_L$. If $\pi > \tilde{\pi}$, then $f^L \preceq_{MLR} f^{\tilde{L}} \preceq_{MLR} f^{\tilde{H}} \preceq_{MLR} f^H$ (intuitively, the posteriors induced by the more informative signal s are more spread out than the posteriors induced by the less informative signal $\tilde{s}$) and the results from section 4 apply.[28] We now provide an example where s is not verifiable, and where the principal truthfully reveals s in equilibrium only when it the signal is sufficiently informative.

Example 3. *For the informative experiment, assume* $q(t) = 0$ *if* $t \in [0, 0.45)$, $q(t) = \frac{1}{4} + \frac{1}{2}t$ *if* $t \in [0.45, 0.55]$ *and* $q(t) = 1$ *if* $t \in (0.55, 1]$. *For the uninformative experiment, assume* $s = s_L$ *and* $s = s_H$ *with equal probability. These imply that the conditional probabilities are* $p(s_H|t) = (1-\pi)\frac{1}{2}$ *if* $t \in [0, 0.45)$, $p(s_H|t) = \pi\left(\frac{1}{4} + \frac{1}{2}t\right) + (1-\pi)\frac{1}{2}$ *if* $t \in [0.45, 0.55]$ *and* $p(s_H|t) = \pi + (1-\pi)\frac{1}{2}$*if* $t \in (0.55, 1]$. *Also, assume the principal's*

[27] This assumption on information structure is analogous to Ottaviani and Sorensen (2006). It is different from the success-enhancing models (e.g. Green and Stokey 2007) that assume whether the experiment has succeeded or failed is observed.

[28] A proof is provided in the appendix.

utility function is $-(a-t)^2$*, the agent's utility function is* $-(a-t-0.08)^2$*, and the common prior on* t *is uniform on* $[0,1]$.

Consider the case in which $\pi = \frac{3}{5}$*. The greatest equilibrium partition in* Γ_L *is* $y^L = (0, 0.21, 1)$ *and the greatest equilibrium partition in* Γ_H *is* $y^H = (0, 0.014, 0.38, 1)$*. Straightforward calculation shows the principal prefers* y^H *to* y^L *independent of her own private signal, and therefore she cannot credibly reveal* s *to the agent. For any less informative signal* $\tilde{s}$ *with* $\tilde{\pi} < \frac{3}{5}$*, the principal cannot credibly reveal her signal either.*

As the principal's signal gets more informative, she may be able to reveal her signal credibly in equilibrium. We see this clearly in the limit as π *goes to* 1*. In the limit, the support of the principal's posterior when she observes* s_L *is* $[0, 0.55]$ *and the support of her posterior when she observes* s_H *is* $[0.45, 1]$*. The greatest equilibrium partition in* Γ_L *is* $y^L = (0, 0.103, 0.55)$ *and the greatest equilibrium partition in* Γ_H *is* $y^H = (0.45, 0.586, 1)$*. Since* $0.586 > 0.55$*, the information contained in partition* y^H *has no value to the principal with signal* s_L *and therefore the principal with signal* s_L *has no incentive to pretend that her signal is* s_H*. Hence there exists an equilibrium in which the principal reveals her signal credibly in the first stage of communication.*

6 Conclusion

This paper contributes to the understanding of strategic communication by introducing a novel relation called "nestedness" among sender-receiver games and linking the amount of information transmitted in equilibrium and the players' welfare to nestedness. Unlike previous results in the literature, our results do not depend on condition M (Crawford and Sobel, 1982). More importantly, we provide new comparative statics results with respect to the prior, not just the payoff functions. The unified approach of "nestedness" allows us to identify what drives the amount of information transmitted in equilibrium. We also provide three applications to illustrate the usefulness of the results.

7 Appendix

Proof of Lemma 1. (i) For any two actions a, a' such that $a < a'$, let $t = \tau(a, a')$ be the unique sender type (if there is any) that is indifferent between actions a and a', i.e. $u^S(a,t) = u^S(a',t)$ and for all a, let $t = \tau(a,a)$ be the unique sender type (if there is any) whose preferred action is a. Concavity of $u^S(a,t)$ in a and the single-crossing condition on u^S imply that the function τ is increasing in its arguments.

Note that for all $k = 1, ..., \kappa - 1$, we have

$$\theta_k^\kappa(z) = \tau\left(a^R(z_{k-1}, z_k), a^R(z_k, z_{k+1})\right).$$

Thus θ^κ is a composite of $a^R(\cdot)$ and $\tau(\cdot)$, which are both increasing mappings. It is therefore an increasing mapping. (ii) Next, let z^* be a κ-equilibrium. Then the sequence (x^n) defined by $x^0 := (0, 1, ..., 1)$ and $x^{n+1} = \theta^\kappa(x^n)$ is well-defined, decreasing and bounded below by z^*, therefore converges to some fixed-point, x^* that is greater than or equal to z^* (and for the same reason also greater than or equal to any other κ-equilibrium). Thus x^* is the greatest κ-equilibrium.

(iii) By (ii), since there are κ-equilibria and $(\kappa+1)$-equilibria, the sequence (x^n) defined in (ii) converges to the greatest κ-equilibrium x^* and the sequence (y^n) in $X_{\kappa+1}$ such that $y^0 := (0, 1, ..., 1)$ and $y^{n+1} = \theta^{\kappa+1}(y^n)$ is well-defined, decreasing and converges to y^*. Then

$$(0, 1, ..., 1) = x^0 \leq \left(y_1^0, ..., y_{\kappa+1}^0\right) = (1, ..., 1).$$

By monotonicity of θ^κ and $\theta^{\kappa+1}$, we obtain $x^n \leq \left(y_1^n, ..., y_{\kappa+1}^n\right)$ for all n, therefore $x^* \leq \left(y_1^*, ..., y_{\kappa+1}^*\right)$, the desired inequality. ■

Proof of Theorem 1. Let θ_A^κ and θ_B^κ be the respective κ-equilibrium mappings of each of the two games. For any z such that θ_A^κ is well-defined at z, the fact that Γ_B is nested into Γ_A implies that θ_B^κ is well-defined at z, and that $\theta_A^\kappa(z) \leq \theta_B^\kappa(z)$, with a strict inequality of the nestedness is strict. In particular, if $\overline{x}^A$ is the greatest κ-equilibrium of Γ_A, the sequence defined by $z^0 = \overline{x}^A$ and $z^{n+1} = \theta_B^\kappa(z^n)$ for all $n \geq 0$ is well-defined, increasing and converges to some κ-equilibrium x^B of Γ_B such that $\overline{x}^A \leq x^B \leq \overline{x}^B$, where the second inequality holds by definition of the greatest κ-equilibrium, which proves both claims. Note the the first inequality is strict if the nestedness is strict.■

The following Lemma is useful in the proof of Theorem 2.

Lemma 5. *Let $\left(u^S, u^R, f^R\right)$ be a game. Let $\kappa \geq 1$ and $y \in D_\kappa \cap X_\kappa$, such that $y < \theta^\kappa(y)$. Then the following holds. (i) $E^R(\theta^\kappa(y)) > E^R(y)$. (ii) If U^S weakly single-crossing dominates U^R, then $E^{S,R}(\theta^\kappa(y)) > E^{S,R}(y)$.*

Proof. Let $i \in \{R, S\}$. Let $y' := \theta^{\kappa}(y)$. For each $h = 0, ..., \kappa - 1$, let $a_h^R := a^R(y_h, y_{h+1})$.

Step 1: First we show that for each h, $U^i\left(a_h^R, y_h, y_h'\right) - U^i\left(a_{h-1}^R, y_h, y_h'\right) \leq 0$.

By definition of y', $u^S\left(a_h^R, y_h'\right) - u^S\left(a_{h-1}^R, y_h'\right) = 0$ for each h. By supermodularity, this implies that $u^S\left(a_h^R, s\right) - u^S\left(a_{h-1}^R, s\right) \leq 0$ for each $s \in [y_h, y_h']$. Thus for each h,

$$U^S\left(a_h^R, y_h, y_h'\right) - U^S\left(a_{h-1}^R, y_h, y_h'\right) \leq 0.$$

Since U^S weakly single-crossing dominates U^i, this implies that for each h,

$$U^i\left(a_h^R, y_h, y_h'\right) - U^i\left(a_{h-1}^R, y_h, y_h'\right) \leq 0.$$

Step 2: Second, we show that for each h, $U^i\left(a_h^R, y_h', y_{h+1}'\right) < U^i\left(a_h^R\left(y_h', y_{h+1}'\right), y_h', y_{h+1}'\right)$.

We know $U^i\left(a, y_h', y_{h+1}'\right)$ is concave in a and reaches a maximum at $a^i(y_h', y_{h+1}')$. Moreover, $a_h^R < a_h^R\left(y_h', y_{h+1}'\right) \leq a^i(y_h', y_{h+1}')$. The first inequality holds because the function $a^R(t, t')$ is increasing in (t, t'). The second inequality holds because U^i weakly single crossing dominates U^R. It follows that

$$U^i\left(a_h^R, y_h', y_{h+1}'\right) < U^i\left(a_h^R\left(y_h', y_{h+1}'\right), y_h', y_{h+1}'\right),$$

for each h.

Step 3:

$$\begin{aligned}
E^i(y) - E^i(y') &= \sum_{h=0}^{\kappa-1}\left[U^i\left(a_h^R, y_h, y_{h+1}\right) - U^i\left(a_h^R\left(y_h', y_{h+1}'\right), y_h', y_{h+1}'\right)\right] \\
&< \sum_{h=0}^{\kappa-1}\left[U^i\left(a_h^R, y_h, y_{h+1}\right) - U^i\left(a_h^R, y_h', y_{h+1}'\right)\right] \\
&= \sum_{h=1}^{\kappa-1} U^i\left(a_h^R, y_h, y_h'\right) - U^i\left(a_h^R, y_{h+1}, y_{h+1}'\right) \\
&= \sum_{h=1}^{\kappa-1} U^i\left(a_h^R, y_h, y_h'\right) - U^i\left(a_{h-1}^R, y_h, y_h'\right) \\
&\leq 0.
\end{aligned}$$

Where the first inequality holds by Step 2 and the last inequality by Step 1.■

Proof of Theorem 2. Let $y^0 := y$. Let θ_A^{κ} and θ_B^{κ} be respectively the κ-equilibrium mappings of Γ_A and Γ_B. Consider the sequence$(y^n)_{n\geq 0}$ of vectors in $X_\kappa \cap D_\kappa$ such that

$y^{n+1} = \theta_B^\kappa(y^n)$ for all $n \geq 0$. We know that $\theta_A^\kappa < \theta_B^\kappa$ by Theorem 1. Thus

$$y^0 = \theta_A^\kappa\left(y^0\right) < \theta_B^\kappa\left(y^0\right) = y^1.$$

Since $\theta_B^\kappa(\cdot)$ is monotone, and $y^0 < y^1$, it follows that $y^n < y^{n+1}$ for all n. Thus the sequence (y^n) converges to some κ-equilibrium y' of Γ_B. By Lemma 5, we have $E^{i,R_B}(y^n) < E^{i,R_B}(y^{n+1})$ for all n and all agent $i \in \{S_B, R_B\}$, characterized by (u^i, f^i). Thus $E^{i,R_B}(y) < E^{i,R_B}(y')$.

In the case, $i = R_B$, this proves claim (i).

In the case $i = S_B$, we obtain $E^{S_B,R_B}(y) < E^{S_B,R_B}(y')$. But for each h,

$$a^{R_A}(y_h, y_{h+1}) < a^{R_B}(y_h, y_{h+1}) < a^{S_B}(y_h, y_{h+1}).$$

The first inequality holds by nestedness and the second because U^{S_B} weakly single-crossing dominates U^{R_B}. Moreover $U^{S_B}(a, y_h, y_{h+1})$ is concave in a and maximized at $a^{S_B}(y_h, y_{h+1})$, therefore $E^{S_B,R_A}(y) < E^{S_B,R_B}(y)$, so that $E^{S_B,R_A}(y) < E^{S_B,R_B}(y')$, which is claim (ii). ■

Proof of Lemma 2. For each $\kappa > 1$, let γ^κ be the κ-equilibrium mapping of the game (u^i, u^i, f^i). First observe that an interior partition y is a critical point for $E^i(y)$ if and only if $(y_1, ..., y_{\kappa-1})$ is a fixed point of the equilibrium mapping $\gamma^{\kappa-2}$. If two distinct interior critical points such that $y_0 = 0$, $y_1 = x_1$, $y_{\kappa-1} = x_{\kappa-1}$ and $y_l = 1$ exist, we have a violation of M_0. Conversely, if M_0 is not satisfied, then for some $\kappa \geq 3$, there are two fixed-points $(y_1, ..., y_{\kappa-1})$ *and* $(z_1, ..., z_{\kappa-1})$ *of* $\gamma^{\kappa-1}$ *in* $D_{\kappa-1}$, *such that* $y_1 = z_1$ *and* $y_2 < z_2$, *and* $y_{\kappa-1} > z_{\kappa-1}$. By continuity, in this case we can find a third fixed point $\left(y_1', ..., y_{\kappa-1}'\right)$ *of* $\gamma^{\kappa-1}$ *in* $D_{\kappa-1}$, *such that* $y_1 = y_1'$ *and* $y_2 < y_2'$, *and* $y_{\kappa-1} = y_{\kappa-1}'$. The partitions $(0, y_1, ..., y_{\kappa-1}, 1)$ and $(0, y_1, ..., y_{\kappa-1}, 1)$ are then two distinct interior critical points of $E^i(y)$ in X_κ with the same coordinates y_1 and $y_{\kappa-1}$.

Last, note that the function $E(y)$ must have at least one interior global maximum, so if condition M_0 holds, the unique critical point is a global maximum.■

Proof of Theorem 3. For any $\kappa \geq 1$ and any nonempty set $X \subseteq T^{\kappa+1}$, let **the supremum of** X, denoted by $\sup[X]$ be the vector in $T^{\kappa+1}$ whose l-th coordinate is the supremum of the set $\{x_l \in T :$ there is $x_{-l} \in T^\kappa$ such that $(x_l, x_{-l}) \in X\}$. A subset

$X \subseteq T^m$ is **complete** if, for each nonempty $Y \subseteq X$, we have $\sup[Y] \in X$.[29] The proof of the Theorem is in five steps.

Step 1: Let $\kappa \geq 2$. Then $Z_{i,\kappa}$ is complete.

Proof. Let Z be an arbitrary nonempty subset of $Z_{i,\kappa}$. Since $Z \subseteq W_\kappa$, and W_κ is complete, then $\sup[Z] \in W_\kappa$. Let z be an arbitrary element of Z. Then, $z \leq \sup[Z]$. Since $\gamma^\kappa(\cdot)$ is nondecreasing, then $\gamma^\kappa(z) \leq \gamma^\kappa(\sup[Z])$. Since $z \in Z_{i,\kappa}$, then $z \leq \gamma^\kappa(z)$. Therefore $z \leq \gamma^\kappa(\sup[Z])$. Since this holds for all $z \in Z$, therefore $\sup[Z] \leq \gamma^\kappa(\sup[Z])$. Therefore $\sup[Z] \in Z_{i,\kappa}$, the desired conclusion.□

Step 2: Let $\kappa > 1$ and let (u^i, f^i) satisfy M_0. Let $x, x' \in W_\kappa$ satisfy $\gamma^\kappa(x) = x$, $\gamma^\kappa(x') \geq x'$ and $(x_0, x_\kappa) = (x'_0, x'_\kappa)$. Then $x' \leq x$.

Proof. As a consequence of Step 1, the set $Z := \{z \in Z_{i,\kappa} : z_0 = x_0 \text{ and } z_\kappa = x_\kappa\}$ is complete. Let x^* be the greatest element of Z. Let $x^{**} := \gamma^\kappa(x^*)$. Let us prove that $x^{**} \in Z$. We already know that $x^{**} \in X_\kappa$. Since $x^* \in Z$, then $x^* \leq x^{**}$. Since $\gamma^\kappa(\cdot)$ is increasing, we have $x^{**} \leq \gamma^\kappa(x^{**})$. Therefore $x^{**} \in Z_{T,\kappa}$. Since $x_0^{**} = x_0^* = x_0$ and $x_\kappa^{**} = x_\kappa^* = x_\kappa$, it follows that $x^{**} \in Z$. Since x^* is the greatest element of Z, then $x^{**} \leq x^*$. Thus $x^{**} = x^*$. Since (u^i, f^i) satisfies M_0, we have $x = x^*$. Since $x' \in Z$, this implies $x' \leq x$.□

Step 3: Let $\kappa > 1$, and let (u^i, f^i) satisfy M_0. Then $Z_{i,\kappa}$ is monotonically connected. That is, for all $y' \leq y'' \in Z_{i,\kappa}$, there is a continuous nondecreasing path $y : [0,1] \to Z_{i,\kappa}$ such that $y(0) = y'$ and $y(1) = y''$.

Proof. For all $\lambda \in [0,1]$, let $g(\lambda) := (1-\lambda)y' + \lambda y''$ and $y(\lambda) := \sup(Z_{i,\kappa} \cap [0_\kappa, g(\lambda)])$. As a consequence of Step 1, the set $Z_{i,\kappa} \cap [0_\kappa, g(\lambda)]$ is complete for all λ. Moreover $y' \in Z_{i,\kappa} \cap [0_\kappa, g(\lambda)]$, therefore this set is nonempty. It follows that for all $\lambda \in [0,1]$, we have $y(\lambda) \in Z_{i,\kappa} \cap [0_\kappa, g(\lambda)]$. For all $\lambda \leq \lambda' \in [0,1]$, we have $Z_{i,\kappa} \cap [0_\kappa, g(\lambda)] \subseteq Z_{i,\kappa} \cap [0_\kappa, g(\lambda')]$. Therefore $y(\lambda)$ is nondecreasing.

It only remains to prove that $y(\lambda)$ is continuous everywhere on $[0,1]$. Since $y(\lambda)$ is nondecreasing, then for all $\lambda \in]0,1]$, the limit $y(\lambda^-) := \lim_{\lambda' \to \lambda^-} y(\lambda')$ exists, and we have $y(\lambda^-) \leq y(\lambda)$. Similarly, for all $\lambda \in [0,1[$, the limit $y(\lambda^+) := \lim_{\lambda' \to \lambda^+} y(\lambda')$ exist, and we

[29] It is easy to see that both W_κ and X_κ are complete. Their respective greatest elements are the vectors $(1, ..., 1)$ and $(0, 1, ..., 1)$. A box $\{y \in T^{\kappa+1} : x \leq y \leq z\}$ is complete, with greatest element z. If two sets are complete, their intersection is also complete.

have $y(\lambda) \leq y(\lambda^+)$. By continuity of $\gamma^\kappa(\cdot)$, we have $\{y(\lambda^-), y(\lambda^+)\} \subseteq Z_{i,\kappa} \cap [0_\kappa, g(\lambda)]$. Since $y(\lambda)$ is the greatest element of this set, then in fact $y(\lambda) = y(\lambda^+)$, for all $\lambda \in [0, 1[$.

It only remains to prove that $y(\lambda^-) = y(\lambda)$ also holds. Suppose, by contradiction, that this is not true, so that $y(\lambda^-) < y(\lambda)$. Then there are indices k, l such that $0 < k \leq l < \kappa$ and satisfying $y_{k-1}(\lambda^-) = y_{k-1}(\lambda)$, $y_{l+1}(\lambda^-) = y_{l+1}(\lambda)$, and for all $h \in \{k, \ldots, l\}$, we have $y_h(\lambda^-) < y_h(\lambda)$. Let h be an arbitrary index such that $k \leq h \leq l$. Since $y_h(\lambda) \leq g_h(\lambda)$, therefore we also have $y_h(\lambda^-) < g_h(\lambda)$. For all $\epsilon > 0$ small enough, we have $y_h(\lambda - \epsilon) < g_h(\lambda - \epsilon)$. The only other constraint that restricts $y_h(\lambda - \epsilon)$ must then bind. Therefore $\gamma_h^\kappa(y(\lambda - \epsilon)) = y_h(\lambda - \epsilon)$. By continuity of $\gamma_h^\kappa(\cdot)$, it follows that $\gamma_h^\kappa(y(\lambda^-)) = y_h(\lambda^-)$ holds, for all h such that $k \leq h \leq l$. Let $x := (y_{k-1}(\lambda^-), \ldots, y_{l+1}(\lambda^-))$ and $x' := (y_{k-1}(\lambda), \ldots, y_{l+1}(\lambda))$. We have $\gamma^{l-k+2}(x) = x$ and $\gamma^{l-k+2}(x') \geq x'$. By Step 2, we conclude that $x' \leq x$, a contradiction.□

Step 4: For all $y \in Z_{i,\kappa}$, and all $k = 1, ..., \kappa - 1$, we have $\frac{\partial E^i}{\partial y_k}(y) \geq 0$. Moreover the inequalities are strict if y is in the interior of $Z_{i,\kappa}$.

Proof. By the envelope Theorem, we have

$$\frac{dE^i}{dy_k}(y) = \left[u^i(a^i(y_{k-1}, y_k), y_k) - u^i(a^i(y_k, y_{k+1}), y_k)\right] f^i(y_k) \geq 0.$$

where the inequality holds because $y \in Z_{i,\kappa}$.□

Step 5: $E^i(y)$ is increasing on $\mathcal{Z}_i$ for the partial order $\leq^*$.

Proof. In the case where $y', y'' \in Z_{i,\kappa}$, and $y' < y''$, the inequality $E^i(y') < E^i(y'')$ follows directly from local monotonicity of E^i on $Z_{i,\kappa}$ (Step 4) and monotone connectedness of $Z_{i,\kappa}$ (Step 3). It remains to prove it in the case where $y' \in Z_{i,\kappa} \cap X_{\kappa'}$ and $y'' \in Z_{i,\kappa} \cap X_{\kappa''}$, with $\kappa' < \kappa''$. It is easily verified that $(0, ..., 0, y')$ (the vector obtained by adding $\kappa'' - \kappa'$ coordinates equal to zero) is an element of $Z_{i,\kappa} \cap X_{\kappa''}$. Thus $E^i(y') = E^i(0, ..., 0, y') < E^i(y'')$, which yields the desired conclusion.□■

Proof of Theorem 4. First, it is immediate that the game Γ_B with parameters b_{R_B}, b_{S_B} and f^{R_B} and the game Γ'_B with parameters b_{R_A}, $b_{S_B} + (b_{R_A} - b_{R_B})$ and f^{R_B} have the same equilibrium partitions. In particular, for all $i \in \{R_A, R_B\}$, the sets

$$\left\{E^i(y) : y \text{ is an equilibrium partition of } \Gamma_B\right\}$$

and

$$\left\{E^i(y) : y \text{ is an equilibrium partition of } \Gamma'_B\right\}$$

are equal. The game Γ'_B is nested into game Γ_A because $b_{S_B} - b_{R_B} \leq b_{S_A} - b_{R_A}$ and f^{R_B} MLR-dominates f^{R_A}. In addition, if either $b_{S_B} - b_{R_B} < b_{S_A} - b_{R_A}$ or f^{R_B} strictly MLR-dominates f^{R_A}, the game Γ'_B is even strictly nested into game Γ_A. Parts 1, 2 and 3 follow from these last two observations and from applying respectively Theorem1, part (i) of Theorem 2 and Theorem 3 to games Γ_A and Γ'_B.■

Proof that if $\pi > \tilde{\pi}$, then $f^L \preceq_{MLR} f^{\tilde{L}} \preceq_{MLR} f^{\tilde{H}} \preceq_{MLR} f^H$.

Since $p(s_H|t) = \pi q(t) + (1-\pi)c$ and $p(s_L|t) = \pi(1-q(t)) + (1-\pi)(1-c)$, we have the conditional density

$$g(t|s_H, \pi) = \frac{f(t)(\pi q(t) + (1-\pi)c)}{\int_0^1 \pi q(x) f(x)\,dx + (1-\pi)c}.$$

To show that $f^{\tilde{H}} \preceq_{MLR} f^H$, it suffices to show that

$$\frac{g(t|s_H,\pi)}{g(t|s_H,\tilde{\pi})} > \frac{g(t'|s_H,\pi)}{g(t'|s_H,\tilde{\pi})}$$

for $t > t'$. Note that

$$\frac{g(t|s_H,\pi)}{g(t|s_H,\tilde{\pi})} = \frac{f(t)(\pi q(t) + (1-\pi)c) / \left(\int_0^1 \pi q(x) f(x)\,dt + (1-\pi)c\right)}{f(t)(\tilde{\pi} q(t) + (1-\tilde{\pi})c) / \left(\int_0^1 \tilde{\pi} q(x) f(x)\,dt + (1-\tilde{\pi})c\right)}$$

and

$$\frac{g(t'|s_H,\pi)}{g(t'|s_H,\tilde{\pi})} = \frac{f(t')(\pi q(t') + (1-\pi)c) / \left(\int_0^1 \pi q(x) f(x)\,dt + (1-\pi)c\right)}{f(t')(\tilde{\pi} q(t') + (1-\tilde{\pi})c) / \left(\int_0^1 \tilde{\pi} q(x) f(x)\,dt + (1-\tilde{\pi})c\right)}$$

Therefore it suffices to show that

$$\frac{(\pi q(t) + (1-\pi)c)}{(\tilde{\pi} q(t) + (1-\tilde{\pi})c)} > \frac{(\pi q(t') + (1-\pi)c)}{(\tilde{\pi} q(t') + (1-\tilde{\pi})c)}.$$

Since $\pi > \tilde{\pi}$, $t > t'$ and $q(\cdot)$ is increasing, the inequality holds. A similar argument shows that $f^L \preceq_{MLR} f^{\tilde{L}}$.

References

[1] Ambrus, A, E. Azevedo and Y. Kamada (2013): "Hierarchical Cheap Talk," *Theoretical Economics*, 8, 233–261.

[2] Blackwell, D (1951): "The Comparison of Experiments." in *Proceedings, Second Berkeley Symposium on Mathematical Statistics and Probability*, 93-102.

[3] Che, Y-K and N. Kartik (2009): "Opinions as Incentives," *Journal of Political Economy*, Vo. 117, No. 5, 815-860.

[4] Chen, Y., N. Kartik and J. Sobel (2008): "Selecting Cheap-Talk Equilibria," *Econometrica*, 76(1), 117–136.

[5] Crawford, V. and J. Sobel (1982): "Strategic Information Transmission," *Econometrica*, 50(6), 1431–1452.

[6] Dessein, W. (2002): "Authority and Communication in Organizations," *Review of Economics Studies*, 69, 811–838.

[7] Goltsman, M., J. Horner, G. Pavlov and F. Squintani (2009): "Mediation, Arbitration and Negotiation." *Journal of Economic Theory*, 144, 1397-1420.

[8] Gordon, S. (2007): "Informative Cheap Talk Equilibria as Fixed Points", Technical report, U. de Montréal.

[9] Gordon, S. (2010): "On Infinite Cheap Talk Equilibria," Technical report, U. de Montréal.

[10] Gordon, S. (2011): "Iteratively Stable Cheap Talk," Technical report, U. de Montréal.

[11] Green, J. and N. Stokey (2007): "A Two-Person Game of Information Transmission," *Journal of Economic Theory*, 135, 90-104.

[12] Harris M. and A. Raviv (2005): "Allocation of Decision-making Authority." *Review of Finance*, 9(3) 353-383.

[13] Ishida, J. and T. Shimizu (2010): "Cheap Talk with an Informed Receiver." Technical report, Osaka University and Kanzai University.

[14] Ivanov, M. (2010): "Communication via a Strategic Mediator." *Journal of Economic Theory*, 145, 869-884.

[15] Lai, E. (2008): "Expert Advice for Amateurs." Technical report, University of Pittsburgh.

[16] Lazzati, N. (2013): "Comparison of Equilibrium Actions and Payoffs across Players in Games of Strategic Complements." *Economic Theory*, 54 (3), 777-788.

[17] Li, W. (2010): "Peddling Influence through Intermediaries." *American Economic Review*, 100 (3), 1136-1162.

[18] Milgrom, P. and J. Roberts (1994): "Comparing Equilibria," *American Economic Review*, Vol. 84, No. 3., 441–459.

[19] Milgrom, P. (1981): "Good News and Bad News: Representation Theorems and Applications." *Bell Journal of Economics*, Vol. 12, No 2, 380-391.

[20] Moreno de Barreda, I. (2010): "Cheap Talk with Two-sided Private Information." Technical report, London School of Economics.

[21] Olszewski, W. (2004): "Informal Communication." *Journal of Economic Theory*, 117, 180-200.

[22] Ottaviani, M and P. Sorensen (2006): "Professional Advice," *Journal of Economic Theory*, 126, 120-142.

[23] Quah, J. and B. Strulovici (2012): "Aggregating the Single Crossing Property," *Econometrica*, 80, Issue 5, 2333–2348.

[24] Seidmann, D. (1990): "Effective Cheap Talk with Conflicting Interests," *Journal of Economic Theory*, 50, 445-458.

[25] Szalay, D. (2012): "Strategic Information Transmission and Stochastic Orders," Technical report, University of Bonn.

[26] Topkis, D. M. (1998): "Supermodularity and Complementarity," Princeton, New Jersey, U.S.A.: Princeton University Press.

[27] Watson, J. (1996): "Information Transmission When the Informed Party is Confused." *Games and Economic Behavior,* 12, 143-161.