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**ERRATUM TO THE ARTICLE**  
**“ZERO MEAN CURVATURE SURFACES IN**  
**LORENTZ–MINKOWSKI 3-SPACE WHICH**  
**CHANGE TYPE ACROSS A LIGHT-LIKE LINE”**  
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In the paper [1] whose title is included in the above title, an error in one estimate was found, although the main results still remain valid. In fact, line 6 of p.292 is incorrect, and the corrected line should read

$$= cM^{k-3}|y|^{k^*} \frac{432c^2}{M^4} \sum_{m=3}^{k-4} \sum_{n=3}^{k-m-1} \frac{k|3n-k+m-1|}{mn(m-1)(n-1)(k-m-n+1)^2}.$$

As a consequence, we have that

$$(1) \quad |kQ_k| \leq cM^{k-3}|y|^{k^*} \frac{432c^2}{M^4} \sum_{m=3}^{k-4} \sum_{n=3}^{k-m-1} \frac{k|3n-k+m-1|}{(m-1)^2(n-1)^2(k-m-n+1)^2}.$$

Theorem 1.1 and Corollary 1.2 of [1] remain true under this correction. To confirm this, it is sufficient to show the inequality at the bottom of [1, p.292]:

$$(2) \quad |kQ_k| \leq \frac{c}{18\tau} M^{k-3}|y|^{k^*} \times 6\tau \leq \frac{c}{3} M^{k-3}|y|^{k^*}.$$

In fact, changing the original inequality in [1, line 6 of p.292] to (1) affects only the proof of (2).

From here on out, we prove (2) assuming (1).

**Lemma 1.** For  $k \geq 7$ , the following inequality holds:

$$\max_{\substack{3 \leq m \leq k-4 \\ 3 \leq n \leq k-m-1}} (k|3n - k + m - 1|) < 2(k - 1)^2.$$

Proof. In fact,

$$\begin{aligned} \max_{\substack{3 \leq m \leq k-4 \\ 3 \leq n \leq k-m-1}} |3n - k + m - 1| &= \max_{(m,n)=(3,3),(3,k-4),(k-4,3)} |3n - k + m - 1| \\ &= \max\{|-k + 11|, |2k - 10|\} \leq 2(k - 5). \end{aligned}$$

In particular, we have

$$\max_{\substack{3 \leq m \leq k-4 \\ 3 \leq n \leq k-m-1}} (k|3n - k + m - 1|) \leq 2k(k - 5) < 2(k - 1)^2,$$

proving the assertion. □

We set  $p := m - 1$ ,  $q := n - 1$  and  $l := k - 1$ . Using (1), Lemma 1 and  $432c^2/M^4 \leq 1/(36\tau)$  (cf. [1, (1.14)]), we have that

$$\begin{aligned} |kQ_k| &\leq \frac{c}{36\tau} M^{k-3} |y|^{k^*} \sum_{m=3}^{k-4} \sum_{n=3}^{k-m-1} \frac{2(k-1)^2}{(m-1)^2(n-1)^2(k-m-n+1)^2} \\ &= \frac{c}{18\tau} M^{k-3} |y|^{k^*} \sum_{p=2}^{l-4} \sum_{q=2}^{l-p-2} \frac{l^2}{p^2q^2(l-p-q)^2} \\ &\leq \frac{c}{18\tau} M^{k-3} |y|^{k^*} \sum_{p=2}^{l-2} \sum_{q=2}^{l-p-2} \frac{l^2}{p^2q^2(l-p-q)^2}. \end{aligned}$$

Thus, for  $k \geq 7$ , it holds that

$$(3) \quad |kQ_k| \leq \frac{c}{18\tau} M^{k-3} |y|^{k^*} \sum_{p=2}^{l-2} \sum_{q=2}^{l-p-2} \frac{l^2}{p^2q^2(l-p-q)^2}.$$

To get (2), we need the following assertion, which is a refinement of [1, Lemma A.2]:

**Lemma 2.** For any integer  $k \geq 4$ , the following inequalities hold:

$$(4) \quad \sum_{p=2}^{k-2} \sum_{q=2}^{k-p-2} \frac{k^2}{p^2q^2(k-p-q)^2} \leq \frac{6}{k} \int_{1/k}^{1-1/k} \frac{du}{u^2(1-u)^2} \leq 6\tau,$$

where  $\tau$  is a positive constant satisfying [1, (A.3)].

Proof. The proof of [1, Lemma A.2] becomes a proof of the inequality (4) simply by replacing the upper limit “ $k - 5$ ” of the sum with “ $k - 2$ ”.  $\square$

By (3) and (4), we have the desired inequality (2).

Finally we note the following typographical errors:

- In line 6 of p.293, “=” should be replaced by “ $\leq$ ”.
- In the third line from the bottom of p.294,

$$\int_{1/k}^{a-1/k} \frac{du}{u^2(a-u)^2}$$

should be

$$\int_{1/k}^{a-1/k} \frac{a^3 du}{u^2(a-u)^2}.$$

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#### References

- [1] S. Fujimori, Y.W. Kim, S.-E. Koh, W. Rossman, H. Shin, M. Umehara, K. Yamada and S.-D. Yang: *Zero mean curvature surfaces in Lorentz-Minkowski 3-space which change type across a light-like line*, Osaka J. Math. **52** (2015), 285–297.

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