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HOMOTOPY REPRESENTATION GROUPS AND SWAN SUBGROUPS

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0. Introduction

Let G be a finite group. A finite dimensional G-CW-complex X is called a homotopy representation of G if the H-fixed point set X^H is homotopy equivalent to a ($\dim X^H$)-dimensional sphere or the empty set for each subgroup H of G. Moreover if X is G-homotopy equivalent to a finite G-CW-complex, then X is called a finite homotopy representation of G and if X is G-homotopy equivalent to a unit sphere of a real representation of G, then X is called a linear homotopy representation of G. T. tom Dieck and T. Petrie defined homotopy representation groups in order to study homotopy representations. Let $V^+(G, h^\infty)$ be the set of G-homotopy types of homotopy representations. We define the addition on $V^+(G, h^\infty)$ by the join and so $V^+(G, h^\infty)$ becomes a semigroup. The Grothendieck group of $V^+(G, h^\infty)$ is denoted by $V(G, h^\infty)$ and called the homotopy representation group. A similar group $V(G, h)$ [resp. $V(G, l)$] can be defined for finite [resp. linear] homotopy representations.

Let $\phi(G)$ denote the set of conjugacy classes of subgroups of G and $C(G)$ the ring of functions from $\phi(G)$ to integers. For a homotopy representation X, the dimension function $\operatorname{Dim} X$ in $C(G)$ is defined by $(\operatorname{Dim} X)(H)=\dim X^H+1$. (If X^H is empty, then we set $\dim X^H=-1$.) Then

$$\operatorname{Dim} X*Y = \operatorname{Dim} X+\operatorname{Dim} Y$$

for any two homotopy representations. ("$*$" means the join.) Hence one can define the homomorphism

$$\operatorname{Dim}\colon V(G, \lambda) \to C(G) \quad (\lambda = h^\infty, h \text{ or } l)$$

by the natural way. The kernel of Dim is denoted by $v(G, \lambda)$. tom Dieck and Petrie proved that $v(G, \lambda)$ is the torsion group of $V(G, \lambda)$ and

$$(0.1) \qquad v(G, h^\infty)\cong \operatorname{Pic} \Omega(G),$$

where $\operatorname{Pic} \Omega(G)$ is the Picard group of the Burnside ring $\Omega(G)$.

There are the natural homomorphisms

$$(0.2)\qquad \begin{aligned} j_G&: v(G, l) \to v(G, h) \\ k_G&: v(G, h) \to v(G, h^\infty)\,. \end{aligned}$$

The homomorphisms j_G and k_G are injective in general and hence we often regard $v(G, l)$ and $v(G, h)$ as the subgroups of $v(G, h^\infty)$ via these injective homomorphisms. We prove the following result in Section 2.

Theorem A. *The homomorphism k_G is an isomorphism if and only if the Swan subgroup $T(G)$ vanishes.*

The definition and properties of the Swan subgroup are mentioned in Section 1. The finite groups with $T(G)=0$ are studied by Miyata and Endo [7]. The Swan subgroups play an important role in the computation of $v(G, h)$. In fact, the computation of $v(G, h)$ for an abelian group G is completely reduced to the computation of the Swan subgroups. By computing the Swan subgroups of some groups, we prove the following result in Section 3.

Theorem B. *Suppose that G is an abelian group. Then j_G is an isomorphism if and only if G is isomorphic to one of the following groups.*

(i) *C a cyclic group*
(ii) *G_2 an abelian 2-group*
(iii) *G_3 an abelian 3-group*
(iv) $\boldsymbol{Z}/2\times G_3$
(v) $G_2\times \boldsymbol{Z}/3$
(vi) $(\boldsymbol{Z}/2)^n\times(\boldsymbol{Z}/3)^m$

In Section 4 we determine the finite groups with $v(G, h^\infty)=0$ by using the results in Section 2.

Theorem C. *The group $v(G, h^\infty)$ vanishes if and only if G is isomorphic to one of the groups:*

$$\boldsymbol{Z}/n\ (n=1, 2, 3, 4 \text{ or } 6),\ D(2n)\ (n=2, 3, 4 \text{ or } 6),\ A_4,\ S_4.$$

Here $D(2n)$ denotes the dihedral group of order $2n$ and $S_n[A_n]$ denotes the symmetric [alternating] group on n symbols.

1. The Swan subgroup

In this Section, we shall summarize the well-known results on the Swan subgroup. Let $\sum_G$ be the sum of elements of G in the integral group ring $\boldsymbol{Z}G$ and $[r, \sum_G]$ be the left ideal generated by r and $\sum_G$, where $r\in\boldsymbol{Z}$ is prime to the order $|G|$ of G. The ideal $[r, \sum_G]$ is projective as a $\boldsymbol{Z}G$-module. Hence $[r, \sum_G]$ decides the element of the reduced projective class group $\tilde{K}_0(\boldsymbol{Z}G)$. From [9], a homomorphism

$$\tilde{S}_G\colon \boldsymbol{Z}/|G|^* \to \tilde{K}_0(\boldsymbol{Z}G)$$

is defined by $\tilde{S}_G(r)=([r, \Sigma_G])$, where $\boldsymbol{Z}/|G|^*$ is the unit group of $\boldsymbol{Z}/|G|$. We put $u(G)=(\boldsymbol{Z}/|G|^*)/\pm 1$. Since $\tilde{S}_G(\pm 1)=0$, $\tilde{S}_G$ induces

$$S_G\colon u(G) \to \tilde{K}_0(\boldsymbol{Z}G)\,,$$

which is called the Swan homomorphism. The image of S_G is called the Swan subgroup of G and denoted by $T(G)$.

The following results are well-known.

Theorem 1.1 ([9]). *If G is a cyclic group, then $T(G)=0$.*

Theorem 1.2 ([11]).

(i) *$T(G)$ is a quotient group of $u(G)$.*

(ii) *If $f\colon G\to G'$ is a surjective homomorphism, then the natural map $\tilde{K}_0(\boldsymbol{Z}G)\to\tilde{K}_0(\boldsymbol{Z}G')$ sends $([r, \Sigma_G])$ to $([r, \Sigma_{G'}])$, hence $T(G)$ onto $T(G')$.*

(iii) *The restriction map $\tilde{K}_0(\boldsymbol{Z}G)\to\tilde{K}_0(\boldsymbol{Z}H)$ sends $([r, \Sigma_G])$ to $([r, \Sigma_H])$, hence $T(G)$ onto $T(H)$.*

(iv) *The exponent of $T(G)$ divides the Artin exponent $A(G)$. (For the Artin exponent, see* [6].)

(v) *$T(D(2^n))=0\,(n\geq 2)$, $T(Q(2^n))=\boldsymbol{Z}/2\,(n\geq 3)$ and $T(SD(2^n))=\boldsymbol{Z}/2\,(n\geq 4)$, where $D(2^n)$ [resp. $Q(2^n)$, $SD(2^n)$] is the dihedral [resp. quaternion, semi-dihedral] group of order 2^n. These groups are called the exceptional groups.*

Theorem 1.3 ([10]).

(i) *If G is a non-cyclic p-group (p: an odd prime), then $T(G)$ is the cyclic group of order $|G|/p$.*

(ii) *If G is a non-cyclic and non-exceptional 2-group, then $T(G)$ is the cyclic group of order $|G|/4$.*

Let G_p denote a p-Sylow subgroup of G.

Corollary 1.4. *If $T(G)$ vanishes, then G_p is cyclic when p is odd and G_2 is cyclic or dihedral.*

2. The inclusion k_G

tom Dieck and Petrie defined the finiteness obstraction map

$$\rho\colon v(G, h^\infty) \to \underset{(H)}{\oplus}\tilde{K}_0(\boldsymbol{Z}WH)\,, \tag{2.1}$$

where $WH=NH/H$ and NH is the normalizer of H in G. They proved that the following sequence is exact.

$$0 \to v(G, h) \overset{k_G}{\to} v(G, h^\infty) \overset{\rho}{\to} \underset{(H)}{\oplus}\tilde{K}_0(\boldsymbol{Z}WH)\,. \tag{2.2}$$

We recall the map ρ. (For details, see [3].) For any element x of $v(G, h^{\infty})$, there exist homotpoy representations X, Y and a G-map $f\colon X\to Y$ such that $x=X-Y$ in $v(G, h^{\infty})$ and $\deg f^H$ is prime to $|G|$ for each subgroup H of G. A function $d\in C(G)$ is defined by $d(H)=\deg f^H$ for any (H) and called the invertible degree function of x. Conversely, any $d\in C(G)$ with $(d(H), |G|)=1$ for any (H) is the invertible degree function of some x in $v(G, h^{\infty})$. The finiteness obstraction map ρ is described as follows. The (H)-component $\rho_H(x)\in\tilde{K}_0(\boldsymbol{Z}WH)$ of $\rho(x)$ is equal to

$$(2.3)\qquad S_{WH}(d(H))-\sum_{\substack{1\neq K\subseteq WH\\ L\subseteq NK}} a_{K,L}\,\mathrm{ind}_L^{WH}\,\mathrm{res}_L^{NK}\,S_{NK}(d(\tilde{K}))\,,$$

where $\tilde{K}$ is the subgroup of G such that $\tilde{K}/H=K$ and $a_{K,L}$ are certain integers and d is the invertible degree function of x.

Proof of Theorem A. For any r which is prime to $|G|$, we take the function $d\in C(G)$ such that $d(1)=r$ and $d(H)=1$ for $(H)\neq(1)$. By (2.3), we have $\rho_1(x)=S_G(r)$ and $\rho_H(x)=0$ for $(H)\neq(1)$, where x denotes the element of $v(G, h^{\infty})$ represented by d. Hence $T(G)=0$ if $\rho=0$. Conversely if $T(G)$ vanishes, then $S_K=0$ for any subquotient group K of G by Theorem 1.2. Hence $\rho=0$ and so k_G is an isomorphism.

Corollary 2.4. *Let G be $D(2^n)$, $Q(2^n)$ or $SD(2^n)$. Then $v(G, h)=v(G, l)$.*

Proof. In the case of $D(2^n)$, we have proved it in [8]. In the cases of $Q(2^n)$ and $SD(2^n)$, $v(G, l)$ is the subgroup of index 2 of $v(G, h^{\infty})$ ([8]). On the other hand $v(G, h)$ is a proper subgroup of $v(G, h^{\infty})$ since $T(G)=\boldsymbol{Z}/2$. Hence $v(G, h)=v(G, l)$.

Remark 2.5. If G is nilpotent, then $\operatorname{Dim} V(G, l)=\operatorname{Dim} V(G, h^{\infty})$ ([3]) and hence $V(G, h)=V(G, l)$ for the above groups.

Corollary 2.6. *If $v(G, h^{\infty})$ vanishes, then $T(G)$ also vanishes.*

3. The inclusion j_G

Let G be an abelian group. Then $v(G, l)$ and $v(G, h^{\infty})$ were computed by Kawakubo [5] and tom Dieck-Petrie [3] respectively and it is known that the following diagram is commutative.

$$(3.1)\qquad \begin{array}{ccc} v(G, l) & \longrightarrow & v(G, h^{\infty}) \\ \big\downarrow\cong & & \alpha\big\downarrow\cong \\ \prod\limits_{\substack{H\\ G/H:\text{ cyclic}}} u(G/H) & \subset & \prod\limits_{H} u(G/H) \end{array}$$

Here $u(G/H)=(\boldsymbol{Z}/|G/H|^*)/\pm 1$.

Furthermore, tom Dieck and Petrie showed the following commutative diagram.

$$
\text{(3.2)} \qquad
\begin{array}{ccc}
v(G, h^\infty) & & \\
\alpha \downarrow \cong & \searrow \rho & \\
\prod_H u(G/H) & \xrightarrow[\prod_H S_{G/H}]{} & \bigoplus_H \tilde{K}_0(\boldsymbol{Z}[G/H])
\end{array}
$$

Hence we obtain

Proposition 3.3. *Let G be an abelian group. Then*

(i) $v(G, h)\cong v(G, l)\times N(G)$,

where $N(G)=\prod_{\substack{H \\ G/H\,;\ \text{non-cyclic}}} \operatorname{Ker} S_{G/H}$. *(If G is cyclic, then we put* $N(G)=1$.*)*

(ii) $v(G, h^\infty)/v(G, h)\cong \bigoplus_H T(G/H)$.

Proof. These are obtained from the exactness of the sequence (2.2) and the fact that $T(G/H)=0$ if G/H is cyclic.

Corollary 3.4. *Let G be an abelian group. Then*

$$V(G, h)\cong V(G, l)\times N(G)\,.$$

REMARK 3.5. For any finite group, one can show that

$$|v(G, h^\infty)/v(G, h)|\geq |\bigoplus_{(H)} T(WH)|\,.$$

From now we shall prove Theorem B. Theorem B is proved by the following lemmas.

Lemma 3.6. *If* $N(G)=1$ *for a non-cyclic abelian group G, then* $|G|=2^n\cdot 3^m$ $(n, m\geq 0)$.

Proof. If a p-Sylow subgroup G_p $(p\geq 5)$ is non-cyclic, then there exists a subgroup L such that G/L is isomorphic to $\boldsymbol{Z}/p\times\boldsymbol{Z}/p$. Since $\operatorname{Ker} S_{G/L}$ is non-trivial by Theorem 1.3, G_p must be cyclic. We may put $G=G_2\times G_3\times C$, where C is a cyclic group with $(|C|, 6)=1$. We prove that C is trivial. Assume that C is non-trivial. Since G is non-cyclic, there exists a subgroup K such that G/K is isomorphic to $\boldsymbol{Z}/q\times\boldsymbol{Z}/q\times\boldsymbol{Z}/p$ ($q=2$ or 3, $p\geq 5$). The Artin exponent $A(G/K)$ is equal to q and so $T(G/K)$ is a q-group by Theorem 1.2. On the other hand, it is easily checked that the exponent of $u(G/K)$ is not equal to q. Hence $\operatorname{Ker} S_{G/K}\neq 1$ and so $N(G)\neq 1$. This is a contradiction. Therefore C is trivial.

Lemma 3.7. *Put* $G=\boldsymbol{Z}/2\times\boldsymbol{Z}/2\times\boldsymbol{Z}/3^m$ $(m\geq 1)$. *Then* Ker $S_G\neq 1$ *if* $m\geq 2$ *and* Ker $S_G=1$ *if* $m=1$.

Proof. Since the Artin exponent $A(G)=2$ and $|u(G)|=2\cdot 3^{m-1}$, the Swan subgroup $T(G)$ is isomorphic to 1 or $\boldsymbol{Z}/2$. Moreover $T(\boldsymbol{Z}/2\times\boldsymbol{Z}/2\times\boldsymbol{Z}/3)=\boldsymbol{Z}/2$ ([4], [7]). Hence $T(G)=\boldsymbol{Z}/2$. Thus the desired result holds.

Lemma 3.8. *Put* $G=\boldsymbol{Z}/2^n\times\boldsymbol{Z}/3\times\boldsymbol{Z}/3$ $(n\geq 1)$. *Then* Ker $S_G\neq 1$ *if* $n\geq 2$ *and* Ker $S_G=1$ *if* $n=1$.

Proof. The proof is similar to the proof of Lemma 3.7. The details are omitted.

Lemma 3.9. *Let* G_2 *be a non-cyclic abelian group of order* 2^n. *We put* $G=G_2\times\boldsymbol{Z}/3$. *Then* Ker $S_{G_2}=1$ *and* Ker $S_G=1$.

Proof. By Theorem 1.3, it is clear that Ker $S_{G_2}=1$. We consider the restriction map

$$R=(\mathrm{res}_{G_2},\ \mathrm{res}_K)\colon T(G)\to T(G_2)\oplus T(K)\,,$$

where K is a subgroup which is isomorphic to $\boldsymbol{Z}/2\times\boldsymbol{Z}/2\times\boldsymbol{Z}/3$. We show that R is surjective. Take any element (a, b) in $T(G_2)\oplus T(K)$. Then there exists $r\in\boldsymbol{Z}$ with $(r, |G|)=1$ such that $\mathrm{res}_{G_2} S_G(r)=S_{G_2}(r)=a$. Put $c=\mathrm{res}_K S_G(r)=S_K(r)$. If $c\neq b$, then take $(2^n-1)r$ [resp. $(2^n+1)r$] instead of r when n is odd [resp. even]. Then

$$\mathrm{res}_{G_2} S_G((2^n\pm 1)r)=S_{G_2}(r)=a$$

and

$$\mathrm{res}_K S_G((2^n\pm 1)r)=\begin{cases} S_K(5)+S_K(r) & \text{if } n \text{ is even} \\ S_K(7)+S_K(r) & \text{if } n \text{ is odd}\end{cases}=b\,.$$

The last equality follows from the facts that $T(K)=\boldsymbol{Z}/2$, $S_K(5)\neq 0$ and $S_K(7)\neq 0$. Hence R is surjective.

The orders of $T(G_2)$ and $T(K)$ are 2^{n-2} and 2 respectively. Since $|u(G)|=2^{n-1}$, $|u(G)|=|T(G)|$. Hence Ker $S_G=1$.

Lemma 3.10. *Let* G_3 *be a non-cyclic abelian group of order* 3^m. *We put* $G=\boldsymbol{Z}/2\times G_3$. *Then* Ker $S_{G_3}=1$ *and* Ker $S_G=1$.

Proof. This follows from the comparison between the orders of $u(G)$ and $T(G)$.

Lemma 3.11. *We put* $G=G_2\times G_3$ *for the above* G_2 *and* G_3. *Then* Ker $S_G=1$.

Proof. The restriction maps $T(G)\to T(G_2\times \boldsymbol{Z}/3)$ and $T(G)\to T(G_3)$ are surjective. Since $|T(G_2\times \boldsymbol{Z}/3)|=2^{n-1}$ by Lemma 3.9 and $|T(G_3)|=3^{m-1}$ by Theorem 1.3, we have $|T(G)|\geq 2^{n-1}\cdot 3^{m-1}$. Hence Ker $S_G=1$.

Proof of Theorem B. Assume that j_G is an isomorphism (i.e. $N(G)=1$). By Lemma 3.6, $G=G_2\times G_3$. If both G_2 and G_3 are cyclic, then G is cyclic. If G_2 is cyclic and G_3 is non-cyclic, then $G_2=1$ or $\boldsymbol{Z}/2$ by Lemma 3.8. If G_2 is non-cyclic and G_3 cyclic, then $G_3=1$ or $\boldsymbol{Z}/3$ by Lemma 3.7. If both G_2 and G_3 are non-cyclic, then $G=(\boldsymbol{Z}/2)^n\times(\boldsymbol{Z}/3)^m$ by Lemmas 3.7 and 3.8. Conversely, if G is one of the groups (i)–(vi), then $N(G)=1$ by Lemmas 3.7–3.11.

4. The finite groups G with $v(G, h^\infty)=0$

In this Section we determine the finite groups with $v(G, h^\infty)=0$. We first show the following result.

Proposition 4.1. *Let C be a cyclic subgroup of G. Then the restriction map*

$$\mathrm{res}\colon v(G, h^\infty) \to v(C, h^\infty)$$

is surjective.

Proof. Let $d\in C(C)$ be an invertible degree function representing $x\in v(C, h^\infty)$. We can choose an integer a_K such that $d(K)+a_K|C|$ is prime to $|G|$ for any subgroup K of C. Then $d'(K)=d(K)+a_K|C|$ is also an invertible degree function representing x. (See [3].) We define $e\in C(G)$ by

$$e(H)=\begin{cases} d'(gHg^{-1}) & \text{if } (H)\in\phi(G) \text{ with } gHg^{-1}\subseteq C \\ 1 & \text{otherwise.}\end{cases}$$

This is well-defined since C is cyclic. Let $y\in v(G,h^\infty)$ be the element represented by e. Then res $y=x$ since d is an invertible degree function of res y.

In the abelian case, we have

Lemma 4.2. *Let G be an abelian group. Then $v(G, h^\infty)=0$. If and only if G is isomorphic to* 1, $\boldsymbol{Z}/2$, $\boldsymbol{Z}/3$, $\boldsymbol{Z}/4$, $\boldsymbol{Z}/6$ *or* $D(4)$ $(=\boldsymbol{Z}/2\times\boldsymbol{Z}/2)$.

Proof. Using the isomorphism $v(G, h^\infty)\cong\prod_H u(G/H)$, one can easily see it.

By Lemmas 4.1 and 4.2, we have

Lemma 4.3. *If $v(G, h^\infty)$ vanishes, then any cyclic subgroup C of G is isomorphic to* 1, $\boldsymbol{Z}/2$, $\boldsymbol{Z}/3$, $\boldsymbol{Z}/4$ *or* $\boldsymbol{Z}/6$.

On the other hand, if $v(G, h^\infty)$ vanishes, then the Swan subgroup $T(G)$

also vanishes (Corollary 2.6) and hence we have the following conclusion by Lemma 4.3 and Corollary 1.4.

Lemma 4.4. *If $v(G, h^\infty)$ vanishes, then a 2-Sylow subgroup G_2 is isomorphic to* 1, $\boldsymbol{Z}/2$, $\boldsymbol{Z}/4$, $D(4)$ *or* $D(8)$ *and a 3-Sylow subgroup G_3 is isomorphic to* 1, *or* $\boldsymbol{Z}/3$ *and a p-Sylow subgroup G_p ($p\geq 5$) is trivial.*

We consider a non-abelian group G. Suppose that $v(G, h^\infty)$ vanishes. Then $|G|=6$, 8, 12 or 24 by Lemma 4.4. If $|G|=6$, then G is isomorphic to $D(6)$. If $|G|=8$, then G is isomorphic to $D(8)$ by Lemma 4.4. If $|G|=12$, then G is isomorphic to A_4, $D(12)$ or $Q(12)$. In the case $|G|=24$, G_2 is isomorphic to $D(8)$ by Lemma 4.4. From Burnside's book ([1] Chap. 9, 126.), G is isomorphic to one of the groups: $D(24)$, $D(8)\times\boldsymbol{Z}/3$, S_4 and $K=\langle a, b, c\,|\,a^4=b^2=c^3=1,\ bc=cb,\ b^{-1}ab=a^{-1},\ a^{-1}ca=c^{-1}\rangle$. However $D(24)$ and $D(8)\times\boldsymbol{Z}/3$ are omitted by Lemma 4.3. Since K has a subgroup which is isomorphic to $\boldsymbol{Z}/2\times\boldsymbol{Z}/2\times\boldsymbol{Z}/3$, the Swan subgroup $T(K)$ is non-trivial and K is also omitted by Corollary 2.6. Therefore, in the non-abelian case, if $v(G, h^\infty)$ vanishes, then G is isomorphic to one of the groups: $D(6)$, $D(8)$, $D(12)$, $Q(12)$, A_4 and S_4.

We proved the following formula in [8]. (See also [2].)

Proposition 4.4. *For any finite group,*

$$|v(G, h^\infty)| = 2^{-n}|\Omega(G)^*|\prod_{(H)}\varphi(|WH|)\,, \tag{4.5}$$

where φ is the Euler function and n is the number of conjugacy classes of subgroups of G.

By computing $|v(G, h^\infty)|$ as in [8], one can see that $|v(G, h^\infty)|=1$ for $G=D(6)$, $D(8)$, $D(12)$, A_4 or S_4 and $|v(G, h^\infty)|=2$ for $G=Q(12)$. Therefore we have

Theorem 4.6. *$v(G, h^\infty)$ vanishes if and only if G is one of the following groups*: $\boldsymbol{Z}/n$ ($n=1$, 2, 3, 4, 6), $D(2n)$ ($n=2$, 3, 4, 6), A_4 *and* S_4.

As a remark, there exist infinitely many groups with $v(G, \lambda)=0$ ($\lambda=h$ or l). Indeed we have

Proposition 4.7. *Let G be an abelian group. Then $v(G, l)$ vanishes if and only if $G=(\boldsymbol{Z}/2)^n\times(\boldsymbol{Z}/4)^m$ or $(\boldsymbol{Z}/2)^n\times(\boldsymbol{Z}/3)^m$ ($n, m\geq 0$).*

Proof. One can see it by using the isomorphism $v(G, l)\cong\prod_{\substack{H\\ G/H:\text{ cyclic}}} u(G/H)$.

By Proposition 4.7 and Theorem B, we have

Corollary 4.8. *Let G be an abelian group. Then $v(G, h)$ vanishes if and* *only if $v(G, l)$ vanishes.*

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