

A NOTE ON THE BOUNDARY BEHAVIOR FOR NON-NEGATIVE FINELY SUPERHARMONIC FUNCTIONS IN THE UPPER HALF SPACE

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1. Introduction

In the previous paper [3], we studied the boundary behavior for non-negative finely superharmonic functions in a Green space at the Martin boundary. In this paper, as an application of Main Theorem in [3] we give a theorem of a Littlewood type (cf. [2, Theorem]) for non-negative finely superharmonic functions in the upper half space \mathbf{H} of the d -dimensional Euclidean space ($d \geq 2$).

2. Theorem of a Littlewood type

Theorem. *Let U be a finely open subset of \mathbf{H} and let $\Delta_1(U) = \{\zeta \in \partial\mathbf{H} : \mathbf{H} \setminus U \text{ is minimally thin at } \zeta\}$. Suppose that u is a non-negative finely superharmonic in U . Then, at almost every point ζ of $\Delta_1(U)$ with respect to the $(d-1)$ -dimensional Lebesgue measure, there exists a polar subset $N(\zeta)$ of the unit sphere with center ζ satisfying the following property: $u(z)$ converges to $f\text{-}\lim_{x \in U \rightarrow \zeta} u(x)$ as a point z in U approaches ζ along every ray issued from ζ which does not meet $N(\zeta)$.*

For the definitions of notations and terminologies used in Main Theorem, we refer to [3] and the word "almost every" will be later used to mean "except for a null set with respect to the $(d-1)$ -dimensional Lebesgue measure".

Proof of Theorem. Let u be a non-negative finely superharmonic function in a finely open subset U in \mathbf{H} . Then, Main Theorem in [3] states that u has a fine limit at almost every point ζ of $\Delta_1(U)$. From Naïm's result [4, Théorème 12], we see that, at almost every point ζ of $\Delta_1(U)$, there exists a finely open subset $V(\zeta)$ of U such that $\mathbf{H} \setminus V(\zeta)$ is minimally thin at ζ and that $u(z)$ converges to $f\text{-}\lim_{x \in V \rightarrow \zeta} u(x)$ as a point z in $V(\zeta)$ approaches ζ in the sense of the Euclidean topology in \mathbf{R}^d . On the other hand, by Lelong-Ferrand [1, Théorème in §8], there exists a polar subset N of the unit sphere with center ζ in \mathbf{R}^d satisfying the following property: for every ray l in \mathbf{H} issued from ζ which

dose not meet N , there exists a positive ρ such that $(I \cap B(\zeta, \rho)) \setminus \{\zeta\} \subset U$, where $B(\zeta, \rho)$ is the ball with center ζ and radius ρ in \mathbf{R}^d . Therefore, we obtained the desired result.

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References

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