A CHARACTERIZATION OF CERTAIN DOMAINS WITH GOOD BOUNDARY POINTS IN THE SENSE OF GREENE-KRANTZ, III

Dedicated to Professor Masaru Takeuchi on his 60th birthday

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Introduction

This is a continuation of our previous papers [9, 10, 12]. For a domain $D$ in $\mathbb{C}^n$, we denote by $\text{Aut}(D)$ the group of all biholomorphic automorphisms of $D$ and write $\partial D$ (resp. $\bar{D}$) for the boundary (resp. closure) of $D$.

Let $D$ be a bounded domain in $\mathbb{C}^n$ and $x \in \partial D$. Assume that $x$ is an accumulation point of an $\text{Aut}(D)$-orbit. Can we then determine the global structure of $D$ from the local shape of $\partial D$ near $x$? Of course, this is impossible without any further assumptions, as one may see in the examples such as the direct product of the open unit disk in $\mathbb{C}$ and an arbitrary bounded domain in $\mathbb{C}^{n-1}$. In the previous papers [2,8,9,10,12], this was exclusively studied in the case where $\partial D$ near $x$ coincides with the boundary of a generalized complex ellipsoid

$$E(n;n_1,\ldots,n_s;p_1,\ldots,p_s)$$

$$= \{(z_1,\ldots,z_s) \in \mathbb{C}^{n_1} \times \cdots \times \mathbb{C}^{n_s}; \sum_{i=1}^{s} \|z_i\|^{2p_i} < 1 \}$$

in $\mathbb{C}^n = \mathbb{C}^{n_1} \times \cdots \times \mathbb{C}^{n_s}$, where $p_1,\ldots,p_s$ are positive real numbers and $n_1,\ldots,n_s$ are positive integers with $n = n_1 + \cdots + n_s$.

The purpose of this paper is to establish the following extension of some results obtained in [2, 9, 10, 12]:

**Theorem.** Let $D$ be a bounded domain in $\mathbb{C}^n$ and $E = E(n;n_1,\ldots,n_s;p_1,\ldots,p_s)$ a generalized complex ellipsoid in $\mathbb{C}^n$. Let $x \in \partial D$. Assume that the following three conditions are satisfied:

1. $p_1,\ldots,p_s$ are all positive integers;
2. $x \in \partial E$ and there exists an open neighborhood $Q$ of $x$ in $\mathbb{C}^n$ such that
$D \cap \Omega = E \cap \Omega$; and

(3) $x$ is a good boundary point of $D$ in the sense of Greene and Krantz [6], that is, there exist a point $b \in D$ and a sequence $\{\varphi_v\} \subset \text{Aut}(D)$ such that $\varphi_v(b) \to x$ as $v \to \infty$.

Then we have $D = E$ as sets. In particular, at least one of the $p_i$'s must be equal to 1.

Note that the existences of a point $b \in E$ and a sequence $\{\tilde{\varphi}_v\} \subset \text{Aut}(E)$ such that $\tilde{\varphi}_v(b) \to x$ as $v \to \infty$ are not assumed in the theorem. Hence, this does not follow directly from the results obtained in [7 or 12]; and also this gives an affirmative answer to Problem 1 in [11; p.62] in the case where $\partial D$ near $x$ is $C^\omega$-smooth. In the special case $n_i=1$ for all $i=1,\cdots,s$, we know by [9, 10] that our theorem holds even for arbitrary $0<p_1,\cdots,p_s \in \mathbb{R}$ (not necessarily integers). And, in its proof, Rudin's extension theorem [16] of holomorphic mappings defined near boundary points of the unit ball $B^*$ in $\mathbb{C}^n$ played a crucial role. Notice that this theorem of Rudin can be applied no longer to the case $n_i \geq 1$ in general. However, employing a recent result due to Dini and Selvaggi Primicerio [3] instead of that due to Rudin and using the same scaling technique as in [12], we can prove the theorem above.

As an immediate consequence of our theorem, we now obtain the following:

**Corollary.** For arbitrary integers $p_1,\cdots,p_s \geq 2$, any bounded domain $D$ in $\mathbb{C}^n$ with a point $x \in \partial D \cap \partial E (n;n_1,\cdots,n_s;p_1,\cdots,p_s)$ near which $\partial D$ coincides with $\partial E (n;n_1,\cdots,n_s;p_1,\cdots,p_s)$ cannot have any $\text{Aut}(D)$-orbits accumulating at $x$.

Clearly this gives an affirmative answer to the following conjecture of Greene and Krantz [6; p. 200]: Let $x$ be a boundary point of the domain $E = \{(z_1,z_2) \in \mathbb{C}^2; |z_1|^4 + |z_2|^4 < 1\}$. Then any weakly pseudoconvex bounded domain $D$ in $\mathbb{C}^2$ with $x \in \partial D$ near which $\partial D$ coincides with $\partial E$ cannot have any $\text{Aut}(D)$-orbits accumulating at $x$.

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1. **Preliminaries**

For later purpose, in this section we shall recall a recent result on localization principle of holomorphic automorphisms of generalized complex ellipsoids due to Dini and Selvaggi Primicerio [3], which plays an essential role in our proof.

For convenience and without loss of generality, in the following we will always assume
and write a generalized complex ellipsoid \( E \) in the form
\[
E = E (n; n_1, n_2, \ldots, n_s; 1, p_2, \ldots, p_s).
\]
Here it is understood that 1 does not appear if \( n_1 = 0 \), and also this domain is the unit ball \( B^n \) in \( C^n \) if \( s = 1 \).

For a generalized complex ellipsoid \( E \) as in (1.2), we denote by \( \mathcal{W}(E) \) the set consisting of all weakly, but not strictly, pseudoconvex boundary points of \( E \). Then it can be seen that
\[
\mathcal{W}(E) = \{ (z_1, z_2, \ldots, z_s) \in \partial E; \| z_2 \| \cdots \| z_s \| = 0 \}
\]
\[
\subset \bigcup_{i=2}^{s} \{ (z_1, \ldots, z_i, \ldots, z_s) \in C^{n_1} \times \cdots \times C^{n_i} \times \cdots \times C^{n_s}; z_i = 0 \}.
\]

We can now state the result due to Dini and Selvaggi Primicerio [3] in the following form:

**Theorem D-S.** Let \( E_1, E_2 \) be generalized complex ellipsoids in \( C^n \) with \( C^\omega \)-smooth boundaries and \( \mathcal{W}(E_1), \mathcal{W}(E_2) \) the sets of weakly pseudoconvex boundary points of \( E_1, E_2 \) respectively, as in (1.3). Let \( x_1 \in \partial E_1, x_2 \in \partial E_2 \) and \( U_1, U_2 \) open neighborhoods of \( x_1, x_2 \) in \( C^n \), respectively. Assume that:

1. \( \mathcal{W}(E_1) \) and \( \mathcal{W}(E_2) \) are contained in the union of finitely many complex linear subspaces of \( C^n \) of codimension at least 2;
2. \( U_1 \cap \partial E_1 \) is a connected open subset of \( \partial E_1 \);
3. \( \Psi: U_1 \cap E_1 \rightarrow U_2 \cap E_2 \) is a biholomorphic mapping that can be extended to a continuous mapping \( \overline{\Psi}: U_1 \cap \overline{E_1} \rightarrow \overline{E_2} \) with \( \overline{\Psi}(x_1) = x_2 \) and \( \overline{\Psi}(U_1 \cap \partial E_1) \subset \partial E_2 \).

Then \( \Psi \) extends to a biholomorphic mapping \( \Phi \) from \( E_1 \) onto \( E_2 \).

As noted by themselves in [3], the assumption (1) cannot be dropped in general; and also, after shrinking \( U_1 \) if necessary, one may further assume that \( \Psi \) is defined on all of \( U_1 \).

We finish this section by the following:

**Definition.** Let \( E_1 = E (n; n_1, n_2, \ldots, n_s; 1, p_2, \ldots, p_s) \) and \( E_2 = E (n; m_1, m_2, \ldots, m_t; 1, q_2, \ldots, q_t) \) be two generalized complex ellipsoids in \( C^n \). Then we say that \( E_1 \) precedes \( E_2 \) if \( s \leq t \) and there exists a permutation \( \sigma \) of the set \{2, \ldots, t\} such that \( (p_2, n_i) = (q_{\sigma(i)} m_{\sigma(i)}) \) for \( i = 2, \ldots, s \).

Note that every generalized complex ellipsoid precedes itself and that the unit ball \( B^n \) in \( C^n \) precedes any generalized complex ellipsoid in \( C^n \).
2. Proof of the Theorem

With the same assumption and notation as in (1.1) and (1.2), we write the given \( E \) and \( \bar{E} \) in the form \( E = E(n; n_1, n_2, \ldots, n_s; 1, p_2, \ldots, p_s) \) and \( x = (x_1, x_2, \ldots, x_s) \in C^{n_1} \times C^{n_2} \times \cdots \times C^{n_s} \).

In order to prove the theorem, we prepare the following:

Lemma. The domain \( D \) is biholomorphically equivalent to a generalized complex ellipsoid \( \tilde{E} \) that precedes \( E \).

Proof. The following proof will be presented in outline, since the details of the steps can be filled in by consulting the corresponding passages in the proof of [12; Theorem I].

If \( s = 1 \), i.e., \( E = B^n \), then \( x \) is a \( C^\infty \)-smooth strictly pseudoconvex boundary point of \( D \); and hence, \( D \) is biholomorphically equivalent to \( B^n \) by Rosay [15].

Assume that \( s > 1 \). According to the form of \( x \), we shall divide the proof into two cases as follows:

Case A. \( x = (x_1, 0, \ldots, 0) \).

In this case, there exists a sequence \( \{ \tilde{\phi}_v \} \subset \text{Aut}(E) \) such that \( \tilde{\phi}_v(0) \to x \) as \( v \to \infty \), where \( 0 \in E \) denotes the origin of \( C^n \). Hence, \( D \) is biholomorphically equivalent to \( E \) by Kodama, Krantz and Ma [12].

Case B. \( x = (x_1, \ldots, x_i, \ldots, x_s) \) with some \( x_i \neq 0 \) \((2 \leq i \leq s)\).

First of all, passing to a subsequence if necessary, one may assume that \( \phi_v(b) \in D \cap Q = E \cap Q \subset E \) for all \( v \). So there exists a sequence \( \{ \psi_v \} \) in \( \text{Aut}(E) \) such that

\[(2.1) \quad \psi_v(\phi_v(b)) = (0, z_2, \ldots, z_s) \quad \text{for} \quad v = 1, 2, \ldots; \]

\[(2.2) \quad \text{each } \psi_v \text{ can be written in the form} \]

\[ \psi_v(z) = \left(\left(A^z_1 + b^z\right) / (c^z_1 + d^z), \right. \]
\[ \left. z_2 / (c^z_1 + d^z)^{1/p_2}, \ldots, z_s / (c^z_1 + d^z)^{1/p_s}\right) \]

for \( z = (z_1, z_2, \ldots, z_s) \in E \subset C^{n_1} \times C^{n_2} \times \cdots \times C^{n_s} \).

Moreover, if we define the holomorphic mappings \( \psi_v^1 : B^n \to C^{n_1} \) by

\[(2.3) \quad \psi_v^1(z_1) = \left(\left(A^z_1 + b^z\right) / (c^z_1 + d^z)\right) \quad \text{for} \quad z_1 \in B^{n_1}, \]

then \( \psi_v^1 \in \text{Aut}(B^{n_1}) \) for all \( v = 1, 2, \ldots \). (For the structure of \( \text{Aut}(E) \), see [12].) Setting \( y_v = \phi_v(b) = (y_1^v, y_2^v, \ldots, y_s^v) \) for \( v = 1, 2, \ldots \), we have now

\[(2.4) \quad \psi_v^1(y_1^v) = 0 \quad \text{for all} \quad v = 1, 2, \ldots. \]
On the other hand, since $\|x_1\|^2 + \sum_{i=2}^{s} \|x_i\|^{2p_i} = 1$ and $x_i \neq 0$ for some $2 \leq i \leq s$, we see that

\[(2.5) \quad \text{the point } x_1 = \lim_{v \to \infty} y_i^v \text{ is contained in } B^{*1}, \]

which implies that $\{y_i^1\}$ lies in a compact subset of $B^{*1}$. This combined with (2.4) guarantees that $\{y_i^1\}$ has a convergent subsequence in $\text{Aut}(B^{*1})$ [13; p.82]. Here we assert that, after taking a subsequence if necessary, $\{\psi_v\}$ converges to some $\psi \in \text{Aut}(E)$. In fact, this can be seen as follows. With the same notation as in section 1 of [12], we can express $\text{Aut}(B^{*1}) = U(n_1, 1) / S^1$, where $U(n_1, 1)$ is a special kind of linear Lie group and $S^1$ is closed normal subgroup of $U(n_1, 1)$. Hence $\text{Aut}(B^{*1})$ is the base space of the principal fiber bundle $\pi : U(n_1, 1) \to U(n_1, 1) / S^1$. Let us assume that $\lim_{v \to \infty} \psi_i^1 = \psi \in \text{Aut}(B^{*1})$. Then there exists a $C^\omega$-smooth local cross section $\gamma$ of $\pi : U(n_1, 1) \to \text{Aut}(B^{*1})$ defined on an open neighborhood $O$ of $\psi_1$. Without loss of generality, we may assume that

$\{\psi_i^1\} \subset O$ and in (2.3) $\gamma(\psi_i^1) = \begin{pmatrix} A^v & b^v \\ c^v & d^v \end{pmatrix}$ for $v = 1, 2, \cdots$.

Then we have

$$\lim_{v \to \infty} \begin{pmatrix} A^v & b^v \\ c^v & d^v \end{pmatrix} = \gamma(\psi_1) \in U(n_1, 1).$$

This combined with (2.2) assures that $\{\psi_v\}$ converges to some $\psi \in \text{Aut}(E)$, as desired. Now, notice here that each $\psi_v$ as well as $\psi$ are defined on $B^{*1} \times C^{n_2} \times \cdots \times C^{n_s}$; and, in fact,

\[(2.6) \quad \{\psi_v, \psi_v^{-1} ; v = 1, 2, \cdots\} \subset \text{Aut}(B^{*1} \times C^{n_2} \times \cdots \times C^{n_s}); \]

\[(2.7) \quad \psi_v(z) \to \psi(z) \text{ (resp. } \psi_v^{-1}(z) \to \psi^{-1}(z)) \text{ uniformly on compact subsets of } B^{*1} \times C^{n_2} \times \cdots \times C^{n_s}.\]

Hence we have $z^0 := \lim_{v \to \infty} \psi(y^v) = \psi(x) \in \partial E$, because the set $\{x, y^v ; v = 1, 2, \cdots\}$ is now compact in $B^{*1} \times C^{n_2} \times \cdots \times C^{n_s}$ by (2.5). Therefore, Case I in the proof of [12; Theorem I] does not occur in our Case B. Once it is shown that there exists a small open neighborhood $U$ of $z^0$ such that $\psi_v^{-1}(E \cap U) \subset E \cap Q = D \cap Q$ for all sufficiently large $v$, the rest of our proof can be done with exactly the same arguments as in the proof (Case II, pp.181–190) of [12; Theorem I] only by setting $\Gamma = \text{id}_{C^n}$ throughout. Therefore, it is enough to prove the existence of such a neighborhood $U$ of $z^0$. To this end, taking (2.5) into account, we choose an open neighborhood $V$ of $x$ with compact closure in $(B^{*1} \times C^{n_2} \times \cdots \times C^{n_s}) \cap Q$. Then, by (2.6) and (2.7) we see that $\psi(V) = \lim_{v \to \infty} \psi(V)$ is an open neighborhood of $z^0$ and
\( \psi^{-1}(\psi(V)) \subset (B^{n_1} \times C^{n_2} \times \cdots \times C^{n_s}) \cap Q \) for all sufficiently large \( v \). Hence, every open neighborhood \( U \) of \( z^* \) with \( U \subset \psi(V) \) satisfies the requirement above. This completes the proof of the lemma in the case \( n_1 > 0 \).

Finally, consider the case \( n_1 = 0 \). Then, setting \( \Gamma = \text{id}_{C^n} \) and also \( \psi = \text{id}_{C^n} \) for all \( v \), and proceeding along exactly the same line as in the proof (Case II, pp.181–190) of [12; Theorem I], we can check that \( D \) is biholomorphically equivalent to some generalized complex ellipsoid \( \tilde{E} \) that precedes \( E \); thereby completing the proof of the lemma.

Q.E.D.

Proof of the Theorem. After relabeling the indices, one may assume that

\[(2.8) \quad n_2 = \cdots = n_k = 1 < n_{k+1}, \ldots, n_s \quad \text{for some integer} \quad k \quad (1 \leq k \leq s).\]

Here it is understood that all \( n_2, \ldots, n_s \geq 2 \) if \( k = 1 \), and also \( n_2 = \cdots = n_s = 1 \) if \( k = s \).

By virtue of the Lemma, \( D \) is now biholomorphically equivalent to a generalized complex ellipsoid \( \tilde{E} \) in \( C^n \) that precedes \( E \). Therefore, remembering the definition of precedence and renaming the indices if necessary, we may assume that \( D \) is biholomorphically equivalent to the generalized complex ellipsoid \( E^* \) in \( C^n \) defined by

\[ E^* = \{ z = (z_1, \ldots, z_s) \in C^{n_1} \times \cdots \times C^{n_s} = C^n ; \rho(z) < 1 \}, \]

where

\[ \rho(z) = \|z_1\|^2 + \sum_{a=2}^{j} |z_a|^2 + \sum_{a=j+1}^{k} |z_a|^2 + \sum_{b=k+1}^{l} \|z_b\|^2 + \sum_{b=l+1}^{s} \|z_b\|^2 \]

for some integers \( j, l \) \((1 \leq j \leq k \leq l \leq s)\), with the natural understanding that some of summands may vanish (for example, \( \sum_{a=2}^{j} |z_a|^2 = 0 \) if \( j = 1 \)). Let us fix a biholomorphic mapping \( F : D \to E^* \) and take a point

\[ z^* = (z^*_1, \ldots, z^*_s) \in Q \cap \partial D \quad \text{with} \quad \|z^*_1\| \cdots \|z^*_s\| \neq 0. \]

There is a sequence \( \{z^t\} \) in \( D \) such that

\[ z^t \to z^* \quad \text{and} \quad F(z^t) \to w^* \quad \text{for some point} \quad w^* \in \partial E^*. \]

Since \( z^* \) is a \( C^\omega \)-smooth strictly pseudoconvex boundary point of \( D \) and since \( w^* \) satisfies Condition \( (P) \) in the sense of Forstnerič and Rosay [5], the inverse mapping \( F^{-1} : E^* \to D \) of \( F \) has a continuous extension \( G : W \cap \tilde{E}^* \to \tilde{D} \) by [5], where \( W \) is a sufficiently small open neighborhood of \( w^* \) in \( C^n \). Clearly \( G(w^*) = z^* \). So there exist open neighborhoods \( U^*, W^* \) of \( z^* \), \( w^* \) in \( C^n \), respectively, such that
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\[ U^* \subset \mathcal{Q} \cap \{ \{ z_1, \ldots, z_s \} \in \mathbb{C}^n; \| z_1 \| \cdots \| z_s \| \neq 0 \}; \]

\[ W^* \subset W \text{ and } G(W^* \cap \overline{E}^*) \subset U^*. \]

Take a point

\[ w^{**} = (w_1^{**}, \ldots, w_s^{**}) \in W^* \cap \partial E^* \text{ with } \| w_1^{**} \| \cdots \| w_s^{**} \| \neq 0 \]

and set \( z^{**} = G(w^{**}) \in U^* \cap \partial D \). Then \( z^{**} \) and \( w^{**} \) are \( C^\omega \)-smooth strictly pseudoconvex boundary points of \( D \) and \( E^* \), respectively. Applying again the extension theorem of Forstnerič and Rosay [5] to the biholomorphic mappings \( F: D \to E^* \) and \( F^{-1}: E^* \to D \), one can find open neighborhoods \( U^{**}, W^{**} \) of \( z^{**} \), \( w^{**} \) respectively in \( \mathbb{C}^n \) such that

\( (2.9) \quad U^{**} \subset U^*, \quad W^{**} \subset W^* \text{ and } U^{**} \cap \partial D \text{ is a connected subset of } \partial D; \)

\( (2.10) \quad F \text{ extends to a homeomorphism } H: U^{**} \cap \overline{D} \to W^{**} \cap \overline{E}^* \text{ with } H^{-1} = G \text{ on } W^{**} \cap \overline{E}^*. \)

Now, define the mappings \( \Pi_1, \Pi_2: \mathbb{C}^n \to \mathbb{C}^n \) by setting

\[ \Pi_1(z) = (z_1, (z_2)^{p_2}, \ldots, (z_k)^{p_k}, z_{k+1}, \ldots, z_s), \]

\[ \Pi_2(z) = (z_1, \ldots, z_j, (z_{j+1})^{p_{j+1}}, \ldots, (z_k)^{p_k}, z_{k+1}, \ldots, z_s) \]

for \( z = (z_1, \ldots, z_s) \in \mathbb{C}^{n_1} \times \cdots \times \mathbb{C}^{n_s} = \mathbb{C}^n \); and consider the generalized complex ellipsoids \( E_1, E_2 \) in \( \mathbb{C}^n \) defined by

\[ E_1 = E(n_1 + n_2 + \cdots + n_k, n_{k+1}, \ldots, n_s; 1, p_{k+1}, \ldots, p_2), \]

\[ E_2 = E(n_1 + n_2 + \cdots + n_1, n_{k+1}, \ldots, n_s; 1, p_{l+1}, \ldots, p_2). \]

Since \( n_2 = \cdots = n_k = 1 \) by (2.8) and since \( 2 \leq p_2, \cdots, p_k \in \mathbb{Z} \) by (1.1), both \( \Pi_1 \) and \( \Pi_2 \) are proper holomorphic mappings from \( \mathbb{C}^n \) onto \( \mathbb{C}^n \) such that

\( (2.11) \quad \Pi_1(E) = E_1 \) and \( \Pi_2(E^*) = E_2; \)

\( (2.12) \quad \Pi_1 \) and \( \Pi_2 \) are injective near \( z^{**} \) and \( w^{**} \), respectively.

After shrinking \( U^{**} \) and \( W^{**} \) if necessary, we can therefore assume that the restrictions \( \Pi_1|U^{**}: U^{**} \to \Pi_1(U^{**}) \) and \( \Pi_2|W^{**}: W^{**} \to \Pi_2(W^{**}) \) are biholomorphic mappings. Consider here the homeomorphism

\[ \Psi := \Pi_2 \circ H \circ (\Pi_1|U^{**} \cap \overline{D})^{-1}: \Pi_1(U^{**}) \cap \overline{E}_1 \to \Pi_2(W^{**}) \cap \overline{E}_2. \]

Then, it is obvious that the hypotheses (2) and (3) of Theorem D-S hold with \( x_1 = \Pi_1(z^{**}), \quad x_2 = \Pi_2(w^{**}), \quad U_1 = \Pi_1(U^{**}) \) and \( U_2 = \Pi_2(W^{**}) \). Moreover, in view of (1.3), the set \( \mathcal{W}(E_1) \) (resp. \( \mathcal{W}(E_2) \)) is contained in the union of finitely many complex linear subspaces of \( \mathbb{C}^n \) of codimension at least 2 if and only if all
\( n_{k+1}, \ldots, n_e \geq 2 \) (resp. \( n_{l+1}, \ldots, n_e \geq 2 \)), which is now guaranteed by (2.8). (Note that \( p_{k+1}, \ldots, p_e \geq 2 \) and \( l \geq k \geq 1 \).) Therefore, Theorem D-S can be applied to obtain a biholomorphic mapping \( \Phi : E_1 \to E_2 \) such that \( \Psi(z) = \Phi(z) \) for all \( z \in \Pi_1(U^{**} \cap E_1) \), or equivalently

\[
\Phi^{-1} \circ \Pi_2 \circ \Gamma(z) = \Pi_1(z) \quad \text{for all } z \in U^{**} \cap D;
\]

consequently \( \Phi^{-1} \circ \Pi_2 \circ \Gamma(z) = \Pi_1(z) \) for all \( z \in D \) by the principle of analytic continuation. This combined with the fact that \( \Phi^{-1} \circ \Pi_2 \circ \Gamma : D \to E_1 \) is a proper mapping yields at once that \( D = E \) as sets.

Finally, since \( \text{Aut}(E) = \text{Aut}(D) \) is now non-compact by the hypothesis (3) of the theorem, one concludes that \( n_1 > 0 \), i.e., at least one of the \( p_i \)'s must be equal to 1. (Recall the understanding made after (1.2).) This completes the proof of the theorem. Q.E.D.

**Remark 1.** In the proof above, one can assume that the continuous extension \( H : U^{**} \cap \tilde{D} \to W^{**} \cap \tilde{E} \) of \( \Gamma \) is the restriction of a biholomorphic mapping from \( U^{**} \) onto \( W^{**} \) (after shrinking \( U^{**} \) and \( W^{**} \) if necessary). In fact, this immediately follows from [4,14] or [1], because by the construction both \( U^{**} \cap \partial D \) and \( W^{**} \cap \partial E \) are \( C^\infty \)-smooth strictly pseudoconvex real hypersurfaces in \( C^n \) and \( H : U^{**} \cap \partial D \to W^{**} \cap \partial E \) is a CR-homeomorphism.

**Remark 2.** In the theorem, assume the following (2)* instead of (2):

(2)* There exist a point \( \tilde{x} \in \partial E \), open neighborhoods \( Q \) of \( x \), \( \tilde{Q} \) of \( \tilde{x} \), and a biholomorphic mapping \( \Gamma : Q \to \tilde{Q} \) such that \( \Gamma(x) = \tilde{x} \) and \( \Gamma(D \cap Q) = E \cap \tilde{Q} \).

Then, a glance at our proof of the theorem tells us that \( D \) is biholomorphically equivalent to \( E \) and that at least one of the \( p_i \)'s must be equal to 1.

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**References**


