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Measurement of Shear Wave Velocity in Soils*

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(With 1 Table and 9 Figures)

Introduction

There are some experimental works to measure deformation constants using dynamic methods, however the results obtained had been scarcely discussed with interparticle bonds of soil. The deformation characteristics have been measured statically and discussed widely in soil mechanics with strain range of several percents in the aspect of failure of soils. To discuss mechanical characteristics of interparticle bonds with deformation characteristics in very small strain range, dynamic method of resonance column has been applied in this study to obtain elastic constants of soil. The deformation characteristics of soil have been recognized to show various wide ranges from elastic, viscoelastic to plastic nature according to stress or strain levels applied.

If the soil is assumed as homogeneous and elastic material for very small strain range, only two independent elastic moduli exist, such as bulk modulus and rigidity. Rigidity of soil is considered due to interparticle bond of the soil structure and in the other hand bulk modulus is considered to depend on the mechanical interaction between solid soil skelton and fluid movement.

In this paper, the author describes the method to obtain elastic constants in small strain range from the resonance column method applied for soils and further discuss the results of elastic constants as well as damping characteristics.

Apparatus

There are two types of method to measure dynamic characteristics of soils in small strain range, one is ultrasonic pulse method, another is resonance method, which have been utilized by ISHIMOTO and IIDA (1937), HARDIN and RICHART (1963), HALL and RICHART (1963), HARDIN and BLACK (1968), HUMPHRIES and WAHLS (1968) and AFIFI and RICHART (1973) *etc.* to measure velocity of some soils. The pulse method may be considered simpler and easier to handle than resonance column method. The shape of pulse, however, changes drastically with propagation due to damping effect, and sometimes the initial S wave pulse is difficult to be identified due to the earlier arrival of P waves associated with S wave.

Although the column method needs more complicated equipment than pulse method, it gives fairly good accuracy to shear wave velocity and damping factor which measure resonance frequency and its sharpness. The author adapted resonance column method for its better accuracy with much simpler equipment than that used by the past researchers.

The bottom of the soils specimen is fixed to the base of the pedestal and torsional

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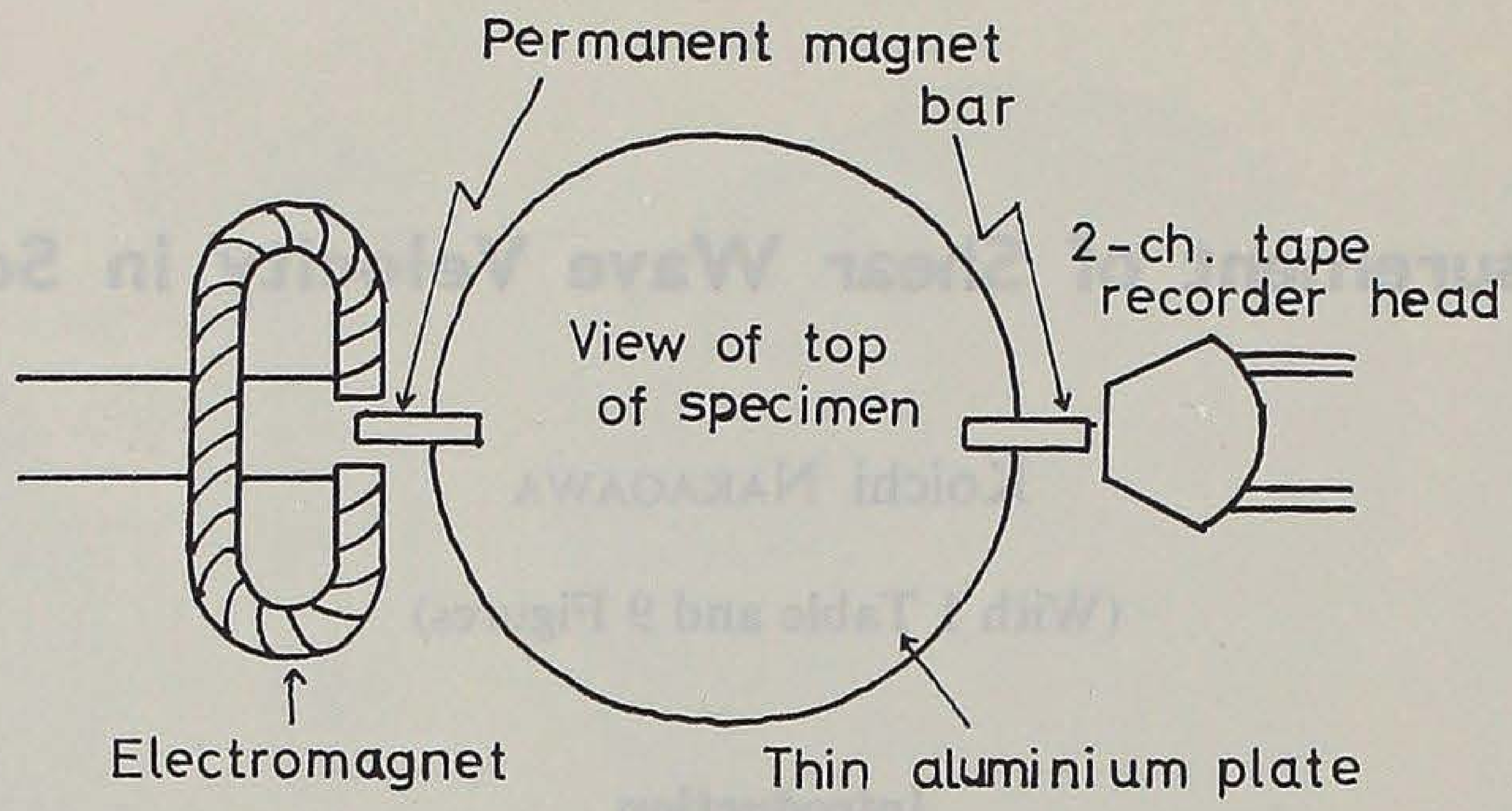

Fig. 1 Schematic diagram of transducer system on the vibration cap. The output power fed to an electromagnetic exicter and the amplitude of the torsional vibration are detected by a 2-ch recorder head. The permanent magnet bars are fastened to the top free end of the specimen with the thin aluminium plate.

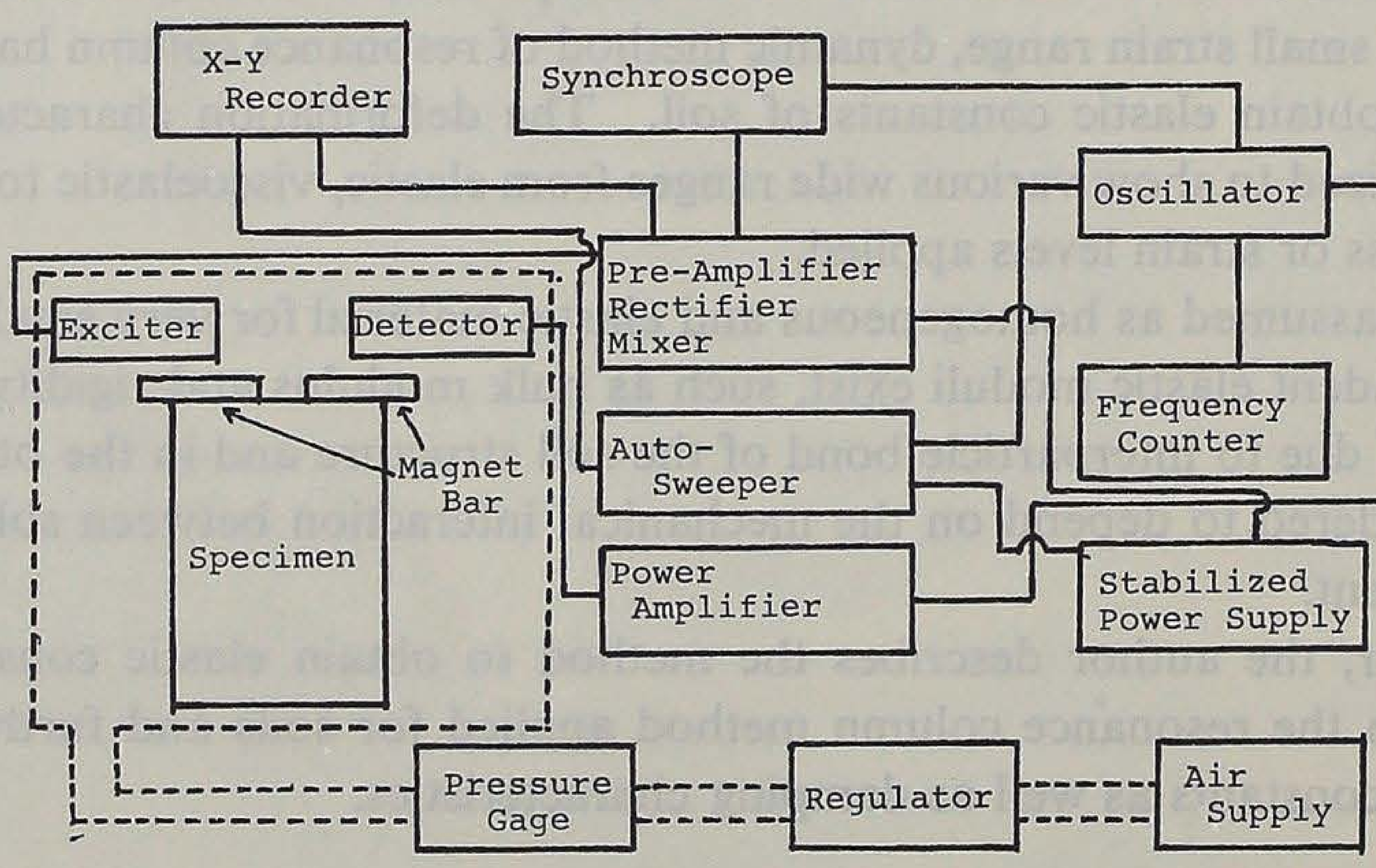

Fig. 2 Schematic diagram of arrangement of the apparatus for resonance column test.

vibrator and detector are attached on the top of the specimen (shown in Fig. 1). One of the special feature of this apparatus is in the way of excitation and detection of vibration shown in Fig. 2. To achieve the unconstrained condition of the top of specimen, the top surface equipped with very small vibrator and senser is completely free from any mechanical contact. The vibrator and senser on the top are consist of two little permanent magnet bars with diameter of 2 mm and length of 10 mm which are fixed to the vibration cap of thin aluminium circular plate. To clarify the effect of the vibration cap to the resonance frequency which is one of the main objective in this study, various diameters of the cap are arranged for the specimens with diameter of 3.0~5.5 cm and length of 0.7~25 cm. To avoid any possible energy loss due to friction, the bottom of the specimen is fixed to pedestal and the top to vibration cap with glue, Alon alpha.

Driving force to give torsional deformation of the column is applied to the one of the permanent magnets of the vibration cap through magnetic coil to which the power is supplied by sine wave generator and power amplifier. The velocity type transducer to obtain the amplitude of torsional vibration consist of a permanent magnet bar and an electromagnetic head of the conventional tape recorder. The electromagnetic head to change the mechanical signals into electric signals is two-channel type in order to cancel out the current of direct electric induction to the head due to the current of driving coil.

The change of the vibrational frequency for driving coil is obtained by frequency sweep control unit automatically or manually. The recording chart of the amplitude of vibration vs frequency is thus obtained using X-Y recorder with monitoring the frequency by counter and the shape of the wave on oscilloscope. This apparatus can be set int othe conventional triaxial cell to investigate soils under various pressure conditions.

Specimen

Specimens used for this study are obtained from Pleistocene and Recent sedimentary clay in Osaka. The specimens are mainly marine clay in the Osaka Group and the later Pleistocene sediments named Ma 1–Ma 12 in ascending order. Recent alluvial clays are sampled from the bottom of the present Osaka Bay. These specimens are trimmed in cylindrical shape with great care and wrapped with saran sheet to prevent change of water content during the measuring.

The experimental results and discussion

The resonance frequency is read directly from the monitoring digital frequency counter, while the resonance curve, the relationship of induced vibrational amplitude and frequency, are obtained automatically in X-Y recorder (Fig. 3). In this resonance method, it is possible to have fundamental deflection mode, however it is easy to identify the torsional mode through observing higher mode. The relationships between resonant frequency and mode number of the torsional vibration are shown in Fig. 4.

If specimen is assumed as homogeneous and linear elastic material, the resonance frequency for torsional vibration with the boundary condition of one end free and the other end fixed is uniquely given by the following equation

$$f=\frac{(2n+1)Vs}{4l}=\frac{(2n+1)}{4l}\sqrt{\frac{G}{\rho}} \qquad \cdots\cdots\cdots\cdots(1)$$

where, Vs : shear wave velocity
G : rigidity
ρ : density
l : length of specimen
n : mode number.

However, in usual testing procedure the condition of the free end is not satisfied due to the effect of some attachment equipped to the "free" end. Consequently, the resonance frequency obtained may be different from the value given by (1) and need to be corrected.

The solution of torsional resonance frequency of the elastic cylindrical column with one end fixed and the other free end having some rigid body with polar momentum of inertia is given by elastic theory as follows,

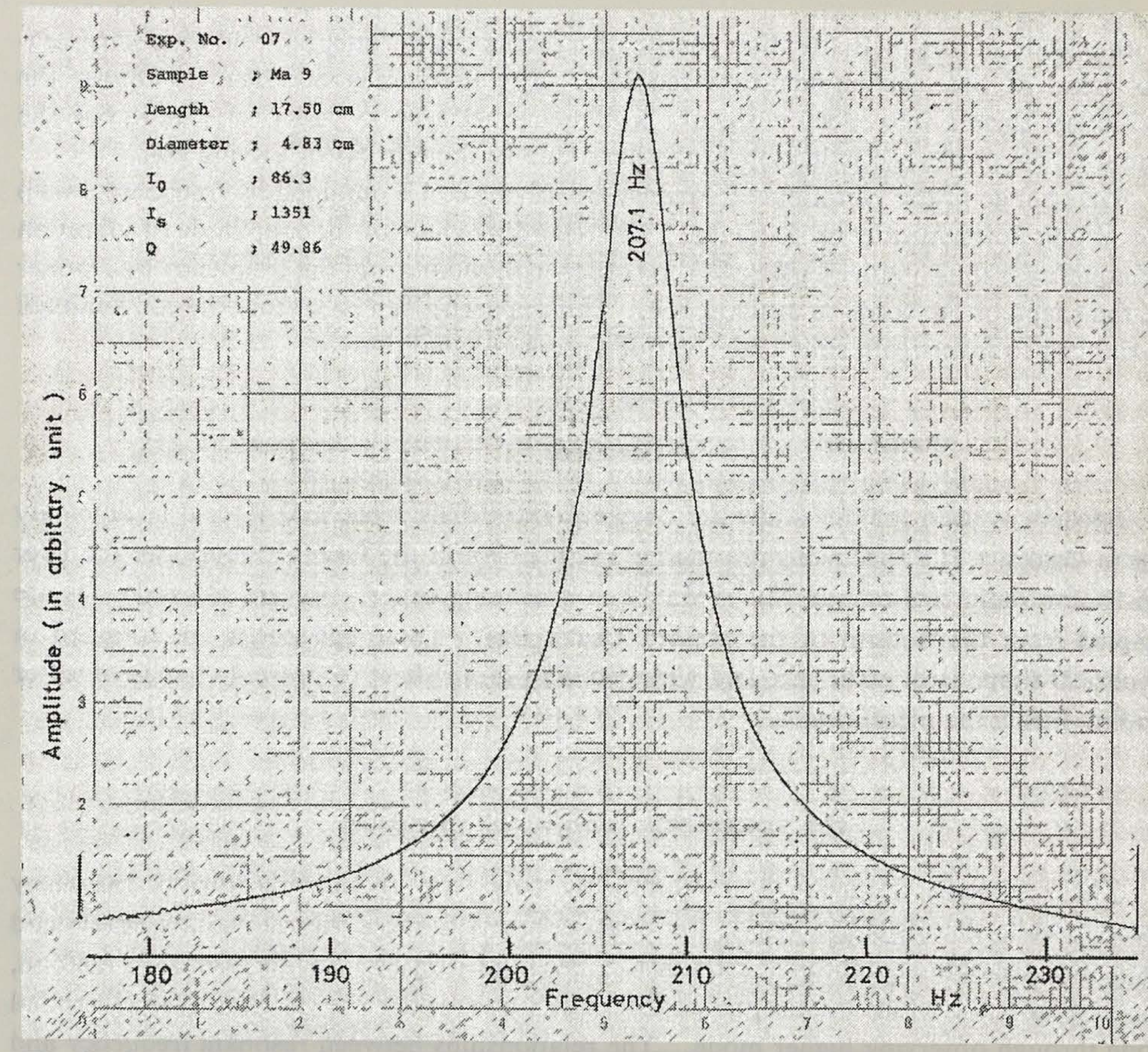

Fig. 3 An example of chart recording resonance curve. The specimen is the Pleistocene sedimentary clay Ma 9.

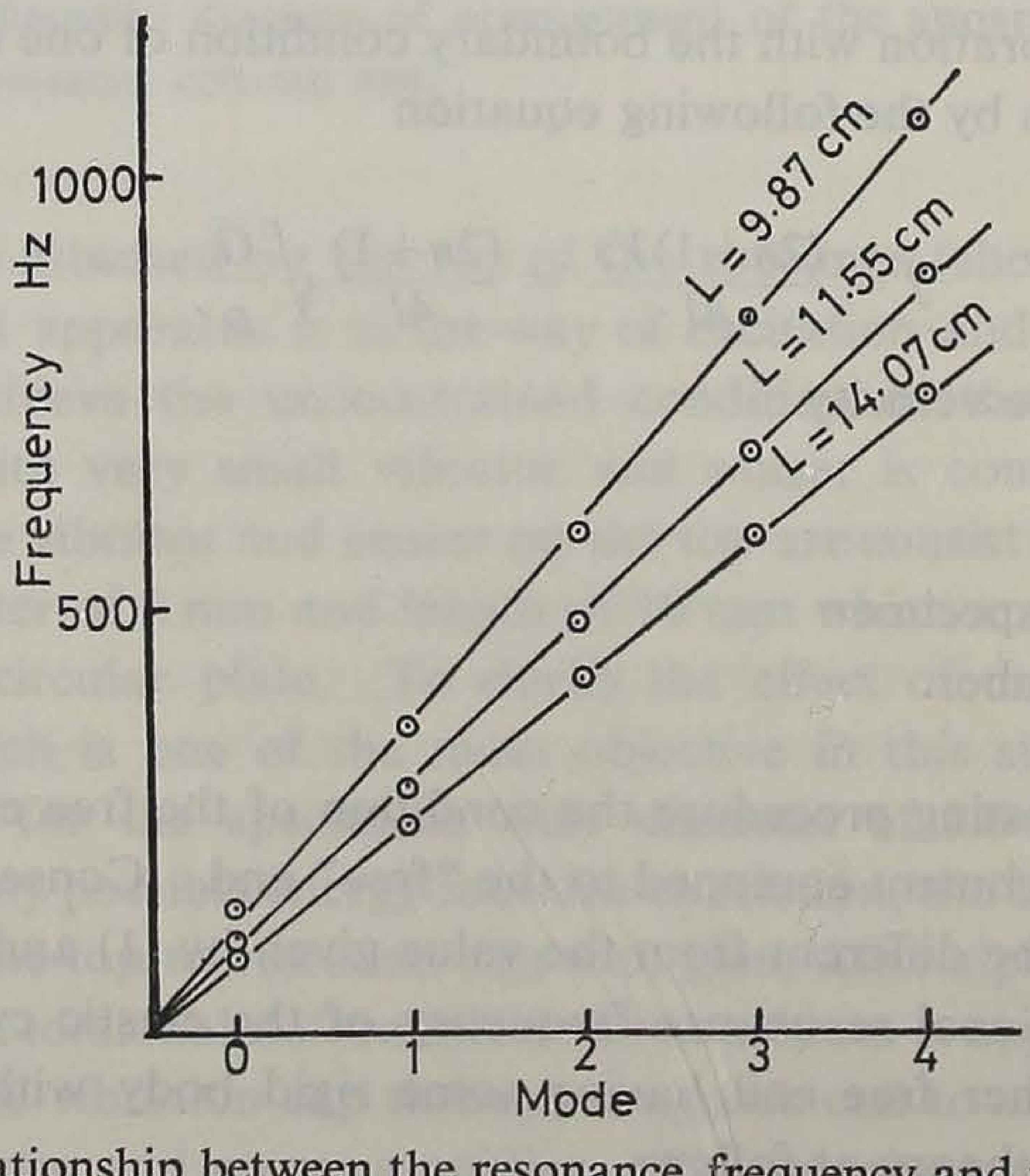

Fig. 4 Relationship between the resonance frequency and torsional mode number when fundamental one is 0.
The specimen is the Pleistocene sedimentary clay Ma 1.

$$\cot \alpha = \alpha\beta \qquad \cdots\cdots\cdots\cdots(2)$$

where, $\alpha = \dfrac{\pi f}{2 f_0}$, $\beta = \dfrac{I_0}{I_s}$

and f, f_0 are frequencies of effective and elastic column only and I_s, I_0 are polar momentum of inertia of attached rigid body and elastic column, respectively.

In the past study the effect of the added polar momentum of inertia I_0 to the resonance frequency is neglected or corrected to obtain the true resonance frequency (HALL and RICHART, 1963) without any argument of the applicability and the accuracy of the measuring method to the soil with assuming soil to be linear elastic.

In this study some of these fundamental aspects of the method are discussed. A serious of test has been carried out with various length of specimens and changing polar momentum of inertia of the attachment to measure the effect of added moment to the apparent resonance frequency. The result of the test shows the nearly linear relationship between the measured resonance period T and the added moment I_0 (Fig. 5).

The true resonance period T_0 is obtained by the extrapolation of this line to the corresponding period for $I_0=0$. In Fig. 6, the true resonance period obtained above procedure are plotted against the length of the specimens, which shows linear relationship leading to a conclusion that the soil can be treated as linear elastic in resonance test with in this frequency range.

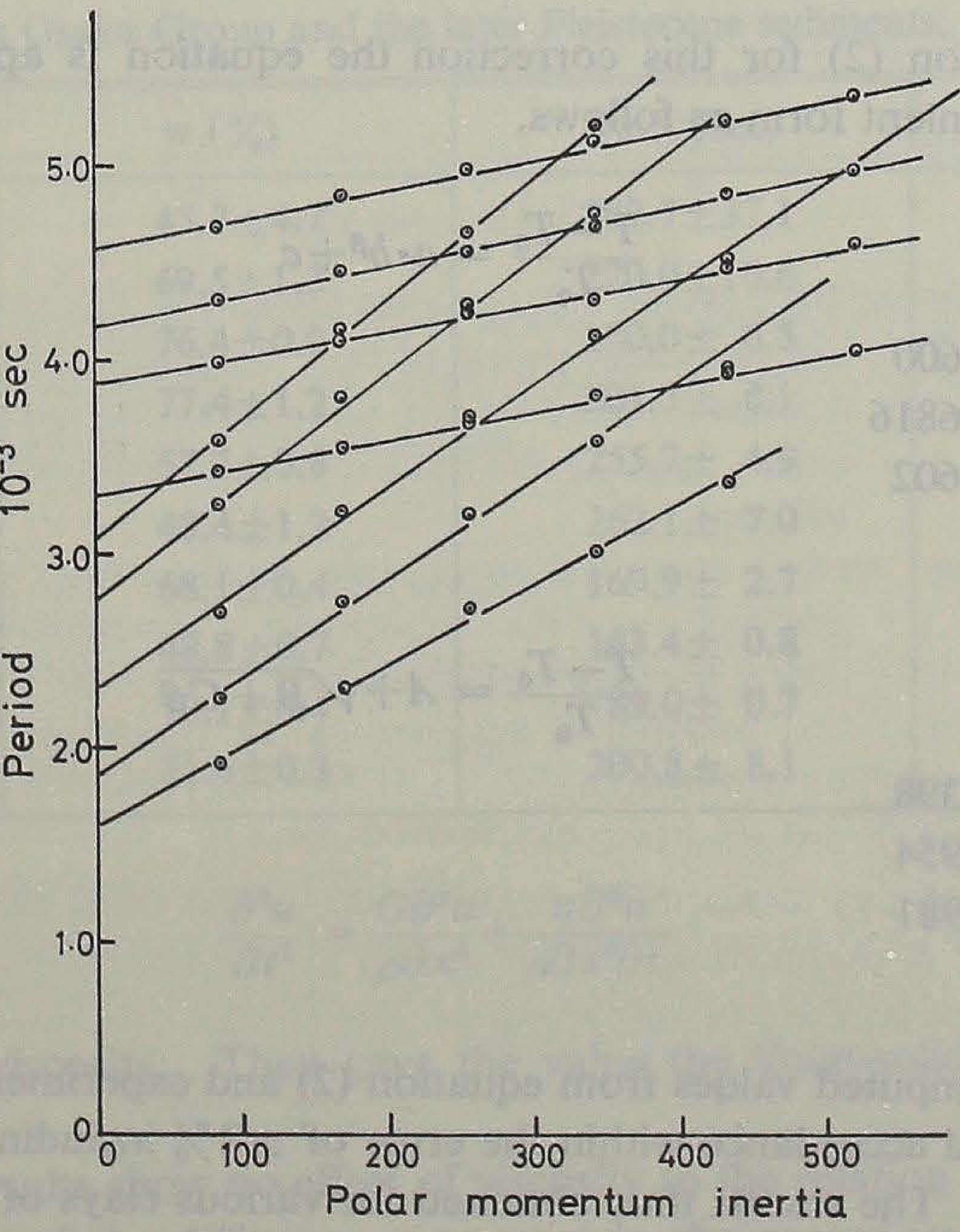

Fig. 5 Relationship between the observed [resonance 'period T and the added polar momentum I_0 of inertia of attachment. The specimen is the Recent Alluvial clay. Lines for the group of larger period, $T_0 > 3.2 \times 10^{-3}$ sec, are of specimen with diameter of about 5.0 cm and lines for lower group are of about 3.3 cm.

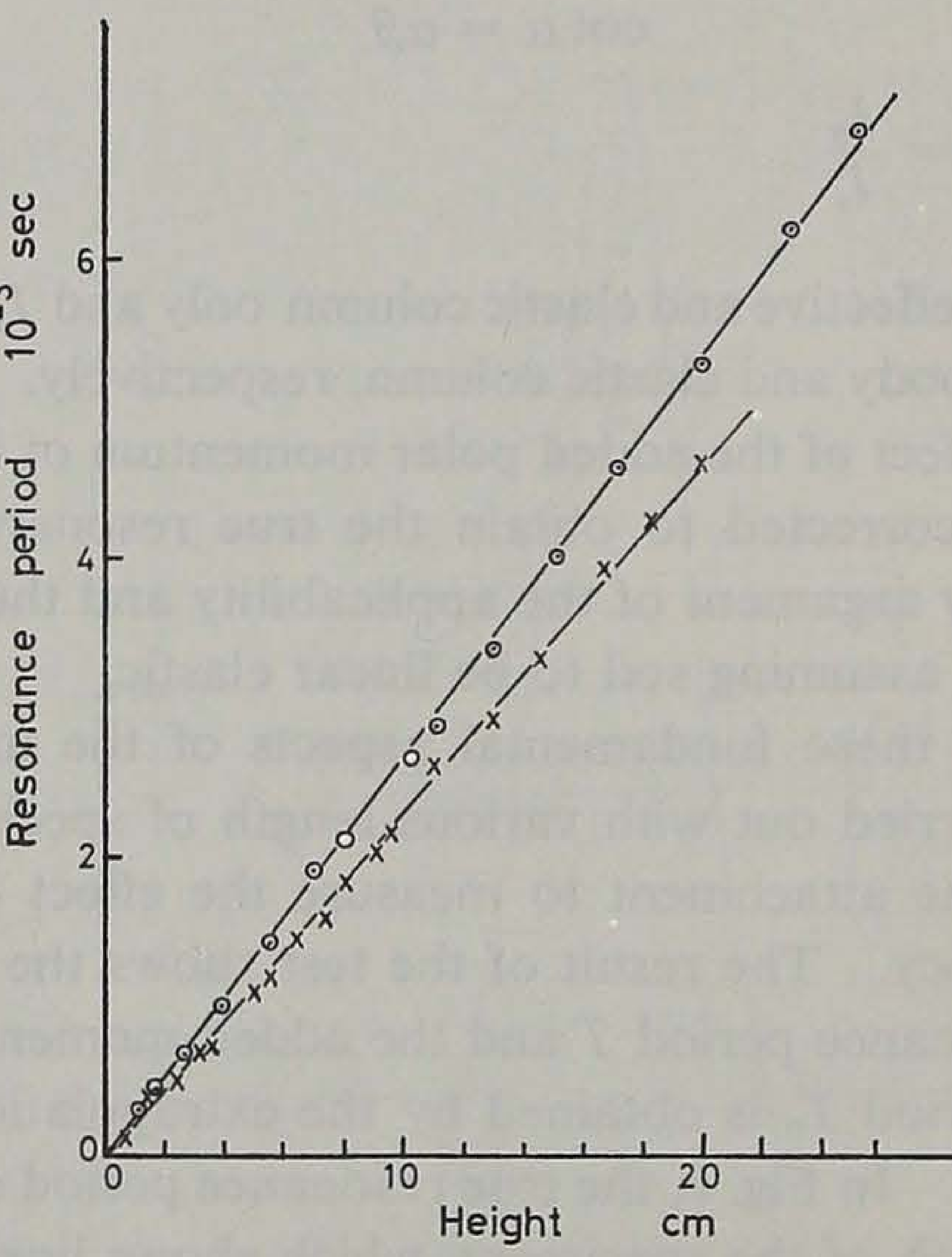

Fig. 6 Relationship between the resonance period and length of specimen.
◉; Ma 9, ×; Recent alluvial clay (remolded)

To apply equation (2) for this correction the equation is approximately expressed rather in much convenient form as follows,

$$\frac{T-T_0}{T_0}=a\cdot b^{\beta}+c \qquad \cdots\cdots\cdots\cdots(3)$$

where a : -2.600
b : 0.6816
c : 2.602
and when $\beta \leq 1$
or

$$\frac{T-T_0}{T_0}=A+\sqrt{B+C\beta} \qquad \cdots\cdots\cdots\cdots(4)$$

where A : -1.398
B : 1.954
C : 2.981
and when $\beta \leq 1$

In Fig. 7, the computed values from equation (2) and experimentally estimated values are compared in good accordance within the error of ±2% including the difference of the specimen measured. The results thus obtained for various clays of the Osaka Group and the later Pleistocene sediments are shown in Table 1.

ISHIMOTO and IIDA (1936 and 1937) obtained the nonlinear relationship between the resonance period and the length of the specimen from resonance column test and discussed as the viscoelastic behaivior due to the frequency effect on the resonance period based on the equation

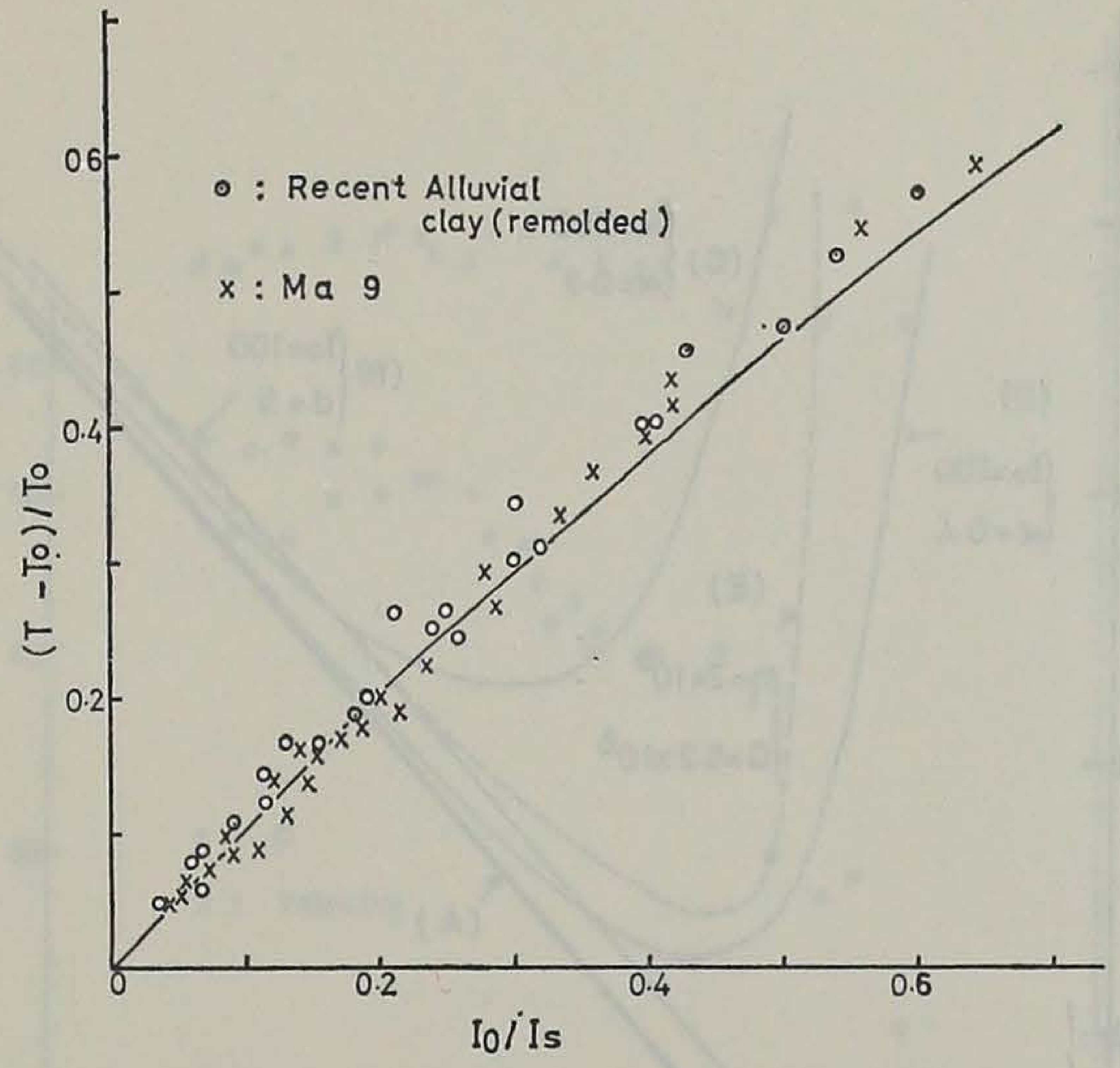

Fig. 7 Relationship between the period ratio and the polar momentum of inertia ratio. The solid line is theoretical curve from equation (2).

Table 1. Shear wave velocities, quality factors and water contents of various sedimentary clays in the Osaka Group and the later Pleistocene sediments.

Sample	w (%)	Vs (m/sec)	Q
Ma 1	45.7±4.7	292.7±37.1	35.1±4.8
Ma 6	69.5±1.3	220.0±10.6	48.6±3.0
Ma 12 (A40)	76.4±0.9	130.0± 6.5	23.0
Ma 12 (A42)	77.4±1.2	201.3± 8.1	38.5±2.9
Ma 10 (A81)	57.7±0.8	255.7± 4.9	28.4±3.0
Ma 10 (A83)	48.4±1.2	262.1± 7.0	36.4±1.5
Ma 6 (K38)	68.1±0.4	169.9± 2.7	41.0±4.1
Ma 5 (K59)	48.8±0.7	163.4± 0.8	33.4±4.7
Ma 4 (K78)	73.1±0.4	89.0± 0.7	32.0±6.8
Freshwater clay	31.6±0.3	200.8± 8.1	19.9±1.3

$$\frac{\partial^2 u}{\partial t^2} = \frac{G\partial^2 u}{\rho\partial x^2} + \frac{\eta\partial^3 u}{\rho\partial x^2\partial t} \qquad \cdots\cdots\cdots\cdots(5)$$

where η is Voigt-solid viscosity. They gave the value the Voigt-solid viscosity of the soil used as $10^4 \sim 10^5$ poise.

The present test results show no effect of viscosity to the relation of period and length as Fig. 6. The reason of the difference may exist in the procedure of analysis used by ISHIMOTO and IIDA (1936 and 1937), who seem to have neglected the effect of the added polar momentum of inertia which lead to the nonlinear relationship between period and length. The feature of effect of the added moment and the Voigt-solid viscosity to the resonance period are shown in Fig. 8.

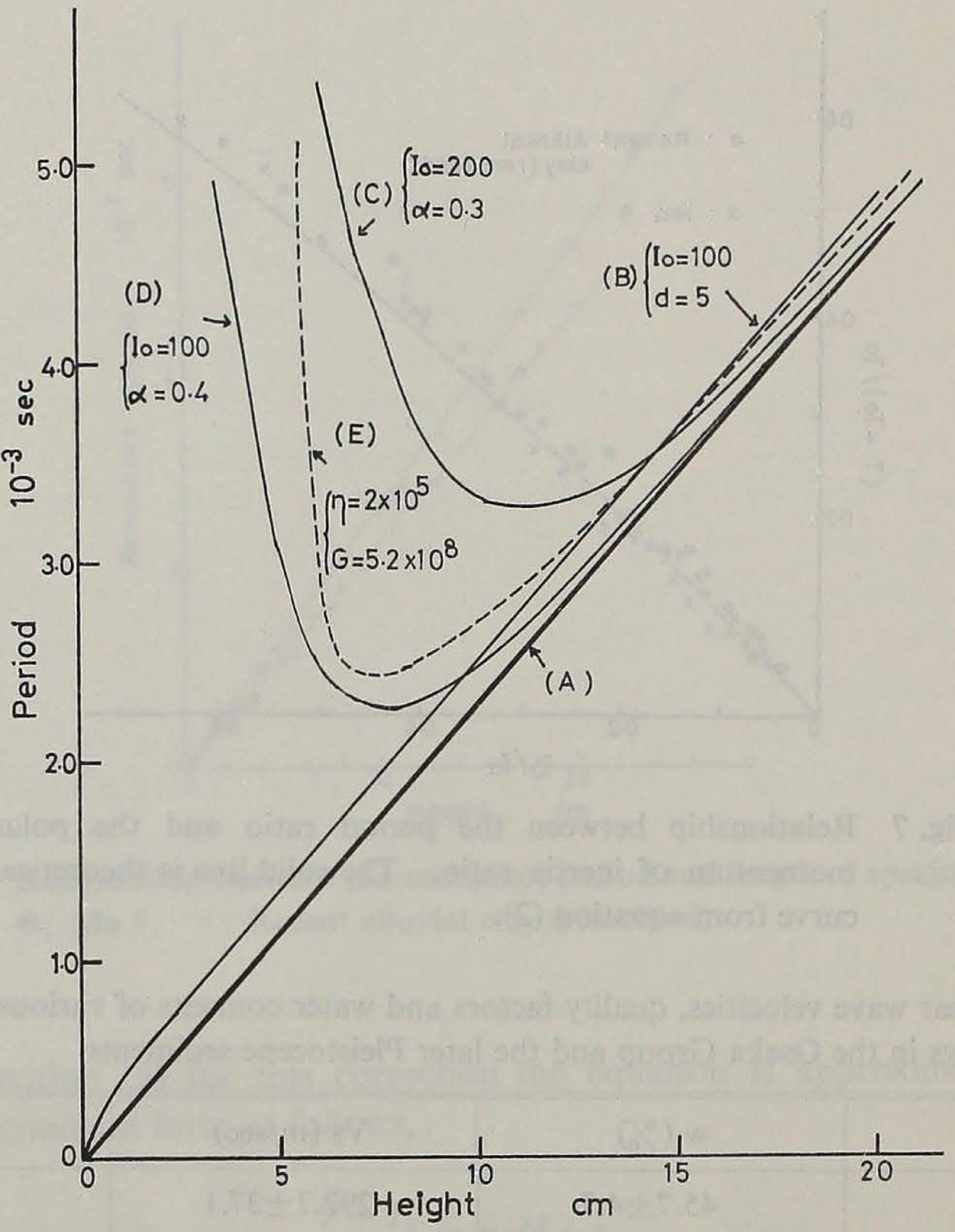

Fig. 8 Relationship between the specimen length and the resonance period.
Line A: true resonance period of Ma 9 specimen after the correction.
Line B: Apparent period of the same specimen with constant diameter, 5 cm. Polar momentum of inertia of attachment is 100 gcm.
Line C and D: Apparent period with constant ratio of diameter by length, 0.3 and 0.4, respectively. The polar momentums of inertia are 100 and 200 gcm.
Line E: The material is assumed as the viscous Voigt-solid based on equation (5).

The viscosity or energy loss coefficient, however, which does not give any effect on the relationship between period and specimen length in present study, are obtained from the sharpness of the resonance curve and some of the results are shown in Fig. 9 as well as shear wave velocity calculated from resonance period with changing frequency. Shear wave velocity of the clays seems not to depend on frequency within the tested frequency range, but the damping factor due to viscosity is shown to increase with frequency increase which may not be expressed through simple Voigt model.

Shear wave velocity or rigidity of soils as well as damping factor is very important as fundamental physical constants which lead not only to the qualitative but also to quantitative discussion of the bonding of soil formation through the physico-chemical aspect which will be followed in the next report.

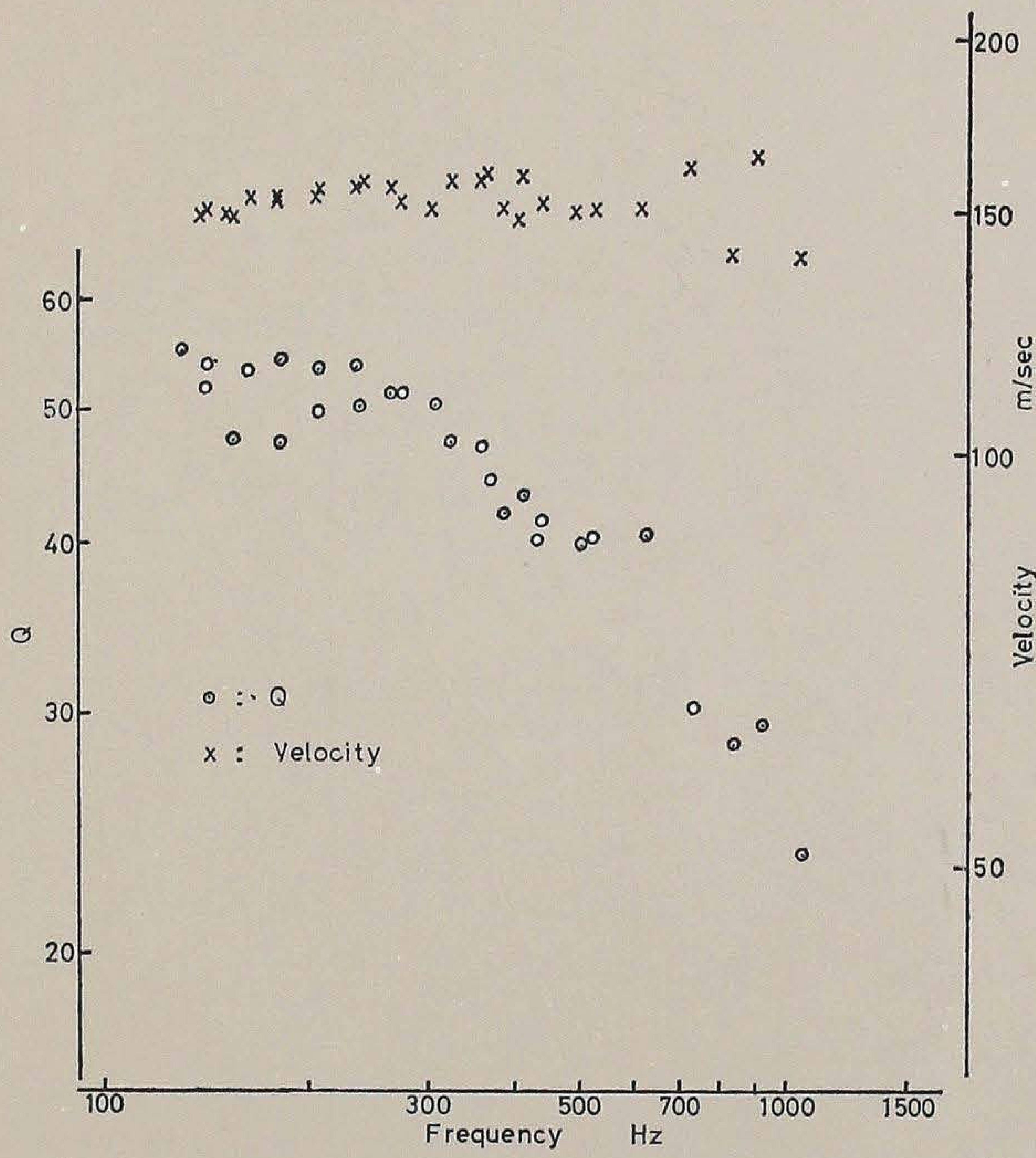

Fig. 9 Variation of the quality factor Q and shear wave velocity with frequency (Ma 9).

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