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ZEROS OF THE ALEXANDER POLYNOMIAL OF KNOT

AKIO NOGUCHI

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Abstract

The leading coefficient of the Alexander polynomial of a knot is the most informative element derived from this invariant, and the growth of orders of the first homology of cyclic branched covering spaces is also a familiar subject. Accordingly, there are a lot of investigations in each subject. However, there is no study which deals with both subjects in the same context. In this paper, we show that the two subjects are closely related in p-adic number theory and dynamical systems.

1. Introduction

The leading coefficient of the Alexander polynomial $\Delta_K(t)$ of a knot K is a well-known invariant for detecting fibered knots. The Alexander polynomial of a fibered knot is always monic [22]. The converse is not always true, but it holds for many knots, for example, alternating knots [21]. Moreover the monic condition characterizes fibered knots in the sense of realization [2, 24].

The leading coefficient of the Alexander polynomial of a knot is also related to the commutator subgroup G'_K of the knot group $G_K = \pi_1(S^3 \setminus K)$. The abelianaization of G'_K is finitely generated if and only if the leading coefficient is ± 1 [4, 25].

The r-fold cyclic covering $X_r(K)$ branched over a knot K is a fundamental object in the knot theory because topological invariants of it are also invariants of the knot. In [8], Gordon studied the growth of the order of $H_1(X_r(K); \mathbb{Z})$ with respect to r and asked whether the growth is exponential in the case where some zeros of $\Delta_K(t)$ are not a root of unity. More than 15 years later, this question was affirmatively answered by Riley [26] and González-Acuña and Short [7] independently.

As Gordon commented, the difficulty in computing the growth of orders arises from the case in which all zeros belong to the unit circle but some are not a root of unity (e.g. $\Delta_{5_2}(t) = 2t^2 - 3t + 2$). In this case, the standard norm is useless. Riley [26] overcomed this difficulty using p-adic analysis. On the other hand, González-Acuña and Short [7] calculated the growth by showing that it is equal to the Mahler measure of the Alexander polynomial. Moreover, in their introduction, González-Acuña and Short remarked on the possibility of interpreting the growth as the entropy of a dynamical system.

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In this paper, we clarify the dynamical system which was predicted by González-Acuña and Short and also explain that the p-adic approach of Riley is useful in essence.

Solenoidal dynamical systems are natural generalizations of toral dynamical systems. The entropy of solenoidal dynamical systems was first calculated by Juzvinskiĭ [11] and latterly re-calculated by Lind and Ward [18] employing p-adic number theory. Furthermore Einseidler and Ward [5] also investigated the relation between the Mahler measure and the entropy of a solenoidal dynamical system associated with a Fitting ideal of a module.

Applying the above results of solenoidal dynamical systems to knot theory, we refine some topics on the Alexander polynomial of a knot. In the following paragraph, we exhibit some of our results, where apparently different topics are closely related. The reason of this connection is explained in Corollary 3 below.

Let $\Delta_K(t) = \sum_{i=0}^{n} a_i t^i$ $(a_0 a_n \neq 0)$ be the Alexander polynomial of a knot K and α_i the zeros (counted with multiplicity) of $\Delta_K(t)$. Then,

(1) the leading coefficient of $\Delta_K(t)$ is

$$\log|a_n| = \sum_{p<\infty} \sum_{|\alpha_i|_p>1} \log|\alpha_i|_p$$

(Corollary 4), and

(2) the growth of order of the first homology of the r-fold cyclic covering branched over K is

$$\lim_{\substack{r\to\infty \\ |H_1(\cdot)|\neq 0}} \frac{\log|H_1(X_r(K);\mathbb{Z})|}{r} = \sum_{p\leq\infty} \sum_{|\alpha_i|_p>1} \log|\alpha_i|_p$$

(Corollary 1).

Here, $|\cdot|_p$ are the p-adic norm normalized with $|p|_p = p^{-1}$ on the field of p-adic numbers $\mathbb{Q}_p$, and $|\cdot|_\infty$ is the standard norm on the field of real numbers $\mathbb{Q}_\infty = \mathbb{R}$. (we assume that the embeddings $\overline{\mathbb{Q}} \to \overline{\mathbb{Q}}_p$ are fixed.)

In our study, we also establish the following:

- The distribution of the zeros measures a "distance" of the Alexander module from being finitely generated as a $\mathbb{Z}$-module. (Section 3.2)
- In [8], the primary interest in investigating the growth (2) might be to study the periodicity of $H_1(X_r(K);\mathbb{Z})$. However, the growth also measures complexity of the Alexander module naturally in a sense of solenoidal dynamical systems. This means that this growth can be interpreted as volume growth in an adele ring. (Section 3.1)

We have a few comments on this study, which might make it a little more attractive:

The Alexander polynomial of a knot is defined as a greatest common divisor of the initial Fitting ideal (elementary ideal) of the Alexander module $H_1(X_\infty(K);\mathbb{Z})$ as a $\mathbb{Z}[t^{\pm}]$-module. Here, the indeterminate t is identified with the meridian action on

$H_1(X_\infty(K);\mathbb{Z})$ where $X_\infty(K)$ is the infinite cyclic cover of $X(K) = S^3 \setminus K$. Then, by tensoring with the rational numbers $\mathbb{Q}$, the Alexander polynomial is also the generator of the Fitting ideal of the module $H_1(X_\infty(K);\mathbb{Q})$ as a $\mathbb{Q}[t^{\pm}]$-module, and hence it is the characteristic polynomial of the meridian action on $H_1(X_\infty(K);\mathbb{Q})$, up to units (see Theorem 6.17 in [16]).

While the rational homology $H_1(X_\infty(K);\mathbb{Q})$ gives a nice explanation of the Alexander polynomial, the information on the leading coefficient a_n is lost in $H_1(X_\infty(K);\mathbb{Q})$ because a_n is a unit in $\mathbb{Q}[t^{\pm}]$. On the other hand, the entropy is advantageous to manage this difficulty because we can replace $H_1(X_\infty(K);\mathbb{Z})$ with $H_1(X_\infty(K);\mathbb{Q})$ with preserving the entropy (cf. Step 1 in the proof of Proposition 1).

In [26], Riley also proved a result on the p-part of $|H_1(X_r(K);\mathbb{Z})|$. He obtained the upper bounds for the p-parts: $|H_1(X_r(K);\mathbb{Z})|^{(p)} < AH^{pE}r^n$, where A, H, E, n are constants depending on a knot and these upper bounds are best possible (up to a constant multiplier). That is, while each p-part grows at most polynomially with respect to r, nevertheless the whole of $|H_1(X_r(K);\mathbb{Z})|$ can grow exponentially. (Silver and Williams [28] also studied this topic. Unfortunately, this part of their results had been already established by Riley. However they discussed this topic under mild conditions.)

In this paper, the p-adic coefficient homology group $H_1(X_\infty(K);\mathbb{Q}_p)$ plays an important role in studying the distribution of zeros (Theorem 1). This approach was motivated by the previous paper [23], in which we discussed an analogy between the reciprocity of Δ_K and the functional equation in the Weil conjecture. Likewise, p-adic (co)homology theory has been also investigated to approach the Weil conjecture (cf. [9]).

We should mention [10] for the pro-p and p-adic method in the Alexander-Fox theory and its application to p-homology groups in analogy with Iwasawa theory, and refer to [12].

2. Solenoidal entropy and Alexander polynomial

A solenoid Σ^d is, by definition, a compact connected finite-dimensional abelian group, which arose from a generalization of the torus $\mathbb{T}^d$. The following theorem, which was given by Lind and Ward [18], plays a key role for our results.

Proposition 1 (Lind and Ward [18]). *Let T be an automorphism of an d-dimensional solenoid Σ^d. Then,*
(1) *the entropy of T is the sum of the entropies of the automorphisms of $\mathbb{Q}_p^d$ derived from T*

$$h(T;\Sigma^d) = \sum_{p\le\infty} h\big(T;\mathbb{Q}_p^d\big),$$

and

(2) *the p-adic entropy is computed by the eigenvalues* $\lambda_1, \ldots, \lambda_d$ *of the induced automorphism in* $GL(d, \mathbb{Q})$ *as follows*:

$$h\big(T; \mathbb{Q}_p^d\big) = \sum_{|\lambda_i|_p > 1} \log|\lambda_i|_p.$$

We shall identify the topological entropy and the measure theoretic entropy by the formula of Bowen [1]:

$$h_d(T) = \lim_{\varepsilon \to 0} \limsup_{n \to \infty} \left[-\frac{1}{n} \log \mu \left(\bigcap_{k=0}^{n-1} T^{-k} B(e, \varepsilon) \right) \right],$$

where $B(e, \varepsilon)$ is an open ε-ball of the identity element with respect to an invariant metric d, μ is a Haar measure and T is a surjective endomorphism on a locally compact abelian group. In other words, we regard the entropy as the topological entropy and also the measure theoretic entropy with respect to a Haar measure.

In [18], Lind and Ward computed the entropy of solenoidal automorphisms with intrinsic arguments. As the proof is helpful in understanding our results, we provide an outline here. However we strongly recommend the reader to refer to the original paper.

Outline of the proof. STEP 1. Because the dual group of Σ^d can be embedded into $\mathbb{Q}^d$, $\varinjlim \Gamma_n \cong \mathbb{Q}^d$ for $\Gamma_n = (1/n!)\hat{\Sigma}^d$. That is, $\hat{\mathbb{Q}}^d \cong \varprojlim \hat{\Gamma}_n$. Hence there are K_n such that $\hat{\Gamma}_n \cong \hat{\mathbb{Q}}^d / K_n$ and consequently $h(T; \hat{\mathbb{Q}}^d) = h(T; \hat{\Gamma}_n) + h(T; K_n)$. Because $h(T; \hat{\Gamma}_n) = h(T; \Sigma^d)$ for any n and $h(T; K_n) \to 0$ as $n \to \infty$, we obtain that

$$h(T; \Sigma^d) = h(T; \hat{\mathbb{Q}}^d).$$

STEP 2. The entropy on the full solenoid $\hat{\mathbb{Q}}^d$ can be lifted to the entropy on the adele ring $\mathbb{A}_{\mathbb{Q}}^d$ because $\mathbb{A}_{\mathbb{Q}}/\mathbb{Q} \cong \hat{\mathbb{Q}}$. So, we have

$$h(T; \hat{\mathbb{Q}}^d) = h\big(T; \mathbb{A}_{\mathbb{Q}}^d\big).$$

STEP 3. Since the adele ring is a restricted direct product space, the entropy on it can be decomposed into the entropies of local components, i.e.

$$h\big(T; \mathbb{A}_{\mathbb{Q}}^d\big) = \sum_{p \le \infty} h\big(T; \mathbb{Q}_p^d\big).$$

STEP 4. Finally, as the p-adic norm is the module on $\mathbb{Q}_p$ (i.e. $\mu(a\,\cdot) = |a|_p \mu(\cdot)$ for any $a \in \mathbb{Q}_p$, where μ is a Haar measure), we have

$$h\big(T; \mathbb{Q}_p^d\big) = \sum_{|\lambda_i|_p > 1} \log|\lambda_i|_p. \qquad \square$$

To connect the Alexander polynomial with a solenoidal automorphism, we need the following lemma.

Lemma 1. *For any knot, the dual group of the first homology group of an infinite cyclic cover $H_1(X_\infty(K);\mathbb{Z})$ is a n-dimensional solenoid. Here, n is the degree of the Alexander polynomial of the knot.*

Proof. It is sufficient to prove that $H_1(X_\infty(K);\mathbb{Z})$ is a discrete torsion-free abelian group which has finite-rank n. Rapaport [25] and Crowell [4] proved that $H_1(X_\infty(K);\mathbb{Z})$ is torsion-free and has finite rank n. (Here, the rank of A means the cardinality of any maximal set of $\mathbb{Z}$-linearly independent elements of A.) □

Because the Alexander polynomial is equal to the characteristic polynomial of the meridian action on $H_1(X_\infty(K);\mathbb{Q})$, up to multiplication by a unit, the following theorem follows immediately from Proposition 1 and Lemma 1.

Theorem 1. *Let α_i be the zeros (counted with multiplicity) of the Alexander polynomial of a knot. Then,*
(1) *the entropy of the meridian action on the p-adic Alexander module; $t_p\colon H_1(X_\infty(K);\mathbb{Q}_p) \to H_1(X_\infty(K);\mathbb{Q}_p)$ is*

$$h(t_p) = \sum_{|\alpha_i|_p > 1} \log|\alpha_i|_p,$$

where $|\cdot|_p$ are the p-adic norms, and
(2) *the entropy of dual action of meridian $\hat{t}$: $\widehat{H_1(X_\infty(K);\mathbb{Z})} \to \widehat{H_1(X_\infty(K);\mathbb{Z})}$ is $h(\hat{t}) = \sum_{p\le\infty} h(t_p)$, that is*

$$h(\hat{t}) = \sum_{p\le\infty} \sum_{|\alpha_i|_p > 1} \log|\alpha_i|_p.$$

3. Applications

3.1. Growth of order of homology of branched cyclic covering spaces. In this section, we study the relation between the p-adic norm of the zeros of Δ_K and the growth of orders of the first homology groups of the r-fold cyclic coverings of S^3 branched over K. The r-fold cyclic covering branched over K is the (Fox) completion of the unbranched covering of $S^3 \setminus K$ which associated with a surjective homomorphism $G = \pi_1(S^3 \setminus K) \to \mathbb{Z}/r\mathbb{Z}$ (for precise definition, see [3, 16]).

The order of the first homology group of this space can be computed by the following formula.

Proposition 2 (Fox [6], Weber [30]). *Let $X_r(K)$ be the r-fold cyclic covering of S^3 branched over K. Then, the order of the first homology group of $X_r(K)$ is given by*

$$|H_1(X_r(K);\mathbb{Z})| = \left|\prod_{d=1}^{r-1} \Delta_K\left(\exp\left(\frac{2d\pi\sqrt{-1}}{r}\right)\right)\right|.$$

By convention, $|H_1(X_r(K);\mathbb{Z})| = 0$ means that $H_1(X_r(K);\mathbb{Z})$ is an infinite group.

DEFINITION 1 (logarithmic Mahler measure [19]). For non-zero Laurent polynomial $f(x)$ with integral coefficients, the logarithmic Mahler measure of f is defined by

$$m(f) = \int_0^1 \log\left|f\left(\exp\left(2\pi t\sqrt{-1}\right)\right)\right| dt.$$

The growth of orders $|H_1(X_r(K);\mathbb{Z})|$ is expressed by the logarithmic Mahler measure of the Alexander polynomial, which was proved by González-Acuña and Short [7].

Proposition 3 (González-Acuña and Short [7]).

$$\lim_{\substack{r\to\infty \\ |H_1(\cdot)|\neq 0}} \frac{\log|H_1(X_r(K);\mathbb{Z})|}{r} = m(\Delta_K).$$

REMARK 1. In [27], Silver and Williams generalized the result of González-Acuña and Short [7] to links and also pointed out that Proposition 3 holds, though the order $|H_1(X_r(K);\mathbb{Z})|$ is replaced with the order of the torsion subgroup $|TH_1(X_r(K);\mathbb{Z})|$.

Note that the Mahler measure is deeply related to the entropy of an algebraic dynamical system, which is found in [17] for example. In this paper, we use a more suitable result which was proved by Einseidler and Ward [5].

Proposition 4 ((Einseidler and Ward [5])). *Let $0 \to F_n \to \cdots \to F_1 \xrightarrow{\phi_1} F_0 \to M \to 0$ be a finite free resolution of the $\mathbb{Z}[t^{\pm}]$-module M and $J(\phi_1)$ the initial Fitting ideal. Let α_t a natural automorphism of M which is induced by the shift of the indeterminate t. Then the entropy of $\hat{\alpha}_t$ is*

$$h(\hat{\alpha}_t) = m(\gcd(J(\phi_1))).$$

By combining Proposition 2 and 4, we can see that the growth of orders gives another method to compute the entropy in Theorem 1. Therefore, we can obtain the following corollary.

Corollary 1.

$$\lim_{\substack{r\to\infty \\ |H_1(\cdot)|\neq 0}} \frac{\log|H_1(X_r(K);\mathbb{Z})|}{r} = \sum_{p\leq\infty} \sum_{|\alpha_i|_p>1} \log|\alpha_i|_p,$$

where α_i are the zeros of the Alexander polynomial $\Delta_K(t)$.

Proposition 3 and Corollary 1 are similar. However, the each terms $\sum_{|\alpha_i|_p>1} \log|\alpha_i|_p$ of the right hand side of Corollary 1 make sense in a dynamical system. Actually, Corollary 1 can be rewritten in the following form:

$$\lim_{\substack{r\to\infty \\ |H_1(\cdot)|\neq 0}} \frac{\log|H_1(X_r(K);\mathbb{Z})|}{r} = \sum_{p\leq\infty} h(t_p),$$

where $h(t_p)$ is the entropy of the meridian action t_p: $H_1(X_\infty(K);\mathbb{Q}_p) \to H_1(X_\infty(K);\mathbb{Q}_p)$. In other words, we resolved the growth into the sum of the p-adic entropies, which are related to the Haar measure on the p-adic numbers fields $\mathbb{Q}_p$ (cf. Step 4 in the proof of Proposition 1).

Similarly to González-Acuña and Short [7], we also obtain the following property as a special case of Corollary 1.

Corollary 2 (Riley [26], González-Acuña and Short [7]). *Let $X_r(K)$ be the r-fold cyclic covering branched over K. Then, if the Alexander polynomial $\Delta_K(t)$ have zeros which are not roots of unity, the finite values of the order of the first homology group $|H_1(X_r(K);\mathbb{Z})|$ grows exponentially with respect to r.*

Proof (Indirect proof). Because all α_i belong to the valuation ring $\mathcal{O}_p = \left\{x \in \overline{\mathbb{Q}}_p \;\middle|\; |x|_p \leq 1\right\}$ by Corollary 1, $f(t) = \prod_i (t-\alpha_i)$, which is $\Delta(t)$ up to constant multiples, belongs to $\mathbb{Z}_p[t] \cap \mathbb{Q}[t]$, where $\mathbb{Z}_p = \{x \in \mathbb{Q}_p \mid |x|_p \leq 1\}$. This holds for any prime p. Hence $f(t) \in \mathbb{Z}[t]$. (Another way to see this is to prove Corollary 4 before the proof of Corollary 2. But in this proof, the condition $\Delta(1) = \pm 1$ is not necessary.) Consequently, the zeros of the Alexander polynomial must be roots of unity, because $|\alpha_i| \leq 1$ and Kronecker's theorem [15]. □

3.2. Leading coefficient of Alexander polynomial. In this section, we apply Theorem 1 to a criterion for the Alexander module being finitely generated as a $\mathbb{Z}$-module. By the corollary below, we can regard the entropies $h(t_p)$ for all primes $p < \infty$ as obstructions for being finitely generated.

Corollary 3. *Let $h(t_p)$ be the entropy of the meridian action on the p-adic Alexander module $H_1(X_\infty(K);\mathbb{Q}_p)$. If the Alexander module $H_1(X_\infty(K);\mathbb{Z})$ is finitely generated as $\mathbb{Z}$-module, then the entropies $h(t_p)$ are equal to zero for all finite primes $p < \infty$.*

Proof. The Alexander module $H_1(X_\infty(K);\mathbb{Z})$ is finitely generated if and only if $\widehat{H_1(X_\infty(K);\mathbb{Z})}$ is isomorphic to the n dimensional torus, which is a quotient of $H_1(X_\infty(K);\mathbb{R})$. By the well-known result for toral automorphisms, the entropy of the meridian action on $\widehat{H_1(X_\infty(K);\mathbb{Z})}$ is

$$h(t) = \sum_{|\alpha_i|>1} \log|\alpha_i|,$$

where α_i are eigenvalues of the meridian action on $H_1(X_\infty(K);\mathbb{R})$.

This entropy must be equal to the entropy in Theorem 1-(2). Therefore, the entropies of the meridian action on $H_1(X_\infty(K);\mathbb{Q}_p)$ are zero for any $p < \infty$, that is

$$h(t_p) = \sum_{|\alpha_i|_p>1} \log|\alpha_i|_p = 0 \quad \text{for} \quad p < \infty. \qquad \square$$

These obstructions give a new viewpoint for the leading coefficient of the Alexander polynomial. The following Corollary means that the entropies $h(t_p)$ are summands of the leading coefficient of the Alexander polynomial.

Corollary 4. *Let α_i be the zeros of $\Delta_K(t) = \sum_{i=0}^n a_i t^i$. Then the leading coefficient of $\Delta_K(t)$ is the sum of the entropies of the meridian action on the p-adic Alexander module $H_1(X_\infty(K);\mathbb{Q}_p)$ for finite prime $p < \infty$. In other words, the following equation holds*:

$$\log|a_n| = \sum_{p<\infty} \sum_{|\alpha_i|_p>1} \log|\alpha_i|_p.$$

Proof. Let $f(t) = \Delta_K(t)/a_n = \prod(t - \alpha_i)$ and s the least common multiple of the denominators of the coefficients of $f(t)$. Then,

$$\sum_{p<\infty} h(t_p) = \sum_{p<\infty} \sum_{|\alpha_i|_p>1} \log|\alpha_i|_p = \log s.$$

Because $\Delta_K(1) = \pm 1$, the coefficients are relatively prime $(a_n, \ldots, a_1) = 1$. Hence $s = |a_n|$. (The above argument is essentially found in the proofs of Theorem 3 in [18] and Theorem 2 in [26].) $\square$

Since $\Delta_K(1) = \pm 1$ for any knot, the Alexander polynomial can be completely determined (up to ± 1) by the zeros. Hence the leading coefficient is also determined. On the other hand, Corollary 4 shows that the leading coefficient is resolved into the p-adic entropy and consequently can be recovered from the zeros.

3.3. Final remarks.

3.3.1. Determining knots by cyclic branched covers. In [14], Kojima showed that prime knots are determined by their cyclic branched covers. So, there might be a method to compute the Alexander module by the data of cyclic branched covers.

The growth of orders $|H_1(X_r(K);\mathbb{Z})|$ does not determine completely the Alexander module $H_1(X_\infty(K);\mathbb{Z})$. However the growth measures complexity of the Alexander module from a dynamical viewpoint, and also it is a similar invariant to the leading coefficient of the Alexander polynomial.

In addition, it still remains open whether infinitely many branched covers are necessary for determining the knot.

3.3.2. Volume conjecture. The volume (or Kashaev) conjecture [13, 20] expects that the asymptotic behavior of the Kashaev invariant (= the specialization of the colored Jones polynomial) implies the hyperbolic volume of the complement of a hyperbolic knot.

In general, topological entropy picks up natural measures by the variational principle. In other words, topological entropy is related to natural measure theory (for more details, see Capter 8 in [29]).

In this paper, the asymptotic behavior of the special values of the Alexander polynomial is related with the entropy of an action on the adele ring of $\mathbb{Q}$. Consequently, the asymptotic behavior is related to the volume growth with respect to the Haar measure on the adele ring.

In general, when an asymptotic behavior is interpreted as an entropy, it can be related naturally to measure theory. Does this strategy work out for the volume conjecture?

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References

[1] R. Bowen: *Entropy for group endomorphisms and homogeneous spaces*, Trans. Amer. Math. Soc. **153** (1971), 401–414.

[2] G. Burde: *Alexanderpolynome Neuwirthscher Knoten*, Topology **5** (1966), 321–330.

[3] G. Burde and H. Zieschang: Knots, de Gruyter Studies in Mathematics **5**, Walter de Gruyter & Co., Berlin, 1985.

[4] R.H. Crowell: *The group G'/G'' of a knot group G*, Duke Math. J. **30** (1963), 349–354.

[5] M. Einsiedler and T. Ward: *Fitting ideals for finitely presented algebraic dynamical systems*, Aequationes Math. **60** (2000), 57–71.

[6] R.H. Fox: *Free differential calculus*. III. *Subgroups*, Ann. of Math. (2) **64** (1956), 407–419.

[7] F. González-Acuña and H. Short: *Cyclic branched coverings of knots and homology spheres*, Rev. Mat. Univ. Complut. Madrid **4** (1991), 97–120.

[8] C.McA. Gordon: *Knots whose branched cyclic coverings have periodic homology*, Trans. Amer. Math. Soc. **168** (1972), 357–370.

[9] A. Grothendieck et al.: Séminaire de Géométre Algebrique du Bois Marie (SGA4), Lecture Notes in Mathematics **269**, **270**, **305**, Springer-Verlag, Berlin, 1972–73.

[10] J. Hillman, D. Matei and M. Morishita: *Pro-p link groups and p-homology groups*, to apper in Contemp. Math. **416**, Amer. Math. Soc., Providence, RI, 2006.

[11] S.A. Juzvinskiĭ. Computing the entropy of a group of endomorphisms, Sibirsk. Mat. Ž. (1967), 230–239, English transl. in Siberian Math. J. **8** (1968), 172–178.

[12] T. Kadokami and Y. Mizusawa. *Iwasawa type formulas for covers of a link in a rational homology sphere*, preprint.

[13] R.M. Kashaev: *The hyperbolic volume of knots from the quantum dilogarithm*, Lett. Math. Phys. **39** (1997), 269–275.

[14] S. Kojima: *Determining knots by branched covers*; in Low-Dimensional Topology and Kleinian Groups (Coventry/Durham, 1984), London Math. Soc. Lecture Note Ser. **112**, Cambridge Univ. Press, Cambridge, 193–207.

[15] L. Kronecker: *Zwei Sätze ueber Gleichungen mit ganzzahligen Coeffichienten*, J. Reine Angew. Math. **53** (1857), 173–175.

[16] W.B.R. Lickorish: An Introduction to Knot Theory, Graduate Texts in Mathematics **175**, Springer-Verlag, New York, 1997.

[17] D. Lind, K. Schmidt and T. Ward: *Mahler measure and entropy for commuting automorphisms of compact groups*, Invent. Math. **101** (1990), 593–629.

[18] D.A. Lind and T. Ward: *Automorphisms of solenoids and p-adic entropy*, Ergodic Theory Dynam. Systems **8** (1988), 411–419.

[19] K. Mahler: *An application of Jensen's formula to polynomials*, Mathematika **7** (1960), 98–100.

[20] H. Murakami and J. Murakami: *The colored Jones polynomials and the simplicial volume of a knot*, Acta Math. **186** (2001), 85–104.

[21] K. Murasugi: *The commutator subgroups of the alternating knot groups*, Proc. Amer. Math. Soc. **28** (1971), 237–241.

[22] L.P. Neuwirth: Knot Groups, Ann. of Math. Stud. **56**, Princeton Univ. Press, Princeton, N.J., 1965.

[23] A. Noguchi: *A functional equation for the Lefschetz zeta functions of infinite cyclic coverings with an application to knot theory*, Topology Proc. **29** (2005), 277–291.

[24] C.V. Quach: *Polynôme d'Alexander des noeuds fibrés*, C.R. Acad. Sci. Paris Sér. A-B **289** (1979), A375–A377.

[25] E.S. Rapaport: *On the commutator subgroup of a knot group*, Ann. of Math. (2) **71** (1960), 157–162.

[26] R. Riley: *Growth of order of homology of cyclic branched covers of knots*, Bull. London Math. Soc. **22** (1990), 287–297.

[27] D.S. Silver and S.G. Williams: *Mahler measure, links and homology growth*, Topology **41** (2002), 979–991.

[28] D.S. Silver and S.G. Williams: *Torsion numbers of augmented groups with applications to knots and links*, Enseign. Math. (2) **48** (2002), 317–343.

[29] P. Walters: An Introduction to Ergodic Theory, Graduate Texts in Mathematics **79**, Springer-Verlag, New York, 1982.
[30] C. Weber: *Sur une formule de R.H. Fox concernant l'homologie des revêtements cycliques*, Enseign. Math. (2) **25** (1979/1980), 261–272.

Department of Mathematics
Tokyo Institute of Technology
Oh-okayama, Meguro-ku, Tokyo 152–8551
Japan
e-mail: akio@math.titech.ac.jp