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# Multi-Mode Resonant Column Technique to Determine Elastic Moduli of Soils

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(with a table and 9 textfigures)

## Introduction

Laboratory measurements of dynamic moduli of soils will give basic information of the mechanical properties of soils leading to an increase in knowledge of the underground structure or ground behavior expected during earthquake excitation. Experimental studies of elastic waves in soils have been made by many investigators. Generally, three different types of laboratory testing methods have been conducted to determine the dynamic properties of soils, i.e. the ultrasonic pulse transmission method, the resonant column soil testing method and, the low frequency cyclic loading method.

For the ultrasonic pulse transmission method, the arrival time of compressional waves in soil is clearly recognizable on a CRT display. However, it is very difficult to identify the pulse form of shear waves because of its large damping ratio in cohesive and especially high plastic soils. For both the resonant column soil testing method and the low frequency cyclic loading method, rigidity can be determined from the response of torsional cyclic loading and Young's modulus can be also found from axial cyclic loading. The resonant column method is mainly used to determine the dynamic modulus under relatively low amplitude strain (less than 0.01 percent). On the contrary, the low frequency cyclic loading method is generally applied to the soil specimen under high amplitude strain as in strong ground motion during an earthquake.

Thus, the resonant column method is rather rapid and simple to determine elastic moduli or velocities and damping ratio of soil under low amplitude strain conditions.

If soil is regarded as a mechanically isotropic material, all of the elastic constants can be deduced by only two moduli. Rigidity and Young's modulus can be directly obtained from the resonant column soil testing appratus. Poisson's ratio or bulk modulus of soil is determined from these two moduli.

Since the elasticity of soils are affected by confining pressure, the testing device is equipped within a triaxial cell surrounded by a known air pressure. With low strains, one specimen can be used for the longitudinal test first and the torsional test next without changing the quality of the specimens. However with higher strains, the hysteretic

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property results in the irreversible change in the specimens and two specimens are required for longitudinal and torsional tests respectively. This paper describes a special type of resonant column test apparatus which uses two vibration modes for two exciters, longitudinal and torsional, resulting in a much easier and a more precise way to determine the elastic constants of soils.

## Apparatus

The device developed in this study is basically the type which has been used in the

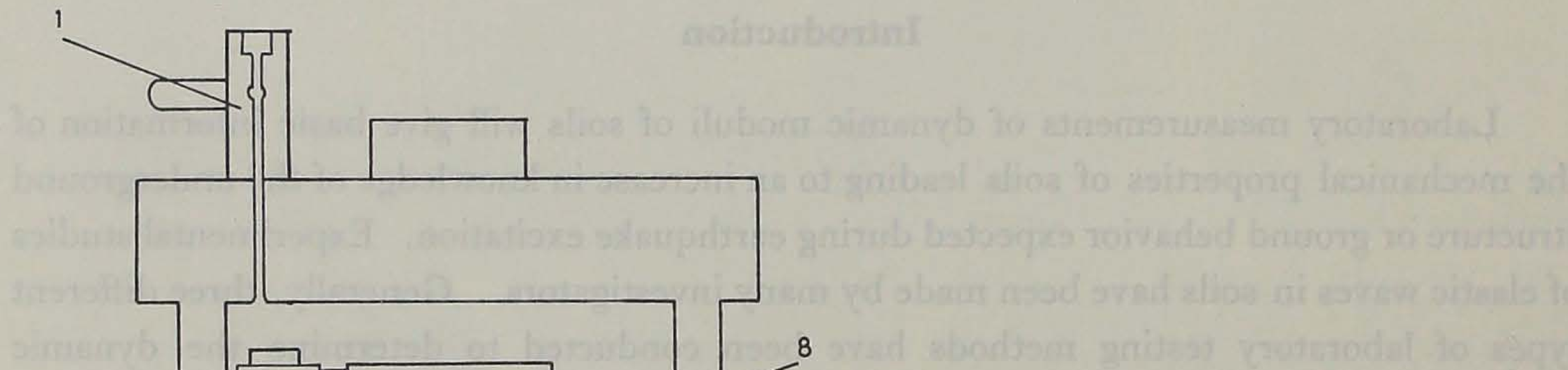

Fig. 1. a Schematic diagram of multi-mode resonant column soil testing device.
1. Air valve 2. Supporting stainless steel rod 3. Pick up of displacement sensor system for torsional mode 4. Pick up of displacement sensor system for longitudinal mode 5. Test specimen 6. Triaxial cell 7. Switching relay 8. Electromagnet 9. Moving coil 10. Tap of drainage line 11. Bar magnet 12. Electromagnet 13. Porous metal 14. O-ring.

b Schematic diagram of top cap with some parts of driving system and diplacement sensor system.
1. Moving coil 2. Taps of moving coil 3. Aluminium plate 4. Fins inducing eddy current for displacement sensor system 5. Permanent bar magnet 6. Rubber membrane 7. O-ring.

system called the resonant column method. The primary investigation on elastic properties of soil by means of this method is presumed to be made by ISHIMOTO and IIDA (1936, 1937). In their experiments, the soil specimen was kept on a specially designed pedestal which was vibrated either axially (longitudinally) in one system, or torsionally in another system. HARDIN and RICHART (1963) reported in detail the study of elastic wave velocities in soils under confining pressure by using two or more kinds of devices. The device used in this study is similar to those of HARDIN and RICHART (1963), HALL and RICHART (1963), NAKAGAWA (1975) etc. except it uses a special driving system to make two vibration modes in a single testing device.

The details of the driving system and the strain sensor system are shown in Figs. 1a and 1b. In this device, the boundary conditions of the soil specimen are fixed at the bottom and free at the top in both modes. The driving system consists of a permanent magnet, two electromagnets and a moving coil. The driving forces are given electromagnetically by feeding the coil with alternating current. A thick stainless steel rod is set vertically on a bottom plate of the triaxial cell in order to install two kinds of driving and displacement sensor systems. The magnitude of each vibration mode is detected by using the eddy current type transducers installed near by the top cap of specimen in two directions. The sensitivity of these displacement sensors is an order of 0.1 $\mu$m. Soil specimens are trimmed into a cylindrical form 5.0 cm in diameter and 10 cm in height. To avoid sliding between the top cap or pedestal and the upper or lower ends of the specimen during twisting, a number of spikes are driven into porous metals of the top cap and the pedestal. Drainage in the specimen is controlled during a test by opening or closing a drainage

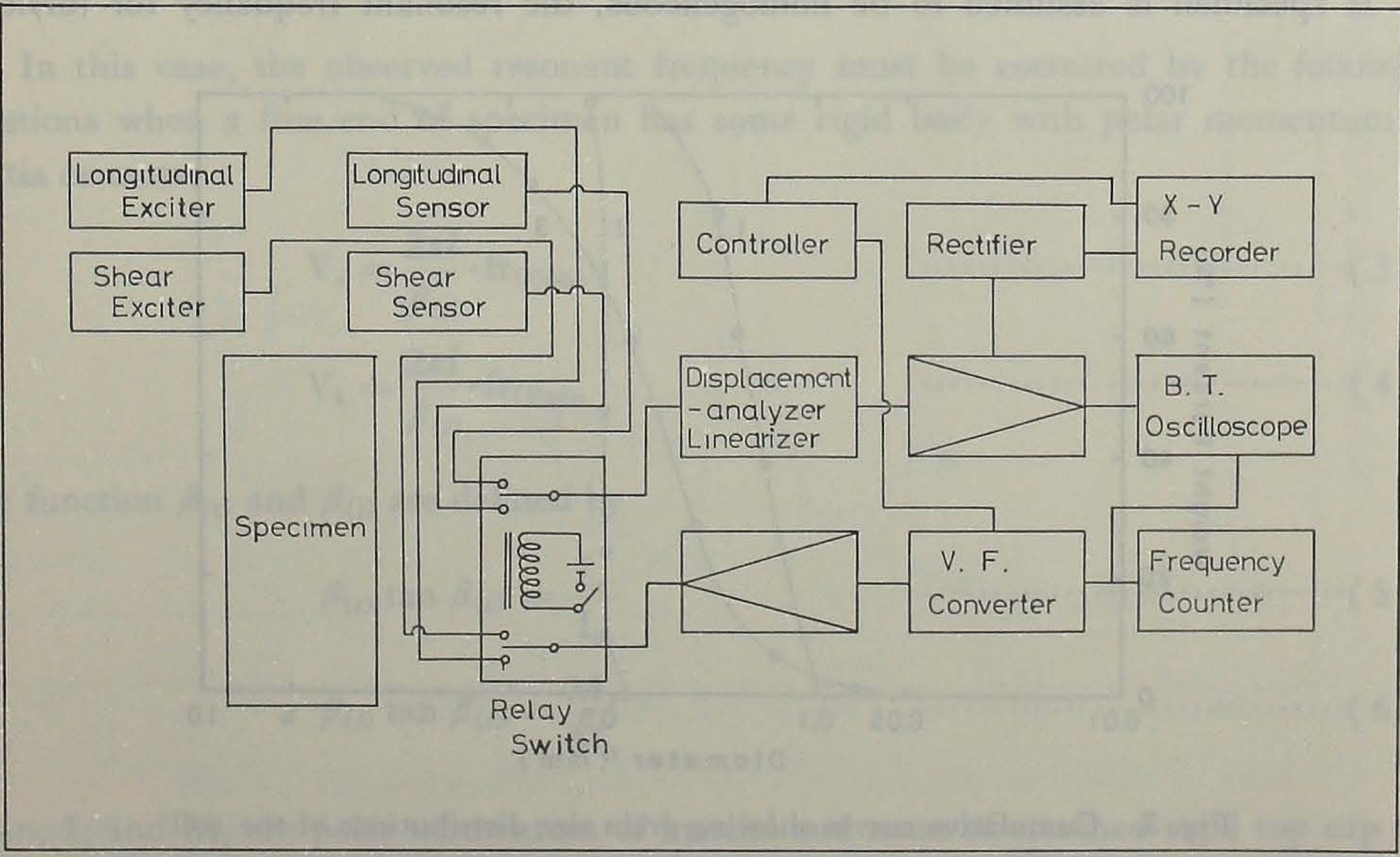

Fig. 2. Schematic block diagram for multi-mode resonant column soil testing device related to the electric circuit.

valve. A drainage tube of the top cap is usually closed except setting up a specimen. The maximum pressure which could be applied in the triaxial cell is 10 bars.

The instrumental block diagram of the testing method related to electrical circuit is shown in Fig. 2. As shown in the figure, the main components of the setup include a signal generator, an oscilloscope, a frequency sweep controller, a voltmeter and an X-Y recorder. The signal generator supplies a sinusoidal voltage to the driving coils via a power amplifier.

The maximum power of the output signal to feed the coil for the vibrating of soil specimen is about 400 watts p-p. The magnitude of the strain signal is monitored on an oscilloscope and given a more accurate value by using the voltmeter and the X-Y recorder. In automatic operation, the sweep controller changes the output frequency proportional to the input voltage. Therefore, the resonance curve is drawn on a chart of the X-Y recorder by both the voltage of the displacement linearizer output and the voltage corresponding to the frequency. The longitudinal or torsional vibration mode can be made by switching each driving or sensor systems alternatively.

## Preliminary experimental results and the discussions

Soils used for this study are two kinds of standard sand of Toyoura and Soma sand and locally available alluvial sand from Umeda in Osaka City. These standard sands were sieved to simplify their grain size distribution. Fig. 3 shows the grain size distribution of the soil specimens. Also, the soil parameters of the specimens used are tabulated in Table 1. All s6il specimens were tested in the drained state.

If specimen is assumed to be homogeneous, the resonant frequency for torsional

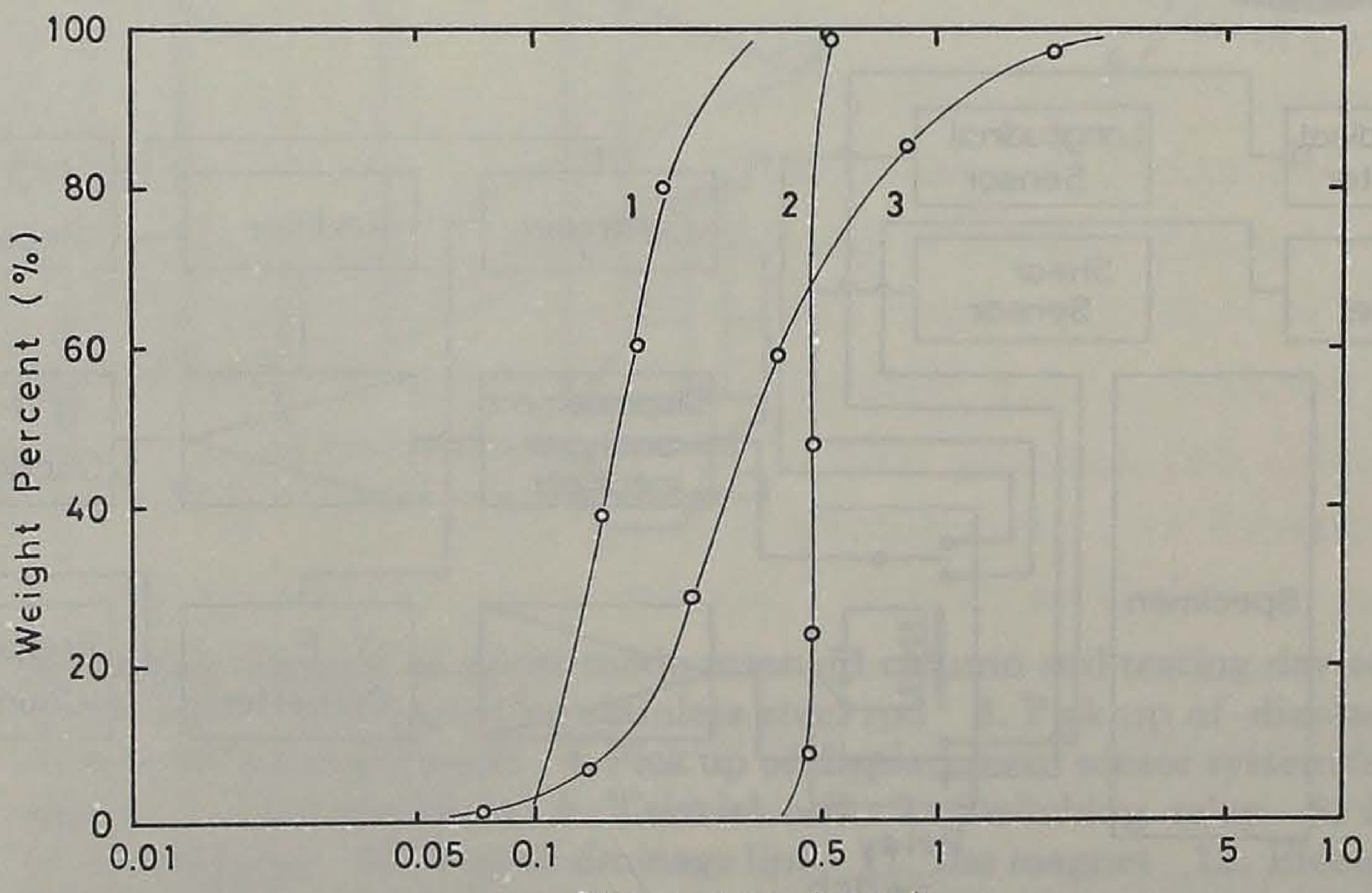

Fig. 3. Cumulative curves showing grain size distributions of the soil specimens.
1. Toyoura standard sand 2. Soma standard sand 3. Umeda alluvial sand.

Table 1 Soil properties and localities of specimen tested.

| Sample | Sampling Loc. | Specific Gravity of Grain | $e_{max}$ | $e_{min}$ | Uniformity Coefficcient |
|---|---|---|---|---|---|
| Soma St. Sand | Soma County Fukushima Pref. | 2.632 | 1.039 | 0.664 | 1.0 |
| Toyoura St. Sand | Toyoura County Yamaguchi Pref. | 2.643 | 0.932 | 0.606 | 1.5 |
| Umeda Alluvial Sand | Kita Ward Osaka Pref. | 2.654 | 1.086 | 0.630 | 2.6 |

or longitudinal vibration with the above boundary condition is given by the following equations,

$$\mathrm{fr}_{(t)} = \frac{(2n+1)V_s}{4l} = \frac{(2n+1)}{4l}\sqrt{\frac{G}{\rho}} \quad \cdots\cdots(1)$$

$$\mathrm{fr}_{(l)} = \frac{(2n+1)V_b}{4l} = \frac{(2n+1)}{4l}\sqrt{\frac{E}{\rho}} \quad \cdots\cdots(2)$$

where, $V_s$ : shear wave velocity
$V_b$ : longitudinal velocity (bar velocity)
G : rigidity
E : Young's modulus
$\rho$ : density
$l$ : length of specimen
n : mode number.

In this case, the observed resonant frequency must be corrected by the following equations when a free end of specimen has some rigid body with polar momentum of inertia or mass,

$$V_s = \frac{2\pi l}{\beta_{(t)}} \cdot \mathrm{fr}_{(t)\mathrm{obs}} \quad \cdots\cdots(3)$$

$$V_b = \frac{2\pi l}{\beta_{(l)}} \cdot \mathrm{fr}_{(l)\mathrm{obs}} \quad \cdots\cdots(4)$$

The function $\beta_{(t)}$ and $\beta_{(l)}$ are defined by

$$\beta_{(t)} \tan \beta_{(t)} = \frac{I_s}{I_0} \quad \cdots\cdots(5)$$

$$\beta_{(l)} \tan \beta_{(l)} = \frac{M_s}{M_0} \quad \cdots\cdots(6)$$

when, $I_s$ and $M_s$ are polar momentum of inertial and mass of attached rigid top cap respectively. To apply equations (5) and (6) to this correction, the equations are approximately expressed rather in much convenient form as follows,

$$fr = fr_{obs}(A+\sqrt{B+C\beta'}) \cdots \cdots\cdots\cdots\cdots\cdots\cdots\cdots (7)$$

where, $fr$ : corrected resonant frequency

$fr_{obs}$ : observed resonant frequency

$\beta'$ : the ratio of polar momentum of inertia or mass of the attached rigid top cap to those of specimen

$A$ : $-0.398$

$B$ : $1.954$

$C$ : $2.981$

The equation (7) is valid when $\beta' \lesssim 1$.

BANCROFT (1938) has theoretically examined the error analysis of Young's modulus which is obtained from the measurement of longitudinal wave velocity in a cylindrical bar. By his investigation, the precise value of Young's modulus depends on Poisson's ratio and ratio of the specimen diameter to the wave length. Applying his results to this present study, the errors of the measured bar velocity of the specimen with an aspect ratio of about 2 become less than 1.0 percent, where the aspect ratio is the ratio of length to diameter of the specimen.

The specimen was prepared as having an appropriate void ratio under distilled water in a membrane on a pedestal. Drainage of the specimen was permitted during the

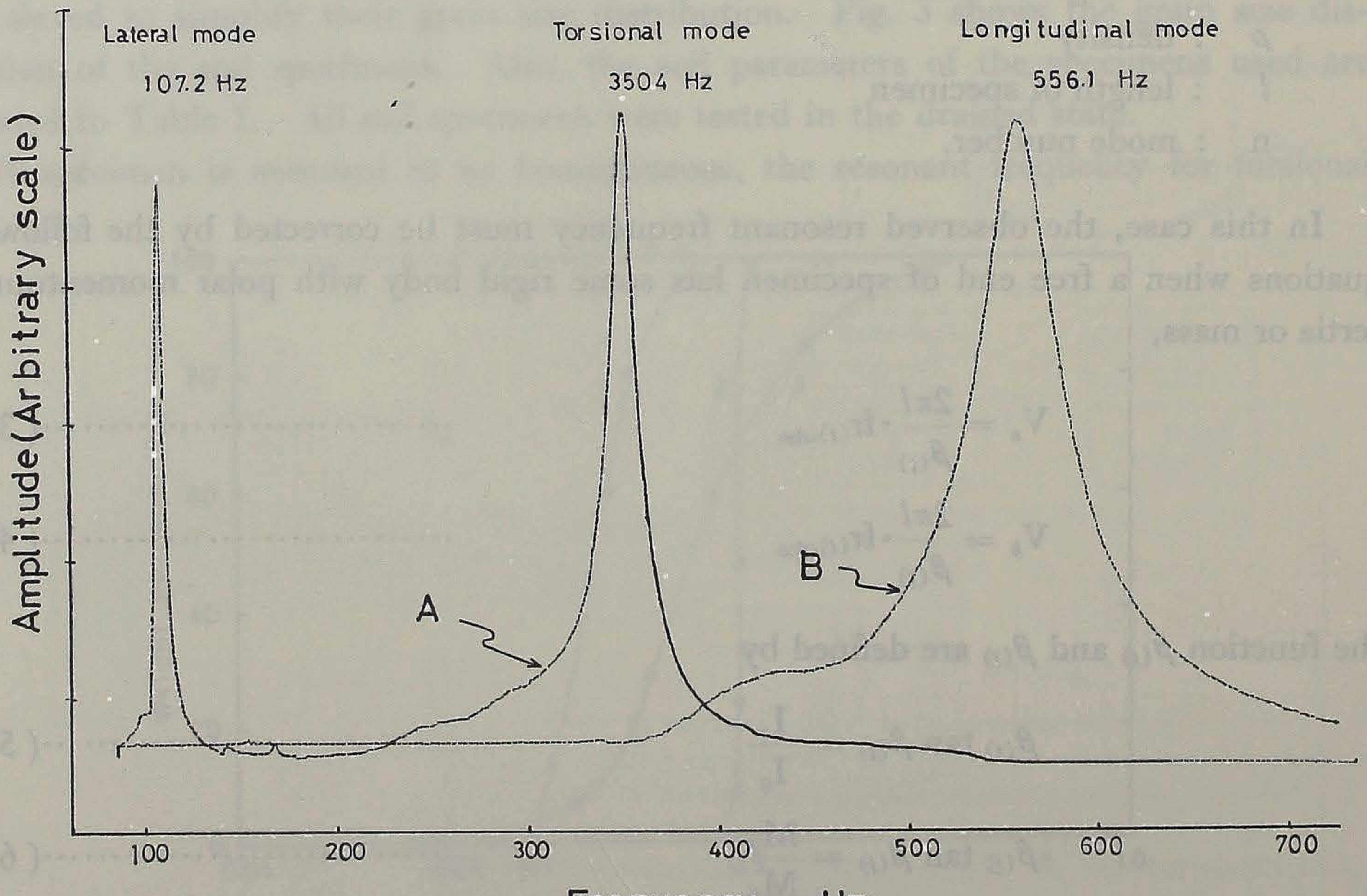

Fig. 4. An example of the resonance curves on recording chart of X-Y recorder. Curves A and B are obtained from torsional and longitudinal vibration respectively. Specimen: Soma standard sand, void ratio: 0.717, Confining pressure: 0.5 kg/cm (ca. 0.05 Mpa).

entire testing sequence. Selection of the driving system and the displacement sensing system can be made manually by using a simple switch as shown in Fig. 2.

An example record of the resonance curves of each mode for the Soma standard sand drawn on an X-Y recorder chart is shown in Fig. 4. It is well known that the dynamic rigidity of soils decreases with an increasing strain amplitude and the degree of the decrease in strain amplitude is greater with high strains. In the present study, Young's modulus of the soil was also found to decrease with an increase in strain amplitude. In Fig. 5, the change of the moduli against induced maximum strain is shown and hysteresis is found in the changes of both moduli at the cycles of increasing and decreasing the strain. This hysteresis seems to be important in measurements greater than $5\times10^{-5}$ strain magnitude. Fig. 6 shows the relation between moduli and strain amplitude for the Soma standard sand. If both rigidity and Young's modulus of the isotropic material are determined, the other elastic moduli can be directly calculated from the elastic theory as follows,

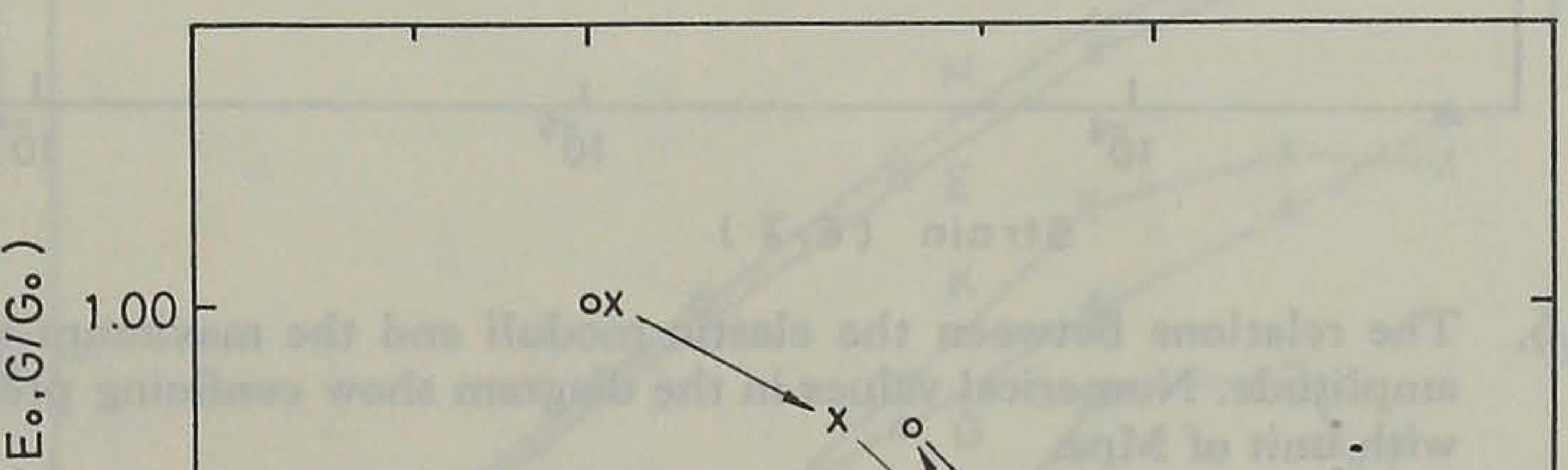

Fig. 5. The relations between the elastic moduli and the maximum strain amplitude

Specimen: Soma standard sand Void ratio: 0.740 Confining pressure: 0.5 Kg/cm$^2$ (ca. 0.05 Map)

1. Young's modulus 2. Rigidity.

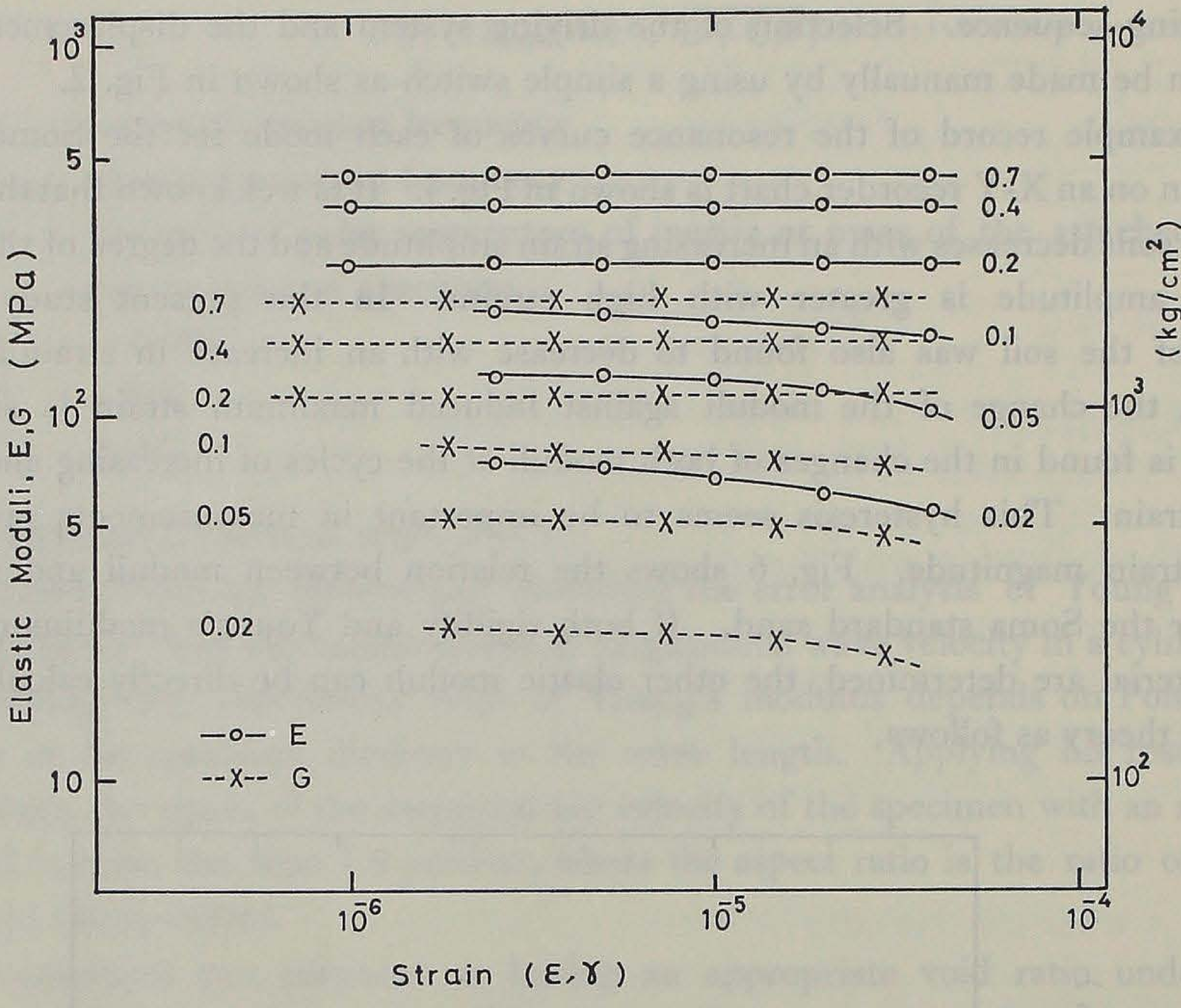

Fig. 6. The relations between the elastic moduli and the maximum strain amplitude. Numerical values in the diagram show confining pressure with unit of Mpa.

$$K = \frac{EG}{3(3G-E)} \qquad \cdots\cdots(8)$$

$$M = \frac{G(4G-E)}{(3G-E)} \qquad \cdots\cdots(9)$$

$$\lambda = \frac{G(E-2G)}{(3G-E)} \qquad \cdots\cdots(10)$$

$$\nu = \frac{E}{2G}-1 \qquad \cdots\cdots(11)$$

where K, M, $\lambda$ and $\nu$ are bulk modulus, constrained modulus, Lame's constant and Poisson's ratio respectively. Fig. 7, 8 and 9 show the hydrostatic confining pressure effect on some elastic moduli of three soils calculated from rigidity and Young's modulus which are determined at the lower strain amplitude level as an order of $10^{-6}$. As shown in these figures, the elastic moduli except Poisson's ratio increase with an increase in confining pressure.

HARDIN and RICHART (1963) experimentally obtained the equations of rigidity of the granular materials as functions of confining pressure and its void ratio. They showed that shear wave velocity varies as the one-quarter power of confining pressure rather than the one-six power predicted by Hertz theory. Similarly, rigidity varies as the one-

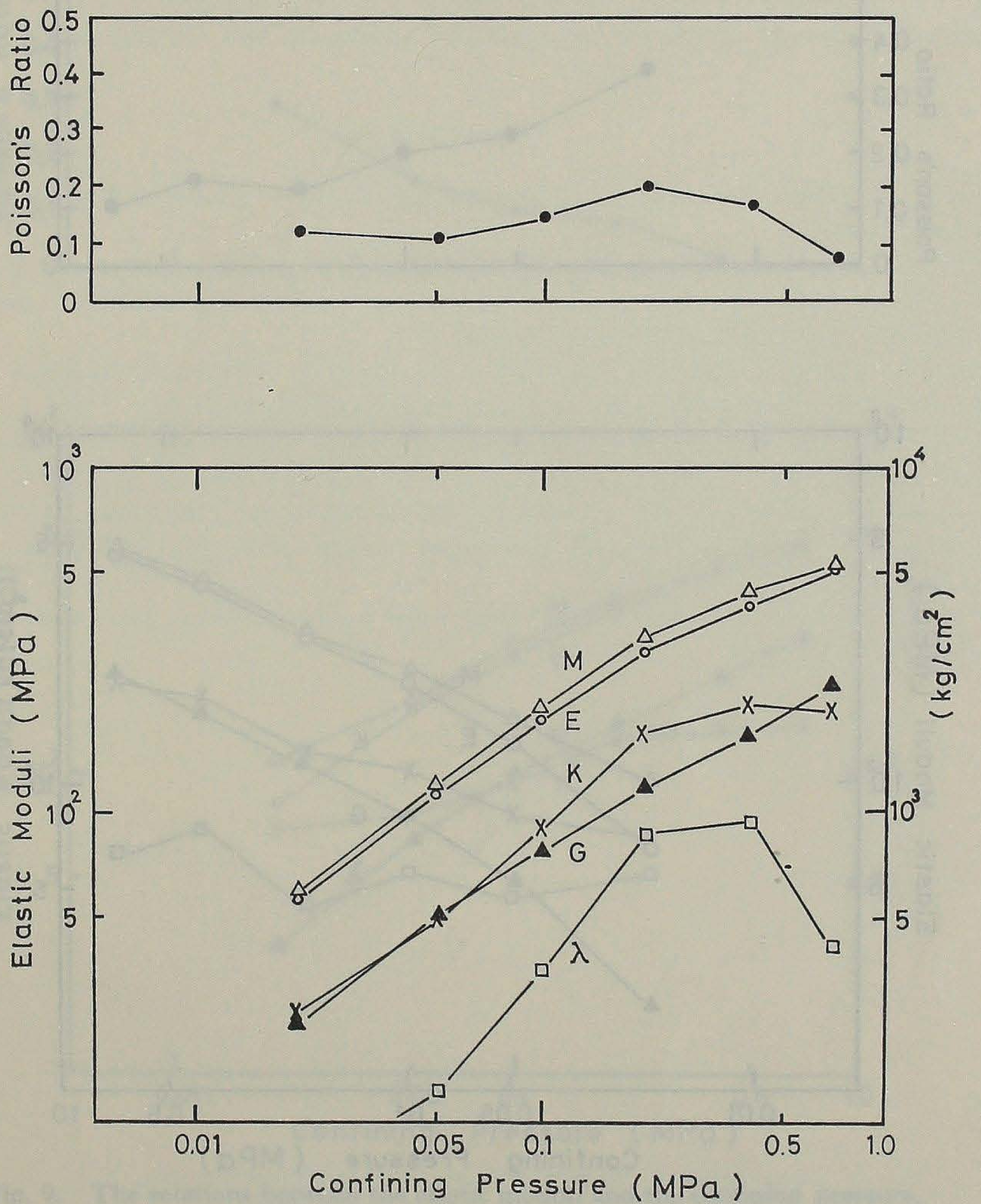

Fig. 7. The relations between the elastic moduli and the confining pressure. See the text for the symbols.
Specimen: Umeda alluvial sand Void ratio: 0.595.

half power of confining pressure rather than the one-third power. The confining pressure dependency of Young's modulus of soils seems to be nearly equal to that of rigidity so far as in this study. The moduli of K, M and $\lambda$ are also found to increase with the confining pressure with nearly or less than one-half power. On the other hand, Poisson's ratio for two kind of standard sand specimens linearly decreases with the increasing confining pressure as shown in Fig. 8 and 9. This tendency was not confirmed for the Umeda alluvial sand specimen as shown in Fig. 8.

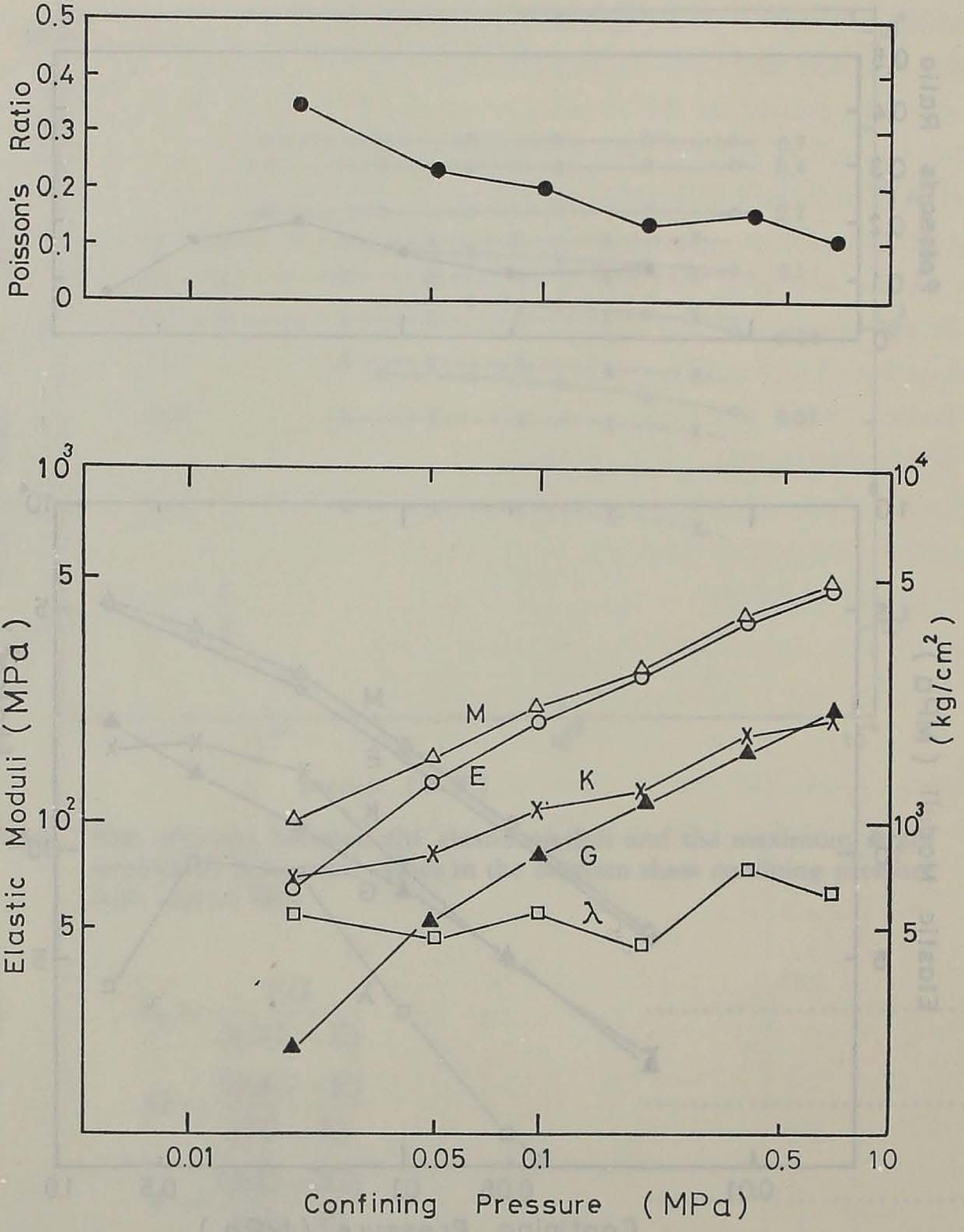

Fig. 8. The relations between the elastic moduli and the confining pressure. See the text for the symbols.
Specimen: Soma standard sand Void ratio: 0.717.

## Conclusion

In this paper, a description of the special device developed for resonant column soil testing technique to determine the elastic moduli of a soil specimen has been explained. The most characteristic point is its ability to determine both rigidity and Young's modulus from a single specimen by using a single device. Some results obtained by using this device were also described for three kinds of sand specimens.

The conclusions obtained in this study are as follows;

(1) The design of the new device to determine the elastic moduli for soil is schematically

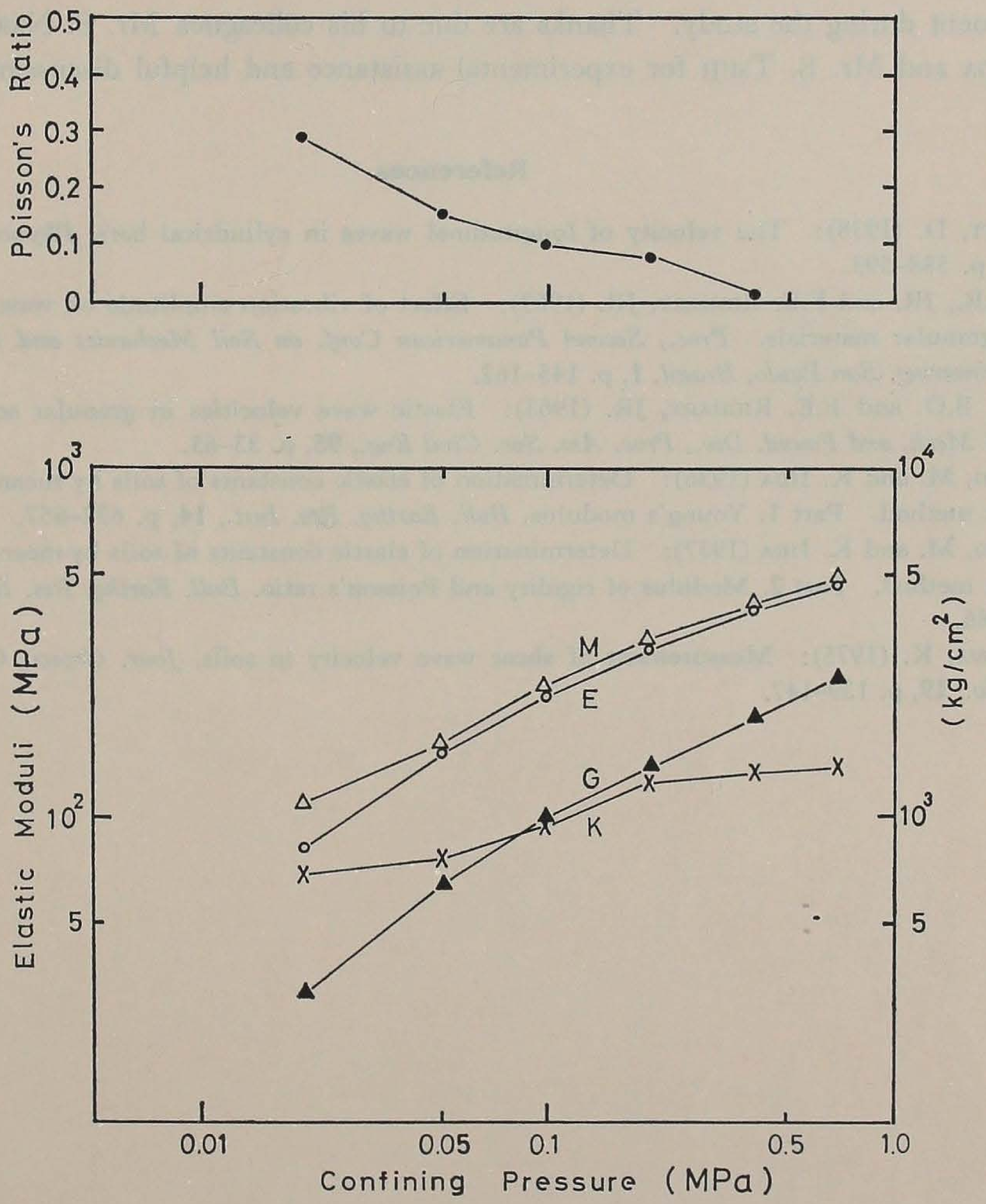

Fig. 9. The relations between the elastic moduli and the confining pressure. See the text for the symbols. Specimen: Toyoura standard sand Void ratio: 0.676.

and pictorially shown in Figs. 1a and 1b.

(2) The block diagram related to electric circuit is shown in Fig. 2.

(3) The effect of maximum strain amplitude under lower strain condition on rigidity and Young's modulus of sand specimen was experimentally confirmed.

(4) The effect of confining pressure under lower strain condition on three kinds of sand specimens was found to increase elastic moduli such as bulk modulus, constrained modulus and Lame's constant, except for Poisson's ratio.

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