Three-dimensional gravity analysis for underground structures accompanying with reverse faults

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Abstract

The Quaternary sedimentary basins in Japan, where there is a compressive stress field, generally have basement structures with reverse fault systems. An understanding of these underground structures is important from the viewpoint of the formation process of the field. This understanding could lead to disaster prevention from earthquakes as well as solving problems with the geological environment. Gravity analysis is one of the powerful techniques for this purpose. However, a few examples of gravity analysis on basement structures containing reverse faults have been presented since the depth from the ground surface is not determined uniquely at locations where a reverse fault exists. It is another reason that complex parameters are required to design a model. In this study, an algorithm is presented to calculate gravity anomalies for the basement structure containing reverse faults, as well as a method is proposed to determine the dip or throw parameters of the fault.

Basement structure is modeled as a set of the triangular prisms, which have half infinity length and inclined upper surface. The calculation is enabled by a set of triangular prisms. In order to avoid inputting many complex parameters, a program was developed that used digital elevation model (DEM) data. Accordingly, the theoretical gravity for reverse fault structures could be easily determined. The technique developed in this study is effective for the structural analysis of basements containing reverse faults.

Key-words: Basement structure modeling, DEM data, triangular forms, computation sequence, FORTRAN program

1. Introduction

The sedimentary basin in Southwest Japan has a basement structure that has been typically formed by the development of a reverse fault system (Ishiyama, 2004). An understanding of this structure is very important to consider the geological developing process and to determine the source parameter for the calculation of strong ground motion. However, it is difficult to determine an underground structure from the result of gravimetric prospecting as an inversion problem because the solution is generally not unique. The model is often judged to be valid if the gravity value is obtained by forward analysis based on a 3D structure model that corresponds to the measurement value (Akamatsu and Komazawa, 2003). However, this modeling becomes complex if someone attempts to strictly calculate the reverse fault; the computation time increases significantly as well. Therefore, the vertical fault model is often used in the analysis.

Götz and Lahmeyer (1988) developed an algorithm and a FORTRAN subprogram to calculate the gravity value. It used triangular prisms that had each inclined the upper surfaces. In this algorithm, a variable of integration is converted from the volume to the surface area by Green's theorem. Ryoki (1996) proposed a method for converting the DEM (digital elevation model) of the depth of a
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The idea devised by Talwani and Ewing is the first epoch-making method. But $\Delta z$, which is thickness of the basement subsurface into a triangular prism in order to use this subprogram easily.

If the order of describing the vertices of an upper triangle is counterclockwise in Götze and Lahmeyer's algorithm, the result becomes a negative value. To calculate the gravity value of the reverse-fault structure without changing the technique represented by DEM, the characteristic of this algorithm can be used. Utilizing the characteristic, a new method is proposed to determine a 3D basement structure from gravity anomalies, and an example of its application is shown in this research.

2. Theory and method

In the early stage, structural analysis of gravity anomalies was based on determining plausible shapes of which gravity effects may be calculated by analytical calculus. Next, some analytical methods were developed to calculate the gravity anomalies from intricate models whose geometry was approximated by a great many of elements, i.e., polygons, prisms or polyhedrons.

Talwani and Ewing (1960) developed a calculation method in which the 3D body is represented by contours. They replaced each contour with a horizontal irregular polygonal lamina. The gravity effect produced by each lamina was determined analytically at any vertex and considered to be a function of the depth of the lamina (Fig. 1). If the body is represented by $m$ contours and each contour is replaced by an $n$-sided polygonal lamina, the gravity effect $g$ is obtained

$$ g = \sum_{i=1}^{m} V_i \cdot \Delta z $$

where $V$ is the anomaly caused by the each lamina per unit thickness; $\Delta z$, thickness of the lamina. $V$ is expressed by a surface integral. Talwani and Ewing obtained $V$ as follows:

$$ V = G \rho \sum_{i=1}^{m} \left[ \frac{1}{r_i} \left( \frac{\sin^{-1} \frac{f_i s}{\sqrt{r_i^2 + z^2}} - \sin^{-1} \frac{z q s}{\sqrt{r_i^2 + z^2}}}{\sqrt{r_i^2 + z^2}} \right) \right] $$

where $G$ is the universal constant of gravitation, $\rho$ is the volume density of the lamina, $z$ is the depth of the lamina, $(x_i, y_i)$ is indicated the coordinates of $i$-th vertex of the lamina,

$$ s = \begin{cases} +1 & \text{if } P_i \geq 0 \\ -1 & \text{if } P_i < 0 \end{cases} $$

$$ w = \begin{cases} +1 & \text{if } m_i \geq 0 \\ -1 & \text{if } m_i < 0 \end{cases} $$

$$ P_i = (\xi - x_{i+1}) \frac{r_i}{r_{i+1}} + (\eta - y_{i+1}) \frac{r_{i+1}}{r_i} $$

$$ q_i = (\xi - x_{i+1}) \frac{r_{i+1}}{r_i} + (\eta - y_{i+1}) \frac{r_i}{r_{i+1}} $$

$$ f = (\xi - x_{i+1}) \frac{r_{i+1}}{r_i} + (\eta - y_{i+1}) \frac{r_i}{r_{i+1}} $$

$$ m_i = (\xi/r_i)(x_{i+1}/r_{i+1}) - (\eta/r_i)(y_{i+1}/r_{i+1}) $$

$$ r_i = \left( x_i^2 + y_i^2 \right)^{1/2} $$

$$ r_{i+1} = \left( x_{i+1}^2 + y_{i+1}^2 \right)^{1/2} $$

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\[
T(x,y,h) = x \ln y + \frac{y(x^2 + y^2 + h^2)^{3/2} + y \ln h}{h(x^2 + y^2 + h^2)^{3/2}} \cdot \tan^{-1} \frac{xy}{h(x^2 + y^2 + h^2)^{3/2}}, \quad h = z_2 - z_1.
\]

\[
\Delta g_{0(0,0,0)} = G \rho \left[ T(x_2, y_2, h) - T(x_1, y_1, h) - T(x_2, y_1, h) + T(x_1, y_2, h) \right]
\]

where

\[
T(x,y,h) = x \ln y + \frac{y(x^2 + y^2 + h^2)^{3/2} + y \ln h}{h(x^2 + y^2 + h^2)^{3/2}} \cdot \tan^{-1} \frac{xy}{h(x^2 + y^2 + h^2)^{3/2}}, \quad h = z_2 - z_1.
\]

\( G \) is the universal constant of gravitation; \( \rho \), the volume density of the vertical square prism.

In their study, the square prism generally has a horizontal surface as the upper boundary. Therefore, it is necessary to have many square prisms to express a basement structure correctly. However, if we introduce a polyhedron having an inclined upper surface, the basement structure can be expressed more adequately (Götze, 1984).

At point \( P \), the gravity anomaly \( Lg(P) \) obtained from the \( m \)-th polyhedron shown in Fig. 3a is given by the following expression.

\[
Lg(P) = G \rho \sum_{j=1}^{m} \left[ n_{zj} \cdot \int_{S_j} \left( \frac{1}{R} \right) ds_j \right]
\]

where \( S_j \) denotes a polyhedron surface \( \left( j=1, \text{and } m, \text{the number of surface} \right) \); \( ds_j \), the surface element on \( S_j \); \( R \), the distance from \( P \) to \( ds_j \); \( n_{zj} \), the direction cosine of \( S_j \) from the \( z \)-coordinate. If \( n_z \) denotes a surface normal of \( S_j \) and \( z \) denotes a unit vector on \( z \)-coordinate, \( n_{zj} \) is obtained as follows:

\[
n_{zj} = \cos \theta = \frac{n_z \cdot z}{|n_z| |z|}
\]

where \( \theta \) is an angle between \( n_z \) and \( z \).

In order to evaluate the surface in Expression (4), Götze required a transformation of the coordinate system where by the new system \( x', y', z' \) should be surface-oriented (Fig. 3b). The \( x' \)-axis runs parallel to \( \nabla \cdot V \); and the \( z' \)-axis to the noticed surface normal \( n_f \). The \( y' \)-axis is orthogonal to the \( x' \) and \( z' \) axes. Next, he converted the surface integral in Expression (4) into a linear integral via polygon \( P_j \), which limits surface \( S_j \). Finally, the expression for the gravity effect of a polyhedron is obtained as follows:
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Point $p_i$ is defined as $V_{i,1} = L, 2, ..., K_i$. Fig. 4 An illustration of the polyhedron surface plane $S_j$ to derive Expression (6) (after Götze, 1984).

$$\Delta g(P) = G \rho \sum_{j=1}^{\infty} n_{ij} \left\{ \left( \frac{h_j}{r_j} \right) \left( \frac{PP^*}{PP^* + r_j} \right)^2 dp_j + 2 \pi PP^* \delta \right\}$$

with $\delta = \begin{cases} 0, & P^* \notin S_j \\ 1, & P^* \in S_j \end{cases}$

where $p_j$ denotes $j$-th polygon of the polyhedron shown as Fig. 4; $P^*$, the projection point of $P$ with respect to the surface; $dp_j$, a differential line on the side; $h_j$, the distance from $P^*$ to the side on which $dp_j$ lies; $r_p$, the distance between $P^*$ and $dp_j$; $\delta$, the distribution coefficient of the exterior angle in the vertex of each side, and $n_{ij}$ is expressed as Expression (5). Since the effect arising from successive surfaces or sides of a polygon is counterbalanced in Expression (6), it needs not necessarily to be calculated when an underground structure is expressed as a set of the polyhedron. This distinctive feature is very useful in numerical calculations. It decreases computer resources or computing time. Therefore, the gravity effect is simply calculated if an underground structure is divided into the polyhedrons, i.e., triangular prisms.

Based on Expression (6), Götze and Lahmeyer (1988) developed a FORTRAN subprogram called “NEWTON” to obtain the gravity effect for a triangular prism having an inclined upper surface. The source code for this subprogram is open to the public, and its use is allowed for scientific studies. Some methods, for instance a method for dividing an area between contour lines into triangles (Nishikawa et al., 2005) or a finite element method (Taniguchi, 1992), were devised to divide the geological boundary surface into triangles. However, if the vertices of the triangles are decided according to each method, the result may be vague because there are many degrees of freedom. Then, Ryoki (1996) proposed a method to convert the DEM data of the basement structure into triangular forms.

Ryoki (1996) assumed that triangles are not piled up on each other in his program called “GRD2TRI.” However, if a triangle overhangs the neighboring one, the overhanging part expresses the reverse fault structure. Fig. 5 shows a cross section of the model of a basement structure along the $y$-axis. The fault is expressed as a normal fault in Fig. 5a. If the model is remade with point $P_i$ in the top of the fault moved to overhang above the footwall $P_{i+1}$, $P_{i+2}$, $P_{i+3}$, ..., the model of the normal-fault structure shown in Fig. 5a becomes like the model of the reverse-fault structure shown in Fig. 5b.

Fig. 5 A method to make the reverse-fault structure model: (a) a model of the normal-fault type expressed by DEM form usually; (b) a model expressed as the reverse fault.
In NEWTON, the output result becomes a positive value if the upper triangular vertices are calculated in counterclockwise sequence, and negative in clockwise sequence. With this characteristic, it is possible to calculate the gravity anomaly of the reverse fault structure.

2-1. Case of normal fault or strike-slip fault

For example, at point P on the ground surface, we think about gravity value \( g(P) \) which five triangular prisms introduce (Fig. 6a). On the upper surface \( \Delta ABC \) of a prism, the gravity value \( g_{\Delta ABC} \) is calculated in counterclockwise sequence of \( A \rightarrow B \rightarrow C \). As such, \( g_{\Delta ABC} \) is a positive value. As other prisms are also calculated similarly, gravity values \( g_{\Delta ABED}, g_{\Delta ADEP}, g_{\Delta SCDF} \) and \( g_{\Delta ABDC} \) are positive. Therefore, \( g(P) \) is given as follows:

\[
g(P) = g_{\Delta ABC} + g_{\Delta ABED} + g_{\Delta ADEP} + g_{\Delta SCDF} + g_{\Delta ABDC} \quad (7)
\]

2-2. Case of reverse fault

To consider a case of the reverse fault structure, point D shown in Fig. 6a is moved to a point above point A. Thus it is that \( \Delta ABC \) becomes a fault plane of the reverse fault (Fig. 6b). Now, \( M_i \) is defined as a volume of the triangular prism \( ABCA'B'C' \). Similarly,

- \( M_2 \), a prism \( BEDB'E'D' \);
- \( M_3 \), a prism \( DEFD'E'F' \);
- \( M_4 \), a prism \( DFCD'F'C' \);
- \( M_5 \), a prism \( BDCB'D'C' \).

As shown in Fig. 6b, the sum of \( M_2, M_3 \) and \( M_4 \) subtracted by \( M_5 \) makes a hanging wall body of the reverse fault. Furthermore, the hanging wall body plus \( M_i \) becomes \( M \), which is a model of the reverse fault structure:

\[
M = M_1 + M_2 + M_3 + M_4 - M_5. \quad (8)
\]

\( g(P) \) can be obtained as the result of subtracting the gravity effect formed by \( M_i \) from the gravity effects formed by \( M_1, M_2, M_3, \) and \( M_4 \). If the gravity effect formed by each triangular prism shown in Fig. 6b is calculated by NEWTON, \( g_{\Delta ABC}, g_{\Delta ABED}, g_{\Delta ADEP} \) and \( g_{\Delta SCDF} \) are positive value as well as the case of 2-1. However, \( g_{\Delta ABDC} \) is a negative value because it is calculated in clockwise sequence of \( B \rightarrow D \rightarrow C \). Therefore, \( g(P) \) is given as follows:

\[
g(P) = g_{\Delta ABC} + g_{\Delta ABED} + g_{\Delta ADEP} + g_{\Delta SCDF} + g_{\Delta ABDC} \quad (9)
\]

Expression (9) is the same as Expression (7), yet \( g_{\Delta ABDC} \) in Expression (7) is positive because it is calculated in counterclockwise sequence of \( B \rightarrow D \rightarrow C \) shown in Fig. 6a. To urge understanding, there is a cross section of a simple conceptual diagram in Fig. 7. If the lower part of fault plane as shown in Fig. 7c is subtracted from an entire volume as shown in Fig. 7b before and the remainder is added to the part of the footwall as shown in Fig. 7d, the result gives a structure of the reverse fault as shown in Fig. 7a. In this way, the gravity value of the overhanging part is
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Fig. 7 A cross section of the reverse-fault model: (a) the reverse-fault model; (b) an entire volume; (c) a lower part of the fault plane; (d) a part of the fault footwall.

given as that of a reversed fault since the value obtained from a triangular prism of ABDC is subtracted from that obtained from a square gimlet DBCFE as shown in Fig. 6b.

Based on the above-mentioned theory, an interactive FORTRAN program to calculate the gravity anomalies is proposed. The program is called “TRIGRV,” and it uses the triangular forms proposed by Ryoki (1996). The source code of the program to use for scientific studies is scheduled to be opened to the public at a homepage of the World Wide Web on the Internet or at another suitable medium. Anyone may ask the author for the source code file through e-mail. The level of compensation to calculate the gravity anomalies may be set at will. To reduce the truncation error, in this program the outer side of the former model can be extended to a wide enough area.

TRIGRV has some default conditions that are easy to use. In one of them, the level of compensation takes the value of the deepest place in the model. And in another, the outer side extension takes five times the average of distance of the center point and each lattice point of the outer side projected on the x-y plane in the model. The output file is in the same format as that of “TERRAMOD,” which is a program for the generation of a 3D terrain model using DEM data (Masumoto et al., 1993). It is necessary that NEWTON is linked to TRIGRV at execution time.

By way of illustration, to correct the input file, the following procedure has to be executed as shown in Fig. 5. The y-coordinate of each point is considered to be the same for simplicity. A normal fault structure can be expressed as shown in Fig. 5a. In order to assume that this fault expresses a reverse fault and not a normal one, point \( P_i = (x_i, y, z_i) \) in the top of the fault plane is moved to point \( P_i' = (x_i', y, z_i) \), as shown in Fig. 5b. A pertinent part from the input file of the main program is corrected to coordinate \( P_i' \). In order to express a basement structure as reverse faults, only the coordinates of the corresponding points are properly retouched in the input file. Therefore, calculations are possible without rewriting TRIGRV.

In order to verify the developed program TRIGRV, the gravity value was calculated for a 2D reverse fault structure shown in Fig. 8. The x-, y- and z-axis values are normalized by the center of the fault plane \( Z_0=1 \) km in depth. For an arbitrarily assumed cross-sectional direction \(-5\leq x\leq 5\) and fault strike \(-5\leq y\leq 5\), a 3D structure model was developed with a spacing of 0.2. A result for a fault dip of \( \theta =30^\circ \) and a vertical slip of \( H=1 \) km is shown in Fig. 9. The program is used with default conditions, and the grid numbers of the calculated result, \( 101 \times 101 \), are the same as that of the model. The result is normalized by the gravity effect calculated as follows:

\[
2 \pi G \Delta \rho Z_0 = 41.9088 \text{ mgal} ,
\]

where \( Z_0 \) denotes a depth of the midpoint of fault plane as 1 km.

No truncation error caused by the finite-size model appears in Fig. 9. It is the reason that the appearance of the
isopleths in the vicinity of the center correspond to the edge \((y=-5\) or \(y=5)\) at any \(x\)-coordinate. Therefore, it is suggested that the use of default conditions is appropriate for this calculation.

Fig. 10 shows a gravity anomaly profile over the interval \(-3 \leq x \leq 3\) at the center line parallel to the \(x\)-axis in Fig. 9 with the theoretical curve of the 2D reverse fault structure. A result for \(\theta = 60^\circ\) is also shown in Fig. 10. The analytical results using TRIGRV are almost corresponding to the theoretical curve. The errors in the mean square are \(1.372 \times 10^{-4}\) for \(\theta = 30^\circ\) and \(4.339 \times 10^{-5}\) for \(\theta = 60^\circ\) over the interval \(-3 \leq x \leq 3\). This amount corresponds to almost \(2-6 \text{ } \mu\text{gal}\) if compared with the result for a standardized reference value. Thus, it is confirmed that TRIGRV can be practically used without any problems.

### 4. Discussion

In order to understand some spatial characteristics of gravity anomalies for a reverse fault structure, the gravity value for the structure shown in Figs. 11a as a plan and 11b as a cross section is calculated. The distribution of gravity anomalies was calculated with a parameter as dip angle \(\theta\). Fig. 12a shows a distribution of the gravity anomalies for \(\theta = 45^\circ\), while Fig. 12b shows the same for \(\theta = 75^\circ\). It is found in the neighborhood of the fault that the density of the isopleths in Fig. 12b is higher than that in Fig. 12a. However, the difference is not clearly observed in these figures. Therefore, a norm distribution about horizontal components of the gravity gradient and a distribution of the second order derivative value with respect to the vertical direction are calculated.

A primary differentiation of the gravity value yields an inclination vector. Thereupon, the norm of the gradient was obtained by the following expression:

\[
\|\text{grad}(g)\| = \left\{ \left( \frac{\partial g}{\partial x} \right)^2 + \left( \frac{\partial g}{\partial y} \right)^2 \right\}^{1/2}
\]  

(11)
where \( g \) denotes the gravity value at point \( P \). The obtained distribution of the norm of the inclination is shown in Fig. 13. The high gradient area in the case of small dip-angle (Fig. 13a) is narrower than in the case of large dip-angle.

On the other hand, according to the potential theory, a potential \( U \) in the space without the material is described by Laplace's Equation. No particular sequence is obtained in the independent variables of higher order partial derivative because the space is isotropic in the free space or in the atmosphere around the surface. Then, by taking the derivative of Laplace's Equation with respect to \( z \), and by making the substitution with \( g = \partial U / \partial z \), the next expression is obtained as follows:

\[
\frac{\partial^2 g}{\partial z^2} = -\left( \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} \right).
\]  \hspace{1cm} (12)

From Expression (12), the value of the second order derivative with respect to the vertical component can be calculated since the value of the second order derivative of gravity with respect to the horizontal components can be easily obtained from the measurement value. Fig. 14 shows a distribution that is obtained from the second order partial derivative of gravity with respect to the vertical component by using Expression (12).

Frequently, it is pointed out that a definite shape difference should not be assumed since the theoretical curves of the gravity anomalies made by 2D fault structures appear similar. However, careful notice reveals that the curvature decreases at low angles or the position of

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**Fig. 11** Test model of a reverse fault structure. The density difference between the sediment and its basement is assumed to be 1 g/cm³. The hanging wall has a depth of 1 km, and the throw is 1 km. (a) ground plan and (b) cross section along \( A-A' \) line in (a).

**Fig. 12** Distribution of gravity anomalies obtained from a test model of a reverse fault structure. These were calculated using \( \theta \) as a parameter, as shown in Fig. 11. The contour interval is 1 mgal. (a) \( \theta = 45^\circ \) and (b) \( \theta = 75^\circ \). Solid and broken lines indicate the top and bottom of the fault, respectively.
maximal absolute value of the curvature deviates from the fault position. These features are clearly found on the curve of the first or second order derivative of gravity anomalies along the horizontal direction. Then, the fault dip can be estimated based on these features if an underground structural model having reverse faults is considered based on the measured gravity profile. Considering these features, the position, where the gradient changes shown in Fig. 13, suggests the location of the fault plane.

Fig. 14 shows the distribution map of the second order partial derivative of gravity with respect to the vertical direction. The area indicated by high values in the case of large dip-angle (Fig. 14b) shifts to the side of the hanging wall of the fault compared with the small dip-angle (Fig. 14a).

The position of a fault plane may be assumed on the position where the second order partial derivative becomes zero or where the intervals of the isopleths change suddenly in the distribution of the second order derivative.
at the 2D fault section. The interval where the isopleths change suddenly is wide when the fault dips at a low-angle ($\theta = 45^\circ$), but narrow at a high angle ($\theta = 75^\circ$). In other words, when analyzing an underground structure by means of a gravity survey to predict the characteristics of the reverse fault, it is possible to predict the dip of fault plane by considering a pattern of the map that shows a distribution of the norm of the gravity gradient or the second order partial derivative of gravity with respect to the vertical direction.

5. Conclusion

In this paper, a method to analyze a reverse-fault structure by means of a gravity survey was proposed as well as its application was confirmed. It was very time-consuming to calculate strictly the theoretical gravity anomalies by using the model of the reverse-fault structure so far. Contrary to this, the theoretical gravity anomalies can be calculated more simply if the presented method is used. The method is a modeling algorithm to calculate the gravity value of the reverse-fault structure. The reverse-fault structure model is proposed to be expressed by combining triangular prisms that have each inclined the upper surfaces. In this method, the DEM data can be used without changing the form. A method to predict the reverse fault structure model was discussed by closely examining the distributions of the norm of the gravity inclination and the second partial derivative with respect to the vertical direction.

In general, it is a fact that there are infinite solutions in an inverse problem requested by the gravity analysis. However, a structural model can be obtained from the evaluation if the geological constraint conditions are examined closely. In the future, it is necessary that an algorithm to express the reverse fault model should be developed.

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