# Hybrid Test for Simulating Seismic Behavior of Steel and Composite Bridge Piers

Hiroshi NAKAI,[1] Toshiyuki KITADA[2] and Katsuyoshi NAKANISHI[3]

( *Received September 30, 1996* )

**Synopsis:** This paper briefly describes a series of the hybrid testing systems, which are developed in Postgraduate School of Engineering, Osaka City University to simulate the seismic response of bridge pier subjected to strong earthquakes and to investigate the ultimate strength and ductility of them. These systems are equipped two actuators of the capacity of 10tf and 50tf controlled by a digital computer of 32 bits, respectively. The contests of the paper are consisted of ; outlines of the systems, logical concept of hybrid tests, analogous rule between actual structures and scaled down specimens, and prediction of the viscous dumping coefficient. These systems are verified through comparison of the simulated results with the numerical results.

***Keywords*** *: hybrid test, pseudo-dynamic test, on-line test, bridge pier, steel and composite pier, seismic response test*

## 1. Introduction

The original concept of the hybrid tests was developed in 1969 by Hakuno, Shidawara and Hara [1], where an actuator was controlled by an analogue computer. In 1974, Takanashi, Udagawa, Seki, Okada and Tanaka [2] applied the Hakuno's idea to much more actual hybrid tests. Then, Yamada and Iemura [3] succeeded in developing a hybrid testing system by using a digital-computer of 8 bits in 1981. After these developments of hybrid technique, however, only a few systems for hybrid tests had been equipped in a few laboratories of universities and companies [4-6] before the Great Hanshin-Awaji Earthquake occurred on early morning of January 17, 1995.

Before the Great Hanshin-Awaji Earthquake, many bridge structures made of reinforced concrete had, of course, been seriously damaged or collapsed by the other earthquakes. However, many bridge structures made of not only reinforced concrete but also steel were damaged or collapsed by the Great Hanshin-Awaji Earthquake. After the shock of this earthquake, the hybrid tests have been needed as the indispensable one in order to investigate the ultimate strength and ductility of steel structures. Particularly, the hybrid tests shall be conducted for the steel bridge piers subjected to as strong earthquakes alike the Great Hanshin-Awaji Earthquake, because it is very difficult to simulate the behavior in the vicinity of the ultimate state and after the ultimate state of steel structures with buckled plate elements theoretically.

In these circumstances, a system for hybrid tests of materials and structures [7] was developed in the Laboratory for Large Sized Structures, Post Graduate School of Engineering, Osaka City University by the supports of the Ministry of Education of Japan in March 1990. Three actuators with the capacity of 10tf, 50tf and 100tf completely controlled by a digital computer of 32 bits are equipped in this system. In 1991, a hybrid testing system by using the actuator of 10tf was developed for investigating of the ultimate strength and ductility of steel bridge piers with steel columns or concrete-filled steel columns subjected to strong earthquakes for the tests of comparatively small scaled specimens. Therefore, another hybrid testing system by using the actuator of 50tf was also developed in 1995, because of the hybrid tests for the tests of large scaled specimens were required for investigating the collapse behaviors of structural details such as plate

[1] Professor, Department of Civil Engineering
[2] Associate Professor, Department of Civil Engineering
[3] Research Associate, Department of Civil Engineering

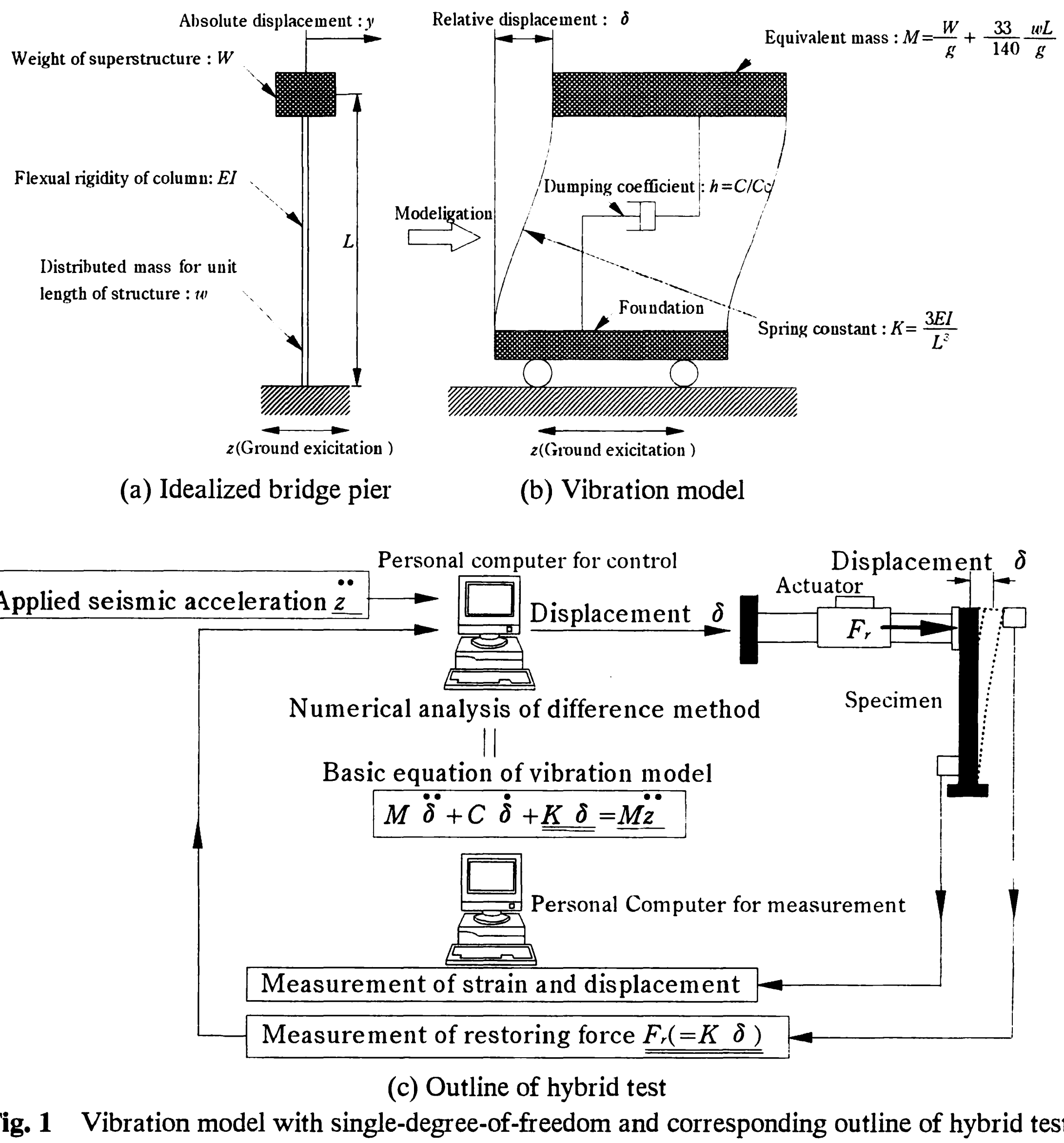
elements and welding parts. After the development of these hybrid testing systems, many studies on the seismic behaviors of bridge piers have already been curried out by using them [8-11].

In this paper, our two hybrid testing systems to simulate the seismic response of bridge piers subjected to earthquakes are briefly described. Also predicated is the logical concept of hybrid tests, analogous rule between full scale models and scaled down models, and prediction of viscous dumping coefficient. The accuracy of these hybrid testing systems are verified through the comparisons with numerical results in elastic region.

## 2. Outline of Hybrid Testing Systems

### *2.1 Mechatronics of Hybrid Testing Systems*

A vibration model with single-degree-of-freedom for simulating the dynamic response of a bridge pier under a seismic excitation and the corresponding scheme of a hybrid test are illustrated in **Figs. 1(a)-(c).** These hybrid tests for simulating the seismic behaviors is also called the pseudo-dynamic test which connects a numerical on-line response analysis in a computer with a loading test to obtain the unknown restoring force




(c) Outline of hybrid test

**Fig. 1** Vibration model with single-degree-of-freedom and corresponding outline of hybrid test

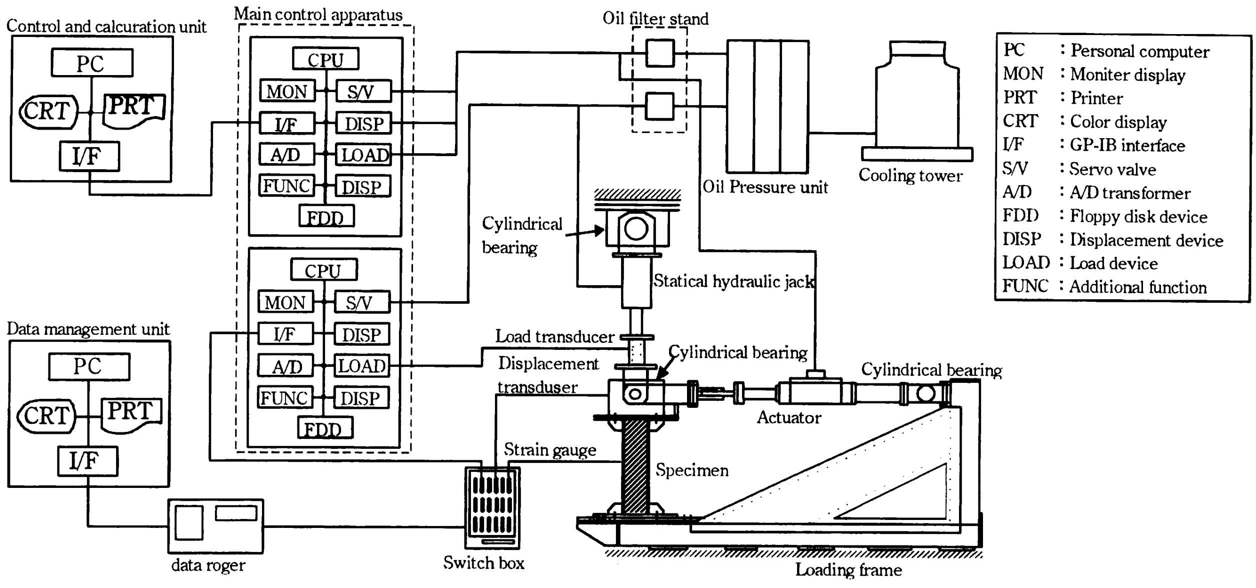



**Fig. 2** Mechatronics in hybrid testing system

of a vibration model for bridge pier, because it is difficult to obtain the restoring force analytically. As an example one of our hybrid testing systems for a simple vibration model by using one actuator controlled through the computer is shown in **Fig. 2.**

Thus, the differential equation of motion for the vibration model under the excitation of a ground acceleration is numerically analyzed in the computer. It is, however, necessary to know the restoring force of the vibration model for analyzing the differential equation of motion. When the vibration model is in the elastic region, it is possible and easy to derive the restoring force of the vibration model analytically. If the vibration model behaves either in the elasto-plastic region, or particularly in the region that the plate elements composing the test models have buckled, it is, nevertheless, very difficult to calculate the accurate restoring force in the post-buckled region theoretically. Therefore, the restoring force is evaluated by the on-line measured experimental one simultaneously in these hybrid testing systems.

### *2.2 Loading Apparatus*

Our loading apparatuses for the hybrid tests are designed so as to be able to apply a transverse load simulating seismic forces and compressive load for the dead load of superstructures into the specimens simultaneously or separately. The compressive load is applied to the specimens at the location of the top of specimens by using the statical hydraulic jack of the capacity of 30tf or 100tf, and the transverse load is generated by using the actuator of the capacity 10tf or 50tf. In addition, cylindrical bearings or boll joint are inserted between the loading frame and the statical hydraulic jack, and between the actuator and the frame not to restrain the rotation of the specimen at the top. For the same reason, a cylindrical bearing is also set at the loading point of the specimen. Then, two loading apparatuses by using 10tf and 50tf actuators are installed in our laboratory.

#### *(1) Loading Apparatus Using 10tf Actuator*

The loading apparatus by using the 10tf actuator is depicted in **Fig. 3**. This loading apparatus consists of the 30tf statical hydraulic jack for the compressive load and the 10tf actuator for the transverse load.

#### *(2) Loading Apparatus Using 50tf Actuator*

The loading apparatus by using the 50tf actuator is sketched in **Fig. 4**, where the loading apparatus consists of the 100tf statical hydraulic jack for the compressive load and the 50tf actuator for the transverse load.

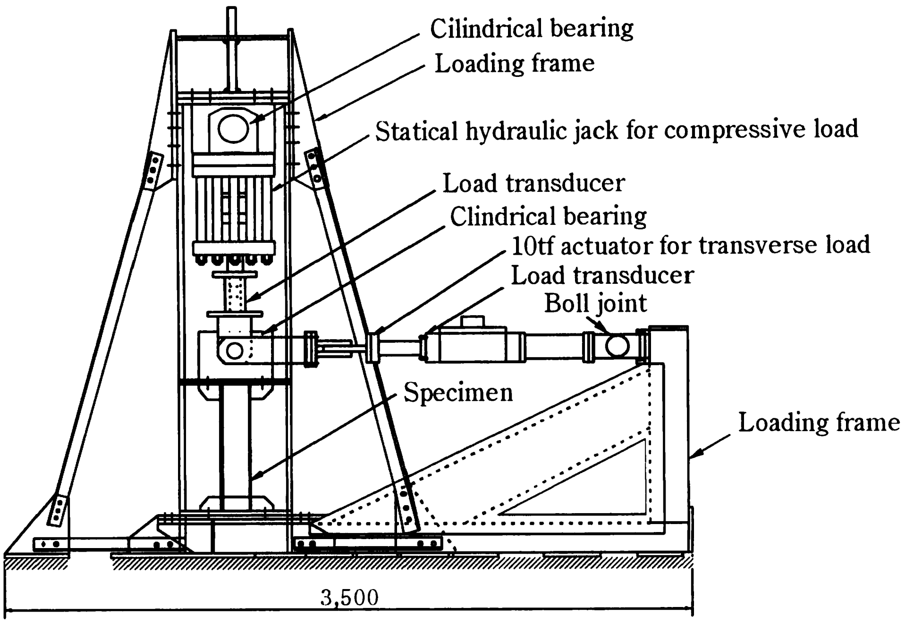



**Fig. 3** Loading apparatus by using 10tf actuator (Unit : mm)

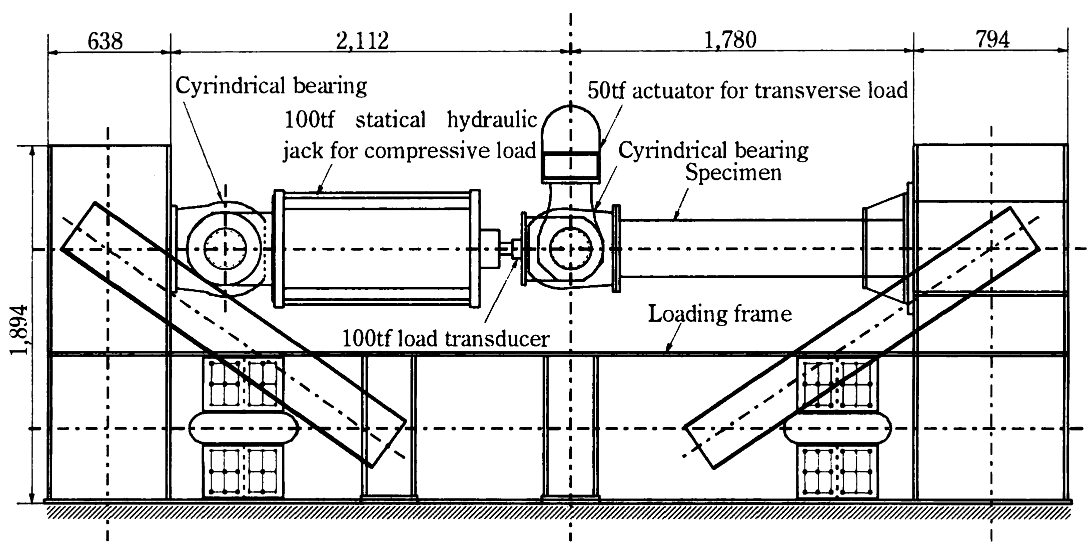



**Fig. 4** Loading apparatus by using 50tf actuator (Unit : mm)

## 3. Logical Concept of Hybrid Test

### *3.1 Flow of Hybrid Test*

Our hybrid testing systems mainly consist of two processes which are connected on-line each other, i.e. one of them is the analytical process and the other is the experimental one. The flowchart of the hybrid tests is shown in **Fig. 5**.

Firstly, data for adjustment of the experimental errors which are obtained through preliminary tests, seismic acceleration records to be applied to test specimens and an initial values of the displacement, velocity and acceleration of the equivalent mass of a vibration model are inputted from the data file.

Secondly, the displacement, $\delta_1$, the velocity, $\dot{\delta}_1$ and the acceleration, $\ddot{\delta}_1$ at the top of specimen on the first step of time increment are calculated by using Newmark's $\beta$ method. The displacement, $\delta_1$ is applied to the top of specimen. Then, the restoring force, $K_1\delta_1$ for the first step which means the horizontal reaction at the top of specimen, is measured at the same time.

Thirdly, the displacement, $\delta_2$, the velocity, $\dot{\delta}_2$ and the acceleration, $\ddot{\delta}_2$ at the top of the specimen on the second step of time increment are calculated by using the central difference method. Then, this displacement is applied to the specimen, and the restoring force, $K_2\delta_2$ is measured at the same time. On and after these processes, the same calculations and measurement are repeated until the time, $t$ becomes longer than $t_{max}$ or the displacement, $\delta_i$ becomes larger than $\delta_{max}$.

*3.2 Analytical Process*

The analytical process to predict the displacement for the next step of time increment by the numerical integration of the differential equation of motion is explained as below.

The central difference method is adopted for the numerical integration in consideration of the stability condition, $\Delta t \leqq 2/p$, except that Newmark's $\beta$ method is used for the first time increment in order to determine the primary conditions for the central deference method, where $\Delta t$ and $p$ ($=\sqrt{K/M}$, $K$ : flexural rigidity of pier column, $M$ : equivalent mass ) are the incremental time and the natural frequency of the specimens, respectively, because the numerical solution by this method can easily be converged even in the elasto-plastic region in comparison with Newmark's $\beta$ method.

*(1) Differential Equation of Motion*

A vibration model with a single-degree-of-freedom, which has the flexural rigidity, $K$ and the equivalent mass, $M$, is taken as illustrated in **Figs. 1 (a)** and **(b)**. When a seismic excitation is applied to the ground which supports the vibration model, the response of the vibration model due to this ground excitation is expressed in terms of the relative displacement, $\delta$ ($=y-z$) of the vibration model according to the principle of d'Alembert as follows :

$$M\left(\ddot{y}-\ddot{\delta}\right)+C\left(\dot{y}-\dot{z}\right)+K\left(y-z\right)=M\ddot{z} \tag{1}$$

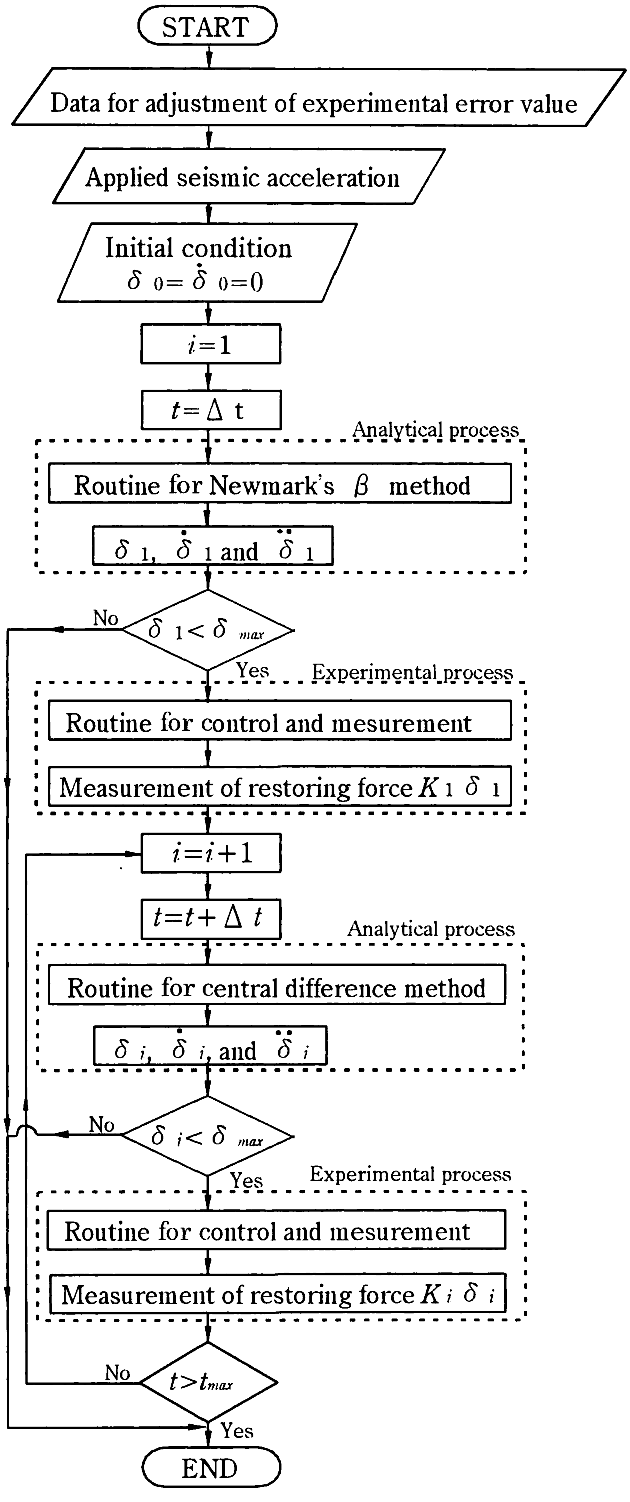



**Fig. 5** Flowchart of hybrid test

Then, it follows that :

$$\ddot{\delta} + \frac{C}{M}\dot{\delta} + \frac{K}{M}\delta = -\ddot{z} \tag{2}$$

where $\delta$ , $\dot{\delta}$ and $\ddot{\delta}$ is the relative displacement, relative velocity and relative acceleration, respectively. Also, $C$ is defined as the viscous dumping constant.

*(2) Central Difference Method*

The basic equations of the central difference method are given by :

$$\ddot{\delta}_i = \frac{\delta_{i+1} - 2\delta_i + \delta_{i-1}}{\Delta t^2} \tag{3}$$

$$\dot{\delta}_i = \frac{\delta_{i+1} - \delta_{i-1}}{2\Delta t} \tag{4}$$

and the following equation is derived by substituting Eqs. (3) and (4) into Eq. (2) :

$$\delta_{i+1} = \frac{\left(C_i \frac{\Delta t}{2} - M\right)\delta_i + 2M\delta_i - \left(K_i\delta_i + M\ddot{z}_i\right)\Delta t^2}{M + C_i\frac{\Delta t}{2}} \tag{5}$$

where $\delta_i$, $\dot{\delta}_i$ and $\ddot{\delta}_i$ are respectively the relative displacement, relative velocity and relative acceleration on the $i$-th step of time increment and $K_i$ is designated as the flexural rigidity on the $i$-th step.

*(3) Newmark's $\beta$ Method*

According to Newmark's $\beta$ method, the relationships between the corresponding response values on the $i$-th step and those on the $(i+1)$-th step can be estimated as follows :

$$\dot{\delta}_{i+1} = \dot{\delta}_i + (1-\gamma)\ddot{\delta}_i^{\,2}\Delta t + \gamma\,\ddot{\delta}_{i+1}\Delta t \tag{6}$$

$$\delta_{i+1} = \delta_i + \dot{\delta}_i\Delta t + \left(\frac{1}{2} - \beta\right)\ddot{\delta}_i\Delta t^2 + \beta\,\ddot{\delta}_{i+1}\Delta t^2 \tag{7}$$

In general, the values of parameters, $\gamma$ and $\beta$ are set as 1/2 and 1/6, respectively.

However, the acceleration $\ddot{\delta}_{i+1}$ on the $(i+1)$-th step is unknown value on the $i$-th step. Therefore, it is assumed that $\ddot{\delta}_{i+1}$ equals to $\ddot{\delta}_i$. Thus, the velocity, $\dot{\delta}_{i+1}$ and displacement, $\delta_{i+1}$ on the $(i+1)$-th step are calculated by using the above Eqs. (6) and (7).

Next, the following equation is newly derived by substituting these values into the Eq. (2) :

$$\ddot{\delta}_{i+1} = f(t_i) - \frac{C_i}{M}\dot{\delta}_i - \frac{K_i}{M}\delta_i \tag{8}$$

The velocity, $\dot{\delta}_{i+1}$ and the displacement, $\delta_{i+1}$ are calculated again by substituting this acceleration, $\ddot{\delta}_{i+1}$, which is obtained from Eq. (8) into Eqs. (6) and (7). These calculations are repeated until the value of $\ddot{\delta}_{i+1}$ converges to a certain constant value with a negligible small error.

*3.3 Experiment Process*

The displacement, $\delta_i$, which is calculated by the computer numerically, is applied to the specimen, and the restoring force, $F_r$ (=$K_i\,\delta_i$) in Eq. (1) or Eq. (2) is measured by the on-line test at the same time. However, undesirable behavior was observed in the relationship between the restoring force, $F_r$ and horizontal displacement, $\delta$ in the vicinity of very small restoring force as shown in **Fig. 6.** This phenomenon is caused by inevitable gab between the actuator and loading frame as well as the specimen and the loading frame at the bases of the specimen.

In order to eliminate the experimental errors caused by this phenomenon, a static test in the elastic region is executed to obtain data on the undesirable behavior before the start of a hybrid test. Then, the measured values in the hybrid test are corrected to eliminate the experimental errors by using the data.

### *3.4 Analogous Rule Between Actual Structures and Experimental Models*

The analogous rule between an actual structure must be investigated and the corresponding experimental model must be considered in simulating the dynamic behavior of the actual structures through the tests by using the experimental models with dimension of reduced scale [12].

When the equivalent mass, the viscous dumping constant and the flexural rigidity of the column of a specimen with the scaled down dimensions is defined by $M_m$, $C_m$, and $K_m$, respectively, and those of the full-scale model for the actual structure is given by $M_a$, $C_a$ and $K_a$, respectively, the differential equations of motion for the specimen and the full-scale model subjected to a seismic acceleration are given as follows :

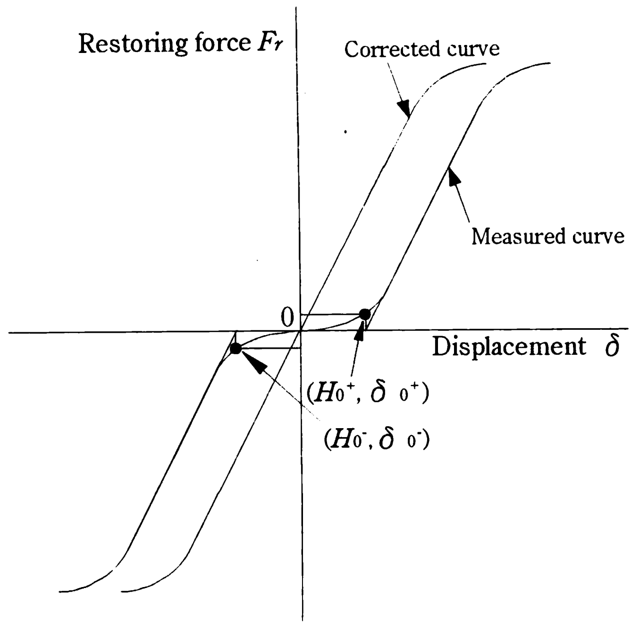



**Fig. 6** Undesirable restoring force-displacement behavior caused by inevitable gab between actuator and loading frame

Scaled down model : $M_m \ddot{\delta}_m + C_m \dot{\delta}_m + K_m \delta_m = -M_m \ddot{z}_m$ (9)

Full-scale model : $M_a \ddot{\delta}_a + C_a \dot{\delta}_a + K_a \delta_a = -M_a \ddot{z}_a$ (10)

where

$\ddot{\delta}_m$ , $\ddot{\delta}_a$ : response accelerations of the specimen and the full-scale model, respectively

$\dot{\delta}_m$ , $\dot{\delta}_a$ : response velocities of the specimen and the full-scale model, respectively

$\delta_m$ , $\delta_a$ : response displacements of the specimen and the full-scale model, respectively

$\ddot{z}_m$ , $\ddot{z}_a$ : seismic accelerations applied to the specimen and the full-scale model, respectively

Firstly, by setting $t_a = S_1 t_m$ and $\delta_a = S_2 \delta_m$ , the relationship of $\dot{\delta}_a = (S_2/S_1)\dot{\delta}_m$ and $\ddot{\delta}_a = (S_2/S_1^2)\ddot{\delta}_m$ are derived according to the unit systems of $\delta_a$ [cm], $\dot{\delta}_a$ [cm/s] and $\ddot{\delta}_a$ [cm/s$^2$], where $S_1$ and $S_2$ are the rate of reduced scale for time and size, respectively. Then, the relationship of $F_{ra} = \delta_a K_a = S_2^2 F_{rm}$ can be derived between the restoring forces of the full-scale and the scaled down model, because the restoring forces are determined by the cross-sectional areas of the specimen and the full-scale model on the assumption that the materials of the specimen and the actual structure are the same. Furthermore , the equation for the flexural rigidity of columns between specimen and actual structure is $K_a = S_2 K_m$ . In addition, the equation $M_a = S_1^2 S_2 M_m$ can be derived by using the equation $M_a = F_{ra}/\ddot{\delta}_a$ . Besides, the equation $C_a = S_1 S_2 C_m$ can be also given by substituting $K_a$ and $M_a$ into the equation $C_p = 2h\sqrt{K_a M_a}$ .

Consequently, a set of relationships between specimen and actual structures are given as follows :

$$\dot{\delta}_a = \frac{S_2}{S_1}\dot{\delta}_m,\ \ddot{\delta}_a = \frac{S_2}{S_1^2}\ddot{\delta}_m,\ K_a = S_2 K_m,\ F_{ra} = S_2^2 F_{rm},\ M_a = S_1^2 S_2 M_m,\ \text{and}\ C_a = S_1 S_2 C_m \quad (11)_{a\sim f}$$

Secondly, the following differential equation can be derived by substituting Eq. (11) into Eq. (10).

$$S_1^2 S_2 M_m (S_2/S_1^2)\ddot{\delta}_m + S_1 S_2 C_m (S_2/S_1)\dot{\delta}_m + S_2 K_m S_2 \delta_m = -S_1^2 S_2 M_m \ddot{z}_a \quad (12)$$

Moreover, Eq. (13) can be obtained from dividing both the sides of this equation by $S_2^2$ .

$$M_m \ddot{\delta}_m + C_m \dot{\delta}_m + K_m \delta_m = -\frac{S_1^2}{S_2} M_m \ddot{z}_a = -M_m \ddot{z}_m \qquad (13)$$

Therefore, the seismic acceleration to be applied to the specimen and the equivalent mass to solve the differential equation of motion in the hybrid test are $S_1^2/S_2$ and $1/(S_1^2 S_2)$ times of those for the full-scale model, respectively.

## 4. Accuracy of Hybrid Testing Systems

### *4.1 Accuracy of Control by Actuator and Measurements*

*(1) Accuracy of Actuator*

Although the actuators of the capacity of 10tf and 50tf have the resolving function of $\pm 1/32{,}768$ for both the load and displacement in controlling them, they have only the resolving function of $\pm 1/2{,}048$ for both the load and displacement in measuring them. For example, the minimum load increment, $\Delta H_c$ of 0.3052kgf can be applied to a specimen, but the applied load is measured with the error of 4.883kgf in every load control range of 10tf. While, the minimum incremental displacement, $\Delta \delta_c$ to be applied is 0.00458mm. Then, the maximum error of measured displacement, $\Delta \delta_m$ is 0.0732mm in the control displacement range of 150mm.

*(2) Error of Load Measurement*

The actuators are able to control the incremental load of about 5kgf in the load control range of 10tf according to the resolving function of the actuators. Strictly speaking, the actuators can, however, control only the incremental load of above 75kgf for the existence of the mechanical error caused by opening and closing the servo valves.

*(3) Error of Displacement Measurement*

The actuators can control the incremental displacement of about 0.07mm in the displacement control range of 150mm according to the resolving function of the actuators. The actuators can, however, control only the incremental displacement of above 0.2mm for the same reason in case of the load measurement.

### *4.2 Verification of Hybrid Testing Systems*

A free vibration test and seismic response vibration test in the elastic region are carried out by the hybrid testing system through the adoption of actuator with capacity 10tf in order to verify the accuracy for our

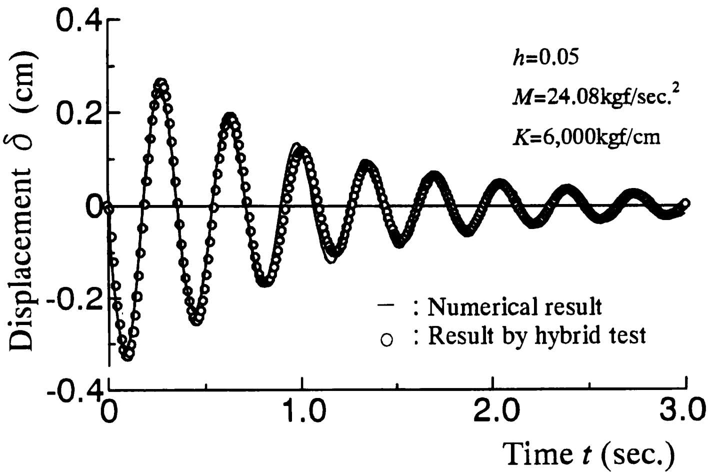



**Fig. 7** Result of free vibration test

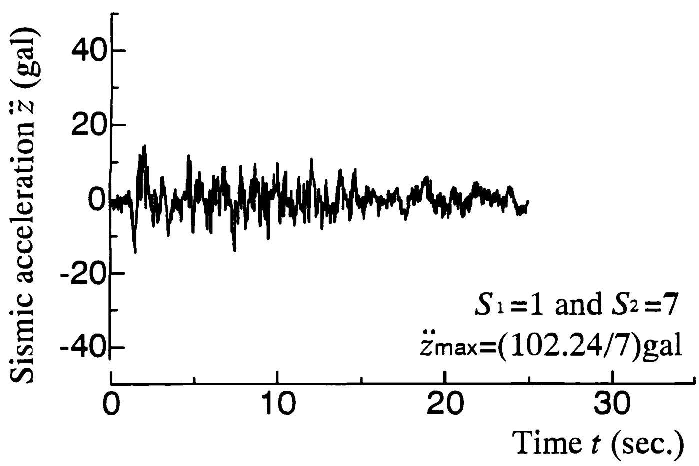



**Fig. 8** Time history of applied seismic acceleration for specimen

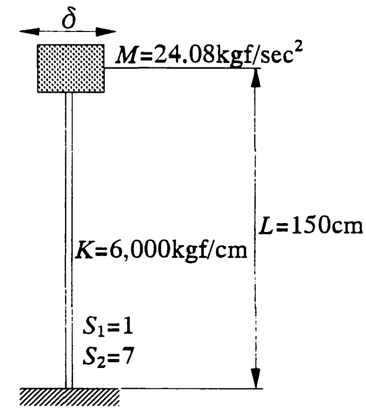



**Fig. 9** Test specimen

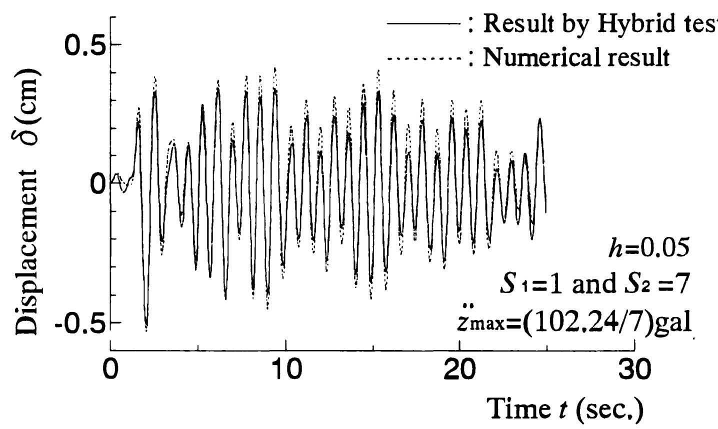



**Fig. 10** Time history of horizontal displacement response

hybrid testing system.

A result of the free vibration test is shown in **Fig. 7** in comparison with the numerical result by computer under the same conditions.

A seismic acceleration record shown in **Fig. 8** is applied to a specimen with the rate of reduced scale, $S_1=1$ and $S_2=7$, as is shown in **Fig. 9**, in the seismic response vibration test. The time history of the response for horizontal displacement, which is obtained by the hybrid test, is shown in **Fig. 10** together with the corresponding numerical result.

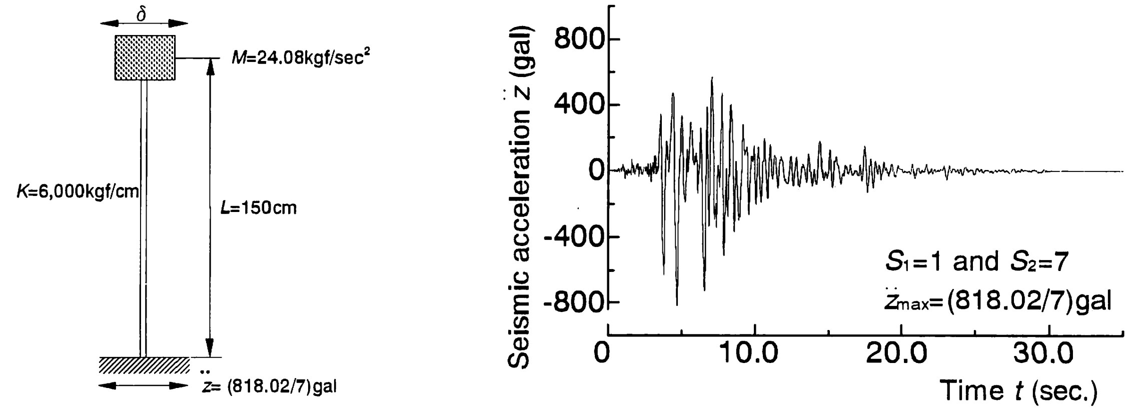
It can be seen from **Figs. 7** and **10** that the results by the hybrid test are in quite well agreement with the numerical results. Thus, the accuracy of our hybrid testing system can be verified through these comparisons of the test results with numerical ones.

### *4.3 Prediction of Viscous Dumping Constant*

For dynamic analyses and hybrid tests, it is an important factor how to decide viscous dumping constant, $C$ in the differential equation of motion of Eq. (2). The constant, $C$ is presently decided by either of the following two methods in the hybrid tests.

i) $C$ is decided through the following equation by using a specified and constant value of dumping coefficient, $h$ and the flexural rigidity of specimens, $K$ in elastic region, and it is also assumed to be constant in elasto-plastic region throughout a hybrid test.

$$C = 2h\sqrt{MK} \tag{14}$$




(a) Model for numerical analysis (b) Time history of applied seismic acceleration ( Record of the Great Hanshin-Awaji Earthquake )

**Fig. 11** Model for numerical analysis and applied seismic acceleration

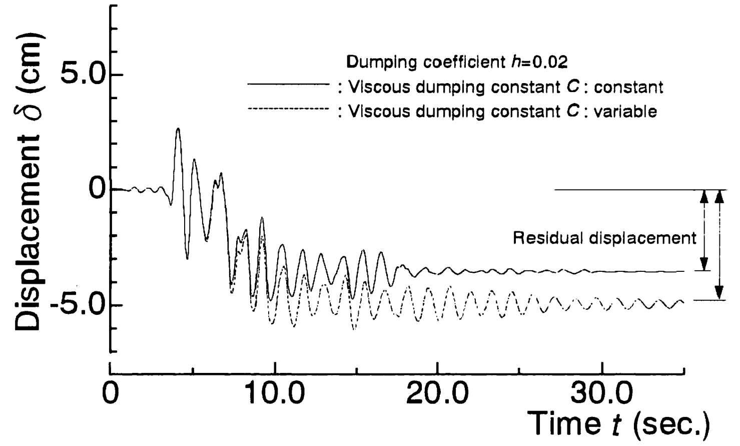



**Fig. 12** Time history of displacement response in case of damping coefficient $h$=0.02

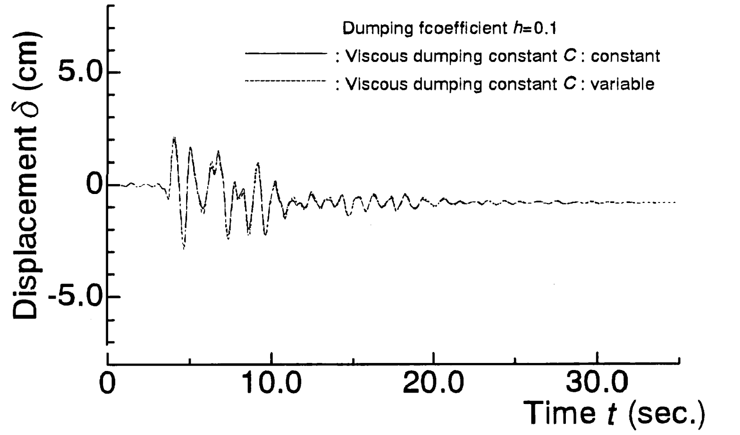



**Fig. 13** Time history for displacement response in case of damping coefficient $h$=0.1

ii) $C$ is calculated from Eq. (14) by using a specified and constant value of $h$ and variable, $K$ measured through the experimental process in a hybrid test. The value of $C$ is, therefore, variable throughout the hybrid test.

In our hybrid tests, this method is adopted to determine the value of $C$.

A vibration model with the ratio of reduced scale $S_2$=7, shown in **Fig. 11**, is numerically analyzed to examine the influence of the methods for deciding the value of $C$ by the seismic behavior of specimens. Numerical results are plotted in **Figs. 12** and **13** for $h$=0.02 and 0.1, respectively. It can be seen from these figures that the residual displacement in case of $h$=0.02 is fairly different by the methods for deciding the value of $C$, whereas the response displacements in case of $h$=0.1 is almost similar tendency to the numerical results by using constant and variable viscous damping constants.

Although the value of $C$ may be variable in the elasto-plastic region as mentioned above, this value is also dependent upon the vibration modes. It is necessary to inquire the validity of these predictions through the comparisons of numerical results with results through the dynamic tests by using a shaking table.

## 5. Conclusion

Two hybrid testing systems, which was respectively equipped the actuators of the capacity of 10tf and 50tf, were developed in Postgraduate School of Engineering, Osaka City University for simulating the seismic behaviors of bridge piers subjected to strong earthquakes alike the Great Hanshin-Awaji Earthquake and for investigating the ultimate strength and ductility of them. Outline of these systems, logical concept of hybrid tests, analogous rule and prediction of the viscous dumping coefficient have been predicated in this paper. These systems were verified through the comparison of the test results with the numerical results.

## Acknowledgments

The authors would like to express our appreciations to Mr. A. Kambayashi of Technical Laboratory of Takenaka Co., Ltd. who was formerly research associate of Osaka City University and Mr. M. Uemura and Mr. J. Matsuno of Saginomiya, Co., Ltd., as well as Mr. K. Bando of NTT Corp. who was formerly under graduate student of Osaka City University for his energetic supports in developing this hybrid testing system.

## References

1. Hakuno, M., Shidawara, M. and Hara, T. : Dynamic Destructive Test of a Cantilever Beam Controlled by an Analog-Computer, Proceedings of JSCE, No. 171, pp. 1-9, November 1969 (in Japanese).
2. Takanashi, K., Udagawa, K., Seki, M., Okada, Y. and Tanaka, H. : Non-linear Earthquake Response Analysis of Structures by a Computer-Actuator On-line System (Part 1, Details of System), Transactions of Architectural Institute of Japan, No. 229, pp. 77-83, March 1975 (in Japanese).
3. Yamada, Y. and Iemura, H. : Hybrid Analysis on Earthquake Response of Deteriorating Hysteretic Structures, Proceedings of Sino-American Symposium on Bridge and Structural Engineering, Part 1, pp. 4.13.1-14, September 1982.
4. Iemura, H. : Development and Future Prospect of Hybrid Experiments, Proceedings of JSCE, No. 356/I-3, pp. 1-10, April 1985 (in Japanese).
5. Niwa, Y., Watanabe, E. and Isami, H. : Automated Structural Testing Using Microcomputer System, Proceedings of JSCE, No. 332, pp. 145-158, April 1983.
6. Takanashi, K. and Nakashima, M. : Japanese Activities on On-Line Testing, Journal of Engineering Mechanics, ASCE, Vol. 113, No. 7, pp. 1014-1032, July 1987.
7. Nakai, H. and Kitada, T. : A System for Hybrid Tests of Materials and Structures, Memoirs of the Faculty of Engineering, Osaka City University, Vol. 31, pp. 181-190, December 1990.
8. Nakai, H., Kitada, T., Yoshikawa, O., Nakanishi, K. and Oyama, T. : Experimental Study on Ultimate Strength and Ductility of Concrete Filled Steel Box Columns, Journal of Structural Engineering, JSCE, Vol. 39A, pp. 1347-1360, March 1993 (in Japanese).
9. Nakai, H., Kitada, T., Nakanishi, K., Sugiyama, I. and Kouno, Y. : Experimental Study on Ultimate Strength and Ductility of Concrete Filled Thin-Walled Steel Box Columns after Receiving Seismic Loading, Journal of

Structural Engineering, JSCE, Vol. 39A, pp. 1347-1360, March 1993 (in Japanese).
10. Nakanishi, K., Kitada, T. and Nakai H. : Experimental Study on Deterioration of Ultimate Strength and Ductility of Damaged Concrete Filled Steel Box Columns, Proceedings of ASCCS International Conference, Vol. 4, pp. 127-130, Kosice, Slovakia, June 1994.
11. Nakai, H., Kitada, T. and Nakanishi, K. : Experimental Study on Ultimate Strength and Ductility of Steel and Composite Bridge Piers under Gradually Increasing and Cyclic Load, Proceedings of JSCE, No. 513/I-1, pp. 89-100, March 1995 (in Japanese).
12. Komatsu, S. and Nakai, H. : Application of the Analogue Computer to the Analysis of Dynamical Response of Curved Girder Bridge, Proceedings of JSCE, No. 178, pp. 11-26, June 1970( in Japanese ).