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Extracting Three-dimensional Information with Complex Amplitude
-Pattern Matching using a Holographic Method-

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Synopsis

We worked on a method for extracting three-dimensional information from holographic interference fringe patterns by using an interferometer. So far, we identified the matching method by using the sum of products and the method that used an optical correlation system, we made a lot of masks for the depth direction and had to exchange them. In the work described in this paper, we used only one mask as complex amplitude distribution instead of the amplitude distribution. Here we present the method that extracts three-dimensional information by sifting the plane of observation phase information data with the improved optical correlation system.

KEYWORDS: holography, three dimensional information extract, optical correlation

Introduction

Recently three-dimensional computer graphics and virtual reality have been studied, and we expect much from three dimensional display. As especially animation developed using holography, which can display natural three dimensional images, three dimensional input and measure technique became more necessary[1]. So, we supposed an optical method for extracting three dimensional information of an object surface using a CCD with holography, which SHIRAI supposed in ultrasound[2].

So far, we identified the matching method by using the sum of products and the method that used an optical correlation system, we made a lot of masks for the depth direction and had to exchange them. In the work described in this paper, we used only one mask as complex amplitude distribution instead of the amplitude distribution.

Here, we present the method that extracts three-dimensional information by sifting the plane of observation phase information data with the improved optical correlation system.

We can collect based patterns to only one by shifting observation data plane with complex amplitude pattern matching. Then, the many memory become needless and the processing time is shortened. Therefore, we can correct the weak point of the former method.

The principle of extracting three-dimensional information

The optical layout for measuring three dimensional information is shown in Figure 1. First, the plane wave incident the object in the observation space. The phase of the object beam and the one of the reference beam interfere as the surface of the object is the point source. On the CCD camera, the light intensity is taken as three dimensional information. On Figure 1, the complex amplitude distribution of the object O(m,n) is;

$$O(m,n) = U(m,n)\exp\{j\phi_o(m,n)\} \tag{1}$$

where m,n is coordinates, U(m,n) is amplitude on the CCD and ϕ o(m,n) is the phase distribution of the point sources. A is the amplitude of the reference beam form the point sources. r(m,n) is the distance from the point sources to the coordinate on the CCD. λ is wave length. Then, U(m,n) is expressed as U(m,n)=A/r(m,n) and ϕ_o(m,n) is done as $\phi_o(m,n)=2\pi r(m,n)/\lambda$. The complex amplitude distribution of the reference beam R(m,n) is;

$$R(m,n) = R_0\exp\{j\phi_R\} \tag{2}$$

where R_0 is the amplitude of the reference beam, ϕ_R is the phase of the reference beam.
A base pattern for detecting three dimensional information S(m,n) is;

$$\begin{aligned} S(m,n) &= I(m,n) - |O(m,n)|^2 - |R(m,n)|^2 \\ &= 2U(m,n)R_o\cos\{\phi_R - \phi_o(m,n)\} \end{aligned} \tag{3}$$

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where l(m,n) is light intensity of the observed interference fringe pattern on CCD except for the intensity of object beam and the one of reference beam.

Next step, we suppose that the surface of the object is the aggregation of the N point sources. The complex amplitude distribution of the object beam is getting by Huygens – Fresnel principle;

$$O(m,n) = \sum_{i=1}^{N} U(i,m,n)\exp\{j\phi_o(i,m,n)\} \quad (4)$$

where U_i, A_i, r_i, ϕ_{oi} is the variable of the each point sources.

The observed data (hologram information) of the object in any place is;

$$\begin{aligned} g(m,n) &= I(m,n) - |O(m,n)|^2 - |R(m,n)|^2 \\ &= 2U(m,n)\, R_o \sum_{i=1}^{N} \cos\{\phi_R - \phi_o(i,m,n)\} \end{aligned} \quad (5)$$

The equation(5) is changed by equation(3) as follow;

$$\begin{aligned} g(m,n) &= 2U(m,n)\, R_o \sum_{i=1}^{N} \cos\{\phi_R - \phi_o(i,m,n)\} \\ &= \sum_{i=1}^{N} S(i,m,n) \end{aligned} \quad (6)$$

We have detected thee dimensional information by the correlation between this observed data and any based pattern.

We propose the method that is to get the three dimensional information by sifting the based pattern of the complex amplitude distribution.

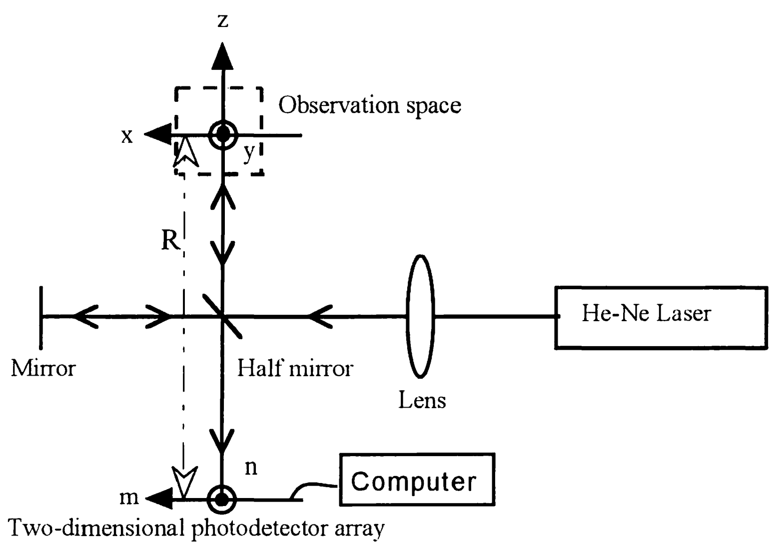

Fig. 1 Optical layout for measureing three dimensional information.

The method of detecting three dimensional information with the complex amplitude distribution pattern

1. The pattern matching with the complex amplitude based pattern

This section shows that the complex amplitude distribution pattern matching as well as the amplitude distribution pattern matching is able to get the three dimensional information.

The observed data on CCD is given by equation(5). This equation is the sum of the phase distribution of the object beam include the constant phase ϕ_R and the conjugate phase distribution. If ϕ_R is zero;

$$g(m,n) = U(m,n)\, R_o \sum_{i=1}^{N} [\exp\{j\phi_o(i,m,n)\} + \exp\{-j\phi_o(i,m,n)\}] \quad (7).$$

On the old method, this observed data was two dimensional amplitude distribution. So, the based pattern was the amplitude distribution as shown equation (3) and we have matched by the amplitude distribution. However, the data of equation(7) include phase information as the complex amplitude distribution. Then, we have used the phased distribution $S_c(l,m,n)$ as shown equation(8) which is expressed for the complex amplitude distribution.

$$S_c(i,m,n) = U(i,m,n)\exp\{j\phi_o(i,m,n)\} \quad (8)$$

We take the thee dimensional information by the complex amplitude pattern matching between the based pattern and the observed data as shown equation(7). In the case of the complex amplitude pattern matching, the absolute value of the complex amplitude increase in the same complex amplitude pattern. So, we evaluate the absolute value as the distribution of the absolute value or light intensity.

2. Optical correlation processing with the complex amplitude based pattern

This section show that we have sifted the objective data without changing the based pattern on the optical correlation system, and get the correlation image by the based pattern of each z coordinates.

The optical correlation system for measuring three dimensional information is shown in Figure 2. On creating the Fourier transformation hologram of based patterns, a based pattern set on lens L1 of the font focus point. Then, the Fourier transformation hologram is made on the P2 plane by the reference beam from b which set on the P1 plane. Also, on the correlation operating, the made hologram is set on the P2 plane. Then, the objective shift data is set on the front focus point. We have tried to set it on the distance d from the lens L1.

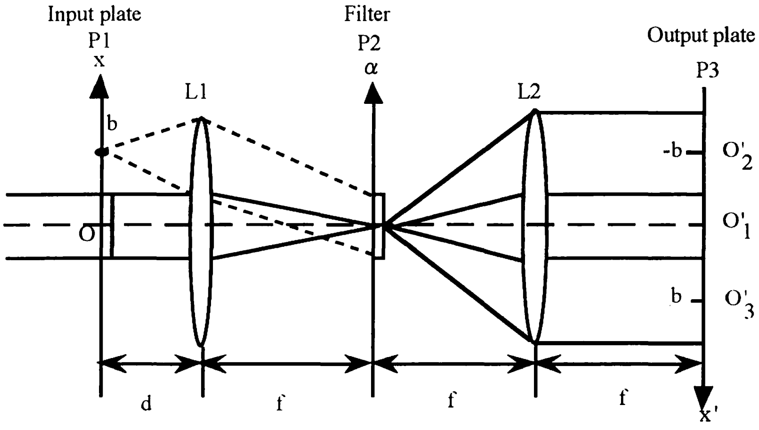

Fig.2 Optical correlation system for measureing three dimensional information.

Then, we think that we take the correlation between a complex amplitude based pattern and an objective dataAn amplitude transmission distribution $T_A(\alpha,\beta)$ of the based Fourier transformation on the P2 plane is ;

$$
\begin{aligned}
T_A(\alpha,\beta) = &\left\{ \left| S_c\left(\frac{\alpha}{\lambda f},\frac{\beta}{\lambda f}\right) \right|^2 + R_{AO}^2 \right\} \\
&+ S_c\left(\frac{\alpha}{\lambda f},\frac{\beta}{\lambda f}\right) \cdot R_{AO} \exp\left\{ 2\pi j b \frac{\alpha}{\lambda f} \right\} \\
&+ S_c^*\left(\frac{\alpha}{\lambda f},\frac{\beta}{\lambda f}\right) \cdot R_{AO} \exp\left\{ -2\pi j b \frac{\alpha}{\lambda f} \right\}
\end{aligned} \tag{9}
$$

where * is complex conjugation, (α,β) is the coordinates on the P2 plane, λ is the wavelength, f is the distance of focus point, $S_c(\alpha/\lambda f, \beta/\lambda f)$ is the Fourier transformation of $s_c(x,y)$, R_{AO} is an amplitude of a reference beam on the P2 plane. The b is x coordinates of the reference beam on creating the Fourier transformation hologram.

Therefore, the light intensity distribution $E(\alpha,\beta)$ through the P2 plane is ;

$$
\begin{aligned}
E(\alpha,\beta) &= G_A\left(\frac{\alpha}{\lambda f},\frac{\beta}{\lambda f}\right) \bullet T_A(\alpha,\beta) \\
&= G_A\left(\frac{\alpha}{\lambda f},\frac{\beta}{\lambda f}\right) \bullet \left\{ \left| Sc\left(\frac{\alpha}{\lambda f},\frac{\beta}{\lambda f}\right) \right|^2 + R_{AO}^2 \right\} \\
&+ G_A\left(\frac{\alpha}{\lambda f},\frac{\beta}{\lambda f}\right) \bullet Sc\left(\frac{\alpha}{\lambda f},\frac{\beta}{\lambda f}\right) \bullet R_{AO} \exp\left\{ 2\pi j b \frac{\alpha}{\lambda f} \right\} \\
&+ G_A\left(\frac{\alpha}{\lambda f},\frac{\beta}{\lambda f}\right) \bullet Sc^*\left(\frac{\alpha}{\lambda f},\frac{\beta}{\lambda f}\right) \bullet R_{AO} \exp\left\{ -2\pi j b \frac{\alpha}{\lambda f} \right\}
\end{aligned} \tag{10}
$$

where $G_A(\alpha/\lambda f, \beta/\lambda f)$ is the Fourier transformation of $g_A(x,y)$.

Then, the mutual correlation image between an objective data and a based pattern is taken by the third term of the equation (10). Therefore, we think about only third term of the equation (10).

$$E_3(\alpha,\beta) = G_A\left(\frac{\alpha}{\lambda f},\frac{\beta}{\lambda f}\right) \bullet S_C^*\left(\frac{\alpha}{\lambda f},\frac{\beta}{\lambda f}\right) \bullet R_{AO}\exp\left\{-2\pi jb\frac{\alpha}{\lambda f}\right\} \qquad (11)$$

When the objective data is set on the P1 plane which is the distance d from lens L1, the light intensity $G'_d(\alpha,\beta)$ on P2 plane is;

$$\begin{aligned} G'_d(\alpha,\beta) &= \exp\left[jk\left(\frac{\alpha^2+\beta^2}{2f}\right)\left(1-\frac{d}{f}\right)\right] \bullet \mathrm{FT}\{g_A(x,y)\} \\ &= \exp\left[jk\left(\frac{\alpha^2+\beta^2}{2f}\right)\left(1-\frac{d}{f}\right)\right] \bullet G_A\left(\frac{\alpha}{\lambda f},\frac{\beta}{\lambda f}\right) \end{aligned} \qquad (12)$$

where F T is Fourier transformation operator, and $k = 2\pi/\lambda$.

R is the distance from the point of origin to objective plane. The based pattern as $S_{c,z=0}$ is a complex amplitude distribution at Z=0. This distribution is equal to the phase distribution which is the distance R from the point sources. Therefore, this distribution $S'_{c,z=0}(\alpha,\beta)$ on P2 plane is;

$$\begin{aligned} S'_{C,Z=0}(\alpha,\beta) &= \exp\left[jk\left(\frac{\alpha^2+\beta^2}{2f}\right)\left(1-\frac{R+f}{f}\right)\right] \bullet FT\{\delta(x,y)\} \\ &= \exp\left[jk\left(\frac{\alpha^2+\beta^2}{2f}\right)\left(-\frac{R}{f}\right)\right] \end{aligned} \qquad (13)$$

This equation is equal to Fourier transformation $S'_{c,z=0}(\alpha/\lambda f, \beta/\lambda f)$ of $S_{c,z=0}$. This recorded distribution as the based pattern is set on P2 plane. Then, on setting the observed data $g_A(x,y)$ at the distance d from lens L1, the light intensity $E_3(\alpha,\beta)$ is given by equation (10);

$$\begin{aligned} E_3(\alpha,\beta) &= G'_d(\alpha,\beta) \cdot \{S'_{C,Z=0}(\alpha,\beta)\}^* \cdot R_{A0}\exp\left\{-2\pi jb\frac{\alpha}{\lambda f}\right\} \\ &= \exp\left[jk\left(\frac{\alpha^2+\beta^2}{2f}\right)\left(\frac{R+(f-d)}{f}\right)\right] \cdot G_A\left(\frac{\alpha}{\lambda f},\frac{\alpha}{\lambda f}\right) \\ &\quad \cdot R_{A0}\exp\left\{-2\pi jb\frac{\alpha}{\lambda f}\right\} \end{aligned} \qquad (14)$$

On setting the incident plane P1 shown as Figure 3 (d=f), the first term of equation (14) is equal to a complex amplitude distribution of the equation (13) as the equation (15). Therefore, the mutual correlation image as shown equation (15) is given by correlation between the based pattern $s_{c,z=0}(x,y)$ and the objective data $g_A(x,y)$.

$$\begin{aligned} E_{3,d=f}(\alpha,\beta) &= \exp\left[jk\left(\frac{\alpha^2+\beta^2}{2f}\right)\left(\frac{R}{f}\right)\right] \cdot G_A\left(\frac{\alpha}{\lambda f},\frac{\beta}{\lambda f}\right) \cdot R_{A0}\exp\left[-2\pi jb\frac{\alpha}{\lambda f}\right] \\ &= \{S'_{C,Z=0}(\alpha,\beta)\}^* \cdot G_A\left(\frac{\alpha}{\lambda f},\frac{\beta}{\lambda f}\right) \cdot R_{A0}\exp\left[-2\pi jb\frac{\alpha}{\lambda f}\right] \end{aligned} \qquad (15)$$

Also, on the case of setting the incident plane P1 shown as Figure 3 (d = f - a), the given distribution is equal to a complex amplitude distribution at Z=a. Therefore, the mutual correlation image as shown equation (16) is given by correlation between the based pattern $s_{c,z=a}(x,y)$ and the objective data $g_A(x,y)$.

$$\begin{aligned} E_{3,d=f}(\alpha,\beta) &= \exp\left[jk\left(\frac{\alpha^2+\beta^2}{2f}\right)\left(\frac{R+a}{f}\right)\right] \cdot G_A\left(\frac{\alpha}{\lambda f},\frac{\beta}{\lambda f}\right) \cdot R_{A0}\exp\left[-2\pi jb\frac{\alpha}{\lambda f}\right] \\ &= \{S'_{C,Z=a}(\alpha,\beta)\}^* \cdot G_A\left(\frac{\alpha}{\lambda f},\frac{\beta}{\lambda f}\right) \cdot R_{A0}\exp\left[-2\pi jb\frac{\alpha}{\lambda f}\right] \end{aligned} \qquad (16)$$

As mentioned above, we have taken the mutual correlation between the based pattern at Z coordinates and the objective data by sifting the incident plane P1. The advantage of this method is the needless to change a based pattern at each Z coordinates, that is to use only one based pattern (Z=0). So, this system is the high utility factor to cut down for making a based pattern. Furthermore, this system is able to reduce a lot of memory for recording based patterns. Therefore, the depth resolution is improved.

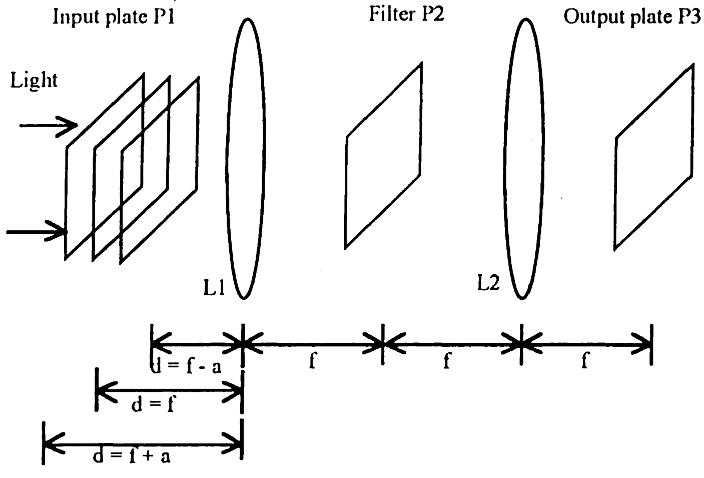

Fig.3 Slide of input plate P1.

Simulation

This section shows the simulation as shown Figure 2. We calculated by giving variation of the value of a. Figure 4 shows the object that is A(2,0,-5), O(0,0,0), B(-2,0,5).
We used the objective data (512X512) which is made in the calculator. The only one based pattern is the complex amplitude distribution at Z=0. The result shows in Figure 5. The normalized light intensity of each points is recognized by the maximum value of the correlation result.

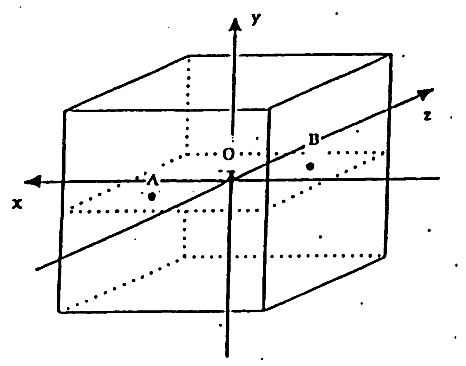

Fig.4 Arrangement of point sources.

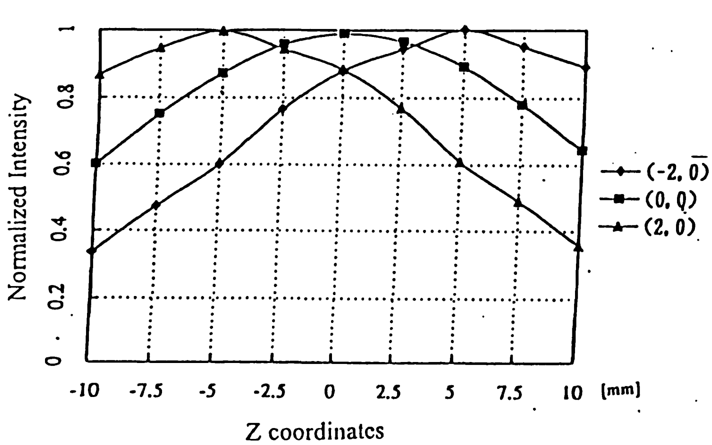

Fig.5 The result of calculator simulation.

Optical simulation with complex amplitude pattern matching

This section shows an optical simulation to evaluate optical correlation system.

The optical measuring system is as shown in Figure 6. We have used He-Ne Laser and lens L1, L2 which focus on 25[cm]. In the case of making a based Fourier transformation hologram, The point source of the reference is made by a pinhole B(PH.B) and composed by a half mirror (HM). The gap between the original point and the reference beam on P1 plane is made by moving a HM in parallel as shown in Figure 6.

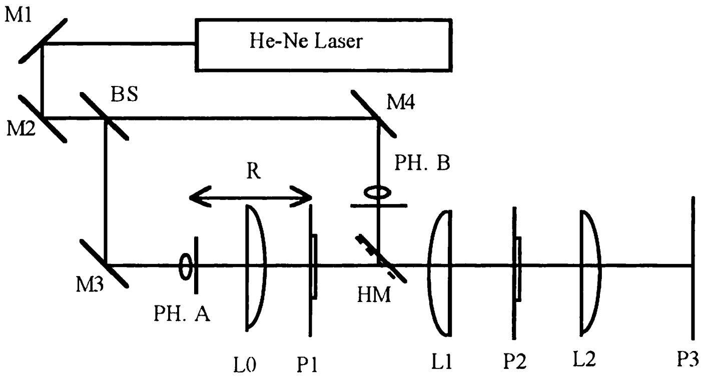

Fig.6 Optical measuring system.

Thus, the incident plane P1 is set with inputting directly the wave front from the point source (PH.A) as the complex amplitude distribution based pattern. The point source is set the locate which is distance R from the P1 plane. Then, the wave front of the point source reach the P1 plane without lens L0. This wave front is an object beam. The Fourier transformation hologram of the based pattern is made by the reference beam of the pinhole (PH.B) and the object beam.

In the case of operate a correlation, The Fourier transformation hologram of the based pattern set on the P2 plane by cutting the reference beam and the objective data set on the P1 plane. Then, lens L0 set to incident a parallel beam on the P1 plane. The result on P3 plane of a correlation between the based pattern each Z coordinates and the objective data on P1 plane is taken by sifting the P1 plane. Then, the three dimensional information of the object is taken by getting the light intensity distribution with CCD camera as shown in Table 1. Also, the pixcel pitch of the object image data shows in Table 2.

Table 1 Specification of CCD camera.

CCD CAMERA	(XC-77 SONY)
Pixcel	768(H)×493(V)
Area	8.8(mm)×6.6(mm)
Translate method	Inter line method
Cell size	11 (μm) ×13 (μm)
Chip size	10.0 (μm) ×8.2 (μm)

Table 2 Specification of flame memory

horizontal	1.33×10^{-2}(mm)
vertical	1.34×10^{-2}(mm)

We have used the object data which is used in the above calculator simulation as three point sources A(2,0,-5), O(0,0,0) and B(-2,0,5). Figure 7, 8 shows the result. Figure 7 shows the change of the light intensity distribution on (2,0), (0,0) and (-2,0). The light intensity distribution is normalized by the maximum value of the given data. The pitch of Z coordinates is equal to the one of sifting (in this case; 2.5[mm]). On the Z coordinates which the point sources exist, the normalized light intensity each Z coordinates take the maximum value. Figure 8 shows the intensity of X coordinate. The light intensity distribution is normalized by the maximum value of the given data. The spread of the point at peak value is 16 pixcels [0.21mm].Therefore, we can detect the three dimensional information by the above methods which is sifting the object data.

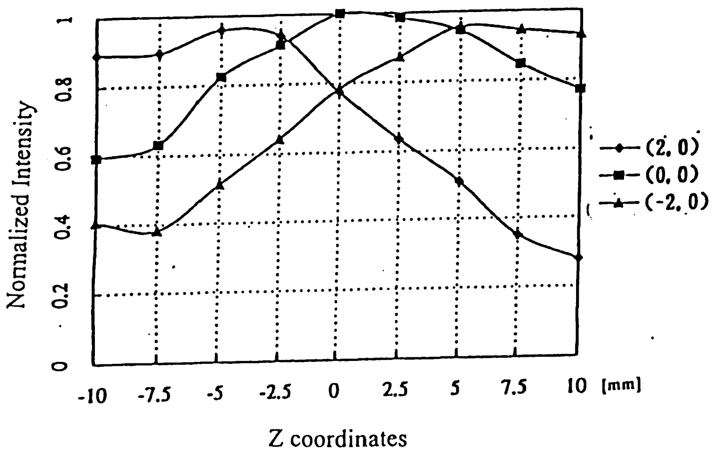

Fig.7 The result of optical detection simulation (1).

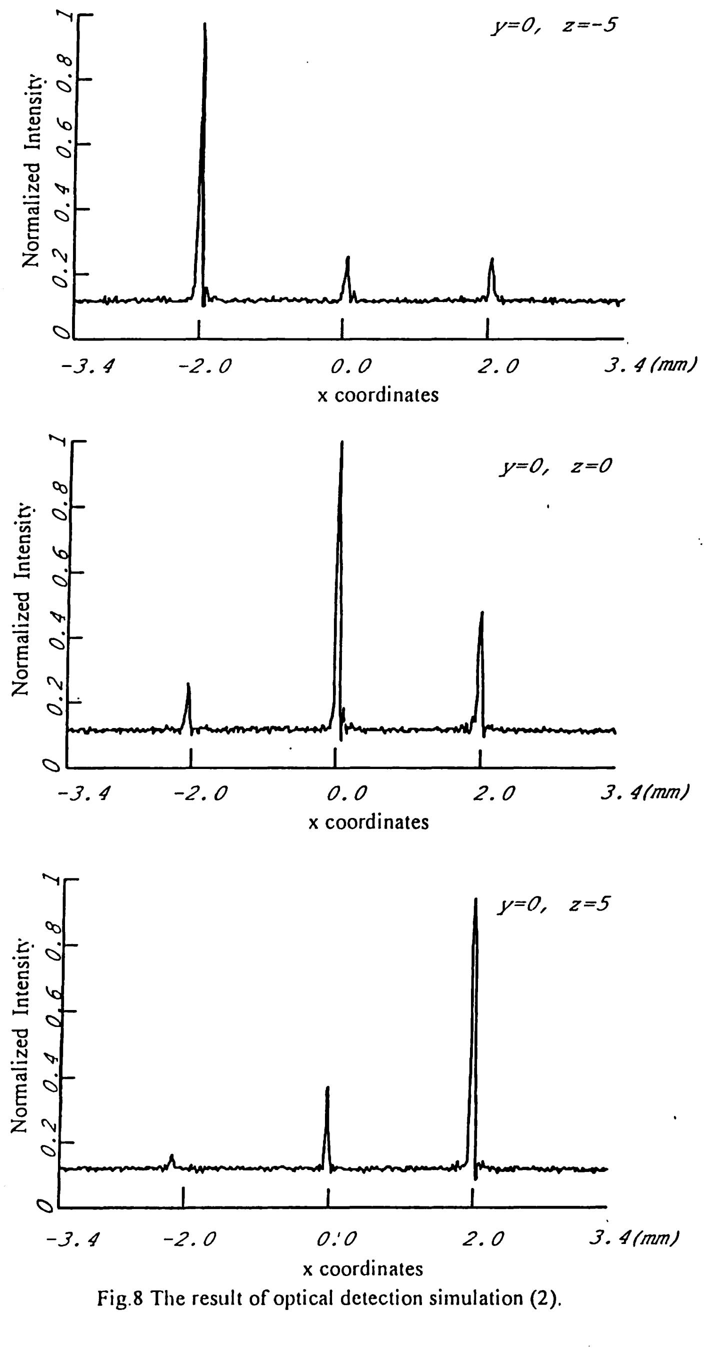

Fig.8 The result of optical detection simulation (2).

conclusion

In this paper, we propose the method that extracts three-dimensional information by sifting the plane of observation phase information data with the improved optical correlation system.

We can collect based patterns to only one by shifting observation data plane with complex amplitude pattern matching. Then, the many memory become needless and the processing time is shortened. Therefore, we can correct the weak point of the former method.

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