

## A study on the precision of the depth measurement with view directions of human eyes

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### Synopsis

Selecting an object in a 3-dimensional space is more useful for a human interface. The depth is easily calculated by stereography with view directions of both eyes. However, human eyes include the vibration. We discussed the affect of the small vibration of human eyes to the depth measurement theoretically and estimated its quantity with the measured value of the small vibration in the experiment. From the experiment result, it is pointed out that the neighboring objects on the depth of 300(mm) are good discriminative, though the neighboring objects on the depth of 3000(mm) are not sufficiently discriminative.

**KEYWORDS:** depth measurement, stereography, human gazing, human interface

### Introduction

Human's glance is one of the desirable interfaces of the computerized systems. Some systems using the glance as human interfaces have been studied recently, on which the glance is used for directly selecting marks or words on the display of the computer<sup>(1,2,3,4)</sup>. There are many advantages in this way because the human does not use his hand troublesomely and give his attention to one of some selective items easily with his eyes. Furthermore, we think that selecting an object in a 3-dimensional space is more useful for a human interface. Thus, the depth to the object can be utilized as the additional information fed to the system. The depth is easily calculated by stereography with view directions of both eyes. Nevertheless, the movement of human eyes includes the small vibration as its special quality, which might affect the precision of the depth measurement with view directions of both eyes. In this paper, we discuss the affection of the small vibration to the precision of the depth measurement and show its estimated values in the experiment.

### Method

#### Principal of stereography

The principal of the depth measurement on stereography with view directions is shown in Fig. 1. The view direction angles of the left eye ( $=\theta_\ell$ ) and the right eye ( $=\theta_r$ ) are used for the calculation of the depth ( $=H$ ) to the object p. Assume that the distance between the left and right eyes is  $2 \cdot L$ , the distance between the left eye and the perpendicular point (h) is  $L_\ell$ , and the distance between the right eye and the point h is  $L_r$ .

$$2 \cdot L = L_\ell + L_r \quad , \quad (1)$$

$$H = L_\ell \cdot \tan(\theta_\ell) = L_r \cdot \tan(\theta_r). \quad (2)$$

From Eq. (1) and Eq. (2),  $H$  is expressed with  $\theta_\ell$  and  $\theta_r$  as follows:

$$H = \frac{2 \cdot L}{\frac{1}{\tan(\theta_\ell)} + \frac{1}{\tan(\theta_r)}}. \quad (3)$$

Then, the distance  $L_\ell$  is obtained as

$$L_\ell = \frac{2 \cdot L \cdot \tan(\theta_r)}{\tan(\theta_\ell) + \tan(\theta_r)}. \quad (4)$$

The distance  $D$  from the central point of the both eyes to the point h is obtained as follows:

$$\begin{aligned}
 D &= L_\ell - L \\
 &= L \cdot \frac{\tan(\theta_r) - \tan(\theta_\ell)}{\tan(\theta_r) + \tan(\theta_\ell)}.
 \end{aligned} \tag{5}$$

As the result, we obtain the equations for the depth of the object as Eq. (3) and its position on leftward or rightward as Eq. (5), using  $L$ ,  $\theta_r$  and  $\theta_\ell$ .

#### Equation with coordinates instead of angles.

As Eq. (3) and Eq. (5) include a trigonometric function ( $\tan$ ), it may be difficult to analyze the affection of the vibration. On the other hand, we use the eye measurement system that offers view direction as coordinate values. From the both reasons, we arrange Eq. (3) and Eq. (5) to be expressed with coordinates instead of angles. The Fig. 2 shows the relation between the angles and the coordinates. Then, the next equation is obtained.

$$\tan(\theta_\ell) = \frac{U}{X_\ell}, \tag{6}$$

$$\tan(\theta_r) = -\frac{U}{X_r}. \tag{7}$$

Where the  $U$  means the distance from eyes to the coordinate line M in the Fig. 2.

Similarly to Eq. (3), the next equation is obtained from Eq. (6) and Eq. (7).

$$H = \frac{2 \cdot L \cdot U}{X_\ell - X_r}. \tag{8}$$

In the same way, the next equation is obtained instead of (5).

$$D = L \cdot \frac{X_\ell + X_r}{X_\ell - X_r}. \tag{9}$$

Therefore, from Eq. (8) and Eq. (9), we can calculate the depth and the position with the coordinates that are confirmed from the experimental system.

#### Affects of small vibration parallel to both eyes

We introduce the variables  $A$  and  $B$  instead of  $X$  and  $-X_r$ , respectively. Furthermore, we use the variables  $a$  ( $>0$ ) and  $b$  ( $>0$ ) as the absolute values of the small vibration of the left and the right eyes, respectively. From Eq. (8) and Eq. (9), we obtain the next equation with using  $A$  and  $B$ .

$$H = \frac{2 \cdot L \cdot U}{A + B}, \tag{10}$$

$$D = L \cdot \frac{A - B}{A + B}. \tag{11}$$

The max discrepancy  $d_\ell$  of left is obtained when selecting  $(A - a)$  and  $(B + b)$  instead of  $A$  and  $B$ , respectively, as shown in Fig. 3(a).

$$\begin{aligned}
 d_\ell &= L \cdot \frac{A - B}{A + B} - L \cdot \frac{A - a - B - b}{A - a + B + b} \\
 &= 2 \cdot L \cdot \frac{A \cdot b + B \cdot a}{(A + B) \cdot (A + B - a + b)}
 \end{aligned}$$

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$$= \left( \frac{H}{U} \right) \cdot \frac{A \cdot b + B \cdot a}{A + B - a + b} \quad (12)$$

We assume that  $a$  is equal to  $b$  and replace them by  $\delta$  ( $a=b=\delta$ ). Then the next equation is obtained.

$$d_\ell = \frac{H \cdot \delta}{U} \quad (13)$$

Furthermore, the depth is not varied when  $a=b$ .

The max discrepancy  $d_r$  of right is obtained when selecting  $(A + a)$  and  $(B - b)$  instead of  $A$  and  $B$ , respectively, as shown in the Fig. 3(b). In the same way, the next equation is obtained when  $a$  is equal to  $b$  ( $a=b=\delta$ ).

$$d_r = \frac{H \cdot \delta}{U} \quad (14)$$

Because the discrepancy  $d_r$  is equal to the discrepancy  $d_\ell$ , we define the value  $d$  as  $d_\ell$  or  $d_r$  ( $d_\ell=d_r=d$ ). Therefore, we obtain the next equation.

$$d = \frac{H \cdot \delta}{U} \quad (15)$$

As the result, the equation for calculating the discrepancy of left or right is obtained.

#### Affects of small vibration parallel to depth direction

Next, we consider the affect of the small vibration parallel to the depth direction. The max discrepancy  $d_-$  on this side is obtained when selecting  $(A + a)$  and  $(B + b)$  instead of  $A$  and  $B$ , respectively, as shown in the Fig. 4(a). From Eq. (10), the discrepancy  $d_-$  is expressed as follows:

$$\begin{aligned} d_- &= \frac{2 \cdot L \cdot U}{A + B} - \frac{2 \cdot L \cdot U}{A + a + B + b} \\ &= \frac{2 \cdot L \cdot U \cdot (a + b)}{(A + B) \cdot (A + a + B + b)} \end{aligned} \quad (16)$$

From Eq. (10) and Eq. (16), the next equation is obtained by deleting the term  $(A + B)$ .

$$d_- = H \cdot \frac{a + b}{\frac{2 \cdot L \cdot U}{H} + a + b} \quad (17)$$

Then, we assume that  $a$  is equal to  $b$  and replace them by  $\delta$  ( $a=b=\delta$ ). The next equation is obtained from Eq. (17).

$$\begin{aligned} d_- &= H \cdot \frac{2 \cdot \delta}{\frac{2 \cdot L \cdot U}{H} + 2 \cdot \delta} \\ &= H \cdot \frac{1}{\frac{L \cdot U}{H \cdot \delta} + 1} \end{aligned} \quad (18)$$

From Eq. (15) and Eq. (18), the next equation is obtained.

$$d_- = H \cdot \frac{1}{\frac{L}{d} + 1} \quad (19)$$

On this case, the position of left or right may move though the depth might not vary.

The max discrepancy  $d_+$  on other side is obtained when selecting ( $A - a$ ) and ( $B - b$ ) instead of  $A$  and  $B$ , respectively, as shown in the Fig. 4(b). In the same way, the discrepancy  $d_+$  is expressed as follows:

$$d_+ = H \cdot \frac{1}{\frac{L}{d} - 1} \quad (20)$$

The Eq. (19) and Eq. (20) mean that the discrepancy for the depth is calculated with two of the depth of the object and the discrepancy of left or right.

## Experiment

### System parameters

In this experiment, we used the eye movement measurement system (made by TAKEI KIKI co.). In this system, the measurement value is obtained on the coordinates. We assume that the view direction angle of  $10^\circ$  is corresponding to the value  $K$  in the system. The value  $K$  was already estimated, and the value from about 1000 to about 1100 was obtained as  $K$  in the preliminary experiment. Assume that the symbol  $u$  is the unit length of the coordinate in the system.

$$\frac{u \cdot K}{U} = \tan 10^\circ . \quad (21)$$

Then, assume that the symbol  $\delta_v$  means the small vibration value using the unit length.

$$\delta = u \cdot \delta_v . \quad (22)$$

Accordingly, the  $\delta/U$  included in Eq. (15) is obtained as follows:

$$\frac{\delta}{U} = \frac{\delta_v}{K} \cdot \tan 10^\circ . \quad (23)$$

So, the discrepancy of left or right is obtained from (15).

$$d = \frac{H \cdot \delta_v}{K} \cdot \tan 10^\circ . \quad (24)$$

### Calculation of the small vibration value measured by the experimental system

The quantity of the small vibration of three subjects was estimated by the experimental system. The measured values showing the view direction were obtained at 30 times for each second. The standard deviations (SD) in every 30 points are averaged in the measurement data. As the result, we obtained that the averaged SD of all three subjects is about 10.

Assume that  $\delta_v$  in Eq. (23) is set to the averaged SD and  $K$  is set to 1100. Then, the value of Eq. (23) is about 0.00169. From the Eq. (24), if  $H$  is set to 1000(mm), the following value is obtained.

$$d = 1.7 \text{ (mm)}.$$

From Eq. (18) and Eq. (19), the next values are obtained.

$$\begin{aligned} d_- &= 50 \text{ (mm)}, \\ d_+ &= 55 \text{ (mm)}. \end{aligned}$$

The Table 1 shows  $d$ ,  $d_-$  and  $d_+$  for several values of the small vibration ( $\delta_V$ ) at several depths ( $H$ ). The Fig. 5 shows  $d$ ,  $d_-$  and  $d_+$  versus the depth  $H$  when  $\delta_V = 1*SD$ . As a whole, the  $d$  is obviously proportional to  $H$ . The variation of  $d_-$  is bigger than the variation of  $d$ , and the variation of  $d_+$  is the biggest.

The depth of 300(mm) means that an object is within a reach, for example, such as a book on the desk. On selecting 1\*SD as  $\delta_V$ , the precision on the horizon and the depth are shown as about 0.5(mm) and about 5(mm), respectively. We think that the values mean discriminative distance between small objects neighboring to each other in the sense of standard deviation of static.

The depth of 3000(mm) is very deeper than the distance between the left and right eyes (about 65mm). Therefore, the precision of the measured depth is not sufficiently at all. Even on selecting 1\*SD as  $\delta_V$ , the precision on the horizon and the depth are shown as about 50(mm) and about 410-560(mm), respectively. We think that the error of the depth could not be acceptable.

Especially, we point out that other objects in the same area defined with the depth to the object and the three discrepancies ( $d$ ,  $d_-$  and  $d_+$ ) are not sufficiently discriminated in the sense of statistics analysis.

**Result**

We discussed the affect of the small vibration of human eyes to the depth measurement theoretically and estimated its quantity with the measured value of the small vibration in the experiment. From the experiment result, it is pointed out that the neighboring objects on the depth of 300(mm) are good discriminative, though the neighboring objects on the depth of 3000(mm) are not sufficiently discriminative. We will utilized these results to develop the image display system with eyes' glance

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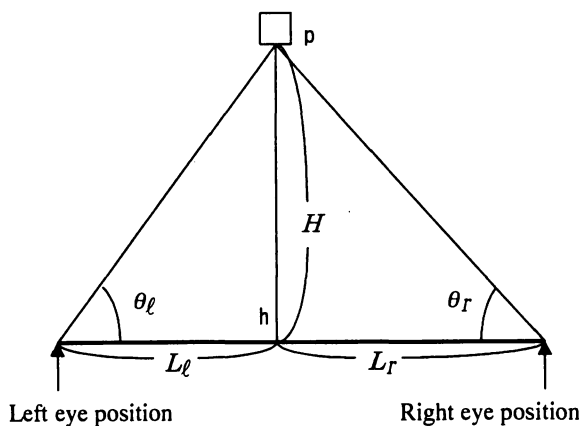


Fig.1 Distance( $H$ ) calculated by stereography with  $\theta_\ell$  and  $\theta_r$ .

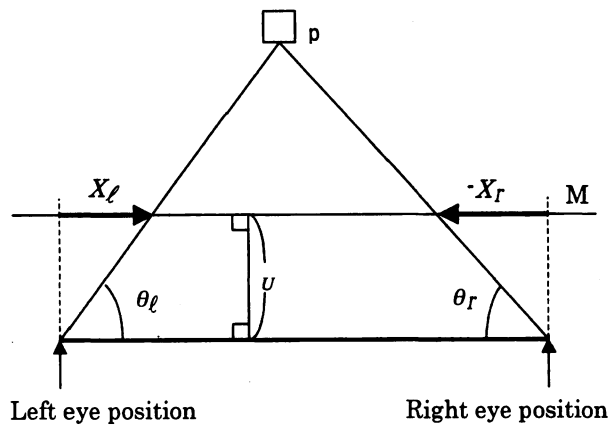
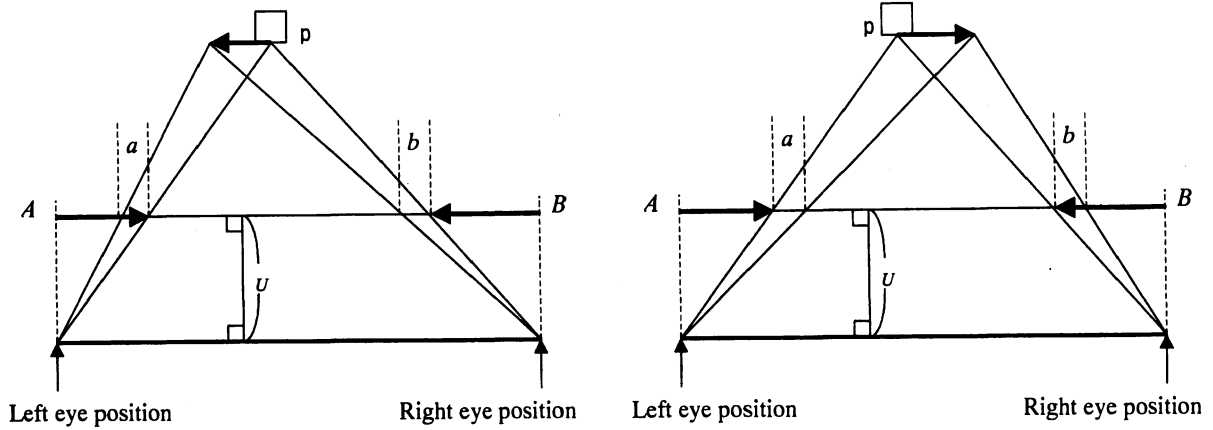


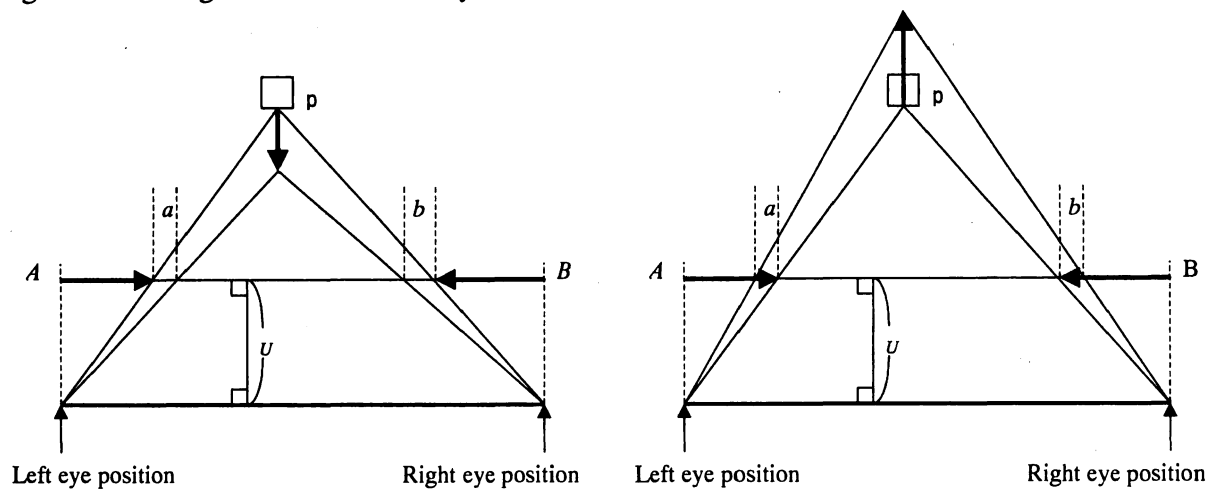
Fig.2 X-coordinate system  $X_\ell$  and  $X_r$  for  $\theta_\ell$  and  $\theta_r$ .



(a) On the case of left-ward movement

(b) On the case of right-ward movement

Fig.3 Left- and right-ward movement by visual fluctuation.



(a) On the case of the movement on this side

(b) On the case of the movement on the other side

Fig.4 The movement on this and the other sides by visual fluctuation.

Table 1  $d$ ,  $d_-$  and  $d_+$  for SD, 2\*SD and 3\*SD of the small vibration ( $=\delta_V$ ) at 30(cm), 1(m), 3(m) of the depth ( $=H$ ).

| $\delta_V$ | H    | $d$<br>( $d/H$ ) | $d_-$<br>( $d_-/H$ ) | $d_+$<br>( $d_+/H$ ) |
|------------|------|------------------|----------------------|----------------------|
|            | 300  | 0.51<br>(0.17 %) | 4.6<br>(1.5 %)       | 4.8<br>(1.6 %)       |
|            | 1000 | 1.7<br>(0.17 %)  | 50<br>(5.0 %)        | 55<br>(5.5 %)        |
|            | 3000 | 5.1<br>(0.17 %)  | 410<br>(14 %)        | 560<br>(19 %)        |
|            | 300  | 1.0<br>(0.33 %)  | 9.1<br>(3.0 %)       | 9.7<br>(3.1 %)       |
|            | 1000 | 3.4<br>(0.34 %)  | 94<br>(9.4 %)        | 120<br>(12 %)        |
|            | 3000 | 10<br>(0.33 %)   | 710<br>(24 %)        | 1400<br>(47 %)       |
|            | 300  | 1.5<br>(0.50 %)  | 13<br>(4.3 %)        | 15<br>(5.0 %)        |
|            | 1000 | 5.1<br>(0.51 %)  | 140<br>(14 %)        | 190<br>(19 %)        |
|            | 3000 | 15<br>(0.50 %)   | 960<br>(32 %)        | 2600<br>(87 %)       |

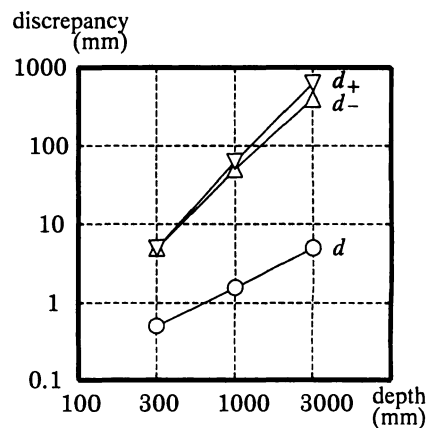


Fig.5  $d$ ,  $d_-$  and  $d_+$  for the small vibration of SD ( $\delta_V=SD$ ).