SVERIGES GEOLOGISKA UNDERSÖKNING
Rapporter och meddelanden nr 27

Intruded and relict groundwater of marine origin

SWIM 81

Proceedings of
Seventh Salt Water Intrusion Meeting
Uppsala, Sweden, 14—17 September 1981

Uppsala 1981

Proceedings of Seventh Salt Water Intrusion Meeting, Uppsala
Sveriges Geologiska Undersökning, Rapporten och meddelanden, 27, 193-219

THE SUBTERRANEAN FLOW OF FRESH AND SALT WATER UNDERNEATH THE WESTERN BELGIAN BEACH

by L. LEBBE[1]

ABSTRACT

On the gently sloping sandy runnel and ridge beach thirty holes were drilled through the unconfined aquifer. The unconfined aquifer consists of a complex of permeable and semi-permeable layers with a thickness of about 30 m. In each of the boreholes a resistivity logging was performed. The resistivity logging thus obtained provide a fairly good idea of the fresh, brackish and salt water distribution underneath the beach. Five resistivity profiles perpendicular to the shore line were drawn. At one of these profiles the hydraulic head pattern has been measured continuously in the upper and the lower part of the aquifer. From these piezometers groundwater has been sampled for determining the mean anion- and kation concentrations. Based on these data a mathematical model, which describes the flow of fresh and salt water has been developed.

INTRODUCTION

In the western Belgian coastal plain a large part of the water demand is supplied by catchments in the dune belt. Because of two major reasons the demand of water has increased continuously. The first reason is the expansion of the tourism since W.W.II. This not only caused an increase in the water demand but the extension of the urbanized area also has reduced the natural infiltration in the dunes. The second reason is the connection of the farms and local communities in the polder area to the public water supply system.

Since 1958 several hydrogeological studies have been undertaken in the Belgian coastal area (J. DE PAEPE & W. DE BREUCK, 1958). The location of the fresh-/salt-water interface in the coastal plain has been mapped by resistivity soundings, resistivity traverses and borings(W. DE BREUCK & G. DE MOOR, 1974). This map shows that the fresh-water supplies in the polder area are limited to small domestic uses. The only parts in the coastal plain which can provide larger water quantities are the dune areas.

In the Westhoek area of the dunes (L. LEBBE, 1978),covering a surface of 9 km² (fig. 1), a hydrogeological study has been carried out. The larger part of the area is occupied by dunes

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bounded in the north by the beach and in the south by the polders. The western part of the area is a natural reserve. The eastern part comprises the urban area and the water catchment. More than eighty borings have provided a detailed picture of the lithology and the stratigraphy of the aquifer. By means of grain-size analyses, geo-electrical well loggings, pumping tests, measurements of tidal and seasonal fluctuations of the hydraulic head the hydraulic parameters have been estimated. They include the horizontal and the vertical permeabilities, the elastic storage coefficient, and the storage coefficient near the watertable.

By means of a mathematical model the hydraulic parameters of the pervious and semi-pervious layers were adjusted and the groundwater flow was calculated. From the hydraulic head configuration three zones of groundwater flow have been deduced (fig. 1), one where the groundwater flows towards the polder area, one where grounwater flows towards the sea. Near the French border the fresh groundwater flow towards the sea is a maximum while it is very much reduced in the eastern part of the area studied.

The chemistry of the groundwater was investigated by means of resistivity logging and chemical analyses of groundwater samples. The sandy deposits below the dunes resting on a clayey substratum only contain fresh water. In the polder area the distribution of salt, fresh and brackish water is complex. In the lower part of the aquifer salt water usually occurs. In the upper part the water is usually fresh and sometimes brackish. Between the salt and the fresh water there is a transition zone of brackish water of variable thickness. At the beach near the high-tide line the upper part of the aquifer contains salt water. The lower part contains fresh water.

Because of the rather unusual occurrence and flow of salt water above fresh water the study was further focused on the groundwater flow underneath the beach. Most of the studies about saltwater intrusion (TODD, D.K., 1980, BEAR, J., 1972) deal with the flow of fresh water on denser brackish and salt water. The Badon Ghyben-Herzberg formula gives the relation between the depth of a sharp fresh-/salt-water interface below mean sea level and the height of the water table. It assumes a static equilibrium and a hydrostatic pressure distribution in the fresh water region with stationary seawater. In the course of time many investigators have improved this formula by taking into consideration the flow, horizontal as well as vertical, in the fresh and/or salt water and the dispersion and diffusion processes. The configuration of the interface between fresh and salt water has also been investigated intensively in groundwater reservoirs with semi-pervious layers. The practical problem which has received most attention in this context is the upconing of brackish water under a pumped well.

CHARACTERISTICS OF THE BEACH AT DE PANNE

The Belgian coast is a sandy runnel and ridge beach. At De Panne the mean slope is 1,1 %. The beach is covered by semi-diurnal tides. The difference between the high and low tide is at spring tide approximately 5 m and at neap tide 3 m. The horizontal distance between the mean high-tide line and the mean low-tide line or the width of the beach ranges from 300 m to 450 m. The sealevel reaches its lowest point at 0 m T.A.W.[1] and its highest point at +5,5 m T.A.W. With a frequency of about eleven cases in ten years the sealevel overreaches +5,5 m T.A.W.level (R. COPPE & L. DE KEYSER, 1967).

From the hydrogeological profile (fig. 2) of the adjacent dune area (L. LEBBE, 1979) we may deduce the constitution of the aquifer. The substratum of the unconfined aquifer is formed by impermeable clay of Eocene age. The lower part of the aquifer is formed by medium to coarse medium sands of Eemian age, with a mean horizontal hydraulic conductivity of 24 m/d. The larger part of the aquifer consists of fine medium sands. Their mean horizontal hydraulic conductivity is 9 m/d. Lenses of silty fine sands with a small hydraulic resistance can occur. The top of the aquifer is formed by medium sands with a estimate horizontal hydraulic conductivity of 12 m/d. Before the urban area of De Panne on the back shore occurs a sandy layer with an accumulation of shells between the fine medium sands and the uppermost medium sands. On some places underneath the beach there can occur a silty fine sand layer between the lower medium to coarse medium sands and the upper fine medium sands which forms a semi-permeable layer.

RESISTIVITY LOGGING

In June, July and August 1980 five rows of borings were made on the beach at De Panne. These rows were situated perpendicular to the coast line, beginning at the French border K0 and continuing with K1, K2, K3 and K4 respectively 1, 2, 3 and 4 km to the north east (fig. 3). Drilling was done by the rotary method; the drilling mud consisted of a suspension of water and organic additives that degrade with time. The borehole had a diameter of approximately 100 mm. Due to this small diameter the weight of the drilling equipment was kept small involving only low costs and speed. Hence it was possible to drill very close to the low tide line. The small borehole diameter provides another advantage. The resistivity of the surrounding sediments, measured with a long normal device, can be compared with measurements in a screened well of diameter 80 mm (L. LEBBE, 1978).

According to the long normal method, a current and a potential electrode 1 m apart are lowered into the borehole. The other current and potential electrodes are placed at the surface at some distance from the borehole and from each other.

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1 T.A.W. Datum level of the second general leveling (National Geografical Institute)

The electrodes lowered into the borehole were ring electrodes with a diameter of 40 mm and a height of 10 mm.

The methods were compared as follows. A long normal log in a borehole Ø 100 mm filled with drilling mud was compared with one performed in screened well of Ø 80 mm at a very short distance.

The mean of 23 ratios between the resistivities in the borehole filled with drilling mud and those in the screened well was about 0,98, the standard deviation being 0,14. The largest value for the ratio was 1,27 and the smallest 0,74. As expected the two kinds of measurements did not differ much because of the ring electrodes and the rather small diameter of the boreholes. Both measured resistivities did not differ much from the true formation resistivity.

In a former hydrogeological study in an adjoining area chemical analyses of groundwater samples and resistivity logging of a screened well have shown that the ratio between the resistivity of the sediments, and the resistivity of the pore water in the sediments has an average value of approximately 2,7. This ratio varies between 2,3 for silty very fine sands and 4,5 for medium to coarse medium sands. According to the resistivity limits of fresh, brackish and salt water as proposed by G. DE MOOR & W. DE BREUCK (1969) (table 1) we have deduced the corresponding resistivity limits of the sediments filled with fresh, brackish and salt water, measured with the long normal device.

Tabel 1. Resistivity of fresh, brackish and salt water and resistivity of the sediments in the area filled with salt, brackish and fresh water measured with a long normal device (AM = 1 m)

Quality	Resistivity of the water ρ_w (in Ωm at 10°C)	Resistivity, ρ_m, measured with the long normal device (in Ωm)
Fresh	$\rho_w > 7,5$	$\rho_m > 20$
Brackish	$0,94 < \rho_w < 7,5$	$2,5 < \rho_m < 20$
Salt	$\rho_w < 0,94$	$\rho_m < 2,5$

The results of the resistivity logging on the beach are represented in five resistivity profiles perpendicular to the coastline (fig. 3). Three zones can be distinguished. A zone where the resistivity is larger than 20 Ωm, consisting of sediments filled with fresh water, a zone where the resisitivity varies between 2,5 and 20 Ωm, containing brackish water, and a zone where the resistivity is smaller than 2,5 Ωm with salt water.

In the resistivity profile KO at the French border salt water rests upon fresh water. There is a brackish transition zone of varying thickness. At the limit of the dune and the beach the transition zone between the fresh and the salt water is nearly vertical. Further under the beach in the direction of the sea the transition zone is nearly horizontal,gently rising towards the sea. Hence the salt-water lens decreases in thickness towards the sea. The resistivity of the sediments with fresh water decreases towards the sea, which indicates that the water becomes more mineralised. This rather simple configuration of fresh, brackish and salt water under the beach is found where the groundwater divide is situated the farthermost inland. This is where a large part of the fresh water underneath the dune drains towards the beach.

From the French border in the direction of the agglomeration of De Panne the water divide approaches the beach so that the fresh groundwater flow towards the sea gradually diminuish.

At 1 km from the French border the resistivity profile K1 shows underneath the back shore the same picture of the resistivity profile, at the French border. Underneath the fore shore the fresh water gradually passes into brackish water. Underneath the fore shore the salt-/fresh-water interface first rises and then descends towards the low-tide line where salt water flows above brackish water.

At 2 km from the French border the resistivity profile K2 shows underneath the back shore approximately the same picture of K0 and K1. Underneath the fore shore it shows a quite different picture. Here the thickness of the salt-water layer is very much reduced while brackish water appears under the fresh water. The resistivity of the fresh water reduces in the direction of the sea.

At 3 km from the French border before the agglomeration of De Panne where the water divide approaches the beach the resistivity profile K3 shows a completely different picture. In the lower part of the aquifer the extension of fresh water is strongly reduced. The fresh water passes into brackish and salt water over a short distance. The largest part of the lower aquifer is here occupied by salt water with a slightly lower salt content than the water in the upper part. At the low-tide line a lens of brackish water appears halfway the aquifer.

At 4 km from the French border at the NE corner of the urban area of De Panne the resistivity profile K4 shows similar to K3. The occurrence of fresh water in the lower part of the aquifer is limited to a small zone under the back shore and the transition from fresh to brackish and salt water occurs within a short distance. A lens of brackish water appears under the low-tide line.

HYDRAULIC HEAD MEASUREMENTS

A first series of hydraulic head measurements was made to determine the mean hydraulic head just underneath the surface of the beach. Because the hydraulic head fluctuates with the tidal movement it was necassary to measure it continuously at different distances from the high-tide line. Six piezometers were installed in a plane perpendicular to the shore line, the same plane of resistivity profile K0. The screens of the piezometers were placed at the -2 m T.A.W. level, in the upper part of the aquifer. They had a diameter of 63 mm and a screen length of 0,5 m. The piezometers P1, P2, P3, P4, and P5 were placed at respectively 360, 245, 165, 90, and 5 m from the landmark (stone marking the border between Belgium and France) in the direction of the sea. The piezometer 6 was placed in the dune area at 70 m from the landmark in the direction of the land. This landmark almost coincides with the high-high-tide line.

In the piezometers the pressure was measured by means of a pressure transducer of the VEGA-TYPE 137 series. By means of a data logger LOGMASTER MDL 1000 of MESS + SYSTEM TECHNIK the pressure was directly converted into a fresh-water head[1]. The results were printed and recorded on a magnetic tape. These measurements were made every ten minutes on the six piezometers. In fig. 4 the fresh-water head is plotted versus time. In table 2 the mean fresh-water head of the six days period of measurements are indicated together with the elevation of the surface.

Table 2 - The mean fresh-water head deduced from the first series of measurements during a six days period

Piezometer	Distance in m from landmark positive in the sea-direction	Surface elevation m T.A.W.	Mean fresh-water head at -2 m T.A.W.
P1	360	+0,6	+2,96
P2	245	+1,8	+3,15
P3	165	+2,6	+3,46
P4	90	+3,1	+3,74
P5	5	+5,3	+4,59
P6	-70	+8	+4,55

These data show that the hydraulic head at the groundlevel is induced by the tidal mouvement of the seawater on the beach. Below the low-low-tide line the salt-water head at the surface of the seafloor equals the mean sea-level. At the fore shore the salt-water head equals the sealevel when that part of the beach is

1 The reference level of the fresh-water heads and the salt-water heads used in this paper is the 0 m T.A.W. level

inundated. When the sea retreats the salt-water head is approximately equal to the elevation of the surface above the reference plane. The mean salt-water head at the surface of the beach can be deduced from the elevation of the surface and the tidal mouvement of the seawater. Because of the depth of the screen of the piezometer below the beach surface there is a slight difference between the hydraulic head measured and the one at the surface. At the seaward part of the fore shore at piezometer P1, the salt-water head at 2,5 m below the surface stood a little above the surface when the sea retreats, which indicates a seepage of saline water. In the piezometer P2, P3 and P4 the salt water head is about equal to or a little smaller than the elevation of the surface. When the sea inundates the beach the salt-water head in the piezometers is a little smaller than the sealevel. This difference increases towards the landward part of the fore shore. The difference is larger in piezometer P4 than in piezometer P2. The difference causes an infiltration of seawater at the upper part of the beach. Under the uppermost part of the beach, between the high-tide line of neap tide and the highest sealevel elevation at spring tide during western or northern storms, the hydraulic head is almost stable and is not influenced by the tidal mouvement . Only in periods when the sea inundates that part of the beach seawater percolated through the unsaturated zone, of which the thickness varies between 0 and 1,5 m;this causes a rise of the hydraulic head under this part of the beach. This is confirmed by the hydraulic head measurements in piezometer P5 during the first two days of measurements with stormy weather. After the retreat of the sea, the hydraulic head decreases very slowly. Under the dune area the hydraulic head just below the watertable does not fluctuate under the unfluence of the tides.

The mean fresh-water head (fig. 4 , tab. 2) indicates that the horizontal hydraulic head gradient is very small underneath the dune. There is even a small landward hydraulic gradient resulting from the high waters of the stormy weather. There is a steep gradient underneath the back shore and the landward part of the fore shore. Further to the lower part of the beach the horizontal hydraulic gradient drops to zero below the low tide line. The pattern of the mean hydraulic head at the surface of the beach, which is deduced here, is a boundary condition which defines the mean flow of salt, brackish and fresh water underneath the beach. The pattern of the mean hydraulic head depends on the amplitude of the tidal movements of the sea and the topography of the beach. The other boundary conditions are the seaward flow of the fresh water underneath the dunes and the hydraulic head pattern in the aquifer underneath the sea. These boundary conditions, and the horizontal and vertical permeability are the input data of the mathematical model which describes the flow of fresh and salt water underneath the beach.

A second series of hydraulic head measurements was made to deduce the mean hydraulic head in the lower part of the aquifer underneath the beach. In the same plane of the resistivity profile K0 two piezometers, P2' and P4', were placed at respectively 265 and 160 m from the landmark in the direction of the sea. The diameter of the piezometer was 63 mm and the length 2 m. During a period of three days the pressure was measured every ten minutes and converted into a fresh-water head which was printed and recorded on a magnetic tape. During the same period these measurements were also made in the piezometers P2, P5 and P6 of the first series. In fig. 5 the fresh-water head is plotted versus time, table 3 shows the mean fresh-water head of the three days period and the elevation of the surface.

Table 3 - The mean fresh-water head deduced from the second series of measurements

Piezometer	Distance in m landmark positive towards the sea	Surface elevation in m T.A.W.	Mean fresh-water head
			at -22 m T.A.W.
P2'	265	+1,7	+3,5
P4'	95	+2,6	+4,1
			at -2 m T.A.W.
P2	245	+1,8	+2,78
P5	5	+5,3	+4,39
P6	-70	+8	+4,47

With this series of measurements it was possible to follow the hydraulic head underneath the seaward part of fore shore in the upper part of the aquifer (piezometer P2) and in the lower part of the aquifer (piezometer P2'). The mean fresh-water head is about 0,7 m higher in piezometers P2' than in piezometer P2. The maxima of the hydraulic head are reached at approximately the same time. The minima in piezometer P2' are reached earlier than in piezometer P2. The difference between the maximum and minimum is larger in piezometer P2 than in piezometer P2'. During the measurements this difference varied between 2 and 2,35 m in piezometer P2', in piezometer P2 between 2,40 and 2,70 m.

The comparison of the measurements in piezometer P4 of the first series and those in piezometer P4' of the second series shows that the mean hydraulic head is about the same in the upper part and in the lower part of the aquifer. The maxima are reached a little later. The minimum in piezometer P4' is reached earlier than in piezometer P4. The difference between the maximum and minimum varies during the measurement between 1,0 and 1,5 m. The results of this second series of measurements can be a helpful tool in the calibration of the mathematical model.

CHEMICAL ANALYSES OF GROUNDWATER

After the hydraulic head measurements groundwater has been sampled from the piezometers. Three samples were taken at the -2 m T.A.W.-level in piezometer P2, P4 and P5 and two at the -22 m T.A.W.-level in the piezometer P2' and P4'. The results of the analyses have been indicated on the resistivity profile K0 (fig. 6) : total salt content, anions (Cl^-, SO_4^{--}, HCO_3^-) and kations (Na^+, K^+, Mg^{++}) are given in mg/l.

The mean concentration of the anions and kations of the fresh water underneath the dune belt was deduced from the chemical analyses of 27 groundwater samples (L. LEBBE, 1978). The mean Cl^- concentration of the fresh water underneath the dune belt is 32 mg/l; the mean Cl^- concentration of the infiltrating seawater on the beach can be estimated at 15.700 mg/l.

From these data and supposing that there is no exchange of Cl^--ions with the deposition one can deduce the mixture of these two primary waters in every piezometer.

The Cl^- concentration in P4' being approximately 91 mg/l one may deduce that the groundwater sample represents a mixture of 99,62 % fresh water and 0,38 % seawater. The Cl^- concentration in P2' is about 512 mg/l so that this sample is a mixture of 96,9 % fresh water and 3,1 % seawater. The Cl^- concentration in P5 is about 3 650 mg/l so that this groundwater sample is a mixture of 77 % of fresh water and 23 % of seawater. According to the well logging the piezometers P2' and P4' are both located in fresh water but one can see that the salinity increases towards the sea. The brackish water of piezometer P5 confirms the well logging data.

The groundwater samples taken in piezometers P2 and P4 at small depth underneath the beach have approximately the same kation and anion composition as the infiltrating seawater.

MATHEMATICAL MODEL

Theory

The flow of groundwater in an aquifer filled with water of different densities is defined by the generalized Darcy equation in vector form :

$$q_i = -\frac{k'_i}{\mu_i} (\nabla p_i + \rho_i g) \qquad (1)$$

where i is an index which defines any point in the groundwater reservoir

q	the seepage velocity	(LT^{-1})
k'	the intrinsic permeability	(L^2)
μ	the dynamic viscosity	$(ML^{-1}T^{-1})$
ρ	the density	(ML^{-3})
∇p	the pressure gradient	$(ML^{-2}T^{-2})$
g	gravitational acceleration	(LT^{-2})

The relation between the hydraulic conductivity and the intrinsic permeability can be written as :

$$k_i = \frac{k'_i \rho_i g}{\mu_i} \qquad (2)$$

where k is the hydraulic conductivity (LT^{-1})

The relation between the pressure and the fresh-water head can be represented as follows :

$$\varphi_{if} = z_i + p_i/\rho_f g \tag{3}$$

where φ_{if} is the fresh-water head (L)
z_i the elevation of point i above the reference plane (L)
p_i the fluid pressure in point i ($ML^{-}T^{-2}$)
ρ_f the density of the fresh water (ML^{-3})

With the relations (1), (2) and (3) one can determine the horizontal and the vertical seepage velocity in a point i of the groundwater reservoir containing waters of different densities, if the fresh-water head and the density in that point are given.

$$q_{hi} = k_{hi}\frac{\rho_f}{\rho_i}\frac{\partial\varphi_{if}}{\partial x} \tag{4}$$

$$q_{vi} = k_{vi}\frac{\rho_f}{\rho_i}\left(\frac{\partial\varphi_{if}}{\partial z}+\frac{(\rho_i-\rho_f)}{\rho_f}\right) \tag{5}$$

where q_{hi} is the horizontal seepage velocity (LT^{-1})
q_{vi} the vertical seepage velocity (LT^{-1})
k_{hi} the horizontal hydraulic conductivity (LT^{-1})
k_{vi} the vertical hydraulic conductivity (LT^{-})

Because of the lack of accuracy by which the horizontal and vertical hydraulic conductivity can be determined the factor ρ_f/ρ_i, which is small, is ignored in the development of the model in this paper.

The law of continuity for steady state flow can be written as :

$$\nabla q = 0 \tag{6}$$

Finite-difference approximation

The groundwater flow is considered to be two-dimensional in a vertical plane. On this plane a grid is superposed (fig. 7). The intersections are called "nodes". To each node a value for the fresh-water head $\varphi(I,J)$ is assigned. The node I,J is numbered in such a way that the coordinates are given : $x_{I,J}$ = I x L and $y_{I,J}$ = J x H. The mean difference of the density of the water between the nodes I,J+1 and I,J and the density of fresh water divided by the fresh-water density is given by the symbol VD(I,J) and can also be called the mean areal buoyancy.
By means of a central finite-difference approximation one can determine the horizontal and the vertical seepage velocity.

$$q_h(I,J) = k(J)(\varphi_f(I+1,J)-\varphi_f(I,J))/L \tag{7}$$

$$q_v(I,J) = (\varphi_f(I,J+1)-\varphi_f(I,J)+VD(I,J)H)/c(J) \tag{8}$$

where the density of fresh water is supposed to equal 1 g/cm³

$q_h(I,J)$	is the horizontal seepage velocity from the node I+1,J to the node I,J	(LT^{-1})
$k(J)$	the horizontal hydraulic conductivity of layer J	(LT^{-1})
$\varphi_f(I,J)$	the fresh-water head in node I,J	(L)
L	the horizontal distance between two adjacent nodes of the same layer	(L)
$q_v(I,J)$	the vertical seepage velocity from the node I,J+1 to the node I,J	(LT^{-1})
$c(J)$	the hydraulic resistance between the nodes of the J+1-layer and the nodes of the J-layer	(T)
VD(I,J)	the mean difference of the density of the water between the nodes I,J+1 and I,J and of the fresh water density divided by the fresh-water density or mean areal buoyancy (dimensionless)	
H	the vertical distance between two adjacent nodes of the same column	(L)

With the equation (7) and (8) one can write the continuity law at the node I,J.

$$K(J)H(\varphi_f(I+1,J)-2\varphi_f(I,J)+\varphi(I-1,J))/L$$
$$+\ L(\varphi_f(I,J+1)-\varphi_f(I,J)+VD(I,J)H)/c(J)$$
$$-L(\varphi_f(I,J)-\varphi_f(I,J-1)+VD(I,J-1)H)/c(J-1)\ =\ 0 \qquad (9)$$

Boundary conditions

The groundwater reservoir is bounded below by an impermeable layer. Because there is no flow through this boundary there is no need for a water quality boundary condition. At the landward vertical boundary we assume a constant horizontal flow of fresh water with a density of 1 g/cm³.

The upper boundary of the groundwater reservoir is partially situated underneath the dunes and partially underneath the beach and the sea. Underneath the dunes we assume a constant vertical flow boundary. The vertical flow equals the mean annual infiltration rate of fresh water in the dune area. Beneath the beach and the sea one assumes a constant hydraulic head boundary. These hydraulic heads have been deduced from measurements on the beach. Where on the beach a downward vertical flow occurs one assumes that there is an infiltration of salt water with a density of 1,025 g/cm³.

The seaward vertical boundary is a constant hydraulic head boundary. On this boundary the salt-water head is held the same over the whole depth. The density of the water which flows through this boundary depends on the flow direction. When the flow is landward through the boundary into the considered area we assume that this inflowing water has the density of seawater. When water flows from the considered area in the direction of the sea the density of the water is defined by the flow of salt and fresh water in the mathematically considered area.

Control of the iteration process

The finite-difference approximation leads to a system of linear algebraic equations. This system is solved by an alternating direction implicit iterative method. In the vertical section initially the water is considered to be of equal density. The accuracy of the calculation of the flow in the system is checked by establishing a water balance of the system. When the iteration process converges the difference between the inflow and the outflow of the system decreases. When the calculations of the hydraulic head and consequently of the flow have reached a sufficient accuracy the matrix VD is deduced. This is done by following the stream line which forms the limit between the flow of the fresh and salt water. The lower end of the fresh-/salt-water boundary is found by following the streamline which begins at the intersection of the reservoir floor and the landward or seaward vertical boundary. The upper salt-/fresh-water boundary can be found by following the streamline which begins at the high-high-tide line. At the landward part of this line fresh water infiltrates and at the seaward part salt water. When a fraction f_s of the area between two nodal points I+1,J and I,J is occupied by salt water and the complementary fraction $(1-f_s)$ is occupied by fresh water the value of the mean areal buoyancy VD(I,J) is calculated as follows :

$$VD(I,J) = f_s \, (\rho_s - \rho_f)/\rho_f \qquad (10)$$

The matrix VD being determined one now computes the fresh-water head configuration and consequently the flow anew. After reaching a sufficient accuracy the matrix VD is computed again. This algoritm is repeated until one instantly obtains a sufficiently accurate flow when applying the newly determined matrix VD. So we obtain a hydraulic head configuration where the flow is in agreement with the density distribution.

Input

The first series of input data determines the dimensions of the nodal grid and of the considered area. The dimensions of the grid are given by the number of colomn x and the number of layers y. The real dimensions of the considered area are defined by the horizontal distance between two adjacent nodes of the same layer, L, and by the vertical distance between two adjacent nodes of the same column, H. The number of the columns, situated underneath the dune area, x1, defines the position of the high-high-tide line.

The hydraulic conductivity of the aquifer is given by an array of horizontal hydraulic conductivities for each layer K(J), where J varies between 1 and y, and by an array of hydraulic resistances between each layer c(J) where J varies between 1 and y-1. The hydraulic resistance c(1) is the hydraulic resistance between layer 1 and 2.

For the boundary conditions one must define an array of horizontal seepages Q(J) where J varies between 1 and y-1. The infiltration N is provided by a constant vertical seepage through the upper boundary of the column underneath the dunes. The boundary condition on the beach is a constant hydraulic head pattern. This pattern of fresh-water heads is deduced from the hydraulic head measurements. This is also done for the vertical constant hydraulic head boundary under the seafloor.

Output

As a result of the calculation we obtain a hydraulic head distribution where the resulting flow is in agreement with the density distribution. For each nodal point one has a value for the fresh-water head and a value for the mean density of the water between the nodal points. These results are represented by lines of equal fresh-water head and stream lines on the vertical plane of the considered area. The horizontal axis is taken positive towards the sea and the origin is located at the high-tide line. The vertical axis is positive in the upward direction and the origin corresponds with the 0 m T.A.W.-level.

The lines of equal fresh-water head are deduced by an interpolation between the value of the fresh-water head of four nodal points within the area of these points.

The constant flow boundary conditions are indicated by a letter, Q, for the horizontal fresh water flow of the dunes in the direction of the sea and N for the vertical fresh water flow equal at the infiltration rate in the dune area. The constant head boundary conditions are represented by the value of the constant fresh water head. The inflow and outflow through the permeable boundary of the mathematically treated area are calculated and represented near these boundaries. The stream lines are drawn in such a way that they begin at the permeable boundary with an inflow of fresh or salt water. The arrows on the streamlines indicate the even years that a waterdrop is underway on the streamline from the beginning at the boundary of the considered area. For the last calculation a water-conducting porosity of 0,30 is taken into account.

Examples of fresh-/salt-water distribution as calculated by the mathematical model

The same nodal grid is used in the three examples. The grid consists of 25 columns and 18 layers (x = 25, y = 18). Each column is 30 m wide and each layer is 1,5 m thick (L = 30 m, H = 1,5 m). The lower boundary is an impermeable substratum. The first column contains the hydraulic heads resulting from the constant flow boundary condition. The next 4 columns have an upper constant flow boundary. The constant flow equals the infiltration rate in the dune area (N = 270 mm/year or $N = 7,39.10^{-4}$ m/d). In this case a dune area of 120 m width is considered. In the mathematical model the beach has a width of 420 m. This is represented by 14 columns with an upper constant hydraulic head boundary. The fresh-water heads have been deduced from the pattern of measured hydraulic heads just below the beach surface. This pattern is a function of the tidal fluctuation of the sealevel and the topography of the beach. Finally 150 m of offshore is considered. This is represented by 5 columns with an upper constant hydraulic head boundary. These constant fresh-water heads are calculated from the salt-water head on the seabottom based on the mean sealevel. The last column contains the constant hydraulic head boundary condition. The salt-water head being the same over the entire depth the fresh-water head consequently increases with a value of 0,025 m per metre increasing depth.

The horizontal hydraulic conductivities are the same for all layers, K(J) = 10 m/d for J = 1 to 18 as well as the hydraulic resistances between the layers, c(J) = 15 d for J = 1 to 17. The horizontal flow through the vertical boundary is the same for every layer. Only the sum of this horizontal flow differs in the three examples.

In the first example the sum of the horizontal flow of fresh water through the vertical boundary is 0,25 m²/d (fig. 8). Because of the considerable seaward flow there is a horizontal hydraulic gradient towards the sea underneath the dune area. Underneath the back shore and the upper part of the fore shore salt water infiltrates. It forms a lens which flows upon the fresh dune water. On the largest part of the fore shore the salt-water flow is directed vertically upward resulting in an outflow at the beach surface. The fresh water only flows out at the seabottom.

In the second example the sum of the horizontal flow of fresh water through the vertical boundary is strongly reduced, namely 0,05 m²/d (fig. 9). Consequently the horizontal seaward hydraulic gradient underneath the dunes. There the vertical hydraulic gradient becomes more important. The upper salt-water lens is enlarged in depth as well as in width. The fresh-water outflow is consequently further offshore on the seabottom. The zone of this fresh-water flow is narrower than in the first example. The lower salt-water tongue has retreated towards the sea. The flow pattern and the fresh-/salt-water distribution are in good agreement with the field data of fresh-water heads and the resistivity profiles K1, K2, K3 before the natural reserve. There is still a fresh water flow towards the sea.

In the third example the sum of the fresh-water flow through the vertical boundary is reversed, namely -0,09 m²/d (fig. 10). Nearly all the infiltrating fresh water in the dune area is flowing towards this boundary. Only a small part of the infiltrating fresh water flows towards the sea. On the upper part of the beach seawater infiltrates and fills the whole aquifer under the beach. In spite of the small fresh-water flow towards the sea there is still a tongue of fresh water underneath the back shore. This is confirmed by the resistivity logging in front of the urban area of De Panne (K3 and K4).

This two-dimensional model can be used to describe the flow of fresh and salt water under a part of the dunes, the beach and a part of the offshore. Through sensitivity analyses one can deduce how the different parameters defines the flow and distribution of the salt and fresh water under the beach. These parameters are the hydraulic head pattern just below the beach surface, the flow of fresh water towards the sea, the horizontal and vertical permeability distribution.

In the mathematical model the process of hydrodynamic dispersion was not incorporated until now. It is in our intention to do so in the future. The former field measurements and chemical analyses of the groundwater can be used to evaluate the longitudinal and transversal dispersion coefficients.

The seaward flow of fresh water and the hydraulic head pattern underneath the beach are time dependent. The seaward fresh-water flow is considerable larger after a period of successive wet years than after a prolonged period of dry years. For example at the end of the dry summer 1976. The horizontal hydraulic gradient was landward over the whole length of the fore dunes. This was only during a short period. At the end of spring 1981, after a number of wet years, there was a seaward hydraulic gradient over the whole length of the dunes although this gradient was many times larger before the natural reserve than before the urban area. The pattern of the hydraulic head underneath the beach can strongly change. North-western to northern storm causes a supplementary rise of the sealevel during the whole period of the storm. These results in the increase of salt-water infiltration on the back shore and the upper part of the fore shore. The upward outflow of salt water on the largest part of the fore shore diminished or reversed. So during these period the salt water amount of the upper salt-water lens can increase. The reverse can also take place during a south-eastern to southern storm which result in a drop of the sealevel during this period. This south-eastern to southern storms are at the Belgian coast restricted in occurrence.

Because of the possible large variation in time of the two important factors which define the distribution of fresh, brackish and salt water there is a need for mathematical models which describe the unsteady-state flow of water with waters of different qualities. Continuous measurements of the time dependent factors are needed for the calibration of such models.

Such studies will indicate the relative importance of the different factors which define the variation of the salt, brackish and fresh water distribution like the hydrodynamic dispersion, the change of the fresh-water flow and of the hydraulic head pattern underneath the beach. This knowledge is necessary for a good management of groundwater reservoirs with water of different qualities like the one in the Belgian coastal plain.

Practical considerations

It is clear that the danger for salt-water intrusion under the seaward part of the dunes comes first of all from the upper salt water lens. In this case it is not from the lower salt water tongue that the danger of salt water intrusion will arise. The location of the lower salt water tongue is defined by the mean sea level and the dispersion (COOPER et al., 1964). The salt water of the upper lens is close to the dunes. The salt-water head is there higher than the mean sealevel. This salt-water head depends on the height and the duration of the high waters and the permeability of the subsurface of the back shore.

At slightly inclining coast with a permeable surface and subsurface and a meaningfull tidal fluctuation an upper salt-water lens can be expected. This can be a potential danger of salt water intrusion. Because of this salt-water infiltration on the beach the lower salt-water tongue in a thick aquifer will be more seaward than can be deduced from older studies which ignore the upper salt-water lens and flow.

In thick aquifers where fresh water wells are placed near the beach one must know the distribution and flow of salt, brackish and fresh water. The distribution consists of an upper salt-water lens an upper salt-/fresh-transition zone, a fresh-water zone, a lower fresh-/salt-transition zone and a lower salt-water tongue. The distribution of the horizontal and vertical permeabilities with the hydrodynamic dispersion coefficients define further the flow to the wells of the waters of different qualities.

CONCLUSION

Resistivity logging in the Western Belgian beach has revealed different salt-/fresh-water distributions. The unconfined aquifer with a thickness of about 30 m consists of permeable layersof fine to medium fine sand with discontinuous semi-permeable lenses of very silty fine sands. Above the fresh groundwater from the dunes which flows seawards, a salt water lens is present. The salt water infiltrates on the back shore and the upper part of the fore shore. Under the lower part of the fore shore the salt water flows upwards resulting in an outflow. The fresh water under the salt water gradually becomes more mineralised. Underneath the low-tide line it is nearly brackish.

In front of the urban area of De Panne, at K3 and K4, the fresh groundwater flow is very much reduced because of the decrease in infiltration and the influence of the water catchment. Only a small fresh-water tongue occurs in the lower part of the aquifer under the back shore. Under the fore shore the whole aquifer is occupied by salt water. The resistivity of the lower part of the aquifer being slightly higher than the upper part one can deduce that the salinity in the upper part of the aquifer is a little higher than in the lower part.

The occurrence of this salt-fresh water distribution underneath the Belgian beach can be explained by means of a two-dimensional mathematical model, taking the density differences into account. One of the boundary conditions which defines the flow of salt and fresh water is the pattern of the mean hydraulic head at the surface of the beach. Hydraulic-head measurements have shown that this pattern depends on the amplitude and the topography of the beach. Because the amplitude and the topography are approximately the same along the resistivity profiles the differences in fresh-/salt-water distribution are mainly due to the difference in seaward fresh-water flow from the dunes. A second, but minor reason for the changes of the salt-/fresh-water distribution under the beach is the occurrence of discontinuous semi-pervious lenses at different levels.

So far hydrodynamic dispersion has not been included in the model. To do so in the future, field measurements and chemical analyses will be used to evaluate the longitudinal and transversal dispersion coefficients.

Some of the parameters which define the salt-/fresh-water distribution under the beach are time dependent. The variability of these time-dependent parameters requires repeated observations and the construction of mathematical models for unsteady flow.

A better knwoledge of all parameters defining the flow will contribute to a better management of reservoirs containing groundwater of differing quality.

ACKNOWLEDGEMENT

The author would like to thank the National Fund of Scientific Research (Belgium) under whose auspices the study was carried out and Prof. Dr. W. DE BREUCK for the help in obtaining the field data and for the constructive discussions and suggestions on an earlier form of this manuscript.

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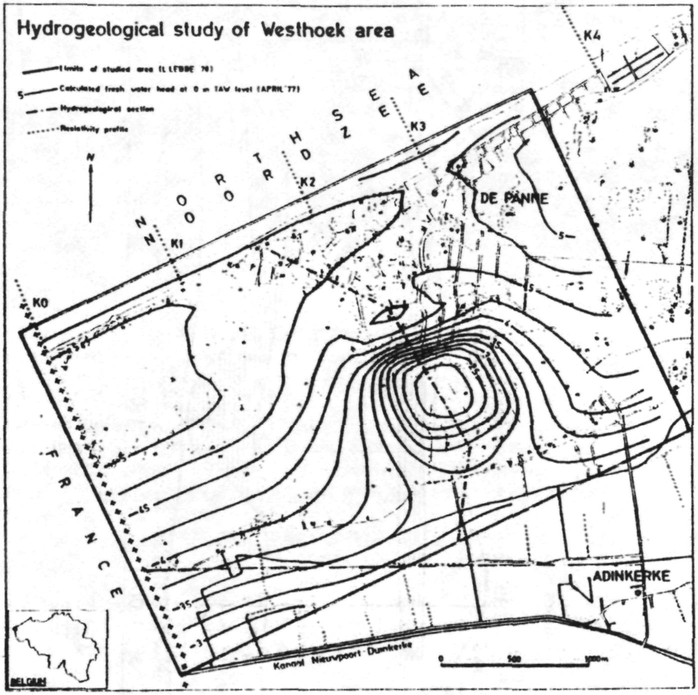


Fig. 1 - Localisation of the hydrogeological study in the Westhoek area with calculated lines of equal hydraulic head, of the hydrogeological section and of the resistivity logging profiles

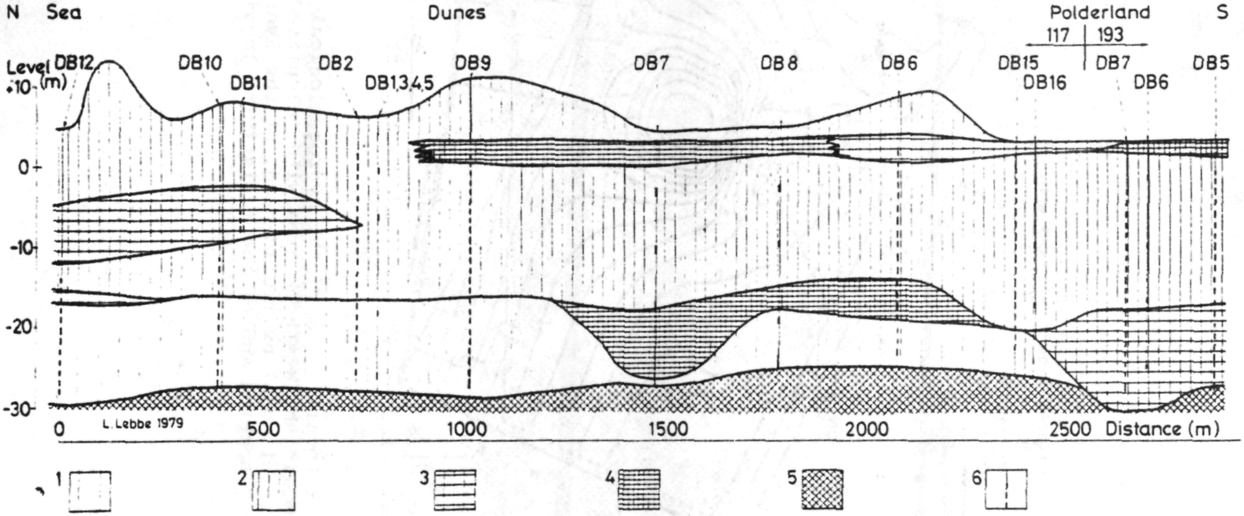


Fig. 2 - North-south hydrogeological section through the Westhoek area (L. LEBBE, 1979)

1. permeable layer with large horizontal hydraulic conductivity
2. permeable layer with small horizontal hydraulic conductivity
3. semi-permeable layer with small hydraulic resistance
4. semi-permeable layer with large hydraulic resistance
5. impermeable layer
6. well screen

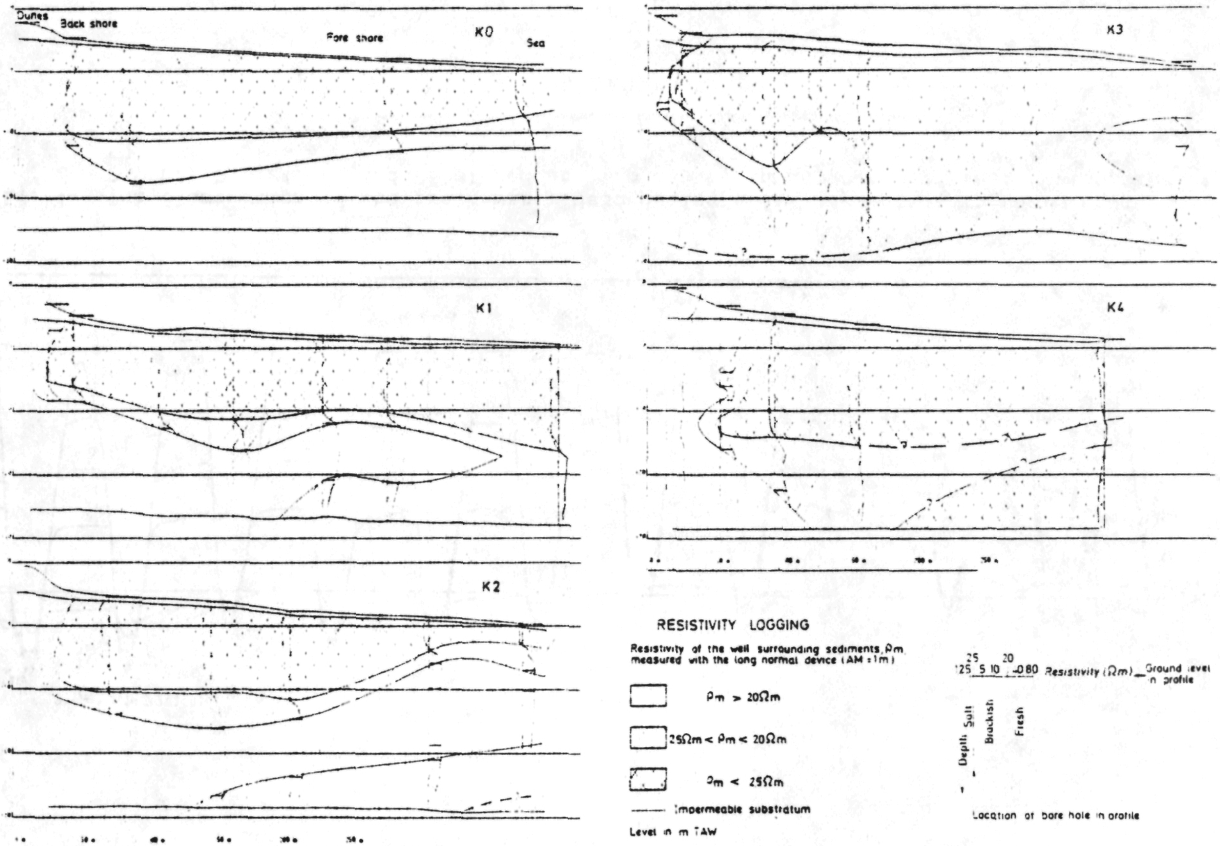


Fig. 3 - The resistivity profiles perpendicular to the shore line of the Westhoek area

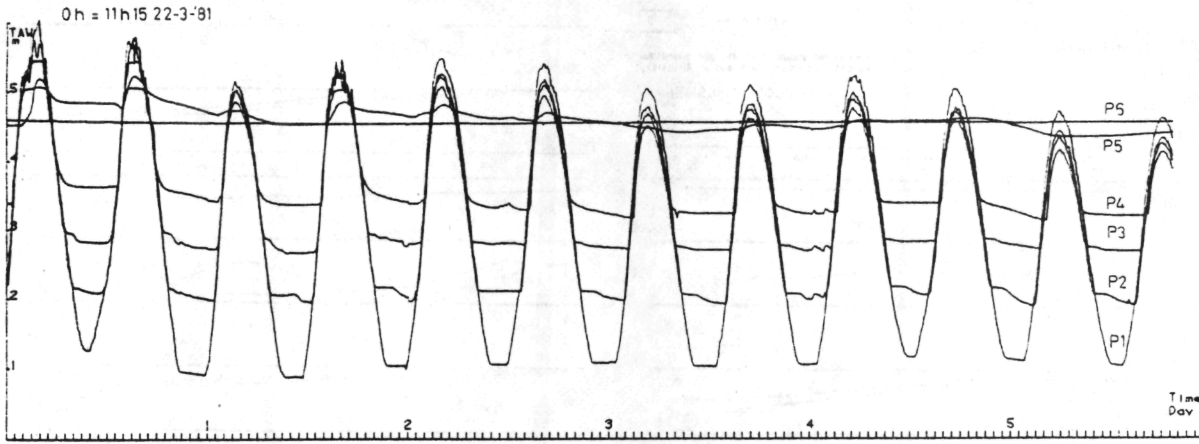


Fig. 4 - The fluctuation of the fresh-water head during a six day period in the piezometers P1, P2, P3, P4, P5 and P6 placed at -2 m T.A.W. in the plane of resistivity profile K0

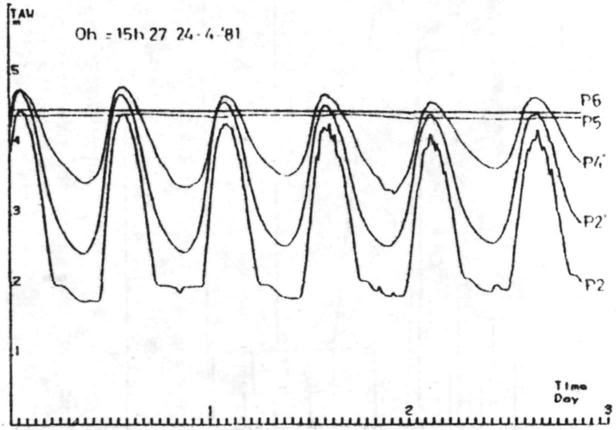
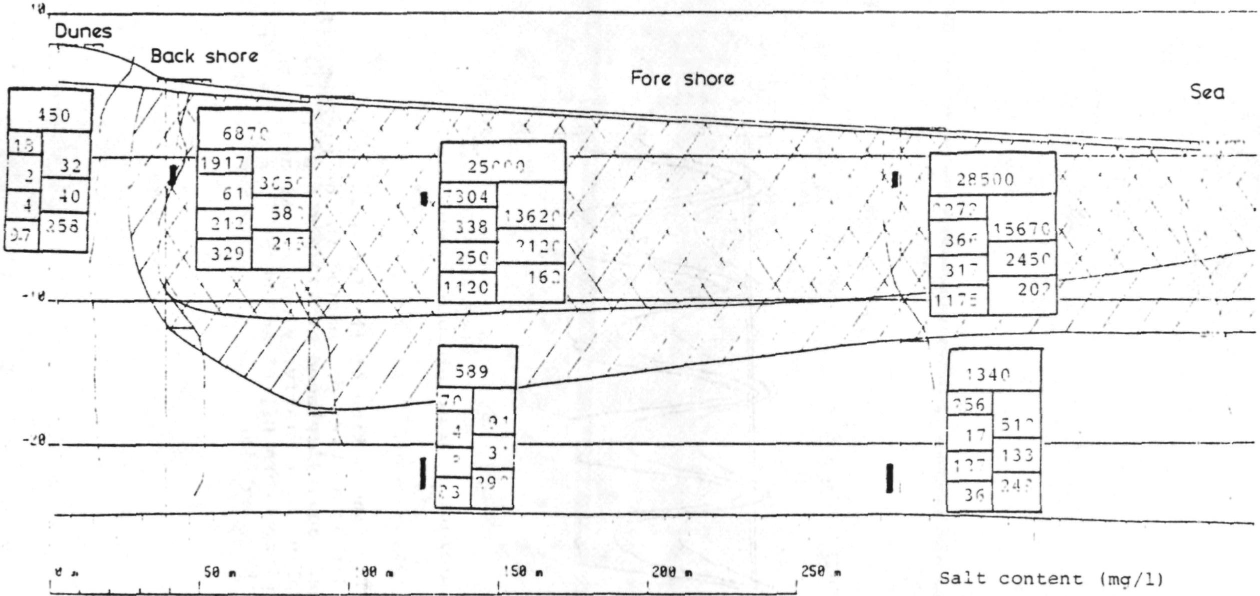


Fig. 5 - The fluctuation of the fresh-water head during a three day period in the piezometers P2' and P4' placed at -22 m T.A.W. in the plane of resistivity profile K0 and in the piezometers P2, P5 and P6 at -2 m T.A.W. in the same plane



Salt content (mg/l)

Total	
Na^+	Cl^-
K^+	SO_4^{--}
Mg^{++}	HCO_3^-
Ca^{++}	

RESISTIVITY LOGGING

Resistivity of the well surrounding sediments, ρ m, measured with the long normal device (AM = 1 m)

ρ ... 20 Ωm

2,5 Ωm<ρm<20 Ωm

ρm<2,5 Ωm

CHEMICAL ANALYSES

Location of screen pumped for collecting groundwater sample

Impermeable substratum

Level in m T.A.W.

Fig. 6 - The results of the chemical analyses of groundwater samples collected in the plane of resistivity profile K0

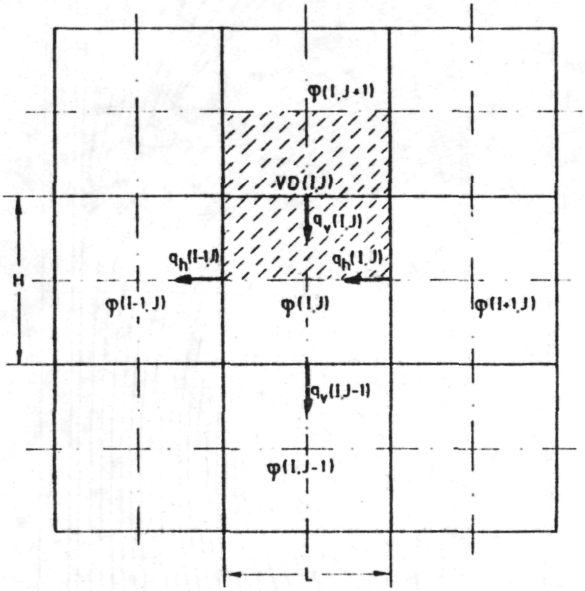


Fig. 7 - The superposed grid on the mathematically considered vertical plane with the fresh-water heads φ(I-1,J), φ(I,J), φ(I+1,J), φ(I,J+1), φ(I,J-1), the horizontal seepage velocities q_h(I-1,J), q_h(I,J), the vertical seepage velocities q_v(I,J), q_v(I,J-1) and the mean areal bouyancy VD(I,J) of the dashed area between the points I,J and I,J+1

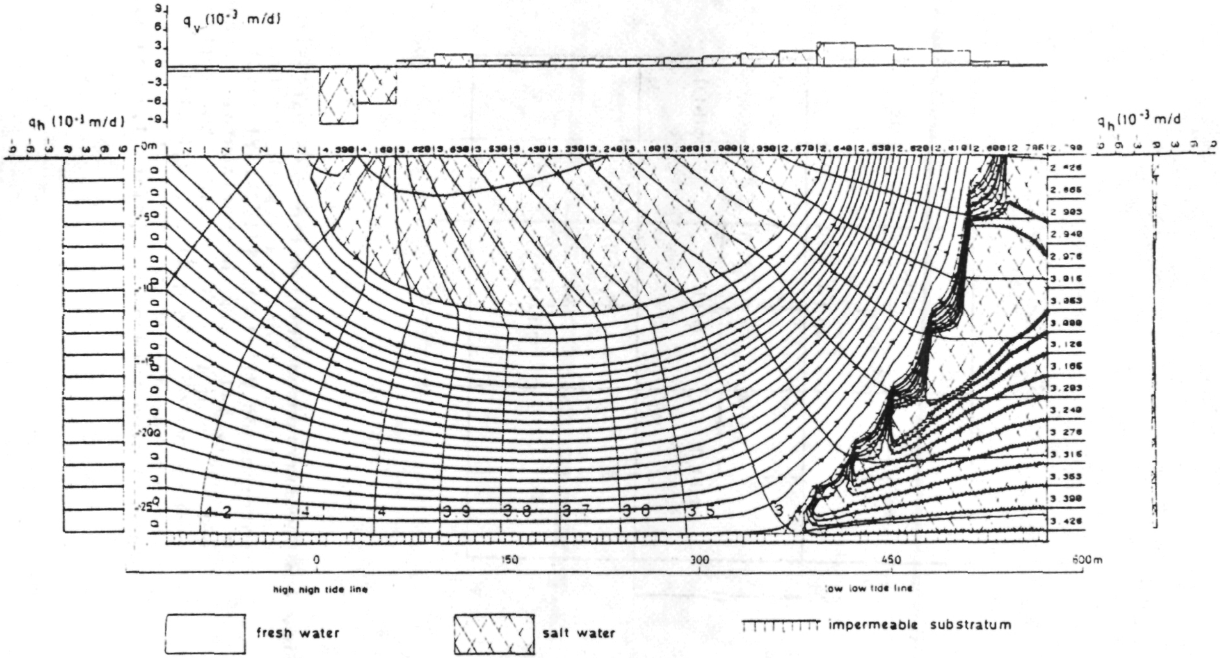


Fig. 8 - The flow of salt and fresh water and the lines of equal fresh-water head with a seaward fresh-water flow of 0,25 m^2/d

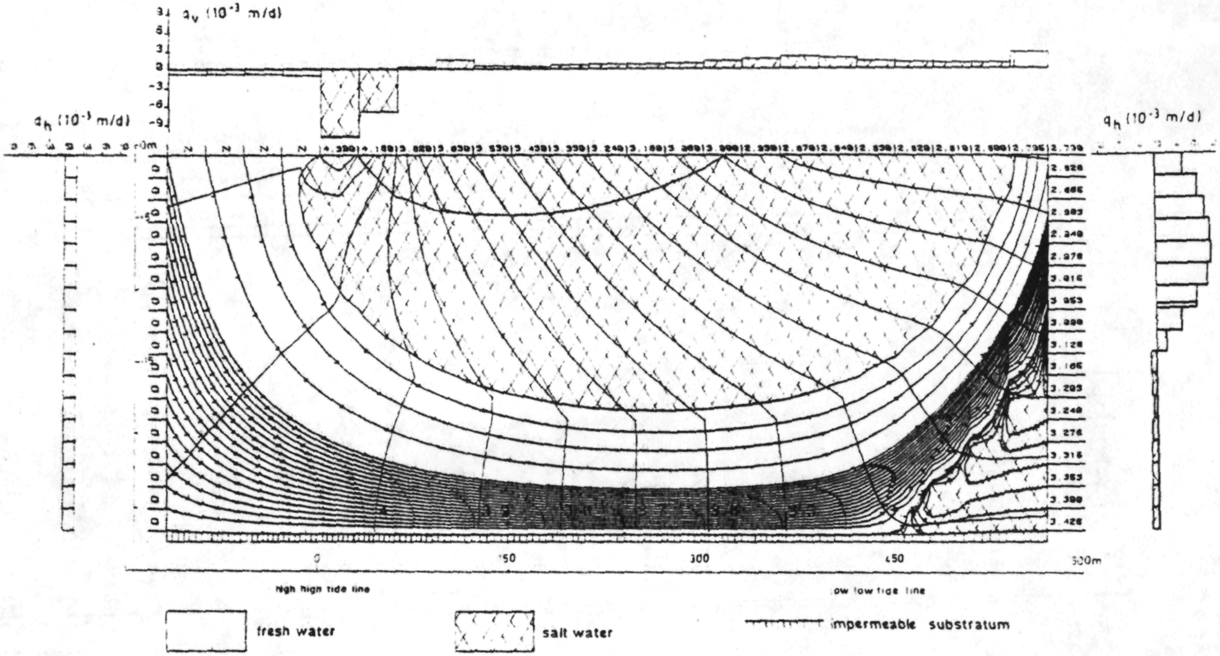


Fig. 9 - The flow of salt and fresh water and the lines of equal fresh-water head with a seaward fresh-water flow of 0,05 m²/d

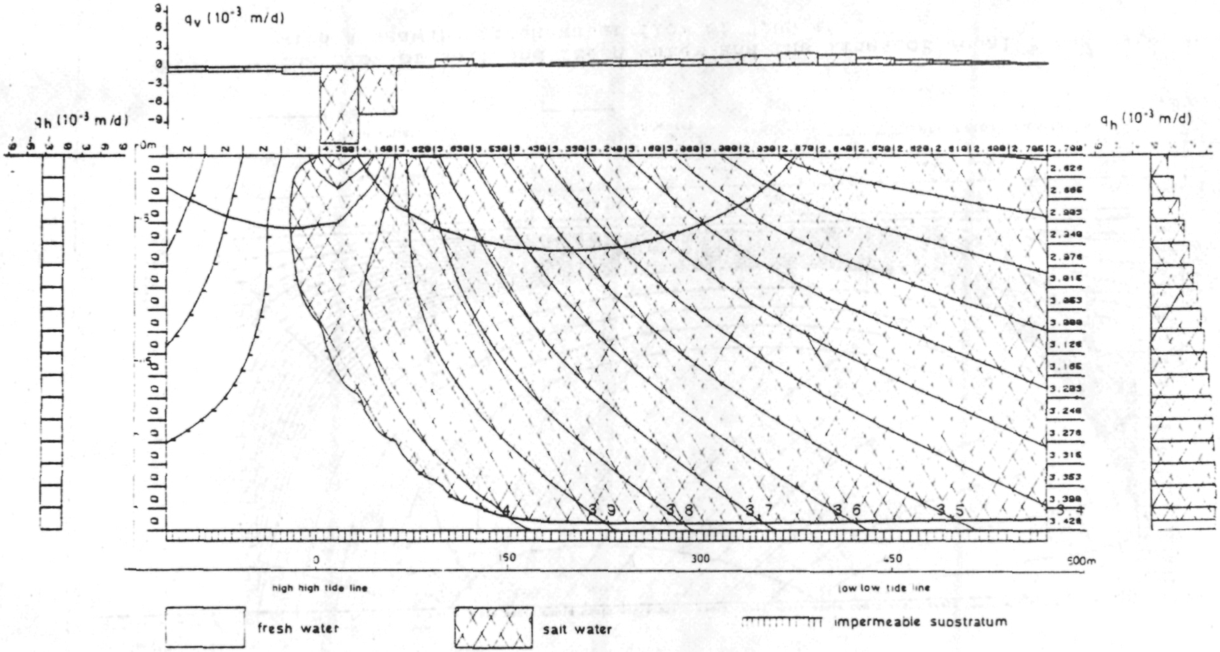


Fig. 10 - The flow of salt and fresh water and the lines of equal fresh-water head with a landward fresh-water flow of 0,09 m²/d