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MODELE MATHEMATIQUE DE LA POLLUTION EN MER DU NORD.

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Contribution of the nuclear power plants at Doel to the temperature profile of the Scheldt estuary

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1. Introduction

A nuclear power plant producing 2000 MW with PWR reactors will have to dissipate 4000 MW ($\sim 10^9$ cal/s) into the environment. The average water flow rate at Doel being approximately 100 m^3/s, one could expect a 10°C mean temperature rise. But at every turning of the tide, the flow stops and even higher temperatures could be reached.

Two phenomena will however limit these temperature changes : heat transfer to the surroundings and turbulent dispersion. On behalf of Pr. Dr. I. Elskens, we have set up a model to predict the effect of the interplay of these phenomena.

2. Hydraulics

The flow pattern in the Scheldt estuary is quite complicated but constantly monitored, so that fairly accurate data are available to represent the water movement. The "Ministerie van Openbare Werken" regularly published the values of the wetted cross-sectional area Ω and of the local velocity u as functions of the distance to the estuary mouth x and of the time during one mean tide t (the mean over some definite period). A one-dimensional non stationary model underlies these useful data.

In the present report, Fourier-series are used to represent the mean data of 1950 [1] in analytical form. For want of time, only the first two harmonics were used.

$$u(x,t) = a_0(x) + a_1(x) \sin wt + a_2(x) \sin 2\,wt + b_1(x) \cos wt + b_2(x) \cos 2\,wt \qquad (1)$$

$$\Omega(x,t) = c_0(x) + c_1(x) \sin wt + c_2(x) \sin 2\,wt + d_1(x) \cos wt + d_2(x) \cos 2\,wt \qquad (2)$$

This yields a fair approximation for u and Ω , as can be seen from fig. 1.

It should be noticed that Ω is measured and u is estimated from the conservation law for water :

$$\frac{\partial \Omega}{\partial t} + \frac{\partial (\Omega u)}{\partial x} = v' \qquad (3)$$

where v' is the flowrate of affluents per unit length of the estuary.

3. Heat transfer phenomena

The energy conservation equation is in this case :

$$D \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} + (u - D\frac{\partial \ln \Omega}{\partial x}) \frac{\partial T}{\partial x} + (\frac{v'}{\Omega} + \frac{q}{hc_p\rho}) (T - T_u) \qquad (4)$$

The eddy diffusivity D has been estimated at 200 m^2/s in several independent measurements [2] . The heat transfer coefficient was calculated from data on evaporation ponds [3] and the term

$$B = \frac{v'}{\Omega} + \frac{q}{hc_p\rho} \qquad (5)$$

was taken as $10^{-7}s^{-1}$.

4. Solution method

The partial differential equation (4) can be cast into a simpler form using the following coordinate transformation :

$$d\xi = dx - (u - D\frac{\partial \ln \Omega}{\partial x}) dt \qquad (6)$$

$$d\tau = dt \qquad (7)$$

$$t=0 : x=\xi$$

Then (4) becomes

$$D \frac{\partial^2 T}{\partial \xi^2} = \frac{\partial T}{\partial \tau} + B(T - T_u) \qquad (8)$$

Since far (see appendix) from Doel $T = T_u$, and a constant heat flow Q' is released at Doel, the solution of (8) is :

$$T(\xi,\tau) - T_u = \frac{Q'}{2\sqrt{\pi D}} \int_0^\infty \frac{e^{-\frac{\xi^2}{4D\sigma} - B\sigma}}{\Omega(0,\tau-\sigma)\sqrt{\sigma}} d\sigma \qquad (9)$$

The problem is thus reduced to the integration of two ordinary differential equations with initial conditions, namely the integration of (6) with $d\xi=0$, and the computation of $T(\xi,\tau)$ from (9) ; these computations were performed on the CDC 6500 of the VUB-ULB Rekencentrum.

5. Results

The temperature profiles at time intervals of a quater period are given in fig. 2. Arrows indicate the sequence of the profiles, the leftmost and rightmost curves obtain right after the turning of the tide and the other two when the velocity is maximum seawards or streamupwards. The average flow rate at Doel was 140 m^3/s and the corresponding maximum temperature, of about 7°C, could have been estimated from a simple stationary heat balance. The details of the temperature profiles are revealed only by this more realistic model : existence of a sharp temperature peak at the turning of the tide and periodic movement of the temperature profile streamupwards from Doel (between 0 and 20 km). For a distance of 10 km, fig. 3 gives the temperature as a function of time : over a full tide, the maximum temperature variation is 3.9°C.

6. Further comments

It would be interesting (and not extremely time consuming) to repeat these computations for other realistic values of the parameters. The values of the heat transfer coefficient q and of the mean velocity $a_0(x)$ are likely to change with meteorological conditions by up to a factor of 10.

It would, for instance, be instructive to compute the temperature profiles for the case where the mean water flow rate has dropped to 25 m^3/s for an extended period of time, and to see how the thermal efficiency of the plant is affected by the warmer cooling water.

7. Appendix

The solution (9) was obtained under the hypothesis that the length of the estuary is effectively infinite. The validity of the hypothesis was checked as follows. A stationary model is obtained by dropping the non stationary term, $\frac{\partial T}{\partial t}$, from (4). The resulting ordinary differential equations is then integrated (on an AD2-64PB analog computer) under the following boundary conditions and for $Q' = 10^9$ cal/s :

a) $T = T_u$ at $x = +\infty$

$T = T_u$ at $x = -\infty$

en b) $T = T_u$ at $x = +\infty$

$T = T_u$ at $x = -61500$ m (Vlissingen).

As expected, the results (fig. 4) are quite similar in the vicinity of Doel.

8. Notations

- B defined by eqn. (5) (s^{-1})
- C_p specific heat (J/kg°K)
- D eddy diffusivity (m^2/s)
- h average depth (m)
- Q' heat input a Doel (W)
- t time, after high tide at VLISSINGEN (s)
- T temperature (°C)
- T_u temperature of surroundings (°C)
- u velocity (m/s)
- v' flow rate of affluents per unit length (m^2/s)
- w angular velocity (s^{-1})
- x distance from DOEL, positive towards land (m)
- ξ distance, eqn. (6)
- τ time, eqn. (7)
- ρ specific mass (kg/m^3)
- Ω cross sectional area (m^2)

9. References

[1] Ministerie der Openbare Werken (Belgium) stormvloeden op de Schelde, deel 4, 1966.

[2] Origine et mécanismes de l'envasement de l'estuaire de l'Escaut. Ch. VII ULB - 1973 - (Pr. R. Wollast et al.).

[3] PERRY, ed. Chemical Engineers Handbook 5th ed. Mc Graw Hill New York 1974.

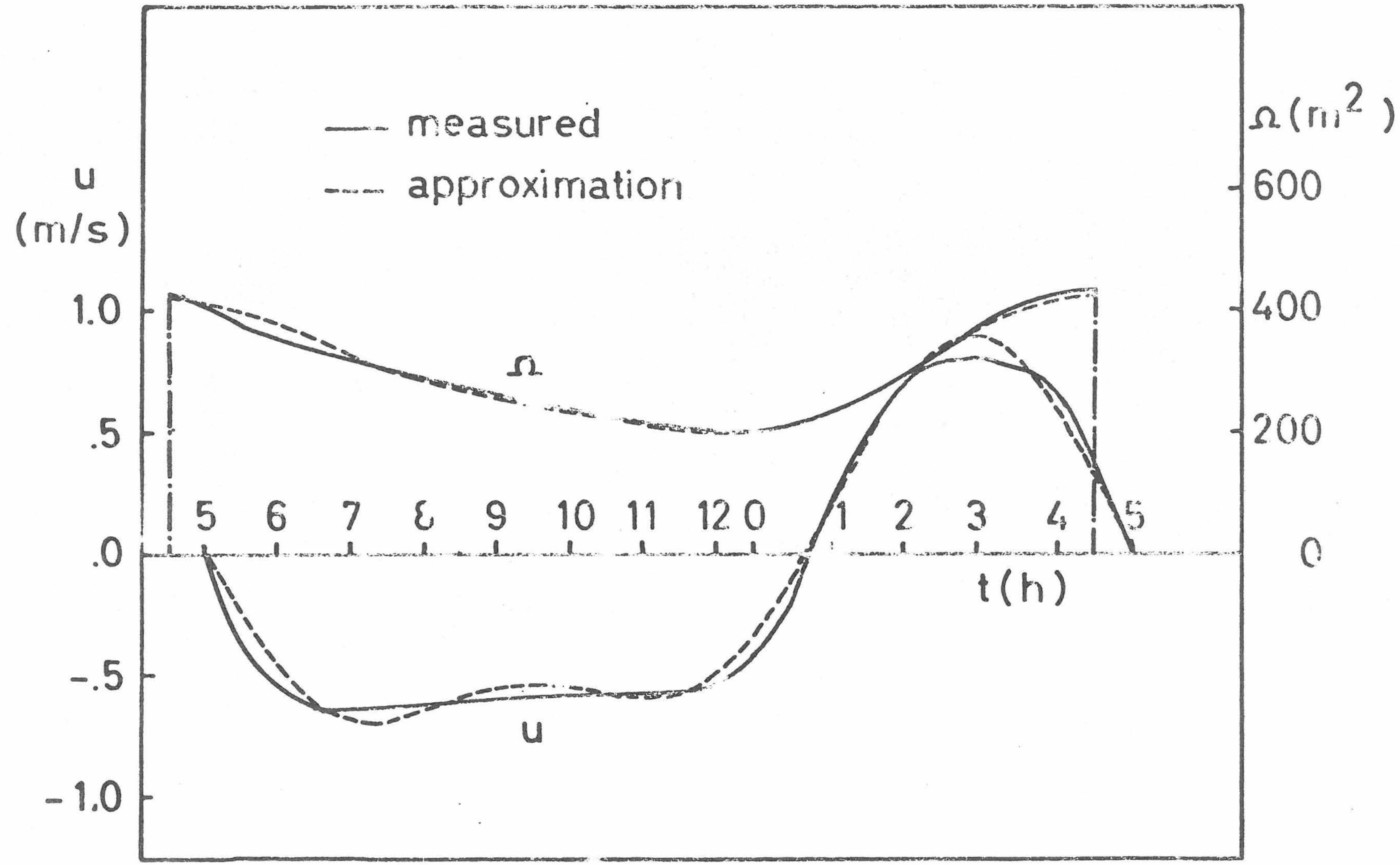
measured
approximation
u
(m/s)
1.0
.5
.0
-.5
-1.0
Ω(m²)
600
400
200
0
5 6 7 8 9 10 11 12 0 1 2 3 4 5
t(h)
Ω
u

Fig.1

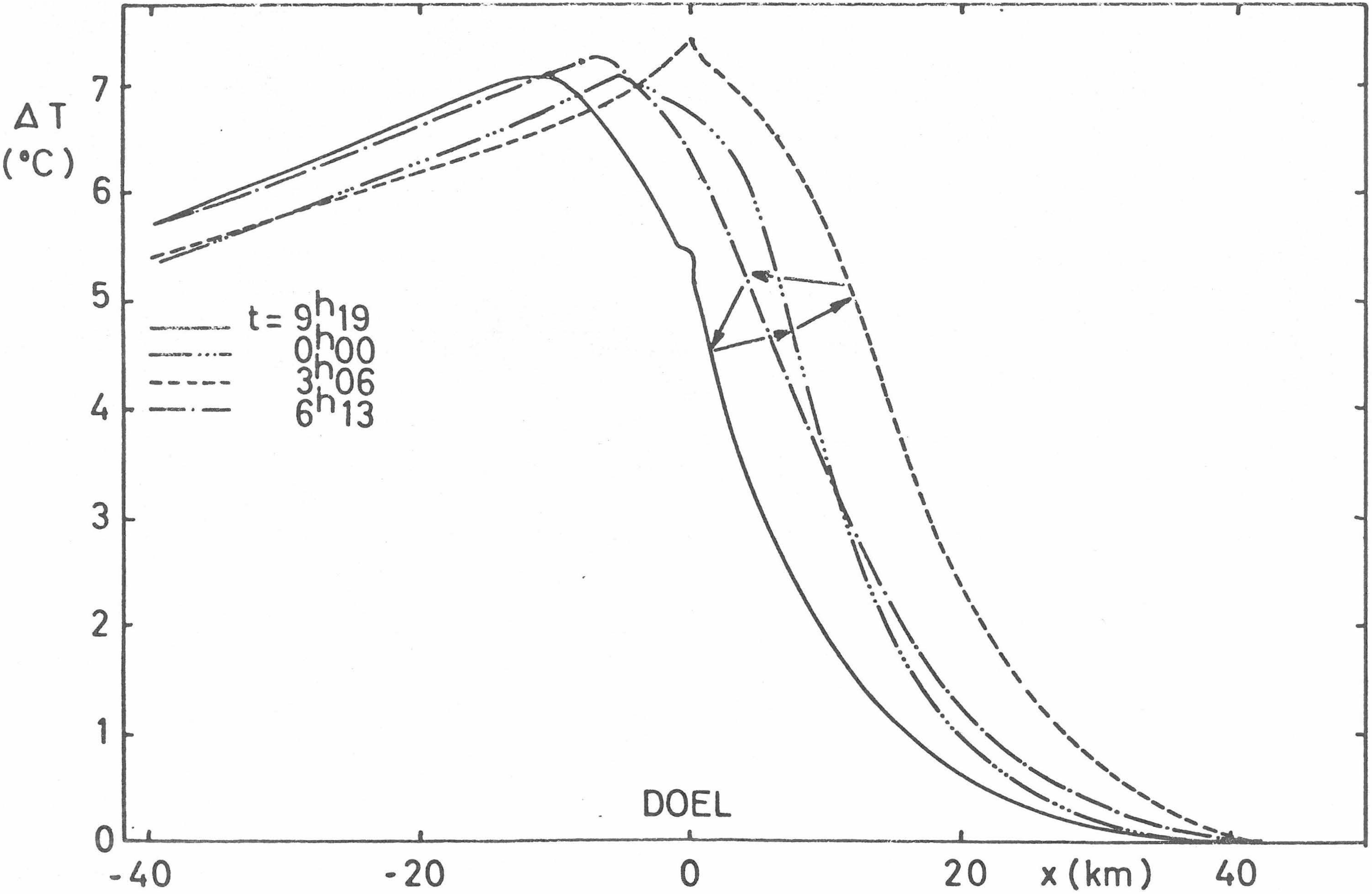

Fig.2

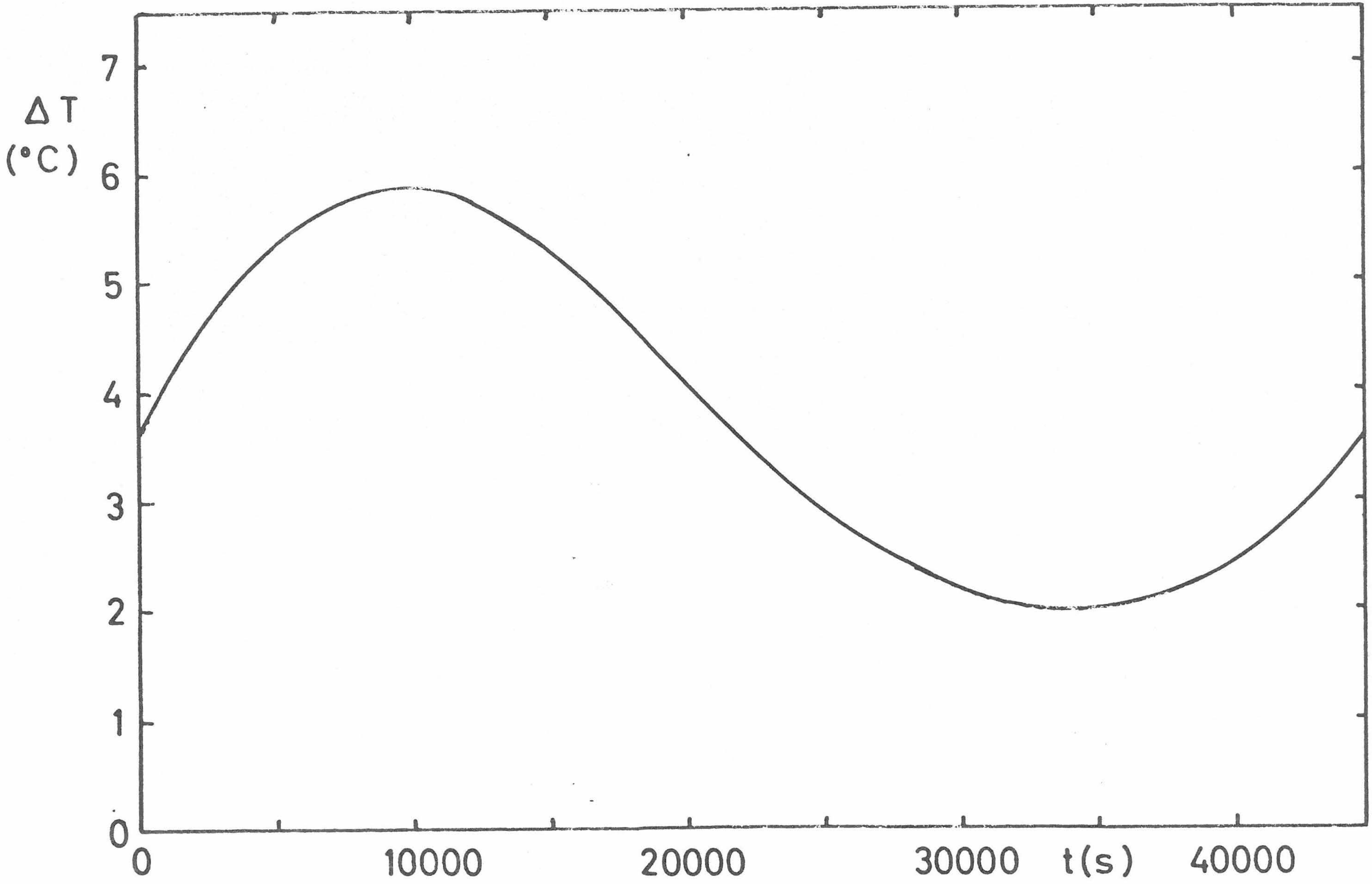

Fig. 3

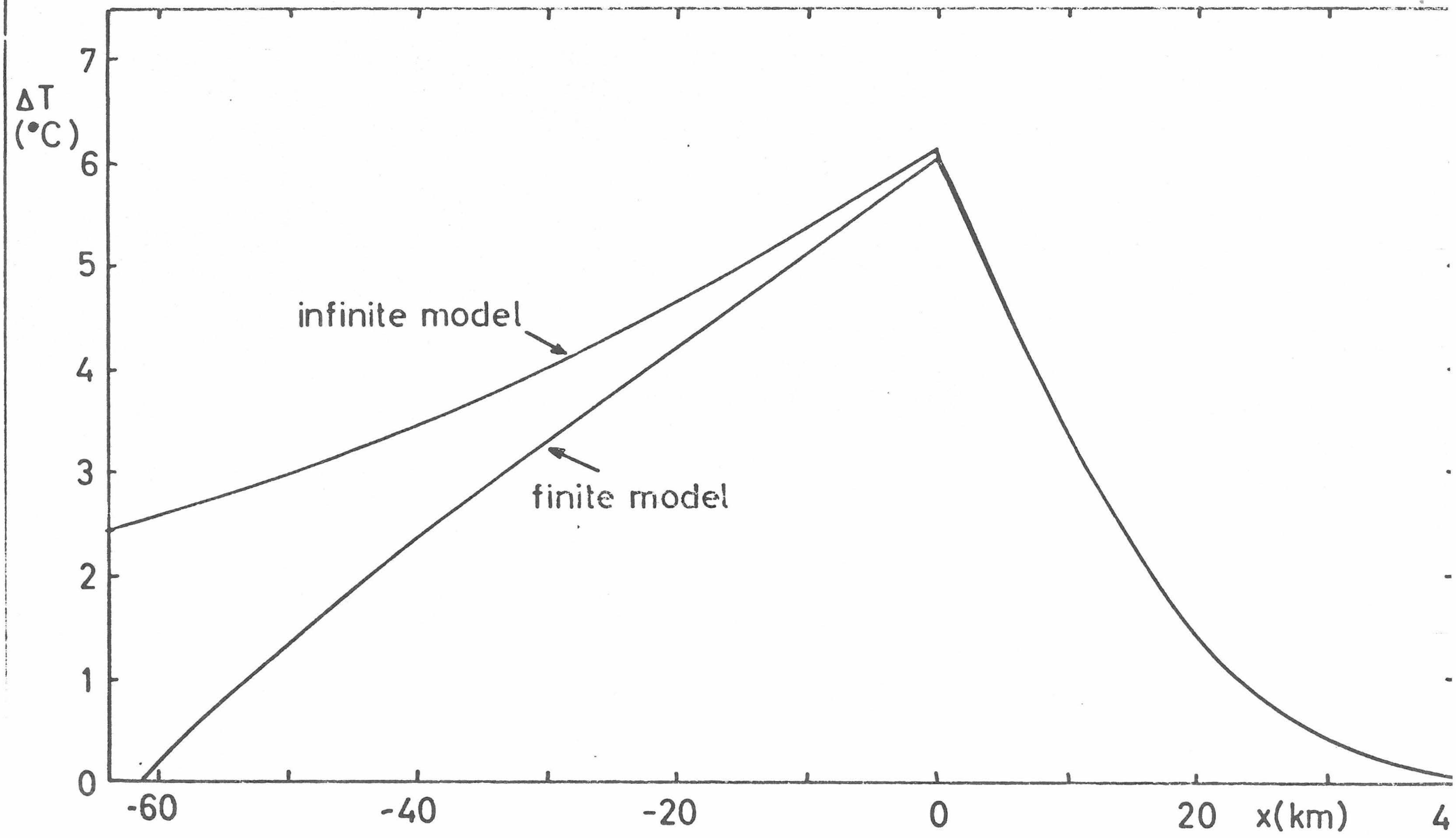

Fig.4