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TOWARDS A LOCAL DIAGNOSTIC METHOD

Grigory Isayev

ABSTRACT

Closed expressions for absolute velocity as a function of density, surface wind stress, and bottom topography are derived from the governing equations of geostrophy, hydrostaticity, incompressibility, and conservation of mass plus the vertical boundary conditions of Ekman pumping at the ocean surface and no-normal-flow at the ocean bottom. These expressions are horizontally local and are lower in order of derivatives than the Needler formula.

1. INTRODUCTION

The steady motion of an incompressible ideal fluid of low Rossby number is described by the geostrophic, hydrostatic equations of motion, continuity, and conservation of mass (Welander, 1971; Needler, 1985; Pedlosky, 1987)

$$\rho \mathbf{f} \times \mathbf{u} = -\nabla P - g\rho \mathbf{k} \qquad (1)$$

$$\nabla \cdot \mathbf{u} = 0 \qquad (2)$$

$$\mathbf{u} \cdot \nabla \rho = 0 \qquad (3)$$

Here $\mathbf{u} = (u, v, w)$ is the three-dimensional velocity, ρ the density, P the pressure, g the gravitational acceleration, $\mathbf{k}$ a unit vector along the vertical axis z, and $\mathbf{f}$ the Coriolis parameter f times $\mathbf{k}$.

As has been shown by Welander (1971), these equations lead to conservation of the large-scale potential vorticity, $q = f(\partial\rho/\partial z)$, along a streamline

$$\mathbf{u} \cdot \nabla q = 0 \qquad (4)$$

As a consequence of (3)–(4) the absolute velocity can be represented in the form (assuming that $\mid \nabla\rho \times \nabla q \mid$ is not zero)

$$\mathbf{u} = \gamma \mathbf{n}, \quad \mathbf{n} = \frac{\nabla\rho \times \nabla q}{\mid \nabla\rho \times \nabla q \mid} \qquad (5)$$

where $\mathbf{n}$ is a unit vector which according to (3), (4) is normal to $\nabla\rho$ and ∇q simultaneously and γ is a scalar coefficient of proportionality. Also available as a consequence of (1)–(3) are the thermal wind relations

$$\frac{\partial \mathbf{u}_H}{\partial z} = -\frac{g}{\rho f}(\mathbf{k} \times \nabla\rho) \qquad (6)$$

where $\mathbf{u}_H = (u, v, 0)$ is a horizontal projection of $\mathbf{u}$. Below the subscripts H and z designate horizontal and vertical projections of a vector).

The fascinating fact that within the governing equations (1) – (3) the constant of integration of the thermal wind relations can be determined from density field was noticed by Stommel and Schott (1977) within the β-spiral method and by Killworth (1979) within the analysis of section data. A closed expression for absolute velocity as a function of density was derived from (1) – (3) by Needler (1985) based on the representation (5). The Needler formula written in terms of notations (5) looks as follows

$$\mathbf{u} = \frac{g}{\rho}\frac{1}{\mid \nabla\rho \times \nabla q \mid}\left(\frac{\mathbf{k}\cdot\mathbf{n}}{\mathbf{n}\cdot\nabla\left(f\frac{\partial q}{\partial z}\right)}\right)\mathbf{n} \qquad (7)$$

This formula shows (Pedlosky, 1987) that for an ideal fluid on a global scale the density field completely determines the absolute velocity field. The Needler formula is local, i.e. the absolute velocity at certain point is determined by the density field in the vicinity of this point. It does not involve horizontal or vertical integration and does not require horizontal or vertical boundary conditions. The Needler formula is of the third order in derivatives of density and contains all three components of $\mathbf{n}$. Vertical component of $\mathbf{n}$ is much smaller than its horizontal components ($n_z / \mid \mathbf{n}_H \mid = w / \mid \mathbf{u}_H \mid \sim 10^{-4}$). Higher derivatives of density and n_z are the troublesome terms that are very sensitive to errors in the input density field (for analysis of errors of vertical component of velocity computed from climatological data see Isayev and Levitus, 1995).

Isayev (1994) formulated a system of linear algebraic equations for computation of absolute velocity from density, surface wind stress, and bottom topography based on the governing equations (1), (2), advection-diffusion balance of density, an assumption that there is a thermocline, and the vertical boundary conditions of Ekman pumping at the surface and no normal flow at the bottom. These equations are applicable in the areas where there is a thermocline and do not contain derivatives of the input data higher than first.

Chu (1994) used combination of equations (5) and (6) to derive a system of linear algebraic equations for computation of horizontal components of absolute velocity from density field. These equations are of the second order in derivatives of density. The results of Chu (1994) computations of the North Atlantic circulation give rise to hope for practical applicability of a local diagnostic method based on the governing equations (1) – (3).

In Section 2 of this work a closed expression for absolute velocity as a function of density is derived from the governing equations (1) – (3) that is of the same order in derivatives of density as the Needler formula, but does not contain n_z. In Sections 3 and 4 the closed expressions for the absolute velocity as a function of density, surface wind stress, and bottom topography are derived from the governing equations (1) – (3) plus the vertical boundary conditions of Ekman pumping at the surface and no-normal-flow at the bottom which are correspondingly of the second and of the first order in derivatives of the input data indicated above.

2. AN ALTERNATIVE TO THE NEEDLER FORMULA

Combination of the thermal wind relations (6) with z derivative of the horizontal projection

$$\mathbf{u}_H = \gamma \mathbf{n}_H \tag{8}$$

of equation (5) leads to

$$\frac{\partial \gamma}{\partial z}\mathbf{n}_H + \gamma\frac{\partial \mathbf{n}_H}{\partial z} = -\frac{g}{\rho f}(\mathbf{k}\times\nabla\rho) \tag{9}$$

Vector product of equation (9) and vector $\mathbf{n}_H$ eliminates the term with $\partial\gamma/\partial z$

$$\gamma\left(\mathbf{n}_H\times\frac{\partial \mathbf{n}_H}{\partial z}\right) = -\frac{g}{\rho f}(\mathbf{n}_H\times(\mathbf{k}\times\nabla\rho)) = -\frac{g}{\rho f}\mathbf{k}(\mathbf{n}_H\cdot\nabla\rho) \tag{10}$$

Assuming that $(\mathbf{n}_H\times(\partial\mathbf{n}_H/\partial z))_z$ is not zero, z-component of equation (10) yields an expression

$$\gamma = -\frac{g}{\rho f}\frac{\mathbf{n}_H\cdot\nabla\rho}{\left(\mathbf{n}_H\times\frac{\partial \mathbf{n}_H}{\partial z}\right)_z} \tag{11}$$

For comparison, following is the Needler expression of γ written in terms of horizontal, H, and vertical, z, components of $\mathbf{n}$ and ∇

$$\gamma = \frac{g}{\rho}\frac{1}{|\nabla\rho\times\nabla q|}\frac{n_z}{\left(\mathbf{n}_H\cdot\nabla_H\left(f\frac{\partial q}{\partial z}\right)+n_z\frac{\partial}{\partial z}\left(f\frac{\partial q}{\partial z}\right)\right)} \tag{12}$$

Both (11) and (12) contain the spatial derivatives of density of up to the third order. However, formula (11) unlike (12) does not contain n_z.

3. FORMULA FOR ABSOLUTE VELOCITY OF THE SECOND ORDER IN DERIVATIVES

It follows from the thermal wind relations (6) that $\mathbf{u}_H$ can be represented in the form

$$\mathbf{u}_H = \mathbf{u}_{Hb} + \mathbf{u}_{HS} \tag{13}$$

where $\mathbf{u}_{Hb}$ is unknown bottom velocity and

$$\mathbf{u}_{HS} = -\int_{-h}^{z}\frac{g}{\rho f}(\mathbf{k}\times\nabla\rho)dz' \tag{14}$$

is the geostrophic shear relative to the bottom determined by ρ and h.

Substitution of (13) and (8) provides

$$\mathbf{u}_{Hb} + \mathbf{u}_{HS} = \gamma\mathbf{n}_H \tag{15}$$

Assuming that $\mathbf{n}_H\cdot\nabla_H Q$ is not zero, scalar product of equation (15) and $\nabla_H Q$, where $Q = f/h$ is the planetary vorticity,

$$\mathbf{u}_{Hb}\cdot\nabla_H Q + \mathbf{u}_{HS}\cdot\nabla_H Q = \gamma\mathbf{n}_H\cdot\nabla_H Q \tag{16}$$

yields an expression for γ

$$\gamma = \frac{\mathbf{u}_{Hb}\nabla_H Q + \mathbf{u}_{HS}\nabla_H Q}{\mathbf{n}_H\cdot\nabla_H Q} \tag{17}$$

In the right hand side of (17) $\nabla_H Q$, $\mathbf{u}_{HS}$, and $\mathbf{u}_{Hb}$, and $\mathbf{n}_H$ are determined by ρ and h, while two components of the horizontal bottom velocity $\mathbf{u}_{Hb}$ remain unknown. However, it is shown below as a consequence of the governing equations (1) – (3) and the vertical boundary conditions

$$w(z=0) = w_e = \mathbf{k}\cdot\nabla x\left(\frac{\tau}{f}\right) \tag{18}$$

$$w(z=-h) = w_b = -\mathbf{u}_{Hb}\cdot\nabla_H h \tag{19}$$

that the combination $\mathbf{u}_{Hb}\cdot\nabla_H Q$ can be expressed through ρ, h and and surface wind stress τ.

The condition of Ekman pumping (18) and the condition of no-normal-flow at the bottom (19) are known to be reliable (see e.g. Bogden et al., 1993; Isayev and Levitus, 1995). Note that (18) is equivalent to $(w+w_E)\,|_{z=O}= 0$, where $w_E(\lambda,\phi,z)$ is the Ekman component of vertical velocity that is small compared with the geostrophic component, w, below the surface Ekman layer, while $w_e(\lambda,\phi)$ is the Ekman pumping determined by the surface wind stress.

Vertical integration of the geostrophic vorticity balance

$$\beta v = f\frac{\partial w}{\partial z} \tag{20}$$

that follows from (1) – (3) and use of the vertical boundary conditions (18) – (19) yields

$$\beta\int_{-h}^{0} v dz = f(w_e + \mathbf{u}_{Hb}\cdot\nabla_H h) \tag{21}$$

On the other hand, according to (13), (14)

$$\int_{-h}^{0} v dz = hv_b + V_S \tag{22}$$

where

$$V_S = \int_{-h}^{0} v_S dz = -\int_{-h}^{0} z\frac{\partial\rho}{R\cos\phi\partial\lambda}dz \tag{23}$$

V_S is the geostrophic shear component of meridional transport determined by ρ and h. Combination of (21) and (22) and regrouping leads to the equation

$$u_b f \frac{\partial h}{R\cos\phi \partial\lambda} + v_b\left(f\frac{\partial h}{R\partial\phi} - \beta h\right) = - f w_e + \beta V_S \quad (24)$$

where λ is the longitude, ϕ the latitude, and R the Earth radius. (24) can be transformed into

$$\mathbf{u}_{Hb} \cdot \nabla_H Q = (1/h^2)(f w_e - \beta V_S) \equiv B \quad (25)$$

The expression in (25) designated by B is of the first order in derivatives of density, surface wind stress, and bottom topography. Note that B is the component of the bottom flow normal to planetary vorticity contours. Equations that are equivalent to (25) were discussed in a different context by Mertz and Wright (1992), Bogden et al., (1993).

A closed expression for horizontal component of absolute velocity that follows from (8), (17), (25) has the form

$$\mathbf{u}_H = \frac{B + \mathbf{u}_{HS} \cdot \nabla_H Q}{\mathbf{n}_H \cdot \nabla_H Q}\mathbf{n}_H = \frac{B + \mathbf{u}_{HS} \cdot \nabla_H Q}{(\nabla\rho \times \nabla q)_H \cdot \nabla_H Q}(\nabla\rho \times \nabla q)_H \quad (26)$$

Expression (26) is horizontally local and is of the first order in derivatives of τ and h and of the second order in derivatives of ρ. Second derivatives of ρ are present in (26) only within $\mathbf{n}_H$ which in the present context is a horizontal unit vector (since $n_z/n_H \mid= w/\mid \mathbf{u}_H \mid\sim 10^{-4}, \mid \mathbf{n}_H \mid= 1-\delta, \delta \sim 10^{-4}$). Direction of $\mathbf{n}_H$ can be determined geometrically as intersection of the surfaces of constant ρ and constant q or, in other words, as direction of the line of constant potential vorticity drawn on isopycnal surface. Plots of such lines known as isopycnic potential vorticity maps are commonly available (e.g., Stammer and Woods, 1987).

For each horizontal location it is enough to determine horizontal components of absolute velocity at one level $z = z_a$. Then, the horizontal components of bottom velocity are determined by (13), (14) and (26)

$$\mathbf{u}_{Hb}(\lambda, \phi) = \mathbf{u}_H(\lambda, \phi, z_a) - \mathbf{u}_{HS}(\lambda, \phi, z_a) \quad (27)$$

and absolute velocities at different levels are deterined by (13), (14), and (27). Therefore, conservation of mass has to be employed only in the vicinity of one surface $z = z_a(\lambda, \phi)$. Such surface can be chosen at mid depths away from boundary layers, say, along the 27.5 kg $\cdot$ m^{-3} isopycnal where representation (5) with unknown γ leads to realistic streamlines (e.g., Stammer and Woods, 1987).

Vertical integration of (20) with the use of (22) leads to

$$\beta(hv_b + V_S) = f(w_e - w_b) \quad (28)$$

$$w_b = w_e - \frac{\beta}{f}(hv_b + V_s) \quad (29)$$

Given the horizontal components of bottom velocity, formula (29) allows one of calculate also its vertical component. Formula (29) is preferable for this purpose to the vertical component of equation (5) with γ determined by (26) or to the bottom boundary condition (19). This is because z-component of (5) involves the troublesome term $\mathbf{n}_z$ and because (19) is a scalar product of nearly parallel vectors $\mathbf{u}_{Hb}$ and ∇h (see discussion of this problem in Isayev and Levitus, 1995). Vertical component of absolute velocity at different levels can be determined from the integrated geostrophic vorticity balance (20)

$$w(z) = \frac{\beta}{f}\int_{-h}^{z}(v_b + v_S)dz' + w_b \quad (30)$$

where v_b, w_b, and v_S are determined by (26), (27); (29), and (14).

4. FORMULA FOR ABSOLUTE VELOCITY OF THE FIRST ORDER IN DERIVATIVES

The condition of no-normal-flow at the bottom (19) can be written in the form

$$\mathbf{u}_b \cdot \nabla h = 0 \quad (31)$$

where $\mathbf{u}_b = (\mathbf{u}_H, w_b)$ and by definition $\nabla h = (\nabla_H h, 1)$. As a consequence of (3) and (31) the bottom velocity can be represented in the form (assuming that $\mid (\nabla\rho)_b \times \nabla h \mid$ is not zero)

$$\mathbf{u}_b = \alpha\ \mathbf{m}_b, \quad \mathbf{m}_b = \frac{(\nabla\rho)_b \times \nabla h}{\mid (\nabla\rho)_b \times \nabla h \mid} \quad (32)$$

where $\mathbf{m}_b$ is a unit vector directed along the bottom velocity streamline which according to (3) and (31) is normal to $(\nabla\rho)_b$ and ∇h simultaneously, and α is a scalar coefficient of proportionally.

Substitution of the horizontal projection of (32)

$$\mathbf{u}_{Hb} = \alpha\ \mathbf{m}_{Hb} \quad (33)$$

into equation (25) leads to

$$\alpha\ \mathbf{m}_{Hb} \cdot \nabla_H Q = B \quad (34)$$

Assuming that $\mathbf{m}_{Hb} \cdot \nabla_H Q$ is not zero, equation (34) yields a closed expression for α and consequently equation (32) yields a closed expression for bottom velocity

$$\mathbf{u}_b = \frac{B}{\mathbf{m}_{Hb} \cdot \nabla_H Q}\mathbf{m}_b = \frac{B}{((\nabla\rho)_b \times \nabla h) \cdot \nabla_H Q}((\nabla\rho)_b \times \nabla h) \quad (35)$$

Formula (35) determines all three components of the reference (bottom) velocity as a horizontally local function of density, surface wind stress, bottom topography, and their first derivatives. Direction of the unit vector $\mathbf{m}_b$ can also be determined geometrically as direction of line of constant potential density drawn on the ocean bottom. As has been discussed in Section 3, it is preferable to calculate w_b through equation (29) rather than through (35).

Absolute horizontal velocities at different levels are determined by (35) and by integrated thermal wind relations (13), (14). Then absolute vertical velocities at different levels are determined by (30).

5. CONCLUSION

The Needler formula for absolute velocity (equations (5), (12)) which is a consequence of the governing equations (1) – (3) is of the third order in derivatives of density and contains the troublesome term $\mathbf{n}_z$. An alternative formula for absolute velocity (equations (5), (11)) derived in this work from the same governing equations is of the same third order in derivatives of density but does not contain n_z. Imposition of the vertical boundary conditions (18), (19) in addition to the same governing equations (1) – (3) allowed us to derive the horizontally local formulae for the absolute velocity of the second order (equations (5), (26)) and of the first order (equations (32), (35), (13), (14)) in derivatives of density, surface wind stress and bottom topography.

The main advantage of a horizontally local diagnostic method based on the formulae discussed above over the established nonlocal diagnostic methods (e.g., Mellor et al., 1982) is that it does not involve horizontal integration and does not require horizontal boundary conditions. For this reason such a method is applicable in the regions without continuous horizontal boundaries and in the areas within closed f/h lines that are characteristics of the basic equation of the Mellor et al. (1982) method.

The absolute geostrophic velocity computed using closed analytical expressions discussed above can be used for initialization of prognostic models. Use for such initialization of the climatological density field and the absolute geostrophic velocity field balanced with it, rather than zero velocity field, may reduce the spin-up time (Chu, 1994). Indeed, a greatly reduced spin-up time in the analogous situation is pointed out by Ezer and Mellor (1994).

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A METHODOLOGY FOR MODEL INTERCOMPARISON: PRELIMINARY RESULTS

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INTRODUCTION

Over recent years the intercomparison of shallow sea model results has been performed in different domains of interest, e.g. the North Sea (Peeck et al., 1983; de Vries, 1992; de Vries et al., 1995), the Adriatic Sea (de Vries et al., 1995), the Aegean Sea (de Vries et al., 1995), the English Channel (Jamart and Ozer, 1989; Werner, 1989, 1995), the plume of the Rhine River (Ruddick et al., 1995) or the Halten Bank area, off Norway (Hackett and Roed, 1994).

All of these exercises followed relatively similar procedures. First, the model runs to be carried out were defined so that the models had sufficiently similar computational domains and forcings. Second, the distance between the model results – and, in most cases, between the model results and the observations – was evaluated according to various norms. Finally, attempts were made to identify the reasons for the discrepancies between the model results.

It seems that the first and second tasks were generally carried out successfully. The third stage, however, did not always lead to detailed or useful conclusions. One of the reasons underlying this partial failure was obviously the difficulty to link cause and effect when dealing with complex, non-linear models. In view of these difficulties, we have decided to seek techniques that could facilitate the understanding of the discrepancies between model results. Here, we provide a preliminary analysis of the potential for intercomparison studies of the "factor separation method", which was devised by Stein and Alpert (1993) in the scope of meso-scale atmospheric modelling.

AN INTERCOMPARISON METHOD

One of the most popular approaches for understanding how a given model is working consists in performing sensitivity studies. The latter may be performed in an extremely elegant and rational way by resorting to the method of Stein and Alpert (1993), which is outlined below.

In the governing equations, or in the forcings, switches may be introduced, so that appropriate terms may be "turned on" or "turned off". Hereafter, the switches are denoted s_i ($i = 1,\ 2,\ \ldots, n$). When a given switch is zero, the corresponding term is disabled, and when the switch is equal to 1, the corresponding term is active.

Here, we assume that switches may act on any kind of terms, be they part of the initial conditions, the boundary conditions or the governing equations. Stein and Alpert (1993), however, seem to adopt a more restrictive way of introducing switches.

Every quantity ψ considered in the analysis of the model results may be regarded as a function of the switches, i.e. $\psi = \psi(s_1,\ s_2,\ \ldots,\ s_n)$. The method may be applied whether or not ψ depends on time or space coordinates.

If there is only one switch, $\psi(1)$ is associated with the results obtained when the phenomenon concerned by the switch is present. Conversely, if the relevant process is disabled by setting the switch to zero, the variable ψ then reads $\psi(0)$. We may write

$$\psi(1) = \psi(0) + \Delta\psi(1), \qquad (1)$$

where $\Delta\psi(1) = \psi(1) - \psi(0)$ is a measure of the role of the process in which we are interested. Obviously, to evaluate the sensitivity of the model results to a given phenomenon, i.e. to compute $\Delta\psi(1)$, 2 runs of the model must be performed.

Computing $\Delta\psi(1)$ is typical of a model sensitivity study where the influence of one single process is examined. If several processes need to be considered at a time, say 2 for simplicity, one may compute $\Delta\psi(1,0) = \psi(1,0) - \psi(0,0)$ and $\Delta\psi(0,1) = \psi(0,1) - \psi(0,0)$. Nonetheless, in general, one may not write $\psi(1,1) = \psi(0,0) + \Delta\psi(1,0) + \Delta\psi(0,1)$, since the response of the model is unlikely to be linear. This is the stumbling block of certain sensitivity studies. Stein and Alpert (1993) suggested overcoming this difficulty by taking into account the "synergistic term", defined to be $\Delta\psi(1,1) = \psi(1,1) + \psi(0,0) - \psi(1,0) - \psi(0,1)$, so that

$$\psi(1,1) = \psi(0,0) + \Delta\psi(1,0) + \Delta\psi(0,1) + \Delta\psi(1,1). \qquad (2)$$

The synergistic term $\Delta\psi(1,1)$ may be regarded as a measure of the interactions between the two processes considered. As was shown by Khain et al. (1993), the synergistic term may be much larger than the classical sensitivity measures, $\Delta\psi(1,0)$ and $\Delta\psi(0,1)$. In such a situation, it would be foolhardy to deal with the classical sensitivity estimates only.

If the influence of 2 phenomena is investigated, 4 model runs are needed. When 3 processes are concerned, it is necessary to produce 8 sets of model results. In general, 2^n runs must be carried out. Whatever the number of phenomena considered, the model response may alway be written in terms of quantities obeying a simple additive principle that holds true in spite of the possible non-linearity of the model. The generalisation of expressions (1) and (2) may be found in Stein and Alpert (1993).

The choice of the processes to be singled out is crucial. Some choices may obviously lead to irrelevant results. In addition, the sensitivity of the model results to a given phenomenon depends on the whole set of phenomena on which switches are acting (Alpert et al., 1995).

We suggest applying the factor separation method in the framework of model intercomparison exercises. One would analyse the sensitivity of each model to a series of processes or forcings. A thorough analysis of the $\Delta\psi$'s would certainly help explaining the reason of the discrepancies between the models considered. Such an approach is illustrated below.

ILLUSTRATION

The usefulness of the factor separation technique for model intercomparison is illustrated with the help of MUMM's operational North Sea model (Adam, 1987). If η and $\mathbf{u}$ represent the sea surface elevation and the depth-averaged horizontal velocity, the equations of the model read

$$\frac{\partial \eta}{\partial t} + \nabla \cdot (H\mathbf{u}) = 0, \qquad (3)$$

$$\frac{\partial \mathbf{u}}{\partial t} + s_1 \mathbf{u} \cdot \nabla \mathbf{u} + f \mathbf{e}_z \times \mathbf{u} = -g\nabla\eta + \frac{\tau^S - s_2 \tau^b}{H} + A\nabla^2 \mathbf{u}, \qquad (4)$$

where t is time; ∇ denotes the "gradient operator"; f, $\mathbf{e}_z$, g and $A(= 10^4 \text{ m}^2 \text{ s}^{-1})$ represent the Coriolis factor, the vertical unit vector, the gravitational acceleration and the horizontal viscosity, respectively; H is the sea depth, while τ^S and τ^b stand for the specific surface and bottom stresses.

In all the simulations carried out here, the wind stress is neglected, i.e., $\tau^S = 0$. The flow in the computational domain is forced by prescribing the M2 tide elevation – which is the dominant tidal component in the North Sea – along the open sea boundaries. Each model run is carried out until a periodic regime is established.

The variable we are going to examine, i.e. ψ, is the amplitude of the M2 tide, obtained from a Fourier analysis of the model results. Thus, ψ is a time-independent quantity ensuing from the post-processing of the model results. It is also worth bearing in mind that ψ is a positive definite quantity, i.e. $\psi \geq 0$.

It is decided to investigate the respective roles of momentum advection and bottom friction. Accordingly, the switches s_1 and s_2 – pertaining to advection and bottom friction, respectively, – are introduced into the momentum equation (4).

To have different models, two versions of MUMM's model – hereafter referred to as A and B – are set up. The model A is that which is used in operational forecasting: the advection of momentum is discretized by a first-order upwind scheme and the bottom stress is computed as

$$\tau^b = C_D \mid \mathbf{u} \mid \mathbf{u}, \qquad (5)$$

where the bottom drag coefficient C_D is 2.40×10^{-3} in the English Channel and the Southern Bight, and is taken to be 2.04×10^{-3} in the rest of the computational domain. The model B is similar to A, except that the momentum advection is computed according to a second-order, centered scheme. In addition, the bottom stress is still parameterized by (5), but the drag coefficient is 2.32×10^{-3}, $g/(25 + H)^2$ and 1.21×10^{-3}, if $H \leq 40$ m, $40 < H \leq 65$ m and 65 m $< H$, respectively (Verboom et al., 1992).

In a "real-world" model intercomparison study, the models dealt with are generally so different that they are unlikely to provide similar results even when all switches are set to zero. To place ourselves in a similar situation, we have further modified the model B, by adding 5 meters to the unperturbed sea depth used in A. In other words, we have resisted the temptation of cheating by preventing $\psi^A(0,0)$ from being equal to $\psi^B(0,0)$...

When analysing the model results, use is made of the operator "<>", denoting the average over the computational domain.

Since

$$\frac{<\mid \psi^A(1,1) - \psi^B(1,1) \mid>}{(<\psi^A(1,1)> + <\psi^B(1,1)>)/2} = 0.19, \qquad (6)$$

the results of the complete models are significantly different. That $<\psi^A(1,1)>$ (= 1.2 m) is somewhat smaller than $<\psi^B(1,1)>$ (= 1.4 m) may indicate that there is more damping in A.

The space structures of ψ^A (1, 1) and ψ^B (1, 1), depicted in Figs. 1 and 2, exhibit some discrepancies, the interpretation of which is quite difficult.

The ratios

$$\left(\frac{<\mid \Delta\psi^A(1,0) \mid>}{<\psi^A(0,0)>}, \frac{<\mid \Delta\psi^B(1,0) \mid>}{<\psi^B(0,0)>} \right) = (0.25, 0.022) \qquad (7)$$

imply that A is much more sensitive to advection than B, which, at first, may seem surprising, since the Rossby number of the flow is of order 0.02 – if the velocity and space scales are taken to be 0.5 m s^{-1} and 250 km, respectively. It may be deemed reassuring that $<\mid \Delta\psi(1,0) \mid> / <\psi(0,0)>$ is of the same order of magnitude as the Rossby number for model B, while it is worrying that this ratio is about

10 times larger for model A. That the model A is overly sensitive to advection is most probably due to the first-order upwind discretization of the momentum advection. Carrying things to extremes, one might say that, in model A, the advection terms are, in fact, dissipative terms. This is somewhat confirmed by $< \Delta\psi^A(1,0) >$ being negative, because advection reduces the amplitude of the tidal elevation.

On the other hand, we have

$$\left(\frac{<\mid \Delta\psi^A(0,1) \mid>}{<\psi^A(0,0)>}, \frac{<\mid \Delta\psi^B(0,1) \mid>}{<\psi^B(0,0)>}\right) = (0.59, 0.42), \qquad (8)$$

implying that the effect of the bottom friction is slightly more important in A than in B, which is not surprising since, at most locations, the bottom drag coefficient is larger in A than in B. Nonetheless, in A and B, C_D is of the same order of magnitude, which is in agreement with the ratios (8) being also of the same order of magnitude.

The synergistic terms are

$$\left(\frac{<\mid \Delta\psi^A(1,1) \mid>}{<\psi^A(0,0)>}, \frac{<\mid \Delta\psi^B(1,1) \mid>}{<\psi^B(0,0)>}\right) = (0.25, 0.017). \qquad (9)$$

The relative smallness of the synergistic term of model B obviously ensues from the advection being truly negligible. For model A, however, it is conceivable that the synergistic term is significant because the bottom stress obeys a non-linear parameterization while the damping effect of advection and bottom friction is of equivalent importance.

CONCLUSION

A more profound analysis of the present numerical experiments will be given in Deleersnijder et al., (1995). Nevertheless, at the present stage, a useful – though preliminary – conclusion may be drawn. It is believed that the factor separation method offers relevant guide lines for performing intermodel sensitivity experiments, which are necessary to identify the very reasons of the discrepancies between the results of different models. In the example above, the discretization of the advection term of model A is shown to be inappropriate – in spite of the smallness of the Rossby number, which might have lead one to believe that advection is, in any case, unimportant. This problem could not have been identified by simply looking at crude model results, i.e. $\psi^A(1,1)$ and $\psi^B(1,1)$ as would have been done in a simple model intercomparison exercise.

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ON THE CONVERSATION OF POTENTIAL VORTICITY FOR VISCOUS CURRENTS

E. Salusti and R. Serravall

1. INTRODUCTION

In this note we discuss a particularly simple generalization of conserved quantities during oceanic processes, as salinity and potential vorticity, nicely discussed by Gill (1973) and Pedlosky (1989).

Let us in general consider a well defined oceanic current with velocities $u_i(x,y,z,t) = u_i(x_i,t)$. A physical quantity φ is conserved if:

$$\frac{d\varphi(x_i,t)}{dt} \equiv \frac{\partial\varphi}{\partial t} + \sum_i u_i \frac{\partial\varphi}{\partial x_i} = 0 \qquad (1)$$

One can easily show that if a quantity $\chi(x_i,t)$ has the property:

$$\frac{d\chi(x_i,t)}{dt} = \alpha(t)\chi(x_i,t) + \gamma(t) \qquad (2)$$

then it gives origin to a conserved quantity, namely φ_χ. Let us indeed consider:

$$\varphi_\chi = \chi(x_i,t)e^{-\int_{t_0}^{t}\alpha(t')dt'} + \beta(t) \qquad (3)$$

one can easily show that:

$$\frac{\partial}{\partial t}\varphi_\chi = -\alpha(t)\chi e^{-\int\alpha(t')dt'} + e^{-\int\alpha(t')dt'}\frac{\partial}{\partial t}\chi + \frac{\partial}{\partial t}\beta \qquad (4)$$

$$u_i\frac{\partial}{\partial x_i}\varphi_\chi = e^{-\int\alpha(t')dt'}\,u_i\frac{\partial\chi}{\partial x_i} \qquad (5)$$

and summing all this, one finally obtains:

$$\frac{d}{dt}\varphi_\chi = \frac{\partial}{\partial t}\beta(t) - e^{-\int\alpha(t')dt'}\alpha\chi + e^{-\int\alpha(t')dt'}\frac{d\chi}{dt}$$

$$\frac{\partial}{\partial t}\beta(t) - e^{-\int\alpha(t')dt'}\alpha\chi + e^{-\int\alpha(t')dt'}\alpha\chi \qquad (6)$$

$$+e^{-\int\alpha(t')dt'}\gamma(t)$$

So the quantity, φ_χ is conserved if we specify $\beta(t)$ such that:

$$\frac{\partial}{\partial t}\beta(t) + \gamma e^{-\int\alpha(t')dt'} = 0 \qquad (7)$$

These considerations give us some freedom in choosing conserved quantities, that indeed can satisfy either the classical equation (1) or the milder equation (2).

2. APPLICATION TO THE POTENTIAL VORTICITY OF OCEANIC CURRENTS

We now apply this viewpoint to the potential vorticity conservation as discussed by Pedlosky (1989), using his formalism and definitions. Indeed calling:

$$\vec{\omega}_a = 2\vec{\Omega} + \nabla\times\vec{u} = 2\vec{\Omega} + \vec{\omega} \qquad (9)$$

$\vec{F}$ the frictional terms

for a regular function λ one has the classical Ertel theorem:

$$\frac{d\Pi_\lambda}{dt} \equiv \frac{d}{dt}\left(\frac{\vec{\omega}_a}{\rho}\cdot\nabla\lambda\right) = \frac{\vec{\omega}_a}{\rho}\cdot\nabla\frac{d\lambda}{dt} + \nabla\lambda\cdot\left(\frac{\nabla\rho\times\nabla p}{\rho^3}\right) + \frac{\nabla\lambda}{\rho}\cdot\left(\nabla\times\frac{\vec{F}}{\rho}\right) \qquad (10)$$

To discuss frictional terms $\vec{F}$ that are well known to be important near the coasts, but also open Ocean currents can feel their effect, the term $\vec{F}$ can be seen in different viewpoints. One of the simplest and most popular approaches assumes:

$$\vec{F} = -\nu\rho\vec{u} \qquad \nu \in R \qquad (11)$$

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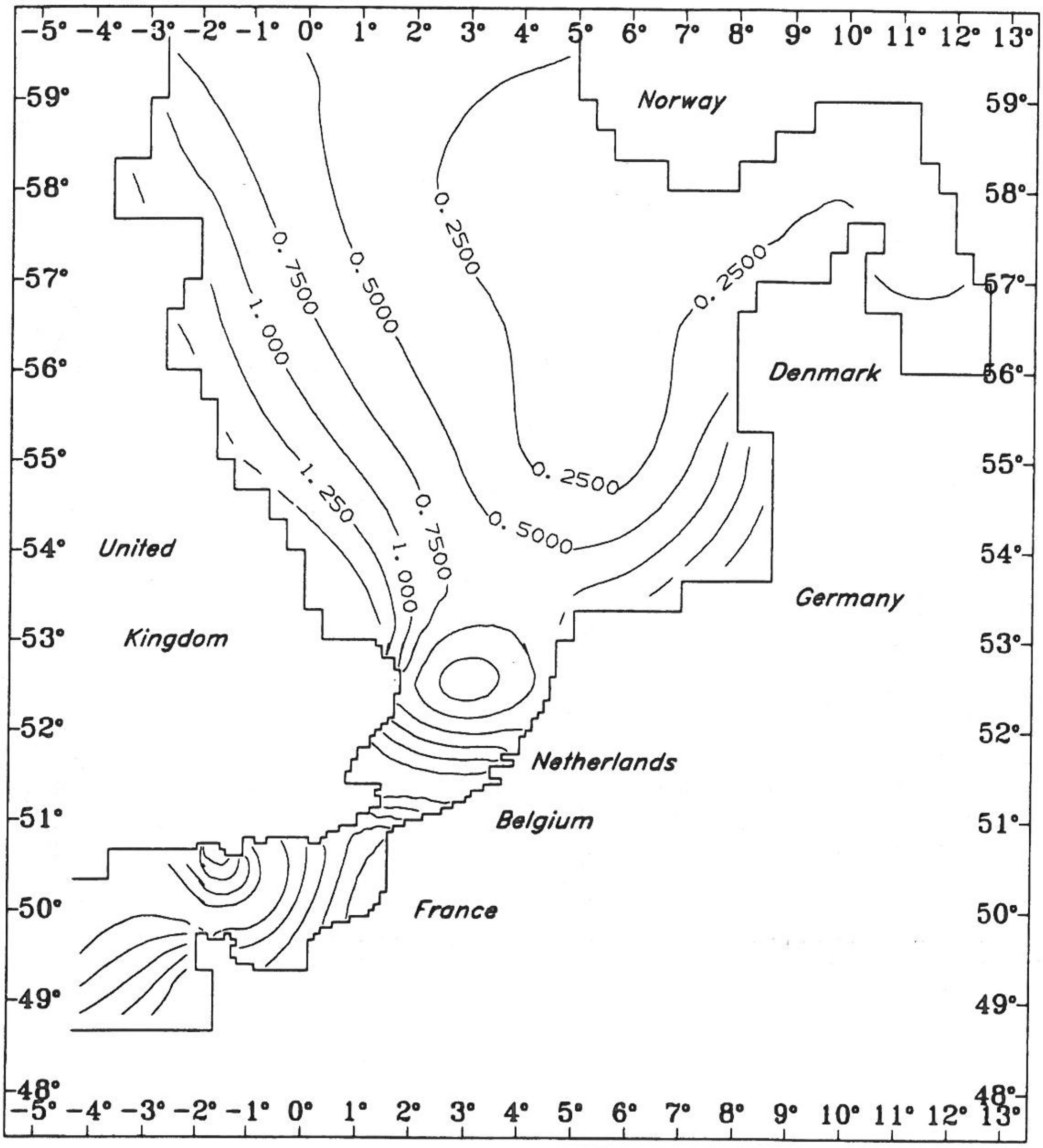

FIGURE 1
Deleersnijder, Ozer and Tartinville

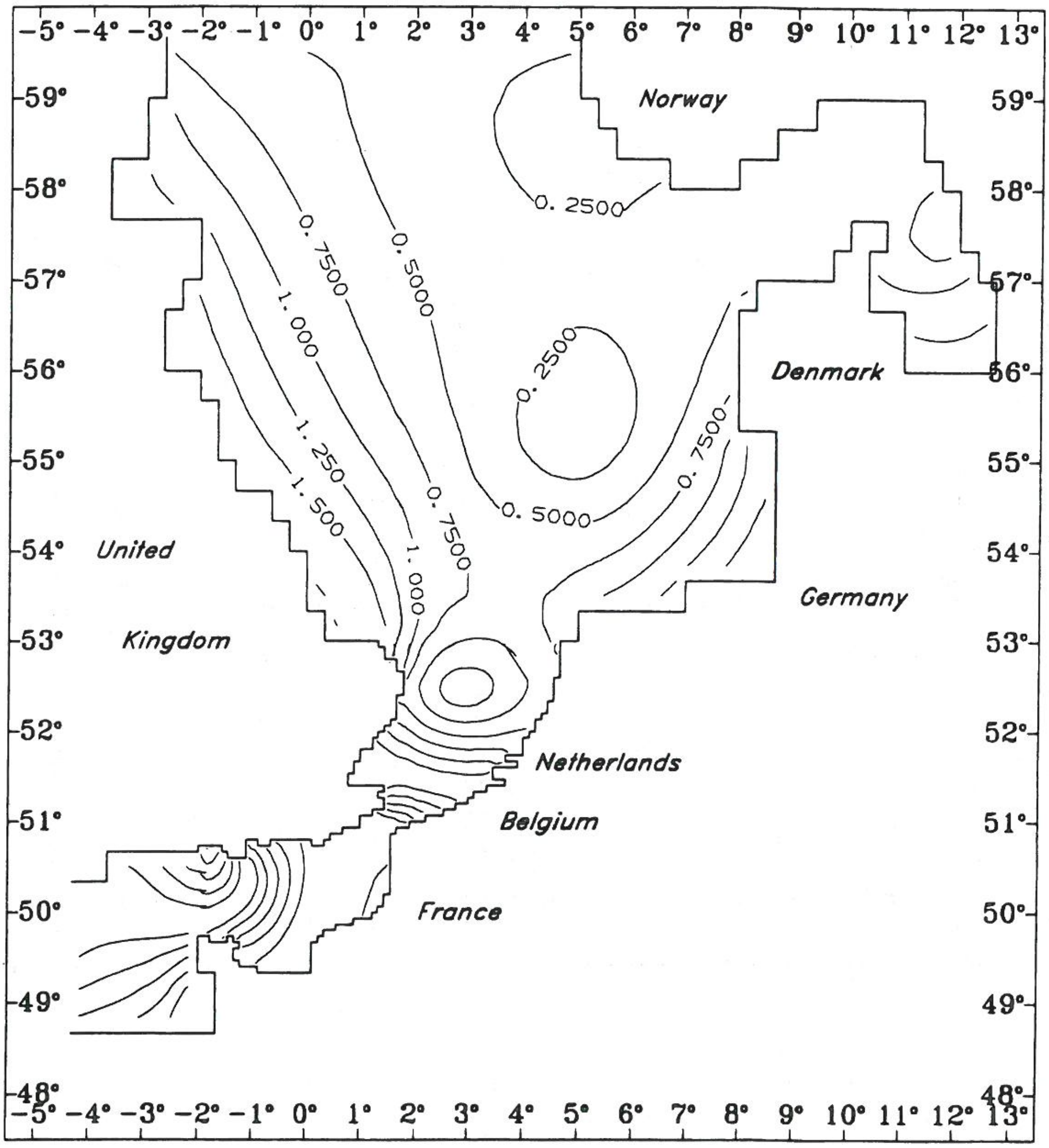

FIGURE 2
Deleersnijder, Ozer and Tartinville