Leptonic mixing angle theta$_{13}$ and ruling out of minimal texture for Dirac neutrinos

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The implications of the recently measured leptonic mixing angle θ_{13} as well as the other two mixing angles have been examined for Fritzsch-like mass matrices with minimal texture for Dirac neutrinos. Interestingly, the existing data seem to rule out this texture-specific case of Dirac neutrinos for normal/inverted hierarchies as well as degenerate scenarios of masses.
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1. Introduction

The recent measurements [1–5] regarding the neutrino mixing angle θ_{13} have undoubtedly improved our knowledge of neutrino oscillation phenomenology. Interestingly, this θ_{13} value, which is unexpectedly 'large', being almost near the Cabibbo angle, would have important implications for flavor physics. Also, it may be mentioned that before the measurement of θ_{13}, assuming it to be zero or nearly equal to zero and considering the canonical values of the other two neutrino mixing angles, the effort was to discover some underlying symmetry [6] in the leptonic sector. The non-zero value of θ_{13} leads to parallelism between the mixings of quarks and leptons as well as signifying the difference between the mixing angles of quarks and leptons, as the leptonic mixing angles are large compared with the corresponding quark mixing angles.

Ever since the observations regarding θ_{13} were made, there has been a good deal of activity on the theoretical front in understanding the pattern of neutrino masses and mixings. Noting that there is a similarity between quark and lepton mixing phenomena [7], it becomes desirable to understand these from the same perspective as far as possible. However, there are some important differences that have to be kept in mind before considering a unified framework for formulating quark and lepton mass matrices on the same footing. For example, one may note that unlike the case of quark mixings, which show a hierarchical structure, the patterns of neutrino mixings do not show any explicit hierarchy. Further, at present there is no consensus about neutrino masses, which may show normal/inverted hierarchy or may even be degenerate. Furthermore, the situation becomes complicated when one realizes that it is not yet clear whether neutrinos are Dirac or Majorana particles.

It may be mentioned that, in the absence of any viable theory for flavor physics, one usually resorts to phenomenological models. In this context, texture-specific mass matrices have got a good deal of attention in the literature; for details and extensive references we refer readers to a recent review article [8]. In particular, Fritzsch-like texture-specific mass matrices seem to be very helpful in understanding the pattern of quark mixings and CP violation [9–18]. Keeping in mind quark–lepton

parallelism [7] and taking a clue from the success of these texture-specific mass matrices in the context of quarks, several attempts [9,19–27] have also been made to consider similar lepton mass matrices. However, noting the above-mentioned complexities of neutrino masses and mixings, it seems necessary to carry out a detailed and case-by-case analysis of texture-specific mass matrices for their compatibility with the mixing data. In particular, for any given texture, the analysis needs to be carried out for all the neutrino mass hierarchies as well as for both Majorana and Dirac neutrinos, since the latter have not yet been ruled out experimentally [28].

Considering neutrinos to be Majorana particles, after the recent measurements of θ_{13}, a few analyses have been carried out for texture-specific mass matrices in the non-flavor basis. In particular, Fukugita et al. [29] have investigated the implications of angle θ_{13} on minimal-texture mass matrices (Fritzsch-like texture 6 zero) for the normal hierarchy of masses. This analysis has been extended further by Fakay et al. [30], wherein, for all the hierarchies of neutrino masses, texture 6 and 5 zero mass matrices have been examined in detail. For the case of Dirac neutrinos, although several authors have examined the possibility of these having small masses [31–40] as well as their compatibility with the supersymmetric GUTs [41], similar attempts have not yet been carried out since the measurements of θ_{13}. In this context, it may be added that the original texture 6 zero Fritzsch mass matrices have been ruled out in the case of quarks; therefore, in the light of the similarity between the mixing patterns of quarks and leptons, it becomes desirable to examine similar mass matrices for the cases of neutrinos.

In the present paper, for the case of Dirac neutrinos, we have carried out detailed calculations pertaining to mass matrices with minimal texture for the three possibilities of neutrino masses having normal/inverted hierarchy or being degenerate. In particular, the analysis has been carried out by imposing Fritzsch-like texture 6 zero structure on Dirac neutrino mass matrices as well as on charged lepton mass matrices. The compatibility of these texture-specific mass matrices has been examined by plotting the parameter space corresponding to the recently measured mixing angle s_{13} along with the other two mixing angles s_{12} and s_{23}. Further, for the normal hierarchy case, the implications of mixing angles on the lightest neutrino mass m_{ν_1} have also been investigated.

The detailed plan of the paper is as follows. To set notations and conventions as well as to make the paper self contained, in Sect. 2 we present some of the essentials regarding texture 6 zero Dirac neutrino mass matrices. Inputs used in the present analysis are given in Sect. 3. The analysis pertaining to inverted and normal hierarchies and the degenerate scenario of neutrino masses are respectively presented in Sects. 4, 5, and 6. Finally, Sect. 7 summarizes our conclusions.

2. Texture 6 zero Dirac neutrino mass matrices

In the Standard Model (SM), the mass terms corresponding to the charged leptons and Dirac neutrinos having non-zero masses are respectively given by

$$- \mathcal{L}_l = \overline{(l)}_L M_l (l)_R + h.c. \tag{1}$$

and

$$- \mathcal{L}_D = \overline{(\nu_a)}_L M_{\nu D} (\nu_a)_R + h.c., \tag{2}$$

where L stands for left-handedness, M_l denotes the charged lepton mass matrix, $M_{\nu D}$ is the complex 3×3 Dirac neutrino mass matrix, and

$$(\nu_a) \equiv \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}, \quad (l) \equiv \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}. \tag{3}$$

The three flavor fields are ν_{aL} ($a = e, \mu, \tau$), and ν_{aR} are the right-handed singlets, which are sterile and do not mix with the active neutrinos. The mass matrices M_l and $M_{\nu D}$ are arbitrary in the SM with a total of 36 real, free parameters, these being quite large in number in comparison with the 10 physical observables. Using the polar decomposition theorem, any general mass matrix M can be expressed as $M = HU$, where H denotes a Hermitian and U a unitary matrix. In the present case, the matrix U can be absorbed by redefining the right-handed singlet neutrino fields, thus enabling one to bring down the number of free parameters from 36 to 18; they are further brought down by considering textures, discussed below.

After defining the charged lepton and neutrino mass matrices, their texture 6 zero Fritzsch structures are given as

$$M_l = \begin{pmatrix} 0 & A_l & 0 \\ A_l^* & 0 & B_l \\ 0 & B_l^* & C_l \end{pmatrix}, \quad M_{\nu D} = \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu^* & 0 & B_\nu \\ 0 & B_\nu^* & C_\nu \end{pmatrix}, \tag{4}$$

M_l and $M_{\nu D}$ respectively corresponding to charged lepton and Dirac neutrino mass matrices. It may be noted that each of the above matrices is texture 3 zero type with $A_{l(\nu)} = |A_{l(\nu)}|e^{i\alpha_{l(\nu)}}$ and $B_{l(\nu)} = |B_{l(\nu)}|e^{i\beta_{l(\nu)}}$.

The formalism connecting the mass matrix to the neutrino mixing matrix [42–45] involves diagonalization of the mass matrices M_l and $M_{\nu D}$; details in this regard can be looked up in Ref. [8]. In general, to facilitate diagonalization, the mass matrix M_k, where $k = l, \nu D$, can be expressed as

$$M_k = Q_k M_k^r P_k \tag{5}$$

or

$$M_k^r = Q_k^\dagger M_k P_k^\dagger, \tag{6}$$

where M_k^r is a real symmetric matrix with real eigenvalues and Q_k and P_k are diagonal phase matrices $\mathrm{Diag}\{e^{i\alpha_k}, 1, e^{-i\beta_k}\}$ and $\mathrm{Diag}\{e^{-i\alpha_k}, 1, e^{i\beta_k}\}$ respectively. The real matrix M_k^r is diagonalized by the orthogonal transformation O_k, e.g.,

$$M_k^{\mathrm{diag}} = O_k^T M_k^r O_k, \tag{7}$$

which on using Eq. (6) can be rewritten as

$$M_k^{\mathrm{diag}} = O_k^T Q_k^\dagger M_k P_k^\dagger O_k. \tag{8}$$

The elements of the general diagonalizing transformation O_k can figure with different phase possibilities; however, these possibilities are related to each other through the phase matrices [8]. For the present work, we have chosen the possibility

$$O_k = \begin{pmatrix} O_k(11) & O_k(12) & O_k(13) \\ O_k(21) & -O_k(22) & O_k(23) \\ -O_k(31) & O_k(32) & O_k(33) \end{pmatrix}, \tag{9}$$

where

$$O_k(11) = \sqrt{\frac{m_2 m_3 (m_3 - m_2)}{(m_1 - m_2 + m_3)(m_3 - m_1)(m_1 + m_2)}}$$

$$O_k(12) = \sqrt{\frac{m_1 m_3 (m_1 + m_3)}{(m_1 - m_2 + m_3)(m_2 + m_3)(m_1 + m_2)}}$$

$$O_k(13) = \sqrt{\frac{m_1 m_2 (m_2 - m_1)}{(m_1 - m_2 + m_3)(m_2 + m_3)(m_3 - m_1)}}$$

$$O_k(21) = \sqrt{\frac{m_1 (m_3 - m_2)}{(m_3 - m_1)(m_1 + m_2)}}$$

$$O_k(22) = \sqrt{\frac{m_2 (m_1 + m_3)}{(m_2 + m_3)(m_1 + m_2)}}$$

$$O_k(23) = \sqrt{\frac{m_3 (m_2 - m_1)}{(m_2 + m_3)(m_3 - m_1)}}$$

$$O_k(31) = \sqrt{\frac{m_1 (m_2 - m_1)(m_1 + m_3)}{(m_1 - m_2 + m_3)(m_1 + m_2)(m_3 - m_1)}}$$

$$O_k(32) = \sqrt{\frac{m_2 (m_2 - m_1)(m_3 - m_2)}{(m_1 - m_2 + m_3)(m_2 + m_3)(m_1 + m_2)}}$$

$$O_k(33) = \sqrt{\frac{m_3 (m_3 - m_2)(m_1 + m_3)}{(m_1 - m_2 + m_3)(m_3 - m_1)(m_2 + m_3)}}, \tag{10}$$

$m_1, -m_2, m_3$ being the eigenvalues of M_k. It may be added that, without loss of generality, we can always choose the phase of one of the mass eigenvalues relative to the other two. For details, we refer the reader to Refs. [8,46].

In the case of charged leptons, because of the hierarchy $m_e \ll m_\mu \ll m_\tau$, the mass eigenstates can be approximated respectively to the flavor eigenstates, as has been considered by several authors [47–52]. Using the approximations $m_{l1} \simeq m_e$, $m_{l2} \simeq m_\mu$, and $m_{l3} \simeq m_\tau$, the first element of the matrix O_l can be obtained from the corresponding element of Eq. (10) by replacing $m_1, -m_2, m_3$ with $m_e, -m_\mu, m_\tau$, e.g.,

$$O_l(11) = \sqrt{\frac{m_\mu m_\tau (m_\tau - m_\mu)}{(m_e - m_\mu + m_\tau)(m_\tau - m_e)(m_e + m_\mu)}}. \tag{11}$$

In the case of neutrinos, for the normal hierarchy of neutrino masses defined as $m_{\nu_1} < m_{\nu_2} \ll m_{\nu_3}$, as well as for the corresponding degenerate case given by $m_{\nu_1} \lesssim m_{\nu_2} \sim m_{\nu_3}$, Eq. (10) can also be used to obtain the elements of diagonalizing transformation for Dirac neutrinos. The first element can be obtained from the corresponding element of Eq. (10) by replacing $m_1, -m_2, m_3$ with $m_{\nu 1}, -m_{\nu 2}, m_{\nu 3}$ and is given by

$$O_{\nu D}(11) = \sqrt{\frac{m_{\nu_2} m_{\nu_3} (m_{\nu_3} - m_{\nu_2})}{(m_{\nu 1} - m_{\nu 2} + m_{\nu 3})(m_{\nu 3} - m_{\nu 1})(m_{\nu 1} + m_{\nu 2})}}, \tag{12}$$

where m_{ν_1}, m_{ν_2}, and m_{ν_3} are neutrino masses.

In the same manner, one can obtain the elements of diagonalizing transformation for the inverted hierarchy case defined as $m_{\nu_3} \ll m_{\nu_1} < m_{\nu_2}$ as well as for the corresponding degenerate case given by $m_{\nu_3} \sim m_{\nu_1} \lesssim m_{\nu_2}$. The corresponding first element, obtained by replacing $m_1, -m_2, m_3$ with $m_{\nu_1}, -m_{\nu_2}, -m_{\nu_3}$ in Eq. (10), is given by

$$O_{\nu D}(11) = \sqrt{\frac{m_{\nu_2} m_{\nu_3}(m_{\nu_3} + m_{\nu_2})}{(-m_{\nu_1} + m_{\nu_2} + m_{\nu_3})(m_{\nu_3} + m_{\nu_1})(m_{\nu_1} + m_{\nu_2})}}. \tag{13}$$

As already mentioned, one can choose the sign of one eigenvalue relative to the other two; therefore, to facilitate calculations for the inverted hierarchy case, we have chosen m_{ν_1} to be positive and both m_{ν_2} and m_{ν_3} to be negative. The other elements of diagonalizing transformations in the cases of neutrinos as well as charged leptons can be similarly found.

After the elements of the diagonalizing transformations O_l and $O_{\nu D}$ are known, the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix [42–45] can be obtained through the relation

$$U = O_l^\dagger Q_l P_{\nu D} O_{\nu D}, \tag{14}$$

where $Q_l P_{\nu D}$, without loss of generality, can be taken as $\mathrm{Diag}\{e^{-i\phi_1}, 1, e^{i\phi_2}\}$. The parameters ϕ_1 and ϕ_2 are related to the phases of mass matrices, i.e., $\phi_1 = \alpha_{\nu D} - \alpha_l$, $\phi_2 = \beta_{\nu D} - \beta_l$, and can be treated as free parameters.

3.   Inputs used for the analysis

In the present analysis, we have made use of the results of the latest global three neutrino oscillation analysis carried out by Fogli et al. [53]. At 1σ C.L. the allowed ranges of the various input parameters are

$$\Delta m_{21}^2 = (7.32 - 7.80) \times 10^{-5}\,\mathrm{eV}^2, \quad \Delta m_{23}^2 = (2.33 - 2.49) \times 10^{-3}\,\mathrm{eV}^2, \tag{15}$$

$$s_{12}^2 = (0.29 - 0.33), \quad s_{23}^2 = (0.37 - 0.41), \quad s_{13}^2 = (0.021 - 0.026), \tag{16}$$

where the Δm_{ij}^2 correspond to the solar and atmospheric neutrino mass square differences and the s_{ij} correspond to the sine of the mixing angle θ_{ij} where $i, j = 1, 2, 3$. At 3σ C.L. the allowed ranges are given as

$$\Delta m_{21}^2 = (6.99 - 8.18) \times 10^{-5}\,\mathrm{eV}^2, \quad \Delta m_{23}^2 = (2.19 - 2.62) \times 10^{-3}\,\mathrm{eV}^2, \tag{17}$$

$$s_{12}^2 = (0.26 - 0.36), \quad s_{23}^2 = (0.33 - 0.64), \quad s_{13}^2 = (0.017 - 0.031). \tag{18}$$

For the purpose of the calculations, the masses and mixing angles have been constrained by the data given in the above equations. In the case of the normal hierarchy, the explored range for the lightest neutrino mass corresponding to m_{ν_1} is taken to be 0.0001–1.0 eV, essentially governed by the mixing angle s_{12} related to the ratio $\frac{m_{\nu_1}}{m_{\nu_2}}$. For the inverted hierarchy case, we have also taken the same range for the lightest neutrino mass corresponding to m_{ν_3}. It may be mentioned that our conclusions remain unaffected even if the range is extended further. In the absence of any constraint on the phases, ϕ_1 and ϕ_2 have been given full variation from 0 to 2π.

4.   Inverted hierarchy of neutrino masses

To examine the compatibility of texture 6 zero Dirac neutrino mass matrices with the recent mixing data, we first discuss the implications of mixing angle θ_{13} for the case pertaining to the inverted hierarchy of neutrino masses. To this end, in Figs. 1(a) and 1(b) we present the plots of the parameter
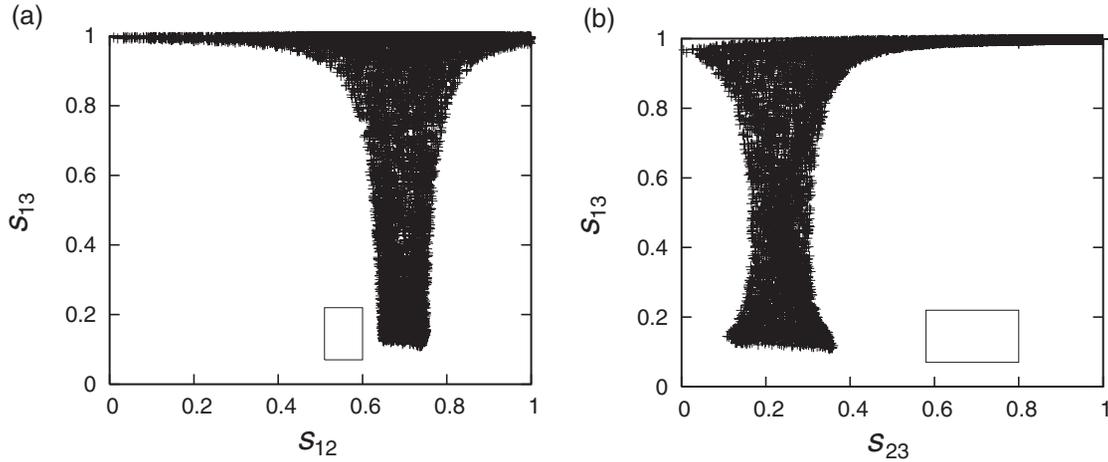
Fig. 1. Plots showing the parameter space corresponding to (a) s_{13} and s_{12} and (b) s_{13} and s_{23}.

space corresponding to s_{13} along with the other two mixing angles s_{12} and s_{23} respectively. Giving full allowed variation to other parameters, Fig. 1(a) has been obtained by constraining the angle s_{23} by its experimental bound given in Eq. (18) and similarly, while plotting Fig. 1(b), the angle s_{12} has been constrained by its experimental limits. Also included in the figures are blank rectangular regions indicating the experimentally allowed 3σ C.L. region of the plotted angles. Interestingly, a general look at these figures reveals that, pertaining to the inverted hierarchy of neutrino masses, the texture 6 zero Dirac neutrino mass matrices are clearly ruled out at 3σ C.L. This can be understood by noting that the plotted parameter space of the two angles has no overlap with their experimentally allowed 3σ C.L. region.

5. Normal hierarchy of neutrino masses

After ruling out texture 6 zero Dirac neutrino mass matrices for the inverted hierarchy, we now examine the compatibility of these matrices for the normal hierarchy case. To this end, in Fig. 2(a) we present the graph of s_{13} versus m_{ν_1}; in the graph, the solid horizontal lines and the dashed lines depict respectively the 3σ C.L. and 1σ C.L. ranges of this angle. The graph depicts the interesting result that the 1σ C.L. range of s_{13} has no overlap with the plotted values of the angle s_{13}, an indication towards the ruling out of texture 6 zero mass matrices at 1σ C.L. for the normal hierarchy of neutrinos. However, a look at the figure also reveals that, corresponding to the 3σ C.L. range of s_{13}, one gets a lower bound on mass $m_{\nu_1} \sim 0.001$ eV. One may add that refinements in the measurement of angle s_{13} would have interesting implications for this case.

To sharpen the above-mentioned conclusions, in Fig. 2(b) we present the graph of angle s_{23} w.r.t. mass m_{ν_1}, with the solid horizontal lines and the dashed lines depicting respectively the 3σ C.L. and 1σ C.L. ranges of this angle. Interestingly, from this figure one can conclude that not only does the 1σ C.L. range of s_{23} again confirm the ruling out of this case of texture 6 zero mass matrices, but also, corresponding to the 3σ C.L. range of this angle, one finds that the ruling out is again largely confirmed. It may be added that, in case we plot a graph of angle s_{12} versus m_{ν_1}, this is an indication of the compatibility of these mass matrices with the data. However, it needs to be noted that, to rule out the matrices, it is sufficient to do so from any one of the mixing angles versus the mass m_{ν_1} graph.
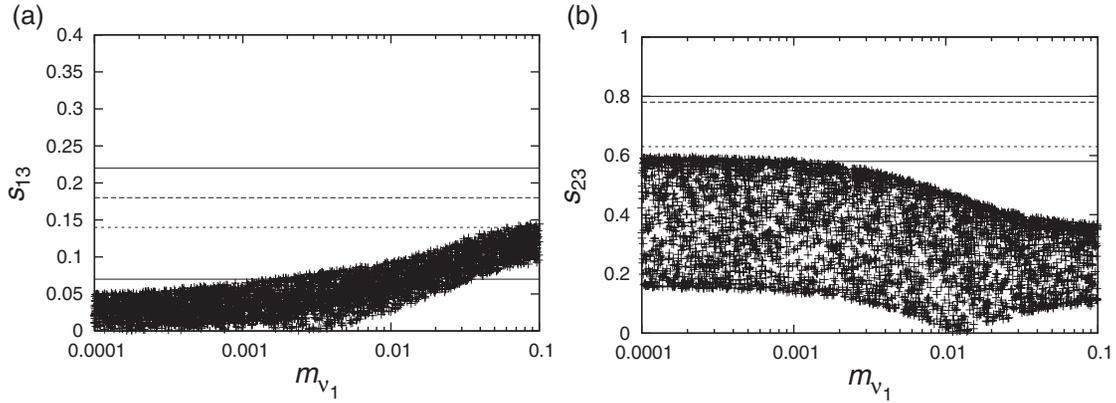
Fig. 2. Plots showing the variation of the lightest neutrino mass m_{ν_1} with (a) s_{13} and (b) s_{23}.

6.   Degenerate scenario of neutrino masses

The degenerate scenario of neutrino masses can be characterized by either $m_{\nu_1} \lesssim m_{\nu_2} \sim m_{\nu_3} \sim 0.1\,\mathrm{eV}$ or $m_{\nu_3} \sim m_{\nu_1} \lesssim m_{\nu_2} \sim 0.1\,\mathrm{eV}$, corresponding to normal hierarchy and inverted hierarchy respectively. As mentioned earlier, the diagonalizing transformations for the above two cases are respectively the same as the ones obtained for the normal hierarchy of masses, Eq. (12), and for the inverted hierarchy of masses, Eq. (13). Therefore, the conclusions regarding the texture 6 zero Dirac neutrino mass matrices corresponding to both normal and inverted hierarchies remain valid for this case too.

This can be understood from Figs. (1) and (2). While plotting Figs. 1(a) and 1(b), the range of the lightest neutrino mass is taken to be 0.0001–1.0 eV, which includes the neutrino masses corresponding to the degenerate scenario; therefore, by discussion similar to the one given for ruling out texture-specific mass matrices for the inverted hierarchy, these are ruled out for the degenerate scenario of neutrino masses as well. Similarly, for the degenerate scenario corresponding to the normal hierarchy of neutrino masses, Fig. 2(b) clearly shows that the values of s_{23} corresponding to $m_{\nu_1} \lesssim 0.1$ eV lie outside the experimentally allowed range, thereby ruling out the mass matrices for the degenerate scenario.

7.   Summary and conclusions

To summarize, for Dirac neutrinos, we have carried out detailed calculations pertaining to minimal texture characterized by texture 6 zero Fritzsch-like mass matrices. Corresponding to these, we have considered neutrino masses having normal and inverted hierarchies as well as degenerate scenarios. The compatibility of these texture-specific mass matrices has been examined by plotting the parameter space corresponding to the recently measured mixing angle s_{13} along with the other two mixing angles s_{12} and s_{23}. Further, for the normal hierarchy case, the implications of mixing angles on the lightest neutrino mass m_{ν_1} have also been investigated.

Interestingly, the analysis reveals that, using 1σ C.L. inputs, all the texture 6 zero cases of Dirac neutrino mass matrices pertaining to normal and inverted hierarchies and degenerate scenarios of the neutrino masses seem to be completely ruled out; for 3σ C.L. inputs, these are also largely ruled out.

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