Editorial
Structured Numerical Linear and Multilinear Algebra: Analysis, Algorithms and Applications

The editors are pleased to secure manuscripts at the leading edge of research on Structured Linear and Multilinear Algebra from the speakers at the conference, and we hope readers of this issue will enjoy the papers.

Methods for the solution of equations in which structured matrices arise are considered in several papers. For example, Noutsos, Serra-Capizzano and Vassalos [10] study the fast solution of Toeplitz linear systems using a preconditioned conjugate gradient method with coefficient matrix $T_n(f)$, where the generating function $f$ is nonnegative and has a unique zero at zero of any real positive order. The Toeplitz structure of a matrix is also considered by Bini, Dendievel, Latouche and Meini [8] for the design of algorithms for the computation of the exponential of large block-triangular block-Toeplitz matrices that arise in fluid queues. Bolten, Huckle and Kravaritis [3] study multigrid methods for the solution of structured linear systems by using sparse matrix approximations to define smoothers and projectors, and to approximate the coarse grid matrix. Numerical results are given, which demonstrate the efficiency and accuracy of the proposed strategies.

Papers on matrix factorizations are included in this volume. In particular, Dewilde, Eidelman and Haimovici [9] design a new algorithm to compute the $LU$-factorization of a matrix represented in a quasiseparable or semiseparable form. The algorithm uses unitary transformations to enhance numerical stability. Lungten, Schilders, and Maubach [5] consider the saddle point problem and design a method for its solution based on the $LDL^T$ factorization with $a$ priori pivoting of a suitably preprocessed matrix. The aim of this preprocessing is a reduction of the original matrix to a form that contains lower trapezoidal blocks. This structure is used to increase the effectiveness of the $LDL^T$ factorization. Mastronardi and Van Dooren [12] develop an algorithm for computing a factorization of a symmetric indefinite matrix $A$ in the form $QMQT^T$, where $M$ is in rank-revealing block anti-triangular form. The computed factorization can be easily updated, when a rank-one matrix, or one row or one column, is appended, in $O(n^2)$ floating point operations for a matrix of order $n$. Numerical results and comparisons
of this method with existing algorithms for symmetric rank-revealing factorizations are included.

Bini and Robol [1] present an algorithm, based on quasipositive matrices, to perform the Hessenberg reduction of a matrix $A$ of the form $A = D + UV^*$, where $D$ is diagonal with real entries, and $U$ and $V$ are of order $n \times k$, $k \leq n$. The cost of the algorithm is $O(kn^2)$, and its application to the solution of polynomial eigenvalue problems and the results of numerical experiments are included. Polynomials are also considered by Bini and Robol [2], who propose a new class of linearizations for an $m \times m$ matrix polynomial $P(x)$ of degree $n$. They also provide a general way to transform $P(x)$ into a strongly equivalent matrix polynomial $A(x)$ of lower degree $l$, a process called $l$-ification.

There are several papers on bilinear forms and numerical integration. Specifically, Fika and Mitrouli [4] estimate bilinear forms of the form $y^* f(A)x$, where $x, y \in \mathbb{C}^p$ and $A \in \mathbb{C}^{p \times p}$, for a suitable function $f$, using extrapolation. They derive families of one-term and two-term estimates by extrapolation of the moments of $A$, and provide results and comparisons for several matrix functions. Reichel, Rodriguez and Tang [13] consider block quadrature rules that can be computed by the symmetric or non-symmetric block Lanczos algorithms. These rules are block generalizations of the generalized average Gauss rules introduced by Spalević and they are more accurate than the standard block Gauss rules. Van Barel [6] considers contour integration for the solution of nonlinear matrix eigenvalue problems. Approximations of the eigenvalues and the corresponding eigenvectors located within a bounded domain in the complex plane are found by solving a generalized eigenvalue problem for a pair of block Hankel matrices generated from the discretized moments. The computation of these moments involves filter functions that are associated with the selected quadrature rule. Optimization techniques are used to design effective filter functions.

Abril Brucero, Bajaj and Mourrain [7] describe a new method to compute cubature formulae based on a corresponding truncated Hankel operator with flat extensions. Jung-hanns, Kaiser and Potts [14] derive necessary and sufficient conditions for the stability of collocation-quadrature methods for Cauchy singular integral equations. It is shown that these methods are simple and fast, and their application to two-dimensional elasticity problems is shown by example.

Regularisation and an example of an ill-conditioned problem are also included in this volume. Noschese and Reichel [15] consider some extensions of Tikhonov regularisation and truncated singular value decomposition (TSVD) for the solution of ill-conditioned linear algebraic equations. Specifically, they derive new regularisation procedures by considering some matrix-nearness problems associated with Tikhonov regularisation. They show that these new procedures share properties with Tikhonov regularisation and TSVD, and that they may give solutions that are better than the solutions obtained by these two methods. Huang, Noschese and Reichel [16] consider the application of Tikhonov regularisation to the solution of large discrete ill-posed problems. The choice of the matrix used for regularisation is important, and it should be chosen to highlight important features of the exact solution. This paper describes a method that requires the
solution of a matrix-nearness problem for the determination of regularisation matrices that possess these properties.

Image deblurring is a typical problem that requires regularisation because of its ill-conditioned nature. Dell’Acqua, Donatelli, Serra-Capizzano, Sesana and Tablino-Possio [17] present an optimal preconditioning strategy for image deblurring when anti-reflective boundary conditions are imposed on a problem that has a non-symmetric point spread function. The form of the optimal preconditioner is considered, and numerical tests that show the effectiveness of the optimal preconditioning strategy are included. Winkler [11] uses polynomial computations for blind image deconvolution when the blur arises from a spatially invariant point spread function because the blurred image can be interpreted as the product of two univariate polynomials in this situation.

Two papers consider other applications of structured matrices. In particular, Fasino and Tudisco [18] consider some properties of various modularity matrices and the spectral properties of generalized modularity matrices. These properties form the basis of various theoretical results and practical spectral-type algorithms for community detection, which is a major problem in modern complex network analysis. Also, Usevich and Markovsky [19] consider the problem of fitting a set of points to an algebraic hypersurface, where the exact hypersurface is defined by a polynomial equation. Each point is corrupted by Gaussian random noise and the least squares estimator is obtained by constructing a quasi-Hankel matrix. The paper also considers the situation in which the variance of the noise is not known.

References


Dario Bini  
*Pisa, Italy*

Marilena Mitrouli  
*Athens, Greece*  
*E-mail address: mmitroul@math.uoa.gr*

Marc Van Barel  
*Leuven, Belgium*

Joab Winkler  
*Sheffield, United Kingdom*

Available online 7 April 2016

* Corresponding author.