Iterative reconstruction of the ultrasound attenuation coefficient from the backscattered radio-frequency signal

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Abstract— Accurate estimation of the local acoustic attenuation based on the backscatter signal has several applications, e.g. ultrasound tissue characterization. Most of the existing techniques determine the attenuation coefficient of the tissue directly from the spectrum of the backscattered signal. In these approaches other effects, such as diffraction, that may influence the attenuation estimation should be corrected for. This correction may be impractical in vivo. In the present study the simulation of ultrasound wave propagation was used for the estimation of the attenuation characteristics. Indeed, the local attenuation coefficient was estimated by iteratively solving the forward wave propagation problem and matching the synthetic backscattered signal to the measured one. The proposed methodology was experimentally validated using tissue-mimicking phantoms with different attenuation characteristics showing promising results.

Index Terms— attenuation estimation, ultrasound simulation, tissue characterization

I. INTRODUCTION

A reliable estimation of the local acoustic attenuation not only provides information about the state of the tissue (e.g. tissue characterization) but its assessment is also essential for correct time-gain compensation and for an accurate evaluation of other acoustic parameters. Several techniques for attenuation estimation from backscatter data exist. Most of these techniques solve the so-called “inverse scattering problem” by estimating the acoustic parameters directly from the recorded backscatter signals and can be classified as time- or frequency-domain methods. For example in the time-domain, a zero-crossing approach has been proposed [1]. Frequency domain methods can be categorized as spectral-difference techniques that calculate the amplitude decay of the backscattered signals [2], [3] and spectral shift techniques that assume a Gaussian-shape of the pulse and estimate the center frequency downshift along the propagation depth [4], [5]. The latter can be achieved using the short-time Fourier analysis [4] or by fitting a Gaussian function to the spectrum in order to determine the mean frequency shift [5]. More recently in this category an approach using a cross-correlation between neighboring power spectra was proposed [6].

While solving the inverse scattering problem, factors that can affect the attenuation estimation, such as diffraction and system-dependent effects, have to be taken into account. Diffraction correction in the time-domain is very difficult. In the frequency-domain, a reference phantom technique is usually used. It consists of comparing the backscattered signal of the sample with the signal recorded from a homogeneous phantom with known attenuation characteristics [7]. However, the use of this method in vivo is not straightforward.

In the present study, we aim to iteratively solve the forward scattering problem through computer simulations in order to match the synthetically generated backscatter signals to experimentally observed ones. During ultrasound wave propagation several phenomena may occur such as reflection, refraction, nonlinear distortion, attenuation, dispersion and diffraction. Ultimately, we aim to optimize all propagation phenomena, but in the present study we concentrate on the attenuation parameter. We consider dispersion-free media and begin with a plane wave approximation to reduce the problem to 1-D and avoid diffraction effects. Moreover, the incidence of the plane wave is normal to the boundary between the tissue layers avoiding oblique refraction. The remaining effects of attenuation, nonlinear distortion, reflection and scattering are taken into account. In this way, radio frequency (RF) signals for media with given acoustic parameters for attenuation and nonlinearity can be simulated. The attenuation parameter can then be iteratively changed to approximate the experimentally observed RF signals in order to determine the attenuation properties of the medium.

The above approach was validated in experiments using homogeneous tissue-mimicking phantoms with different attenuation characteristics. Attenuation estimates, obtained using our method, were compared to the attenuation values that were determined using a through-transmission substitution method. The error of our method for the majority of the estimates was below 10%.

II. METHOD

A. Plane wave propagation model

The effects of attenuation and nonlinear distortion can be modelled using the operator splitting approach in which the individual effects are considered to be independent of each other over small propagation distances. Thus, each effect can
be modelled individually and the combined effect is considered to be the sum of the individual effects [8].

Attenuation was modelled using a power-law in the frequency domain:

\[ S(z+\Delta z, \nu) = S(z, \nu) \cdot e^{-a\Delta z}, \]

where \( S(z, \nu) \) is the spectrum of the signal at depth \( z \), \( a \) is the attenuation coefficient of the medium and \( \nu \) is the frequency.

Nonlinearity was modelled using a time-domain operator [9]:

\[
v_{m+1}^n = \begin{cases} 
v_m^n + \left( 1 + \frac{\beta \Delta z}{c^2 T_s}(v_{m+1}^n - v_m^n) \right), & \text{for } v_m^n \geq 0, \\
v_m^n + \left( 1 + \frac{\beta \Delta z}{c^2 T_s}(v_m^n - v_{m-1}^n) \right), & \text{for } v_m^n < 0, 
\end{cases}
\]

where \( v_m^n \) is the particle velocity at time instance \( \tau = m \cdot \Delta \tau \) and position \( z = n \cdot \Delta z \), \( \beta = 1 + \frac{\eta}{2c^2} \) – the nonlinear parameter of the medium, \( c \) – the speed of sound and \( T_s = \tau_m^m - \tau_{m-1}^n = \Delta \tau - \) the sampling period.

Propagation in a heterogeneous medium was modeled by the introduction of subsequent layers with different characteristics. The reflections from the interfaces between subsequent layers were taken into account and the amplitude transmission and reflection coefficients were calculated as:

\[
T = \frac{2Z_2}{Z_2 + Z_1}, \\
R = T - 1,
\]

where \( Z_1 \) and \( Z_2 \) are the acoustic impedances of the 1st and 2nd medium respectively [10].

Finally, to model scattering, a random distribution of point scatterers (at a predefined density) was placed on the propagation axis. The pressure field was then calculated at each scatterer position including the above effects and was propagated back to the position of the source. Finally, the signals from all scatterers were summed and plotted on the time axis. The amplitude of the scattered wave was considered sufficiently small so that the nonlinear effects could be neglected during the back-propagation. Moreover, multiple scattering was assumed negligible. In case of a high-density distribution of independent point scatterers, the backscattered signal can be approximated as the signal reflected from a single scatterer placed in the middle of the considered window. Doing so, significant reduction in computation time was obtained.

Using this model, RF signals for media with given attenuation and non-linear characteristics can be simulated. These model parameters can then be iteratively changed to approximate experimentally observed RF signals in order to determine the acoustic properties of the medium.

**B. Phantom Preparation**

To test the above approach, tissue-mimicking cylindrical phantoms (4.5 cm in length; 3.5 cm in diameter) were used. Three types of phantoms were prepared: gelatin-, agarose- and poly(vinyl alcohol)-based testing samples.

During the preparation of gelatin-based phantoms, dry-weight gelatin was mixed with distilled water, graphite powder, n-propanol and 40%-formalin solution as described in [11]. Agarose-based phantoms were made by mixing dry agar with distilled water and n-propanol as in [12]. Finally, PVA-based phantoms were prepared from a mixture of dry PVA-powder and distilled water as described in [13]. Adding n-propanol produces a speed of sound in gelatin- and agarose-based materials comparable to that of soft tissue. Addition of a formalin solution to gelatin-based material increases its melting point to 100 °C [11]. Different concentrations of graphite powder were used during the preparation in order to modulate the attenuation characteristics of the materials and to achieve sufficient scattering.

Six phantoms were used in this study: three gelatin-based phantoms: “Phantom A”, “Phantom B” and “Phantom C”, - one agarose-based - "Phantom D", and two PVA-based phantoms: “Phantom E” and “Phantom F”.

**C. Data Acquisition**

Acoustic parameters of the phantoms were first measured using the through-transmission substitution method. This method consists in a comparison of the signal amplitudes in a medium with known parameters (i.e. distilled water) with those obtained during the actual propagation through the sample. First, a reference measurement was made between an emitting and a receiving transducer in a tank filled with distilled water. Then, a phantom was placed in the water in-between the two transducers. All measurements were done at room temperature (22 °C).

Fig. 1 shows a schematic diagram of the through transmission setup used for these measurements. Flat unfocused single-element 0.5” transducers with 65% fractional bandwidth and 10 MHz center frequency (V311-SU, Panametrics NDT, Inc., Waltham, MA) were used. Successive sinusoidal bursts, produced by a waveform generator (AWG NI PXI 5412, National Instruments Corporation, Austin, TX) and controlled by LabVIEW were sent in the form of a discrete frequency sweep (from 0.5 till 20 MHz with 250 kHz step) [14]. At each frequency, the waveform consists of 120 cycles. This signal was amplified (150A100B Amplifier Research, Souderon, PA) and sent to the emitting transducer. An average of 64 signals recorded at the receiving transducer were digitized on a data acquisition card (DAQ PXI NI 5122, 14 bit, 100 MHz sampling rate, National Instruments Corporation, Austin, TX) and was stored on the PC.
The attenuation coefficient of the phantom was calculated from the ratio of the fundamental pressure amplitudes of the reference and sample signals [14], while the nonlinear parameter was calculated from the ratio of the second-harmonic pressure amplitudes. The acoustic parameters of the phantoms determined in the above described through-transmission measurements were considered as ground-truth and were used for the comparison with the results from the presently proposed reconstruction method based on the backscatter measurements and iterative simulation.

Subsequent to the through-transmission experiments, pulse-echo measurements were performed. Fig. 2 shows the schematic diagram of the experimental setup used for the pulse-echo measurements. A single transducer operated as emitter as well as receiver. Transducers used for the measurements were flat unfocused single-element, 0.5" transducers: V306-SU with a 2.25 MHz center frequency and 60% bandwidth, A306-SU with 2.25 MHz center frequency and 50% bandwidth and V309-SU with 5 MHz center frequency and 65% bandwidth (Panametrics NDT, Inc., Waltham, MA). During the measurement, a phantom was placed in the water tank in the far-field of the emitting/receiving transducer in order to avoid near-field diffraction effects. For each phantom, 10 signals were acquired, slightly moving the transducer in the plane parallel to the surface of the phantom after each acquisition. The movement was done by linear motion stages (Velmex Bislides, Velmex Inc., Bloomfield, NY) controlled by a stepper motor drive (NI MID-7604) connected to a motion controller (NI PXI 7334, National Instruments Corporation, Austin, TX). A negative impulse was generated on a Pulser/Receiver (5058PR, Panametrics Canada NDT, Quebec) and sent to the emitting/receiving transducer. An average of 16 received signals was then digitized on a data acquisition card and stored on the PC for further analysis.

Since the actually released pulse is required for realistic simulations, reflections of the emitted pulses from a metal needle were measured. The needle was chosen as a good approximation of a point scatterer and was placed at the same distance from the transducer as the phantoms. The recorded pulses were used as input signals for the simulations.

D. Spectral Comparison

In order to compare the observed RF signals with the simulated ones, a 2 cm-long sliding window approach with a 75% window overlap was used. A 2 cm-long window corresponds to 2500-2700 time-samples of the signal and was chosen as an appropriate window size for robust spectral estimation. The Fourier spectra of 10 measured windowed signals were calculated and averaged for each window. The average measured spectra were used for the comparison with the simulation. Simulated signals were obtained by modeling the propagation of the emitted pulse in a medium with given acoustic characteristics. As mentioned above, the spectrum of the windowed backscattered signal is calculated as a spectrum of the signal reflected from a single scatterer positioned in the middle of the window. The input attenuation coefficient in the simulation was discretely changed in the interval between 0 and 2 dB/cm/MHz with the step of 0.01 dB/cm/MHz in order to match the simulated spectrum to the experimentally observed one. A (-6 dB) frequency range of all spectra was selected for the comparison as the most sensitive range of the transducer. The first window was used for the amplitude calibration of the simulated and measured spectra. The following windows were considered independent of each other and the attenuation coefficient corresponding to the minimal distance between simulated and measured spectra was stored for each window.

The distance between the spectra was calculated by fitting a curve of the simulated spectrum to the noisy spectrum of the measured backscatter signal as shown in the Fig. 3. The distance between the measured and simulated spectra at each frequency point in the selected frequency interval was calculated as:

\[ D = \frac{1}{2} \sum_f |S_{\text{meas}}(z,f) - S_{\text{sim}}(z,f)|, \]

where \(S_{\text{meas}}(z,f)\) is the amplitude of a measured spectrum at depth \(z\) and \(S_{\text{sim}}(z,f)\) is the amplitude of the simulated spectrum at the same position.
The global attenuation coefficient of the phantom was determined as an average of the coefficients determined for all windows excluding the first one.

### III. RESULTS

The results of the through-transmission substitution measurements and the attenuation coefficient estimation for all phantoms are presented in Table 1. The estimated attenuation coefficients appeared to be in a good agreement with the values obtained in the through-transmission experiment.

<table>
<thead>
<tr>
<th>Phantoms</th>
<th>Transducer</th>
<th>Attenuation coefficient, through-transmission, dB/cm/MHz</th>
<th>Attenuation coefficient, dB/cm/MHz</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phantom A</td>
<td>V306-SU</td>
<td>0.71</td>
<td>0.77</td>
<td>8.5%</td>
</tr>
<tr>
<td>Phantom B</td>
<td>V306-SU</td>
<td>0.61</td>
<td>0.65</td>
<td>6.6%</td>
</tr>
<tr>
<td></td>
<td>V309-SU</td>
<td>0.60</td>
<td>0.60</td>
<td>1.6%</td>
</tr>
<tr>
<td>Phantom C</td>
<td>V309-SU</td>
<td>0.63</td>
<td>0.73</td>
<td>15.9%</td>
</tr>
<tr>
<td>Phantom D</td>
<td>V306-SU</td>
<td>0.84</td>
<td>0.83</td>
<td>1.2%</td>
</tr>
<tr>
<td></td>
<td>A306-SU</td>
<td>0.84</td>
<td>0.84</td>
<td>0%</td>
</tr>
<tr>
<td>Phantom E</td>
<td>V306-SU</td>
<td>0.39</td>
<td>0.42</td>
<td>7.7%</td>
</tr>
<tr>
<td></td>
<td>V309-SU</td>
<td>0.46</td>
<td>0.46</td>
<td>17.9%</td>
</tr>
<tr>
<td>Phantom F</td>
<td>V309-SU</td>
<td>0.63</td>
<td>0.66</td>
<td>4.8%</td>
</tr>
</tbody>
</table>

### IV. DISCUSSION AND CONCLUSION

A plane wave propagation model, that includes the effects of attenuation, nonlinear distortion, reflection and scattering, was used to iteratively solve the forward wave propagation model to solve the inverse problem. The proposed approach was validated in a phantom study. Six phantoms of different materials with different attenuation characteristics were used. The nonlinear parameter $\beta$ of all phantoms was very close to water (which is approximately 3.5 at 22 °C [15]) and was not optimized for in the present study. The experimental setup was organized in such a way that a plane wave approximation assumption was satisfied. The plane wave propagation model was used for the iterative estimation of the attenuation coefficient. Hereto, the synthetically generated ultrasound data was fit to the experimentally observed ones by changing the input attenuation parameter of the model. A 2 cm-long sliding window with a 75% overlap was used for the spectral estimation that consisted in fitting a curve of the simulated spectrum to the measured one. The comparison was done in the most sensitive frequency region of the transducer (-6 dB). Estimated attenuation coefficients for all phantoms were compared with the results of the insert-substitution experiment. The error of the estimation for the majority of the measurements was below 10%. In two measurements where the error exceeded 15%, the difference between the measured and estimated attenuation coefficients, nevertheless, did not exceed 0.1 dB/cm/MHz. The estimated values were thus close to the reference values. Ongoing work involves the validation in an in-vitro liver experiment and in multi-layered phantoms. Moreover, future work will also include the experimental validation of the estimation of the non-linearity parameter using this iterative approach.

### V. REFERENCES