Modeling emergency departments using discrete-event simulation: A real-life case study including patient boarding

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1 Introduction

This article provides details on the modeling and validation of a discrete-event simulation study carried out at the emergency department (ED) of a large regional hospital in Belgium. The ED has 21 beds, and a volume of about 30,000 patients per year of which approximately 33% need to be admitted to the hospital. Like many other hospital EDs all over the world (Pines et al., 2011b), the ED we consider in this case study is struggling with a phenomenon called (over)crowding, especially in the late afternoon. Following Moskop et al. (2009), we will consistently use the term “crowding” in this article. While there is no single agreed-upon definition of crowding in the literature, it can be understood in general as “the situation where the demand for emergency services exceeds the ability of an ED to provide quality care within appropriate time frames” (Higginson, 2012). The crowding problem is aggravated by the inability of the ED to transfer patients that need to be admitted to the inpatient wards, due to lack of available inpatient beds. This is referred to as “access block” or “patient boarding” (Crawford et al., 2013; Gilligan et al., 2008; Moskop et al., 2009); by occupying valuable ED space, time, and resources, boarding patients have a negative impact on the length-of-stay (LOS) of patients that still require treatment. The model developed in this article reflects patient boarding using time-dependent boarding times and boarding probabilities, which may vary across
patient types and are estimated from real-life data. While, in reality, the boarding behavior is determined by the time-dependent status of beds at the inpatient units, this approach avoids a detailed modeling of these units. Although some articles have applied queueing theory (Bekker & Koeleman, 2011; Bretthauer et al., 2011; Cochran & Roche, 2008; Gallivan & Utley, 2011; Koizumi et al., 2005; Lin et al., 2014; Shi, 2013; Thompson et al., 2009) to settings in which both the ED and the inpatient unit are being considered, it has been recognized in the literature that simulation is often the preferred tool to study ED operations (Saghafian et al., 2014). The blocking or boarding phenomenon in health care has been studied using simulation from the perspective of the inpatient wards (for instance; Bagust et al. 1999; Bountourelis et al. 2011; El-Darzi et al. 1998; Mustafee et al. 2012) or with a focus on the ED (for instance; Bair et al. 2010; Crawford et al. 2014; Khare et al. 2009; Kolb et al. 2007, 2008; Medeiros et al. 2008; Pines et al. 2011a). None of these studies, however, model the ED in much detail. As will be shown, the general dynamic behavior in the ED is triggered by typical patterns and protocols that have been recognized in the literature, and can be acceptably modeled using ED patient record data (thus avoiding detailed data on the inpatient units).

Section 2 provides an overview of the available data, inputs, and assumptions used in the simulation model, while Section 3 discusses model validation. Section 4 summarizes the main findings. The model was built using the Arena® simulation software (V.14) by Rockwell Automation.

2 Model inputs

In this section, we describe the three main types of inputs to the simulation model: arrival data (Section 2.1), processing and routing data (Section 2.2), and admissions and boarding data (Section 2.3).

2.1 Arrival data

Two main categories of patients can be distinguished in the ED: No-injury (NI) versus Trauma (T) patients. The distinction is important, as these categories are treated in different zones of the ED (the NI zone versus the T zone: more details are given in Section 2.2 below). The NI category includes patients with no apparent trauma and intoxication patients, while the T category includes patients suffering from injuries such as burns, fractures, bruises, open wounds, luxation, and sprains. The patient arrivals are modeled according to a non-stationary Poisson distribution, as is common
in ED simulation studies (Kim & Whitt, 2014; Sinreich & Marmor, 2005; Zeltyn et al., 2011). Patients can either arrive by ambulance, or by walk-in. The main difference between these two patient types is that ambulance patients already undergo triage while they are being transported. The hourly arrival rate per patient type is based on historical data of the year 2013. As shown in Figure 1, these arrival rates fluctuate significantly throughout the day, with a peak period roughly between 10:00 a.m. and 9:00 p.m. No significant seasonal or day-of-week effect could be detected in the data but the model can be easily adjusted to account for these effects if necessary. Upon arrival, patients are categorized further into five urgency classes according to the Manchester triage system (FitzGerald et al., 2010; Ganley & Gloster, 2011): red, orange, yellow, green and blue (in decreasing order of urgency). In general, the red and orange patients are seen as ‘urgent’, while the remaining classes are categorized as ‘non-urgent’. Figure 2 displays the actual hourly patient mix based on historical data of the year 2013; it shows that the majority of the patients visiting the ED in both the NI and T categories are in fact non-urgent. In particular the yellow and green categories show noticeable variability throughout the day. Since patient urgency impacts patient priority as well as patient treatment and required resources, it was decided to implement a time-varying urgency mix for both T and NI patients over the day; as shown by the dotted lines in Figure 2, the urgency mix changes in blocks of 4 hours in the simulation model. The most notable change in patient mix occurs around 8 a.m. when the relative number of green patients increases significantly.
Figure 1: The historically observed hourly arrival rate of (a) NI patients and (b) T patients
2.2 Processing and routing data

The ED consists of two main zones, which each contain a specified number of treatment rooms (referred to as boxes). T patients are treated in the T zone, which has 3 private boxes and 1 shared box (containing 4 beds) for non-urgent patients, 2 private boxes for urgent patients, and an isolation box for aggressive patients (the latter can in fact be used by all urgency categories, in case the other boxes are all occupied). NI patients are treated in the NI zone, which has 9 private boxes for non-urgent patients and 2 private boxes for urgent patients. The ED also contains an imaging box, which is shared between the NI and T patients. Urgent patients can seize non-urgent boxes within their own zone in case all urgent boxes are occupied; the reverse is not allowed, as urgent boxes need to be kept available for urgent patients.

Figure 3 summarizes the patient flow through the ED. We largely distinguish four stages: (1) triage, (2) waiting for an ED bed, (3) treatment, and (4) boarding (if the patient does not leave the ED immediately after being finished). Patients arriving by ambulance are already triaged on their way to the hospital, so they immediately enter the ‘wait for bed’ queue. Walk-in patients go through triage first to determine their urgency. Within each shift, a specific nurse (the triage nurse) takes care of the triage process.
for each newly arriving patient; the remainder of the time, she serves as a
regular ED nurse for other processes within the ED. Once the patient has
seized a bed, he enters the treatment phase. The treatment phase consists
of different process steps, depending on patient type and patient urgency.
While in treatment, the patients alternates between 2 states: he is either
receiving treatment for a given process step, or he is ‘waiting for staff’ (i.e.,
waiting for a doctor and/or nurse to receive the next step in his treatment
plan). We distinguish 9 process steps and each of these steps may require
specific resources (Table 1).

![Figure 3: A simplified patient flow diagram of the ED](image)

<table>
<thead>
<tr>
<th>Process Steps</th>
<th>Doctor</th>
<th>Nurse</th>
<th>Logistics Personnel</th>
<th>External Doctor</th>
<th>External Personnel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clinical Examination</td>
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<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parameters Monitoring</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Blood Sampling</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Internal Medical Imaging</td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>External Medical Imaging</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Internal and External Consulting</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Medication</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other Examination and Treatment</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discharge</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Processing steps in ED and the required resources

Doctors, nurses and logistics personnel are ED resources; external doc-
tors and external personnel are not (e.g., the ED can call upon a doctor from
an inpatient unit for clinical examinations or consulting; external medical
imaging personnel can be called upon when needed for specific imaging
tasks). Logistics personnel is responsible for transporting patients within
the ED (for instance to/from the imaging area). It is the patient’s type
and urgency that determines the probability that he needs to undergo a
given step, as well as the actual resources involved and the process time required from each of these resources. Routing probabilities could be retrieved from internal patient data but as reliable data on the processing times could not be retrieved from historical databases, estimates for the process time requirements were developed based on judgment from an expert team (comprising doctors and nurses). The resulting routing matrix is shown in the Electronic Companion of this paper. The expert team agreed to model the process times as triangular distributions, as in Ahmed & Alkhamis (2009) and Gunal & Pidd (2006). Minimum and maximum values for the triangular distributions were estimated by the expert team; the most likely value was set at $minimum + \frac{1}{3} \times (maximum - minimum)$ to reflect the positive skewness that is generally present in processing times (Holm & Barra, 2011).

Although the use of a simplified distribution such as the triangular has some drawbacks (Holm & Barra, 2011), we opted for this distribution assumption since the meaning of the three parameters required for distribution is relatively easy to understand, which considerably facilitated communication with the expert team. We went through several iterations of adjustments to calibrate the processing times to a level that yielded realistic simulation results and was truthful according to the expert panel.

Patients with higher urgency levels get priority over patients with lower urgency levels when calling upon resources (nurses, doctors, beds, etc.) for treatment. For blue and green patients, however, the priority is not always strict: blue patients are sometimes given priority over green patients in practice. Since it was difficult for the hospital personnel to define a protocol for prioritizing blue patients over green patients, and the number of blue patients is very small relative to green patients (see Figure 2), we opted to give both blue and green patients the same priority in the model. It has been observed in other case studies as well that the 5 color triage system may not always be working well; when the distinction between green & blue and orange & yellow patients is not clear enough, patients are actually triaged into just 3 categories in practice (minor, major, and life threatening) (Gunal & Pidd, 2006). Urgent patients (red and orange) may preempt ED personnel that is treating a non-urgent patient, effectively interrupting the treatment of that non-urgent patient. The only process that cannot reasonably be interrupted is internal medical imaging. Consequently, urgent patients may only have to wait for resources at triage (as their urgency is supposed to be unknown at that time), and at the imaging box. Urgent patients that need more than one resource at a given process step require collaboration from these resources, implying that all resources (for instance, both a doctor and a nurse) need to be available simultaneously before the process can start.
Non-urgent patients do not require such collaboration; resources can be called upon sequentially. Patients that have been discharged from the ED either leave the ED, or need to be admitted to the hospital: in the latter case, they are redirected to an inpatient unit. If there is no bed available in the inpatient unit, the patient remains ‘blocked’ or ‘boarded’ in the ED bed, preventing other patients to start treatment. A very small percentage of patients are initially supposed to be admitted and board in the ED for a while, but are sent home afterwards when their situation improves.

2.3 Admissions and boarding

We identified two possible strategies to model the boarding patients based on ED patient records;

- A first strategy is to use a time-varying rate at which patients are transferred to the inpatient units, based on historical data (as in Khare et al. 2009 and Medeiros et al. 2008).

- An alternative strategy is to introduce a probability of boarding to the different patient types, and fit a time-varying distribution to the actual ‘boarding time’; the timespan between the moment that a patient is finished in the ED and the time that he physically leaves the ED. These boarding times were logged during 3 months (February 8, 2014 until May 8, 2014) by the ED personnel, resulting in 7600 patient records.

The problem with the first approach is that the observed departure rates do not necessarily reflect the availability of beds in the inpatient wards. For instance, no departures will be observed when the boarding patient census is zero; even when there are plenty of beds available in the wards. We thus opt for the second approach. From the ED patient records, we determine the probability that a patient needs to be admitted\(^1\): as shown in Table 2, this depends on the patient type and urgency. As the urgency goes down, the admission probability also tends to go down (as also observed in Khare et al. 2009 and Peck et al. 2013), except for the NI Red category for which ‘only’ 45.45% is admitted. This, however, is due to the high decease probability in this category. We obtain an aggregate admission percentage of 33% (weighted according to the number of patients in each category), which is close to the reported aggregate admission percentages observed in the literature (between 26% and 32%; see Armony et al. 2011; Peck et al. 2013; Shi et al. 2012b).

\(^1\)Note that we do not keep track of the specific inpatient ward to which the patient is transported.
Table 2: Admission probability for each of the patient categories

<table>
<thead>
<tr>
<th>Patient category</th>
<th>Admission probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>NI Red</td>
<td>45.45%</td>
</tr>
<tr>
<td>NI Orange</td>
<td>78.99%</td>
</tr>
<tr>
<td>NI Yellow</td>
<td>60.62%</td>
</tr>
<tr>
<td>NI Green</td>
<td>34.88%</td>
</tr>
<tr>
<td>NI Blue</td>
<td>21.71%</td>
</tr>
<tr>
<td>T Red</td>
<td>93.75%</td>
</tr>
<tr>
<td>T Orange</td>
<td>53.25%</td>
</tr>
<tr>
<td>T Yellow</td>
<td>17.84%</td>
</tr>
<tr>
<td>T Green</td>
<td>2.66%</td>
</tr>
<tr>
<td>T Blue</td>
<td>4.72%</td>
</tr>
</tbody>
</table>

The boarding time will depend on the patient type (T versus NI) and the time that the patient finished treatment in the ED. While there was some evidence that patients with high urgency levels were also prioritized in the admission process to the inpatient units (Table 3), urgency-specific boarding time distributions could not be fit reliably, especially for the red and blue category (which are very few in number). Overall, the average boarding time for patients that have to be admitted is 1.24 hours, which is considerably less than the 2.5 hours observed by Shi et al. (2012a) and the 3.2 hours reported by Armony et al. (2011). Since we focus very strongly on the ED, our “boarding time” ends when the patient physically leaves the ED, similar to the definition used in Shi et al. (2012a). In contrast, Armony et al. (2011) consider the elapsed time from patient assignment to an inpatient unit until receiving the first treatment in the inpatient unit. This may explain part of the difference in average boarding time.

Table 3: Average boarding time for the patients that are admitted

<table>
<thead>
<tr>
<th>Patient urgency</th>
<th>Average boarding time (hours)</th>
<th>Number of observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>0.1429</td>
<td>28</td>
</tr>
<tr>
<td>Orange</td>
<td>1.1069</td>
<td>478</td>
</tr>
<tr>
<td>Yellow</td>
<td>1.2882</td>
<td>1332</td>
</tr>
<tr>
<td>Green</td>
<td>1.2752</td>
<td>646</td>
</tr>
<tr>
<td>Blue</td>
<td>1.4484</td>
<td>43</td>
</tr>
<tr>
<td>Overall average</td>
<td>1.2415</td>
<td>2527</td>
</tr>
</tbody>
</table>

Unlike Armony et al. (2011) and Shi et al. (2012a), who reported the longest boarding times for patients that are finished in the early morning,
the boarding times in our ED are longest for patients that finish treatment around midnight (see Figure 4a). Furthermore, apart from the often-observed peak in boarding patient census in the afternoon (Powell et al., 2012), we discovered a second peak around 6:00 a.m. (see Figure 4b). The expert panel in the hospital confirmed that this is due to two different protocols that influence patient transfer to the wards (see Figure 4):

- The hospital tries to limit the transfer of patients to the inpatient wards during the nights (i.e., between 10:00 p.m. and 7:00 a.m.). Even if there is room in an inpatient unit, a patient may have to wait until 7:00 a.m., when regular transportation of patients from the ED to the inpatient wards resumes. Exceptions on the no-night-transport policy are often made in practice for urgent patients or when the ED is very crowded.

- As is common in many hospitals (Khanna et al., 2011, 2012; Powell et al., 2012; Shi et al., 2012b), the hospital starts discharging patients from the inpatient wards after the morning round of the physicians, causing a peak of discharges around 2:00 p.m., which leads to increased admissions to the inpatient wards. This explains why boarding census steeply increases before 2:00 p.m., and diminishes considerably afterwards.
(a) Expected boarding time of patients that need to be admitted ((un)conditional on whether the patients has a positive boarding time), depending on the time that the patient finished treatment in the ED

(b) Boarding patient census

Figure 4: Historically observed boarding metrics: (a) expected boarding times, and (b) boarding patient census
Estimates of the boarding time distributions of admitted patients\textsuperscript{2} were derived from the historical data (February 8 - May 8, 2014), depending on the time instant at which a patient finished treatment in the ED: (1) between 7:00 a.m. and 2:00 p.m. (which implies that he might have to wait for a bed until 2:00 p.m.); (2) between 2:00 p.m. and 10:00 p.m.; and (3) between 10:00 p.m. and 7:00 a.m. (which implies that he might have to wait for transport until 7:00 a.m.). For timeframe (2), data were sufficient to allow for boarding time estimates dependent on patient type (T versus NI); for the other timeframes, the data for NI and T patients were aggregated. The result is shown in Figure 5.\textsuperscript{3}

For patients that were admitted only after reaching a given threshold (either 7:00 a.m. or 2:00 p.m.), the beta distributions shown in Figure 5 reflect the additional boarding time incurred after reaching that threshold.

Unfortunately, the ED patient records did not allow us to gain insight in the further care needed by boarding patients from the ED doctors and/or nurses. The model thus assumes that boarding patients do not put any workload on these ED resources during their boarding time, and only impact the ED by occupying beds. It is thus likely that the model slightly underestimates the workload of the staff (see Figure 7 of Armony et al. 2011).

\textsuperscript{2}As described earlier, some patients who were eventually not admitted nevertheless incurred a boarding time. The probability of this happening is quite low: more precisely, this occurs with 1.8\% of the non-admitted patients. The distribution $1060 \ast BETA(0.31, 0.734)$ was fit to these historical observations.

\textsuperscript{3}In the simulation model, the boarding times that involved Lognormal distributions were altered to min (X, MAX), where X refers to the random variate generated by the distribution, and MAX refers to the maximum observed boarding time for that category of patients in the historical data. Since $Pr(X > MAX) \leq 2.5\%$ for all patient categories, this rough approach to truncation was deemed adequate.
Figure 5: Boarding time estimates obtained from historical observations, depending on the time instant at which the patient finished treatment in the ED.

2.4 Shift schedule

Table 4 gives an overview of the shift schedule for ED personnel (recall that external doctors and other external personnel are not considered to be ED resources, see Section 2.2). Doctors work in 2 shifts (Day and Night) and nurses in 3 shifts (Early, Late and Night). Nurse shifts tend to overlap by
15 or 30 minutes: this overlap is used for briefing. Note that there is no logistics personnel available from 10:00 p.m. until 6:30 a.m.; their tasks are carried out by the nurses at night.

<table>
<thead>
<tr>
<th>Resource type</th>
<th>Shift type</th>
<th>Hours</th>
<th>number resources following this shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>Doctor</td>
<td>Day Shift</td>
<td>08:00-20:00</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Night Shift</td>
<td>20:00-08:00</td>
<td>1</td>
</tr>
<tr>
<td>Nurse</td>
<td>Early Shift</td>
<td>06:30-14:30</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Late Shift</td>
<td>14:00-22:00</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Night Shift</td>
<td>21:45-06:45</td>
<td>4</td>
</tr>
<tr>
<td>Logistics personnel</td>
<td>Early Shift</td>
<td>06:30-14:30</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Late Shift</td>
<td>14:00-22:00</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Day Shift</td>
<td>08:00-16:30</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4: Current shift schedule of ED personnel

3 Validating the simulation model

To validate the simulation model, we compare the following key outcomes of the simulation model to outcomes observed in the historical data (or acknowledged by ED personnel): differences in patient length-of-stay across patient types, outflow of patients (time-dependent patient completion rate and patient departure rate), and boarding metrics (time-dependent boarding patient census and expected boarding times). All results have been obtained by running 20 replications of 200 days each. Although the warm-up period is not significant in our ED (since it tends to become almost completely empty early in the morning each day; an observation that Gunal & Pidd (2006) also confirmed), the model includes a short warm-up period of 10 days.

3.1 Length-of-stay

We measure LOS as the time elapsed between the moment that a patient arrives in the ED and the time that he finishes all ED processes. As boarding times are largely caused by factors that are not within the control of the ED (such as discharge policies and elective planning in the inpatient wards), boarding time is not included in the LOS measure. Figure 6 displays the LOS for each patient category, broken down into 8 different components. Evidently, the share of the LOS that total waiting takes up is larger for less urgent patients. “Waiting for bed” (WFB) represents
the time that a patient has to wait before he gets assigned to a bed or a box in the ED and actual treatment can be initiated (see also Figure 3). An important observation is that NI patients seem to experience much more WFB than the T patients do. This is consistent with observations from the historical data: these revealed that the probability that all 11 NI beds are occupied is 21.0% while the probability that all 10 T beds are full is only 2.9%. Blue bars represent “waiting for staff” (WFS) while the patients are in the treatment phase of Figure 3. Currently, the doctor seems to be the largest bottleneck since patients have to wait the longest for him on average. Logistics personnel, on the other hand, poses no problem at all. ‘Preemption time’ only appears with the non-urgent categories and represents the average time that a non-urgent patient is preempted by an urgent patient. As expected, the average preemption time is negligibly small, as preemption hardly ever occurs.

![Decomposition of the length of stay](image)

Figure 6: LOS for each of the 10 patient categories (simulation outcomes)

### 3.2 Outflow of patients

While the arrival rate is a direct input into the simulation model, the outflow of patients is the result of the arrival process and the dynamic behavior of the model, following the rules and assumptions that were implemented. Apart from a slight underestimation of outflow between 8:00 p.m. and 10:00 p.m., and an overestimation during the remaining hours of the night, the outflow
of patients matches reality fairly well, as shown in Figure 7.

Figure 7: Outflow of patients (simulation model versus reality)

The sudden drop in outflow at 8:00 p.m. observed in the model might be due to a slight (but systematic) overestimation of doctor process times relative to nurse process times (recall that these process times were estimated based on the advice of the expert team, and are thus likely subject to small errors). This results in the doctors being a larger bottleneck in our model than they probably are in reality, and might explain why the model exhibits the explicit drop in outflow at 8:00 p.m. (when the number of available doctors is reduced from 2 to 1) while in reality, this drop is more gradual over time.

3.3 Boarding metrics

Figure 8 shows that the boarding patient census and the expected boarding times observed in the model correspond reasonably well with the historical outcomes. The overestimation of boarding patient census during the night is partly due to the overestimation of the nightly outflow (as discussed above), which causes more patients to wait for transportation than in reality. Nevertheless, as pictured in Figure 8a, the overall trend over the day in the boarding patient census is true to reality. Likewise, Figure 8b shows that the expected boarding time is also reasonably well approximated.
Figure 8: Comparison of simulation model outcomes to historical observations, for (a) the boarding patient census and (b) the expected boarding time.
4 Conclusion and future research

This article provides details on the modeling and validation of a discrete-event simulation study carried out at the ED of a large regional hospital in Belgium. The model reflects patient boarding using time-dependent boarding times and boarding probabilities estimated from ED patient records, and thus avoids a detailed modeling of the inpatient units. The results show that the general dynamic behavior in the ED is triggered by typical patterns and protocols, and can be acceptably modeled using our approach, in spite of the relatively limited amount of data available.

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