The vehicle routing problem: state of the art classification and review

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Abstract: Over the past decades, the Vehicle Routing Problem (VRP) and its variants have grown ever more popular in the academic literature. Yet, the problem characteristics and assumptions vary widely and few literature reviews have made an effort to classify the existing articles accordingly. In this article, we present a taxonomic review of the VRP literature published between 2009 and 2013. Based on an adapted version of the comprehensive taxonomy suggested by Eksioglu et al. (2009), we classify 144 articles and analyze the trends in the VRP literature. This classification is the first to categorize the articles to this level of detail.

Keywords: vehicle routing, taxonomy, dynamic vehicle routing, time-dependent vehicle routing

JEL code: M1,M2

1. Introduction

Dantzig and Ramser (1959) were the first to introduce the "Truck Dispatching Problem", modelling how a fleet of homogeneous trucks could serve the demand for oil of a number of gas stations from a central hub and with a minimum travelled distance. Five years later, Clarke and Wright (1964) generalized this problem to a linear optimization problem that is commonly encountered in the domain of logistics and transport: i.e., how to serve a set of customers, geographically diffused around the central depot, using a fleet of trucks with varying capacities. This became known as the 'Vehicle Routing Problem' (VRP), one of the most widely studied topics in the field of Operations Research.

The current VRP models, however, are immensely different from the one introduced by Dantzig and Ramser (1959) and Clarke and Wright (1964), as they increasingly aim to incorporate real-life complexities, such as for instance time-dependent travel times (reflecting to traffic congestion), time windows for pickup and delivery, and input information (e.g., demand information) that changes dynamically over time. These features bring along substantial complexity. As the VRP is an NP-hard problem (Lenstra and Rinnooy Kan, 1981), exact algorithms are only efficient for small problem
instances. Heuristics and metaheuristics are often more suitable for practical applications, because real-life problems are considerably larger in scale (e.g., a company may need to supply thousands of customers from dozens of depots with numerous vehicles and subject to a variety of constraints).

The number of solution methods introduced in the academic literature (for old as well as new variants of the VRP) has grown rapidly over the past decades. Moreover, the processing speed and memory capacity of current computers has increased exponentially, enabling to solve larger instances of the VRP which spurs the progression in the research field and the development of commercial software for the VRP. According to a recent survey (Hall, 2012), thousands of companies, among others Coca-Cola Enterprises and Anheus-Bush Inbev, nowadays use VRP software.

The study by Eksioglu et al. (2009) revealed 1021 journal articles with VRP as the main topic, published between 1959 and 2008. A small number of books and a considerable amount of proceedings have also contributed to the VRP literature that exists today. According to Eksioglu et al. (2009), the VRP literature has been growing exponentially at a rate of 6% each year. This popularity makes it difficult to keep track of the developments in the field, and to have a clear overview of which variants and solution methods are relatively novel. The taxonomies and surveys of existing literature are limited: often only specific variants of the VRP are discussed, and differences in assumptions are not always studied in detail.

The purpose of this article is to classify the academic literature on the VRP, based on the detailed characteristics of each article. As we base our classification on the taxonomy by Eksioglu et al. (2009), we restrict our analysis to articles published over the period 2009-2013 and as such do not intend to provide an exhaustive overview of the VRP literature. To the best of our knowledge, this article provides the first structured classification of VRP literature. Although previous literature reviews on VRP have been published (Drexl 2013, Pillac et al. 2013, Drexl 2012, Baldacci et al. 2012, Eksioglu et al. 2009, Laporte 2009), none of these classify literature according to the VRP characteristics. The resulting classification table enables future researchers to find relevant literature by eliminating or selecting characteristics in the taxonomy, leaving only articles tailored to their interests. Additionally, the classification allows to analyze which characteristics and VRP variants are most popular, and which are promising topics for future research.

Section 2 defines the scope of the survey, and Section 3 introduces general notations for popular VRP variants. The taxonomy we apply is discussed in Section 4. We highlight our findings in Section 5, discussing a number of relatively novel topics (i.e., the Open VRP, the Dynamic VRP and the Time-dependent VRP) in further detail. Section 6 provides an overview of the results and avenues for future research.
2. Scope of the survey

We structure the recent literature, published between 2009 and 2013, using a taxonomic framework. The classification is followed by a survey that uses the taxonomy to evaluate the trends in the field, and which articles contribute to these trends. Furthermore, it analyzes which extensions or solution methods deserve more attention.

We restrict the reviewed literature as follows: relevant articles were selected using the LIMO database, requiring "vehicle routing" as title word, and requiring the source type to be English-language journal articles. Accordingly, no books, conference proceedings or dissertations are considered. To extract only the actual relevant literature, all available journals were ranked according to their impact factor. From the 21 suggested journals, 9 journals remained because of their importance in the domain of either Operational Research or Transportation (see Table 1). The abstracts of the resulting 297 articles were read to determine their relevance to the subject. Since the VRP already is an extensive problem, the decision was made not to include any related problems, such as inventory routing (see Bertazzi et al. (2008)), combinations of scheduling and routing (see Ullrich 2013 and Chen et al. 2009), multi-echelon routing (see Hemmelmayr et al. 2012), multi-dimensional loading problem (see Tarantilis et al. 2009) and routing with cross-docking (see Liao et al. 2010).

This resulted in a final set of 144 articles (see Appendix A for an overview of the selected references, according to publication year). Table 1 shows that the European Journal of Operational Research and Computers & Operations Research published most of the surveyed articles.

<table>
<thead>
<tr>
<th>Journal</th>
<th>Impact factor</th>
<th>Number of selected articles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expert systems with applications</td>
<td>2.203</td>
<td>28</td>
</tr>
<tr>
<td>Transportation Research</td>
<td>1.957</td>
<td>16</td>
</tr>
<tr>
<td>European Journal of Operational Research</td>
<td>1.815</td>
<td>36</td>
</tr>
<tr>
<td>Computers &amp; Operations Research</td>
<td>1.720</td>
<td>34</td>
</tr>
<tr>
<td>Decision Support systems</td>
<td>1.687</td>
<td>4</td>
</tr>
<tr>
<td>Computers &amp; Industrial Engineering</td>
<td>1.589</td>
<td>10</td>
</tr>
<tr>
<td>Transportation science</td>
<td>1.507</td>
<td>15</td>
</tr>
<tr>
<td>Applied Mathematics and Computation</td>
<td>1.317</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1: Overview of the number of selected articles per journal

3. Terminology and notations

The classical VRP, also known as the Capacitated VRP (CVRP), designs optimal delivery routes where each vehicle only travels one route, each vehicle has the same characteristics and there is only one central depot. The goal of the VRP is to find a set of least-cost vehicle routes such that each
customer is visited exactly once by one vehicle, each vehicle starts its route from the depot and ends in the depot, and the capacity of the vehicles is not exceeded.

It is possible to extend this problem by varying the capacities, which results in the Heterogeneous Fleet VRP (HFVRP), also known as the Mixed Fleet VRP. Another popular extension, the VRP with Time Windows (VRPTW), assumes that deliveries to a given customer must occur in a certain time interval, which varies from customer to customer. Time windows are defined as hard (or strict) when it is not allowed to deliver outside of the time interval (Agra et al., 2013; Vidal et al., 2013). Soft time windows on the other hand allow deliveries outside the boundaries against a penalty cost (Tas et al., 2013; Figliozzi, 2010).

In the VRP with Pickup and Delivery (VRPPD), goods need to be picked up from a certain location and dropped off at their destination. The pick-up and drop-off must be done by the same vehicle, which is why the pick-up location and drop-off location must be included in the same route (Tasan & Gen, 2012). A related problem is the VRP with backhauls (VRPPB), where a vehicle does deliveries as well as pick-ups in one route (Pradenas et al., 2013). Some customers require deliveries (referred to as linehauls) and others require pick-ups (referred to as backhauls). The combination of linehauls and backhauls has been proven very valuable to the industry. The well-known ‘milk run’ concept is derived from the successes reached with VRPPB: by employing milk runs, transportation costs and total distance travelled can be decreased significantly and the vehicle loading rate increases (Brar & Saini, 2011).

The Multi Depot VRP (MDVRP) assumes that multiple depots are geographically spread among the customers (Stenger et al. 2013; Kuo et al., 2012). The Periodic VRP (PVRP) is used when planning is done over a certain period and deliveries to the customer can be done in different days (Gulczynski et al., 2011b; Yu & Yang, 2011). For the PVRP, customers can be visited more than once, though often with limited frequency.

4. Taxonomy

The proposed taxonomy is an adapted version of the taxonomy proposed by Eksioglu et al. (2009). We distinguish five main topics (type of study, scenario characteristics, problem physical characteristics, information characteristics and data characteristics), each with its own detailed categories and sub-categories (see Table 2). The categories indicated in bold are adapted from Eksioglu et al. (2009); the corresponding topics will be addressed in further detail in the following paragraphs.
1. Type of study
  1.1. Theory
  1.2. Applied methods
  1.2.1. Exact methods
  1.2.2. Classical Heuristics
  1.2.3. Metaheuristics
  1.2.4. Simulation
  1.2.5. Real-time solution methods
  1.3. Implementation documented
  1.4. Survey, review or meta-research

2. Scenario Characteristics
  2.1. Number of stops on route
    2.1.1. Known (deterministic)
    2.1.2. Partially known, partially probabilistic
  2.2. Load splitting constraint
    2.2.1. Splitting allowed
    2.2.2. Splitting not allowed
  2.3. Customer service demand quantity
    2.3.1. Deterministic
    2.3.2. Stochastic
    2.3.3. Unknown
  2.4. Request times of new customers
    2.4.1. Deterministic
    2.4.2. Stochastic
    2.4.3. Unknown
  2.5. Onsite service/waiting times
    2.5.1. Deterministic
    2.5.2. Dependent
    2.5.3. Stochastic
    2.5.4. Unknown
  2.6. Time window structure
    2.6.1. Soft time windows
    2.6.2. Strict time windows
    2.6.3. Mix of both
  2.7. Time horizon
    2.7.1. Single period
    2.7.2. Multi period
  2.8. Backhauls
    2.8.1. Nodes request simultaneous pickups and deliveries
    2.8.2. Nodes request either linehaul or backhaul service, but not both
  2.9. Node/Arc covering constraints
    2.9.1. Precedence and coupling constraints
    2.9.2. Subset covering constraints
    2.9.3. Recourse allowed

3. Problem Physical Characteristics
  3.1. Transportation network design
    3.1.1. Directed network
    3.1.2. Undirected network
  3.2. Location of addresses (customers)
    3.2.1. Customer on nodes
    3.2.2. Arc routing instances
  3.3. Number of points of origin
    3.3.1. Single origin
    3.3.2. Multiple origin

3.4. Number of points of loading/unloading facilities (depot)
  3.4.1. Single depot
  3.4.2. Multiple depots

3.5. Time window type
  3.5.1. Restriction on customers
  3.5.2. Restriction on depot/hubs
  3.5.3. Restriction on drivers/vehicle

3.6. Number of vehicles
  3.6.1. Exactly \( n \) vehicles
  3.6.2. Up to \( n \) vehicles
  3.6.3. Unlimited number of vehicles

3.7. Capacity consideration
  3.7.1. Capacitated vehicles
  3.7.2. Uncapacitated vehicles

3.8. Vehicle homogeneity (Capacity)
  3.8.1. Similar vehicles
  3.8.2. Load-specific vehicles
  3.8.3. Heterogeneous vehicles
  3.8.4. Customer-specific vehicles

3.9. Travel time
  3.9.1. Deterministic
  3.9.2. Function dependent (a function of current time)
  3.9.3. Stochastic
  3.9.4. Unknown

3.10. Objective
  3.10.1. Travel time dependent
  3.10.2. Distance dependent
  3.10.3. Vehicle dependent
  3.10.4. Function of lateness
  3.10.5. Implied hazard/risk related
  3.10.6. Other

4. Information Characteristics
  4.1. Evolution of information
    4.1.1. Static
    4.1.2. Partially dynamic
  4.2. Quality of information
    4.2.1. Known (Deterministic)
    4.2.2. Stochastic
    4.2.3. Forecast
    4.2.4. Unknown (Real-time)
  4.3. Availability of information
    4.3.1. Local
    4.3.2. Global
  4.4. Processing of information
    4.4.1. Centralized
    4.4.2. Decentralized

5. Data Characteristics
  5.1. Data used
    5.1.1. Real-world data
    5.1.2. Synthetic data
    5.1.3. Both real and synthetic data

5.2. No data used

Table 2: The proposed taxonomy (adapted from Eksioglu et al. 2009)
4.1. Type of study

The ‘type of study’ defines the analyzed article according to the content, and can be divided into four categories. The first category, consisting of theoretical articles, contributes to the general understanding of the VRP in all its aspects. In case of a theoretical article in VRP literature, researchers generally investigate the effects of changing certain problem characteristics or formulate strategies to reduce the computational time of heuristics.

The second category refers to the methods that have been applied. The exact solution methods allow the global optimum to be found. However, they are often computationally expensive because the VRP (and many of its extensions) are shown to be NP-hard (Lenstra & Rinnenoy Kan, 1981). As a result, many authors propose heuristics. In addition to what Eksioglu et al. (2009) proposed in their taxonomy, we suggest to differentiate between classical heuristics and metaheuristics. Laporte (2009) defines classical heuristics as heuristics that do not allow the intermediate solution to deteriorate during the process of finding better (optimal) solutions. As a result, they often get trapped in local optima. Metaheuristics, on the other hand, include mechanisms that avoid getting trapped in local optima. Simulation refers to those methods that incorporate what-if analyses to generate multiple scenarios and then select the best one to use these in the main algorithm. The articles that propose real-time solution methods, useful for solving dynamic or online VRPs, are classified in the last subcategory. Here, the solution method continues to solve the problem as more information gets revealed or updated when a vehicle already embarked on its predefined route.

The third category comprises all articles that have documented the real-world implementation of the approach, whereas the last category collects surveys, reviews and meta-researches that concern the VRP.

4.2. Scenario Characteristics

This topic lists all characteristics of the problems that are part of the problem scenario (Eksioglu et al., 2009) and thus indirectly affect the solution.

The number of stops on the route deals with the question how many customers need to be served. In most VRP variants, the number of customers is deterministic: i.e., it is known beforehand and does not change during the route of the vehicle. Whenever new customer orders still arrive during a vehicle’s route, as in the on-line VRP (Liao & Hu, 2011), part of the number of stops is known (all orders that were known before constructing the route), while another part is probabilistic.

Load splitting occurs when a vehicle can serve the customer demand in multiple trips by, for example, serving half of the demand, going back to the depot, filling up the vehicle and serving the other half of the demand. This is the case for the VRP with split deliveries. When customers can only be served once by a single vehicle, load splitting is not allowed.
Customer service demand quantity classifies articles according to the quality of the information on the demand size (see category 4.2.). Most often the size of the demand is known beforehand (i.e., it is deterministic). Demand is stochastic when its size is a random variable with a known distribution. An unknown demand occurs in the dynamic VRP or in the VRP with fuzzy demand, where information is revealed in real-time. The demand quantity is especially crucial for VRP with capacitated vehicles.

The request times of new customers define when new customers are placing their orders. In most VRPs, all the customer orders will be known in advance and no new customer requests will come in. In cases where new requests might arrive, such as the dynamic VRP, the request times are deterministic, stochastic or unknown.

Onsite service or waiting times indicate the exact time a vehicle has to wait at a customer before it can start the service or the amount of time it takes to perform the service. This is particularly relevant when dealing with time windows. In this category, we made an adaptation to the taxonomy provided by Eksioglu et al. (2009) by merging the two categories ‘time dependent’ and ‘vehicle type dependent’ into one category ‘dependent’, as the service time can be dependent on many more aspects (such as the number of personnel in the vehicle, see Pureza et al. (2012), or the delivery quantity, see Gulczynski et al. (2011b)).

A time window is an interval in which the customer has to be served, and can be divided into soft and strict windows, or a mix of both. This taxonomy also differentiates between single and multiple period time horizons. In most VRP articles a vehicle routing plan is calculated for one day only, which is defined as ‘single period’. Some articles, however, take into account that vehicle routes have to be planned for multiple days and the workload of the drivers has to be balanced. In this case the article is classified as ‘multi period’.

The next category classifies the articles that not only do deliveries, or linehauls, but also pickups, called backhauls. In case of simultaneous pickups and deliveries, each customer requests a certain delivery quantity as well as a certain quantity to be picked up. On the other hand, it is also possible that only a subset of customers request deliveries, while the remaining customers request pickups. Node/arc covering constraints show whether customers need to be served together (precedence and coupling constraints), or whether not all customers need to be served, allowing the scheduler to prioritize customers that deliver the highest profit (subset covering constraints), or whether the vehicle is allowed to return to the depot to restock in order to fulfill a customer’s demand (recourse allowed).

4.3. Problem Physical Characteristics

Problem physical characteristics are defined by Eksioglu et al. (2009) as the factors that are embedded in the constraints and therefore directly affect the solution.
The transportation network of the VRP can be designed as a directed graph or an undirected graph. A network is directed whenever the costs associated with the arcs are asymmetric; an undirected network occurs when the cost matrix is symmetric.

Customers can be located on the nodes or the arcs of the graph. Often, when customers are located on the arcs, the VRP is called an Arc Routing Problem (ARP).

As authors mostly test their method on randomly dispersed customers or on clustered customers (e.g. Tarantilis et al. (2012a), Azi et al. (2010), Bräysy et al. (2009) and Solomon (1987)), we decided to remove the characteristic ‘geographical location’ from the taxonomy of Eksioglu et al. (2009).

The number of points of origin and the number of depots go hand in hand. When all vehicles start at the same point (typically the depot) we refer to it as ‘single origin’. In other situations, the vehicles are allowed to start from different locations (multiple origins), either from the customers or from multiple depots.

The time window type classifies articles depending on the party that is restricted by the window (either customers, depots or drivers). Eksioglu et al. (2009) also included a time window restriction on roads. Since this is rare, it was excluded.

VRP articles have also been classified according to the number and type of the vehicles. In case of capacitated vehicles, vehicles can all have exactly the same capacity; this is called ‘similar vehicles’ (the opposite are heterogenous vehicles. Load-specific can only accommodate a specific load (e.g., multi-compartment vehicles where each compartment is dedicated to one specific good). Articles that assume vehicles which can only serve specific customers belong to the ‘customer-specific’ category. Articles without capacity constraint are listed as ‘uncapacitated’.

The travel time can be classified according to the quality of the information (see category 4.2.). Additionally, the time-dependent VRP belongs to the category with function dependent travel time. This means that the travel time is dependent on the time of departure or arrival, which allows to take into account traffic congestion.

The objective category is an adapted version of the original taxonomy and categorizes all the articles according to the selected objective function (either focusing on travel time, distance, number of vehicles, costs related to lateness, costs related to risks or hazards, or any other objective type).

4.4. Information Characteristics

This category distinguishes the evolution, quality, availability, and processing of information.

The evolution of information deals with how the input of the VRP is revealed during the routing process. When all information is known beforehand, the VRP is defined as static; often, however, input is revealed or updated during the process (partially dynamic).

The quality of information depends on the moment the decision has to be made. Information can either be known with certainty (deterministic), or can be a random variable with a known probability distribution (stochastic). When the information is based on information of the past, it is classified as a
forecast. It is also possible that the input is a random variable on which no information is available at the moment of the decision (unknown or real-time). Note that some specific characteristics of the problem, such as travel time or service time, are accounted for separately in the taxonomy (see 2.3. and 3.9. in taxonomy).

Information is locally available when the information is only revealed when arriving at the node; otherwise, it is globally available.

Information can be processed in a centralized way (e.g., in case of a single dispatcher) or decentralized way (for instance, when drivers individually decide which customers to serve and in which sequence).

4.5. Data Characteristics

Solution methods can be tested either on real-world data, synthetic data (such as benchmark data or instances created by the author), or a combination. For theoretical papers or reviews in VRP literature (see 1.1. and 1.4. in taxonomy), often no data is used.

5. Results of the classification

In this section the results of the classification are discussed. The detailed classification results of the 144 articles is shown in the electronic Appendix of this manuscript. The .xlsx format allows the user to select any given (combination of) identifier(s) in order to retrieve the relevant articles. In the remainder of this section, we discuss our main findings.

Table 3 gives an overview of the different VRP variants in the past five years. As multiple assumptions are usually combined (e.g., time windows, multiple depots and capacitated vehicles), categories are not mutually exclusive. The obtained results given an indication of the importance of the given characteristics in the literature.

<table>
<thead>
<tr>
<th>Variant</th>
<th>Number of articles (total = 144)</th>
<th>Relative presence</th>
</tr>
</thead>
<tbody>
<tr>
<td>CVRP (Capacitated)</td>
<td>128</td>
<td>88.89%</td>
</tr>
<tr>
<td>VRPTW (Time Window)</td>
<td>57</td>
<td>39.58%</td>
</tr>
<tr>
<td>HVRP (Heterogeneous)</td>
<td>27</td>
<td>18.75%</td>
</tr>
<tr>
<td>MDVRP (Multi Depot)</td>
<td>18</td>
<td>12.50%</td>
</tr>
<tr>
<td>VRPPB (Backhauls)</td>
<td>17</td>
<td>11.81%</td>
</tr>
<tr>
<td>SDVRP (Split Deliveries)</td>
<td>16</td>
<td>11.11%</td>
</tr>
<tr>
<td>DVRP (Dynamic)</td>
<td>15</td>
<td>10.42%</td>
</tr>
<tr>
<td>PVRP (Periodic)</td>
<td>14</td>
<td>9.72%</td>
</tr>
<tr>
<td>VRPSD (Stochastic Demands)</td>
<td>13</td>
<td>9.03%</td>
</tr>
<tr>
<td>VRRSPD (Simultaneous Pickup and Delivery)</td>
<td>12</td>
<td>8.33%</td>
</tr>
<tr>
<td>OVRP (Open)</td>
<td>9</td>
<td>6.25%</td>
</tr>
<tr>
<td>TDVRP (Time Dependent)</td>
<td>7</td>
<td>4.86%</td>
</tr>
<tr>
<td>MCVRP (Multi-Compartment)</td>
<td>5</td>
<td>3.47%</td>
</tr>
<tr>
<td>CCVRP (Cumulative)</td>
<td>3</td>
<td>2.08%</td>
</tr>
</tbody>
</table>

Table 3: Overview of variants in absolute and relative numbers
Clearly CVRP remains the most common variant. Vehicles are rarely assumed uncapacitated, except in cases where one unit of demand is considered negligible in size (e.g. Ferrucci et al., 2013). Since the CVRP has been popular since the existence of the VRP, which is apparent from the section dedicated to it by Toth and Vigo (2002), we are not going to discuss further details: for references of the latest research on the CVRP, we refer to Marinakis (2012), Baldacci et al. (2012), Jin et al. (2012), Szeto et al. (2011), Liu et al. (2010), Lysgaard (2010), Chen et al. (2010), Ai and Kachitvichyanukul (2009b) and Lin et al. (2009). In the past five years, the Cumulative CVRP has shown up as a new CVRP variant; instead of minimizing the total distance (or travel time) as an objective, it minimizes the sum of the arrival times at the customers (Ke & Feng, 2013; Mattos Ribeiro & Laporte, 2012; Ngueveu et al., 2010). From the 144 articles, three articles address this variant (2% of the classified articles).

Another popular variant is the VRP with time windows. Time windows are usually restricted on the customers (57 articles) and depots (30 articles). Hard time windows remain most popular (44 articles, versus 13 articles assuming soft time windows). For a fairly recent survey on the VRPTW, see Bráysy and Gendreau (2005a, 2005b) and Toth and Vigo (2002). Gendreau et al. (2008) also provide a categorized bibliography where the most important metaheuristics for the VRPTW can be found.

Load-specific or customer-specific vehicle types are not popular in recent literature, although some authors have considered VRPs with multiple compartments. In general, we see a trend towards VRP variants that include real-life assumptions, such as the Open VRP, Dynamic VRP and time-dependent VRP. Given their importance in modelling real-life problems, we pay specific attention to these variants in Section 5. Often, real-life settings such as cash transportation (Yan et al., 2012), small package shipping (Stenger et al., 2013), garbage collection (Kuo et al., 2012) or social legislation for drivers’ working hours (Rancourt et al., 2013; Kok et al., 2010; Goel, 2009), motivate researchers to develop a specific mathematical formulation for which a solution approach is then suggested. Unfortunately, these approaches are usually tailored to the problem at hand, and even specifically adapted to the corresponding benchmark instances.

Table 4, which analyses the type of proposed solution methods, indicates that metaheuristics are used most often to obtain a (sub)-optimal solution. Exact methods and classical heuristics are occur less often, probably due to their disadvantages (the former is computationally expensive for complex and large instances and the latter can get stuck in local optima). Simulation and real-time solution methods are rarely used. Given their importance in solving realistic VRPs, the further development of these methods provides an opportunity for further research. In what follows, we further zoom in on the Open VRP, the Dynamic VRP and the time-dependent VRP.
<table>
<thead>
<tr>
<th>Applied Method</th>
<th>Number of articles (total = 144)</th>
<th>Relative presence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metaheuristic</td>
<td>104</td>
<td>72.22%</td>
</tr>
<tr>
<td>Classical Heuristic</td>
<td>26</td>
<td>18.06%</td>
</tr>
<tr>
<td>Exact Method</td>
<td>20</td>
<td>13.89%</td>
</tr>
<tr>
<td>Simulation</td>
<td>7</td>
<td>4.86%</td>
</tr>
<tr>
<td>Real-time solution methods</td>
<td>6</td>
<td>4.17%</td>
</tr>
</tbody>
</table>

**Table 4: Overview of applied methods in absolute and relative numbers**

5.1. Open VRP (OVRP)

In the Open VRP (OVRP) vehicles are not required to return to the central depot after visiting the last customer. If they do return, they must visit the same customers in the reverse order. Additionally, the OVRP often has two optimization objectives: minimizing the number of vehicles used and (given this number of vehicles) minimizing the total distance (or sometimes time) travelled. In practice, the problem occurs when the vehicle fleet is not owned by the company itself or when the available vehicle fleet is unable to satisfy the demand of its customers, such that (part of) the distribution activities is contracted to a third party logistics (3PL) provider (Repoussis et al., 2010). The OVRP solution then indicates the amount of vehicles that is needed. In addition, the OVRP might be used in case of pick-up and delivery, when after delivering goods to given customers, the vehicles pick up goods from the same customers, but in reverse order (Salari et al., 2010). In real-life the OVRP occurs for instance with home delivery of packages and newspapers (Repoussis et al., 2010), school bus routing (Salari et al., 2010), routing of coal mines material (Yu et al., 2011b) or shipment of hazardous materials (Liu & Jiang, 2012).

Fleszar et al. (2009) observe that Sariklis and Powell (2000) were the first to propose a solution method, a heuristic based on a minimum spanning tree with penalties procedure, for solving the OVRP. Since 2000, researchers have gained interest and have proposed several heuristics and metaheuristics, such as tabu search, deterministic annealing, large neighborhood search and branch-and-cut, to solve the OVRP (Repoussis et al., 2010).

Table 5 shows the nine classified articles (6.25%) that have OVRP as main subject. Overall, all articles include capacity constraints for the vehicles; additionally, more than half of the articles include distance (or time) constraints. All articles, except for one (Li et al., 2012), assume a homogeneous fleet of vehicles and all but one (Cao & Lai, 2010) assume deterministic demands. All proposed solution methods are metaheuristics, except for one (Salari et al., 2010), and all but one (Yu et al., 2011b) are tested on benchmark instances, either gathered from literature or generated by the authors.
MirHassani and Abolghasemi (2011) | Open VRP | No | Particle swarm optimization
---|---|---|---
Yu et al. (2011b) | Open VRP | No | Genetic algorithm and tabu search
Zachariadis and Kiranoudis (2010a) | Open VRP | No | Local Search Algorithm with static move descriptor concept
Fleszar et al. (2009) | Open VRP | Yes | Variable neighborhood search algorithm
Repoussis et al. (2010) | Open VRP | Yes | Hybrid evolution strategy
Salari et al. (2010) | Open VRP | Yes | ILP Improvement Procedure
Liu and Jiang (2012) | Close-Open Mixed VRP | Yes | Memetic Algorithm
Li et al. (2012) | Heterogeneous Fixed Fleet | Yes | Adaptive memory-based tabu search algorithm
Cao and Lai (2010) | Open VRP with fuzzy demands | Yes | Differential evolution algorithm

Table 5: Overview of OVRP articles

5.2. Dynamic VRP (DVRP)

The evolution of real-time technologies, such as Intelligent Transformation Systems (ITS), Advanced Fleet Management Systems (AFMS) and Global Positioning Systems (GPS) has made the DVRP a relatively hot topic in recent years (Psaraftis, 1995): 10.42% of the classified articles discuss the DVRP. In the DVRP (also referred to as online or real-time VRP), the inputs are revealed or updated continuously (such as new customer requests arriving at any point during the vehicle’s route, Pillac et al. 2013). Pillac et al. (2013) recently proposed a comprehensive review of the DVRP.

Table 6 gives an overview of the DVRP articles in our classification. As evident from the table, uncertainty in demand, travel times and request times is most prevalent in the DVRP. Uncertainty in onsite service times or waiting times is not commonly studied. Strikingly, for the DVRP, no benchmark instances are available to test and compare the proposed solution methods objectively.
5.3. Time-dependent VRP (TDVRP)

Most VRPs assume that the travel times between depots and customers are deterministic and constant (e.g. Kok et al., 2010) or equal to the distance between customers (e.g. Li et al., 2012; Lei et al., 2011). In real life, variable travel times (due to congestion) are prevalent, which impacts transportation cost because of increased fuel consumption (Kuo, 2010). The TDVRP assumes that the travel times are a function of current time. As such, the effects of congestion on the total route duration, the number of vehicles and transportation cost can be determined. All TDVRP articles in our classification (see Table 7) satisfy the non-passing property, also known as the First-In First-Out (FIFO) property (Ichoua et al., 2003), which states that a vehicle that leaves earlier from some customer will arrive earlier at its destination. The time-dependent travel are modelled following the example of Ichoua et al. (2003), where the workday is partitioned into several periods and a constant travel speed is assigned to each time interval, resulting in speed being a step function of the departure time for all the arcs. The higher the number of time intervals, the more realistic the model will be because the travel speeds will change more gradually instead of abruptly (Kok et al., 2012). The travel time between two customers is then dependent on the departure time from the first customer and the time-dependent speed on the associated arc between the two customers.
Table 7: Overview of TDVRP articles

<table>
<thead>
<tr>
<th>Article</th>
<th>Variant of TDVRP</th>
<th>Objective (Min.)</th>
<th>Solution method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kuo et al. (2009)</td>
<td>TDVRP</td>
<td>Total route duration</td>
<td>Tabu search</td>
</tr>
<tr>
<td>Kuo (2010)</td>
<td>TDVRP</td>
<td>Fuel consumption</td>
<td>Simulated annealing</td>
</tr>
<tr>
<td>Dabia et al. (2013)</td>
<td>TDVRP with hard TW</td>
<td>Total route duration</td>
<td>Branch-and-price algorithm</td>
</tr>
<tr>
<td>Kok et al. (2012)</td>
<td>TDVRP with hard TW</td>
<td>Total route duration</td>
<td>Dijkstra algorithm and restricted dynamic</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>programming heuristic</td>
</tr>
<tr>
<td>Balseiro et al. (2011)</td>
<td>TDVRP with hard TW</td>
<td>First number of routes, second total route</td>
<td>Ant colony algorithm with insertion heuristics</td>
</tr>
<tr>
<td></td>
<td></td>
<td>duration</td>
<td></td>
</tr>
<tr>
<td>Figliozzi (2012)</td>
<td>TDVRP with hard and soft</td>
<td>First number of routes, second total route</td>
<td>Iterative route construction and</td>
</tr>
<tr>
<td></td>
<td>TW</td>
<td>duration</td>
<td>improvement heuristic</td>
</tr>
<tr>
<td>Lorini et al. (2011)</td>
<td>Dynamic TDVRP with soft</td>
<td>Total route duration and lateness at customers</td>
<td>Insertion and improvement heuristic</td>
</tr>
<tr>
<td></td>
<td>TW</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

All TDVRP articles in our classification assume time windows, either soft (2 articles) or hard (5 articles). Most of these time windows are restricted on the customers, while some are restricted on the depot. All TDVRP are deterministic except for the variant proposed by Lorini et al. (2011), who also take unforeseen events into account. All articles address single period problems with a single depot. The algorithms were either tested on synthetic data (e.g., Kuo et al. 2009, 2010, Balseiro et al. 2011) or a combination of synthetic and real data (Kok et al. 2012). Dabia et al. (2013) are the first and only authors so far to solve the TDVRP with time windows using an exact method. Their branch-and-price method was tested on the Solomon instances with speeds derived from real life.

6. Conclusions and insights

This article classifies 144 VRP articles published between 2009-2013 according to an adapted taxonomy based on Eksioglu et al. (2009). The resulting classification table enables future researchers to find relevant literature by eliminating or selecting characteristics in the taxonomy, leaving only articles tailored to their interests. Additionally, the classification allowed to analyze which characteristics and VRP variants are most popular. As evident from the results, the OVRP, DVRP and TDVRP have gained increasing attention during recent years.

Although the recent literature increasingly incorporates real-life constraints, most authors still propose highly problem-tailored methods that are not applicable to other variants, and in which
parameters are manipulated to provide good performance for the given instance (or for benchmark instances). Consequently, many of the proposed solution methods cannot be easily applied in real-life. In our classification, only Cordeau and Maischberger (2012) and Vidal et al. (2013) propose a “general” algorithm that can solve multiple variants of the VRP. The metaheuristic in Cordeau and Maischberger (2012) is applicable to the classical VRP, PVRP, MDVRP and the site-dependent VRP; the genetic search algorithm in Vidal et al. (2013) can solve large scale VRPTW and extensions such as the periodic VRPTW, the multiple depot VRPTW and VRPTW with vehicle-site dependencies). In conclusion, the further development of such general approaches seems highly worthwhile.

**Bibliography**


### Appendix A: Overview of classified literature per year

<table>
<thead>
<tr>
<th>Year of publication</th>
<th>References</th>
<th>Number of articles per year</th>
</tr>
</thead>
<tbody>
<tr>
<td>2013</td>
<td>Pradenas et al. (2013); Drexl (2013); Almoustafa et al. (2013); Baños et al. (2013); Ahmadi et al. (2013); Agra et al. (2013); Derigs et al. (2013); Pillac et al. (2013); Ferrucci et al. (2013); Wy et al. (2013); Ke and Feng (2013); Hu et al. (2013); Belfiore and Yoshizaki (2013); Rancourt et al. (2013); Stenger et al. (2013); Fleming et al. (2013); Ts et al. (2013); Vidal et al. (2013); Dabia et al. (2013)</td>
<td>18</td>
</tr>
<tr>
<td>2012</td>
<td>Moon et al. (2012); Pillac et al. (2012); Jin et al. (2012); Kuo et al. (2012); Zhang et al. (2012a); Tarantilis et al. (2012a); Subramanian et al. (2012); Cordeau and Maischberger (2012); Rodríguez and Ruiz (2012); Lu and Yu (2012); Yan et al. (2012); Drexl (2012); Liu and Jiang (2012); Zhang et al. (2012b); Kuo and Wang (2012); Marinakis (2012); Chardy and Klopfenstein (2012); Figliozzo (2012); Pureza et al. (2012); Kok et al. (2012); Baldacci et al. (2012); Tasan and Gen (2012); Tarantilis et al. (2012b); Pandelis et al. (2012); Goodson et al. (2012); Mattos Ribeiro and Laporte (2012); Hong (2012); Moghaddam et al. (2012); Zachariadis and Kiranoudis (2012); Anbudayasankar et al. (2012); Erdoğan and Miller-Hooks (2012); Qi et al. (2012); Li et al. (2012)</td>
<td>33</td>
</tr>
<tr>
<td>2011</td>
<td>Lei et al. (2011); Xu et al. (2011); Szeto et al. (2011); Macedo et al. (2011); Lin (2011); Gulczynski et al. (2011a); MirHassani and Abolghasemi (2011); Liao and Hu (2011); Yücenur and Demirol (2011); Pang (2011); Gulczynski et al. (2011b); Salani and Vacca (2011); Yu et al. (2011b); Aras et al. (2011); Juan et al. (2011); Bettinelli et al. (2011); Minis and Tatarakis (2011); Mendoza et al. (2011); Archetti et al. (2011); Bektas et al. (2011); Alabas-Uslu and Dengiz (2011); Lorini et al. (2011); Valle et al. (2011); Balseiro et al. (2011); Felipe et al. (2011); Santos et al. (2011); Yu and Yang (2011); Yu et al. (2011a); García-Najera and Bullinari (2011); Brandão (2011)</td>
<td>30</td>
</tr>
<tr>
<td>2010</td>
<td>Salari et al. (2010); Zachariadis and Kiranoudis (2010a); Benjamin and Beasley (2010); Li et al. (2010); Ren et al. (2010); Mendoza et al. (2010); Subramanian et al. (2010); Ngueveu et al. (2010); Erera et al. (2010); Prescott-Gagnon et al. (2010); Kok et al. (2010); Çatay (2010); Figliozzo (2010); Repoussis and Tarantilis (2010); Muylermans et al. (2010); Gutiérrez-Jarpa et al. (2010); Gulczynski et al. (2010); Wen et al. (2010); Lysgaard (2010); Kuo (2010); Tang et al. (2010); Azi et al. (2010); Liu et al. (2010); Yurtkuran and Emel (2010); Müller (2010); Zachariadis et al. (2010); Bolduc et al. (2010); Nagata et al. (2010); Zachariadis and Kiranoudis (2010b); Cao and Lai (2010); Chen et al. (2010); Marinakis and Marinaki (2010); Tütüncü (2010); Repoussis et al. (2010); Reif et al. (2010)</td>
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<tr>
<td>2009</td>
<td>Belfiore and Yoshizaki (2009); Gaipal and Abad (2009a); Qureshi et al. (2009); Battarra et al. (2009); Moretti Branchini et al. (2009); Eksioglu et al. (2009); Kuo et al. (2009); Laporte (2009); Imran et al. (2009); Novoa and Storer (2009); Yu et al. (2009); Gaipal and Abad (2009b); Jozefowiez et al. (2009); Flészár et al. (2009); Brandão (2009); Cheng and Wang (2009); Bräysy et al. (2009); Liu et al. (2009); Ai and Kachitvichyvanukul (2009a); Janssens et al. (2009); Wang and Lu (2009); Lin et al. (2009); Zachariadis et al. (2009); Hoff et al. (2009); Mendoza et al. (2009); Ai and Kachitvichyvanukul (2009b); Goel (2009); Ceselli et al. (2009)</td>
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