Title: Modelling scale-dependent runoff generation in a small semi-arid watershed accounting for rainfall intensity and water depth

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Abstract:

Observed scale effects of runoff on hillslopes and small watersheds derive from complex interactions of time-varying rainfall rates with runoff, infiltration and macro- and microtopographic structures. A little studied aspect of scale effects is the concept of water depth-dependent infiltration. For semi-arid rangeland it has been demonstrated that mounds underneath shrubs have a high infiltrability and lower lying compacted or stony inter-shrub areas have a lower infiltrability. It is hypothesized that runoff accumulation further downslope leads to increased water depth, inundating high infiltrability areas, which increases the area-averaged infiltration rate. A model was developed that combines the concepts of water depth-dependent infiltration, partial contributing area under variable rainfall intensity, and the Green-Ampt theory for point-scale infiltration. The model was applied to rainfall simulation data and natural rainfall – runoff data from a small sub-watershed (0.4 ha) of the Walnut Gulch Experimental Watershed in the semi-arid US Southwest. Its performance to reproduce observed hydrographs was compared to that of a conventional Green-Ampt model assuming complete inundation sheet flow, with runon infiltration, which is infiltration of runoff onto pervious downstream areas. Parameters were derived from rainfall simulations and from watershed-scale calibration directly from the rainfall – runoff events. The performance of the water depth-dependent model was better than that of the conventional model on the scale of a rainfall simulator plot, but on the scale of a small watershed the performance of both model types was similar. We believe that the proposed model contributes to a less scale-dependent way of modelling runoff and erosion on the hillslope-scale.

Keywords:

Runoff, runon, infiltration, Green-Ampt, Walnut Gulch, modelling, water depth
1 Introduction

The scale-dependency of runoff and infiltration has been recognized widely. In studies that compare runoff at different scales of a hillslope, scale effects appear as reduced runoff coefficients with increasing scale [1-7]. Models that try to reduce scale-dependency in the calculation of runoff generation have concentrated on various effects. Among the explanations for scale effects, variability and duration of a rainstorm are probably the most uncontroversial ones: even on a perfectly homogeneous surface runoff coefficients can be expected to decrease with slope length (or area) due to the infiltration of runoff water in pervious downstream areas (runon infiltration) during periods of low rainfall intensity and/or after the rainfall event. Stomph et al. [8] have quantified the effect of storm duration and slope length on artificial slopes in the laboratory, finding that shorter rainfall durations and longer slopes produce the largest scale effects. Wainwright and Parsons [9] simulated runoff coefficients with a simple storage model, allowing runon infiltration, which is defined as infiltration of water produced by rainfall excess further upslope [10]. They found that the average rainfall intensity and variability of rainfall strongly influenced runoff coefficients, while the reduction in runoff coefficients with slope length of the simulated hillslope was dependent on slope gradient and the hydraulic roughness coefficient. The effect of spatial variation in infiltration capacity on scale effects is more controversial than the temporal effect. Modelling studies have created randomly distributed hydraulic variables, very often saturated hydraulic conductivities ($K_s$), with or without spatial correlation, and have computed the sensitivity of runoff production to the coefficient of variance and/or the spatial structure of the variance [10-12]. If runon infiltration is allowed, which means that runoff produced upstream infiltrates if it flows over unsaturated areas, substantial scale effects appear [13-15].
It has been suggested that heterogeneity in the context of runon infiltration increases scale
effects, because the likelihood that runoff flows onto areas of very high infiltration capacity is
increased with increasing scale, thereby reducing runoff connectivity [7, 16]. However, the
effect is dependent on rainfall intensity and duration, the pattern of runoff concentration and
typical scale of heterogeneities [17], and it has not been quantified yet.

Runon infiltration is usually modelled as sheet flow entering unsaturated modelling units
(cells or pixels), supplying water for infiltration in the same way as rainfall on the whole
surface of the modelling unit [e.g. 13]. In such a conceptualization a constant hydraulic
conductivity for the whole modelling unit is assumed. Runon infiltration has also been
modelled to increase with inundation of a microtopography, albeit without a systematic
variation of hydraulic conductivity within the microtopography [10]. However, in
environments with a clear microtopographical pattern, the local distribution of hydraulic
conductivities is not completely random nor constant, but dependent on the relative elevation
within the microtopography. Lyford and Qashu [18] found hydraulic conductivities to be 2.6
times higher under creosote bushes (Larrea tridentata) and Palo Verde (Cercidium
microphyllum), compared to the lower lying inter-shrub area. Similarly, Johnson and Gordon
[19] have found 2 – 2.5 times higher infiltration rates under sagebrush (Artemisia tridentata)
as compared to the inter-shrub areas. For the Mediterranean semi-arid rangeland
environments it has been described how increased infiltration rates occur closer to plants as a
consequence of mound build-up and improvement of the physical and chemical soil properties
around the plant [e.g. 20, 21]. Dunne et al. [22] have proposed a model for steady state
infiltration that accounts for these systematic patterns. In their model infiltration per unit area
increased with increasing runoff water depth towards the foot of a slope. This increase is
achieved by inundation of parts of an elevation-dependent distribution of hydraulic
conductivities, where higher conductivities are associated with higher elevation of the
microtopography. The typical distance between mounded plants may influence runoff
thresholds and scale effects [23]. Dunne et al. [22] combined the concept of water depth-
dependent infiltration with the concept of partial contributing area for runoff production in
order to explain the convex shape of shrub-covered hillslopes in Kenya. However, they did
not account for temporal variations in rainfall intensity and/or infiltration capacity. Also in
agricultural environments the spatial variation in infiltration capacity shows clear structure,
leading to water depth dependent infiltration rates. Bresson and Valentin [24] showed that on
tilled fields a sedimentary crust with a low hydraulic conductivity builds up in the micro-
valleys while the micro-slopes and ridges are covered by a structural crust with much higher
conductivity. Fox et al. [25] have subsequently shown in a laboratory study that this
phenomenon can lead to a fourfold increase in average hydraulic conductivity with increasing
water depth. Langhans et al. [26-28] have shown that under varying inflow and rainfall
intensity rates on small rainfall simulation plots infiltration rates increase as inflow, rainfall
intensity, water depth and inundated area increase. They have accounted for these effects in a
water-depth dependent infiltration model based on Green-Ampt [27]. Clearly, if runon
infiltration is water depth-dependent this may produce scale effects as, generally, water depths
can be expected to increase with increasing contributing area. While both temporal variation
in rainfall (and infiltration) and the depth-dependency of infiltration may cause scale effects, a
modelling framework integrating both effects has not yet been empirically tested on both the
rainfall simulation plot-scale and the hillslope or small catchment-scale under natural rainfall
conditions.

The model first described in Langhans et al. [27] for a steady state case, integrates
temporal, water depth and rainfall intensity effects and enables the exploration of the relative
importance of these effects on scale effects. In the present study, behaviour and performance
are compared to a similar baseline model without water depth-dependency and with a sheet
flow assumption. Data for testing the models were rainfall simulations on plots, and small

catchment runoff measurements, constituting two typical scales where we would expect

runoff generation rates to decrease from plot- to catchments-scale. We chose to form

infiltration expressions with parameters that are effective at the local scale (resolution) of

approximately 1 m² [29]. The actual resolution of a model unit in a distributed model should

be similar to the typical scale length of microtopography to be able to capture the full process

of interaction between runoff and microtopography. The typical scale length therefore

dePENDS on land use: in agricultural systems, tillage implements determine the width of the

microtopography (~0.3 m), while in semi-arid rangeland of the study area shrubs on mounds

are organized in intervals of ~1.5 m. An advantage of conceptualizing processes at the local

scale is that effective parameters can be readily found by parameterizing the model with

rainfall simulation data, because the scale of modelling units and experimental plots are

similar and it can be assumed that processes within the experimental plot resemble those of

the modelling unit.

In this study we seek to answer the following questions: (1) can we observe scale effects

in a data set of rainfall simulations and runoff data from small catchments? (2) Can both

watershed-scale infiltration models be successfully calibrated using rainfall simulation data or

is calibration from watershed-scale runoff data necessary? (3) Does the incorporation of the

water depth-dependency and runon infiltration into a Green-Ampt based model improve

performance?
2 Methods

This section describes the site and the rainfall simulation data that were used to parameterize the baseline model and the new model to be tested. Further, this section describes the natural rainfall data that were used to validate the models’ results when parameterized with rainfall simulation data and to obtain model parameters by calibration and validation against runoff hydrographs. The parameterization is described in detail in section 3.4.

2.1 Site description

The Walnut Gulch Experimental Watershed (WGEW) of the United States Department of Agriculture (USDA) is a semi-arid watershed located in Southeast Arizona, southwest United States in the transition zone between the Sonoran and Chihuahua desert (Figure 1). Runoff occurs mainly in the summer months during and shortly after major convective storms. Storms of lower intensity during winter months rarely produce runoff. Within the WGEW, the Lucky Hills Site is a 0.4 ha sub-watershed (LH- 106) with an average slope of 7.6 %, and equipped with an H-flume [30]. Figure 2 is a contour map of the watershed, under-laid with a panchromatic satellite image from 7 July 2010 (Quickbird, 0.6 m resolution). The darker gray areas represent single shrubs or agglomerations of a few shrubs (1 to 2 m in diameter), within a network of gravely soil or scattered forbs and grasses, mostly with a stony upper soil layer (light grey areas). The soil is classified as very gravely sandy loam without a topsoil [31]. Dominant shrubs species within the watershed include Creosote (Larrea tridentata) and Whitethorn (Acacia constricta) on mounds of looser, mulched soil with fewer stones (Figure 3).

[location of Figure 1, 2 and 3]
2.2 Rainfall simulations and runoff measurements

Rainfall simulations with rainfall intensities between 25 and 216 mm h\(^{-1}\) were conducted between 2004 and 2008 during summer months about 100 meters south of the outlet of LH-106. Rainfall simulations were conducted with a central oscillating boom rainfall simulator with four nozzles (Veejet 80100) attached at 1.52 m intervals, mounted 3 m above ground, while the plot was protected with a wind shield [32]. Details on the experimental procedure are given in Stone et al. [33]. A dry run at initial soil moisture was followed 45 minutes later by a wet or very wet run. The experiments chosen as the calibration set were wet and very wet runs from 2007 and 2008 on 6 large plots (LP) of 6.1 by 2 m, with apparent steady state runoff reached after application of at least 5 minutes of rainfall. These experiments were chosen as the calibration set, because orthogonal photos of the plot during experiments and dye velocity measurements \((u, \text{ LT}^{-1})\) existed for them which were required for the parameterization of the model (Table 1). The photos covered the lower 3 m of the runoff plot, and on three transects along the plot width, \(w_p\) (L), at lengths of approximately 3.75, 4.5 and 5.25 m from the upper plot border, the width fraction of inundation was measured by visual interpretation. Usually, it could easily be seen which parts were covered by water or not, however, stone pavements protruding through a layer of water occasionally created uncertainty in the visual interpretation. Runoff alternated between more concentrated faster flow around vegetated mounds and broader sheetflow in between. Water often flowed along the lateral plot borders. A trend towards more inundation at the lower end of the plot was not present. Consequently, all three transect measurements were averaged to form a single inundated area fraction \((A_i)\), representative for the plot. Mean flow velocity was computed using the centroid of the electrical resistivity curve measured at the outlet of the plot [34].
Rainfall simulations on large plots from 2004 (four plots), where no photos were taken, and small plots (SP, 0.75 m$^2$, 3 plots) from 2008, were used as validation (Table 1).

Natural rainfall – runoff events were selected from a continuous period of runoff and rainfall measurements from 2001 to 2010. Rainfall depth was digitally recorded at 1 min increments during periods of rainfall at a gage about 100 m from the outlet of LH-106. The runoff hydrograph was measured also as breakpoint data at the H-Flume (Figure 2). During this period 718 rainfall events were recorded of which 60 resulted in runoff. All runoff events except for one (23.07.2008) had incident rainfall totals of more than 4.5 mm. Thus, this value was chosen as a minimum threshold for the selection of a dataset of rainfall events to be analysed. Volumetric soil moisture content at 5 cm depth was measured at a meteorological station 270 meters north of LH-106. The maximum value was taken as the effective saturated soil moisture content or effective porosity, $\eta_e$ (L$^3$L$^{-3}$) = 0.29. To obtain a relevant estimate of initial soil moisture content, $\theta_i$ (L$^3$L$^{-3}$) a value just before rainfall started was selected for each event. Five rainfall events, of which one was producing runoff (07.07.2001), had no soil moisture measurements. The deletion of these events and the use of the rainfall threshold resulted in a total of 147 rainfall events with 58 producing runoff (Table 1). Selecting rainfall events with a threshold rather than selecting only runoff events reflects the fact that in a rainfall – runoff model rainfall is an independent variable while runoff is the quantity to be predicted.
3 Model descriptions

Two contrasting models for the calculation of infiltration and runoff hydraulics at the local scale of a modelling unit are presented. Both were applied in a distributed model for the calculation of runoff at the small watershed-scale.

3.1 Conventional Green-Ampt model with sheet flow (CON)

The model to which the new proposed model is compared is termed conventional, because it uses equations that are commonly used to model runoff on hillslopes. Infiltrability \( f_t \) at the point-scale, assuming piston-type flow is given by the Green-Ampt Equation [35] [e.g. Equation 5.4.1 in 36]:

\[
f_t = K_s \left( \frac{\psi \Delta \theta}{F_t} + 1 \right)
\]  (1)

where \( K_s \) is the point-scale hydraulic conductivity (LT\(^{-1}\)), \( \psi \) (L) is the suction across the wetting front and \( \Delta \theta \) (L\(^3\)L\(^{-3}\)) is the difference between \( \eta_e \) and \( \theta_i \). \( F_t \) is the cumulative infiltration at the end of a time step, which is given by Equation 5.4.2 in Chow et al. [36]:

\[
F_t = F_{t-\Delta t} + K_s \Delta t + \psi \Delta \theta \ln \left( \frac{F_t + \psi \Delta \theta}{F_{t-\Delta t} + \psi \Delta \theta} \right)
\]  (2)

The use of Equations 1 and 2 for the application to variable intensity rainstorms has been described by Chu [37]. Under the assumption of zero heterogeneity, point-scale \( K_s \) equals the
effective hydraulic conductivity $K_e$ (LT$^{-1}$). When the surface becomes saturated at ponding time, infiltration is assumed to occur below a thin continuous water layer covering the whole modelling unit. Equally, runon that enters a modelling unit is effectively distributed over the whole surface area and is added to rainfall to form a common water influx.

For modelling runoff on a hillslope, a roughness coefficient is required for the estimation of flow depth. We chose to use Manning’s $n$, as it can be assumed that flow either becomes turbulent as it concentrates downslope, or it is laminar or intermediate, but much disturbed by roughness elements such as stones or litter. The average flow depth $D$ (L) of the whole modelling unit is calculated as:

$$D = \left( \frac{n}{S^{1/2} q_e} \right)^m$$  \hfill (3)

where $q_e$ (L$^2$T$^{-1}$) is the effective or average unit width discharge of a modelling unit, $S$ is the gradient of the water surface, assumed to be equal to the slope gradient, and $m$ is an exponent which is 0.6 for turbulent flow [38].

At its core, the CON model contains Chu’s variable rainfall intensity Green and Ampt infiltration model [37], with the main addition of the possibility of runon infiltration ($r_{in}$) (Figure 4)

[location of Figure 4]
3.2 Water depth-dependent Green-Ampt model with partial inundation (WDD)

Contrary to the sheetflow assumption, the WDD model acknowledges that there is microtopography. This means that for every level of inundation within the microtopography, there is an average water depth positively correlated to the inundation fraction $A_i$. The type of this relationship will depend on the microtopography. As a first approximation we used a linear relationship between the inundation fraction and water depth.

$$A_i = bD$$  \hspace{1cm} (4)

with $b$ an empirical constant. Equation 4 does not imply that there is a single channel per modelling unit; it is purely empirical and can contain any amount of micro-channels that can take a tortuous route. These surface and flow characteristics (e.g. flow through protruding stones in this study) are accounted for by Manning’s $n$. As there are inundated and not inundated areas, $q_e$ in Equation 3 should apply for the inundated area only. The average effective plot unit runoff $r_e$ (LT$^{-1}$) is computed by averaging the inflow $r_{in}$ (LT$^{-1}$) from upslope area and the outflow $r_{out}$ (LT$^{-1}$) from a modelling unit:

$$r_e = (r_{in} + r_{out}) / 2$$  \hspace{1cm} (5)

Unit width discharge in the WDD model is for width of the inundated area only, so it is defined in relation to $r_e$ as:
\[ q_e = l_p r_e A_j^{1} \]  

(6)

where \( l_p \) (L) is the length of the modelling unit. The depth-discharge relationship can now be adapted for the WDD model, by substituting Equation 4 into Equation 6, which was then substituted into Equation 3:

\[ D = \left( \frac{n l_p r_e}{S^{1/2} b} \right)^{m} \]  

(7)

Hydraulic conductivities in the WDD model are assumed to have an exponential distribution, an assumption that has been successfully tested elsewhere [33, 39, 40]:

\[ g(K_s) = \frac{1}{\mu_K} \exp\left(-\frac{K_s}{\mu_K}\right) \]  

(8)

where \( g(K_s) \) is the probability distribution of hydraulic conductivities on the scale of the microtopography, within a modelling unit, with \( \mu_K \) (LT\(^{-1}) \) their mean, which is assumed constant. We did not assume a random distribution but one that is monotonically increasing with relative elevation of the microtopography, which is in line with field observations (see introduction). This assumption has been made before, but not in conjunction with the exponential distribution [22, 25]. Hawkins [39] formalized the partial contributing area approach: at any rainfall intensity \( i, \) \( \text{LT}^{-1} \), there is always some area where rainfall intensity is limiting infiltration and some area where the soil’s infiltration capacity is limiting infiltration, the latter termed runoff contributing area fraction \( (A_c) \). In the small-scale context
of the microtopography, \( A_c \) can be interpreted as the area where water flows erratically, depending on actual raindrop impact towards an inundated area (Figure 5a). When rainfall starts, \( A_c \) is infinitely small, but once soil suction decreases and rainfall intensity increases, a substantial area fraction can contribute to runoff production (Figure 5a). When water accumulates and concentrates further downstream, or when there is a sudden decrease in rainfall intensity during or at the end of a storm, the accumulated water can inundate an area bigger than \( A_c \) (Figure 5b). In this case runon infiltration will occur on the inundated area. Figure 5a and b depict a single channel for clarity of the illustration, but the generalized model (Equation 4) does not require a specific microtopography form.

In the case of an exponential distribution with a single mean parameter, and when \( \psi \) and \( \Delta \theta \) are spatially constant, the mean infiltration capacity \( \mu_f \) (LT\(^{-1}\)) is proportional to \( \mu_K \), just like \( f_t \) is proportional to \( K_s \) (Equation 1) on the point-scale:

\[
\mu_f = \mu_K \left( \frac{\psi \Delta \theta}{F_t} + 1 \right)
\]  

Equation 9 assumes that there is sufficient horizontal redistribution of infiltrated water to form a single infiltration front. It has been shown that an average Green-Ampt infiltration front produced results close to a three-dimensional finite difference solution [41]. \( F_t \), the cumulative infiltration of the time step is not known when it is required in Equation 9, so the cumulative infiltration of the previous time step \( F_{t-\Delta t} \) (with \( \Delta t \) being the time interval) is used
as an initial guess (Figure 6). During subsequent iterations within the time step the value of \( F_t \) is updated.

Equation 9 yields \( \mu_f \) which is necessary for the calculation of \( A_c \). As \( A_c \) is defined as the area fraction where the infiltration capacity is less than the rainfall intensity \( i \), it is obtained by integrating the exponential distribution of infiltration capacities \( g(f) \) from 0 to \( i \) which yields:

\[
A_c = \int_0^i g(f)df = 1 - \exp\left(-\frac{i}{\mu_f}\right) \tag{10}
\]

where \( g(f) \) is the same as \( g(K_s) \) in Equation 8, only that \( \mu_K \) is substituted with \( \mu_f \). Like \( A_c \), \( A_i \) is also a function of the exponential distribution:

\[
A_i = \int_0^{K_s(A_i)} g(K_s) dK_s = 1 - \exp\left(-\frac{K_s(A_i)}{\mu_K}\right) \tag{11}
\]

where \( K_s(A_i) \) is the hydraulic conductivity at the edge of the water level of inundation. Similarly, \( K_s(A_c) \) is the hydraulic conductivity at the edge of the runoff contributing area.

\[
K_s(A_c) = -\ln(1 - A_c) \mu_K \tag{12}
\]
Where $A_{ci}$ is either $A_c$ or $A_i$. The average hydraulic conductivity of the runoff contributing area $K_c$ and the average hydraulic conductivity of the inundated area $K_i$ are obtained by integrating hydraulic conductivities from the exponential distribution $g(K_s)$ from 0 to $K_s(A_{ci})$, which yields:

$$K_{ci} = \int_0^{K_s(A_{p})} K_s g(K_s) dK_s = \mu_K - \left(1 - A_{ci}\right) (\mu_K + K_s (A_{ci})),$$

(13)

Where $K_{ci}$ is either $K_c$ or $K_i$. $F_i$ can now be calculated as:

$$F_i = F_{t-\Delta t} + i\Delta t \left(1 - A_{ci}\right) + A_{ci} \left[K_{ci} \Delta t + \psi \Delta \theta \ln \left(\frac{F_i + \psi \Delta \theta}{F_{t-\Delta t} + \psi \Delta \theta}\right)\right],$$

(14)

with $A_{ci} = A_i$ and $K_{ci} = K_i$ if $A_i > A_c$ (case Figure 5b), and else $A_{ci} = A_c$ and $K_{ci} = K_c$ (case Figure 5a). Equation 14 is derived from Equation 2, splitting the cumulative infiltration into a part where rainfall infiltrates directly (the second term) and a part where infiltration during a time step under $A_i$ or $A_c$ is limited by the soil (the third term). The resulting value of $F_i$ is still an approximation and the calculation of Equations 9 to 14 are repeated until $F_i$ used in Equation 9 differs from the outcome in Equation 15 by less than a very small threshold value.

The effective infiltration rate $f_e$ (LT$^{-1}$) of the modelling unit is now given by:

$$f_e = i \left(1 - A_{ci}\right) + K_{ci} \left(\frac{\psi \Delta \theta}{F_i} + 1\right) A_{ci}.$$

(15)
The first term is infiltration equal to rainfall intensity on the non-inundated and no runoff producing part of the soil and the second term describes infiltration according to the Green-Ampt equation on the remaining inundated or runoff producing area.

### 3.3 2D distributed Model

Both infiltration models are implemented in MCST [47], a spatially distributed runoff-erosion model that originally used a modified curve number approach. The model routes runoff using the kinematic wave equation in conjunction with a flux decomposition algorithm [43] [48].

A digital elevation model of the watershed with a 1 m horizontal resolution from a RTK GPS survey with 426 elevation points was used [42]. Infiltration and runoff calculation was changed so that runon infiltration could be accounted for in each grid cell and at each time step. Following the logic of the linear scheme of the kinematic wave algorithm, \( r_e \) in Equation 5 was calculated with \( r_{in} \) as inflow into a modelling unit during the present time step and \( r_{out} \) as the outflow calculated on the same modelling unit during the previous time step. In the CON model the estimate of \( r_e \) was added to the rainfall intensity to calculate the amount of water which was available for infiltration on the whole surface of the modelling unit in accordance with the sheet flow assumption. In the WDD model, \( r_e \) was used to calculate \( D \) (Equation 7) and \( A_i \) (Equation 4) and then infiltration as described above. For a given set of parameter values controlling infiltration the WDD model will always predict lower runon infiltration for the same runon amount, unless there is full inundation. Once infiltration is calculated in both models, an estimate for the net lateral inflow into a modelling unit during a time step, \( \Delta r \) (LT\(^{-1}\)), is obtained, which can become negative during runon infiltration, and is given by the difference between rainfall intensity and infiltration. It relates to the kinematic wave equation as:
\[ \frac{\partial (r_{out} l_p)}{\partial l_p} + \frac{\partial D}{\partial t} = \Delta r = i - f_e \] \tag{16}

Equation 17 was solved using a discrete non-linear scheme [36]. So for each time step and modelling unit, runoff, infiltration, water depth, and in the case of the WDD model, inundated area fraction are estimated. For a single rainfall event, three main output quantities were calculated that could be compared to measured quantities: the total discharge, \( Q_t \) (L\(^3\)), which is the sum of discharge of all time steps at the modelling unit (cell) that represents the outlet of the watershed. The steady state effective discharge, \( Q_e \) (L\(^3\)T\(^{-1}\)), is a quantity that is used in erosion modelling, also in the MCST model, and is given by [43]:

\[ Q_e = \left( \frac{\sum_{t=t_1}^{t_n} Q_t^{1.4}}{\sum_{t=t_1}^{t_n} Q_t} \right)^{2.5} \] \tag{17}

where \( Q \) (L\(^3\)T\(^{-1}\)) is the discharge during one time step \( t \), with \( t_1 \) the first and \( t_n \) the last time step of an event. The third quantity is peak discharge, \( Q_p \) (L\(^3\)T\(^{-1}\)). All three quantities can be modelled and also derived from observational hydrographs, measured at the H-flume at the watershed outlet.
3.4 Model parameterization and evaluation

In accordance with the second objective, the model was parameterized using the calibration set of rainfall simulations to find values for \( K_e \) and \( \mu_K \) for the CON and the WDD model, respectively, and values of Manning’s \( n \) and the parameter \( b \), the latter for the WDD model. Secondly, \( K_e, \mu_K, \psi \) and \( n \) were calibrated for natural rainfall-runoff events by optimizing an objective function consisting of the Nash-Sutcliffe model efficiency (ME) and the coefficient of determination (R²) of predictions of \( Q_t, Q_e \) and \( Q_p \), where R² was defined as:

\[
R^2 = ME = 1 - \frac{\sum_{i=1}^{n} (X_{\text{observed}} - X_{\text{predicted}})^2}{\sum_{i=1}^{n} (X_{\text{observed}} - X_{\text{mean}})^2}
\]

(18)

where \( X \) is the quantity of interest. When equation 19 is used to calculate ME, \( X \) is the instantaneous runoff rate during a time step. All parameters were assumed to be constant within the catchment.

3.4.1 Parameterization using rainfall simulations

An estimate of average plot width flow depth on the rainfall simulation plots (assuming sheet flow) was obtained for the CON model using measured average velocities:

\[
D = \frac{r \cdot l_p}{u}
\]

(19)
where \( r_e \) was calculated from Equation 5 with measured \( r_{out} \) and zero \( r_{in} \). This allowed to find a value for Manning’s \( n \) that minimized the square of the residuals between predicted and measured \( D \), using observed \( D, r_e \) and \( S \) data (least squares method) (Equation 3).

For the WDD model, \( D \) is defined for the inundated area only, so Equation 20 becomes:

\[
D = \frac{r_e l_p}{u A_i} 
\]

(20)

\( A_i \) values were measured as described in section 2.2. \( D \) in Equation 21 was substituted with \( A_i/b \) (Equation 4) and then \( b \) was optimized with the least squares method for predicted and measured \( r_{out} \). Similarly to the CON model, Manning’s \( n \) could now be found with the least squares method (Equation 7).

As \( K_e \) was assumed to approximately equal measured final infiltration rates during the (wet run) rainfall experiments, for the CON model an estimate of average \( K_e \) is simply derived by averaging final infiltration rates of the calibration dataset. For the WDD model, \( \mu K \) was optimized as described and depicted in Figure 9, dashed area, in Langhans et al. [27] for the steady state case of the rainfall simulation plots.

3.4.2 Calibration on multiple rainfall events

The dataset of 147 rainfall events (Table 1) was randomly split into a calibration set of 74 events and a validation set of 73 events. In accordance with objective 3, optimized parameter sets were found for the CON and the WDD model from the calibration set to assess the water depth-dependency effect, and for both models parameter sets were found that assess the effect
of runon infiltration. For the CON model this was done by forcing $r_m$ to be zero during infiltration calculation, and for WDD this was done by forcing $A_i$ to be zero during infiltration calculation. The objective function (OBJ) was defined as:

$$OBJ = \left( ME + \left( R^2(Q_T) + R^2(Q_e) + R^2(Q_p) \right) / 3 \right) / 2$$  \hspace{1cm} (21)$$

In words, during optimization, the mean $R^2$ of the predictions of total, effective, and peak discharge was given equal weight as ME. For CON, $\psi$, $K_e$ and Manning’s $n$, and for WDD $\psi$, $\mu_K$ and Manning’s $n$ were simultaneously optimized using the following simple set search method. Starting values within realistic bounds were chosen, and parameter combinations were added that were half the starting value’s step in each direction of the parameter space (with three calibration parameters, this results in a 3D grid, when visualized). For each parameter combination in the set, OBJ was calculated, and the combination with the largest value was chosen. This combination was used as a new starting point for another set search, but with half a step size compared to the previous iteration. The number of iteration was limited by computation time but usually exceeded 4. It was ensured that OBJ were indeed maximized within the search space by plotting OBJ. The accuracy was between 1-5 mm for $\psi$, 0.5 mm/h for $K_e$ and $\mu_K$, and 0.002 for Manning’s $n$.

4 Results

4.1 Rainfall simulations

Water depth of the inundated area show has a clear positive relationship with effective discharge (WDD, Figure 7), with little scatter for the year 2007. Also, 2007 water depth...
values are much higher than values of plots from 2008, while the latter level off at 0.002-
0.003 m at high flow rates. The \( D - r_e \) relationship under the sheet flow assumption (CON, Figure 7) shows more scatter and lower depth estimates at higher flow rates, as can be expected, because flow is spread over the whole plot. The relationship of \( A_i - r_e \) (not shown) gives the same picture as WDD depths in Figure 7, because \( A_i \) and \( D \) are related only with the proportionality parameter \( b \). The range of measured inundation fractions was 0.1 to 0.7. Values for Manning’s \( n \) were 0.26 and 0.57 for CON and WDD, respectively.

Final infiltration rates (\( K_e \)) in the years 2007 and 2008 were much higher than they were in 2004, which were used as validation data (Figure 8). \( K_e \) on the small plots in 2008 were more closely in the range of the infiltration values on the long plots. Moreover, \( K_e \) showed a stronger dependency on rainfall intensity in 2007 and 2008 than in 2004. Average \( K_e \), used in the CON model is a constant at 57.2 mm h\(^{-1}\), while \( \mu_k \) (WDD) was optimized at 62.4 mm h\(^{-1}\) (Table 2). In order to clarify the models’ behaviour on the rainfall simulation plot-scale, final infiltration rates were modelled for a range of rainfall intensities for the 6 m plot, with an average slope of 0.14. The WDD model predicts that the inundation effect (infiltration in area where \( A_i \) exceeds \( A_c \)) is strongest around the mean of the distribution on the 6 m plot (solid line departing from dash-dotted line in Figure 8). For the 1 m plots the curve for the average predicted \( K_e \) lies much closer to the curve of the dashed line, because, relatively, less runon infiltration occurs on a shorter slope (line not shown). Calculated complete inundation at higher rainfall intensities is a bias produced by the assumption of a linear relationship
between average water depth and inundation. Actually, there are always some parts of the plot, such as mounds under creosote bushes that remain un-inundated.

[location of Figure 8] [location of Table 2]

Final infiltration rates on long plots were much lower during 2004 (validation), and both models had negative $R^2$. For the small plots (2008), however, the WDD model yielded better predictions ($R^2 = 0.86$) compared to the CON model ($R^2 = 0.10$).

4.2 Calibration with rainfall-runoff events

Parameter values derived from rainfall simulations were used to model runoff hydrographs of all 147 rainfall events. With hardly any runoff predicted, both models performed poorly, with negative $R^2$ (Equation 19) (results not shown). Calibration on multiple events at field-scale yielded much lower optimized values for $K_e$ and $\mu K$, both for model runs with and without runon infiltration (Table 2). Also, Manning’s $n$ values were much smaller than the rainfall experiments suggested. Optimized $\psi$ values were much higher for the WDD than the CON model. The optimization process during calibration yielded clearly defined maxima and smooth contours for OBJ in the parameter space for both models (Figure 9).

For the calibration set 37% of rainfall-runoff events for both, CON and WDD, had ME values above 0.75 (Table 3). The WDD model however had a higher proportion of events above 0.75 for the validation set. Coefficients of determination for $Q_r$, $Q_p$, and $Q_e$ were good...
for all models and OBJ values were slightly higher for the WDD model than the CON model.

The CON model performed slightly better in validation with runon infiltration than without runon infiltration. Comparing observed vs. predicted $Q_t$ revealed that CON had too many events predicting no runoff where there was runoff observed, and, conversely, the WDD model always predicted some runoff, leading to slight overestimation at low runoff rates (figure not shown).

Comparing observed vs. predicted $Q_t$ revealed that CON had too many events predicting no runoff where there was runoff observed, and, conversely, the WDD model always predicted some runoff, leading to slight overestimation at low runoff rates (figure not shown).

[location of Table 3]

### 4.3 Model behaviour

Model behaviour was further studied by comparing predictions of hydrographs with actually observed hydrographs, and by exploring the effects of water depth-dependency and runon infiltration through calculating infiltration under a steady rain (Figure 10). For a 40 mm h$^{-1}$ synthetic rainstorm of 30 minutes, the average watershed-scale infiltration rate for the CON model was equal to rainfall intensity for the first 10 minutes, after which it dropped to below 23 mm h$^{-1}$ at 30 minutes. When allowing runon infiltration, substantial runon infiltration occurred with the CON model between 30 and 40 minutes. For the WDD model infiltration rates dropped immediately after onset of rainfall to below 20 mm h$^{-1}$ at 30 minutes. When accounting for runon infiltration, infiltration rates of WDD are only less than 0.5 mm h$^{-1}$ larger from minute 8 onwards compared to WDD without runon infiltration, and after 30 minutes runon infiltration drops immediately to 1.5 mm h$^{-1}$ only. Water depth-dependent infiltration under such a scenario is thus negligible, whereas runon infiltration in the CON model is important.
For qualitative analysis of the goodness of hydrograph prediction with the watershed-scale calibration parameters, six events were chosen and displayed: The 90th, 50th, and 10th percentile event, based on descending ranking of ME, both for CON and WDD, with runon (Figure 11).

For the 90th percentile events, the timing of the peak and peak discharge were better predicted for CON than WDD (Figure 11 a and b). For the 50th percentile events, peak discharge was predicted better with the WDD model (Figure 11, c and d), but timing was either worse (Figure 11 c) or better (Figure 10 d) than CON. For the 10th percentile, WDD had a stronger over estimation of peak discharge than CON (Figure 11 e and f), and timing was not apparently wrong. Regarding the form of the hydrographs, WDD generally has a steeper rise than CON, and follows rainfall intensity more closely. CON hydrographs are usually slightly better timed. Generally, however, over- or under-predictions of total or peak discharge not systematic, except, perhaps a tendency of WDD to over-predict during small events and under-predict during larger events.
5 Discussion

5.1 The role of rainfall simulations in watershed-scale models

Rainfall simulation is a widely used tool to infer local scale infiltration parameters and characteristics. Orthogonal photos are an excellent way to gather additional information about runoff characteristics, like inundation and flow paths relative to microtopography. The method of deriving inundated area fractions is particularly useful, because they can be directly linked to depth-discharge relationships (Equations 4 and 7). Despite some inter year variability, calculated depths and effective runoff show a clear relationship (Figure 7). The range of average depth values (< 1 – 7 mm) lies within the range that Abrahams and Parsons [44] measured at a similar site on the WGEW. Direct, high precision measurements of runoff water depth [45] or combined inundation and microtopographical measurements for the calculation of water depth are surely superior to inference from flow velocity. However, even in the absence of more direct depth measurements, taking inundation into account will yield more realistic average water depths than the sheet flow assumption, because flow on a small scale hardly ever covers the whole surface where microtopography is present [46].

Values of Manning’s $n$ calibrated from the natural rainfall-runoff events were lower with a factor of 3 for the CON model and 19 for the WDD model compared to values derived from rainfall simulations, and $K_e$ and $\mu_K$ were four to five times lower (Table 2). There are several possible explanations for this discrepancy, but none taken on its own is sufficient. Regarding hydraulic roughness, it was observed from the orthogonal photos that plot borders impeded flow around mounds of creosote bushes. This must have led to more inundation and higher water depths than would occur in a natural setting without plot borders, where flow can follow unimpeded low lying channels between mounds. This means that on the plot average flow velocities were lower and Manning’s $n$ higher than in a natural setting. Certainly, the
plot borders increase ponding and with it final infiltration rates during rainfall simulations, but it is unlikely that the effect can explain a four to five-fold difference. Runoff experiments in 2004 still had roughly 2.5 times higher final infiltration rates than the calibrated values. Higher infiltration rates are a common observation for rotating or oscillating boom rainfall simulators. Risse et al. [47] and Nearing et al. [48] have reported 2-4 times higher $K_e$ for rainfall simulations than calibrated values from runoff measurements on larger scale for humid environments. Burns [49] reported for the WGEW site 3-6 times higher rainfall simulation derived $K_e$ than were calibrated for the small watershed-scale. The source of this discrepancy has not been reported. Beside the plot border effect, we can briefly point out three areas of explanation that can contribute to the difference. First, the oscillating boom rainfall simulator has intermittent, very high rainfall intensities with no rainfall in between. According to the partial area response concept, during the high rainfall intensity bursts, high infiltration rates can be achieved, while only time-averaged rainfall intensity is reported that would lead to a much lower infiltration rate. Secondly, there could be a bias during rainfall simulation site selection towards higher infiltration ‘interrill’ areas. Finally, during rainfall simulations air can escape laterally, which is not possible under natural rainfall which covers the whole area.

As infiltration was higher on the small rainfall simulation-scale, scale effects were the inverse of what one would expect from runon-infiltration theory. But given the above-mentioned differences between rainfall simulations and watershed-scale natural rainfall runoff measurements that can possibly affect infiltration rates, it can be concluded that the two types are incomparable for the purpose of scale enquiry, at least under the studied conditions and environment. This implies that rainfall simulations cannot simply be used for the parameterization of any of the two predictive models, which is confirmed by their bad performance when parameterized with rainfall simulation data in this study. Nevertheless,
rainfall simulations can be a valuable tool to study depth-discharge relationships, and relative
differences of infiltration between sites and environments, or indeed to study any sub-
processes of infiltration in more detail.

5.2 Importance of water depth-dependency and runon infiltration

Both models can predict hydrographs satisfactorily when calibrated on the watershed-
scale (Table 3). The water depth-dependent model (WDD) performs slightly better than the
conventional sheet flow model (CON), but this improvement is not related to the inundation
effect, as the effect is probably very small when parameterized with a relatively low
Manning’s $n$ of 0.028 (Figure 10). Possibly, the main improvement above CON stems from
the responsiveness of runoff to rainfall, because WDD contains a distribution of hydraulic
conductivities, where low conductivities already respond with runoff production at low
rainfall intensities. For the CON model, the inclusion of runon infiltration improves
performance slightly (Table 3). This supports the theoretical importance of runon infiltration
pointed out already by other researchers [10, 13].

There are three model structural reasons and one physical reason why water depth-
dependent infiltration and runon infiltration does not show up to be important in the WDD
model in this study. First, Langhans et al. [27] have shown in a synthetic study that scale
effects due to the water depth effect can be significant, but they used a relatively high
hydraulic roughness value derived from rainfall simulations. In this study, hydraulic
roughness is relatively low, and significant water depths that cause water depth-dependent
infiltration do not build up. The physical reason, related to this, is that the macrotopography in
the study catchment is very rugged so that runoff concentrates very quickly in channels and
the effective hillslope lengths are very short; water depth cannot build up sufficiently on such
short slopes. The second model structural reason is that the exponential distribution assumed here (Equation 8) is such that only at high inundation levels significant infiltration can occur. If a different distribution had been chosen the water depth effect could have been more pronounced, such as the log-normal distribution which had been suggested before for variation of $K_{sat}$ at larger scales [50-52]. But the analytical solution of the concept described here (i.e. Equations 8 – 16) is not applicable to all distributions and it would require a more stochastic, quasi-empirical approach to formulate the model with empirical or more complex distributions. The third model structural reason is that the assumption that hydraulic conductivities are strictly monotonously increasing within the microtopography is probably too strict. Some randomness within a modelling cell, and between would increase the chance of runon infiltration. A full study of various model structure-related effects, such as the choice of distribution, cell resolution, or randomness would be desirable but is beyond the scope of this study, as most would require a significant reformulation of the model. Consequently, and despite strong theoretical justification of the water depth-dependent infiltration process as pointed out in the introduction, the evidence for the water depth effect remains inconclusive from this study. Future research into the water depth effect and runon infiltration should therefore occur under more controlled field conditions that are tailored to quantify locations and flow pathways during infiltration. Only then it is reasonable to include these effects into models for prediction purpose.

6 Conclusions

The pattern of high infiltrability areas on mounds underneath shrubs and lower lying low infiltrability areas in the inter-shrub area has previously been demonstrated for semi-arid
rangeland. Within the concept of water depth-dependent infiltration, runoff accumulation further downslope leads to increased water depth inundating high infiltrability areas, which increases the area average infiltration rate, one of many explanations for observed scale effects in runoff and erosion. A model was developed that combines the concepts of water depth-dependent infiltration, partial contributing area under variable rainfall intensity, and the Green-Ampt theory for point-scale infiltration. The model was applied to rainfall simulation data and natural rainfall – runoff data from a small semiarid watershed (0.4 ha) in the US Southwest. The model performance was compared to the performance of a conventional Green-Ampt model with the sheet flow assumption. Both models were parameterized with rainfall simulation data, and by calibration from watershed-scale runoff measurements. Parameters of hydraulic roughness and conductivity were larger by a high multiple for the former compared to the latter. This implies that scale effects were the inverse of the expectation, and rainfall simulation plot-scale data could not be used to successfully predict hydrographs. Calibrated models both had good performance, with the water depth-dependent model being slightly better. While runon infiltration was important in improving performance of the sheet flow assumption model, water depth effect was negligible in the new model. This was explained with rigid assumptions in the model structure, low hydraulic roughness, and short hillslopes. The proposed model makes some theoretical advancement towards a less scale-dependent way of modelling runoff and erosion on the hillslope-scale, but when calibrated on the watershed-scale, these structural changes bring little predictive gain.
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Notation

679 $A_c$ rainfall excess contributing area fraction, -
680 $A_i$ flow inundated area fraction, -
681 $b$ empirical factor of the inundation-depth relationship, L$^{-1}$
682 CON conventional Green-Ampt model with sheet flow
683 $D$ average water depth, L
684 $\Delta r$ unit area net inflow, LT$^{-1}$
685 $\Delta t$ time step, T
686 $\Delta \theta$ difference between initial and saturated volumetric moisture content, L$^3$L$^{-3}$
687 $\eta_e$ effective soil porosity, L$^3$L$^{-3}$
688 $f_e$ effective infiltration rate, LT$^{-1}$
689 $f_t$ potential infiltration rate or infiltrability, LT$^{-1}$
690 $F_t$ cumulative infiltration at the end of a time step, L
691 $i$ rainfall intensity, LT$^{-1}$
692 $K_c$ average hydraulic conductivity of runoff contributing area, LT$^{-1}$
693 $K_e$ effective hydraulic conductivity, LT$^{-1}$
694 $K_i$ average hydraulic conductivity of inundated area, LT$^{-1}$
695 $K_s$ point-scale saturated hydraulic conductivity, LT$^{-1}$
696 $l_p$ plot length or modelling unit length, L
697 LP Large plot
698 $m$ exponent of depth-discharge relationship representing turbulence of flow, -
699 ME Model efficiency, -
700 $\mu_f$ mean of distribution of infiltrabilities, LT$^{-1}$
701 $\mu_K$ mean of distribution of $K_s$, LT$^{-1}$
Manning’s roughness coefficient, -

objective function, -
total precipitation of an event, L
suction across the wetting front and Δθ, L
discharge during time step, L^3T^-1
unit flow width discharge, plot length–averaged, L^2T^-1
steady state effective discharge, L^3T^-1
peak discharge, L^3T^-1
total discharge, L^3
Coefficient of determination, -
rainfall and runon excess or unit area flow rate, plot-averaged, LT^-1
unit area inflow rate at upper plot boundary, LT^-1
bulk density, ML^-3
unit area outflow rate at lower plot boundary, LT^-1
slope gradient, -
small plots
time,
initial volumetric soil moisture content, before start of rain, L^3L^-3
volumetric soil moisture content, L^3L^-3
average runoff velocity, LT^-1
water depth-dependent Green-Ampt model with partial inundation
plot width, L


Doi 10.1029/2003wr002858.


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36
Figure captions

Figure 1: Location of the study site in the southwest United States

Figure 2: Contour map of watershed LH-106 in the Lucky Hills Study Area (Figure 1) with a 0.2 m contour interval. The background is a panchromatic Quickbird image from 7 July 2010. Geographic coordinates of the centre of the catchment are 31°44’32’’N, 110°03’14’’W

Figure 3: Creosote bush on mound and stony inter-shrub area.

Figure 4: Flow chart of the CON model and the calibration procedure with multiple rainfall events. Numbers in brackets refer to Equations.

Figure 5a: Schematic representation of a surface unit during the beginning and middle phase of a rainstorm, when $A_c > A_i$. Rainfall excess erratically flows down the microtopographic slope towards the low conductivity area where inundation starts.

Figure 5b: Schematic representation of a surface unit during the recession or directly after a rainstorm in an area where runoff concentrates or accumulates, where inundated area ($A_i$) dominates infiltration.

Figure 6: Flow chart of the WDD model and the calibration procedure with multiple rainfall events. Numbers in brackets refer to Equations.

Figure 7: Relationship between average water depth and effective discharge for 2007 and 2008 on the rainfall simulation plots. In the CON model water depth is calculated for the whole plot width, while in the WDD model it is calculated for the inundated area only.
Figure 8: Relationship between observed final infiltration rates and rainfall intensity under simulated rainfall. Lines are modelled averages of the CON model (dotted), the WDD model without runon infiltration (dashed), and the WDD model with runon infiltration (solid).

Figure 9: Contour plots of the OBJ value in the parameter space: a) Manning’s $n$ vs. $K_e$ for CON, b) $\psi$ vs. $K_e$ for CON, c) Manning’s $n$ vs. $\mu K$ for WDD, d) $\psi$ vs. $\mu K$ for WDD.

Figure 10: Catchment average infiltration rate vs. time for a synthetic 30 min storm with 40 mm h$^{-1}$ rainfall intensity. Infiltration rates of the WDD model with runon infiltration (solid) and without runon infiltration (dotted), and of the CON model with runon infiltration (dashed) and without runon infiltration (dash-dotted) are shown.

Figure 11: Modelled and observed hydrographs of the 90th (a: CON, b: WDD) 50th (c: CON, d: WDD) and 10th (e: CON, f: WDD) percentile events based on descending ranking of ME for the CON and the WDD models. Observed hydrographs are solid-shaded and modelled hydrographs are dashed (WDD) and dash-dotted (CON). Rainfall intensity is displayed as stepped, solid line with an inverted scale on the right.
Plots: $\mu_K$, $n$ (and $h$) from rainfall simulations

Validation: $\mu_K$, $n$ of calibration outcome

Calibration: initial guess

Calibration: adapt $\mu_K$, $n$ until the average $R^2$ of $Q_t$, $Q_e$, $Q_p$ is maximised

$\mu_K, n$

Multiple rainfall events

Flux decomposition runoff routing

$\Delta t$

$\Delta t$

$r_{in}(t)$

$\Delta t$

$r_{out}(t-\Delta t)$

$\Delta t$

WDD model

initial: $F_{i-\Delta t}$

$F_i$ (input)

input: $i, \psi, \Delta \theta$

$\mu_i (9)$

$A_e (10)$

$K_e (11)$

$F_i (15) = F_i$ (input)?

Yes

$F_i (15) \rightarrow f_e (16)$

Kinematic wave approximation: $r_{out} (t)$ (17)

For single event (observations): $Q_t$, $Q_e (18)$, $Q_p$

For single event (model): $Q_t$, $Q_e (18)$, $Q_p$

For multiple events (21): $R^2$ of $Q_e$, $Q_c$ and $Q_p$
Figure 11c

The graph illustrates the relationship between rainfall intensity and runoff rate over time. The graph shows:
- The observed rainfall intensity (hanging) indicated by a shaded area.
- The observed runoff rate shown by a solid line.
- The modified runoff rate (WDD) represented by a dotted line.
- The modified runoff rate (CON) depicted by a dashed line.

The x-axis represents time in hours and minutes, while the y-axis represents the rates in mm h⁻¹.
Table 1: Basic statistics of the rainfall simulation and rainfall events data set

<table>
<thead>
<tr>
<th></th>
<th>Rainfall simulations</th>
<th>Rainfall events</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cal. LP</td>
<td>Val. LP</td>
<td>Val. SP</td>
</tr>
<tr>
<td>Number$^b$</td>
<td>29 (5)</td>
<td>23 (4)</td>
<td>13 (3)</td>
</tr>
<tr>
<td>Mean</td>
<td>125.5</td>
<td>102.7</td>
<td>127.3</td>
</tr>
<tr>
<td>STDV</td>
<td>41.1</td>
<td>45.9</td>
<td>52.3</td>
</tr>
<tr>
<td>Min</td>
<td>61.5</td>
<td>25.4</td>
<td>46.7</td>
</tr>
<tr>
<td>Max</td>
<td>179.5</td>
<td>177.8</td>
<td>216.2</td>
</tr>
</tbody>
</table>

$^a$LP: large plots (6 x 2 m); SP: small plots (1.22 x 0.61 m); $P$: total precipitation of an event; Max $i$: maximum one minute interval rainfall intensity; $\theta_i$: pre-storm (initial) volumetric water content of soil. $^b$ Value in brackets is number of plots. $^c$ Statistics refer to applied rainfall intensity (mm h⁻¹). $^d$ $Q$: total runoff, $Q_e$, effective discharge, $Q_p$: peak discharge; statistics apply to runoff producing events only.
Table 2: Optimized parameter values calibrated for the watershed scale from runoff hydrographs for both models (CON and WDD), with and without runon infiltration, and parameters optimized for steady state rainfall simulation plot infiltration (‘Plot’)

<table>
<thead>
<tr>
<th></th>
<th>CON</th>
<th>WDD</th>
<th>Plot</th>
<th>CON</th>
<th>WDD</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>With runon infiltration</td>
<td>Without runon infiltration</td>
<td>With runon infiltration</td>
<td>Without runon infiltration</td>
<td></td>
</tr>
<tr>
<td>$\Psi$ (mm)</td>
<td>31</td>
<td>22</td>
<td>139</td>
<td>175</td>
<td>-</td>
</tr>
<tr>
<td>$\mu K$ (mm h$^{-1}$)</td>
<td>-</td>
<td>-</td>
<td>16.5</td>
<td>14.5</td>
<td>-</td>
</tr>
<tr>
<td>$K_e$ (mm h$^{-1}$)</td>
<td>8</td>
<td>14</td>
<td>-</td>
<td>-</td>
<td>57.2</td>
</tr>
<tr>
<td>Manning’s $n$</td>
<td>0.085</td>
<td>0.05</td>
<td>0.028</td>
<td>0.026</td>
<td>0.27</td>
</tr>
</tbody>
</table>
Table 3: Fraction of modelled events with a Nash-Sutcliffe Model Efficiency (ME) in a given class, coefficients of determination ($R^2$) for measured vs. modeled $Q_t$, $Q_e$ and $Q_p$ for all rainfall events, and objective measure (OBJ, Equation 21). Parameters were optimized to maximize OBJ, given the CAL dataset and validated with the VAL dataset (Table 1). Both models were calibrated with and without runon infiltration.

<table>
<thead>
<tr>
<th>ME</th>
<th>CON With runon</th>
<th>CON Without runon</th>
<th>WDD With runon</th>
<th>WDD Without runon</th>
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<tbody>
<tr>
<td>&gt; 0.75</td>
<td>0.37</td>
<td>0.11</td>
<td>0.37</td>
<td>0.32</td>
</tr>
<tr>
<td>0.75 &gt; x &gt; 0.5</td>
<td>0.1</td>
<td>0.14</td>
<td>0.07</td>
<td>0.23</td>
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<tr>
<td>0.5 &gt; x &gt; 0</td>
<td>0.13</td>
<td>0.21</td>
<td>0.16</td>
<td>0.07</td>
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<tr>
<td>&lt; 0</td>
<td>0.4</td>
<td>0.54</td>
<td>0.4</td>
<td>0.33</td>
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<table>
<thead>
<tr>
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<th>CON</th>
<th>CON</th>
<th>WDD</th>
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<tr>
<td>$Q_t$</td>
<td>0.82</td>
<td>0.62</td>
<td>0.88</td>
<td>0.76</td>
</tr>
<tr>
<td>$Q_e$</td>
<td>0.91</td>
<td>0.67</td>
<td>0.94</td>
<td>0.69</td>
</tr>
<tr>
<td>$Q_p$</td>
<td>0.93</td>
<td>0.78</td>
<td>0.94</td>
<td>0.74</td>
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<table>
<thead>
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<th>WDD</th>
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</thead>
<tbody>
<tr>
<td>0.66</td>
<td>0.48</td>
<td>0.66</td>
<td>0.55</td>
<td>0.70</td>
</tr>
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</table>
Highlights

- Development of new infiltration model based on Green and Ampt
- Combining water depth-dependent infiltration and partial contributing area concepts
- Good runoff predictions at rainfall simulator and small catchment scales
- Runon infiltration on hillslopes is an important process in semi-arid environments