Efficient Dynamic Network Loading Modeling: The fixed point link transmission model

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1. Problem and research questions
At the core of dynamic traffic assignment (DTA) we find the Dynamic Network Loading (DNL). The goal of the DNL is to find consistency between the propagation of traffic flows on the network and the constraints imposed by roads (e.g., maximum throughput, speed limits,…) or intersections (e.g. turning restrictions, obstructions by crossing traffic,…) by inflicting delays in the network. For solving practical DTA (or DNL) problems, a numerical discretization of the variables (traffic, space, time) is required. In our context:

- Traffic: Individual vehicles are aggregated into a continuous vehicle flow represented by cumulative vehicle numbers (CVN) from which the fundamental traffic characteristics (speed, flow and density) can be derived (Newell, 1993).
- Space: The link transmission model (LTM, Yperman, 2007) allows one to have discrete space intervals the size of homogeneous stretches of roads (links).
- Time: The time discretization is depended on the level of the problem: for standard LTM and most other DNL’s typically short (less than 1 min), for route choice models and origin destination flows a lot larger (typically around 15 min).

For standard numerical schemes that solve DNL sequentially through the time domain, time steps are typically small (Courant Fraichant Lewy or CFL-condition), meaning that for each link in the network the update interval of the corresponding CVN cannot be larger than the minimum time it takes to propagate information over that specific link. This is of major importance for the efficiency of LTM (and other DNL) implementations, as computational resources are almost linearly dependent on the time update frequency. In this paper, we describe a variation on the basic LTM that avoids the CFL-conditions, which as a result allows for inherently faster numerical evaluation.

2. Methodology, research strategy
We adopt the fixed point solution algorithm introduced by Gentile (2007) to the LTM formulation resulting in a model that finds consistent network loadings with much larger update time intervals (up to 15 minutes). Rather than solving the DNL sequentially through the time domain, fixed point LTM iterates over the entire time domain until traffic flow complies with simplified (first order) kinematic wave theory. Each iteration step is composed of a forward phase and a backward phase. In the forward phase, flow is propagated downstream with given time depended link travel time profiles, resulting in CVN’s for each link entering or exiting a node. In the backward phase the travel time profiles are updated by recalculating the CVN at each node that is constrained by a downstream restriction, resulting in an upstream movement of restrictions and congestion.

In the full paper we will describe in detail each of the two phases and propose a numerical algorithm which performs an automatic descretization of the algorithms time intervals that minimize numerical errors and optimizes computational resources. The evaluated time steps are carefully chosen to reduce redundant calculations. This means that only the regions where changes in traffic occur are densely sampled in time. A simple rule is used to update the time mesh. If flow is changing rapidly, points are added to the mesh to decrease interpolation errors whereas if flow is almost not changing, points are removed from the mesh to decrease
calculation time. Next this rule is combined with a smart node activation system that only recalculates those nodes that need updating in the forward or backward phase. The used methodology largely overlaps with the marginal DNL of Corthout (2011).

3. Major findings
In the following section, the fixed point LTM and basic LTM are compared on the dataset of a real network. The test bed is the highly congested E40 highway between Leuven and Brussels in Belgium. It covers a distance of around 30km. Only one typical morning peak period is modeled (60% increase in total travel time). The fixed point LTM loading is preformed for varying time intervals. Table 1 shows that with increasing time intervals the number of node updates decreases rapidly. The calculation time of both algorithms dependents linearly on the number of node updates. From which we can conclude that the calculation time of the fixed point algorithm with large time steps is significantly less than that of the basic LTM. The maximum update time allowed by the CFL-condition is used for the basic LTM. This corresponds to a time interval around 30 seconds (depending on link length).

Table 1: Number of node updates

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>#Node Updates</th>
</tr>
</thead>
<tbody>
<tr>
<td>LTM</td>
<td>234000</td>
</tr>
<tr>
<td>1 min FP LTM</td>
<td>227511</td>
</tr>
<tr>
<td>5 min FP LTM</td>
<td>49245</td>
</tr>
<tr>
<td>10 min FP LTM</td>
<td>24725</td>
</tr>
<tr>
<td>15 min FP LTM</td>
<td>8733</td>
</tr>
</tbody>
</table>

Figure 1 illustrates the errors that are made by increasing the update interval. The travel time is plotted for the 15km long route from Leuven towards Brussels. Origin-Destination flows have been calibrated for 5 minute intervals. The result for the fixed point LTM with 1 minute time interval coincides with the basic LTM. For larger time steps small errors are noted because of the interpolations that are made. These interpolations cause small deviations in the bottleneck activation and deactivation. Overall results are satisfying compared to the decrease in calculation time.

Figure 1: Travel time for various time intervals
In figure 2 the densities along the same route are plotted. It can be observed that congestion patterns remain roughly the same as time steps are increased. For update time intervals larger than 1 minute the high frequent changes in congestion are no longer visible. This hardly influences aggregated travel times as illustrated in figure 1.

Figure 2: Density for various time intervals along route Leuven – Brussels

4. Takeaway
In the previous section the Leuven - Brussels case was presented to illustrate interpolation errors and calculation gains of using a DNL with larger interval times. With respect to these findings the solution of a DNL with large time intervals can serve two purposes: one as a quick-and-dirty method to make a fast/cheap analysis of a congested system, the other as a rough intermediate approximation of the solution in the early steps of an iterative process. This will reduce the calculation time in the early steps to increase overall convergence speed and stability.

Finally the proposed methodology can also be easily exploited to efficiently update an existing solution to small adaptations. The fixed point LTM scheme is designed to find a consistent network loading from any starting solution. If adaptations only influence flow or constraints locally, the forward and backward phase will run only for these local changes. This results in very efficient system for performing sensitivity analysis, calibrating parameters or calculating optimization directions.
5. Keywords
Dynamic network loading, fixed point, moving mesh, link transmission model

References