Nonresonant electromagnetic instabilities in space plasmas: interplay of Weibel and firehose instabilities

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Abstract. In coronal outflows and solar winds, the presence of the interplanetary magnetic field and heat fluxes combined with jets and shock waves give rise to important thermal anisotropies and energetic counterstreaming motions of plasma shells. Such anisotropic structures of plasma quickly lead to the onset of the kinetic electromagnetic instabilities which are dependent solely on bulk properties of the plasma and not on resonant interaction with charged particles. Here the interplay between the Weibel and firehose instabilities, both driven by an excess of kinetic energy in the direction of the ambient magnetic field, is considered. Their growth rates and thresholds are evaluated and compared for the electron temperature anisotropies in the solar wind. It is shown that the instability of the Weibel-type, which is improperly known as the “oblique firehose” instability, is the most efficient mechanism of isoprotisation limiting the increase of particle velocity anisotropy and thus confirming the observations. These instabilities can explain the origin of interplanetary magnetic field fluctuations, which are expected to enhance along the temperature anisotropy thresholds.

Keywords: space plasmas – temperature anisotropy – Firehose instability – Weibel instability
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INTRODUCTION

The solar wind plasmas are magnetized and low or non-collisional, and the electrons and ions can easily develop heat fluxes and temperature anisotropies, and their velocity distribution functions (VDFs) become skewed along the ambient magnetic field [1, 2, 3]. Direct measurements of thermal electron parameters (using data from Helios, Ulysses and Wind) in the solar wind [2, 4, 5] have confirmed the constant presence of temperature anisotropies, $T_\parallel > T_\perp$, where $\parallel$ and $\perp$ denote the directions relative to the interplanetary magnetic field, $B_0$.

The electron temperature anisotropy is, however, not so strong as predicted by collisionless (exospheric) or low-collisional models implying that some constraints, e.g., instabilities or collisions, should limit these anisotropies and explain the observations [3]. Thus, in solar flares, the electron firehose instability (FHI), which is driven by an excess of parallel kinetic energy and propagates along the magnetic field lines ($k_{\text{FHI}} \parallel B_0$), is considered to be an efficient mechanism of temperature isotropization constraining the increase of the electron temperature anisotropy [3, 6]. This instability develops only in the presence of an ambient magnetic field, and can transfer the electron kinetic energy to protons. Furthermore, the limits observed for the electron and proton temperature anisotropies in the solar wind are in agreement with the threshold of the FHI [3, 7, 8, 9].

A surplus of parallel kinetic energy (whether it is a relative directed motion of plasma streams or a temperature anisotropy) may also drive a Weibel-like instability (WI) of the ordinary mode [10, 11] propagating perpendicular to the magnetic field ($k_{\text{WI}} \perp B_0$). The Weibel instability is purely growing (i.e. with a vanishing real part of the frequency, $\omega_r = 0$) and it has originally been described in [12, 13] for a nonmagnetized plasma. This instability could be at the origin of the magnetic field fluctuations [14] frequently observed in the solar wind [15]. In a magnetized plasma with $T_\parallel > T_\perp$, the electron WI seems to be very competitive growing much faster than the electron FHI [16]. Both these instabilities are nonresonant with plasma particles, but their generation mechanisms are different (see [16]), and we therefore consider that the Weibel-like instability is improperly called the “oblique firehose” instability in the literature. Making a clear distinction between these two instabilities is probably more important for the relaxation of the proton anisotropy, when the growth rates of these instabilities are of the same order of magnitude [17].

Suprathermal populations with high energy tails deviated from the Maxwellian and described by power-law distributions are widely present in the interplanetary dilute plasma, and recently it was shown that both the firehose and Weibel instabilities are more likely to develop in such anisotropic plasmas with bi-kappa distributions [18, 19].

These instabilities are both inhibited by the ambient magnetic field in a sense that the temperature anisotropy ($T_\parallel / T_\perp$) must be sufficiently large, larger than specific thresholds, to boost the growing waves, and because the
WI admits a larger threshold, under that, the FHI remains the principal mechanism of relaxation [16]. These limits are analyzed here for anisotropic electron distributions typically encountered in the solar wind. It is shown that the WI can introduce new constraints for the plasma temperature anisotropy as expected from the observations.

**CONSTRAINTS: WI VERSUS FHI**

There is observational evidence that the electron velocity distributions in the core and halo are anisotropic but only small temperature anisotropies $T_{\parallel,e}/T_{\perp,e} \sim 1-2$ are detected, and this ratio only occasionally increases to 4. The interplanetary magnetic field is also small enough, e.g., $B_0 \sim 10^{-1}-10^{-2}$ G (at 1 AU or higher altitudes) [20], so that we can use the recent developed theories [11, 16, 19] to calculate exactly numerically the growth rates of the WI and FHI when these are driven by either a bi-Maxwellian or a bi-kappa anisotropic distribution.

From a linear Vlasov-Maxwell formalism one can find the dispersion relation for perpendicular modes ($k_{\perp} \neq 0$) [11, 16]

$$\frac{\omega^2 - k^2 c^2}{\omega_{p,e}^2} = 1 - \sum_{n=0}^{\infty} \frac{J_n^2(z)}{z^2} - \frac{v_\perp^2}{\Omega \perp} \partial F_0 \Omega \perp \frac{\partial F_0}{\partial v_\perp}$$

(1)

in terms of the modified Bessel function $J_n(u)$ with $z = k_\perp v_\perp / \Omega \perp$, plasma frequency $\omega_{p,e} = (4\pi n_0 e^2/m_e)^{1/2}$, gyrofrequency $\Omega \perp = eB_0/(m_e c)$, and the anisotropic distribution, $F_0(v_\perp, v_\perp)$. The WI growth rates are displayed in Fig. 1 for a solar wind plasma [19] with small temperature anisotropy according to [16]. The maxima reached at the saturation are of the order of the electron gyrofrequency but less than that, $\omega_i \sim 3 \times 10^{-3} \omega_{p,e} \sim 0.3\Omega \perp$. While for a large temperature anisotropy, $w_{\perp,e} \gg w_{\parallel,e} \rightarrow 0$, where $w_{\perp,e} = 2k_B T_{\perp,e}/m_e$ are thermal velocities, the instability threshold is simply given by $\beta_{\parallel} > 2$ [16], for a finite thermal spread $w_{\perp,e} \neq 0$, the threshold is more restrictive [10]

$$\beta_{\parallel} \geq b_1 = \frac{2}{1 - 1.5^{1/2} a_e} > 2,$$

(2)

where $a_e = (T_{\parallel,e}/T_{\perp,e})^{1/2} = w_{\parallel,e}/w_{\perp,e}$.

For parallel modes ($k_{\parallel} \|$ $B_0$), we consider only bi-Maxwellian distributions because the FHI growth rate does not change significantly in kappa-distributed plasmas [18]. The dispersion relation simply reads [18]

$$\omega^2 - \omega_p^2 = \frac{1}{\omega_{p,e}^2} \left[ \frac{k_{\parallel}^2}{\omega_{p,e}} Z(f_p) - 1 \right] + \frac{\omega}{\omega_{p,e}} Z(f_p),$$

(3)

in terms of plasma dispersion function, $Z(f)$ with $f_a = (\omega - \Omega_a)/(k_{\parallel} a)$ ($a = e$ for electrons and $a = p$ for protons). In the present calculations, the ions are assumed isotropic; nevertheless, a proton anisotropy could be introduced, since it exists as well in solar wind, may be due to the resonant interaction with the low-frequency FHI field [6]. Although protons do not provide any free energy to the electron FHI, they absorb the wave energy and further contribute to the instability saturation. The equation (3) admits unstable solutions with a growth rate ($\omega_i \geq 0$) [18]

$$\omega_i \Omega_p = \sqrt{\pi} \frac{k_{\parallel}^2 c^2}{\omega_{p,e}^2} \left[ \frac{\beta_{\parallel}^2}{2} (1 - a_e^2) - 1 \right] \left( \frac{\omega_{p,e}}{\Omega_p} + \pi \frac{\Omega_p}{k_{\parallel}^2 c^2} \right)^{-1},$$

(4)
FHI must be correctly understood as a Weibel-like insta-
tation for one of these constraints: the so-called oblique
ature anisotropies [3, 7]. Here we provide a new descrip-
tion for one of these constraints: the so-called oblique
existence of a stable solar wind plasma to small temper-
represented an efficient constraint at large
low frequency Alfvén waves excited by this instability is
eters can be easily determined when the presence of the
narrow band of solar wind plasmas at 1 AU, the FHI is effective in a
β
stable only for small temperature anisotropies, above L2.

only for
\[
\beta_\parallel \geq b_2 = \frac{2}{1 - \alpha_\parallel^2} > 2. \tag{5}
\]

In order to evaluate the (maximum) growth rates of the FHI we calculate exactly numerically the solutions of Eq. (3) for the same electron-proton plasma model used in Fig. 1. The FHI growth rates are displayed in Fig. 2, and one can simply observe that the maxima are comparable to but less than the proton gyrofrequency, \( \omega_\parallel \lesssim \Omega_p \).

Now, comparing the growth rates of these instabilities, and taking into account that \( \Omega_e \sim 2 \times 10^3 \Omega_p \), we conclude that the WI will grow much faster than the FHI. However, their thresholds are different, \( b_1 > b_2 \), and the FHI can still be effective in the interval \( b_2 < \beta_\parallel < b_1 \). For a very large \( T_{\perp,e}/T_{\parallel,e} \gg 1 \) (no perpendicular thermal spread, \( T_{\perp,e} \rightarrow 0 \)), or in the limit case of a cold beam propagating along the magnetic field, this interval vanishes, \( b_1 = b_2 = 2 \), and the FHI does not grow. On this basis, the effectiveness of the FHI has recently been analyzed for a variety of solar plasmas, from corona to solar winds in the near Earth environment, and within diagrams of \( \beta_\parallel \) vs. \( \beta_\perp \) or \( w_\parallel/w_\perp \) vs. \( \beta_\perp \) [16]. Thus, for solar wind plasmas at 1 AU, the FHI is effective in a narrow band of \( \beta_\parallel \) (or \( w_\parallel/w_\perp \)) so that plasma parameters can be easily determined when the presence of the low frequency Alfvén waves excited by this instability is confirmed by the observations [21].

Recent reports have shown that these instabilities represent an efficient constraint at large \( \beta_\parallel > 2 \) limiting the existence of a stable solar wind plasma to small temperature anisotropies [3, 7]. Here we provide a new description for one of these constraints: the so-called oblique FHI must be correctly understood as a Weibel-like insta-
bility, which is much faster and has a threshold, given in Eq. (2), that seems to provide a new possible con-
strain for small \( \beta_\parallel < 2 \) solar wind plasma. We rewrite the thresholds of these two instabilities in terms of \( T_{\perp,e}/T_{\parallel,e} \) vs \( \beta_\parallel \). Thus, the WI threshold from Eq. (2) transforms to
\[
a_\parallel^2 = \frac{T_{\perp,e}}{T_{\parallel,e}} \leq \frac{2}{3} \left( 1 - \frac{2}{\beta_\parallel} \right)^2, \tag{6}
\]
and the FHI threshold from Eq. (5) to
\[
a_\parallel^2 = \frac{T_{\perp,e}}{T_{\parallel,e}} \leq 1 - \frac{2}{\beta_\parallel}, \tag{7}
\]
which are displayed in Fig. 3 by the limits \( L_1 \) (solid line) and \( L_2 \) (dashed line), respectively. The effectiveness of FHI is limited in between \( L_1 \) and \( L_2 \), in the light gray area [16], and the WI operates below \( L_1 \) relaxing the electron temperature anisotropy.

It is important to note that these two limits, \( L_1 \) and \( L_2 \), are rigorously provided by the theory and they are consistent with the observations, see, for example, in [3], where the large temperature anisotropies, \( T_{\perp,e}/T_{\parallel,e} \ll 1 \), are assumed to be constrained by the fastest unstable modes, known as the oblique firehose modes and here correctly renamed as purely growing Weibel-like modes (with a large growth rate of the order \( \sim (1 - 5) \times 10^{-1} \Omega_e \)). The other isocontours displayed in [3] for finite constant growth rates of the WI, here in Fig. 3 should lie in the dark gray region, under the border \( L_1 \) defined by (6), and for the FHI the isocontours lie under \( L_2 \) defined by Eq. (7). The analytical forms provided by the linear dispersion theory for these isocontours (constant growth rates) are however very complicated so that approximate forms similar to Eq. (6) and Eq. (7) have been proposed [22]
\[
a_\parallel^2 = \frac{T_{\perp,e}}{T_{\parallel,e}} \leq 1 + \frac{a}{\beta_\parallel b}, \tag{8}
\]
where the appropriate values for the parameters \( a \) and \( b \) are given in [22].

In the opposite case with \( T_{\perp,e} > T_{\parallel,e} \), it is the resonant instability of the whistler mode that confines the high beta plasma [3], but for small \( \beta_\parallel < 2 \) no mechanism is known to limit plasma temperature anisotropies. There is therefore a new interesting skill of the WI that must be pointed out in Fig. 3: the WI threshold \( L_1 \) as it is given by Eq. (6) has also a branch for small values of \( \beta_\parallel < 2 \), which is shown here for the first time, and which could prove that, whether parallel beta is small or large, the same WI can constrain the excess of parallel kinetic energy and confirm the observations (see in [3]). The left branch of \( L_1 \) shows us, hypothetically, that the WI can be effective even for a small \( \beta_\parallel < 2 \), but most probably in the vicinity of \( \beta_\parallel \lesssim 2 \) of this limit \( L_1 \) because a

\[
\beta_\parallel \geq b_2 = \frac{2}{1 - \alpha_\parallel^2} > 2. \tag{5}
\]
small $\beta_\parallel < 1$ means negligible temperature effects and/or a strong magnetic field, which certainly suppresses the WI [14, 19]. (The electrostatic loss-cone instability is expected to constrain anisotropy at very small $\beta_\parallel \ll 2$.) We can therefore assert that the WI can be effective at moderately small $\beta_\parallel \lesssim 2$ in the vicinity of $L_1$, thus providing the main constraint for these anisotropies and explaining the occurrence rates for the core population in the solar wind with a dominance in the opposite region of large $\beta_\parallel \gtrsim 2$ [3].

**CONCLUSIONS AND PERSPECTIVES**

We have studied the interplay of the nonresonant instabilities driven by an excess of parallel kinetic energy in the solar wind electron plasmas. Such a thermal anisotropy, $T_\parallel /T_\perp > 1$, is constantly observed in the core populations, which represent $\sim 95\%$ of the total solar wind electrons. Despite the low collision rates of the electrons, these deviations from isotropy are small implying that some plasma instabilities should constrain these anisotropies and explain the observations [2, 3]. These instabilities are the FHI and the so-called oblique FHI, which must be correctly understood as an instability of Weibel-type [3, 16]. The growth rates of these instabilities have been calculated exactly numerically in Figures 1 and 2 for plasma parameters typically encountered in the solar wind. The WI is much faster with a growth rate several order of magnitudes larger than the FHI. The FHI is much more stable with a growth rate $\lesssim 95\%$ of the total solar wind in the vicinity of $L_1$, thus providing the main constraint for these anisotropies and explaining the occurrence rates for the core population in the solar wind with a dominance in the opposite region of large $\beta_\parallel > 2$ [3].

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**REFERENCES**