UNIVERSIDADE ABERTA
www.uab.pt

22 ENGINEERING ECONOMICS: LECTURE NOTES

JOÃO ZAMBUJAL OLIVEIRA
MANUEL MOUTA LOPES

eUAb | Coleção Universitária

ENGINEERING ECONOMICS: LECTURE NOTES

JOÃO ZAMBUJAL OLIVEIRA | MANUEL MOUTA LOPES

UNIVERSIDADE ABERTA | 2020

Palácio Ceia • Rua da Escola Politécnica, 147
1269-001 Lisboa – Portugal
www.uab.pt

Coleção: eUAb | Coleção Universitária; N.º 22
ISBN: 978-972-674-835-9
DOI: https://doi.org/10.34627/e79m-d128

TABLE OF CONTENTS

INTRODUCTION

1. MAJOR FINANCIAL STATEMENTS
   - 1.1 Balance Sheet
   - 1.2 Income Statement by Nature of Expense
   - 1.3 Income Statement by Function of Expense

2. FINANCIAL AND ECONOMIC INDICATORS
   - 2.1 Financial Ratios
     - 2.1.1 Liquidity Ratios
     - 2.1.2 Leverage Ratios
   - 2.2 Economic Indicators
     - 2.2.1 Profitability ratios
     - 2.2.2 Efficiency Ratios

3. PRINCIPLES OF FINANCIAL CALCULUS
   - 3.1 Time value of money: present and future value
   - 3.2 Interest rates
     - 3.2.1 Simple and compound interest rates
     - 3.2.2 Nominal and real interest rates
     - 3.2.3 Equivalent interest rates and financial flows
     - 3.2.4 Effective and nominal interest rate
     - 3.2.5 Banking rates
   - 3.3 Annuities and perpetuities
     - 3.3.1 Annuity
     - 3.3.2 Perpetuity
   - 3.4 Valuation of stocks and bonds
     - 3.4.1 Stocks
     - 3.4.2 Bonds

4. CASH FLOW AND COST OF CAPITAL

4.1 Free Cash Flow to Firm (FCFF)

4.2 Free Cash Flow to Equity (FCFE)

4.3 Discount rate

4.4 Return demanded by shareholders (CAPM)

4.5 Cost of Debt

4.6 Weighted Average Cost of Capital

5. INVESTMENT CRITERIA METHODS

5.1 Net Present Value

5.1.1 Weighted Average Cost of Capital

5.2 Internal Rate of Return

5.3 Modified Internal Rate of Return

5.4 Payback Period

5.5 Profitability Index or Benefit/Cost Ratio

5.6 Adjusted Present Value

5.7 Average Accounting Return

6. INVESTMENT SELECTION UNDER SPECIFIC RESTRICTIONS

6.1 Selecting between long and short-lived equipment

6.2 Mutually exclusive investments

6.3 Incremental Cash-Flows

6.4 Least Common Multiple

6.5 Equivalent Annual Value (EAV)

6.6 The profitability index with capital rationing

6.7 Problems with Prioritization

6.8 Mathematical programming approach

6.9 Optimal timing of investment

7. EXCEL FINANCIAL FUNCTIONS

7.1 Present Value

7.2 Net Present Value

7.3 Future Value

7.4 Annuity

7.5 Number of Periods

BIBLIOGRAPHY

TABLE OF FIGURES

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7

TABLE OF TABLES

Table 1

Table 2

Table 3

Table 4

Table 5

Table 6

Table 7

Table 8

Table 9

Table 10

Table 11

Table 12

Table 13

Table 14

Table 15

Table 16

Table 17

Table 18

Table 19

Table 20

Table 21

Table 22

Table 23

Table 24

Table 25

Table 26

Table 27

Table 28

Table 29

Table 30

Table 31

Table 32

Table 33

Table 34

Table 35

Table 36

Table 37

Table 38

Table 39

Table 40

Table 41

Table 42

Table 43

INTRODUCTION

This book appeared after several editions of the course in a master's degree, in which the idea of extending the entire experience of master's classes to a universe of interested in really knowing how the economic concepts of companies apply to an engineering practice was solidified, for reasons such as an appealing title for master's students or the recognition that the chosen program contents were the appropriate ones.

The work's purpose is to present a mainly financial, but also economic, view of the functioning of organizations, with theoretical and conceptual basis for the practice of financial project analysis capable of sustaining investment decisions and increasing the value of organizations. It also intends to able the engineer to use their practical need toward informed financial decisions in a range of engineering areas: mechanical, civil, industrial or manufacturing, among others.

The book targets master's students, and even bachelor's degree students, with a non-economic basic training but with an interest in perceiving the real functioning of companies. The numerous solved examples also appeal to professionals from various sectors of activity with management responsibilities in corporations, because decisions involve money and the business success depends upon the expertise of economic and financial issues.

The structure of the book begins with a more general view of the corporations, with accounting documents – Balance sheets and Income Statements – included in a company's mandatory divulgence of information on its financial situation.

Chapter two moves into more specific aspects of its current functioning: economic and financial indicators to asses a company's financial health, its strengths or weaknesses, the connection between the operating activities and the financial performance.

Chapter three introduces the principles of financial calculus with interest rates, annuities and perpetuities, or valuation of stocks and bonds.

Chapter four uses discount cash flow techniques for understand the investment decisions with temporal horizon cash flows, and stands out the importance of the weighted average cost of capital for the return demanded by shareholders.

Chapter five is about investment criteria methods, such as the net present value and internal rate of return to support the decision to invest or to postpone it.

Chapter six includes restrictions increasing the complexity in investment decisions.

There is a final chapter to assist with the use of spreadsheet for the construction of maps and indicators presented throughout the book. The pre-build formulas and tools of the spreadsheet allow us to increase the complexity of problems adjusting then to the real world.

1. MAJOR FINANCIAL STATEMENTS

The major financial statements are the Balance Sheet and the Income Statement. The Balance Sheet represents the company's property at a given time, and consists of the Assets, the Liabilities and Equity. The Income Statement represents the progress of the company's activity during a given time period (usually an year), and whose result influences the assets of the company through equity.

1.1 BALANCE SHEET

The fundamental equation of the Balance Sheet is:

Assets = Liabilities + Equity

Therefore, the Net Property (Equity) is given by:

Equity = Assets - Liabilities

In financial terms we have only Investments and Capital Financing:

Investments = Debts + Equity

ASSETS	SHAREHOLDERS' EQUITY
Non current assets	Capital
Financial Investments	Legal reserve
Tangible fixed assets	Retained earnings
Intangible fixed assets	Net profit
	LIABILITIES
	Non current liabilities
	Long term provisions
	Long-term debts (bank loans and bonds)
	Other non credit liabilities
Current assets	**Current liabilities**
Inventory	Short-term debts (bank loans and bonds)
Accounts receivable	Suppliers
State and other public entities	State and other public entities
Other current assets	Other current liabilities
Cash and bank deposits	Deferrals
Deferrals	

1.2 INCOME STATEMENT BY NATURE OF EXPENSE

The income statement can be presented by its nature method:

Result = Income - Expenses

In an income statement by nature, the expenses are presented according to their nature such as, depreciation cost, transports costs, rent expense, wages and salaries, etc. The Net Income (Net Result) can be calculated by its function nature as:

INCOME STATEMENT by Nature of Expense
Sales and services rendered
+ Production inventories Variation
– Cost of Goods and Raw Materials
– External Supplies and Services
– Staff Costs
– Other
EBITDA (Earnings Before Interest, Taxes, Depreciation and Amortization)
– Depreciation Costs
OPERATING INCOME (EBIT: Earnings Before Interest and Taxes)
+ Interest revenues
– Interest paid
EARNINGS BEFORE TAXES (EBT)
– Taxes
NET PROFIT/LOSS

1.3 INCOME STATEMENT BY FUNCTION OF EXPENSE

In an income statement by function, expenses are disclosed according to their functions such as, cost of goods sold, selling expenses, administrative expenses, other expenses, etc.

Gross Income = Revenue – Direct Costs

Operating Income = Gross Income – Fixed Costs

Result = Operating Income – Financial Costs

Net Income = Result – Current Taxes

INCOME STATEMENT by Function of Expense
Sales and services rendered
– Cost of sales and services provided
GROSS INCOME
+ Other operating income and gains
– Distribuition costs
– Administrative expenses
– Other operating costs and loses
OPERATING INCOME (EBIT: Earnings Before Interest and Taxes)
+ Interest revenues
– Interest paid
EARNINGS BEFORE TAXES (EBT)
– Taxes
NET PROFIT/LOSS

Problem: Considering the following accounting records for JZ, Ldt. on 2018, prepare the income statement for 2018:

Accounts	**€uros**
Sales comissions	15 000
Beginning merchandise inventory	16 000
Ending merchandise inventory	9 000
Sales	185 000
Advertising	10 000
Purchase of merchandise	85 000
Employees' salaries	20 000
Other operating expenses	30 000

Solution: Considering the given accounting records, the income statement for 2018 results in the following:

Sales			**185 000**
(-)	Cost of goods sold		
	Beginning inventory	16 000	
	Purchases	85 000	
	Cost of goods available for sale	101 000	
	Ending inventory	9 000	
	Cost of goods sold		92 000
Gross income			**93 000**
(-)	Selling and administrative expenses		
	Sales comissions	15 000	
	Advertising	10 000	
	Employees' salaries	20 000	
	Other operating expenses	30 000	75 000
Operating Income			**18 000**
Net Income			18 000

2. FINANCIAL AND ECONOMIC INDICATORS

The economic and financial indicators measure the company activity using indexes whose operation is independent of the absolute values existing in financial documents, allowing for their annual comparison in company and their competitors, both at the individual level and at sectoral level. This method should consider the following aspects:

a. The firm performance is relative.

b. It's advisable to:
   i. Compare to its historical trend (internal).
   ii. Compare with competitors (positioning the company against the competition).
   iii. Compare with database financial reports (company's positioning in the sector).
   iv. Compare the return with cost of capital or opportunity cost.

Concerning the analysis of several functions of the firm, we can enumerate several types of ratios:

- **Financial ratios**: related to financial aspects (financial structure, debt capacity, solvency).
- **Economic ratios**: related to economic situation (cost structure and profit margins)
- **Economic/financial ratios**: the relative economic and financial aspects (return on equity, asset rotation).

As an example, consider the following balance sheet:

Table 1

ASSETS		SHAREHOLDERS' EQUITY	
NON CURRENT ASSETS		Capital	650 000
Financial Investments	998	Legal reserve	13 145
Tangible fixed assets	1 752 356	Other reserves	1 206 578
Intangible fixed assets	113 445	Retained earnings	- 229 510
		Net profit	229 510
CURRENT ASSETS		EQUITY	2 026 253
Inventory	1 747 280	**LIABILITIES**	
Accounts Receivable		NON CURRENT LIABILITIES	
MLT	0	Long term provisions	0
ST	3 575 620	Debt	
Marketable securities	102 815	MLT	1 739 790
Deferrals	60 279	CURRENT LIABILITIES	
Cash and bank deposits	720 444	ST - Suppliers	2 055 897
		ST - Other current liabilities	1 702 536
		Deferrals	548 761
		TOTAL LIABILITIES	6 046 984
TOTAL ASSETS	8 073 237	**TOTAL LIABILITIES + EQUITY**	8 073 237

and the Income Statement:

Table 2

Sales and services rendered	14 367 563
Cost of sales and services provided	-6 667 327
GROSS INCOME	**7 700 236**
Other operating income and gains	31 993
Distribuition costs	-3 267 384
Administrative costs	-3 285 128
Other operating costs and losses	-771 205
OPERATING INCOME / EBIT	**408 512**
Interest revenues	50 000
Interest paid	-108 248
EBT (EARNINGS BEFORE TAXES)	350 264
Taxes	-120 754
NET PROFIT/LOSS	**229 510**

2.1 FINANCIAL RATIOS

When we refer to the solvency of the firm's overall financial position, we are saying that a liquid firm can easily meet its short-term financial obligations. Liquidity might also be viewed as the concept of converting an asset into cash with small or no loss in value.

2.1.1. Liquidity Ratios

Computing the Currents Assets and Current Liabilities with the following formulas:

- Current assets (CA)

$$\mathrm{CA} = \mathrm{Cash} + \mathrm{Marketablesecurities} + \mathrm{Receivables} + \mathrm{Inventory}$$
$$\mathrm{CA} = 720\ 444 + 163\ 094 + 3\ 575\ 620 + 1\ 747\ 280 = 6\ 206\ 438$$

- Current liabilities (CL)

$$\mathrm{CL} = \mathrm{Short} - \mathrm{termdebt} + \mathrm{Othercurrentliabilities}$$
$$\mathrm{CL} = 2\ 604\ 658 + 1\ 702\ 536 = 4\ 307\ 194$$

We can calculate the Net Working Capital in the following way:

- Net Working Capital (NWC)

$$\mathrm{NWC} = \mathrm{CurrentAssets} - \mathrm{CurrentLiabilities}$$
$$\mathrm{NWC} = 6\ 206\ 438 - 4\ 307\ 194 = 1\ 899\ 244$$

Thus, the **Current Ratio** might be obtained by the following way:

- Current Ratio (CR)

$$\mathrm{CR} = \frac{\text{Current assets}}{\text{Current liabilities}} = \frac{6\ 206\ 438}{4\ 307\ 194} = 1.441$$

Taking the term from the nitric acid test for gold, the quick ratio (acid test ratio) is a strictest test of liquidity:

- Quick Ratio (QR)

$$\mathrm{QR} = \frac{\text{Cash} + \text{Marketable securities} + \text{Receivables}}{\text{Current Liabilities}} = \frac{4\ 459\ 158}{4\ 307\ 194} = 1.035$$

which measures the ability to obtain near cash or quick assets to pay immediately the current liabilities.

The **Cash Ratio**, sometimes designated by cash asset ratio, corresponds to the ratio between cash or cash equivalent assets and current liabilities.

- Cash Ratio

$$\text{CashRatio} = \frac{\text{Cash} + \text{Marketable Securities}}{\text{Current Liabilities}} = \frac{883\ 531}{4\ 307\ 194} = 0.205$$

Problem: The balance sheet at December 31 shows the following:

Table 3

Current Assets	**€**
Cash	4 000
Marketable Securities	8 000
Accounts Receivable	100 000
Inventories	120 000
Prepaid Expenses	1 000
Total Current Assets	**233 000**
Current Liabilities	
Notes Payable	5 000
Accounts Payable	150 000
Accrued Expenses	20 000
Income Taxes Payable	1 000
Total Current Liabilities	**176 000**
Long-Term Liabilities	**340 000**

1. Determine the following ratios: (a) net working capital, (b) current ratio, and (c) quick ratio.
2. Considering industry benchmarks for current ratio of 1.29 and for quick ratio of 1.07, what level of liquidity does this company present?

Solution

1. Using the balance sheet, the following ratios were obtained.

$$\text{NetWorkingCapital} = 232\ 000 - 156\ 000 = 76\ 000$$

$$\text{Current ratio} = \frac{232\ 000}{156\ 000} = 1.49$$

$$\text{Quick ratio} = \frac{112\ 000}{156\ 000} = 0.72$$

2. In what of concern to the Current Ratio, the value obtained for the company

surpasses the industry average and the Quick Ratio is significantly below the industry average. Therefore, the liquidity level of the firm is poor because it has more current liabilities than highly liquid assets.

2.1.2 Leverage Ratios

Leverage ratios indicate the level of debt incurred by a company against other accounts in the balance sheet, income statement, or cash flow statement. These ratios constitute an indication of how the company's assets and company operations were being financed (by debt or equity). Thus, leverage ratios analyse not only, if debt allows the generation of profits using other people (creditors) money, but also creates claims on earnings with a higher priority than those of the firm's owners. Below is an illustration of the total debt ratio, considering the previous balance sheet example:

- Total debt ratio (TD)

$$\mathrm{TD} = \frac{\text{Total Liabilities}}{\text{Total Assets}} = \frac{6\ 046\ 984}{8\ 073\ 237} = 0.75$$

When the firm needs to quantify the degree of indebtedness or the ability to service its debts, it can use the following debt measure:

- Debt to Equity (DTE)

$$\mathrm{DTE} = \frac{\text{Total Liabilities}}{\text{Equity}} = \frac{6\ 046\ 984}{2\ 026\ 253} = 2.98$$

2.2 ECONOMIC INDICATORS

2.2.1 Profitability Ratios

Profitability ratios allow to assess the firm's ability to operate efficiently, being analysed in detail by the firm owners, creditors, and management. The return on sales (ROS) is a ratio employed to quantify the proportion of profits generated from sales, giving the ability of management to generate profits from given levels of sales.

- Return on Equity (ROE)

$$\mathrm{ROE} = \frac{\text{Net Income}}{\text{Average Shareholders Equity}} = \frac{229\ 510}{2\ 026\ 253} = 11.3\%$$

- Return on Assets (ROA)

$$\text{ROA} = \frac{\text{Operating Income}}{\text{Total Assets}} = \frac{408\ 512}{8\ 073\ 237} = 5.1\%$$

- Return on Sales (ROS)

$$\text{ROS} = \frac{\text{Operating Income}}{\text{Sales and Services Rendered}} = \frac{408\ 512}{14\ 367\ 563} = 2.8\%$$

- Net profit margin (NPM)

$$\text{NPM} = \frac{\text{Net Profit}}{\text{Sales and Services Rendered}} = \frac{229\ 510}{14\ 367\ 563} = 1.6\%$$

The net profit margin is expressed as a percentage and shows how much of a company's sales are kept as net profit.

2.2.2 Efficiency Ratios

The efficiency indicators measure the operating efficiency of the firms, regarding their production costs in income (sales) and assets (fixed or inventories and debtors) in cash, and its profitability in relation to the results obtained vis-à-vis capital funders (own and others). An analysis of the firm's liquidity, evaluating the speed with which certain accounts are converted into sales or cash.

- Asset Turnover Ratio (ATR)

$$\text{ATR} = \frac{\text{Net Sales}}{\text{Total Assets}} = \frac{14\ 367\ 563}{8\ 073\ 237} = 1.8$$

- Inventory Turnover Ratio (ITR)

$$\text{ITR} = \frac{\text{Cost of Goods Sold}}{\text{Average Inventory}} = \frac{6\ 667\ 327}{1\ 747\ 280} = 3.8$$

- Average Collection Period (ACP)

$$\text{ACP} = \frac{\text{Receivables}}{\text{Net Sales}} \times 360 = \frac{3\ 575\ 620}{14\ 367\ 563} \times 360 = 89.6$$

- Average Payment Period (APP)

$$\text{APP} = \frac{\text{Accounts Payable}}{\text{Purchases}} \times 360 = \frac{6\ 667\ 327}{2\ 055\ 897} \times 360 = 111.0$$

3. PRINCIPLES OF FINANCIAL CALCULUS

3.1 TIME VALUE OF MONEY: PRESENT AND FUTURE VALUE

Suppose that someone promises you $1\ 000$. Do you prefer to receive the money now or one year from now? Thus, what is the future value of the $1\ 000$, one year from now, considering a 5% interest rate?

$$FV = 1000(1 + 0.05)^1 = 1\ 050$$

$$\left(\frac{FV}{PV}; i = 0.05; n = 1\right) = 1\ 050$$

$(FV/PV); i; n$: compound amount factor; PV : present value; FV : future value; i : interest rate.

For two years from now (with an interest rate of 5% and compound interest regime), the future value would be:

$$FV = 1000(1 + 0.05)^2 = 1000\left(\frac{FV}{PV}; i = 0.05; n = 2\right) = 1102.5$$

If we ask how much should we invest in order to have $1\ 102.5$ in two years, considering a 5% interest rate, the answer would be:

$$PV = FV \times \left(\frac{P}{F}; i; n\right) = 1\ 102.50 \times \left(\frac{1}{1.05^2}\right) = 1\ 000$$

Problem: What is the current value of 100, obtained in three years, considering an annual interest rate of 10%? If the interest rate has the following values in each year $i_1 = 5\%, i_2 = 7\%, i_3 = 10\%$, what would be the current value C_0 ?

Current value (1):

$$C_0 = \frac{C_n}{(1 + i)^n} = \frac{100}{(1 + 0.1)^3} = \frac{100}{1\ 331} = 75.13$$

Current value (2):

$$C_0 = \frac{C_n}{(1+i_1)(1+i_2)(1+i_3)} = \frac{100}{(1+0.05)(1+0.07)(1+0.10)} = \frac{100}{1\,235} = 80.92$$

3.2 INTEREST RATES

Interest is the cost of borrowing money. It should integrate the following components:

- **Real interest rate** to compensate lenders for postponing their own spending during the term of the loan.
- **Inflation premium** to offset the possibility that inflation may erode the value of the money during the term of the loan.
- **Risk premium** to compensate the lender for risky loans.

3.2.1 Simple and compound interest rates

Considering C_0 the present value and C_n the future value:

Simple interest scheme

- **Accumulated capital** (considering interest) after n periods, when the interest rate is equal to i :

$$\boldsymbol{C_n = C_0 \times (1 + ni)}$$

- Value of the **interest rate**, given the number of periods, ending balance C_n and starting balance C_0:

$$\boldsymbol{i = \frac{\frac{C_n}{C_0} - 1}{n} = \frac{C_n - C_0}{nC_0}}$$

- **Number of periods**, given starting and ending balances, and interest rate:

$$\boldsymbol{n = \frac{C_n - C_0}{iC_0}}$$

Compound interest scheme

- **Accumulated capital** after n periods, when the interest rate is equal to i :

$$\boldsymbol{C_n = C_0 \times (1 + i)^n}$$

- **Interest rate**, given the number of periods, ending balance C_n and starting balance C_0:

$$\boldsymbol{i = \left(\frac{C_n}{C_0}\right)^{\left(\frac{1}{n}\right)} - 1}$$

- **Number of periods**, given starting and ending balances, and interest rate i:

$$\boldsymbol{n = \frac{log\left(\frac{C_n}{C_0}\right)}{log(1+i)}}$$

3.2.2 Nominal and real interest rates

The nominal interest rate may have two meanings:

1. rate that includes **the inflation effect**, as opposed to **real interest rate** – current prices, as opposed to constant prices; this is also the case when we refer to nominal GDP versus real GDP;

2. interest rate involving **infra-annual payments** or incomes, that is not the effective annual rate.

Considering the first meaning, the nominal interest rate is given by the formula:

$$i_n = (1 + i_r)(1 + i_i) - 1$$

i_n: nominal interest rate; i_r: real interest rate; i_i: inflation rate.

Example: Consider a nominal interest rate i_n of 2% (at current prices analysis) and an inflation rate i_i equal to 1.5%. Find the real interest rate i_r.

Using the above expression: $(1 + 0.02) = (1 + i_r)(1 + 0.015)$, the real interest rate would be:

$$i_r = \frac{(1 + 0.02)}{(1 + 0.015)} - 1 = 0.0049$$

Approximately, with low inflation rate environment, the real interest rate would be:

$$i_r = i_n - i_i = 0.02 - 0.015 = 0.005$$

Example: Using the previous rates $i_n = 2\%, i_i = 1.5\%, i_r = 0.49\%$, how much would $1\,000$ of today accumulate in a year, in nominal terms, at current prices? How much would accumulate in real terms? (at year 0 constant prices).

- In nominal terms:

$$V_n = 1000 \times (1 + 0.02) = 1\,020.00$$

- In real terms:

$$V_r = 1000 \times (1 + 0.0049) = 1\,004.90$$

Example: Inversely, if we are working at current prices and having the amount of $1\,000$, expressed at year 1 prices, which present value corresponds to an amount of $1\,000$ to be received in 1 year? If the amount of $1\,000$ is already expressed at year 0 constant prices, what would be its present value?

- In nominal terms:

$$V_n = \frac{1000}{(1 + 0.02)} = 980.39$$

- In real terms:

$$V_r = \frac{1000}{(1 + 0.0049)} = 995.12$$

3.2.3 Equivalent interest rates and financial flows

- **Nominal Interest Rates** Banks and other institutions employ the word nominal (annual percentage rate - APR) when infra-annual payments are made at a proportional rate that is not equivalent to the annual effective interest rate.
- **Proportional Interest Rates** Under the simple interest scheme, we only employ proportional rates. For example: consider an annual rate and a quarterly rate. As the relationship between these rates is 4 to 1 is the same as the relationship between one year and four quarters, the rates are proportional.
- **Equivalent Interest Rates** When the interest rates are equivalent, the same amount applied to the same amount and over the same period of time, it yields the same interest.

Example: The equivalent monthly interest rate (i_m) to an annual interest rate (i_a) of 10% is given by:

$$(1 + i_m)^{12} = (1 + r_a)$$

Hence,

$$i_m = (1 + i_a)^{\left(\frac{1}{12}\right)} - 1 = (1 + 0.1)^{\frac{1}{12}} - 1 = 0.00797$$

Under compound interest scheme, effective interest rates do not correspond to proportional rates.

Example: Suppose an investor that made a deposit account with a quarterly interest period. The nominal annual rate specified by the bank is 4%. In this case:

- The effective quarterly rate (i_{eq}) will be:

$$i_{eq} = \frac{0.04}{4} = 0.01$$

- The effective annual rate (i_{ea}) will be:

$$i_{ea} = \left(1 + \frac{i_n}{4}\right)^4 - 1 = 0.041$$

Example: For an APR equal to 12%, with monthly payments, the effective interest rate applied to each month is equal to $12\%/12{=}1\%$. There is no equivalence between paying each month and at the end of the year, in terms of effective interest rate. Actually, paying 1% each month corresponds to $(1.01)^{12} - 1 = 12.683\%$ at the end of the year.

3.2.4 Effective and nominal interest rate

Whenever the interest payment occurs with a frequency inferior to year and the interest scheme is compound, the effective annual rate (i_e) exceeds the nominal one (i_n). The effective annual rate depends on the nominal annual interest rate (i_n) and on the frequency of interest payment (k):

$$i_e = \left(1 + \frac{i_n}{k}\right)^k$$

Example: What is the semi-annual interest rate that is equivalent to an effective annual interest rate of 12%? What is the equivalent effective annual interest rate to a semi-annual 6% rate?

Employing the expression $(1 + i_s)^2 = (1 + i_a)$, the semi-annual interest rate is equal to

$$i_s = (1 + 0.12)^{\frac{1}{2}} - 1 = 0.0583$$

3.2.5 Banking rates

There are two types of interest rates:

- **Passive** interest rates – on deposits
- **Active** interest rates – on loans

Sometimes the banks offer interest rates that contains a component designated by spread.

- **Spread** "margin". Banks often announce interest rates that are indexed to interbank market rates (such as Euribor, base rate, etc.) and to which they add a spread.
- **Euribor (European Interbank Offered Rate)** published every day at 11:00 AM in Brussels, by the European Banking Federation, which selects, from 3,000 banks in the E.U., a 64 representative sample, including 58 in the Euro Zone and 6 out of it. It corresponds to the arithmetic average of all interest rates employed by these 64 banks in the day before, in their interbank market transactions.

The main factors behind interest rates are:

- **Inflation Rate** the higher the inflation rate, the higher the nominal interest rate must be, in order to yield a positive real interest rate.
- **Monetary Policy** Central Banks are able to influence market rates by increasing or decreasing reference interest rates.
- **Money Demand** as in any other market, demand is one of the main price drivers; considering that the interest rate is the price paid for using money, a rising demand for money, all else constant, will raise interest rates.
- **Borrower's risk** the stronger the borrower's risk, the higher the likelihood that he does not repay, so the higher the interest rate.

3.3 ANNUITIES AND PERPETUITIES

Annuities and perpetuities: A stream of cash-flows corresponding to a fixed sum each period or having a pre-specified growth rate.

- **Annuity** – the number of flows is finite.
- **Perpetuity** – the stream of cash-flows is perpetual, and initial capital is not returned to investors.

Annuities examples are housing or car loans with constant monthly payments. Perpetuity examples are perpetual income payments, perpetual pension payments, and perpetual subsidies. The perpetuity technique is widely employed for determining the value of a firm or the salvage value in case of unknown project life.

3.3.1 Annuity

Considering a finite stream of cash-flows, the present value of that stream of cash flows is given by the expression:

$$PV(A) = A\frac{1-(1+i)^{-n}}{i}$$

where A is the annuity, i is the interest rate and n indicates the number of periods. If the cash flow occurs in the beginning of the period, we have an annuity with anticipated payments. In that case, for an annuity that begins on the moment 0, the present value of the stream of cash-flows will be:

$$PV_d(A) = A\frac{1-(1+i)^{-n}}{i}(1+i)$$

where A is the annuity-due, i is the interest rate and n indicates the number of periods. When the stream of cash flows begins at a period, different from the first one, we are in the presence of an annuity with deferred payments. Deferred payment arrangements are used in retail settings where a person buys an item with a contract to begin making payments at a future date. The present value for this type of annuity can be calculate as follows:

$$PV_{def}(A) = A\frac{1-(1+i)^{-n}}{i}\frac{1}{(1+i)^t}$$

where A is the deferred annuity, i is the interest rate, n indicates the number of periods and corresponds to the number of deferred periods.

3.3.2 Perpetuity

Considering a perpetual stream of cash-flows or a stream of cash-flows with uncertain maturity, the present value of that stream of cash flows is given by the expression:

$$PV(P) = P\lim_{n\to\infty}\frac{1-(1+i)^{-n}}{i} = P\frac{1}{i}$$

where P is the perpetuity and i is the interest rate.

Example (Insurance): Find the Present Value of a pension insurance that pays you a monthly amount of 200, starting next month, until you die (consider effective annual rate of return $=4\%$).

Monthly rate:

$$i_m = (1 + 0.04)^{\frac{1}{12}} - 1 = 0.0033$$

Insurance value:

$$PV(P) = \frac{200}{0.0033} = 61\,092.21$$

Constant Growth Perpetuity (growth rate $g < i$)

When we consider a perpetuity that is not constant but that grows at a given growth rate g, we have the following cash flow stream:

0	1	2	...	n
C	$\frac{A}{(1+i)}$	$A\frac{(1+g)}{(1+i)^2}$	...	$A\frac{(1+g)^{n-1}}{(1+i)^n}$

In this case the present value of this growing perpetuity is:

$$PV(P) = P\frac{1}{(i-g)}$$

where P is the perpetuity, i is the interest rate and g indicates the growth rate of the perpetuity.

3.4 VALUATION OF STOCKS AND BONDS

3.4.1 Stocks

Basic concepts about equity capital of companies:

- **Common stocks (common shares)** – Ownership shares in a publicly held corporation.
- **Preferential stocks** – stocks paying a pre-defined dividend rate, higher than

the one for common stocks and with priority over these; however, they usually do not endow a voting right in General Assemblies.

- **Dividend** – Periodic cash distribution from the firm to the shareholders.

Considering a stock bought at moment 0 by the price P_0 and sold at moment 1 by the price P_1, receiving a dividend Div_1 at the moment 1, the expected return r of this transaction will be:

$$r = \frac{\text{Div}_1 + P_1 - P_0}{P_0}$$

Example (Common Stocks Valuation): If Fledgling Electronics is selling for 100 per share today and it is expected to sell for 110, one year from now, what is the expected return if the dividend, one year from now, is forecasted to be 5?

$$r = \frac{5 + 110 - 100}{100} = 0.15$$

The expected return formula may be decomposed into the sum of the following two expressions:

$$r = \frac{\text{Div}_1}{P_0} + \frac{P_1 - P_0}{P_0}$$

where $\frac{Div_1}{P_0}$ represents the dividend yield and $\frac{P_1 - P_0}{P_0}$ indicates the capital gain or loss.

Profitability Measures

- Dividend yield:

$$\frac{\text{Div}_1}{P_0}$$

- Return on Equity (ROE):

$$\frac{\text{Earning per Share}}{\text{Book Value of Equity per Share}}$$

Dividend Discount Model

Compute the present stock price, admitting the stock value corresponds to the present value of all future expected dividends.

$$P_0 = \frac{\text{Div}_1}{(1+r)^1} + \frac{\text{Div}_2}{(1+r)^2} + \cdots + \frac{\text{Div}_n + P_n}{(1+r)^n}$$

where n represents the time horizon for the investment, Div_1 indicates the dividend at the end of the first period and r is the discount rate.

Example: A company expects to pay dividends of 3, 3.24, and 3.50 over the next three years. At the end of three years you anticipate selling your stock at a market price of 94.48. What is the price of the stock given a 12% expected return?

$$P_0 = \frac{3.00}{(1+0.12)^1} + \frac{3.24}{(1+0.12)^2} + \cdots + \frac{3.50 + 94.48}{(1+0.12)^3} = 75.00$$

If the investor forecasts no dividend growth and plans to hold out stock indefinitely, the stock P_0 can be valued as a perpetuity:

$$P_0 = \frac{\mathrm{Div}_1}{r} + \frac{\mathrm{EPS}}{r}$$

assuming all earnings (EPS) are paid to shareholders. Considering the dividends growing indefinitely at a constant rate g, the stock price is given by the Gordon Growth Model that corresponds to the expression:

$$P_0 = \frac{\mathrm{Div}_1}{r-g}$$

Example: If the same stock of the previous example is selling for 100 in the stock market, what might the market be assuming about the growth in dividends?

$$100 = \frac{3.00}{0.12 - g} \rightarrow g = 0.09$$

Therefore, the market is assuming the dividend is growing annually and indefinitely 9%.

3.4.2 Bonds

Common Bonds

Classified as medium and long term debt. A debt security which represents equal fractions of a loan. Entitles the holder to a fixed set of payments (periodic interest=coupons). The holder can receive interest until maturity and at maturity, the holder receives back the face value of the bond (redemption).

The common bonds are quoted as a percentage of face value and the main characteristics are: nominal or coupon interest rate, face value (or nominal value), issue price, redemption value and repayment scheme. In the fixed income securities, the income is independent of the company's performance.

Example (Valuing a Bond): This is January 2011. What is the value of a bond that pays a 5.375% yearly interest (coupon) over 6 years, and whose redemption value is the bond's nominal value 100?

Table 4

2102	2013	2014	2015	2016	2017
5.375	5.375	5.375	5.375	5.375	105.375

Considering there are other securities with similar risk that offer a 3.8% rate of return, the bond's present value would be:

$$P_0 = \frac{5.375}{(1+0.038)^1} + \frac{5.375}{(1+0.038)^2} + \frac{5.375}{(1+0.038)^3} + \frac{5.375}{(1+0.038)^4} + \frac{105.375}{(1+0.038)^5} = 108.31$$

Alternatively, if the return required by the shareholder (opportunity cost) was just 2.0%, the bond's present value would rise to:

$$P_0 = \frac{5.375}{(1+0.02)^1} + \frac{5.375}{(1+0.02)^2} + \frac{5.375}{(1+0.02)^3} + \frac{5.375}{(1+0.02)^4} + \frac{105.375}{(1+0.02)^5} = 118.90$$

Inversely, if the bond's market price is:

- 108.31, the implicit rate of return (Yield to Maturity) is 3.8%;
- 118.90, the implicit rate of return (Yield to Maturity) is 2%.

Explanation for the rise in the Portuguese Treasury bonds yields:

- Demand and supply forces push price down;
- Lower price corresponds to a higher Yield to Maturity (YTM).

When we use the return rate of equally risky alternative bonds as discount rate for determining the present value of a given bond, we are implicitly employing the notion of opportunity cost. This concept is widely applied in the evaluation of investment and engineering projects.

4. CASH FLOW AND COST OF CAPITAL

The cash flow of investment projects is different from the accounting cash flow because it:

- Includes opportunity costs as an investment cost.
- Doesn't include irreversible costs.
- Includes incremental flows resulting from adjacent projects (+) and from anticipated end of products (−).

4.1 FREE CASH FLOW TO FIRM (FCFF)

- Investment Cash Flow (ICF)

Investment Cash Flow (ICF) =

(−) Δ Capital Investment (Tangible, Intangible)

(−) Δ Net Working Capital =

Δ (Cash reserve+Accounts Receivable+Inventory+State and other public bodies)

Δ (Accounts Payable+State and other public bodies)

(+) Residual Value of Fixed Assets =

Market Value-tax rate (Market Value – Book Value)

(+) Residual Value of Net Working Capital =

Net Working Capital at the end of the project

- Operating Cash Flow (OCF)

Operating Cash Flow (OCF)=

(+) EBIT (earnings before interests and taxes) × (1-tax rate)

(+) Depreciation & Amortization

Therefore, the expression for calculating the cash-flows used in investment projets is given by:

$$FCF(F) = ICF + OPC$$

FCF: Free Cash Flow; ICF: Investment Cash Flow; OPC: Operating Cash Flow.

Example: Consider an investment project of a trading company with an initial investment of $10\ 000$:

Table 5

	Period						
	0	1	2	3	4	5	6
1 Capital Investment	10 000						-1 949
2 Accumulated depreciation		1 583	3 167	4 750	6 333	7 917	9 500
3 Year-end book value	10 000	8 417	6 833	5 250	3 667	2 083	500
4 Working capital		550	1 289	3 261	4 890	3 583	2 002
5 Total book value (3+4)	10 000	8 967	8 122	8 511	8 557	5 666	2 502
6 Sales		1 396	12 887	32 610	48 901	35 834	19 717
7 Cost of goods sold		837	7 729	19 552	29 345	21 492	11 830
8 Other costs	4 000	2 200	1 210	1 331	1 464	1 611	1 772
9 Depreciation		1 583	1 583	1 583	1 583	1 583	1 583
10 Pre-tax profit (6-7-8-9)	-4 000	-3 224	2 365	10 144	16 509	11 148	4 532
11 Taxes at 35%	-1 400	-1 129	828	3 550	5 778	3 902	1 586
12 Profit after taxes (10-11)	-2600	-2 096	1 537	6 593	10 731	7 246	2 946

From which, we can obtain the Investment Cash Flow

Table 6

	Period						
	0	1	2	3	4	5	6
Investment Cash Flow							
Capital Investment	-10 000						1 442
Change in Net Working capital		-550	-739	-1 972	-1 629	1 307	3 583
Total Investment CF	-10 000	-550	-739	-1 972	-1 629	1 307	5 025

and the Operating Cash Flow

Table 7

	Period						
	0	1	2	3	4	5	6
Operating Cash Flow							
Sales		1 396	12 887	32 610	48 901	35 834	19 717
Cost of goods sold		-837	-7 729	-19 552	-29 345	-21 492	-11 830
Other costs	-4 000	-2 200	-1 210	-1 331	-1 464	-1 611	-1 772
Depreciation		-1 583	-1 583	-1 583	-1 583	-1 583	-1 583
EBIT	-4 000	-3 224	2 365	10 144	16 509	11 148	4 532
Taxes (35%)	-1 400	-1 129	828	3 550	5 778	3 902	1 586
EBIT (1-t)	-2 600	-2 096	1 536	6 593	10 731	7 246	2 946
Depreciation	0	1 583	1 583	1 583	1 583	1 583	1 583
Total Operating CF	**-2 600**	**-512**	**3 120**	**8 177**	**12 314**	**8 829**	**4 529**
FCCF	**-12 600**	**-1 062**	**2 381**	**6 205**	**10 685**	**10 136**	**9 554**

4.2 FREE CASH FLOW TO EQUITY (FCFE)

- Operating Cash Flow (OCF)

Operating Cash Flow (OCF) =

(+) Net Income

(+) Depreciation & Amortization

- Financing Cash Flow

Net Borrowings (NB) =

(+) Debt Issued

(−) Debt Repaid

In this case, the FCF(E) for investment projets consists in:

$$FCF(E) = ICF + OPC + NB$$

FCF: Free Cash Flow; ICF: Investment Cash Flow; OPC: Operating Cash Flow.

Table 8

	Period						
	0	1	2	3	4	5	6
1 Capital Investment	10 000						-1 949
2 Accumulated depreciation		1 583	3 167	4 750	6 333	7 917	9 500
3 Year-end book value	10 000	8 417	6 833	5 250	3 667	2 083	500
4 Working capital		550	1 289	3 261	4 890	3 583	2 002
5 Total book value (3+4)	10 000	8 967	8 122	8 511	8 557	5 666	2 502
6 Sales		1 396	12 887	32 610	48 901	35 834	19 717
7 Cost of goods sold		837	7 729	19 552	29 345	21 492	11 830
8 Other costs	4 000	2 200	1 210	1 331	1 464	1 611	1 772
9 Interest		420	336	252	168	84	
10 Depreciation		1 583	1 583	1 583	1 583	1 583	1 583
11 Pre-tax profit (6-7-8-9-10)	-4 000	-3 644	2 029	9 892	16 341	11 064	4 532
12 Taxes at 35%	-1 400	-1 276	710	3 462	5 719	3 872	1 586
13 Profit after taxes (10-11)	-2600	-2 369	1 319	6 430	10 621	7 191	2 946

Table 9

	Period						
	0	1	2	3	4	5	6
Cash Flow from operations	-2 600	-785	2 902	8 013	12 205	8 775	4 529
Changes in Net Working Capital		550	739	1 972	1 629	-1 307	-3 583
Capital Expenditures	10 000						-1 442
Net Debt issued	7 000	-1 400	-1 400	-1 400	-1 400	-1 400	
FCFE	**-5 600**	**-2 735**	**763**	**4 641**	**9 176**	**8 682**	**9 554**

Table 10

	Period						
	0	1	2	3	4	5	6
FCCF	-12 600	-1 062	2 381	6 205	10 685	10 136	9 554
Interest × (1-tax)		273	218	164	109	55	0
Net Debt issued	7 000	-1 400	-1 400	-1 400	-1 400	-1 400	0
FCFE	**-5 600**	**-2 735**	**763**	**4 641**	**9 176**	**8 682**	**9 554**

4.3 DISCOUNT RATE

For calculating the NPV, we need to consider differently the discount rate:

- If cash flow is FCFF, the discount rate should be equal to the weighted average cost of capital.
- If cash flow is FCFE, the discount rate should be equal to equity cost of capital.

The Weighted Average Cost of Capital (WACC) might be calculate with the following expression:

$$WACC = \frac{\text{equity}}{\text{equity} + \text{debt}} r_e + \frac{\text{equity}}{\text{equity} + \text{debt}} r_d(1-t)$$

with r_e meaning the equity cost of capital,

For calculating the equity cost of capital r_e, can be considered the following alternative and complementary ways of estimating it:

- In non-public firms
  - (absent from stock exchange) the return on equity:

$$ROE = \frac{\text{Net Income}}{\text{Equity}}$$

- In public firms
  - Gordon Model

$$r = \frac{\text{Div}(t_1)}{P} + g$$

r: equity cost of capital, $Div(t_1)$: dividends distributed by the firm;

P: spot price of the stock; g: growth rate of dividends.

  - Government Bonds + Risk Premium

4.4 RETURN DEMANDED BY SHAREHOLDERS (CAPM)

$$E[r] = r_f + \beta\left(E[r_M] - r_f\right)$$

r_f=risk-free interest rate; β=risk measure; r_M=expected market interest rate.

Example (Dividend Valuation Model): The dividends on a common stock over the past four years are as follows:

Table 11

Year	Dividend per Share (€)
2002	1.37
2003	1.38
2004	1.40
2005	1.40

We can calculate the growth rate of dividends using the following expression:

$$Div_{2005} = Div_{2002} \times (1+g)^3$$
$$1.40 = 1.37 \times (1+g)^3$$
$$g = 0.0072$$

For estimating the next period's dividend (2006), we apply the growth rate g to the dividend of year 2005. Since the most recent dividend $Div_{2005} = 1.40$, the estimate of the next period's dividend results from:

$$Div_{2006} = 1.40 \times (1 + 0.0072\ ) = 1.41$$

As the price of the stock at the end of 2005 is $P_0 = 42.51$, we can calculate the equity cost of capital:

$$r_e = \frac{1.41}{42.51} + 0.0072 = 0.0040$$

Solving an illustration example by the Capital Asset Pricing Model requires the following information:

- To estimate the risk-free rate of interest r_f, we assume an yield on 30-year treasury bonds of 5%.
- To estimate the expected return on the market r_m, we assume the average annual return on an stock index for 10 years (11%)
- To estimate the stock's return's sensitivity to changes in the market's return β there are several alternatives:
  - Assume that the regression of the returns on company stock against the returns on the market produced a slope of this line: $\beta = 0.54$

- Obtaining an estimate of β from financial services media:
  - Value Line Investment Survey: $\beta = 0.95$
  - Yahoo! Finance: $\beta = 0.7$

Considering the small number of β values, we can consider the median (0.7) of the three values. Thus, the return required by the investors r_e corresponds to:

$$r_e = 0.05 + 0.7 \times (0.11 - 0.05) = 0.092$$

4.5 COST OF DEBT

Cost of debt refers to the effective rate a company pays on its current debt. In most cases, this phrase refers to after-tax cost of debt, but it also means the company's cost of debt before taking taxes into account. The difference in cost of debt before and after taxes lies in the fact that interest expenses are deductible. As an example to calculate cost of debt, a company must figure out the following yields:

- On recent debt offerings with similar risk: 0.071;
- On recent debt offerings made by the company (It did not issue debt securities recently).
- On outstanding debt of the company: 0.064 to 0.084 (slightly lower than coupon rates on current debt which ranges from: 0.065 to 0.085.)

Assuming the yield of the market (0.071) with similar risk, as the standard and knowing that the outstanding debt is 1 bps below the average coupon rate, we adopted a cost of debt of $r_d = 0.07$.

4.6 WEIGHTED AVERAGE COST OF CAPITAL

The weighted average cost of capital (WACC) of company is the overall cost of capital, considering all funding sources.

- The market-value proportions for capital structure equal 13.44% for debt and 86.6% for common equity.
- Since the book-value proportions for the capital, 32.9% debt (D) and 67.1% equity (E) are quite different from the correspondent market-value proportions,

the error of considering the book-value proportions would be considerable. Therefore, the $WACC$ can be calculated as follows:

$$WACC = \%E \times r_e + \%D \times r_d \times (1 - t) = 0.866 \times 0.092 + 0.134 \times 0.07 \times (1 - 0.25) = 0.087$$

5. INVESTMENT CRITERIA METHODS

The main issues of profitability analysis are:

- Accepting or rejecting investment projects in function of a rejection threshold;
- Ranking investment projects and, eventually, selecting among them.

When we need to select between two or more projects, two relevant questions arise:

- Are the projects mutually exclusive?
- Are the projects comparable in size and duration?

5.1 NET PRESENT VALUE

The NPV represents the net present value of an investment employing a discount rate and a stream of expected cash flows. For calculating the NPV, we employ the following expression:

$$NPV(r) = \sum_{(t=0)}^{n} \frac{CF_t}{(1+r)^t}$$

Where n is number of periods of the project, r is the opportunity cost of capital of the investor and CF_t is the cash flow of the period t.

5.1.1 Weighted Average Cost of Capital

$$NPV = \sum_{(i=0)}^{n} \frac{FCF\ F_i}{(1+wacc)^i} \geq 0$$

Flow to Equity

$$NPV = \sum_{(i=0)}^{n} \frac{FCF\ E_i}{(1+r_e)^i} \geq 0$$

Example: If we pay now for a stock (C_0 = Investment = 7) that later yield 1 dividend in periods 1 and 2 (C_1 and C_2) and then sell it at a price of 10 in period 3, the Net Present

Value (NPV) would be:

$$NPV(r) = -\frac{7}{(1+r)^0} + \frac{1}{(1+r)^1} + \frac{1}{(1+r)^2} + \frac{10}{(1+r)^3}$$

Example: What is the NPV of building a house in two years, with an investment in moment 0 ($C_0 = 100\ 000$) and an investment in moment 1 ($C_1 = 150\ 000$) and sell it two years later for $350\ 000$ ($C_2 = 0$ and $C_3 = 350\ 000$)?

$$NPV(r) = -\frac{(100\ 000)}{(1+r)^0} - \frac{150\ 000}{(1+r)^1} + \frac{0}{(1+r)^2} + \frac{(350\ 000)}{(1+r)^3}$$

The **Rejection Threshold** allows to calculate the value created by the investment project value comparing the internal return to the opportunity cost of capital, expressed by the discounting rate.

When the average internal rate of return equals the discounting rate, ; when it exceeds the discounting rate, $NPV(r) > 0$; when it is inferior to the discounting rate, $NPV(r) < 0$.

When $NPV(r) > 0$, the project creates value and should not be rejected. This means that we should only proceed with the investment project when its return is greater than the employed discounting rate.

When we compare two investment projects (1 and 2) and $NPV_1(r) > NPV_2(r) 0$, then the project P_1 is better than the project P_2.

Example Considering an initial investment of $1\ 000$ (C_0) and positive cash flows of 100 during 5 years, one can calculate the net present value for different costs of capital (5%, 10% e 15%) of different investors profiles.

Table 12

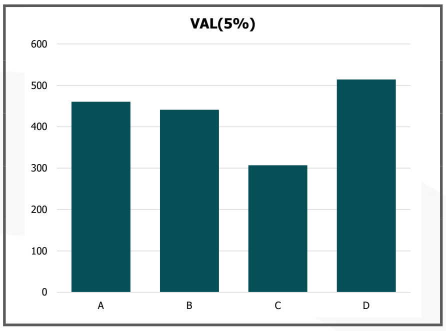
Period	0	1	2	3	4	5	NPV	r
CF (€)	-1 000.00	100.00	100.00	100.00	100.00	1 100.00		
$\frac{CF_t}{(1+r)^t}$	-1 000.00	90.91	82.64	75.13	68.30	683.01	0.00	10%
	-1 000.00	95.24	90.70	86.38	82.27	861.88	216.47	5%
	-1 000.00	86.96	7561	65.75	57.18	546.89	-167.61	15%

- If $r = 5\%$; $NPV(5\%) = 216.47$ (see column Sum) the project should be executed;
- If $r = 15\%$; $NPV(15\%) = -167.61$ the investment project should not be accepted.

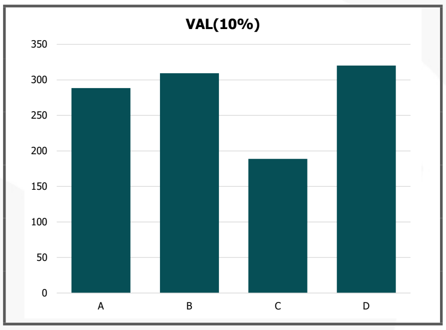
Example: Consider the following tables:

Table 13

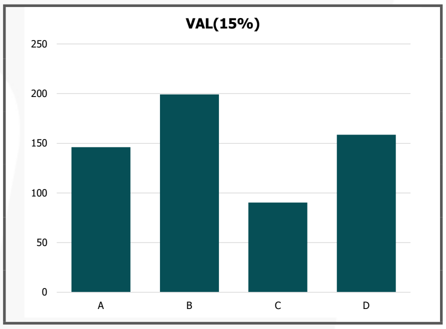
Projeto	Ano					VAL(5%)
	0	**1**	**2**	**3**	**4**	
A	-1000	300	320	500	550	460,3689
B	-800	350	350	350	350	441,0827
C	-750	240	300	320	340	306,8271
D	-1200	400	420	550	580	514,1829



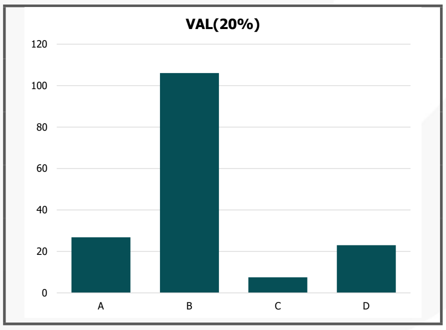
	Ano					VAL(10%)
	0	**1**	**2**	**3**	**4**	
A	-1000	300	320	500	550	288,5049
B	-800	350	350	350	350	309,4529
C	-750	240	300	320	340	188,761
D	-1200	400	420	550	580	320,1147



	Ano					VAL(15%)
	0	**1**	**2**	**3**	**4**	
A	-1000	300	320	500	550	146,0579
B	-800	350	350	350	350	199,2424
C	-750	240	300	320	340	90,34005
D	-1200	400	420	550	580	158,6572



	Ano					VAL(20%)
	0	**1**	**2**	**3**	**4**	
A	-1000	300	320	500	550	26,81327
B	-800	350	350	350	350	106,0571
C	-750	240	300	320	340	7,484568
D	-1200	400	420	550	580	22,99383



The NPV ranking of the investment projects, that maintain the same cash flows, can be changed, simply by employing different costs of capital of several investors. The tables above show how the ranking of investments projects based on their NPV varies with the discount rate. For example, the best project for a discount rate of 5% would be project D, but for a discount rate of 20% the best project would be the project B.

Another important aspect, which should not be forgotten, is being consistent when comparing and ranking investment projects with different start and final dates.

Example: Consider three projects that begin in different moments.

Table 14

Year	2010	2011	2012	2013	2014
Project A	-1 000	300	420	500	
Project B		-500	250	300	
Project C			-600	340	420

For a cost of capital of 10%, we might obtain for each project (A, B, C) the following NPV (€):

$$NPV_A(10\%) = -1000 + \frac{300}{(1+0.1)} + \frac{420}{(1+0.1)^2} + \frac{500}{(1+0.1)^3} = -4.51$$

$$NPV_B(10\%) = -\frac{500}{(1+0.1)} + \frac{250}{(1+0.1)^2} + \frac{300}{(1+0.1)^3} = -22.54$$

$$NPV_C(10\%) = -\frac{600}{(1+0.1)^2} + \frac{340}{(1+0.1)^3} + \frac{420}{(1+0.1)^4} = 46.44$$

Therefore, considering the NPV values, the investment project C should be chosen among them all.

Example: A new plant to produce steel tubing requires an initial investment of $10M$. It is expected that after three years of operation, an additional investment of $5M$ will be required, and after six years of operation, another investment of $3M$. Annual operating costs will be $3M$ and estimated annual revenues will be $8M$. The estimate life of the plant is 10 years. If the discount rate is 15% per year, how much should be the net present value of this plant?

$$NPV = -10 + 5 \times (P/A\,; 15\%; 10) - 5 \times (P/F\,; 15\%; 3) - 5 \times (P/F\,; 15\%; 6) = 10.51$$

$$(P/A\,; i; n) = \frac{1-(1+r)^{-n}}{i}$$

$$(P/F\,; i; n) = \frac{1}{(1+r)^n}$$

5.2 INTERNAL RATE OF RETURN

The internal rate of return (IRR) is the rate employed in capital budgeting to evaluate and compare the investment return. The IRR is the discount rate that makes NPV equal to 0. It can be computed iteratively and in the vicinity of the point where $NPV = 0$ (to the left

$NPV > 0$ and to the right $NPV < 0$), by linear interpolation. It's given by the expression:

$$NPV(IRR) = 0 = \sum_{(t=0)}^{n} \frac{CF_t}{(1 + IRR)^t}$$

where t is the period and n is the total number of periods. The decision criterion works in the following way:

a. If the IRR is greater than the opportunity cost of capital, we might accept the investment project.

b. Otherwise, we should reject the investment project.

Concerning the decision criterion, the internal rate of return is called the MARR (minimum acceptable rate of return) or Hurdle Rate. There are some pitfalls when calculating and using the IRR:

1. The first one is that when non-conventional investments can imply multiple IRR's.

Example: Considering the following non-conventional cash flows:

Periods	0	1	2
Cash Flow	-4 000	25 000	25 000

The figure shows 3 internal rates of return. This situation turns the measurement useless as a feasible investment criterion for evaluating this investment project.

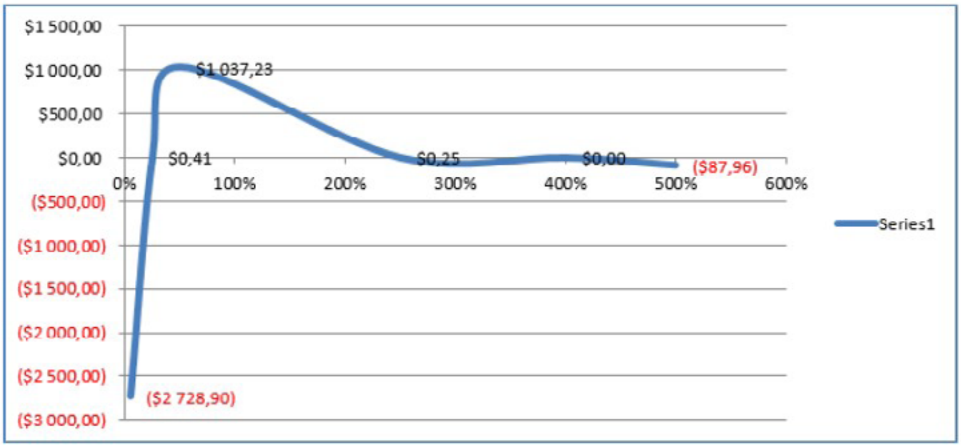


Figure 1

2. The second one is that for some investment projects there are none internal rate of return.

Example: Considering the following non-conventional cash flow:

Table 15

Periods	0	1	2
Cash Flow	1 000	-3 000	2 000

The figure produced by this investment project is similar to the third picture. The curve never crosses the axis of the interest rate i.

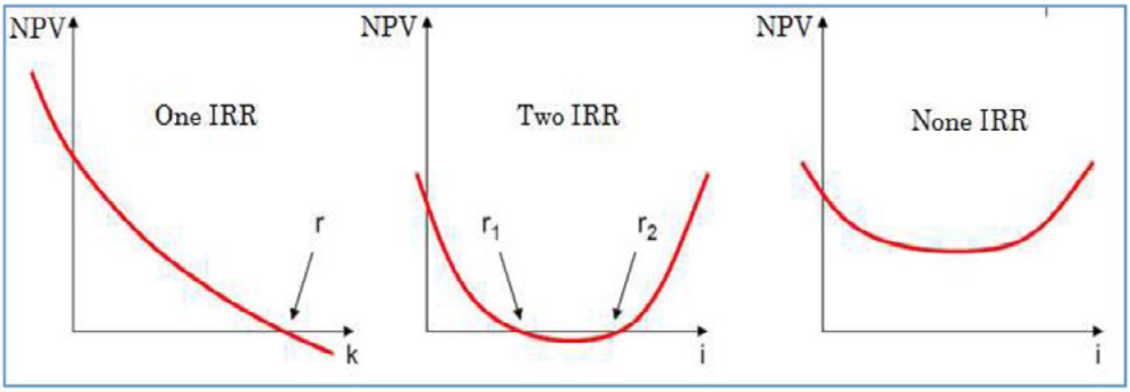


Figure 2

3. The IRR does not permit the investor choosing correctly between mutually exclusive projects, with different invested amounts.

Example: Considering two mutually exclusive projects (A and B):

Table 16

Project	CF_0	CF_1 to CF_{10}	IRR
A	-40 000	8 000	15%
B	-20 000	5 000	21%
A-B	-20 000	3 000	8%

If we are incapable of investing the remaining in an alternative project with an IRR higher that 8%, net value will be higher by choosing A rather than B, even knowing that $IRR_A < IRR_B$.

4. The IRR calculation method assumes that all cash flows (also the cash inflows generated during the life of the project) can be always reinvested at the same interest rate – the IRR. Usually this is not true. Instead, NPV uses the cost of capital to discount cash flows. A given investment project may have a higher IRR than another one and also a lower NPV at the prevailing cost of capital – this depends on the temporal distribution of cash flows.

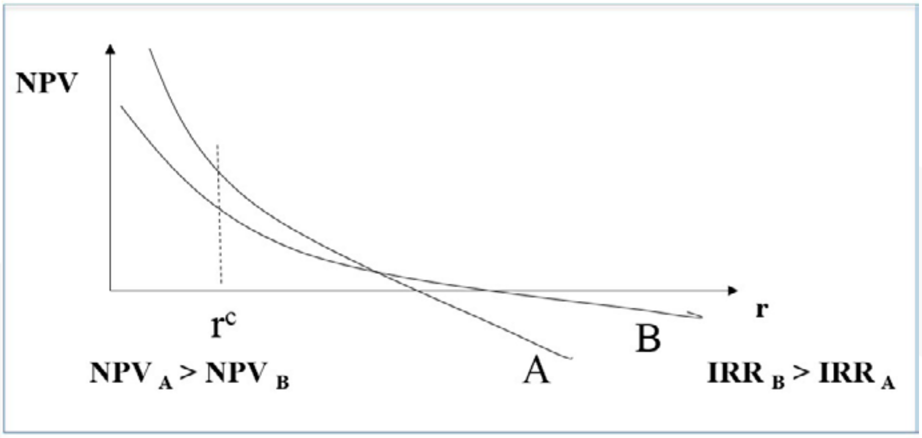


Figure 3

Example: Two alternative deposits (mutually exclusive), with and without compounding. They have the same IRR, but different NPV's at other discounting rates – ex: 5%.

Table 17

Deposit	CF_0	CF_1	CF_2	IRR	NPV(5%)
A	-1 000	0	1 210	10%	97.51
B	-1 000	100	1 100	10%	92.97
Dif.					4.54

5. The IRR is also inadequate for ranking financial flows that correspond to loans or to any other sequence of cash flows in which inflows precede outflows.

Example: Considering the loan of the next table, the $IRR_A(10\%)$ is higher the $IRR_B(5\%)$ but the $NPV_A(10\%)$ is much lower than NPV_B.

Table 18

Projects	CF_0	CF_1	CF_2	NPV(10%)	IRR
A	1 000	-100	-1 100	0	10%
B	1 000	0	-1 100	91	5%

$NPV_B(10\%) > NPV_A(10\%) \rightarrow$ Proj.(B) is preferable to Proj.(A).

Example: Consider a capital structure composed by 50% of debt. Calculate the NPV (FCFF and FCFE) for a levered cost of equity equal to 15%.

First, we need to calculate the weighted average cost of capital (WACC):

$$WACC = \frac{500}{1\,000} \times 0.15 + \frac{500}{1\,000} \times 0.08 \times (1 - 0.25) = 0.105$$

then,

- Net Present Value to Firm NPV_f, using FCFF:

$$NPV_f = -1\,000 + \frac{800}{1.105} + \frac{800}{1.105^2} = 379.17$$

- Net Present Value to Equity NPV_e, using FCFE:

$$NPV_e = -1\,000 + \frac{770}{1.15} + \frac{270}{1.15^2} = 373.72$$

5.3 MODIFIED INTERNAL RATE OF RETURN

The Modified IRR (MIRR) constitutes an alternative to Internal Rate of Return (IRR) and intends to be an answer to:

a. The existence of multiple IRRs, by previously discounting all negative flows to period 0 and all positive to period n;

b. The reinvestment of positive cash flows at a given interest rate, different from the IRR.

The expression to calculate the value of MIRR is the folllowing:

$$NPV(r, MIRR) = \sum_{t=0}^{n} \frac{CF_t^-}{(1+r_c)^t} + \sum_{t=0}^{n} CF_t^+ (1+r_r)^{n-t} \frac{1}{(1+MIRR)^n} = 0$$

where r_c is the cost of capital and r_r is the reinvestment rate.

5.4 PAYBACK PERIOD

The payback period corresponds to the number of periods it takes until the forecasted cash flow accumulated being equal to the initial investment. It can be also be designated as the period required to recover the capital expenses in an investment project in order to reach the break-even point. There are two methods to calculate the payback period:

a. No discounting method

$$\sum_{t=0}^{PBP} CF_t = 0$$

PBP is the payback period; CF_t is the cash flow from the period t.

b. Discounting method

$$\sum_{t=0}^{PBP} \frac{CF_t}{(1+r)^t} = 0$$

PBP is the payback period; CF_t is the cash flow from the period T, and r is the cost of capital.

Example (PBP): Consider the following cash flows:

Table 19

Period	0	1	2	3	4	5	6
Cash Flow	-1 000	200	300	400	420	500	700
Sum (CF)	-1 000	-800	-500	-100	320	820	1 520

All values in monetary units

$$PBP(\text{notdiscounted}) = 3 + \frac{100}{\big(320-(-100)\big)} = 3.24 \text{ years}$$

Example (PBP): The discount rate is equal to 5%. Calculate the payback period.

Table 20

	0	1	2	3	4	5	6	
	-1 000	200	300	400	420	500	700	
Cum. Value	-1 000	-800	-500	-100	320	820	1 520	
Payback Period				3	0,24			3,24
Disc. Value	-1 000	190.48	272.11	345.54	345.54	391.76	522.35	5.00%
Cum. Disc. Value	-1.000	-809.52	-537.41	-191.88	153.66	545.42	1067.77	
Disc. Payback Period				3	0,56			3.56

Payback is also risk criteria: A lower value of the PBP means a more rapid recovery of capital and a lower risk.

- More widely employed in countries suffering from high inflation rates or subject to political or social turbulence;
- As profitability criterion, this method fails, primarily because it ignores the last cash flows of the investment project. Examples of its misuse are nuclear or strategic investments.

Example: Determine the payback period and the net present value for each proposal in the table below, using an interest rate of 10% per year, compounded annually.

Table 21

Projects	CF_0	CF_1	CF_2	CF_3	CF_4	Payback	$NPV(10\%)$
Proj. A	-75 000	25 000	25 000	25 000	25 000	3.00	4 247
Proj. B	-75 000	20 000	25 000	30 000	35 000	**3.00**	10 288
Proj. C	-75 000	0	0	0	130 000	3.58	**13 792**

As can be seen in the table, the project chosen by PBP method (Proj. B) is different from the one, chosen by NPV method (Proj. C).

5.5 PROFITABILITY INDEX OR BENEFIT/COST RATIO

Profitability Index allows a meaningful comparison between two projects whose initial investments have very different values.

$$PI = \frac{NPV}{|PV(Inv)|} > 0 \text{ or } PI = \frac{PV}{|PV(Inv)|} > 1$$

- NPV considers all the cash flows of the project;
- PV (Inv) considers only the investment cash-flows; and
- PV considers the generated cash flows and the residual value of the Investment.

Example (PI): Considering the following investment project, calculate its profitability index.

Table 22

Period	0	1	2	3	4	5 to 15	NPV
Investment	-10 000	-960	-640				-11 600
Cash Flow		400	960	2 048	2 050	8 234	13 692

Considering the only the investment cash flows, $NPV(Inv)$, the profitability index is given by:

$$PI = \frac{PV}{NPV(Inv)} = \frac{13692}{11600} = 1.18 > 1$$

As the profitability index is greater than 1, the project creates value over the initial investment. In that case, the initial investment can even increase by without call into question the investment profitability. In a safety perspective, the generated cash-flows can even decrease $15\%(81 - \frac{1}{1.18}) = 0.15$ without affecting project acceptance.

Disadvantages and advantages

- Drawbacks: The ratio can be manipulated, as the calculation method permits that reduce the denominator in order to increase the profitability index ratio. One of the problems with the profitability index is its behavior for the case of mutually exclusive projects.
- Advantages: Provides a quick estimate of the possible increase of investment costs included in the denominator without jeopardizing the project acceptance.

Example: Consider the mutually exclusive projects A and B with the following cash flows (columns C_0 and C_1):

Table 23

Investment	CF_0	CF_1	$NPV(10\%)$	$PI\,(NPV/\text{Inv})$	$PI\,(PV/\text{Inv})$
A	-1	3.3	2	2	2
B	-10	22	10	1	2
A-B	9	-18.7	-8	-0.89	-1.89
B-A	-9	18.7	8	0.89	1.89

When considering the selection between projects A and B, it occurs the following situation:

- Using PI: we might select the project A, but
- Using NPV: we might select the project B.

As the profitable index does not constitute an acceptable decision criterium for mutually exclusive projects, we should calculate the differential project $B - A$, ensuring that $NPV > 0$ and $PI > 1$ and select the project B.

Example: A proposed public works project that has total present-worth benefits of $75\ M€$ and total present-worth costs of $55\ M€$. For deliberating about this proposal, some town members council have suggested that the project has a total present-worth non--benefit of $15\ M€$ and some other members feel the $15\ M€$ should be treated as a cost. How should the proposal be evaluated?

To answer this question, we should think about how the expense of $15\ M€$ should be considered (nonbenefit or cost).

Non-benefit:

$$PI = \frac{(75 - 15)}{55} = 1.09 > 1$$

Cost:

$$PI = \frac{75}{(55 + 15)} = 1.07 > 1$$

Additionally, we can calculate the Net Benefit Value as:

$$NBV = 75 - 15 - 55 = 5 > 0$$

As the $NBV > 0$, the investment project is acceptable.

5.6 ADJUSTED PRESENT VALUE

Adjusted present value (APV), corresponds to the sum of the net present value of a project financed by equity with the present value of financing benefits. Considering the free cash flows to firm (FCFE), discounting them at a rate equal to the unlevered equity cost of capital and adding the impact of external financing (EF):

$$APV = \sum_{i=0}^{n} \frac{FCFE_i}{(1 + r_{eu})^i} + \sum_{i=0}^{n} \frac{EF_i}{(1 + r_d)^i}$$

$$APV = NPV_e + PV_d$$

The investment's acceptance determines that $APV \geq 0$.

Example: Consider a loan of $1\ 000$, payable at the end of year 2, with the following cash flows:

$CF_0 = -1\ 000, CF_1 = 800, CF_2 = 800$

The financial structure of the company integrates 50% in equity and the equity cost of capital corresponds to 10%. Knowing the tax rate is 25% and the interest rate is 8%, calculate the adjusted present value.

If the company did not have any debt, the Net Present Value would be calculate only with equity cost of equity:

$$NPV_e = -1\ 000 + \frac{800}{1.1} + \frac{800}{(1.1)^2} = 388.43$$

If the loan is completely repayable at the end of year 2:

- Present value of external financing:

Table 24

Period	0	1	2	Interest rate	Tax rate
Loan and repayment	500		-500		
Financial expenses		-40	-40	0.08	
Tax shield benefits		10	10		0.25
Financial CF	500	-30	-530		

Therefore, the present value of external financing is:

$$PV_d = +500 - \frac{30}{1.08} - \frac{30}{1.08^2} = 17.83$$

which corresponds to the present value of tax shield benefits:

$TS_1 = TS_2 = 5000 \times 0.08 \times 0.25 = 10$

$$PV_d = \frac{10}{1.08} + \frac{10}{1.08^2} = 17.83$$

Thus, the Adjusted Present Value can be calculated as:

$$APV = NPV_e + PV_d = 388.43 + 17.83 = 406.26$$

If the loan can be repayable in two parts: 50% at end of 1st year and 50% at end of 2nd year.

- Present value of external financing:

Table 25

Period	0	1	2	Interest rate	Tax rate
Loan and repayment	500	-250	-250		
Financial expenses		-40	-20	0.08	
Tax shield benefits		10	5		0.25
Financial CF	500	-280	-265		

As before, the present value of external financing is:

$$PV_d = +500 - \frac{280}{1.08} - \frac{265}{1.08^2} = 13.55$$

which can also be achieved, calculating the present value of tax shield benefits:

$$TS_1 = 500 \times 0.08 \times 0.25 = 10$$
$$TS_2 = 250 \times 0.08 \times 0.25 = 5$$

$$PV_t = \frac{10}{(1.08)} + \frac{5}{(1.08)^2} = 13.55$$

Therefore, the Adjusted Present Value is given by:

$$APV = NPV_e + PV_d = 388.43 + 13.55 = 401.98$$

Example: Consider a capital structure composed by 50% of debt. Calculate the NPV (FCFF and FCFE) for a levered cost of equity equal to 15%.

First, we need to calculate the weighted average cost of capital ($WACC$):

$$WACC = \frac{500}{1\,000} \times 0.15 + \frac{500}{1\,000} \times 0.08 \times (1 - 0.25) = 0.105$$

then,

- Net Present Value to Firm NPV_f, using FCFF:

$$NPV_f = -1\,000 + \frac{800}{1.105} + \frac{800}{1.105^2} = 379.17$$

- Net Present Value to Equity NPV_e, using FCFE:

$$NPV_e = -1\,000 + \frac{770}{1.15} + \frac{270}{1.15^2} = 373.72$$

5.7 AVERAGE ACCOUNTING RETURN

Average Accounting Return (AAR) consists of the percentage of average accounting profit earned from an investment in comparison with the average accounting value of investment over the period. The expression to calculate it is given by:

$$AAR = \frac{\overline{NI}}{\bar{I}}$$

$\overline{NI}$: net income; $\bar{I}$: investment average accounting value.

The average net income after depreciation and taxes (NI_t):

$$\overline{NI} = \frac{\sum_{t=0}^{n} NI_t}{n}$$

The average value of investment ($\bar{I}$):

$$\bar{I} = \frac{I_i + RV}{2}$$

I_t: Initial Investment; RV: Residual value at the end of useful life of project

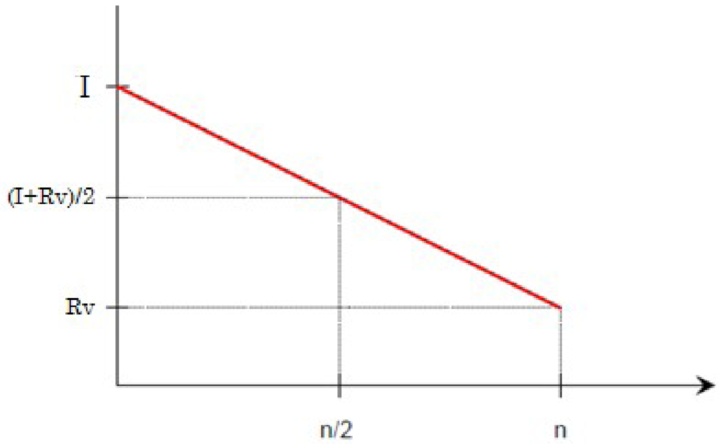


Figure 4

Some problems with this investment measurement should be considered:

- It is obtained directly from the accounting data, taken from the balance sheet, instead of being based in project cash flows;
- As it is the ratio of two average figures, it does not take time into consideration (the moment in which the amounts are obtained);
- Generally, there is no reference rate for comparison purposes.

Usage Rules

- **Rejection criteria** If AAR is less than a standard value;
- **Acceptance criteria** Select the project with the largest AAR.

Example (AAR): An investment project consists in the purchase of an equipment of $380\ 000$ € with a useful life of seven years and a residual value of $30\ 000$ €. Determine the accounting rate of return.

Table 26

Period	0	1	2	3	4	5	6	7
Net Income		120	105	125	110	130	95	85
Investment	-380							30
Depreciation		50	50	50	50	50	50	50

Considering the figures from the table, the average return rate should be:

$$AAR = \frac{\frac{\sum_{i=0}^{n} NI_i}{n}}{\frac{I_i + RV}{2}} = \frac{\frac{770}{7}}{\frac{410}{2}} = \frac{110}{205} = 0.54$$

The strong points of this method are:

- It is a simple method for applying;
- It can be used for comparative studies among low value projects and with relatively short duration.

Example A: company is considering a project with an abandonment option. There is an initial cost of 5000 and net cash flows of 2000 for next three years. The expected abandonment cash flows for years 0, 1, 2, and 3 are 5000, 3000, 2000 and 0. The firm's cost of capital is 10%. When should the company abandon the project?

In order to decide among these options, we need to calculate the NPV for following three scenarios:

1. If investment project is maintained for all the 3 years (C_1);
2. If investment project is abandoned after year 1 (C_2);
3. If investment project is abandoned after year 2 (C_3);

Table 27

$Scenario\ 1\ (C_1)$	CF_0	CF_1	CF_2	CF_3
CF (net)	-5 000	2 000	2 000	2 000
CF (abandoned)		20 000		
CF (sum)	-5 000	2 000	2 000	2 000
NPV (10%)	-26.30			
$Scenario\ 2\ (C_2)$	CF_0	CF_1	CF_2	CF_3
CF (net)	-5 000	2 000		
CF (abandoned)		3 000		
CF (sum)	-5 000	5 000		
NPV (10%)	-454.55			
$Scenario\ 3\ (C_3)$	CF_0	CF_1	CF_2	CF_3
CF (net)	-5 000	2 000	2 000	
CF (abandoned)			2 500	
CF (sum)	-5 000	2 000	4 500	
NPV (10%)	537.19			

Therefore, considering a discount rate of 10%, the projet should be maintained for 2 years because scenario (C_3) is the one with higher NPV.

6. INVESTMENT SELECTION UNDER SPECIFIC RESTRICTIONS

6.1 SELECTING BETWEEN LONG AND SHORT-LIVED EQUIPMENT

If there is no need to reinvest in a given period, one can choose the project with higher net present value. So, considering the following stream of cash flows:

$$\mathbf{CA_0} \quad \mathbf{CA_1} \quad \ldots \quad \mathbf{CA_M} \quad \mathbf{CA_N}$$

$$\mathbf{CB_0} \quad \mathbf{CB_1} \quad \cdots \quad \mathbf{CB_M}$$

Considering the absence of reinvestment after period M, one may directly compare the NPV(A) and NPV(B). Thus, assuming that both NPV are positive and NPV(B) is higher than NPV(A), one should select project B as the project that creates more value. However, since one are assuming that projects are not alternative, both projects could be implemented if the budget were enough.

Example: Consider the following machines with different initial and operating costs. Using the NPV criterion, which of the two machines should be selected, considering that the projects are non-alternative projects?

Table 28

Project	A	B
Initial Cost (€)	10 000	15 000
Useful Life (years)	5	10
Residual Value (€)	2 000	1 000
Annual Operating Costs (€)	1 755	1 052
Annual Opportunity Cost of Capital (%)	12%	12%

As the available data about the machines are its costs, the purpose of the analysis should be the selection of the machine with lower costs:

$$NPV(A) = 10\,000 + 1\,755 \times a_{5]0.12} - 2\,000 \times (1+0.12)^{-5} = 15\,192$$

$$NPV(B) = 15\,000 + 1\,052 \times a_{10]0.12} - 1\,000 \times (1+0.12)^{-10} = 20\,622$$

As we are considering the costs' perspective, we should selected machine A because it is the one with lower potential costs:

$$Min[NPV(A), NPV(B)] = NPV(A)$$

6.2 MUTUALLY EXCLUSIVE INVESTMENTS

In mutually exclusive investments, the acceptance of one project automatically excludes the acceptance of other projects, some times for technical reasons. In this cases, the NPV and the IRR methods might get contradictory results, being difficult to choose the most valuable project. The conditions in which contradictory results might occur are:

a. Projects with different life expectancies.

b. Projects with different sizes of investment.

c. Projects whose cash flows differ over time. For example, the cash flows of one project increase over time, while those of another project decrease.

The contradictions result from different assumptions respecting the investment rate over cash flows from the projects:

- The NPV method discounts all cash flows at the cost of capital;
- The IRR method implies a reinvestment at an internal rate constant overtime even when the external conditions change.

Example: Consider two projects (A and B) with the following cash flows and a discount rate of 10%. Which intervals of the opportunity cost of capital are more advantageous to each project?

Table 29

Project	CF_0	CF_1	CF_2	**NPV (10%)**	**IRR**	r_d
A	-100	130		18.18	30%	10%
B	-200	120	144	28.10	20%	10%

The analysis of the previous table shows there is a discrepancy between the two criteria:

- As $IRR(A) > IRR(B)$, the project A would be select by criterion IRR;
- As $NPV(10\%, B) > NPV(10\%, A)$, the project B would be approved by NPV.

The NPV method generally gives the correct ranking, since the cost of capital is a more realistic rate.

Considering these contradictory signs, the differential values constitute a feasable solution to choose between the two projects:

Table 30

Discount Rate	NPV(A)	NPV(B)	NPV(B) − NPV(A)
0%	30.00	64.00	34.00
5%	23.81	44.90	21.09
10%	18.18	28.10	9.92
15%	13.04	13.23	0.19
20%	8.33	0.00	-8.33
25%	4.00	-11.84	-15.84
30%	0.00	-22.49	-22.49
35%	-3.70	-32.10	-28.40
40%	-7.14	-40.82	-33.67
45%	-10.34	-48.75	-38.41
50%	-13.33	-56.00	-42.67

The evolution of NPV (A and B), according to different discount rates can be described as:

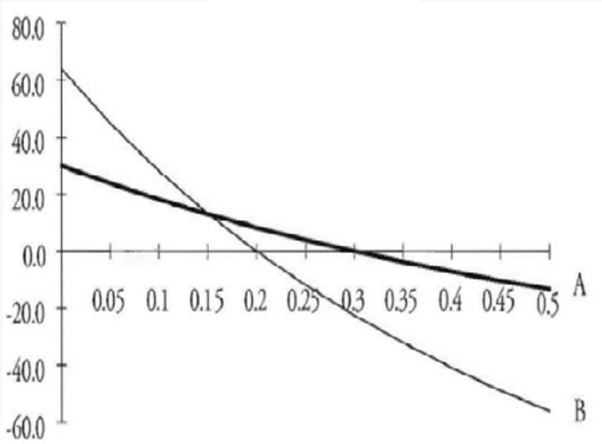


Figure 5

1. For a cost of capital (discount rate) of 10%, project B should be preferred to A because $NPV(B - A) > 0$;

2. For discount rates between 15% and 30%, the $NPV(B - A) < 0$, thus project A creates more value than the project B;

3. For discount rates below 15%, the incremental $NPV(B-A)>0$. Therefore, the creation of value by the project B is superior to the loss of value caused by the abandon of project A;

4. Higher interest rates tend to be associated with projects with lower initial investment, lower capital intensity and faster recovery.

Example: Consider the two following mutually exclusive projects have the following CF and IRR. For a discount rate of 5%, which project will be chosen?

Table 31

Project	CF_0	$CF_1 \cdots CF_{10}$	IRR
A	-40 000	8 000	15%
B	-20 000	5 000	21%
A-B	-20 000	3 000	8%

For a discount rate of the choice will fall to project A because $IRR(A-B)=8\%$.

6.3 INCREMENTAL CASH-FLOWS

The differential/incremental cash flows correspond to the relevant cash flows from the occurrence of a particular investment. They are differential because their value comes from the difference between making or not an investment. It is the appropriated method for situations with absence of information about the revenues of the project. It evaluates sequences of cash flows calculated from the difference between costs of options, not forgetting the tax effects originated by the depreciation costs.

Under this procedure, the profitability analysis is undertaken for the difference between alternative projects. This procedure is particularly useful when there is need but no possibility to obtain all data. Additionally, one must take into account the tax shield of depreciation and other operating costs.

The incremental cash flows only consider the changes in after taxes cash flows. Initial cash flows (investment) include: purchase price of the asset, shipping and installation; after-tax sale of an asset to be replaced if applicable; additional required investments in net working capital (e.g., increases in accounts receivable and inventory less any spontaneous increases in accounts payable and accruals); and any other cash flows necessary to ensure the asset in working order. For the operational cash flows, incremental cash flows consists of differential operational FCFF and are given by the FCFF formula:

$$DCF = \Delta\text{Operating income after taxes} + \Delta\text{Depreciation costs} =$$
$$(\Delta Revenue - \Delta\text{Operating expenses})(1 - \text{tax rate}) + \Delta\text{Depreciation costs.}$$

When choosing between alternatives projects, the following methods are susceptible of being used:

a. NPV with a Least Common Multiple maturity;

b. Equivalent Annual Cost or Equivalent Annual Value;

c. Differential Cash Flows.

6.4 LEAST COMMON MULTIPLE

For alternative (mutually exclusive) projects: Using the NPV criterion for evaluating mutually exclusive projects imply the following steps:

1. Determine the lowest common multiple of the economic life of projects;
2. Consider the reinvestment of project A (project A'), assuming that the economic life of project A+A' is equal to the economic life of project B;
3. Calculate the aggregate NPV (A+A');
4. Compare NPV(B) with NPV(A+A').

Example: Considering the machine's costs available in the following table, the method allows the creation of the following cash flows:

Table 32

Project	0	1	...	5	6	...	10
A	10 000	1 755		-245			
B	15 000	1 052		1 052	1 052		52
A'				10 000	1 755		-245
A+A'	10 000	1 755		9 755	1 755		-245

The net present cost for project B is:

$$NPV(B) = 15000 + 1052 \times \frac{1 - (1 + 0.12)^{-9}}{0.12} + \frac{53}{(1 + 0.12)^{10}} = 20622$$

In this example, the maturity of B is 10 years and the maturity of A is 5 years. As the least common multiple between 10 years and 5 years is 10 years, we need to reinvest project A one more maturity in order to achieve a global maturity of 10 years. In that case, the net present cost for project (A+A′) is given by:

$$NPV(A+A') = 10000 + 1755 \times \frac{1-(1+0.12)^{-4}}{0.12} + \frac{9755}{(1+0.12)^5} +$$
$$+1755 \times \frac{1-(1+0.12)^{-4}}{0.12 \times (1+0.12)^5} + \frac{-245}{(1+0.12)^{10}} = 23\,812$$

As we have the same maturity for the two projects $A{+}A'$ and B, we can apply the NPV criterion for choosing between mutually exclusive projects. Knowing that $Min[NPV(A+A'), NPV(B)] = NPV(B)$, we should select machine B.

$$Min[NPV(A+A'), NPV(B)] = NPV(B)$$

The selected machine is B. When we are choosing between alternative projects it might be considered:

a. Equivalent Annual Cost or Equivalent Annual Value.

b. Differential Cash Flows.

6.5 EQUIVALENT ANNUAL VALUE (EAV)

The Equivalent Annual Cost (EAC) or Equivalent Annual Value (EAV) expresses the costs/ values of alternative investments in comparable terms, translating them into an annual basis at equivalent terms. The assumptions for applying this method to machines with different useful lives are: a) the needs are infinite or common multiple of the useful lives; and b) everything that occurs in the first cycle will occur in the following cycles. The equivalent annual value (EAV) comes from the formula:

$$EAV = \frac{NPV}{a(n,i)}$$

Where $a(n,i)$ corresponds to the annuity factor, employed to transform the present value into annuities:

$$a(n,i) = (P/A,i,n) = [(1+i)^n - 1]/[(1+i)^n i]$$

i : interest rate; n : total number of payments

Example (EAV): Considering the cash flows of the following alternative projects with different economic lives, apply the Equivalent Annual Value criterion to choose one of the projects.

Table 33

Project	0	1	2	NPV(10%)
A	-10	7	7	2.15
B	-10	13		1.82

Knowing that $NPV = EAV \times a(i,n)$, the EAV of each project can be calculated in the following way:

Project A

$$EAV(A) = \frac{NPV(A)}{a_{n|i}} = \frac{2.15}{a_{2|0.1}} = 1.24$$

Project B

$$EAV(B) = \frac{NPV(B)}{a_{n|i}} = \frac{1.82}{a_{1|0.1}} = 2.00$$

NPV : net present value; $a_{n,i}$: annuity factor

Thus, as the annualized created value of B is higher than its correspondent A $\left(EAV(B) > EAV(A)\right)$, the project B should be chosen project.

Example (EAC): Considering the following alternative projects with different economic lives (5 and 9) for the discount rate of 15%, employ the equivalent annual cost criterion to selected one of the projects.

Table 34

Project	A	B
Initial Cost ()	10 000	14 000
Useful Life (years)	5	9
Residual Value ()	1 800	2 250
Annual Operating Costs ()	795	520
Annual Opportunity Cost of Capital (%)	15%	15%

The first step comprehends the calculation of the NPV for both projects at the same discount rate of 15%. The second step consists in annualizing the net present cost.

Project A

$$EAC(A) = \frac{NPV_{15\%}(A)}{a_{\overline{5}|0.15}} = \frac{12\,770.05}{3.35} = 3\,809.50$$

Project B

$$EAC(B) = \frac{NPV_{15\%}(B)}{a_{\overline{9}|0.15}} = \frac{15\,836.35}{4.77} = 3\,319.99$$

The previous formula shows that equivalent annual cost of B, EAC(B), comes from dividing the net present cost of B, NPV(B), by the annuity factor considering a maturity of 9 years and a discount rate of 15%. Thus, for a discount rate of 15%, $EAC(B) < EAC(A)$ implies the selection of project B.

Example: A firm needs to take a replacement decision that comprises to replace or not the current equipment A by an new equipment B. Assuming a tax rate of 30%, a discount rate of 10%, a depreciation rate of 25% and a maturity of 4 years analyze the following problem and indicate the adequate decision.

- Equipment A: Current market value: € $10\,000$; Current book value: $5\,000$ (current age: 2 years old); Annual operating costs: $10\,000$; Market value in 4 years: 0;
- Equipment B: Acquisition Price: $20\,000$; Annual operating costs: $7\,500$; Market value in 4 years: $4\,000$.

First calculate the cash flows for the differential project $B - A$:

$$CF_0 = -20000 + 10000 = -10000$$

$$CF_1 = CF_2 = (10000 - 7500)(1 - 0.3) + \left(\frac{20000}{4} - 2500\right)(0.3) = 2500$$

$$CF_3 = (10000 - 7500)(1 - 0.3) + \left(\frac{20000}{4}\right)(0.3) = 3250$$

$$CF_4 = CF_3 + 4000(1 - 0.3) = 6050$$

These cash flows represent the decision of implementing project B and not investing in project A, losing its value. Discounting the differential cash flows at a discount rate of 10%, one obtains a positive net present cost of €913. Therefore, the decision associated with $NPV(B - A) = 913 > 0.1$ is acquiring machine B and selling machine A.

6.6 THE PROFITABILITY INDEX WITH CAPITAL RATIONING

Remembering the expressions to calculate the profitability index:

$$PI = \frac{NPV}{NPV(Inv)}$$

$$PI = \frac{PV}{NPV(Inv)}$$

PI: profitability index; NPV: net present value; PV: present value; Inv: Capital Expenses (CAPEX)

The acceptance criteria are:

$$PI(NPV) > 0 \text{ or } PI(PV) > 1$$

The profitability index cannot be employed in dependent investments and in multi-annual investments subject to budget constraints.

Example: Consider the following mutually exclusive investment projects (A and B) with the correspondent net present value (NPV) and profitability index (PI):

Table 35

Project	CF_0	CF_1	Discount Rate	NPV	PI
A	-1	3,3	10%	2	3
B	-10	22	10%	10	2
B-A	-9	18,7	10%	8	1.89

As the projects are mutually exclusive, only one of them should be accepted and the other should be rejected. In this case, we have to ensure that the project $(B - A)$ corresponding to the difference between the accepted project and the rejected project has a profitability index higher than 1.

Associated with the profitability index, the common approach to select a portfolio of investment projects is their prioritization. The steps to achieve that prioritization are the following:

a. Compute the Profitability Index (Benefit-Cost ratio) for each project:

$$PI = \frac{PV}{NPV(Inv)}$$

b. Rank the projects according to their PI ratios;

c. Accept (include in the portfolio) all the projects as long as the budget accommodates it.

Example: A company with a budget of $250\ 000$ wants to select the set of acceptable projects:

Table 36

Project	I()	PV()	NPV()	Profitability Index
A	70 000	112 000	42 000	1.6
B	100 000	145 000	45 000	1.45
C	110 000	126 500	16 500	1.15
D	60 000	79 000	19 000	1.32
E	40 000	38 000	-2 000	0.95
F	80 000	95 000	15 000	1.19

The ranking that results from the application of the profitability index shows that projects A, B and D should be the selected projects.

Table 37

Project	Profitability Index	Ranking	Inv	PV
A	1.6	1	70 000	112 000
B	1.45	2	100 000	145 000
C	1.15	5		
D	1.32	3	60 000	79 000
E	0.95	6		
F	1.19	4		
Sum			230 000	336 000

Inv: CAPEX; PV: present value; NPV: net present value.

Therefore, the global net present value of the selected projects can be calculated as

$$NPV = 336\ 000 - 230\ 000 = 106\ 000$$

Unfortunately, the profitability index method has several limitations. One of the most serious is that it doesn't work well whenever more than one resource is rationed.

6.7 PROBLEMS WITH PRIORITIZATION

This section shows how to handle problems related with prioritization of projects such as:

a. Multiple constraints, namely multi-period constraints.

b. Ignores (non-convex) efficient portfolios and leads to sub-optimal choices.

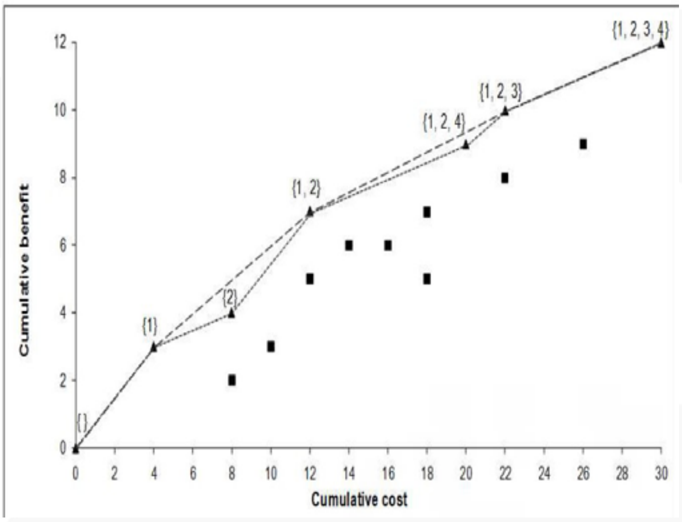


Figure 6

Projects j	v_j	c_j
1	3	4
2	4	8
3	3	10
4	2	8

Legend:
▲ **efficient portfolio**
■ **non-efficient (or dominated) portfolio**

Figure 7

Example: Consider a company with a budget of 250000 that needs to appraisal the following projects. Ranking these investment projects employing the profitability index, the result is as follows:

Table 38

Project	I()	PV()	NPV()	PI	Ranking
A	70 000	112 000	42 000	1.60	1
B	100 000	145 000	45 000	1.45	2
C	110 000	126 500	16 500	1.15	5
D	60 000	79 000	19 000	1.32	3
E	40 000	38 000	-2 000	0.95	6
F	80 000	95 000	15 000	1.19	4

The three most interesting projects from the point of view of the profitability index are projects A, B and D.

Table 39

Project	Inv.	PV()
A	70 000	112 000
B	100 000	145 000
D	60 000	79 000
Sum	230 000	336 000

which corresponds to a global net present value of: $NPV = 336\,000 - 230\,000 = 106\,000$. In this case, the company might only select these 3 projects because accepting the fourth one (E) would not allow to respect the budget constraint. The project E implies the lower initial investment ($40\,000$) but overpasses the margin between the budget limit and the accumulated initial investments ($250\,000 - 230\,000 = 20\,000$). Unfortunately, the profitable index method has some limitations. One of the more serious, it is failing whenever more than one resource is rationed.

Example: Considering the following projects and an investment budget of $10\,000$, which projects might be carried on?

Table 40

Project	0	1	2	NPV(10%)	PI
A	-10 000	20 000	112 000	10 000	2.6
B	-5 000	12 750	145 000	11 343	3.3
C	-5 000	7 500	79 000	3 884	1.7

NPV: Net present Value; PI: Profitability Index

Employing the profitability index, we would select the project A because it's the one with higher value. However, the $NPV(10\%)_A < NPV(10\%)_B + NPV(10\%)_C$. Thus, the investor should implement the projects B and C. Therefore, in environments with budget constraints, the profitable index can lead to not optimal choices.

6.8 MATHEMATICAL PROGRAMMING APPROACH

The goal of the mathematical programming approach is maximizing an objective function, using linear programming. The objective function is given by the sum of the NPV of the projects and multiplied by the implementation degrees. These implementation degrees result from the constraints inherent to capital restriction in each period. When it's not possible doing partial projects, the implementation' degree of a project corresponds to a binary variable of type $(0,1)$. Considering a portfolio of investment projects where V_i is the net present value of a project i and $C_{(i,j)}$ indicates the cash flow of the project i in the period j, the maximization problem can be defined as:

$$\max \sum_{i=1}^{n} = V_i w_i;$$

$$\text{subject to} \sum_{i=1}^{m} C_{i,j} w_i \leq B_j$$

where n is the total number of projects available to participate in the portfolio, m designates the maximum value of periods given by the projects in the portfolio, w_i assumes the value 1 when the project belongs to the optimal portfolio and the value 0 when the model rejected the project.

Example: Consider the following projects with following cash flows. Assume a budget constraint in period 0 and another in period 1. Choose the investment projects that maximize the value created by the projects available (values given in M).

Table 41

Project	CF_0	CF_1	CF_2	NPV(10%)	PI
A	-100	100	100	73.55	1.74
B	-100	50	250	152.07	2.52
C	0	-200	400	148.76	1.82
Budget Restriction	-100	-100			

The objective function of the linear programming problem consists in the following expression:

$$Max(73.55A + 152.07B + 148.76C)$$

which is subjected to the following constraints:

$$100A + 100B + C \leq 100$$

$$100A - 50B + 200C \leq 100$$

with the binary variables A, B and C.

6.9 OPTIMAL TIMING OF INVESTMENT

Most of the investments in real assets have the following characteristics:

- They are irreversible.
- They can be postponed.
- They are subject to one or more sources of uncertainty: prices, costs or investment expenditures.

Therefore, risks can affect, not only the value of the investment opportunity but also timing of the optimal investment strategy and globally the investment level in the economy. The timing to wait becomes a valuable option because after accepted, and once made, the majority of investments are not reversible and could not be postponed. However, some investments are irreversible but can often be postponed. Thus, performing the investment corresponds to extinguish the option to delay.

Considering a deterministic environment with 3 projects (P_1, P_2 and P_3):

Table 42

	0	1	2	3	4	n	NPV
P_1	$-CF_0$	CF_1	CF_2	CF_3	CF_4	CF_n	NPV_1
P_2		$-CF_{1'}$	$CF_{2'}$	$CF_{3'}$	$CF_{4'}$	$CF_{n'}$	NPV_2
P_3			$-CF_{2''}$	$CF_{3''}$	$CF_{4''}$	$CF_{n''}$	NPV_3

This situation implies that to choose the optimal timing to invest, we must be choose the highest NPV among the three projects:

$$NPV = max[NPV_1, NPV_2, NPV_3]$$

Alternatively, assuming a probabilistic environment where each NPV is associated to a given probability.

Table 43

	0	PV	Prob.
P_1	$-CF_0$	PV_1	p_1
P_2		PV_2	p_2
P_3		PV_3	p_3

The expected NPV corresponds to the weighted average of the NPV of the three projects:

$$E[NPV] = p_1 \times (-CF_0 + PV_1) + p_2 \times (-CF_0 + PV_2) + p_3 \times (-CF_0 + PV_3)$$

Example: A company has to decide about an investment project with an initial investment of million in a probabilistic environment. For this project, three equiprobable scenarios can occur with the corresponding present values of $500M$, $1\ 200M$ and $1\ 400M$.

Without delay:

$$E_1[NPV] = 0.333 \times (-1000 + 500) + 0.333 \times (-1000 + 1200) + 0.333 \times (-1000 + 1400) = 33.33M$$

Example: Postponing the decision to invest brings a reduction of $100M$ in each of the Present Values. The use of the option to delay would eliminate the uncertainty about the scenario that will face in future. The cost of capital is 10% per year. Thus, with a timing delay, the expected NPV would be:

$$E_2[NPV] = \frac{\big(0.333(0) + 0.333(-1000 + 1100) + 0.333(-1000 + 1300)\big)}{1.10} = \frac{133.33}{1.10} = 121$$

As $E_1[NPV] < E_2[NPV]$, the company should wait one year for investing.

7. EXCEL FINANCIAL FUNCTIONS

7.1 PRESENT VALUE

The formula for the Present Value is:

$$PV(rate; nper; pmt; [fv]; [type])$$

$rate$: interest rate; $nper$: number of periods; pmt: payment made each period (negative sign if money is receive); fv: [optional] future value after the last payment; if pmt is omitted the fv argument must be include; $type$: [optional] indicates when payments are due (0 payments at the end of the period; 1 - payments at the beginning of the period).

Example (PV): Suppose you're thinking in buying an insurance annuity that pays at the end of every month, for the next 20 years. The cost of the insurance is $60{,}000$, and the money paid out will earn 8%. You need to determine whether this would be a good investment.

If you were asked to pay for the insurance you would determine that this would not be a good investment because the present value of the annuity $(59{,}777.15)$ is less than what you are asked to pay.

7.2 NET PRESENT VALUE

The function NPV in Excel just corresponds to the present value of uneven cash flows. Thus, the initial investment is not included.

$$NPV(rate; value1; [value2]; \cdots)$$

$rate$: interest rate; $value1$: payment or income in period 1;

Example (NPV): Considering a project investment with an initial payment of $40{,}000$ with expected payments of 8000, 9200, 10000, 1200 and 14500. The minimum return rate you accept is 8%. You want to determine whether this would be a good investment.

7.3 FUTURE VALUE

The formula for the Future Value is:

$$FV(rate; nper; pmt; [pv]; [type])$$

$rate$: interest rate; $nper$: number of periods; pmt: payment made each period (negative sign if money is receive).

Example (FV): Compute the accumulated capital in 10 months for a bank account with an initial amount of 500 and monthly deposits of 200, considering an annual interest rate of 10%.

$$FV(40\%/12\,; 10; -200; -500) = 2{,}547.19$$

7.4 ANNUITY

Financial function that returns the periodic payment for a loan, assuming constant payments and a constant interest rate.

$$PMT(rate; nper; pv; [fv]; [type])$$

$rate$: interest rate; $nper$: number of periods; p_v: present value; fv: future value; $type$: [optional] indicates when payments are due; 0 or omitted payments at the end of the period; 1 payments at the beginning of the period

Example (PMT): Calculate the monthly mortgage of an $180{,}000$ loan, with 30 years maturity and a gross nominal annual interest rate of 6%.

$$PMT(6\%/12\,; -30 \times 12; 180000) = 1{,}079.19$$

7.5 NUMBER OF PERIODS

Function that calculates the number of periods required to pay off a loan, for a constant periodic payment and a constant interest rate.

$$NPER(rate; pmt; pv; [fv]; [type])$$

Example (PMT): Say that you have a personal loan of 2500. You've agreed to pay 175 a month with a 7.5% annual interest. How long would it take to pay off that loan?

$$NPER(7.5\%/12\,; -175; 2500) = 15.01 \text{ (15 months and a few days)}$$

BIBLIOGRAPHY

Arnold, G. (2008). *Corporate financial management. Pearson Education*.

Ayyub, B. M. (2014). *Risk analysis in engineering and economics*. Chapman and Hall/CRC.

Baum, A. E., & Crosby, N. (2014). *Property Investment Appraisal*. John Wiley & Sons.

Belli, P. (2001). *Economic Analysis of Investment Operations: Analytical Tools and Practical Applications*. World Bank.

Benninga, S. (2014). *Financial modeling*. MIT press.

Berk, J., DeMarzo, P., Harford, J., Ford, G., Mollica, V., & Finch, N. (2013). *Fundamentals of corporate finance*. Pearson Higher Education AU.

Blank, L. T., & Tarquin, A. (2017). *Engineering Economy*. McGraw-Hill Education.

Bodie, Z., Kane, A., & Marcus, A. J. (2013). *Essentials of investments*. McGraw-Hill/Irwin Taipei.

Brueggeman, W. B., & Fisher, J. D. (2011). *Real estate finance and investments*. McGraw-Hill Irwin New York, NY.

Cassimatis, P. (2013). *A Concise Introduction to Engineering Economics*. CRC Press.

Couto G, Crispim J, Mouta Lopes M, Pimentel P, Sousa, F. (2014). *Avaliação de Investimentos*. Áreas Editora.

Curry, S., Weiss, J., & others. (1993). *Project analysis in developing countries.* Springer.

Damodaran, A. (2010). *Applied corporate finance*. John Wiley & Sons.

Damodaran, A. (2012). *Investment Valuation: Tools and Techniques for Determining the Value of Any Asset*, Third Edition, John Wiley & Sons. Wiley.

DeFusco, R. A., McLeavey, D. W., Pinto, J. E., Runkle, D. E., & Anson, M. J. P. (2015). *Quantitative Investment Analysis*. Wiley.

Edwards, S. (1986). Country risk, foreign borrowing, and the social discount rate in an open developing economy. *Journal of International Money and Finance, 5*, S79--S96.

Ellison, M., & Seitz, N. (1999). *Capital budgeting and long-term financing decisions*. Harcourt Brace College Publishers.

Elton, E. J., Gruber, M. J., Brown, S. J., & Goetzmann, W. N. (2009). *Modern Portfolio Theory and Investment Analysis*. John Wiley & Son

Eschenbach, T. (2010). *Engineering Economy: Applying Theory to Practice*. Oxford University Press.

Esty, B. C., & Matysiak, G. A. (2004). *Modern project finance: A casebook*. Wiley New York, NY.

Finnerty, J. D. (2013). *Project financing: Asset-based financial engineering*. John Wiley & Sons.

Finnerty, J. D., & Stowe, J. D. (1997). *Corporate financial management*. Prentice-Hall International.

Fisher, J., & Martin, R. S. (1994). *Investment Analysis for Appraisers*. Real Estate Education Company.

Fleischer, G. A. (1994). *Introduction to Engineering Economy*. PWS Publishing Company.

Froot, K. A., Scharfstein, D. S., & Stein, J. C. (1993). Risk Management: Coordinating Corporate Investment and Financing Policies. *Journal of Finance, 48*(5), 1629-1658.

Goedhart, M., Koller, T., & Wessels, D. (2015). *Valuation: Measuring and managing the value of companies*. JohnWiley & Sons.

Gordon, M. J. (1962). *The investment, financing, and valuation of the corporation*. RD Irwin Homewood, IL.

Götze, U., Northcott, D., Schuster, P., & others. (2008). Investment appraisal. *Methods and Models, Berlin, Heidelberg 2008*.

Harberger, A. C. (1984). Basic needs versus distributional weights in social cost-benefit analysis. *Economic Development and Cultural Change, 32*(3), 455-474.

Higgins, R. C., & Reimers, M. (1995). *Analysis for financial management*. Irwin Chicago.

Hillier, D., Ross, S. A., Westerfield, R., Jaffe, J. F., & Jordan, B. D. (2016). *Corporate Finance*. McGraw-Hill Education.

Hirschman, A. O. (2014). *Development projects observed*. Brookings Institution Press.

Hirshleifer, J. (1970). *Investment, interest, and capital*.

Hirt, G, & Block, S. B. (2011). *Fundamentals of Investment Management*. McGraw-Hill Publishing.

Hirt, Geoffrey, & Block, S. C. (1990). *Fundamentals of investment management*.

IST. (2015). *Lectures in Engineering Economics*. Lisboa: Universidade de Lisboa - IST.

Jenkins, G P, & Harberger, A. C. (n.d.). *Program on Investment Appraisal and Management: Manual: Cost-benefit Analysis of Investment Decisions*. Harvard Institute for International Development.

Jenkins, Glenn P. (1997). Project analysis and the World Bank. *The American Economic Review, 87*(2), 38-42.

Kruschwitz, L., & Löffler, A. (2006). *Discounted Cash Flow.: A Theory of the Valuation of Firms*. John Wiley & Sons.

Logue, D. E., & Francis, J. C. (1973). Investments: Analysis and Management. In *The Journal of Finance* (Vol. 28). https://doi.org/10.2307/2978201

Merton, R. C. (1980). On Estimating the Expected Return on the Market: An Exploratory Investigation. *Journal of Financial Economics, 8*(4), 323-361.

Modigliani, F., & Miller, M. H. (1958). The cost of capital, corporation finance and the theory of investment. *The American, 1*, 3.

Newnan, D. G., Eschenbach, T., & Lavelle, J. P. (2004). *Study Guide for Engineering Economic Analysis*. Oxford University Press.

Park, C S. (2004). *Fundamentals of Engineering Economics*. Pearson/Prentice Hall.

Park, Chan S, Kim, G., & Choi, S. (2007). *Engineering economics* (Vol. 22). Prentice Hall: Upper Saddle River, NJ, USA.

Parkinson, M. (1980). The Extreme Value Method for Estimating the Variance of the Rate of Return. *The Journal of Business, 53*(1), 61-65.

Philippe, J. (2001). *Value at risk: the new benchmark for managing financial risk*. NY: McGraw-Hill Professional.

Pinheiro-Alves, R., & Zambujal-Oliveira, J. (2012). The Ease of Doing Business Index as a tool for investment location decisions. *Economics Letters, 117*(1), 66-70.

Ranganatham, M. (2006). *Investment Analysis and Portfolio Management*. Pearson Education/Dorling Kindersley (India).

Ross, S. A., Westerfield, R., Jordan, B. D., & Firer, C. (2000). *Fundamentals of corporate finance*. Irwin/McGraw-Hill Boston, MA.

Ruppert, D. (2011). *Statistics and data analysis for financial engineering* (Vol. 13). Springer.

Shapiro, A. C. (2005). *Capital budgeting and investment analysis*. Prentice Hall.

Sjaastad, L. A., & Wisecarver, D. L. (1977). The social cost of public finance. *Journal of Political Economy, 85*(3), 513-547.

Smith, N. J. (2003). *Appraisal, Risk and Uncertainty*. Thomas Telford.

Solow, R. M. (1963). *Capital theory and the rate of return*. North-Holland Publishing Company Amsterdam.

Vajpayee, S. K., & Sarder, M. D. (2019). *Fundamentals of Economics for Applied Engineering*. CRC Press.

Wallingford, B. A., & Reilly, F. K. (1979). Investment Analysis and Portfolio Management. In *The Journal of Finance* (Vol. 34). https://doi.org/10.2307/2327255

White, J. A. (1998). *Principles of engineering economic analysis*. Wiley.

Yates, J. K. (2016). Engineering economics. In *Engineering Economics*. https://doi.org/10.1201/9781315368283

Young, D. (1993). *Modern engineering economy*. J. Wiley.

Zambujal-Oliveira, J. (2012). Tax competition for foreign direct investment under information uncertainty. *Economic Modelling, 29*(6), 2269-2273.

Zambujal-Oliveira, J., & Serradas, C. (2014). Valuation of Technology-Based Companies: The Case of Activision Blizzard. In *Approaches and Processes for Managing the Economics of Information Systems* (pp. 108-122). IGI Global.

Zimmerman, J. L., & Yahya-Zadeh, M. (2011a). Accounting for decision making and control. *Issues in Accounting Education, 26*(1), 258-259.

Zimmerman, J. L., & Yahya-Zadeh, M. (2011b). Accounting for Decision Making and Control. In *Issues in Accounting Education* (Vol. 26). https://doi.org/10.2308/iace.2011.26.1.258

Produção

UNIVERSIDADE ABERTA

edição eletrónica, 2020

João Zambujal Oliveira

Engenheiro de sistemas e informática, mestre em economia, mestre e doutor em gestão, investigador e professor universitário com trabalho publicado em revistas científicas internacionais.

Manuel Mouta Lopes

Doutorado em Gestão pelo ISEG-ULisboa e Mestre em Matemática Aplicada à Economia e à Gestão também pelo ISEG-ULisboa. Docente da Universidade Aberta. Tem trabalho publicado em revistas científicas internacionais. É co-autor do livro "Avaliação de Investimentos".

22 ISBN | 978-972-674-835-9