IDENTIFICATION OF STRUCTURE-BORNE SOUND PATHS OF SERVICE EQUIPMENT IN BUILDINGS USING STRUCTURAL-ACOUSTIC RECIPROCITY

Pieter Schevenels
Belgian Building Research Institute, Avenue Pierre Holoffe 21, 1342 Limelette, Belgium
Katholieke Universiteit Leuven, Laboratory of Acoustics and Laboratory of Building Physics, Celestijnenlaan 200D, bus 02416, 3001 Leuven, Belgium
e-mail: pieter.schevenels@bbri.be

Arne Dijckmans and Gerrit Vermeir
Katholieke Universiteit Leuven, Laboratory of Acoustics and Laboratory of Building Physics, Celestijnenlaan 200D, bus 02416, 3001 Leuven, Belgium

Peter J. G. van der Linden
Qsources BVBA, Ketelwinning 38, 3293 Diest, Belgium

Low-frequency noise from service equipment in buildings tends to be of a structure-borne nature rather than an airborne nature. The transmission between different building elements can be determined on a power basis. The injected structure-borne sound power by an installation can be determined by multiplying velocities and contact forces in each of the installation’s contact points. The velocity can be measured approximately by use of accelerometers close to the contact points, but the measurement of the force is less obvious. Force sensors should be inserted between the installation and the building element, which is often difficult or even impossible. Even when it’s not that tedious, it is advisable not to alter the path of sound transmission, since small changes can imply large differences in injected structure-borne sound power. In this paper, contact forces of a fitness vibration plate are estimated through a structural-acoustic reciprocal method. First, the installation in a source room is put into operation and sound pressures in a receiver room are measured at multiple receiver points. Next, a dedicated volume sound source is placed in the receiver points in order to determine the transfer functions of the sound source’s volume acceleration to the acceleration of the building element at the position of the contact points. A dedicated volume sound source was used that approximates point source behaviour and produces a high noise level down to very low frequencies (> 20 Hz). This enables accurate measurement of the structure-borne sound transmission. And, in a second step, it allows inverse estimation of the structural-acoustic contact forces down to these low frequencies. The estimated forces are compared with directly measured forces.
1. Introduction

There is a lot of interest in determining the force between an installation and a building element. Multiplication of this force with the velocity at the contact points of the installation yields the complex power flow between installation and building. When the real part of this power is taken, the injected structure-borne sound power is known. This quantity is governing the structure-borne noise propagation of installations in buildings\(^1\).

Determining the force between an installation and a building directly is a rather cumbersome task. Force sensors would have to be inserted between installation (source) and building element (receiver), which is difficult or even impossible for large and heavy installations. Even if it is possible, one must be careful not to alter the vibro-mechanical behaviour of the source-receiver system in the frequency range of interest.

In this paper, a structural-acoustic method is used to determine the force between an installation with a tonal character and a concrete floor in a reciprocal way, down to very low frequencies.

2. Theoretical principles

2.1 One contact point

For an installation with only one contact point \(C\) with the surroundings where only one force component is present, the force \(F_C\) can be reciprocally determined in the following way\(^2,3,4\):

\[
P_X = -\frac{a_c}{Q_X} F_C
\]

where \(p_X\) is the pressure that is measured in receiving point \(X\) in the building while the installation is operating. The transfer function \(a_c/Q_X\) is determined by putting an acoustic volume source in a receiving point \(X\) and measuring its volume acceleration \(Q\) (in m\(^3\)/s\(^2\)), while the acceleration in contact point \(C\) is measured without presence of the installation.

2.2 Multiple contact points

To determine the force between the contact points of an installation with multiple contact points – say three points \(a\), \(b\) and \(c\) – in an inverse way, the pressure needs to be measured in three receiving points 1, 2 and 3. For each path between the points \(X\) in the room and the contact points \(C\) on the building element, a transfer function \(a_c/Q_X\) needs to be determined. Figure 1 shows a schematic overview of direct pressure measurement and reciprocal transfer function measurement, assuming other force components than vertical ones are negligible. The following matrix formulation applies:

\[
\begin{bmatrix}
F_{C,a} \\
F_{C,b} \\
F_{C,c}
\end{bmatrix} = -\begin{bmatrix}
a_{c,a} & a_{c,b} & a_{c,c} \\
a_{c,a} & a_{c,b} & a_{c,c} \\
a_{c,a} & a_{c,b} & a_{c,c}
\end{bmatrix}^{-1} \begin{bmatrix}
Q_{X,1} \\
Q_{X,2} \\
Q_{X,3}
\end{bmatrix} \begin{bmatrix}
P_{X,1} \\
P_{X,2} \\
P_{X,3}
\end{bmatrix}
\]

which can also be written in matrix symbols for a general multi-dimensional case:

\[
F_c = -H^{-1} p_X
\]
Figure 1. Measurement of pressures in the room under an operating installation on a floor (left) and determination of transfer functions $a_c/Q_{x,3}$ when the installation is removed from the building element (right).

3. Experimental setup

3.1 Installation and floor

The installation under study is a vibration plate that is used for muscle training in fitness centres and for physical therapy in clinics. The plate, a Fitvibe 600®, generates vertical vibrations with an amplitude of 1.5 mm (“low intensity”) or 3 mm (“high intensity”) and frequencies between 20 and 60 Hz, adjustable in steps of 1 Hz. The vibrations are created by two motors with eccentric masses on their axles that rotate in phase but in different senses, producing a dominant vertical vibration. The vibration plate not only produces a sine at the rotation frequency, but also produces some higher harmonics. The harmonics are the result of minor imperfections in the motor mechanism: rub between rotor and stator, rub inside ball bearings and mechanical looseness in general.

The floor on which the installation is put into operation is a 7.4 cm thick reinforced concrete floor with a 2.5 cm thick epoxy mortar and an epoxy based self levelling layer of about 0.3 cm on top. The total thickness is therefore about 10 cm. The floor’s dimensions are 2 x 2 m². It is located in a horizontal opening between two transmission rooms in the acoustical laboratory of the Katholieke Universiteit Leuven in Belgium.

Figure 2 shows a picture of the vibration plate placed on the concrete floor and a construction scheme of the vibration plate. The manufacturer’s intention of the bottom plate is to minimise structure-borne vibrations to the supporting building element. The machine’s total weight is about 91 kg and the 3 contact points consist of rubber vibration isolators, designated $a$, $b$ and $c$ in Figure 2.

Figure 2. Vibration plate on concrete floor and construction scheme.

3.2 Volume point source

To determine the transfer functions $a/Q$, a prototype volume source of the company Qsources BVBA was used (see Figure 3). This volume source has two special speakers which are driven in phase to approximate a point source behaviour at low frequencies. The prototype source has rela-
tively large dimensions to allow a level of volume acceleration excitation which is sufficient to be able to measure a clear vibration response on the concrete structure from 20 Hz upwards. Still, in the frequency range of interest the wavelength is much larger than the critical dimensions of the volume source. In this way the source can approximate point source behaviour anywhere in the sound field, except in the immediate vicinity.

![Figure 3. Volume point source in lower transmission room.](image)

The source is internally instrumented with sensors that give a real-time signal which is linearly proportional to volume acceleration between 20 and 160 Hz. The volume acceleration sensitivity (or calibration) has been determined in a separate measurement in semi-free field conditions.

The volume point source is placed in the lower transmission room. The positioning was such that the acoustic centre of the source approximately coincided with the used microphone positions in the receiver room.

### 3.3 Direct measurement of the force

To verify the reciprocally measured force, the force is also measured directly. This is done by inserting force sensors between the contact points and the building element. Because of practical limitations, only 1 sensor per contact point is used. Each sensor is mounted between two aluminium cylinders of 2 cm thickness and with a diameter of 8 cm, about the same as the diameter of the rubber vibration isolators at the contact points of the vibration plate. Figure 4 shows the mounting of the force sensors and their arrangement under the vibration plate.

The configuration of the sensor mounting is chosen so that the resonance frequencies of the so constructed “sandwich” are lying well above the frequency range of interest (> 5000 Hz). Calculations show that the first mass-spring-mass-resonance frequency of the sandwich is 11 kHz when the boundaries of the cylinders are assumed to be constrained (worst case scenario). The lowest natural bending frequency is also calculated to be 11 kHz, when the cylinders are assumed to be free at their edges (worst case scenario). The latter was calculated using the formulas Warburton\(^6\) derived in 1951, assuming a rectangular plate of size 8 x 8 cm\(^2\).

![Figure 4. Mounting of the force sensors between two aluminium cylinders and arrangement under the vibration plate.](image)
4. Measurements and analyses

4.1 Verification of reciprocity

First of all, the basic reciprocity of Eq. (1) on the path between the concrete floor and a receiving point in the lower transmission room is verified. The transfer function \( F/p \) is determined by exciting the floor with an impact hammer which registers the force, while the pressure in the room underneath is measured simultaneously. The volume acceleration \( Q \) in transfer function \( a/Q \) is measured by aid of an accelerometer which is incorporated in the volume source and registers the movement of the loudspeaker’s membrane. The relation between acceleration of the membrane and volume acceleration is established through a calibration factor that is determined by the relation between volume acceleration and sound pressure in a semi-anechoic field.

In Figure 5, the absolute values of both transfer functions are compared to each other for a certain point on the concrete floor and a receiving point in the lower transmission room. The correspondence is very satisfying. For frequencies higher than 180 Hz, the assumption of a point source is starting to be violated, which explains the larger deviations.

![Figure 5. Absolute values of the transfer functions p/F and a/Q for the path between a point on the concrete floor and a point in the transmission room underneath.](image)

Of course, also the direct measurement of the transfer function \( p/F \) could be used for inverse determination of the contact forces of an installation. However, the reciprocally measured transfer function \( a/Q \) is preferred when the contact point is difficult to reach, which is often the case for machinery installations. It is also better to use reciprocity when the surface at the contact points is too brittle, such that an impact force \( F \) would deform the surface locally prior to exciting the whole building element. Multiple support locations and directions are another reason to apply reciprocity, because then all support-directions can be measured simultaneously.

4.2 Condition number of transfer matrix

For the inverse problem, numerical problems can be expected when the condition number of the transfer matrix \( H \) in Eq. (3) is too high. The condition number can be improved by over-determination by putting the volume source in more than 3 receiving points in the lower transmission room. If 5 points are taken, the transfer matrix would extend to a 5 x 3-matrix. Of course, while the vibration plate is operating, the pressure has to be measured in these 5 points as well.

Figure 6 shows the condition number in function of the frequency for 3, 4 and 5 points. When the number is larger than 50, the matrix is considered to be ill-conditioned. The condition number
is unfavourably large at the lowest resonance frequencies of the floor and the room (both around 45 Hz), due to the matching behaviour between different points on the floor or in the room, no matter the distance between them. In general, there is a great improvement when 4 instead of 3 points are taken. However the improvement is less obvious when 5 instead of 4 points are taken. Due to practical limitations only 3 pressures can be measured simultaneously, so in the further calculations $\mathbf{H}$ is always a 3 x 3-matrix.

4.3 Comparison of directly measured and reciprocally measured force

In Figure 7, the directly measured force through contact point $a$ is compared with the reciprocally measured force when the vibration plate is operating at high intensity and a frequency of 24 Hz. The response is characterised by a peak at the operating frequency of 24 Hz and by peaks at the harmonics of 24 Hz.

![Figure 6](image_url)

**Figure 6.** Condition number of the transfer matrix $\mathbf{H}$ in function of the frequency for 3, 4 and 5 points in the lower transmission room. Condition numbers higher than 50 (light blue line) indicate ill-conditioned matrices.

![Figure 7](image_url)

**Figure 7.** Comparison of the reciprocally measured and directly measured force through contact point $a$ with the vibration plate operating at high intensity and a frequency of 24 Hz.
For the first 3 maxima in the spectrum, at the operating frequency and its 1st and 2nd harmonic, the correspondence between both measurement techniques is quite good.

The correspondence in-between the peaks is not good because of the high condition number of the matrices involved in the calculation of Eq. (3) (see Figure 6). Another way to circumvent ill-conditioned matrices, apart from taking more receiving points which was practically not possible, is to determine a total force over all contact points instead of a force per contact point. This can be done by averaging the transfer functions between one receiving point and the three contact points to calculate the total force \( F_{C,Tot} \) in the following way:

\[
F_{C,Tot} = -\left( \frac{a_c}{Q_x} \right)^{-1} \langle a_{(a,b,c)} \rangle \langle p_x \rangle = -\langle H^{-1} \rangle_{(a,b,c)} \langle p_x \rangle \tag{4}
\]

This reciprocally measured total force is compared to the sum of the directly measured forces through all contact points in Figure 8. The correspondence in-between the peaks is much better. The correspondence at the operating frequency and its 1st and 2nd harmonic, which was already quite good, has also improved. The condition number near the first resonance frequency of the floor cannot be improved greatly by taking more receiving points. However, the condition number near the first resonance frequency of the room can be improved by also taking receiver points in other rooms with different resonance behaviour, but this was not verified experimentally.

![Figure 8. Comparison of the reciprocally measured and directly measured force through all contact points with the vibration plate operating at high intensity and a frequency of 24 Hz.](image)

The correspondence in-between the peaks in Figure 8 is still not perfect because of noise in the pressure measurement that is used to determine the force reciprocally. Therefore, it is important not to take the receiver points too far away from the installation, otherwise the low signal-to-noise ratio in the receiver points would corrupt the quality of the reciprocally measured forces.

At the peaks from the 3rd harmonic on (\( > 110 \) Hz), the reciprocally measured force is overestimating the directly measured force. This is because for those frequencies, the measured pressure is suspected to be dominated by airborne sound, even in the lower transmission room. The reciprocal method assumes that only structure-borne sound components are present. Therefore, the reciprocally measured force is higher than the real force. A solution to this problem in practice could be to take the receiving points in a room that is not adjacent to the excited building element, so that the structure-borne sound transfer is dominating the sound pressure in the receiving points. However, this requirement might conflict with the requirement of taking receiving points close enough to have a sufficient signal-to-noise ratio of the pressure when the installation is operating.
5. Conclusions

The structural-acoustic reciprocal method to determine forces between service equipment and a building element has been applied on an operating vibration plate generating vertical low-frequency vibrations into a concrete floor in a laboratory environment. The method is based on the transfer function of the volume acceleration of a dedicated volume source at a receiving point in the building over the acceleration of the building element at the contact points of the installation. The volume source is designed to send out low-frequency waves as a point source, down to 20 Hz. The method yields satisfying results as a good agreement was found between the direct measurement and the inverse technique. The method is preferred above use of the transfer function of force over pressure when the contact point is difficult to reach and when the surface of the building element is brittle.

Still, the method has to be used with care. Like all inverse methods, the accuracy of the identified multiple forces is sensitive to the matrix condition. The condition number of the transfer function matrix can be improved by taking more receiving points than strictly necessary, say twice the number of contact points. A further improvement could be to take the receiving points in different rooms of the building with different resonance behaviour, but this was not validated experimentally. At the first resonance frequency of the building element though, the condition number cannot be improved greatly.

Also, it can be possible that the pressure at the receiving points is dominated by airborne sound transfer. The reciprocal method assumes that only structure-borne sound pressure is present and therefore the reciprocally measured forces will overestimate the real forces. A solution is to take the receiving points in a room that is not adjacent to the excited building element. This room cannot be too far away because of a possible insufficient signal-to-noise ratio of the pressure while the installation is operating.

Finally, it is also possible that other force components than vertical ones, like moments, play an important role. These would also raise the pressure at the receiving points, again leading to an overestimation of the vertical forces when the method does not take these other components into account. However, the method can be extended to determine other force components as well, but the transfer matrix has to be enlarged.

REFERENCES